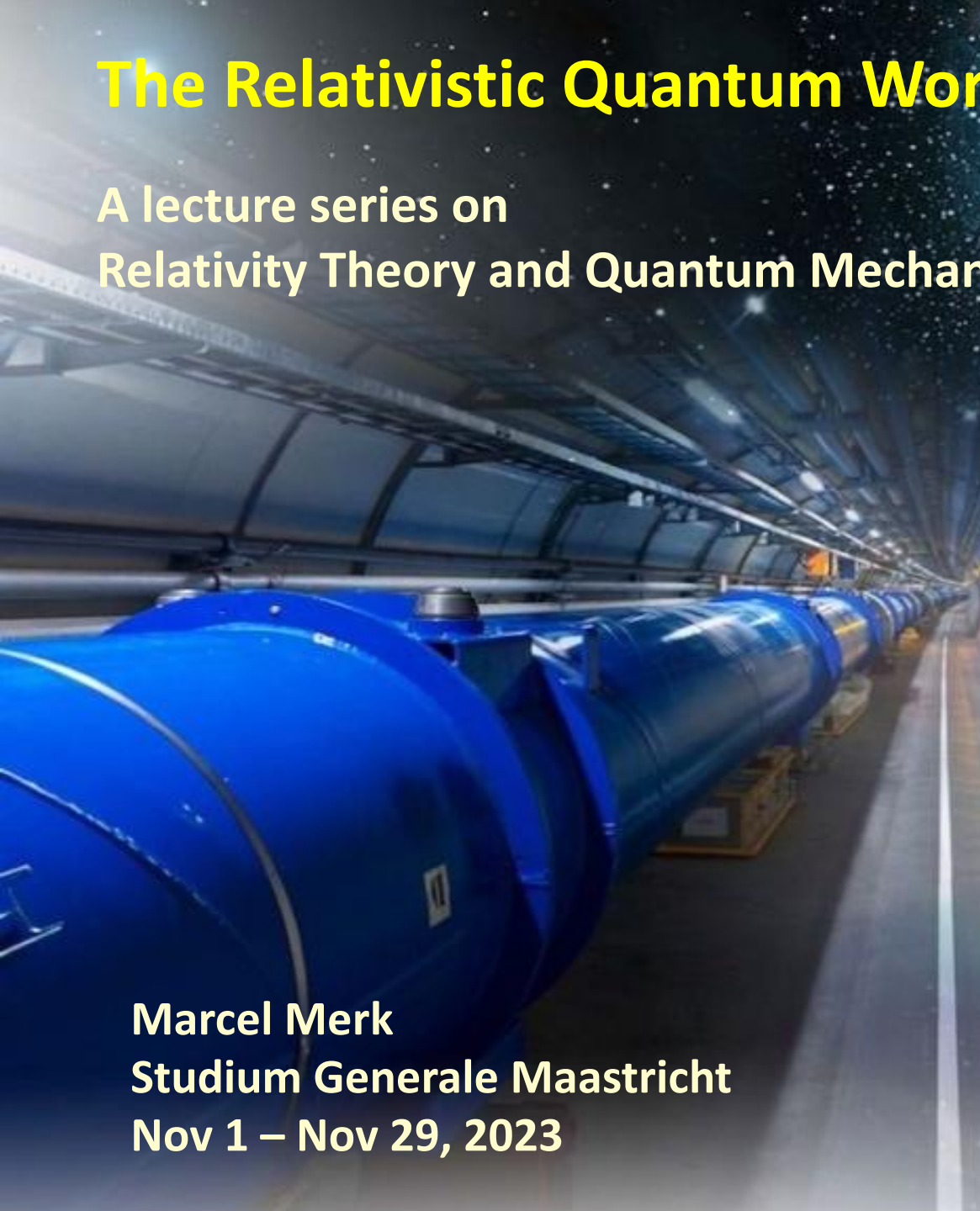
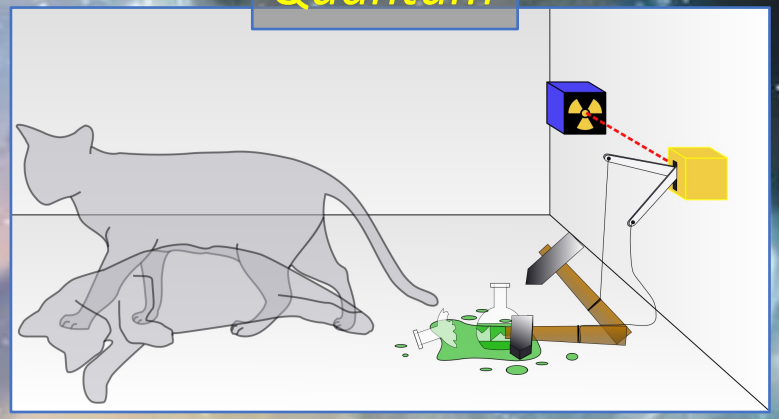
The Relativistic Quantum World

A lecture series on
Relativity Theory and Quantum Mechanics



Relativity

Quantum



Marcel Merk
Studium Generale Maastricht
Nov 1 – Nov 29, 2023

The Relativistic Quantum World

Relativity	Nov. 1:	Lecture 1: The Principle of Relativity and the Speed of Light Lecture 2: Time Dilation and Lorentz Contraction
	Nov. 8:	Lecture 3: The Lorentz Transformation and Paradoxes Lecture 4: General Relativity and Gravitational Waves
Quantum Mechanics	Nov. 15:	Lecture 5: The Early Quantum Theory Lecture 6: Feynman's Double Slit Experiment
	Nov 22:	Lecture 7: Wheeler's Delayed Choice and Schrodinger's Cat Lecture 8: Quantum Reality and the EPR Paradox
Standard Model	Nov. 29:	Lecture 9: The Standard Model and Antimatter Lecture 10: Why is there something rather than nothing?

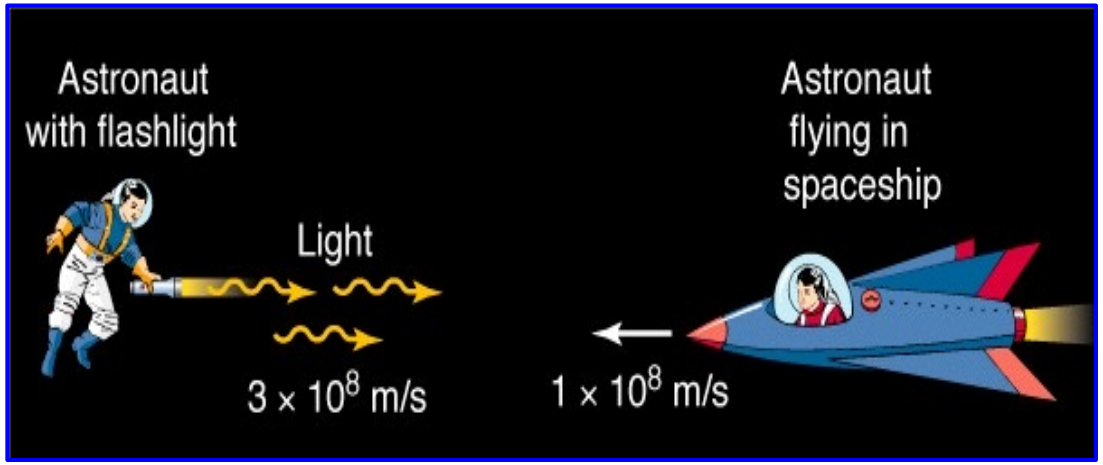
Lecture notes, written for this course, are available: www.nikhef.nl/~i93/Teaching/
Prerequisite for the course: High school level physics & mathematics.

Relativity Theory

Special Relativity

All observers moving in inertial frames:

- Have identical laws of physics,
- Observe the same speed of light: c.

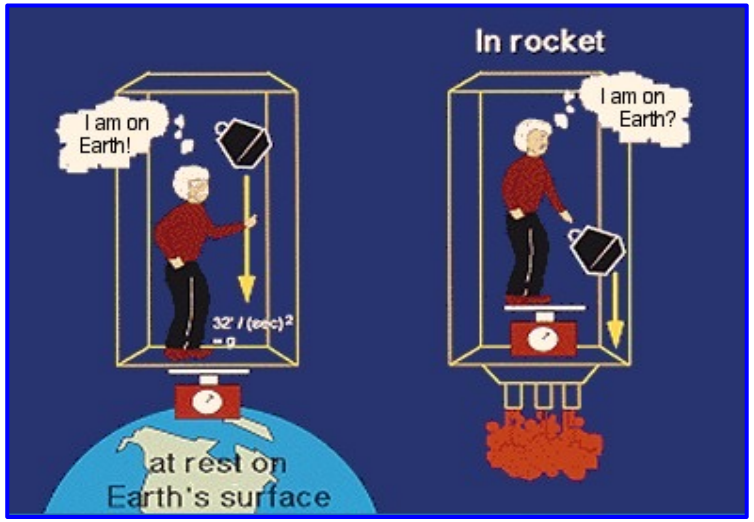


Consequences:

- Simultaneity is not the same for everyone,
- Distances shrink, time slows down at high speed,
- Velocities do not add-up as expected.

General Relativity

- A free falling person is also inertial frame,
- Acceleration and gravitation are equivalent: Inertial mass = gravitational mass

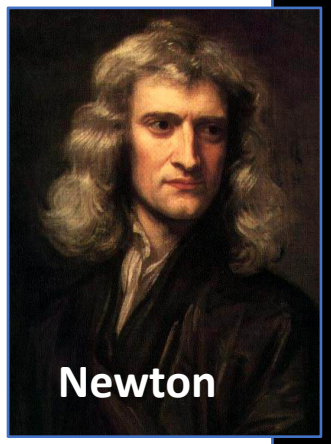


Consequences:

Space-time is curved:

- Light bends around a massive object,
- Time slows down and space shrinks in gravitational fields,
- Gravitational radiation exists.

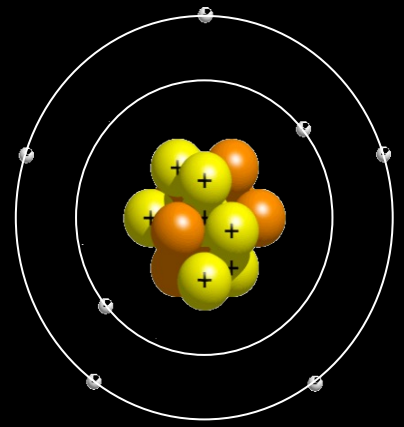
Relativity and Quantum Mechanics

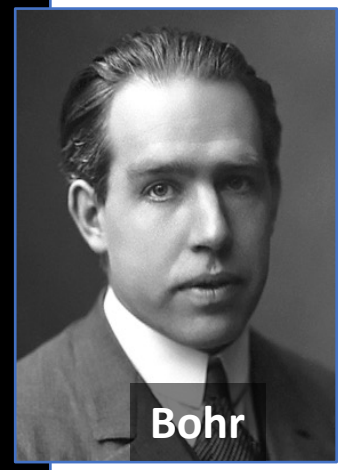


Classical Mechanics

Quantum Mechanics

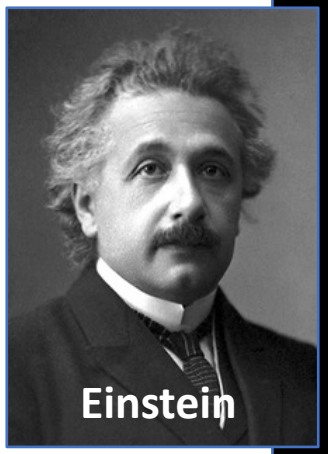
Smaller Sizes (ħ)



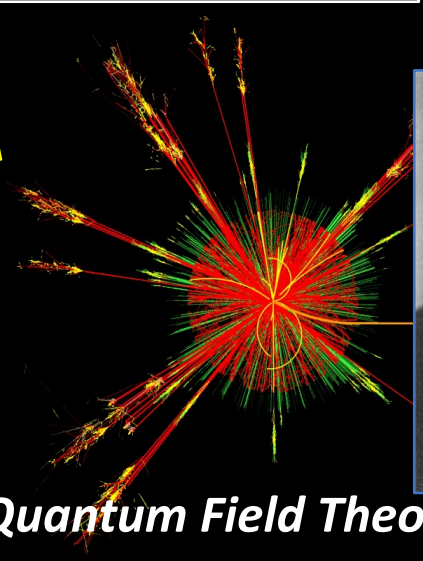


Higher Speed (c)

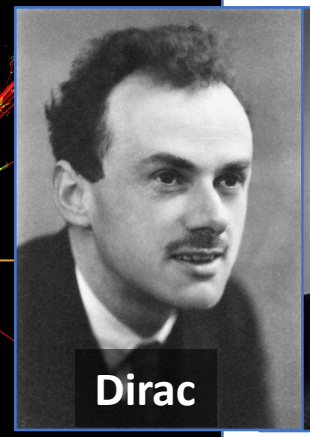
From our experience we are not used to relativistic and quantum mechanical phenomena. That's why we find it counterintuitive.

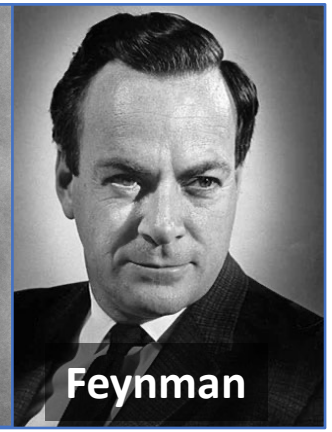


Relativity Theory



Quantum Field Theory





Lecture 5

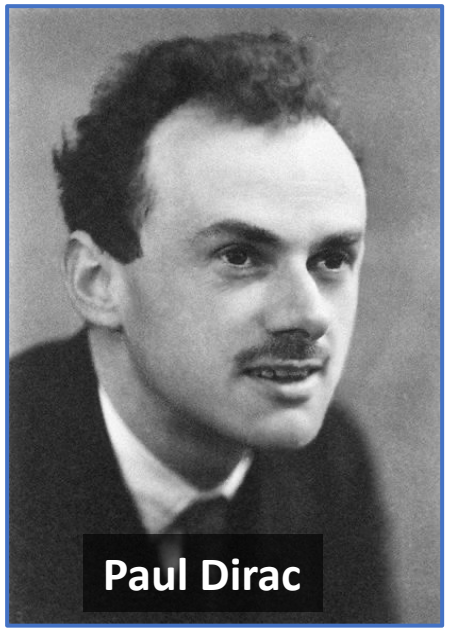
The Early Quantum Theory

“If Quantum Mechanics hasn’t profoundly shocked you, you haven’t understood it yet.”
- Niels Bohr

“Gott würfelt nicht (God does not play dice).”
- Albert Einstein

”Einstein, stop telling God what to do!”
- Niels Bohr

Key Persons of Quantum Mechanics



Niels Bohr:	Nestor of the ”Copenhagen Interpretation”
Erwin Schrödinger:	Inventor of the quantum mechanical wave equation
Werner Heisenberg:	Inventor of the uncertainty relation and “matrix mechanics”
Paul Dirac:	Inventor of relativistic wave equation: Antimatter!
Max Born:	Inventor of the probability interpretation of the wave function

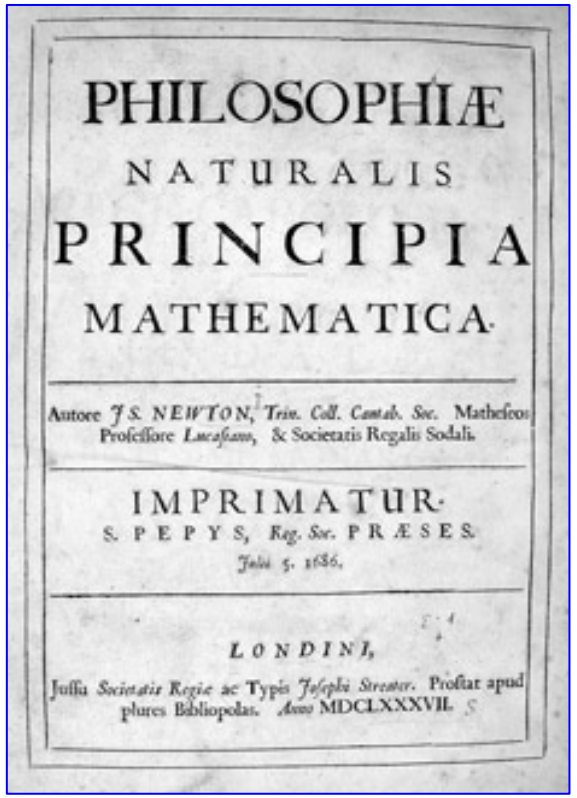
We will focus of the Copenhagen Interpretation and work with the concept of Schrödinger’s wave-function: ψ

