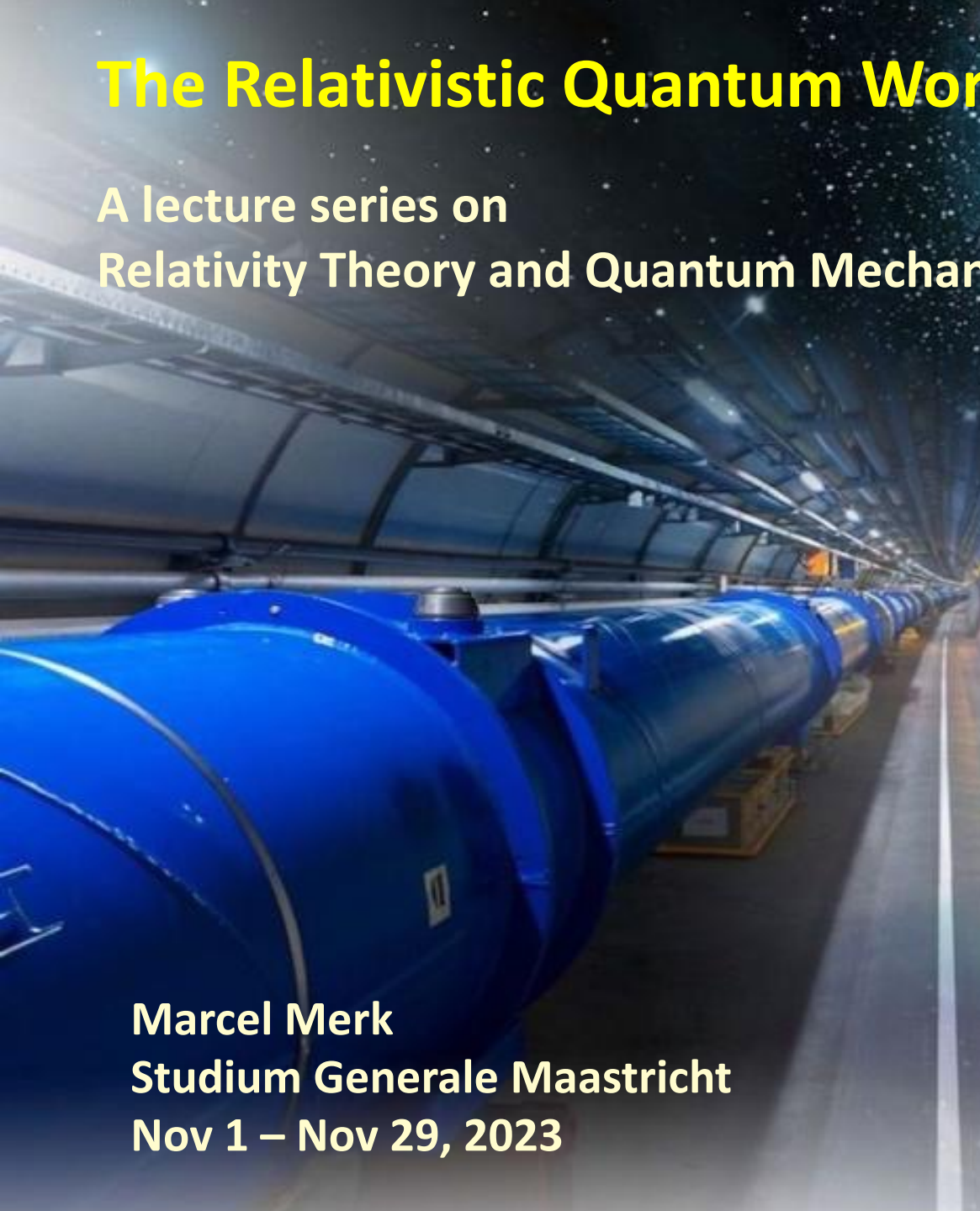
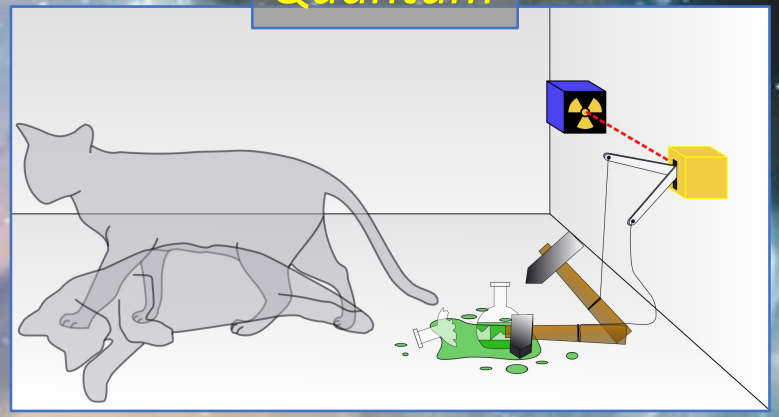
The Relativistic Quantum World

A lecture series on
Relativity Theory and Quantum Mechanics



Relativity

Quantum



Marcel Merk
Studium Generale Maastricht
Nov 1 – Nov 29, 2023

The Relativistic Quantum World

Relativity	Nov. 1:	Lecture 1: The Principle of Relativity and the Speed of Light Lecture 2: Time Dilation and Lorentz Contraction
	Nov. 8:	Lecture 3: The Lorentz Transformation and Paradoxes Lecture 4: General Relativity and Gravitational Waves
Quantum Mechanics	Nov. 15:	Lecture 5: The Early Quantum Theory Lecture 6: Feynman's Double Slit Experiment
	Nov 22:	Lecture 7: Wheeler's Delayed Choice and Schrodinger's Cat Lecture 8: Quantum Reality and the EPR Paradox
Standard Model	Nov. 29:	Lecture 9: The Standard Model and Antimatter Lecture 10: Why is there something rather than nothing?

Lecture notes, written for this course, are available: www.nikhef.nl/~i93/Teaching/
Prerequisite for the course: High school level physics & mathematics.

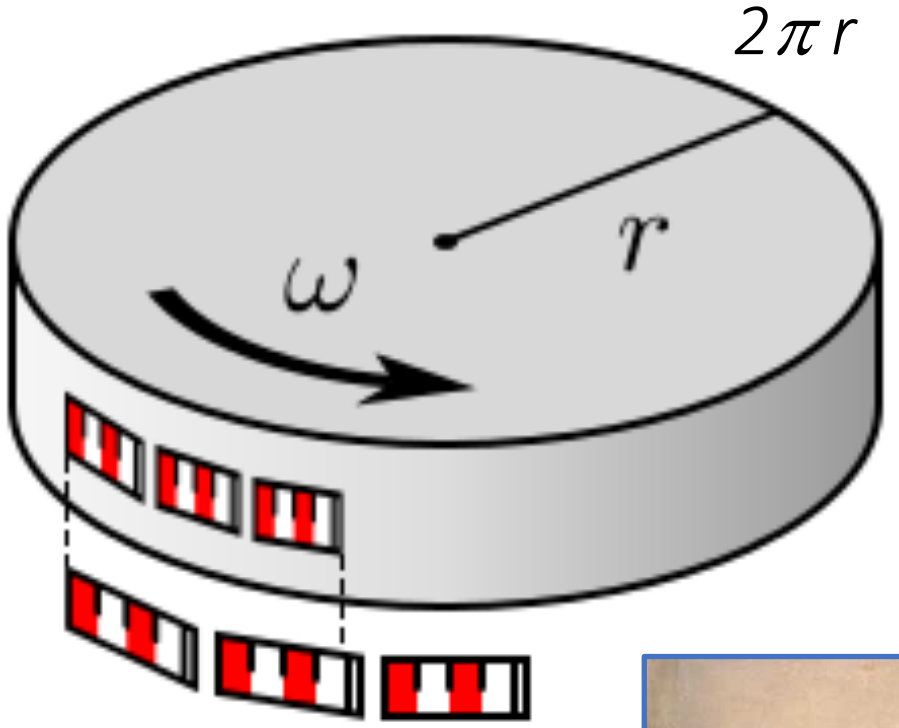
Lecture 4

General Relativity and Gravitational Waves

“Do not worry about your difficulties in mathematics.
I can assure you mine are still greater.”
- Albert Einstein