1927 Solvay Conference Brussels

Deterministic Universe

Mechanics Laws of Newton:

1. The law of inertia: a body in rest moves with a constant speed
2. The law of force and acceleration: F= m a
3. The law: Action = - Reaction



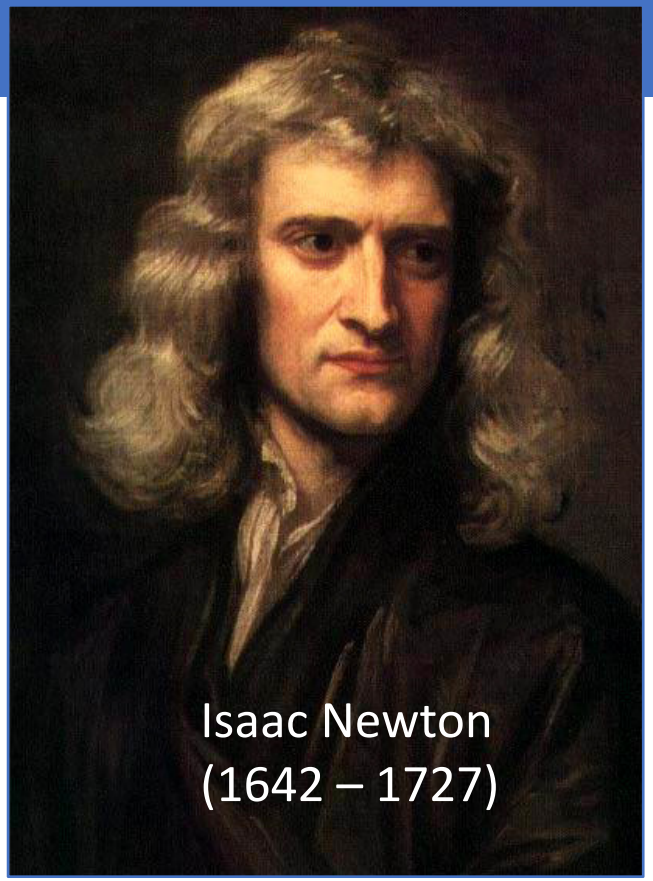
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore *J.S. NEWTON*, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Justu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

"Principia" (1687)



Isaac Newton
(1642 – 1727)

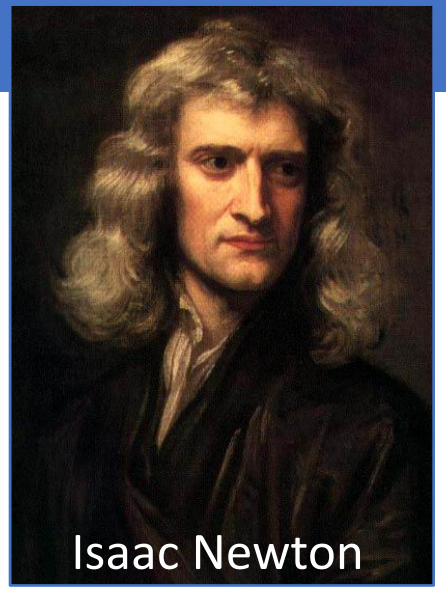
- Classical Mechanics leads to a deterministic universe.
  - From ***exact initial conditions future can be predicted***.
- Quantum mechanics introduces a fundamental element of chance in the laws of nature: Planck's constant: h.
  - ***Quantum mechanics only makes statistical predictions***.

The Nature of Light

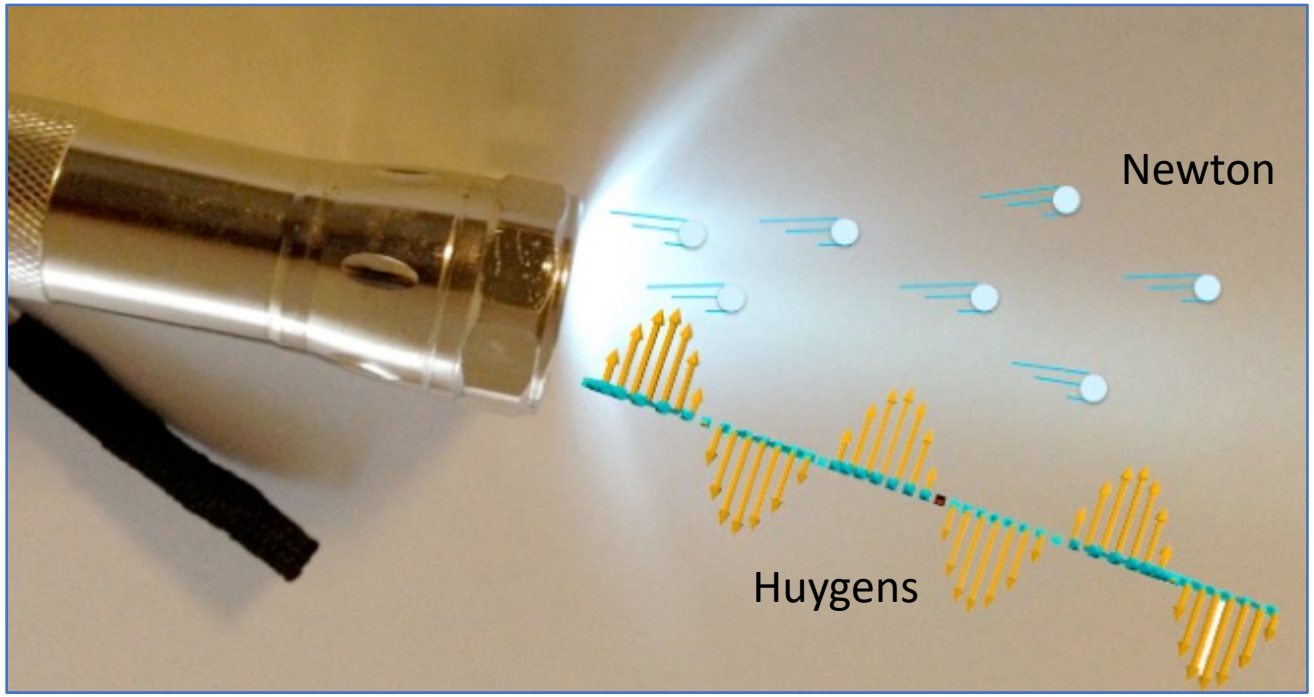
Isaac Newton (1642 – 1727):
Light is a stream of particles.

Christiaan Huygens (1629 – 1695):
Light consists of waves.

Thomas Young (1773 – 1829):
Interference observed: Light is waves!


Isaac Newton

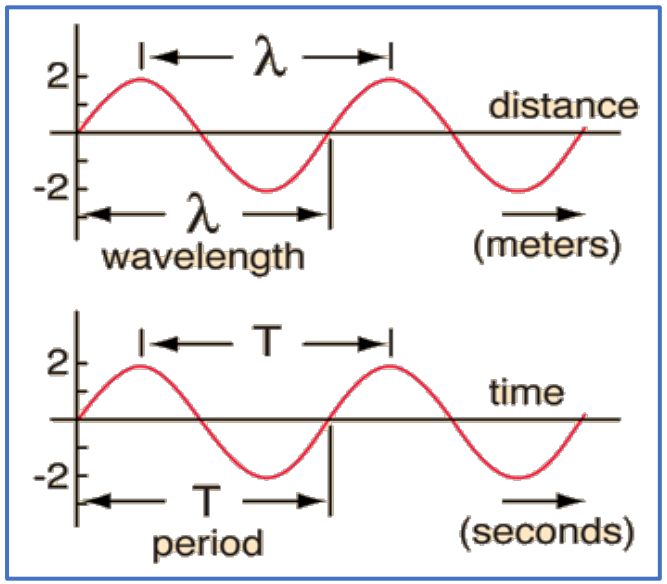

Christiaan Huygens



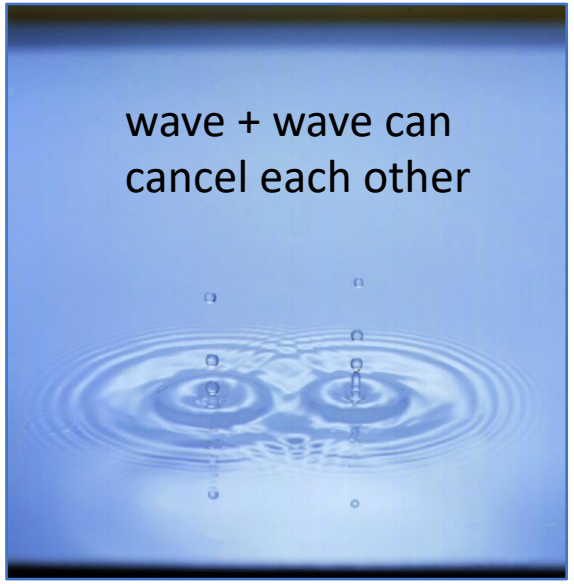

Thomas Young

Waves & Interference : water, sound, light

Principle of a wave

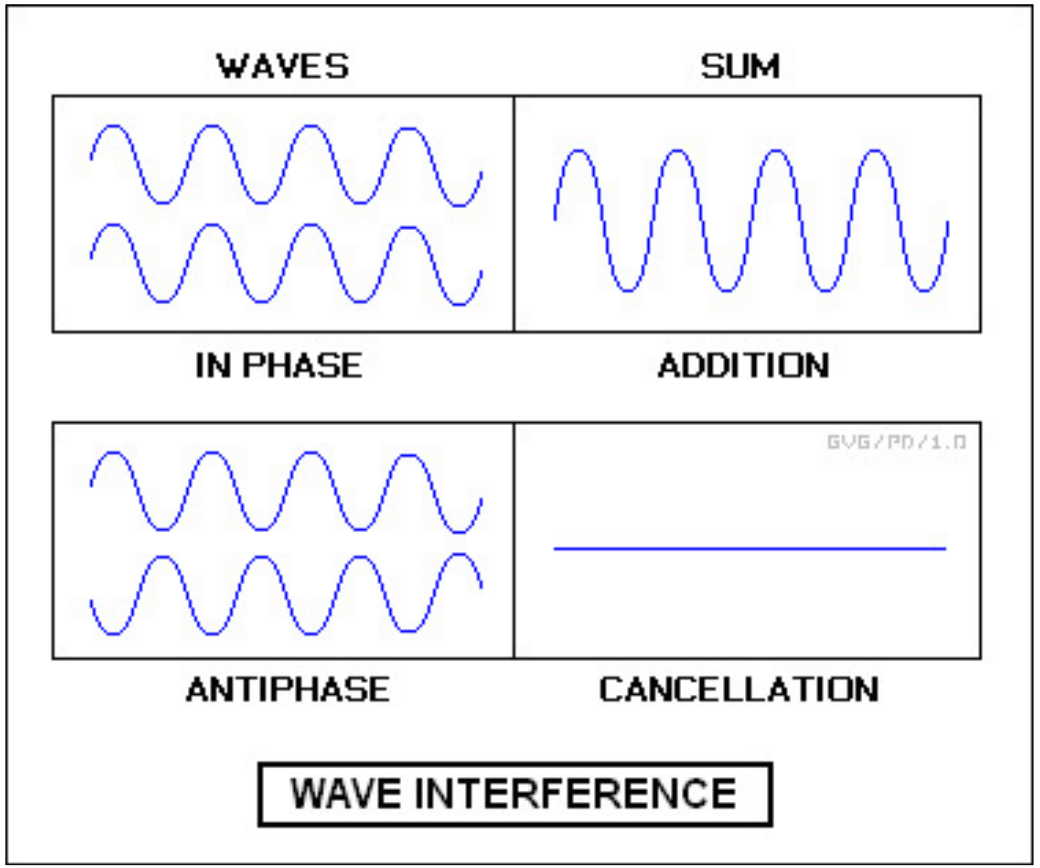


Water: Interference pattern:

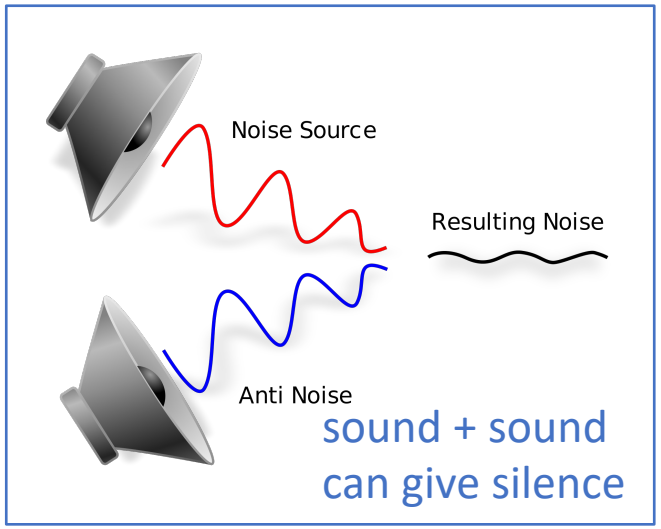


$$\lambda = v/f$$

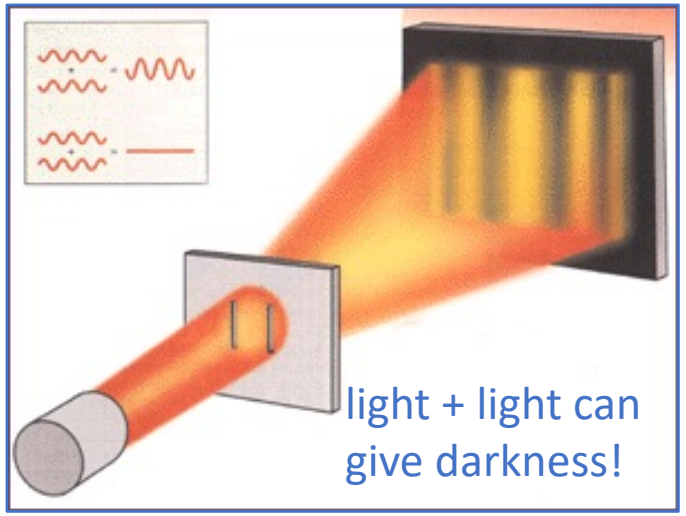
$$f = 1/T$$



Sound: Active noise cancellation:



Light: Thomas Young experiment:

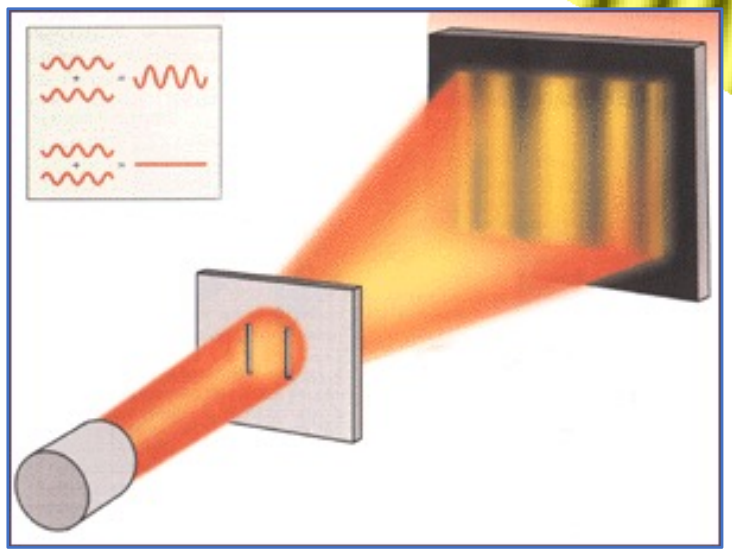


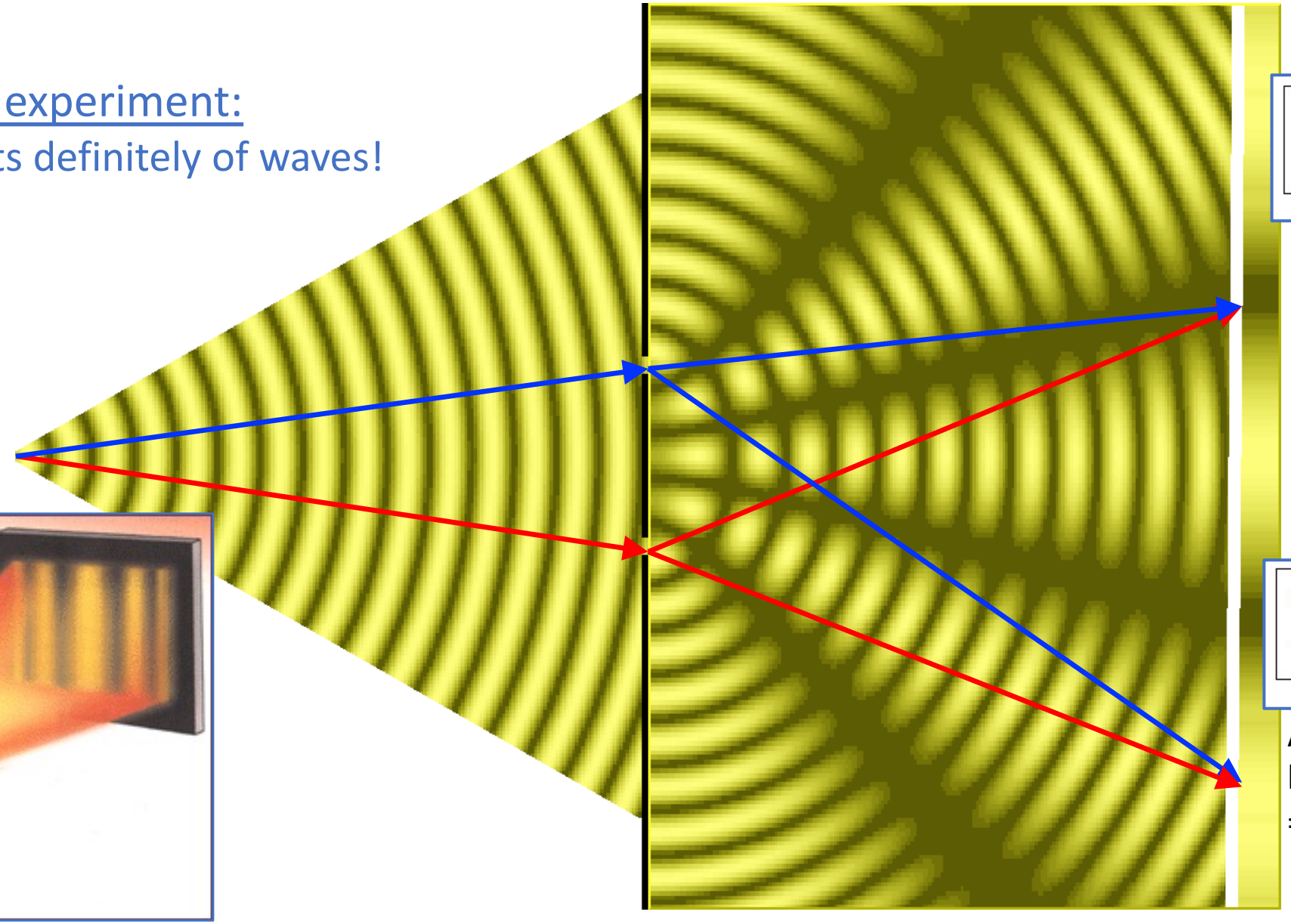
Interference with Water Waves

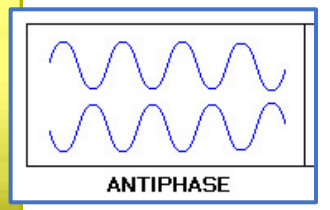
Interfering Waves

Double slit experiment:

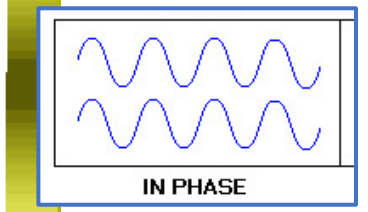
Light consists definitely of waves!





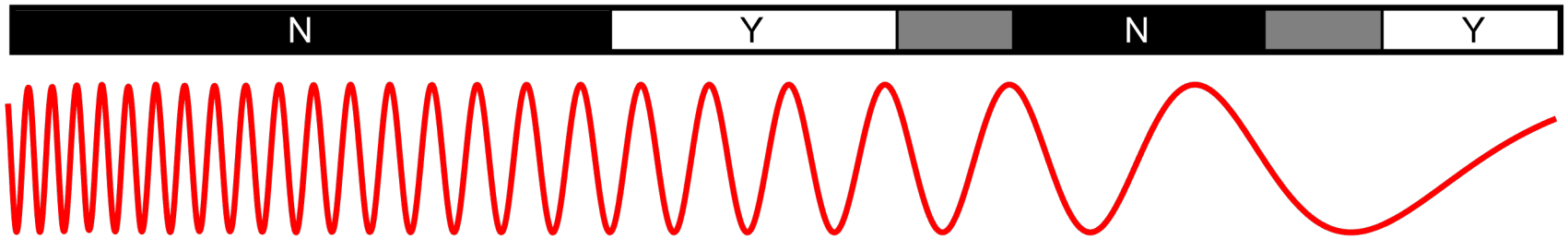


Cancellation:
light + light
= dark



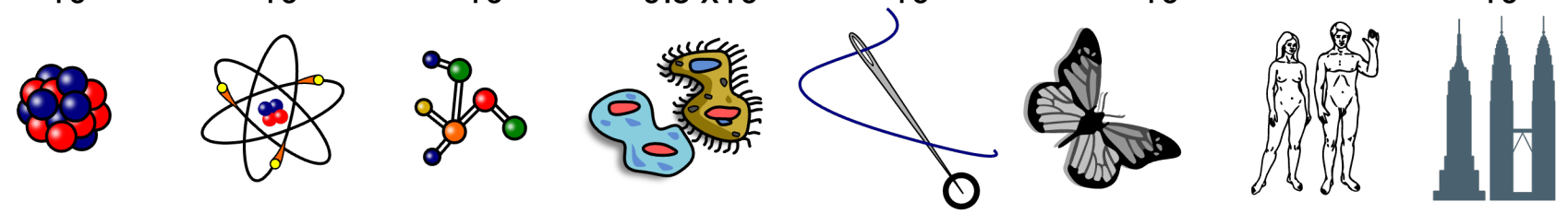
Amplification:
light + light
= more light

What can you see with light waves?



Penetrates Earth's Atmosphere?

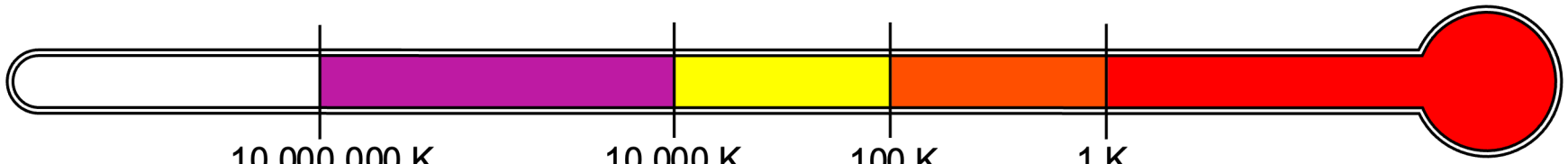
Gamma ray	X-ray	Ultraviolet	Visible	Infrared	Microwave	Radio	Radiation Type
10^{-12}	10^{-10}	10^{-8}	0.5×10^{-6}	10^{-5}	10^{-2}	10^{3}	Wavelength / m



Approximate Scale of Wavelength

Atomic Nuclei	Atoms	Molecules	Protozoans	Needle Point	Butterflies	Humans	Buildings

10^{20}	10^{18}	10^{16}	10^{15}	10^{12}	$^{8}10$	10^{4}	Frequency / Hz



10,000,000 K ~10,000,000°C	10,000 K 9,726°C	100 K -173°C	1 K -272°C	Temperature of objects at which this radiation is the peak wavelength emitted

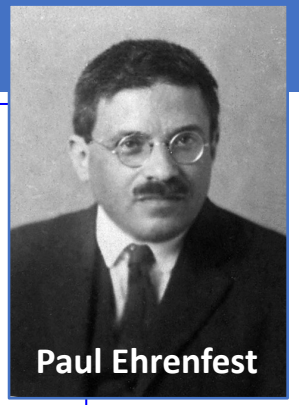
The surface of the sun is approximately 6000K.

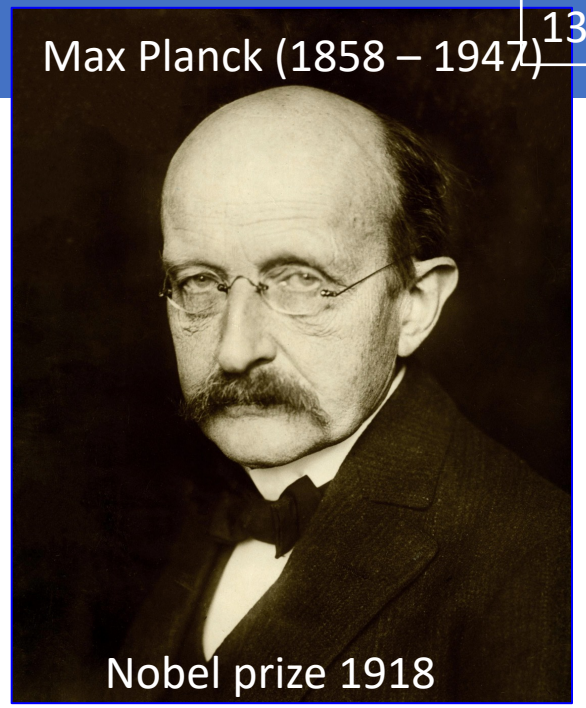
Particle nature: Quantized Light

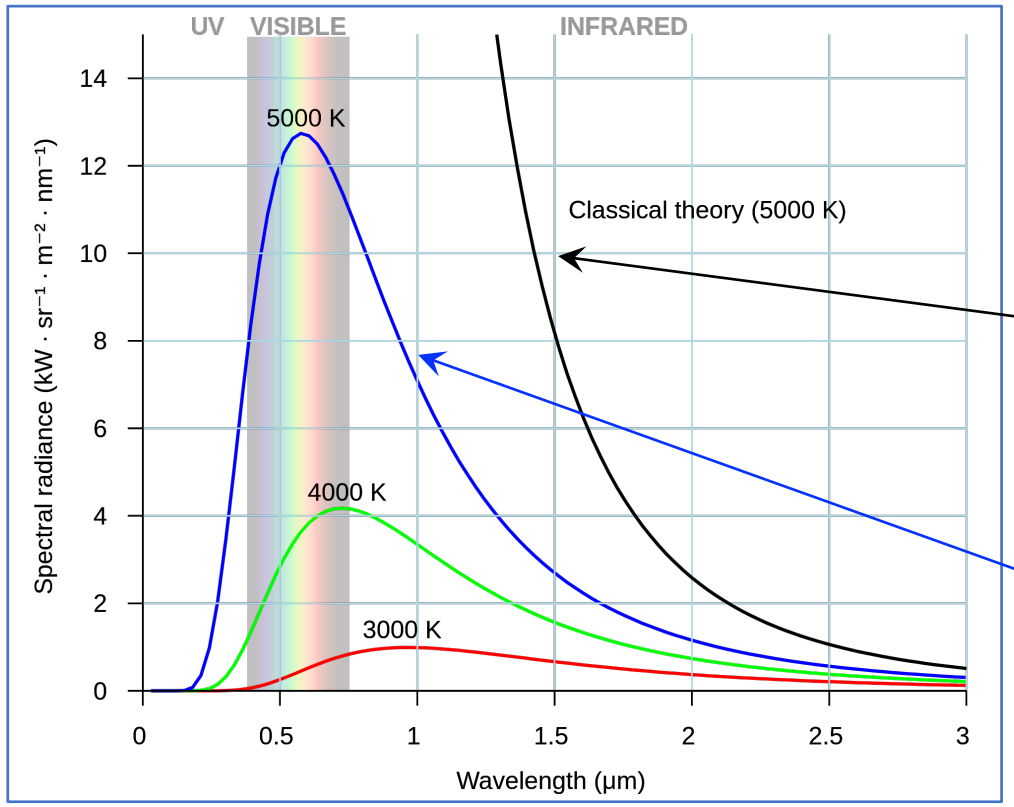
"UV catastrophe" in Black Body radiation spectrum:

If you heat a body it emits radiation. Classical thermodynamics predicts the amount of light at very short wavelength to be infinite!

Planck invented an ad-hoc solution:
For some reason material emitted light in "packages".


Paul Ehrenfest


Max Planck (1858 – 1947)
Nobel prize 1918

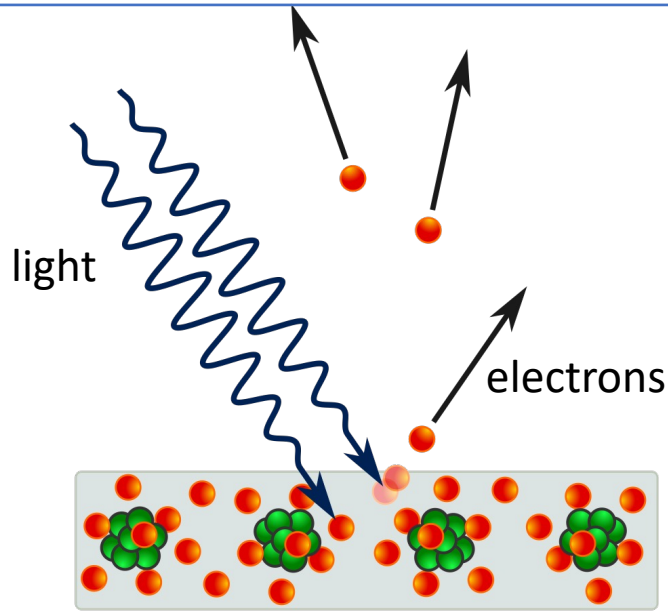


Planck's constant:
$h = 6.62 \times 10^{-34}$ Js

Classical theory:
There are more short wavelength "oscillations" of atoms than large wavelength "oscillations".

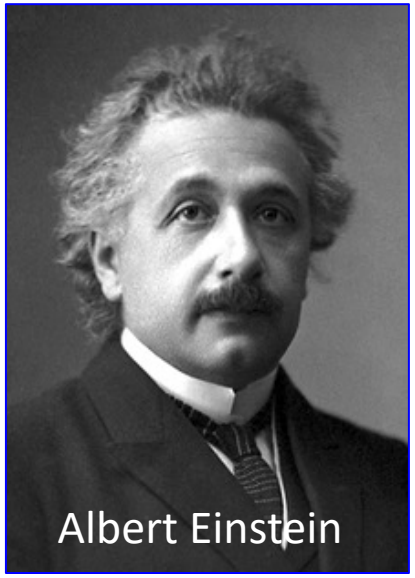
Quantum theory:
Light of high frequency (small wavelength) requires more energy: $E = hf$ (h = Planck's constant)

Photoelectric Effect



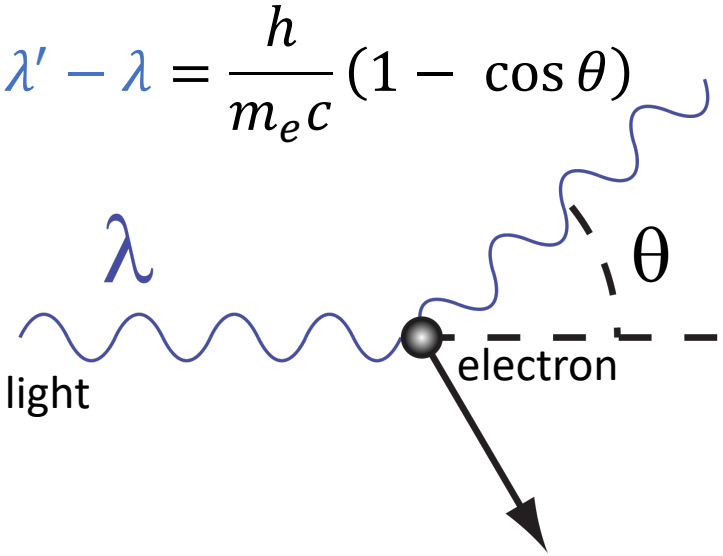
Photoelectric effect:
Light kicks out electron with ***E = h f***
(Independent on light intensity!)