Ehrenfest Paradox

Rotating disk with ruler on the edge:
Circumference: $C = 2 \pi r$

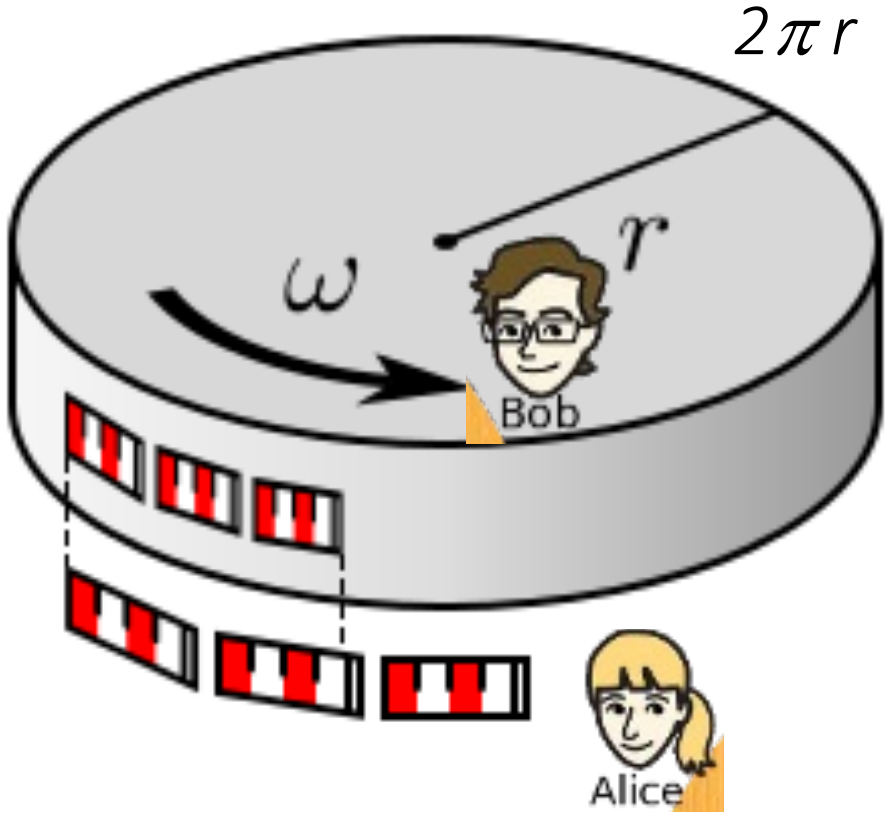


Ehrenfest Paradox

Rotating disk with ruler on the edge:
Circumference: $C = 2 \pi r$

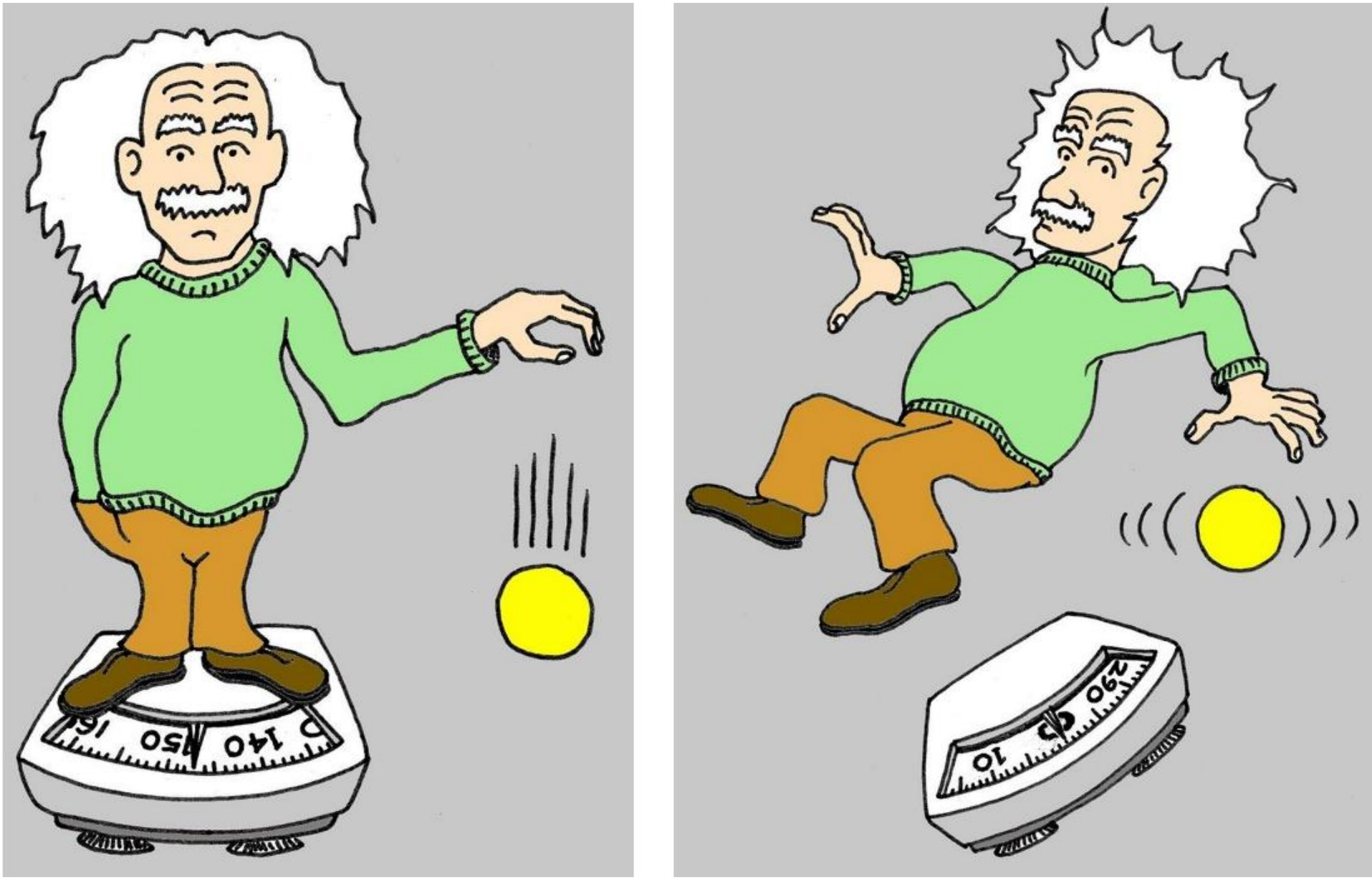
Alice stands next to the disk and sees *rulers on disk* Lorentz contracted:
$C = 2 \pi r / \gamma$
➔ Circumference is smaller!

Bob moves on the disk and sees *rulers next to disk* contracted:
$C = 2 \pi r \cdot \gamma$
➔ Circumference is larger!

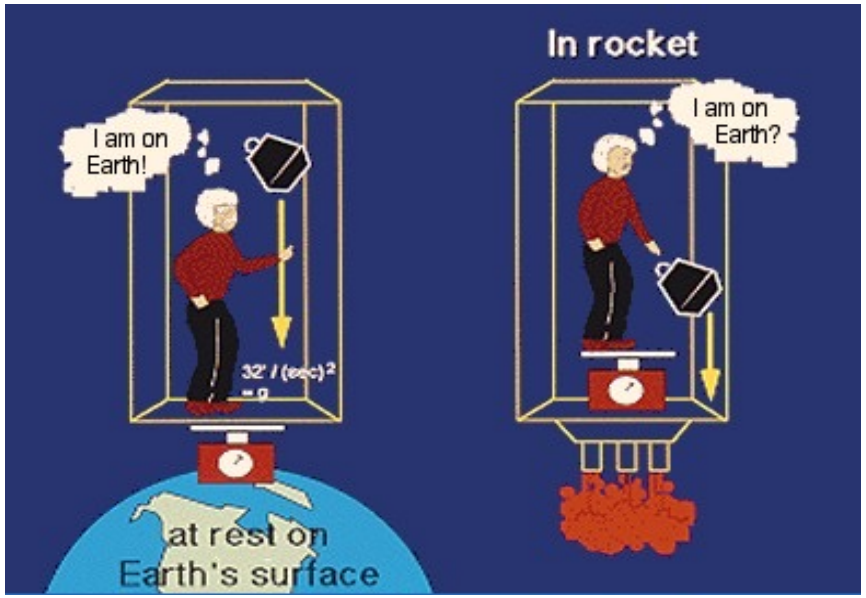


A rotating object is ***not an inertial*** frame:
- Postulate of relativity only worked for ***inertial frames***
- Need to adapt the postulates: special relativity → ***general relativity***

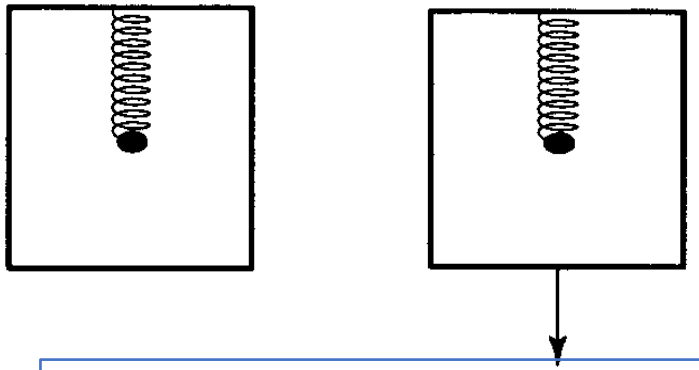
Einstein’s “happiest thought”

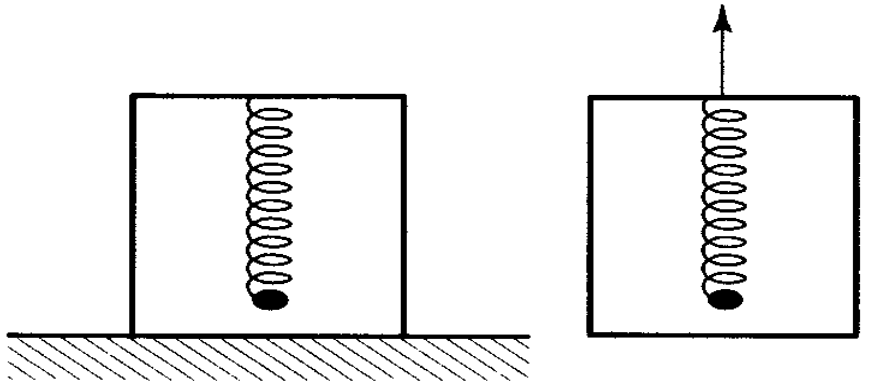


The Equivalence Principle



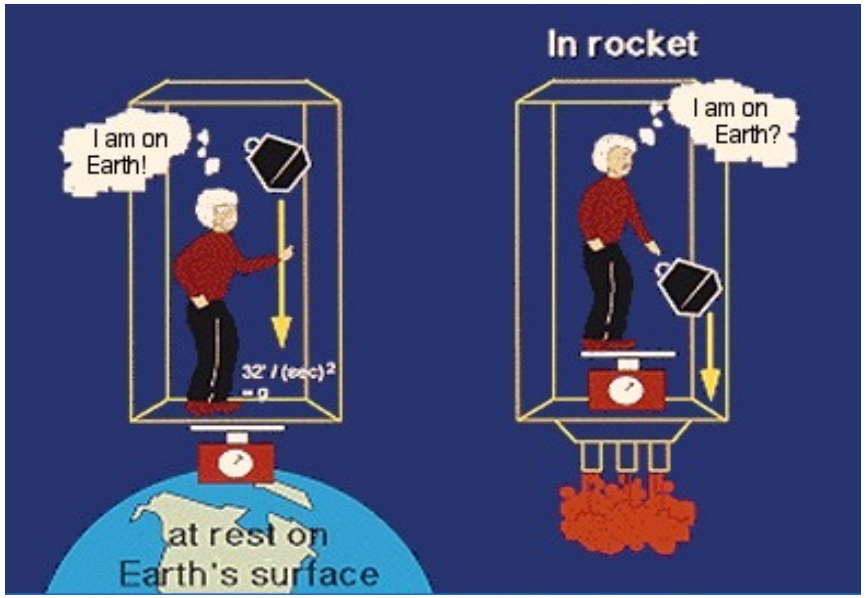
(Inertial Frame)



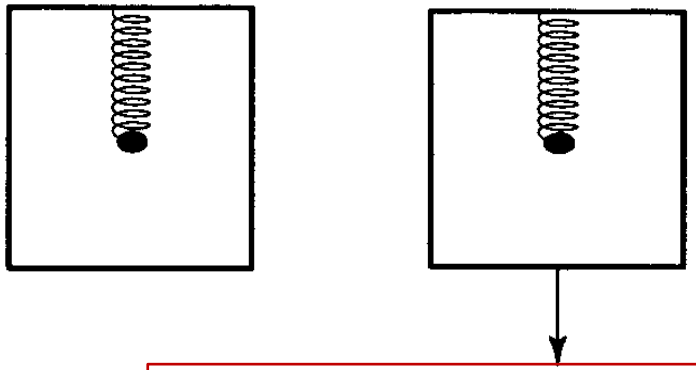


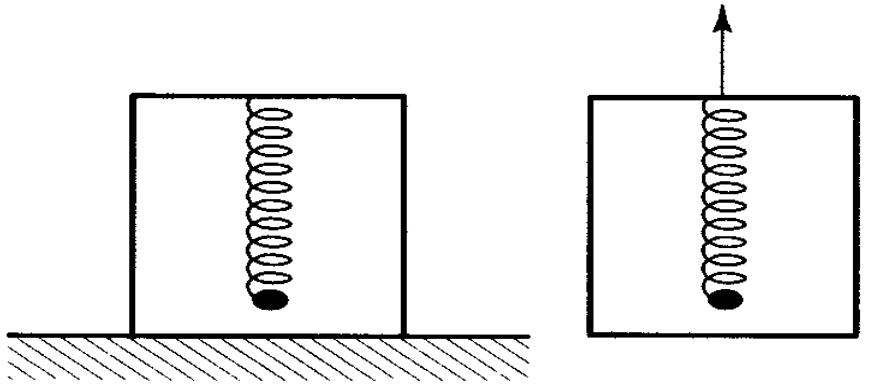
Einstein's "happiest thought": there is ***no way*** to determine whether you are standing on the earth or accelerating upwards in a rocket in space!

The Equivalence Principle



(Inertial Frame)



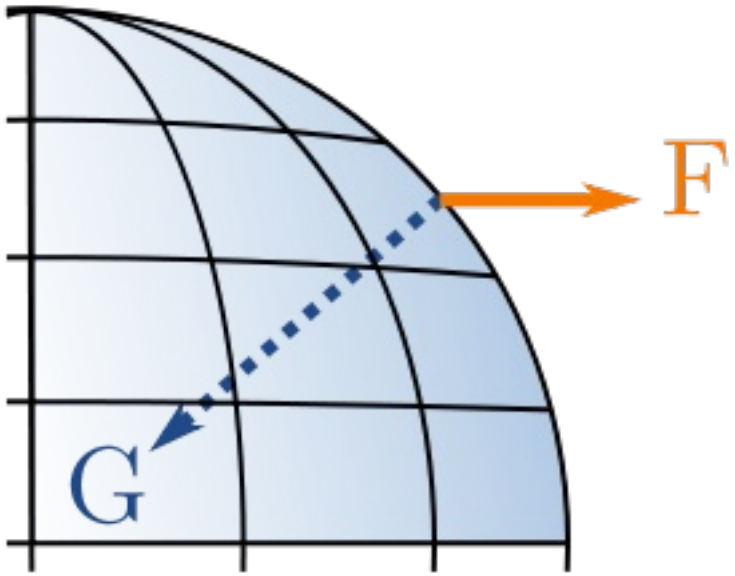


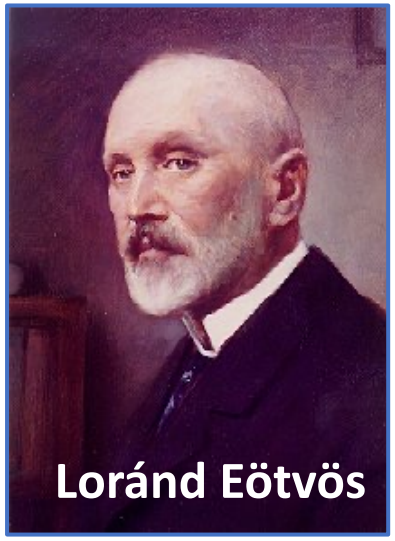
There is no difference between acceleration force and gravitation.