Light ***consists*** of quanta.
(Nobelprize 1921)



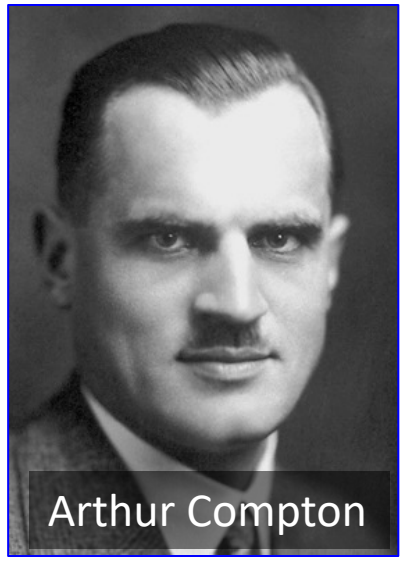
Wave: $E = hf = hc/\lambda$ → $\lambda = hc/E$
Momentum: $p = mv = mc = E/c$ → $E = pc$
It follows that: $\lambda = h/p$

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

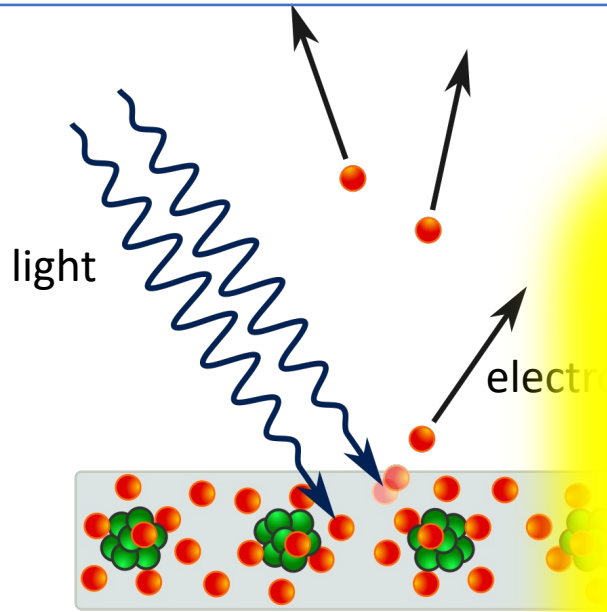


Compton Scattering:
"Playing billiards" with light quanta and electrons.

Light behaves as a particle with: ***λ = h / p***
(Nobelprize 1927)



Photoelectric Effect



Photoelectric effect:
Light kicks out electron with ***E = h f***
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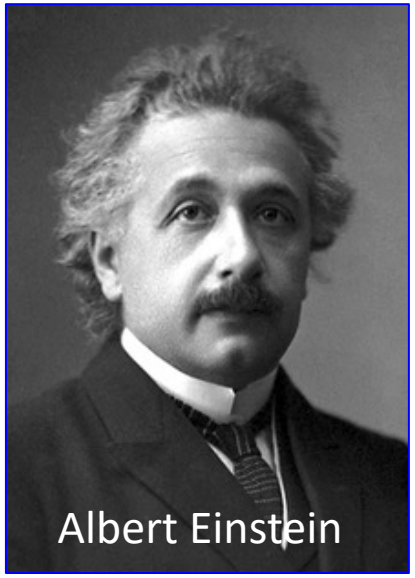
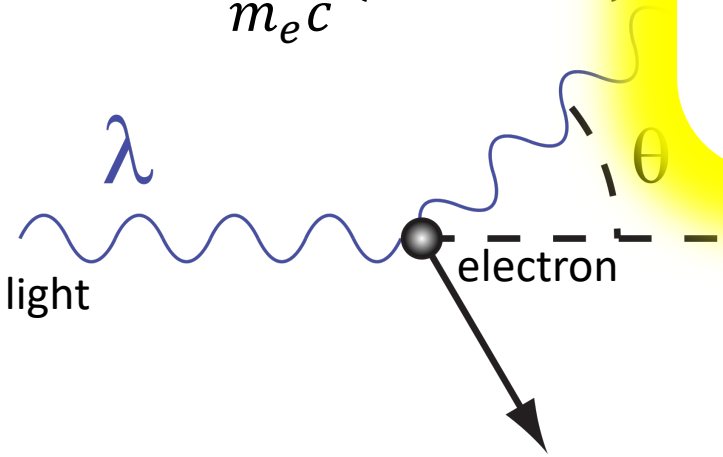
LIGHT IS A
WAVE!

$\lambda = hc/E$
$E = pc$


Albert Einstein

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$



"Playing billiards" with light quanta and electrons.

Light behaves as a particle with: ***λ = h / p***
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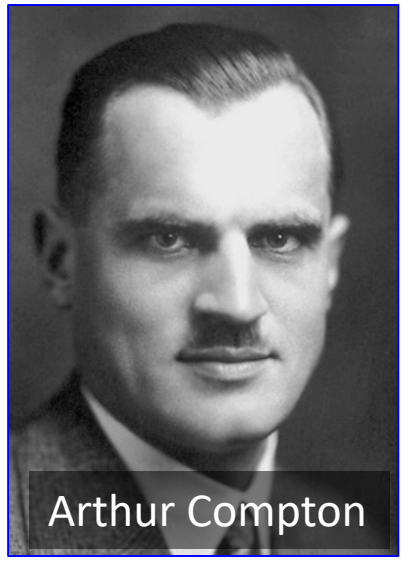
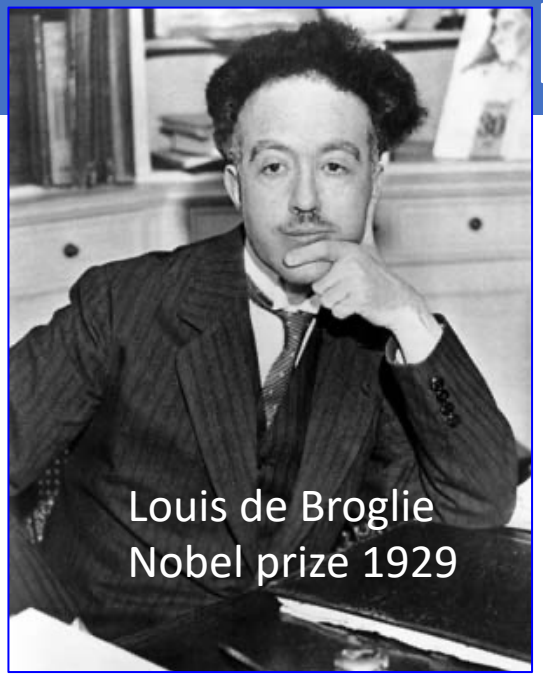

Arthur Compton

"Once and for all I want to know what I'm paying for. When the electric company tells me whether light is a wave or a particle I'll write my check."

Matter Waves

Louis de Broglie - PhD Thesis(!) 1924 (Nobel prize 1929):
If light are particles incorporated in a wave, it suggests that particles (electrons) “are carried” by waves.

Original idea: a physical wave ➔ Quantum mechanics: probability wave!

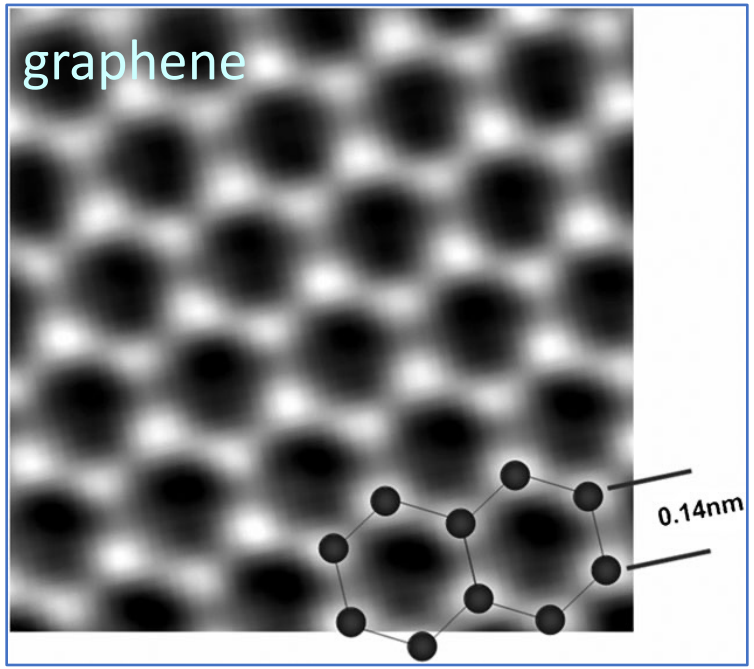


Particle wavelength: $\lambda = h/p \rightarrow \lambda = h/(mv)$

Wavelength visible light:
400 – 700 nm

Use $h = 6.62 \times 10^{-34}$ Js to calculate:

- Wavelength electron with v = 0.1 c:
  0.024 nm
- Wavelength of a fly (m = 0.01 gram, v = 10 m/s):
  0.000000000000000000062 nm



Matter Waves

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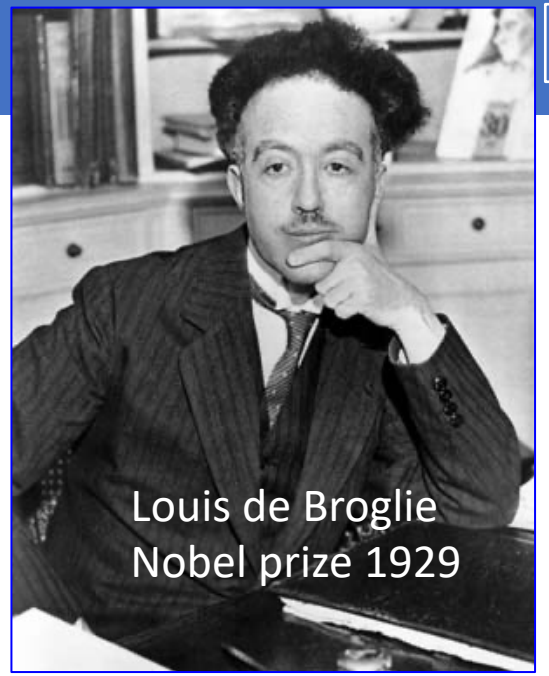
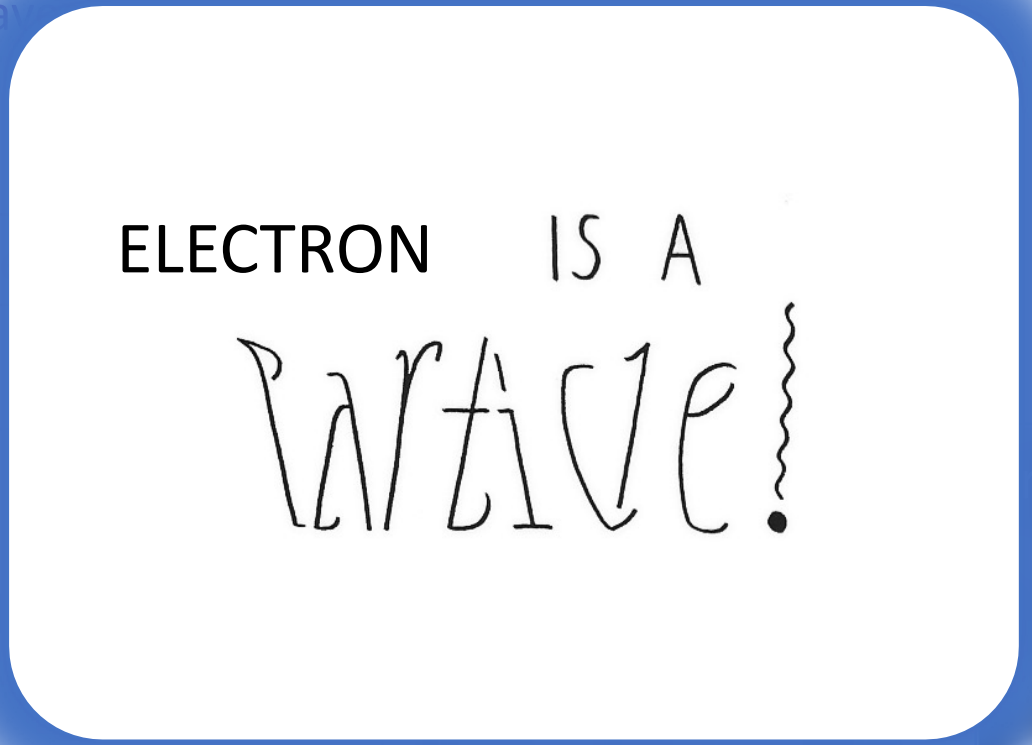
Original idea: a physical wave

Particle wave





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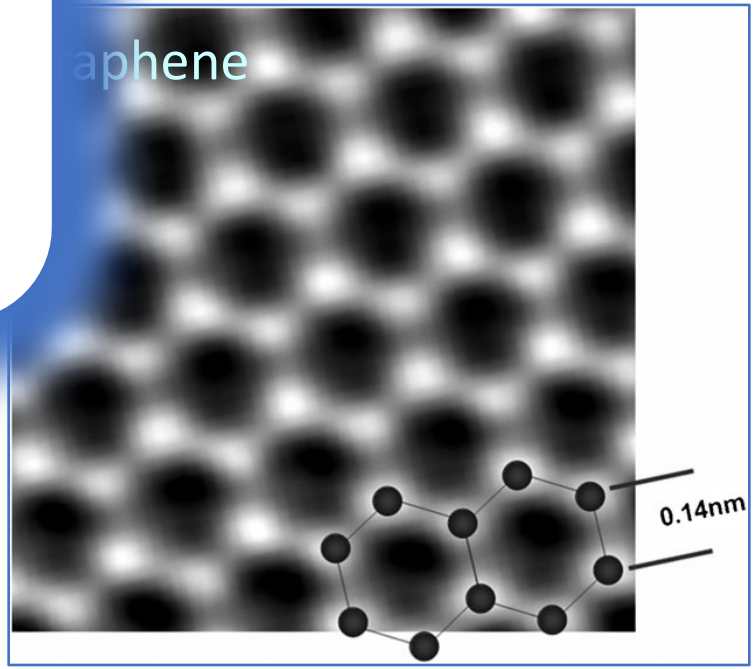
aphene



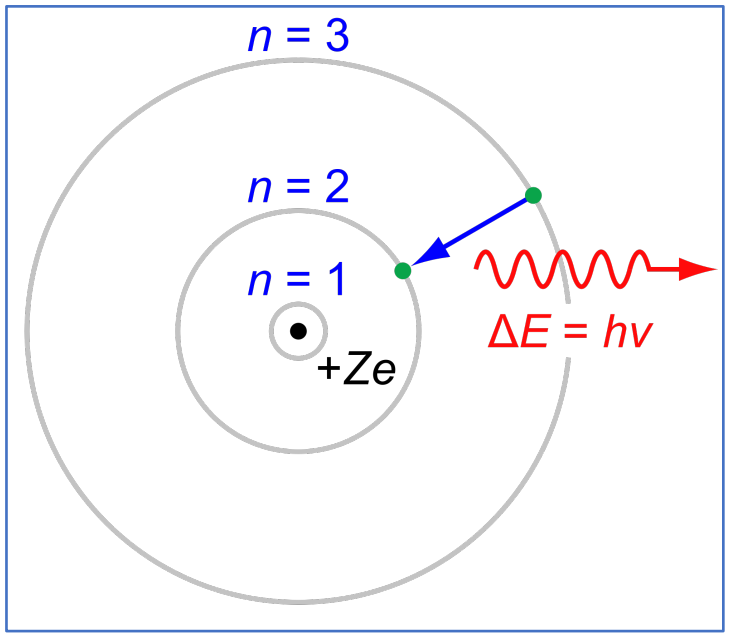
Speaking about Microscopes…

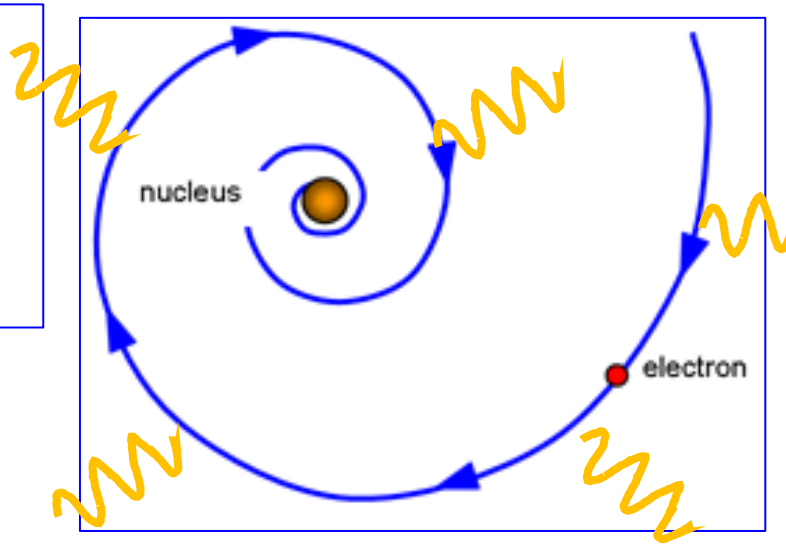
The Quantum Atom of Niels Bohr

The classical Atom is unstable!

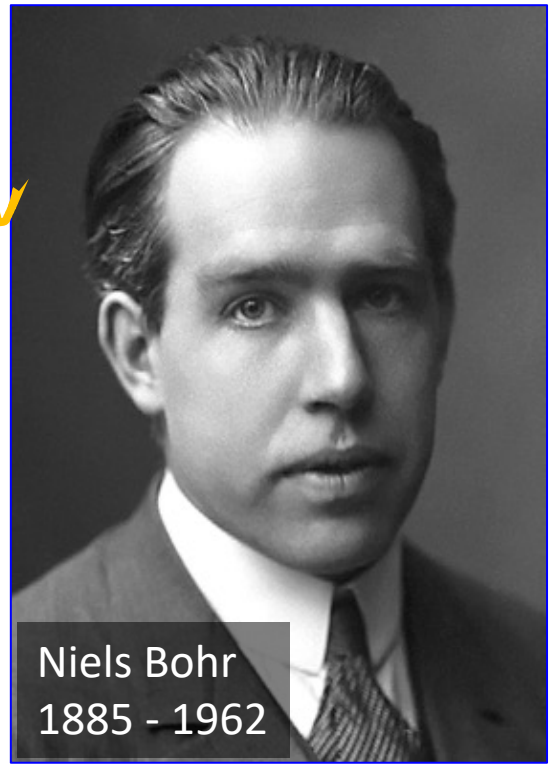
Expect: $t < 10^{-10}$ s

Niels Bohr: Atom is only stable for specific orbits: “energy levels”.





An electron can jump from a high to lower level by emitting a light quantum with corresponding energy difference.

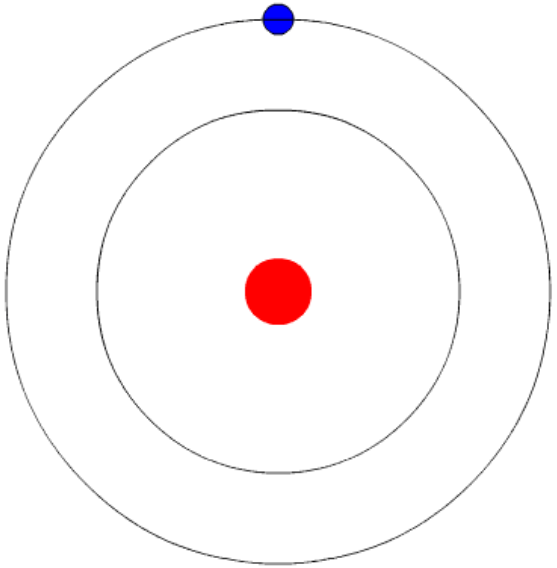

Niels Bohr
1885 - 1962

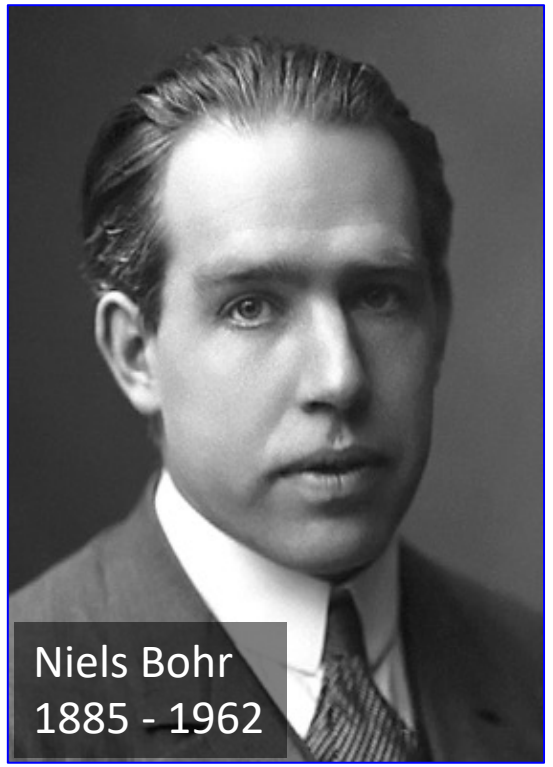
The Quantum Atom of Niels Bohr

The classical Atom is unstable!

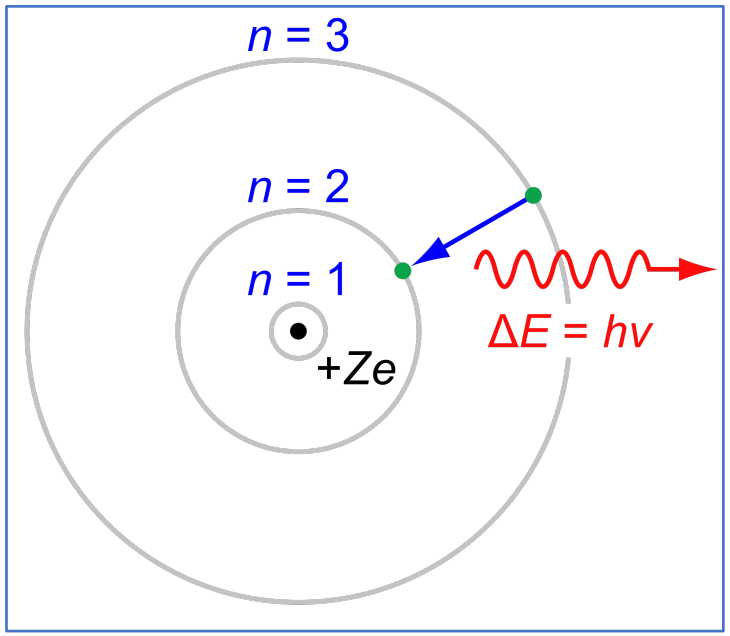
Expect: $t < 10^{-10}$ s

Niels Bohr: Atom is only stable for specific orbits: “energy levels”.





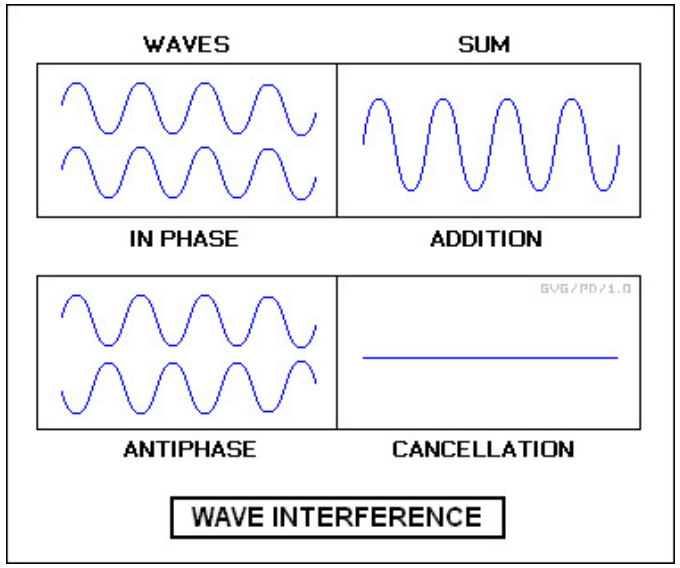
Niels Bohr
1885 - 1962

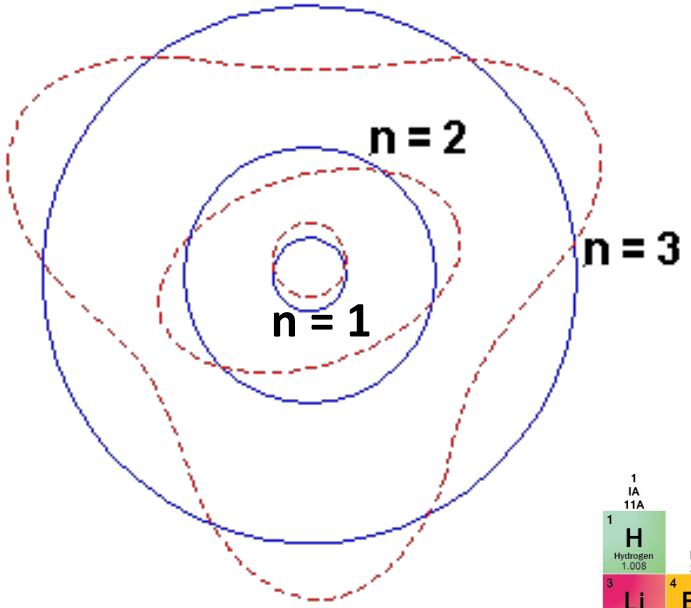


An electron can jump from a high to lower level by emitting a light quantum with corresponding energy difference.

Hydrogen “Balmer” spectrum of wavelengths:

Schrödinger: Bohr atom and de Broglie waves



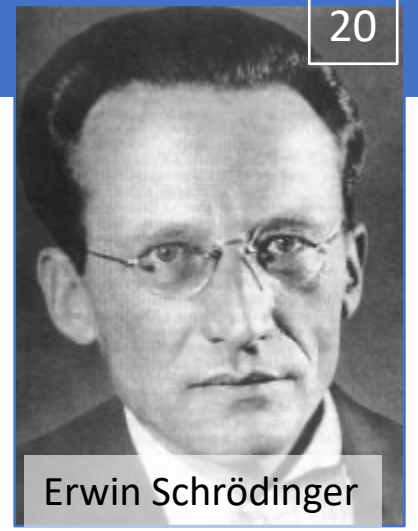


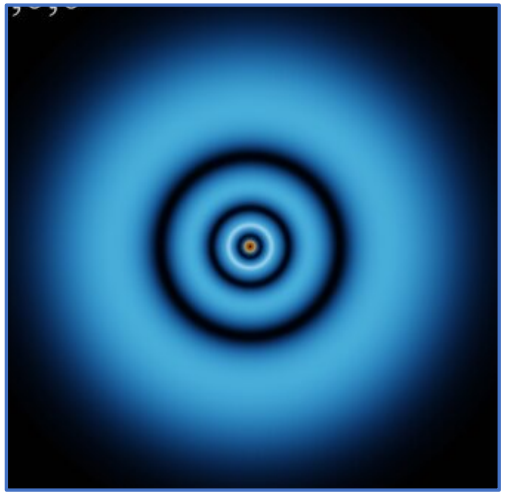
If orbit length "fits":

$2\pi r = n\lambda$ with $n = 1, 2, 3, \ldots$

The wave positively interferes with itself!

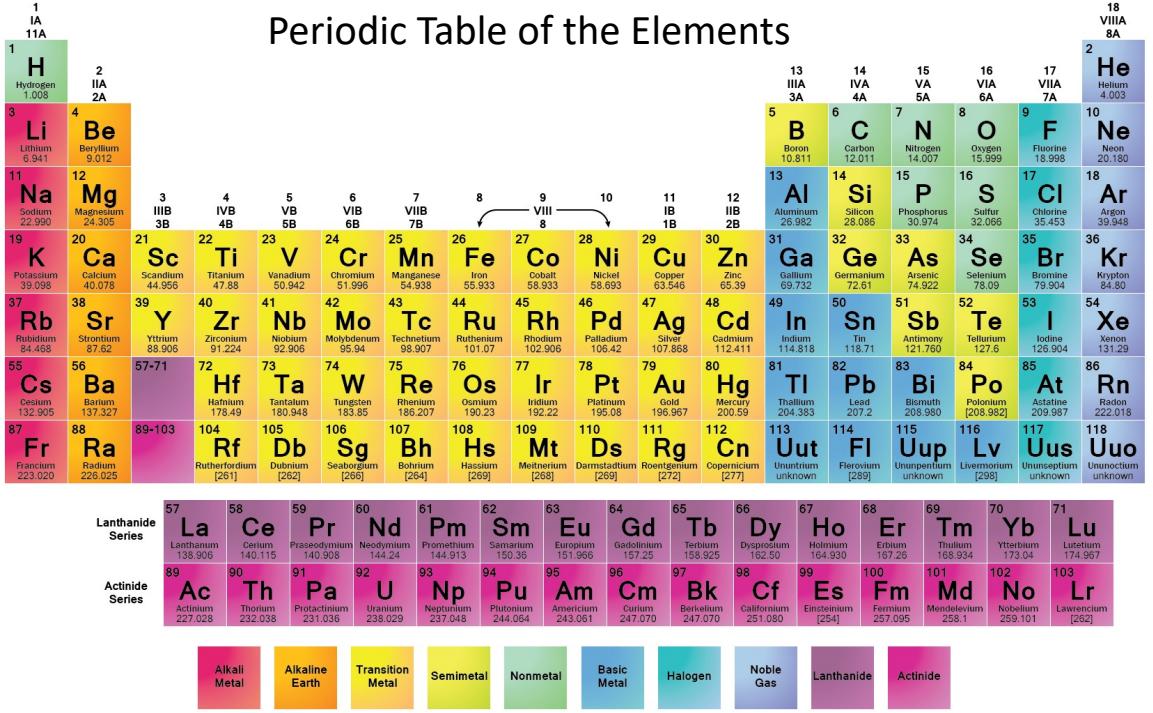
➔ Stable orbits!





More realistic atom: probability waves

Energy levels explained
→ atom explained
Outer shell electrons
→ "chemistry explained"



Not yet explained

The Periodic Table Of Scotch Whisky

Malt Whisky Distilleries (Active)

Lowlands	East of Speyside	Keith	Dufftown	Rothes	Elgin	Ben Rinnes	Centr. Speyside	South Speyside	West Speyside	NE Highlands	Inverness	Centr. Highlands	South Highlands	West Highlands	Islands	Islay
	Ga 1797 3 1.00 Glen Garioch															Kh 2005 2 0.12 Kilchoman
	Gh 1875 2 1.10 Glenglassaugh	Sr 1891 4 2.30 Strathmill			Gs 1876 6 1.80 Glenlossie										Ad 2008 2 0.02 Abhainn Dearg	Ar 1815 2 1.15 Ardbeg
	Cd 1825 2 1.30 Glencadam	St 1786 4 2.40 Strathisla	Gd 1897 6 3.70 Glendullan	Gp 1878 4 1.40 Glen Spey	Ge 1898 6 2.50 Glen Elgin	Gf 1836 6 3.40 Glenfarclas		Sy 1976 2 0.60 Speyside					Le 2004 2 0.6kl Loch Ewe		An 1993 2 0.75 Arran	Br 1881 4 1.50 Bruichladdich
Df 2005 2 0.07 Daftmill	Dr 1839 4 1.40 Glendronach	In 1871 4 2.90 Inchgower	Mo 1823 6 3.80 Mortlach	Sb 1837 2 2.50 Speyburn	Bi 1897 4 2.80 Benriach	Bs 1826 6 3.50 Benrinnes	Kn 1898 4 1.40 Knockando	Bm 1824 6 2.80 Balmenach		Wb 2013 2 0.12 Wolfburn	Da 1839 4 3.70 Dalmore	Ed 2 0.09 Edradour	Tu 1775 2 0.34 Glenturret	Ki 2004 2 0.75 Glengyle	Tm 1798 4 1.00 Tobermory	Bw 1779 3 2.00 Bowmore
Bd 1817 2 0.25 Bladnoch	Kd 1893 2 1.75 Knockdhu	Au 1896 3 3.00 Aultmore	Kv 1990 9 4.80 Kininvie	Cg 1891 4 4.00 Craigellachie	Mr 1897 4 3.30 Glen Moray	Ab 1826 4 3.70 Aberlour	Cr 1869 4 2.20 Cragganmore	Tt 1964 4 3.30 Tomintoul	Bc 1898 2 0.50 Benromach	Pu 1826 2 1.80 Pulteney	Te 1817 6 4.40 Teaninich	Rl 1845 2 0.45 Royal Lochnagar	Gy 1833 3 1.10 Glengoyne	Gc 1832 2 0.75 Glen Scotia	Sc 1885 2 1.50 Scapa	Lg 1816 4 2.35 Lagavulin
Au 1823 3 1.75 Auchentoshan	Fe 1824 4 2.30 Fettercairn	Tc 1897 6 4.20 Glentauchers	Ba 1892 11 5.60 Balvenie	Gr 1878 10 5.60 Glenrothes	Ma 1971 6 3.45 Mannochmore	Al 1975 4 4.00 Allt-a-Bhainne	Ca 1824 6 3.40 Cardhu	Bl 1973 6 4.00 Braeval	Gb 1810 4 4.20 Glenburgie	Bb 1790 2 2.00 Balblair	Rb 1812 4 4.00 Royal Brackla	Dh 1897 2 2.20 Dalwhinnie	Tb 1949 4 2.70 Tullibardine	Sp 1828 3 0.75 Springbank	Ju 1810 4 2.20 Jura	Bn 1881 4 2.50 Bunnahabhain
Gk 1837 2 2.50 Glenkinchie	Md 1962 5 3.34 Macduff	Ak 1974 8 5.00 Auchroisk	Du 1896 6 5.80 Dufftown	Gg 1840 10 5.90 Glen Grant	Li 1821 6 3.75 Linkwood	Ac 1967 4 4.20 Glenallachie	Td 1896 6 4.00 Tamdhu	Tv 1966 6 4.00 Tamnavulin	Mi 1824 6 5.50 Milltonduff	Cy 1967 6 4.80 Clynelish	Go 1838 6 5.00 Glen Ord	Bh 1798 4 2.50 Blair Athol	De 1965 4 3.00 Deanston	Ob 1794 2 0.78 Oban	Hp 1798 4 2.50 Highland Park	Lp 1810 7 3.30 Laphroaig
Ai 2007 8 6.25 Ailsa Bay	Am 1898 8 5.20 Ardmore	Ke 1957 6 6.00 Glen Keith	Gl 1886 28 12.00 Glenfiddich	Mc 1824 21 9.40 Macallan	Ln 1894 8 4.40 Longmorn	Dl 1852 6 5.20 Dailuaine	Tr 1958 8 4.90 Tormore	Gv 1837 14 10.50 Glenlivet	Ro 2009 14 12.50 Roseisle	Gm 1843 12 6.00 Glenmorangie	To 1897 12 5.00 Tomatin	Af 1896 4 3.50 Aberfeldy	Lo 1965 0 4.00 Loch Lomond	Bv 1825 4 1.80 Ben Nevis	Ta 1830 5 2.60 Talisker	Cl 1846 6 6.50 Caol Ila

Distillery/Brand: Di — Est. / Clsd. / Stills / Cap.