"Gravitational mass" = "Inertial mass"

The Eötvös Experiment

Direction of gravity and centrifugal force on earth



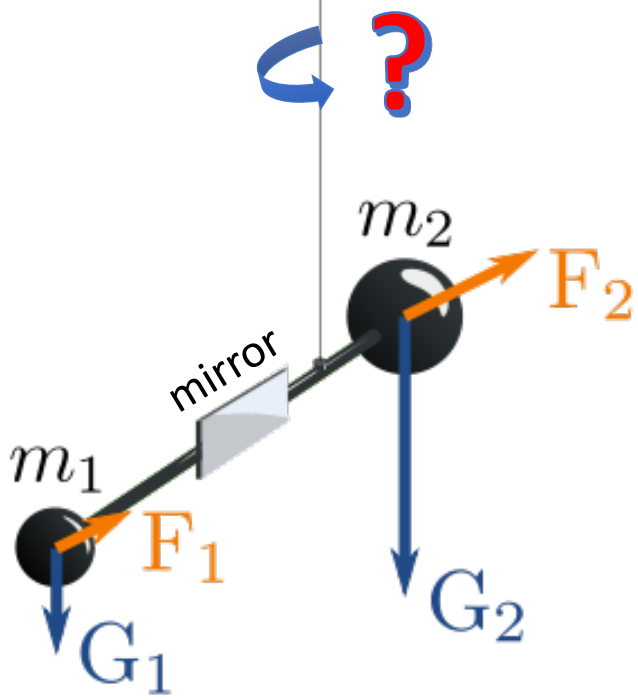


$F = m_i a$ with $a = \frac{v^2}{R}$

$G = m_g g$ with $g = \frac{GM_\oplus}{R^2}$

➔ $m_i = m_g$

Small (m_1) and big (m_2) mass on a rod suspended by a thin fiber



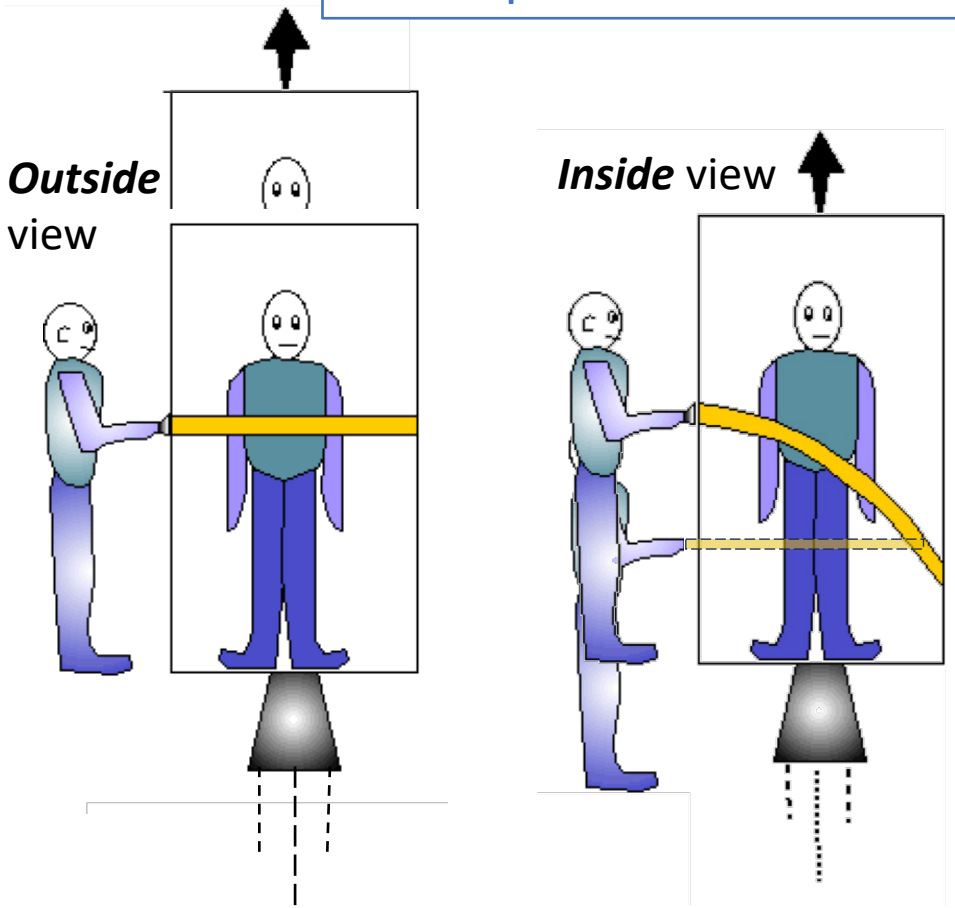
Gravity force **G** depends on Newton's law of gravity: ***gravitational mass***
Centrifugal force **F** depends on Newton's law of motion inertial mass: ***inertial mass***

The system did **not** rotate. ➔ $F_1/F_2 = G_1/G_2$

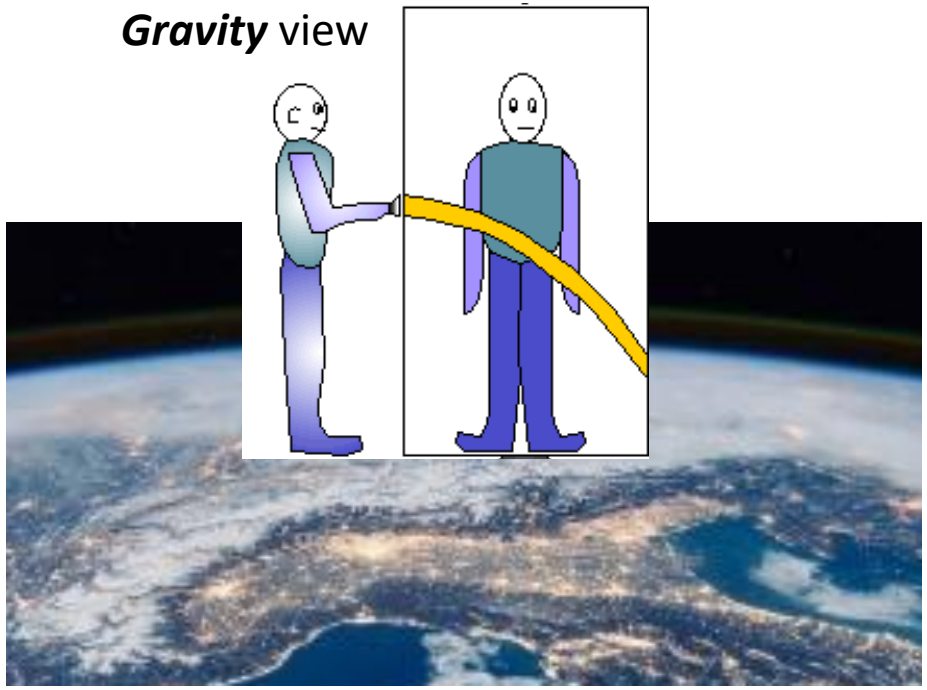
➔Experimental proof that indeed gravitational mass is equivalent to inertial mass.

Bending of Light

Consequence of acceleration and gravity being equivalent



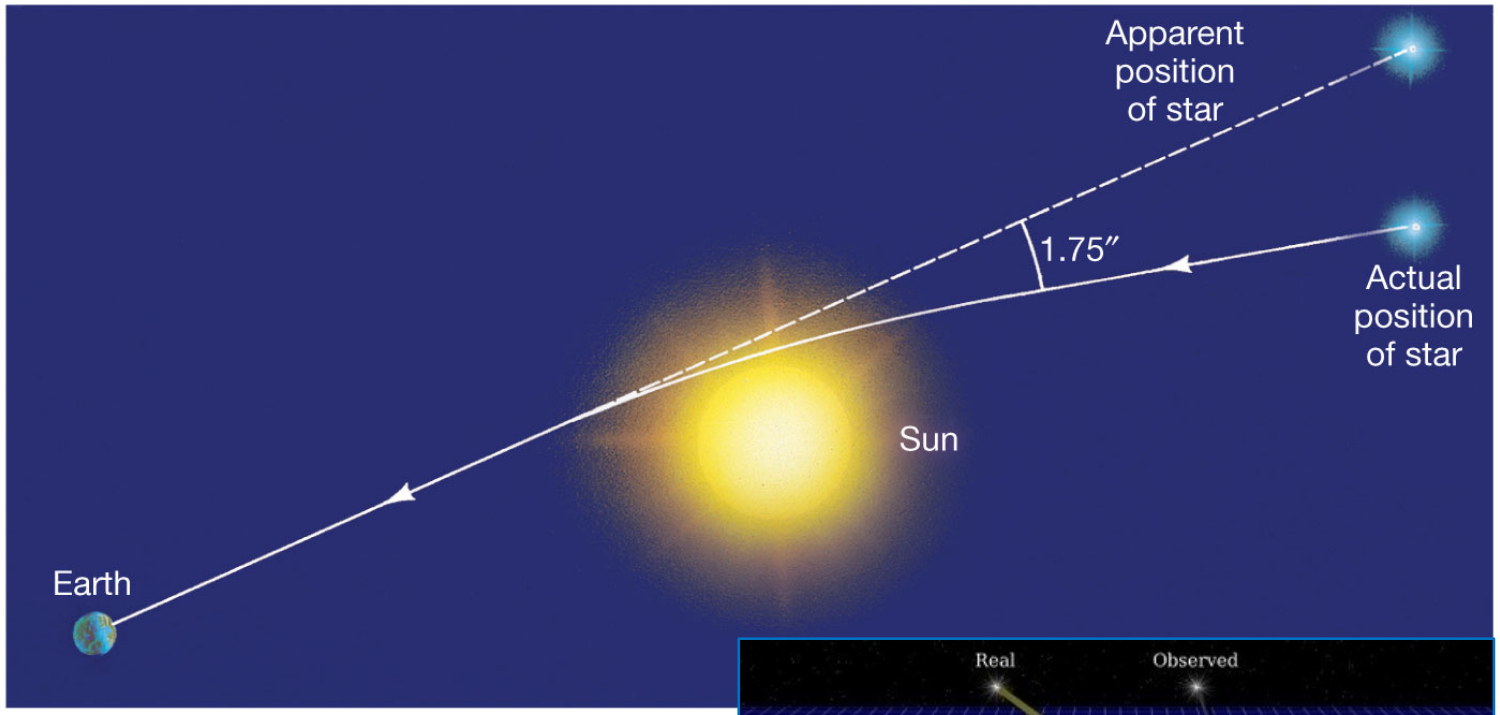
A: Lightbeam in accelerating rocket

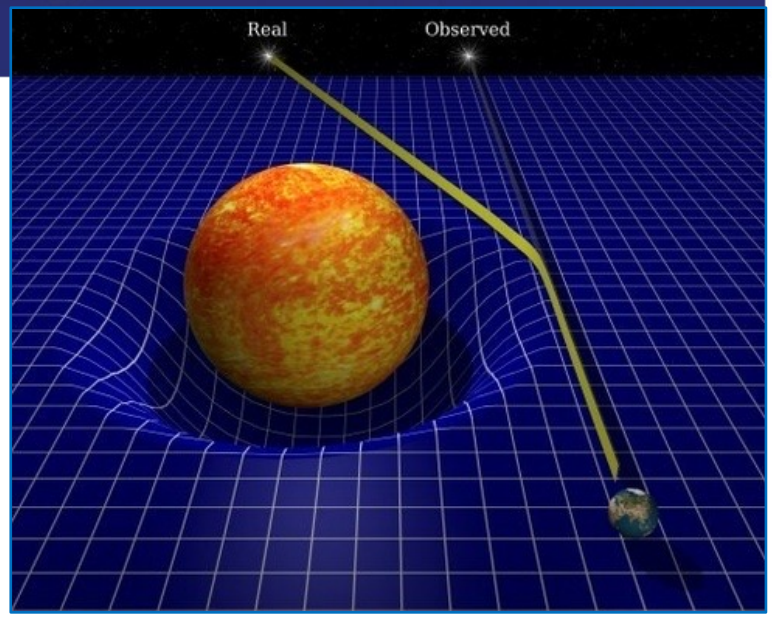


B: Lightbeam in gravitational field

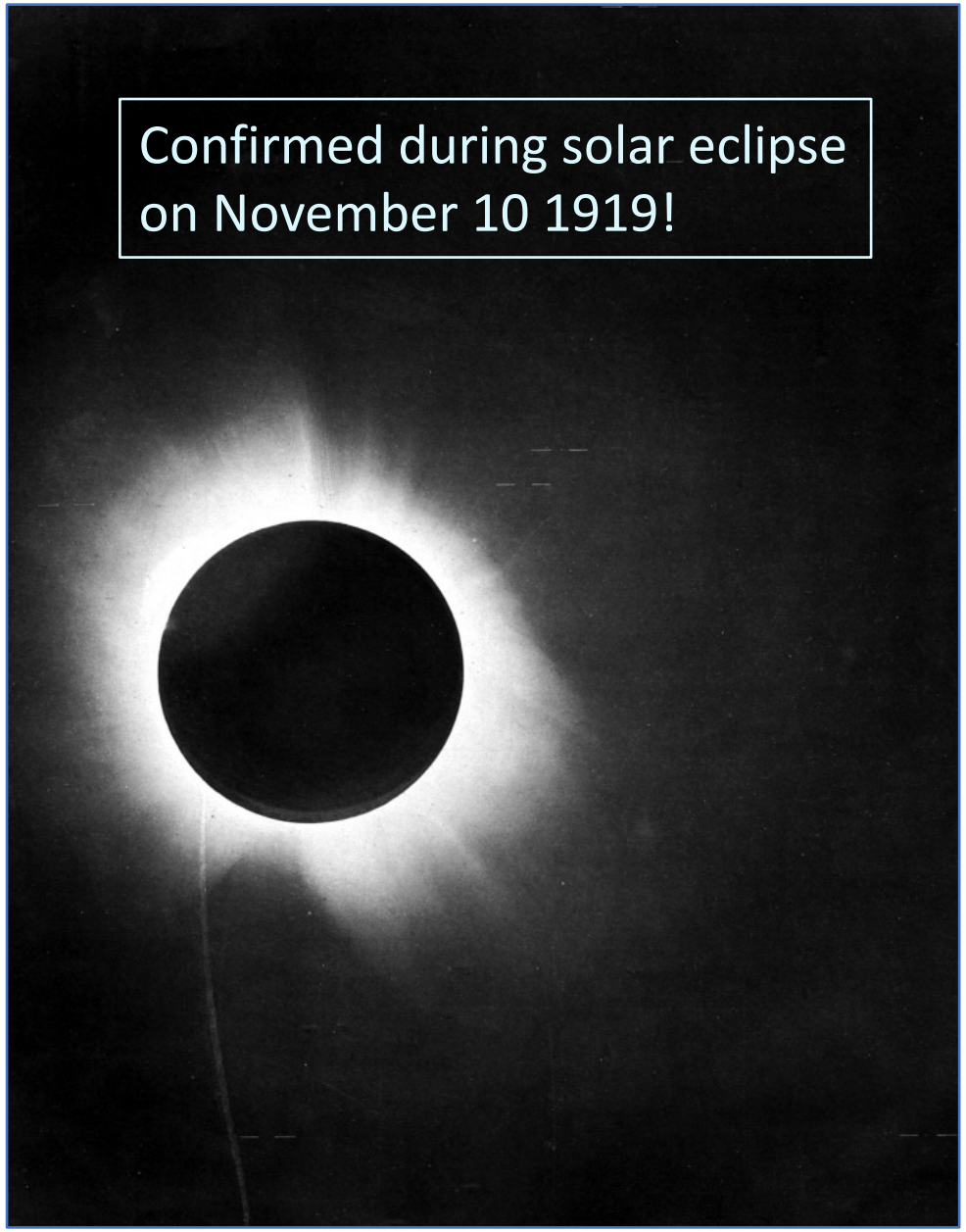
Prediction of Einstein: light beam bends under gravity!