Malt Whisky Distilleries (Closed)

Bf 1824 1983 2 Banff	By 1965 1976 2 Ben Wyvis	Bo 1819 1983 2 Brora	Cp 1897 2002 4 Caperdonich	Co 1897 1985 4 Coleburn	Cv 1894 1985 4 Convalmore	Dd 1898 1983 2 Dallas Dhu	Ay 1844 1983 2 Glen Albyn	Fl 1974 1985 4 Glen Flagler	Mh 1892 1983 2 Glen Mhor	Es 1897 1985 4 Glenesk	Ly 1898 1983 2 Glenlochy	Gu 1831 1983 4 Glenugie	Ur 1825 1983 4 Glenury Royal	Im 1897 1998 4 Imperial	Iv 1938 1991 2 Inverleven	Kc 1957 1975 2 Kinclaith
La 1966 1975 4 Ladyburn	Lm 1772 1996 2 Littlemill	Ls 1957 1992 4 Lochside	Mb 1807 1985 2 Millburn	Np 1820 1982 2 North Port	Pi 1974 1993 4 Pittivaich	Pe 1825 1983 4 Port Ellen	Rs 1798 1993 2 Rosebank	Sm 1795 1983 4 Saint Magdalene								

Grain Whisky Distilleries (Active / Closed)

Cm 1813 100.0 Cameronbridge	Gi 1963 75.0 Girvan	Ig 1961 40.0 Invergordon	Nb 1885 64.0 North British	Sl 2011 25.00 Starlaw	Sd 1927 40.0 Strathclyde											
Distillery capacity may also include other spirits (vodka, gin etc.)																
						Cb 1806 1993 Cambus	Cn 1855 1988 Caledonian	Cs 1799 1983 Carsebridge	Gt 1964 1986 Garnheath	Db 1938 2002 Dumbarton	Ns 1957 1980 North of Scotland	Pd 1811 2010 Port Dundas				

Blended Whisky Brands (Selected)

Aq 1880 Antiquary	Bt 1910 Ballantine's	Be 1904 Bell's	Bk 1884 Black & White	Bb 1897 Black Bottle	Ch 1857 Chivas Regal	Cu 1923 Cutty Sark	Dw 1899 Dewar's	Fa 1896 Famous Grouse	Gn 1887 Grant's	Hg 1888 Haig	Jb 1930 J&B	Jw 1865 Johnnie Walker	Us 1850 Usher's	Vt 1882 VAT 69	Wh 1861 White Horse	Wy 1882 Whyte & Mackay

Owners

- Diageo
- Pernod Ricard
- Bacardi
- Wm Grant & Sons
- Edrington
- Burn Stewart
- United Spirits
- Int. Beverage
- Suntory
- LVMH
- Benriach
- Beam
- Loch Lomond
- Angus Dundee
- Mitchell's
- Campari
- Nikka
- La Martiniquaise
- Remy Cointreau
- Small/Indep.

DRAMMING EVERYTHING WHISKY

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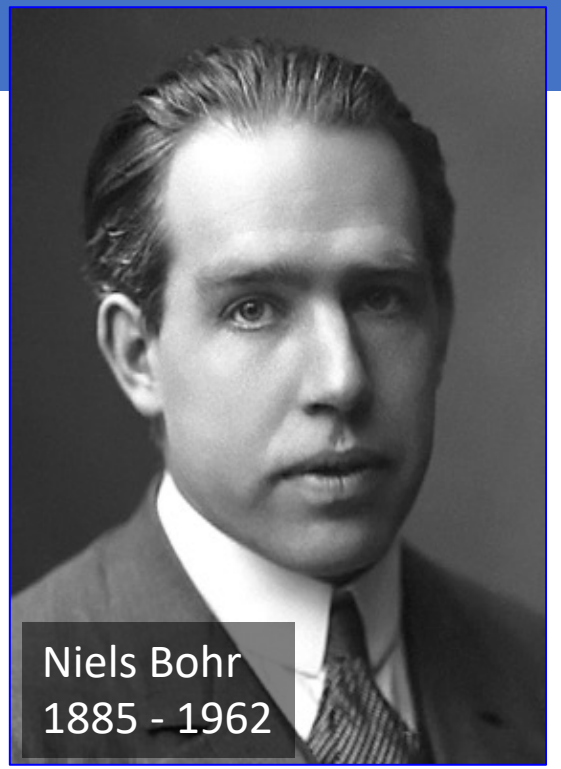
Version 1.4 - 27 May 2013

Dist. capacity in million litres p.a. except Le

Complementarity

Subatomic matter is not just waves and it is not just particles.
It is nothing we know from macroscopic world.


Niels Bohr
1885 - 1962

Copenhagen Interpretation (Niels Bohr):

One can observe wave characteristics or particle characteristics of quantum objects, ***never both*** at the same time.

Particle and Wave aspects of a physical object are ***complementary***.

Similarly one can never determine from a quantum object at the same time: ***energy and time, position and momentum*** (and more).

Heisenberg Uncertainty Relation

A measurement of a characteristic of quantum matter affects the object.
Heisenberg's "non-commuting" observables:

You cannot know ***position*** and ***momentum*** at the same time:

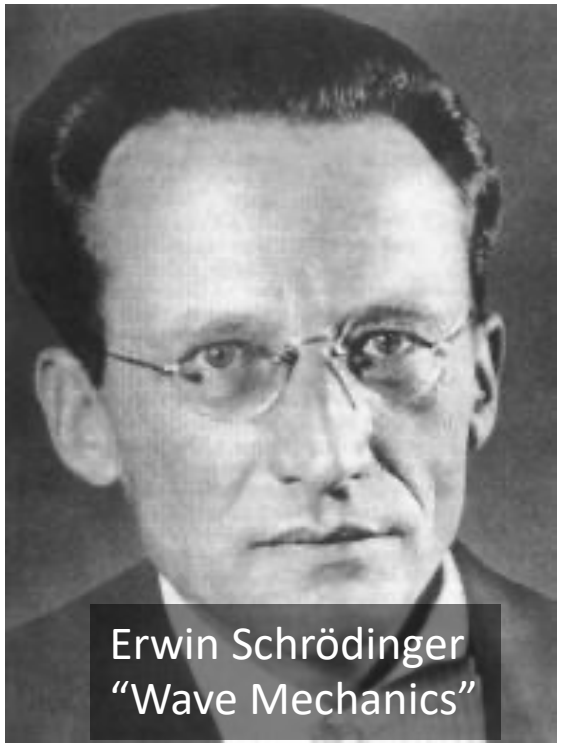
$$\Delta x \ \Delta p \ \geq \ \hbar/2$$

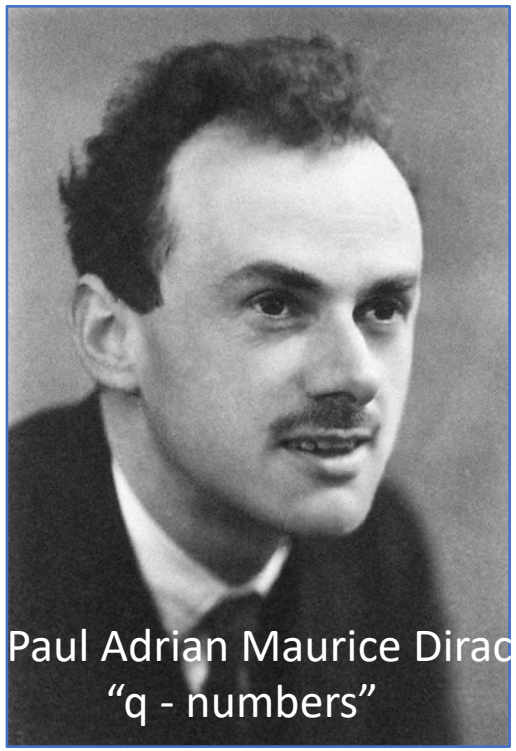
(think of $x \rightarrow \lambda = h/p$)

You cannot know both **energy** and **time**:

$$\Delta E \ \Delta t \ \geq \ \hbar/2$$


Werner Heisenberg
"Matrix mechanics"


Erwin Schrödinger
"Wave Mechanics"


Paul Adrian Maurice Dirac
"q - numbers"

It is a fundamental aspect of nature.

Not related to limited technology!

Heisenberg Uncertainty Relation

Energy and time:

In quantum mechanics energy is the "rate of change of a wavefunction": $E\psi \rightarrow i\hbar \frac{\partial}{\partial t}\psi$

It fundamentally requires time to measure energy, ie. change in time.

Consequence: $\Delta E\ \Delta t \geq \hbar/2$ Measuring energy means locality in time is lost.

Position and momentum:

In quantum mechanics energy is the "rate of change of a wavefunction": $p\psi \rightarrow i\hbar \frac{\partial}{\partial x}\psi$

It fundamentally requires space to measure momentum, ie. change in space.

Consequence: $\Delta x\ \Delta p \leq \hbar/2$ Measuring momentum means locality in space is lost.

It is a fundamental aspect of nature.

Not related to limited technology!

Heisenberg Uncertainty Relation

It is a fundamental aspect of nature.

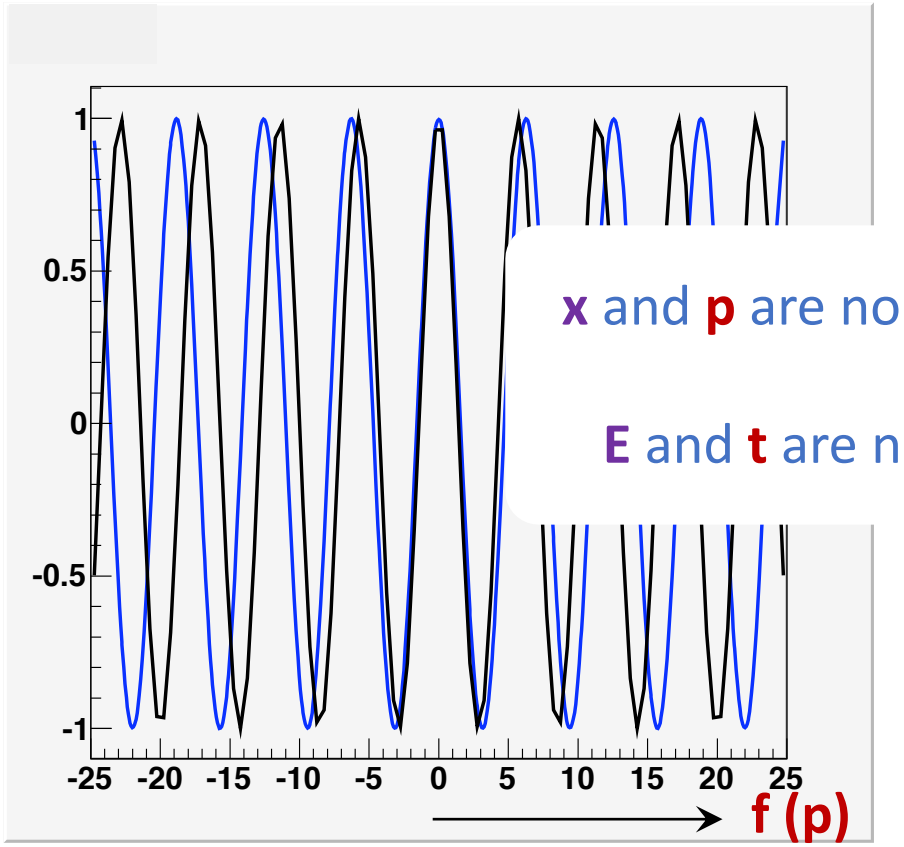
Not related to limited technology!

Waves and Uncertainty

Use the "wave-mechanics" picture of Schrödinger

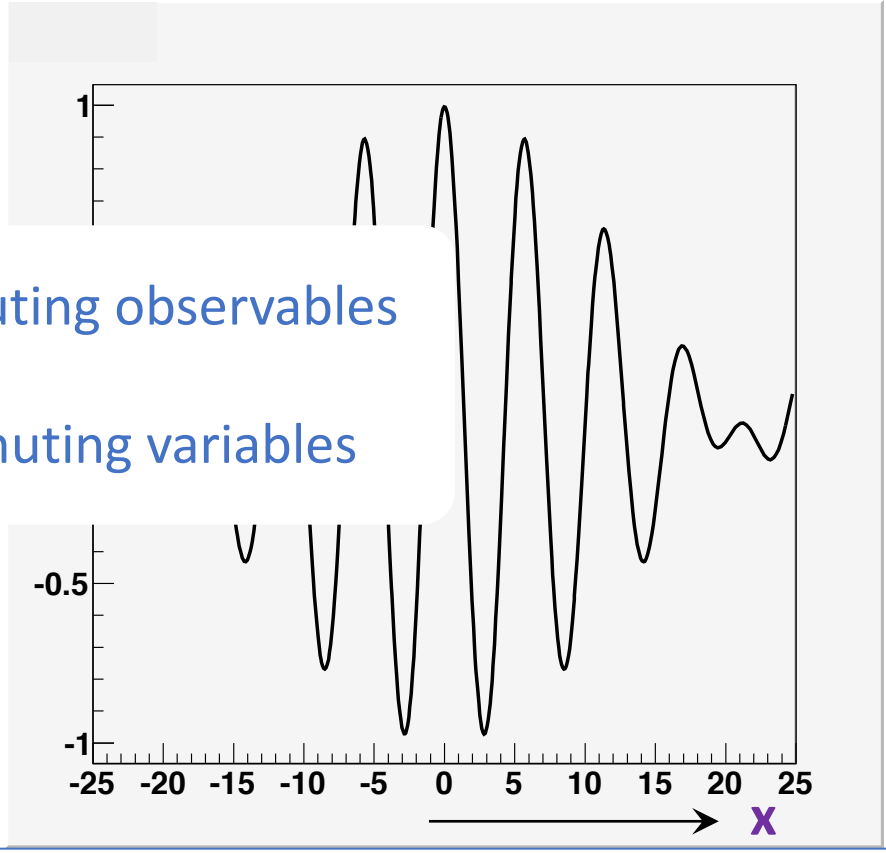
A wave has an exactly defined frequency.

Two waves: $p_1 = hf_1/c$, $p_2 = hf_2/c$



A particle has an exactly defined position.

Wave Packet: sum of black and blue wave



x and **p** are non-commuting observables

also

E and **t** are non-commuting variables

The more waves are added, the more the wave packet looks like a particle, or,

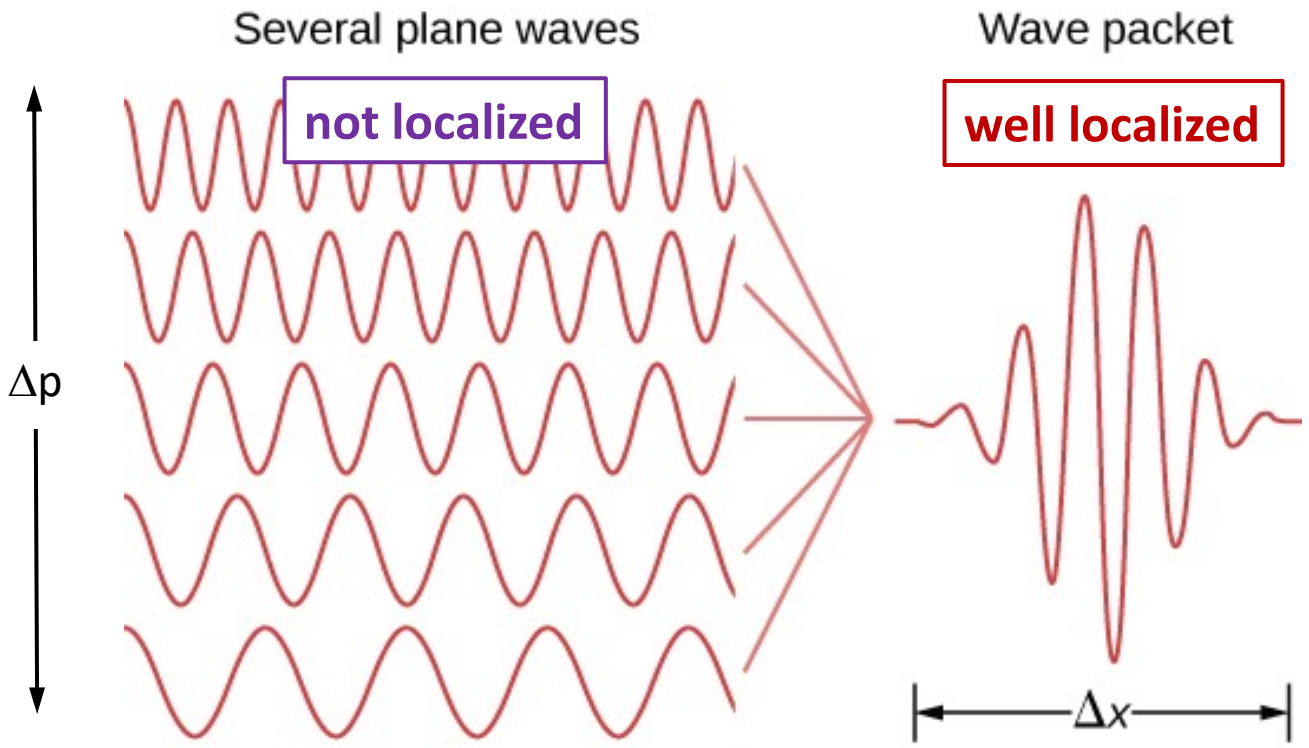
If we try to determine the position **x**, we destroy the frequency/momentum **p** and vice versa.