Bending of light in gravitation field of the Sun

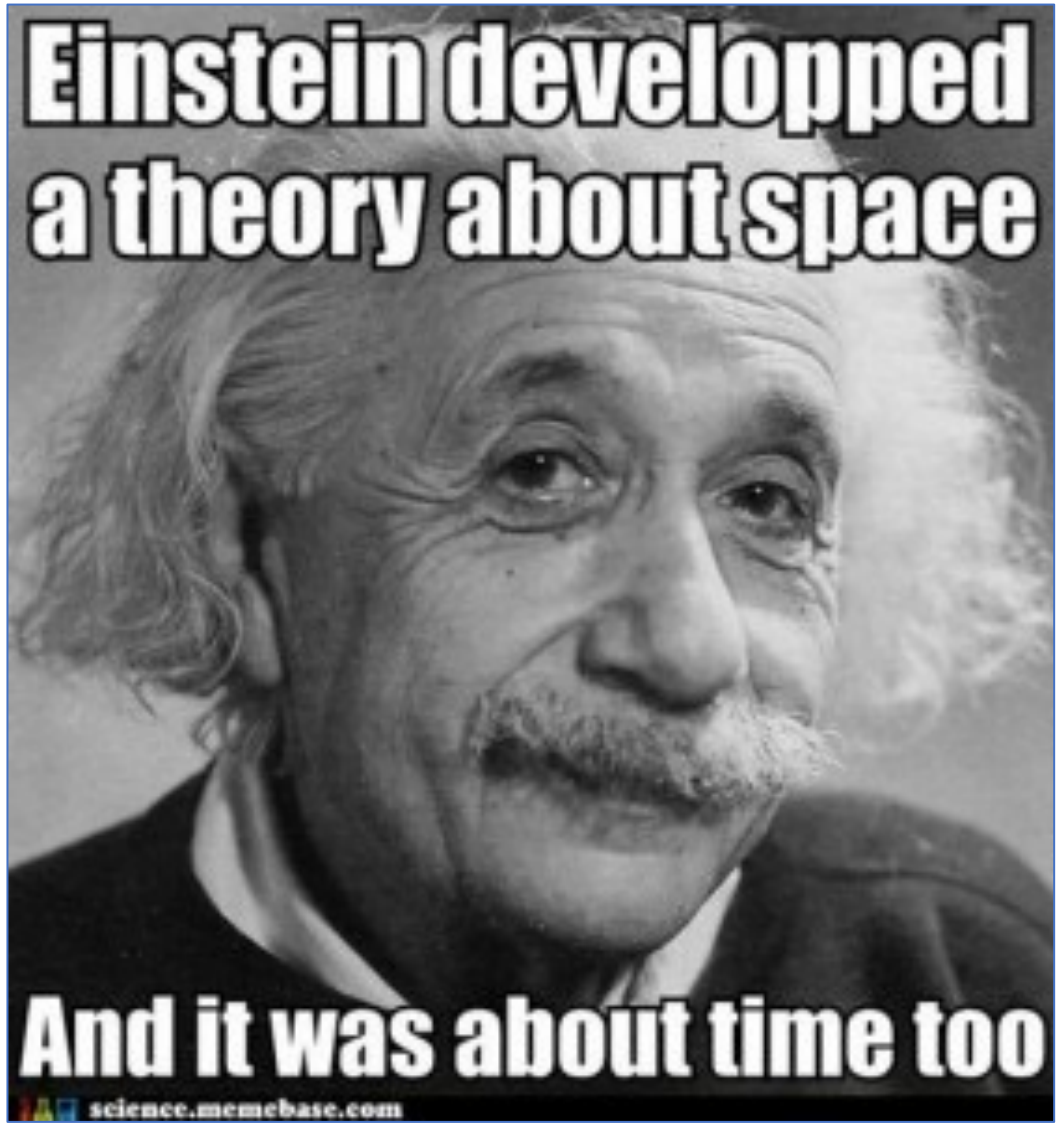




Light wants to go straight but space is curved!

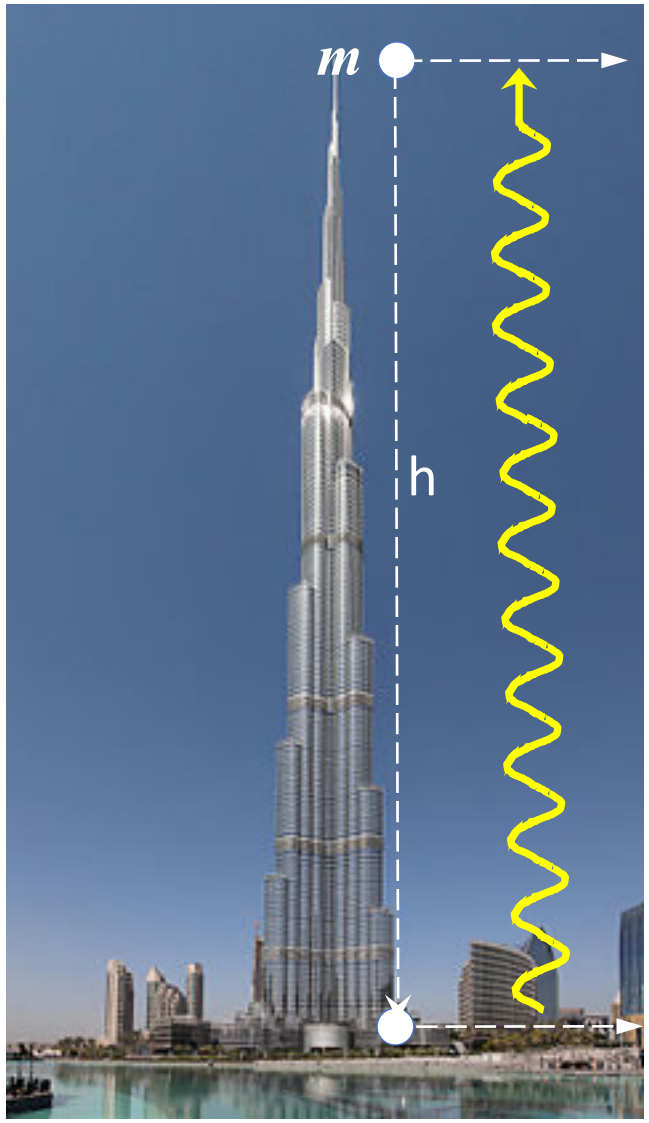


Confirmed during solar eclipse on November 10 1919!


Einstein developped
a theory about space
And it was about time too
science.memebase.com

Einstein's next thought experiment on light

Particle with mass ***m*** falling from tower:



$$E = mc^2 \quad \Rightarrow \quad E' > E$$

From quantum mechanics we know that the energy of light is related to frequency (and wavelength): $E = hf = hc/\lambda$

$E' > E$??

Perpetuum mobile? ➔ No!

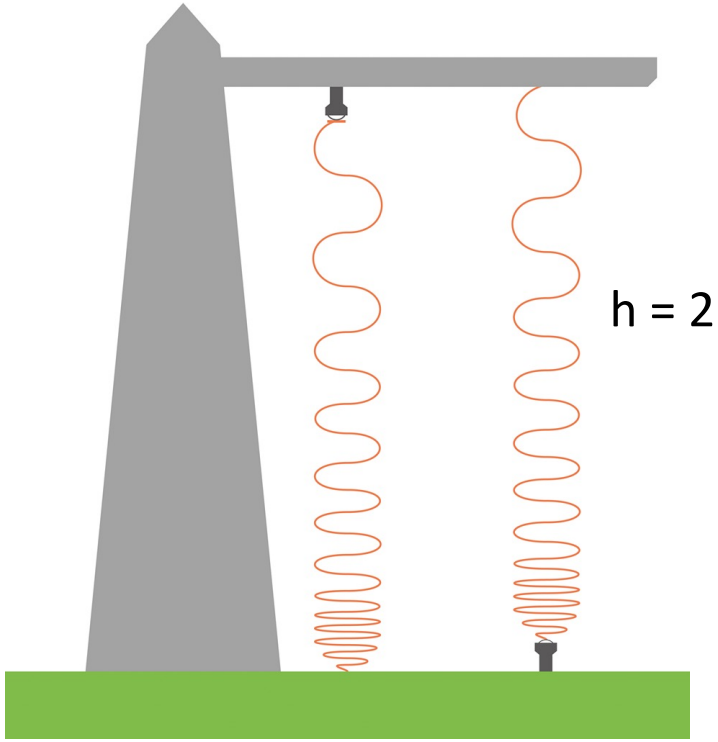
Photon **loses energy** gh/c^2 as it travels up the gravitational field!
→ Wavelength red-shift

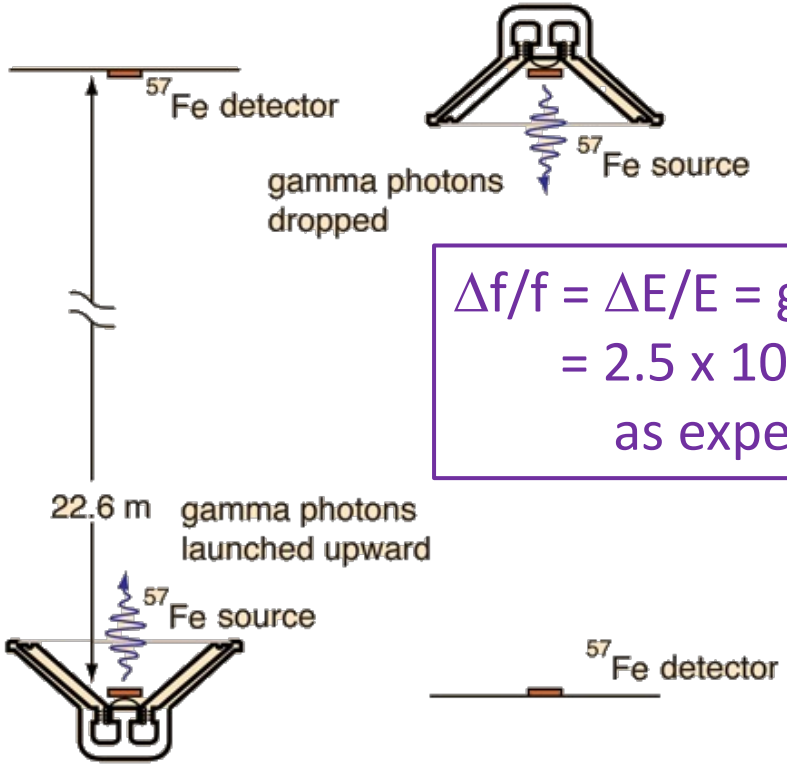
$$E' = mc^2 + \frac{1}{2}mv^2 = mc^2 + mgh$$
$$= mc^2\,(1 + gh/c^2) \quad \Rightarrow \quad E' = hf'$$

$(E_{kin} = E_{pot})$

The Harvard Tower Experiment

Harvard Tower Experiment (Pound-Rebka)
at Jefferson lab in Harvard (1960):
Measure red-shift of photons in earth gravitational field.





$\Delta f/f = \Delta E/E = gh/c^2$
$= 2.5 \times 10^{-15}$
as expected!