Waves and Particles

"Particle:"

Pure waves of different frequency, i.e. different momentum $p = hf/c$

Wave package, i.e. "particle"



Measure precise momentum
→ You force the quantum to ***have*** a specific frequency
→ You lose the locality of the quantum
→ No position information.

Measure precise position
→ You force the quantum to ***have*** a specific position
→ You lose the momentum (frequency) information of the quantum

The uncertainty relation at work

Shine a beam of light through a narrow slit which has a opening size Δx.
The light comes out over an undefined angle that corresponds to Δp_x.

$$\Delta x \, \Delta p_x \sim \hbar/2$$

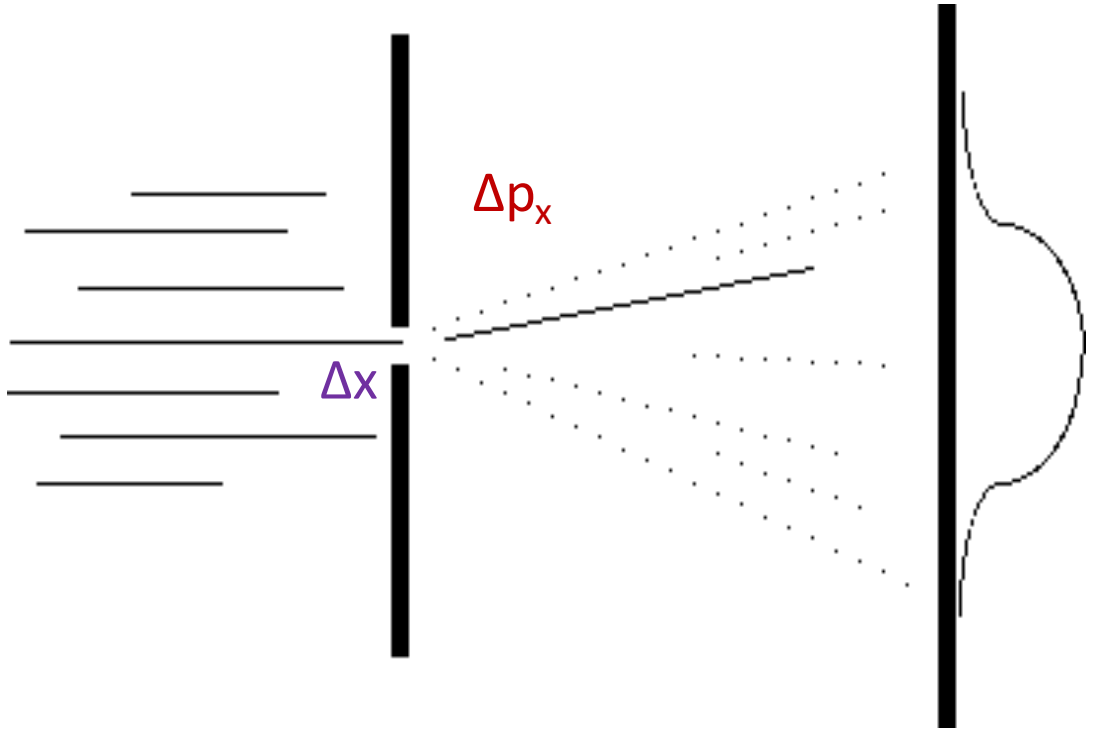
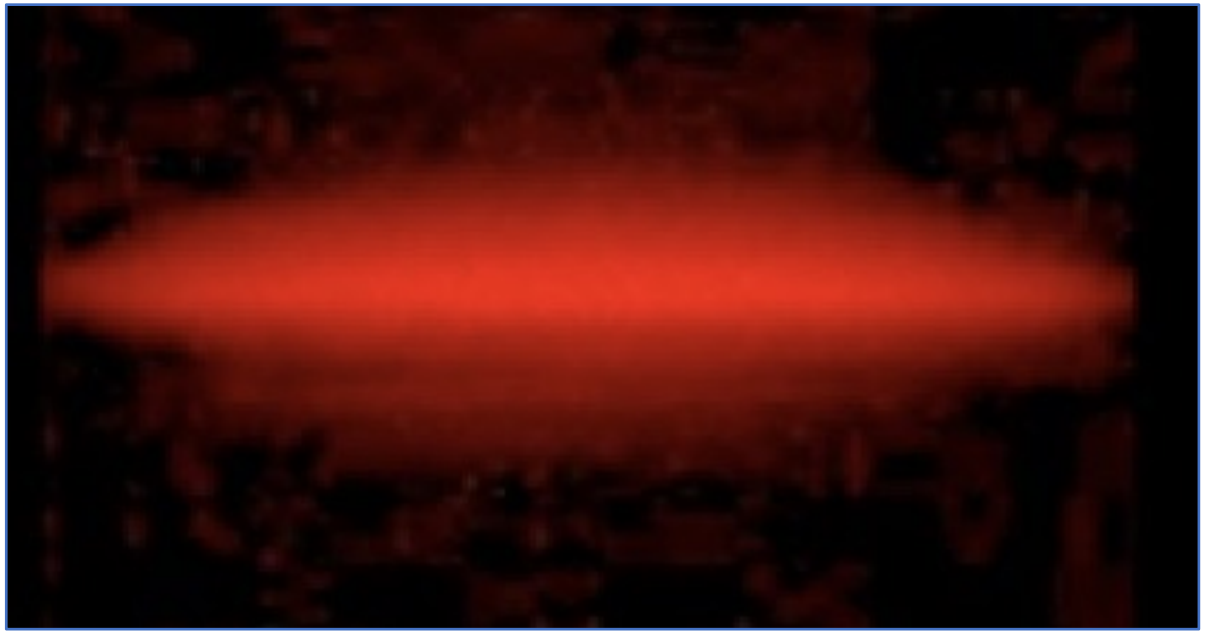
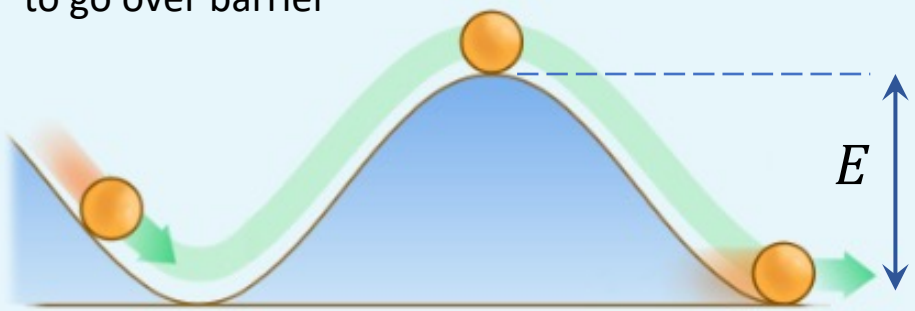


Image of a laser pointer after passing through a slit:

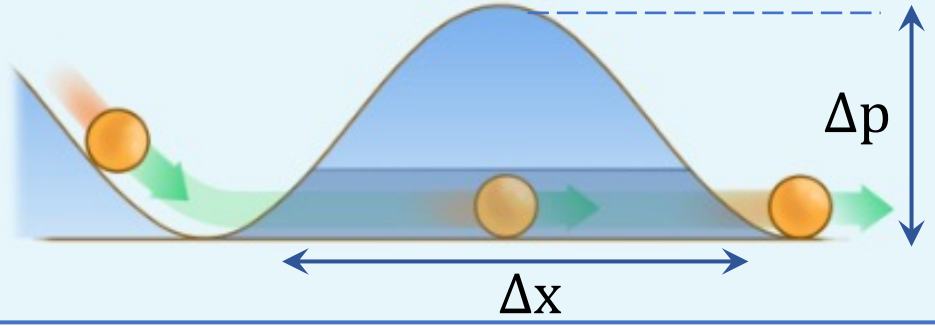


Consequence: Particles can quantum tunnel through a wall!

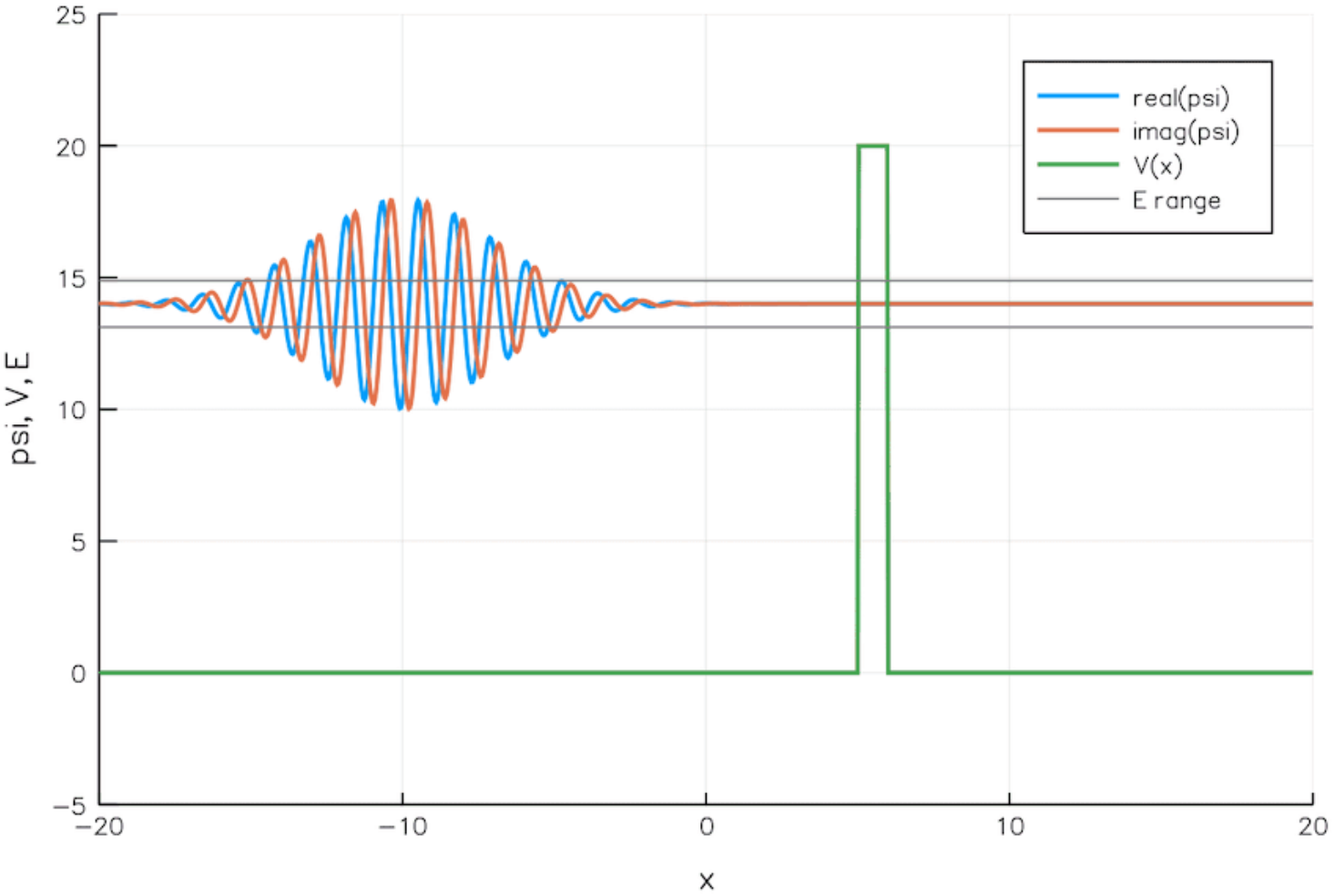
Classic: particle needs enough energy $E = \frac{1}{2}mv^2$ to go over barrier



Quantum: particle can tunnel through the barrier. Even if its energy is not enough.

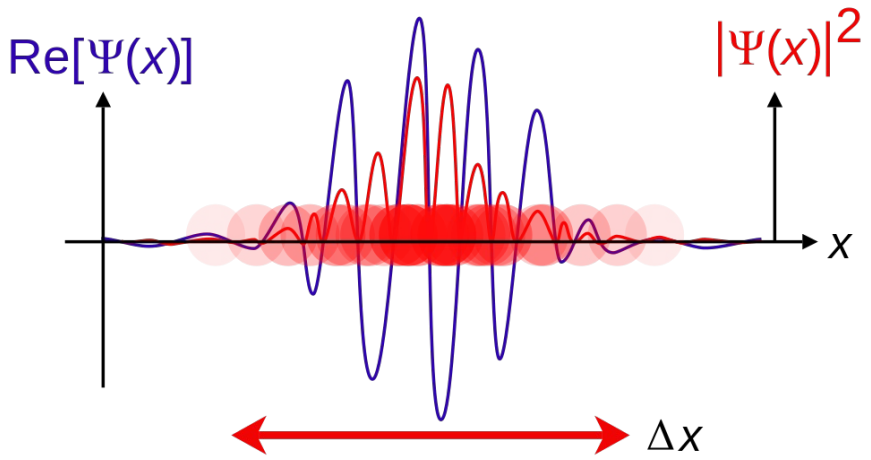


Quantum explanation: particles are waves packets: "The particles energy (or momentum) is uncertain enough to pass if the time (or space) is short enough."

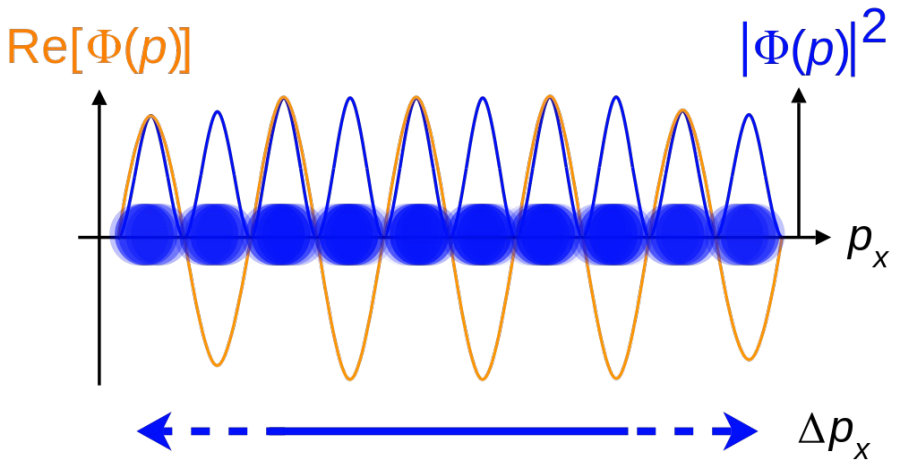


The wave function ψ

Position fairly known

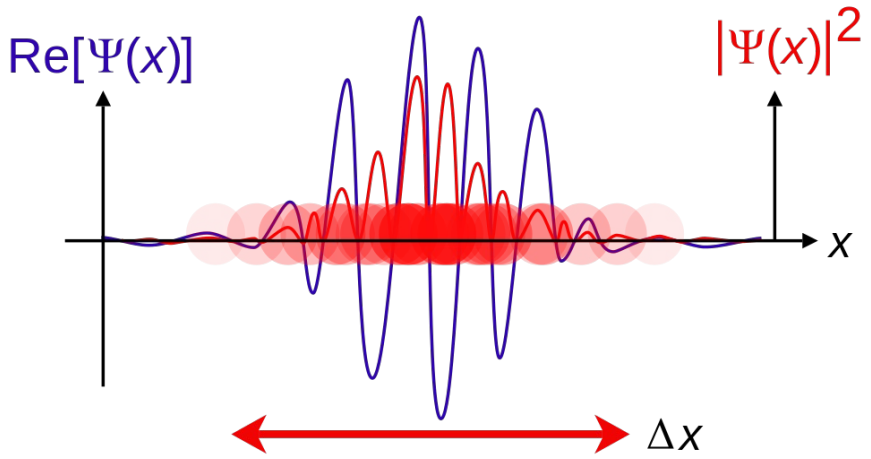


Momentum badly known

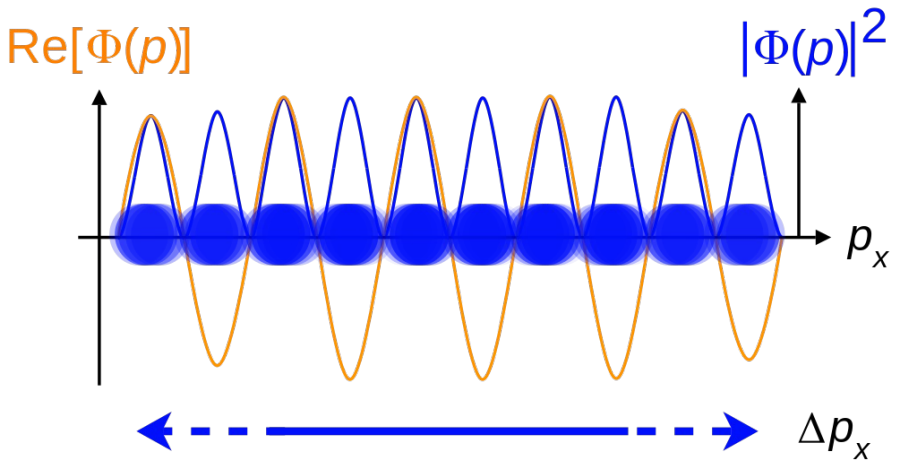


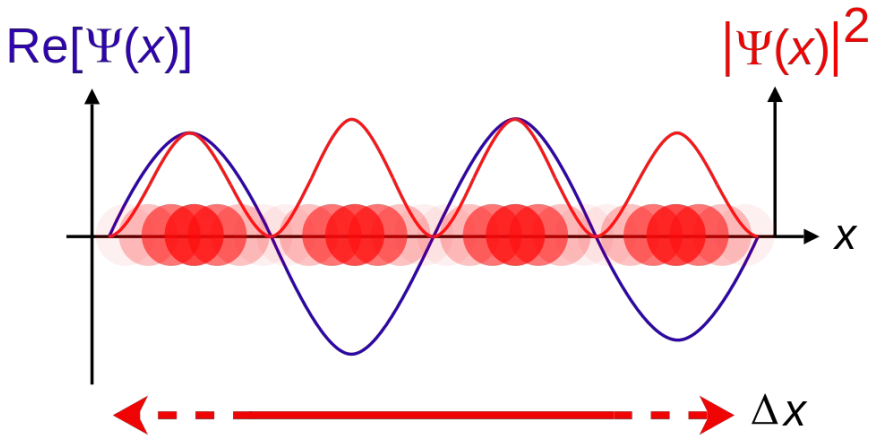
The wave function ψ

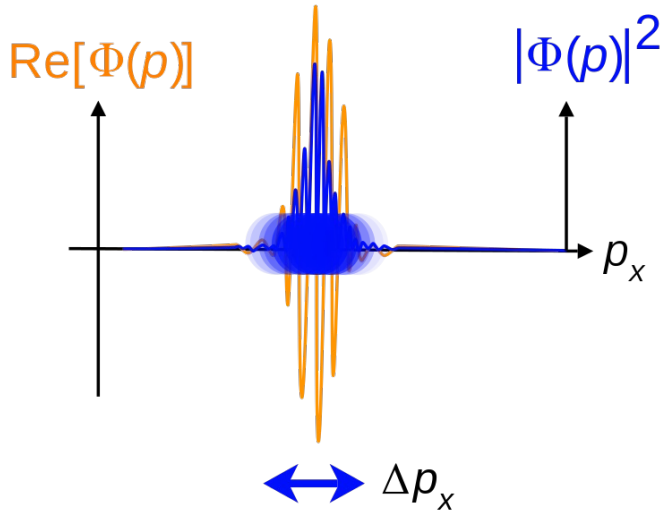
Position fairly known



Momentum badly known



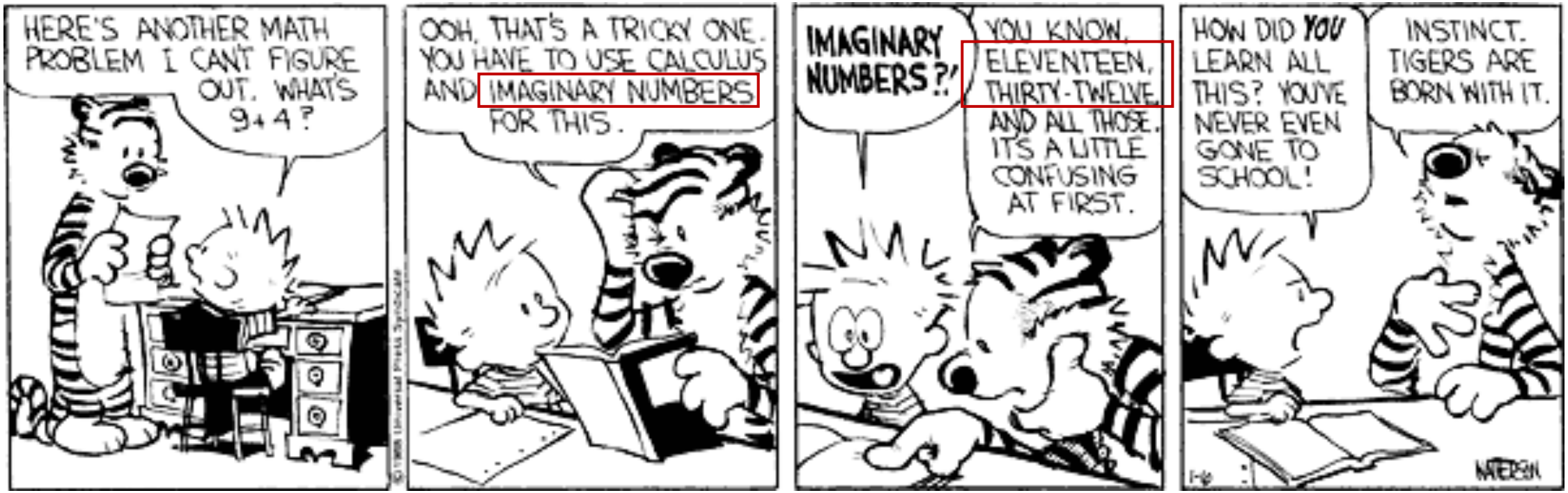




Position badly known

Momentum fairly known

Imaginary Numbers



The Copenhagen Interpretation

The wave function ψ is not a real object.
The only physical meaning is that its square gives the probability to find a particle at a position x and time t.

$$\text{Prob}(x,t) = |\psi(x,t)|^2 = \psi\,\psi^*$$


Niels Bohr

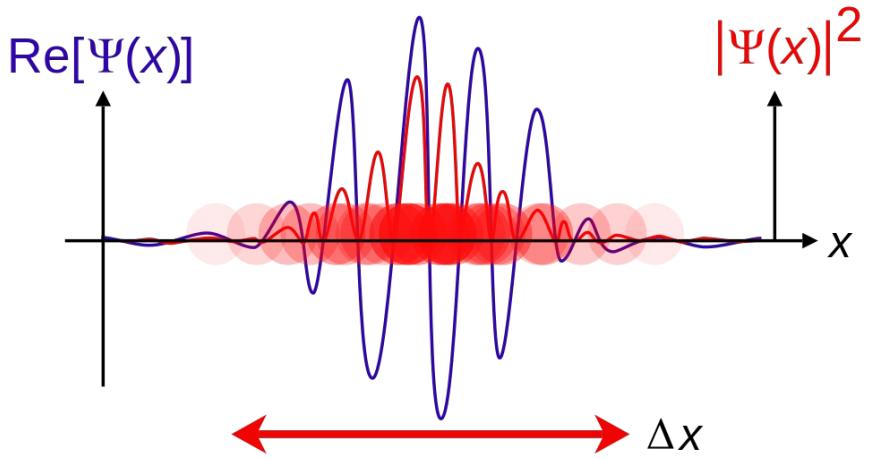

Max Born

Quantum mechanics allows only to calculate ***probabilities*** for possible outcomes of an experiment and is non-deterministic, contrary to classical theory.
Einstein: "Gott würfelt nicht."

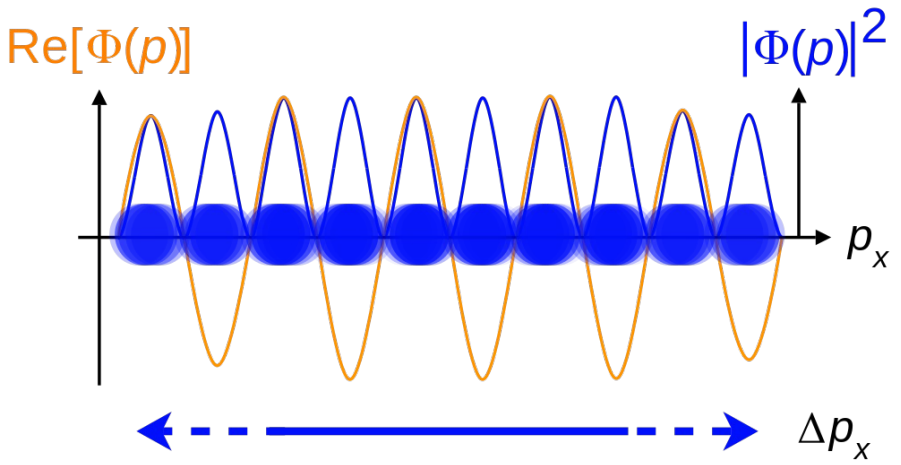
The mathematics for the ***probability*** of the quantum wave-function is the same as the mathematics of the ***intensity*** of a classical wave function.

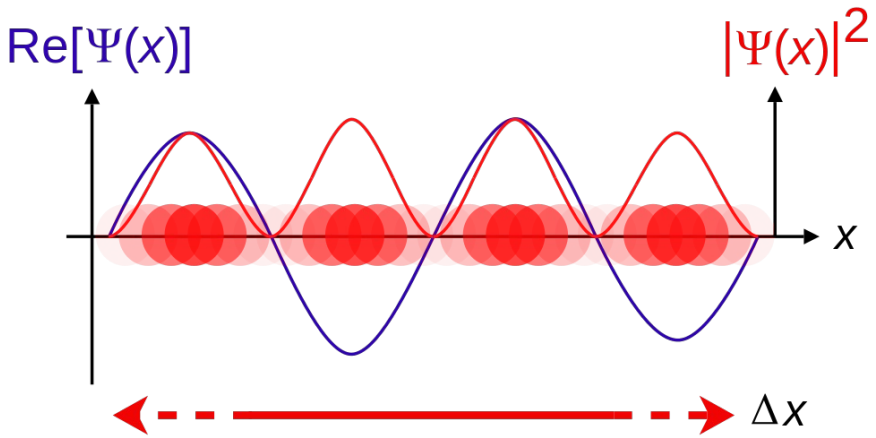
The wave function ψ

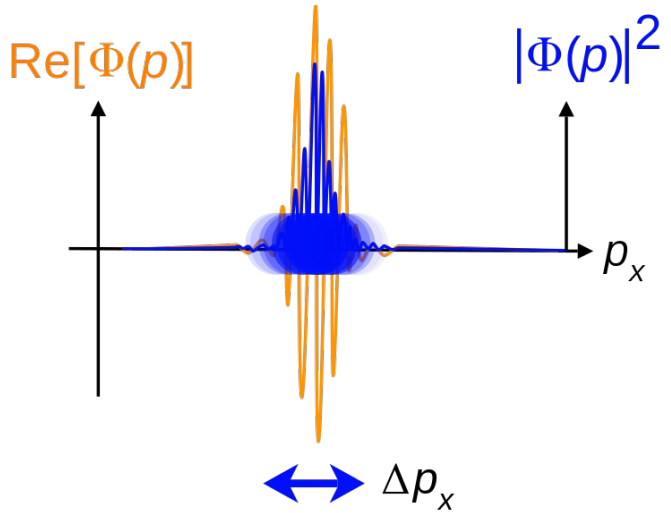
Position fairly known



Momentum badly known



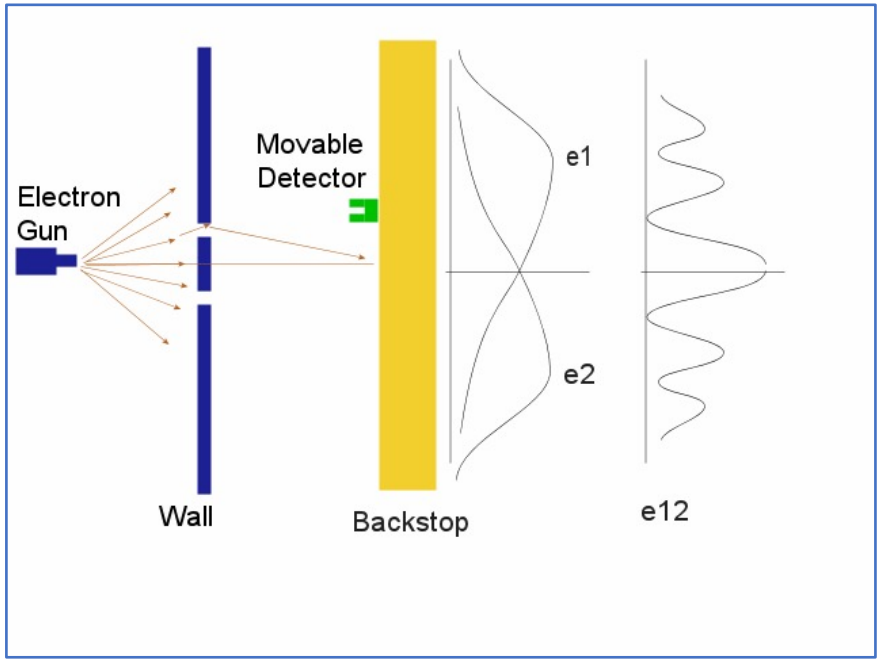


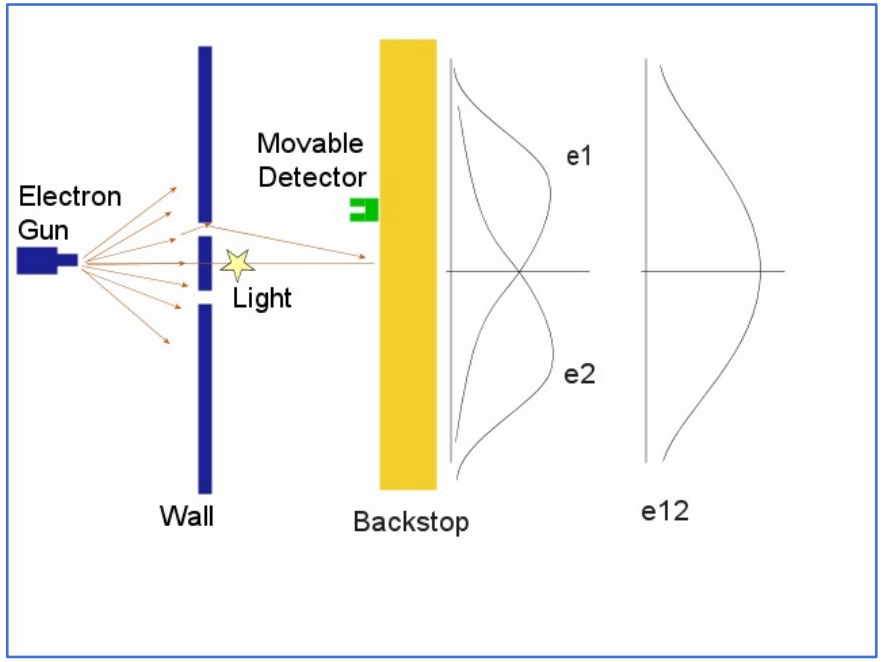


Position badly known

Momentum fairly known

Next Lecture: double slit experiment





The core of quantum mechanics illustrated by Feynman.
Einstein and Schrödinger did not like it.
Wheeler later took it to the extreme.
Even today people are debating its interpretation.