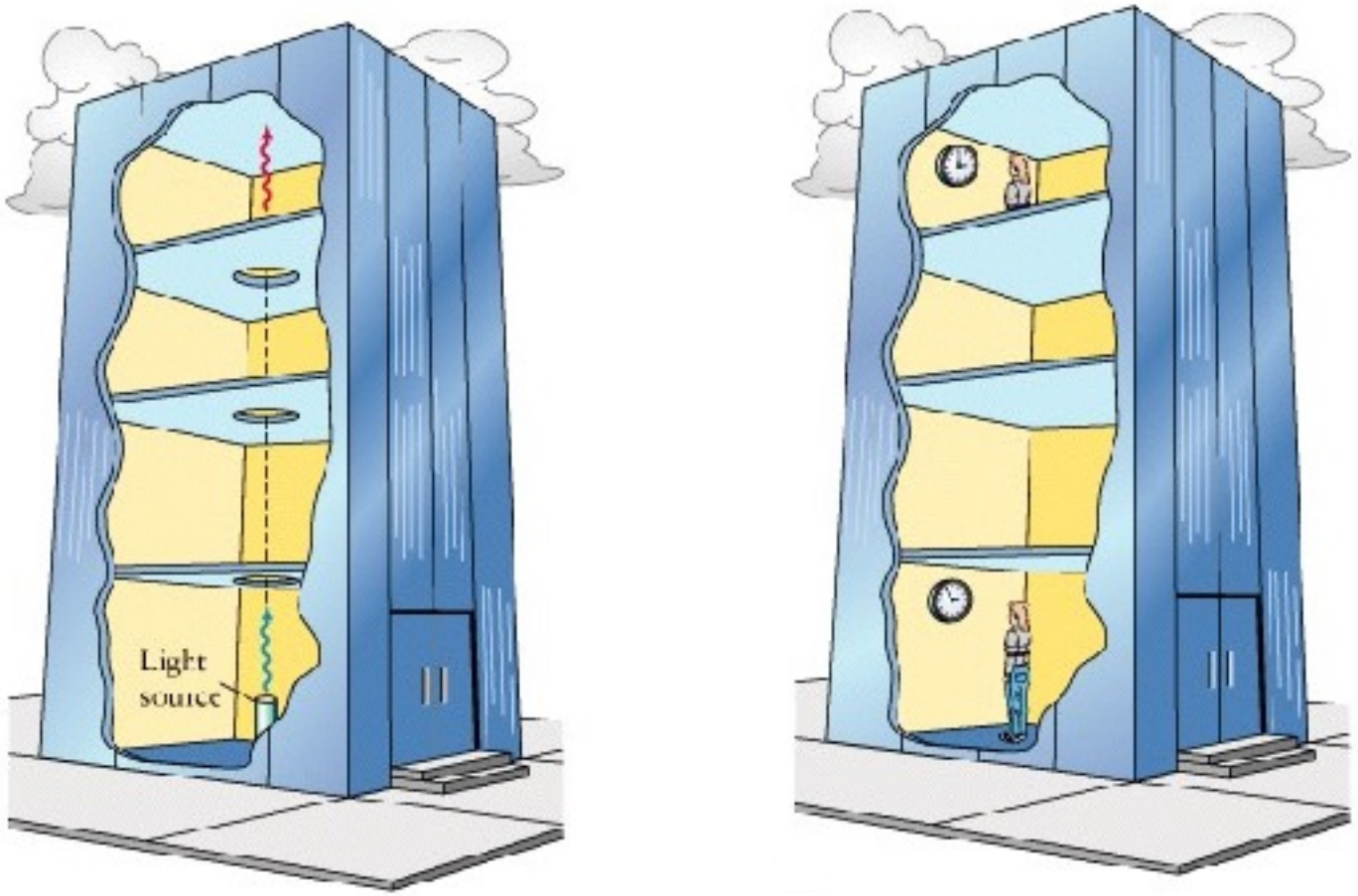
Gravitational Time Dilation

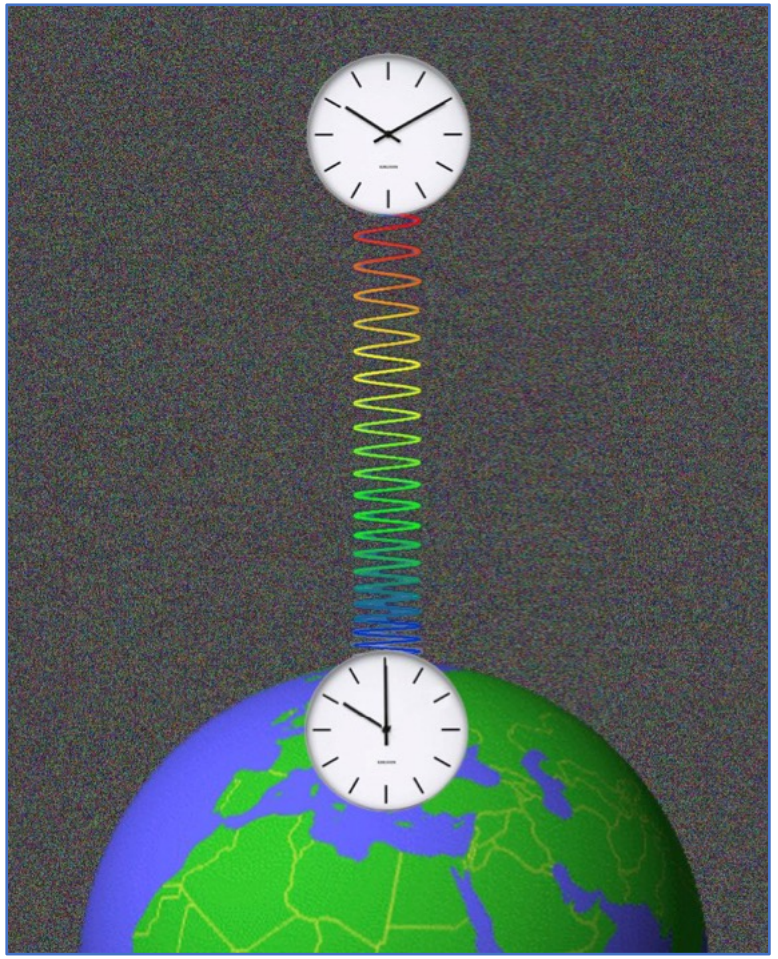
The photon loses energy as it climbs the gravitational field.

$$c = \lambda f$$

Longer wavelength

Lower frequency



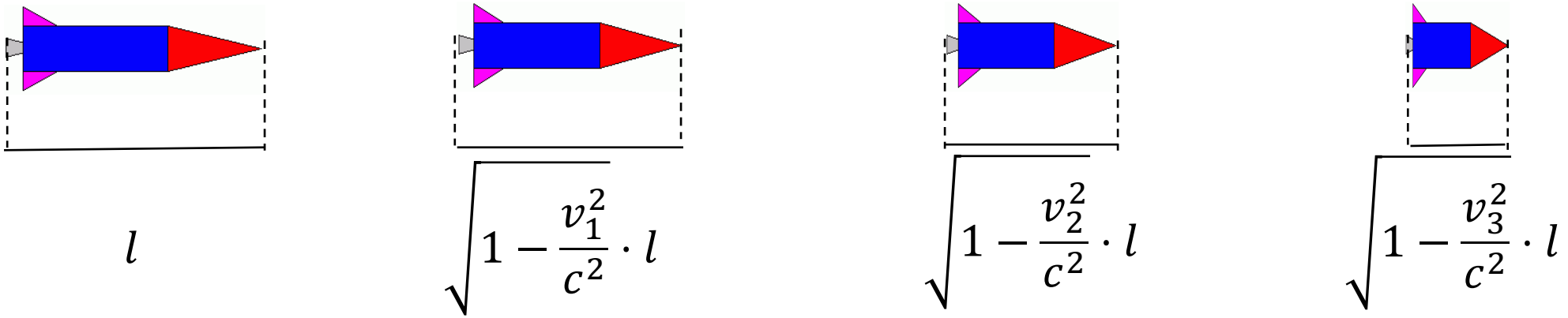


➔Time ticks faster at higher altitude.

Accelerating Rocket

From special relativity we know that space contracts at high velocity

Velocity : $0 < v_1 < v_2 < v_3$



Space is seen to shrink further and further with increasing velocity!

$$1/\gamma = \sqrt{1-\frac{v^2}{c^2}}$$

Free falling object and Einstein's Equivalence Principle

$$E_{pot} = G\frac{mM_\oplus}{R}$$

↓

$$E_{kin} = E_{pot}$$

$$\frac{1}{2}mv^2 = G\frac{mM_\oplus}{R}$$

↓

$$v^2 = 2\frac{GM_\oplus}{R}$$

↓

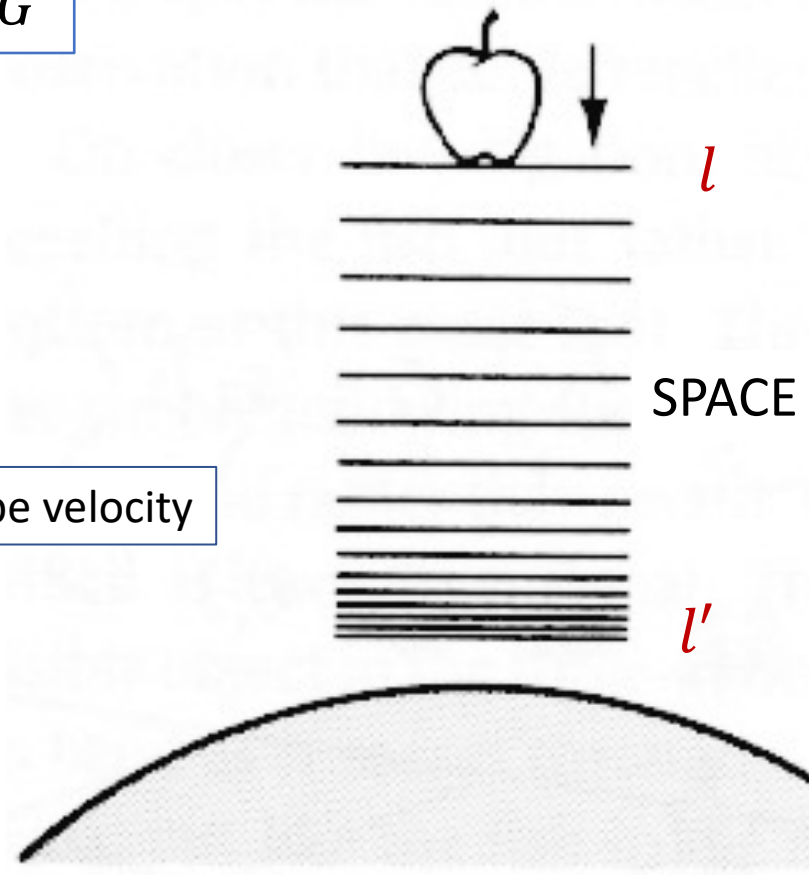
$$\sqrt{1-\frac{v^2}{c^2}} = \sqrt{1-2\frac{GM_\oplus}{Rc^2}}$$

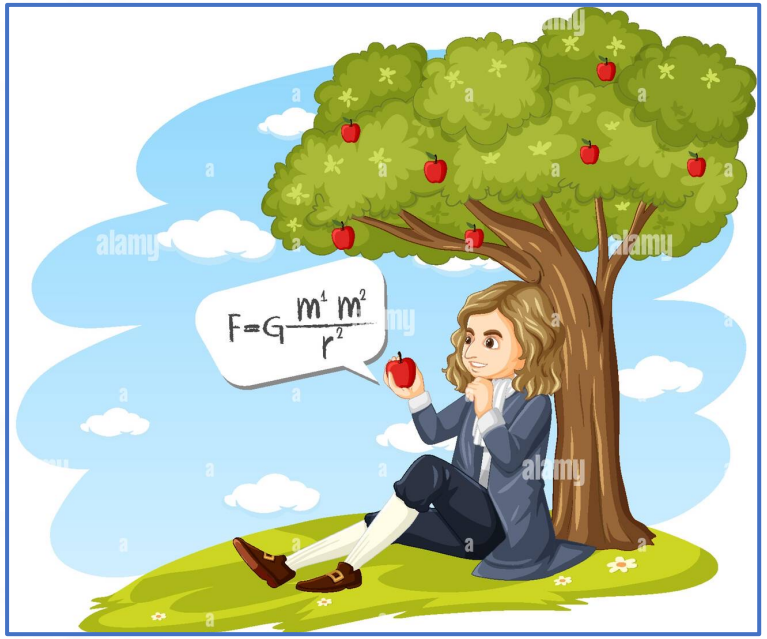
Newton's Constant G

v =Escape velocity

Lorentz factor

Falling apple:





Space shrinkage ("curvature"):

$$l' = \sqrt{1-2\frac{GM_\oplus}{Rc^2}} \cdot l$$

Free falling object and Einstein's Equivalence Principle

$$E_{pot} = G\frac{mM_\oplus}{R}$$

$$\downarrow$$

$$E_{kin} = E_{pot}$$

$$\frac{1}{2}mv^2 = G\frac{mM_\oplus}{R}$$

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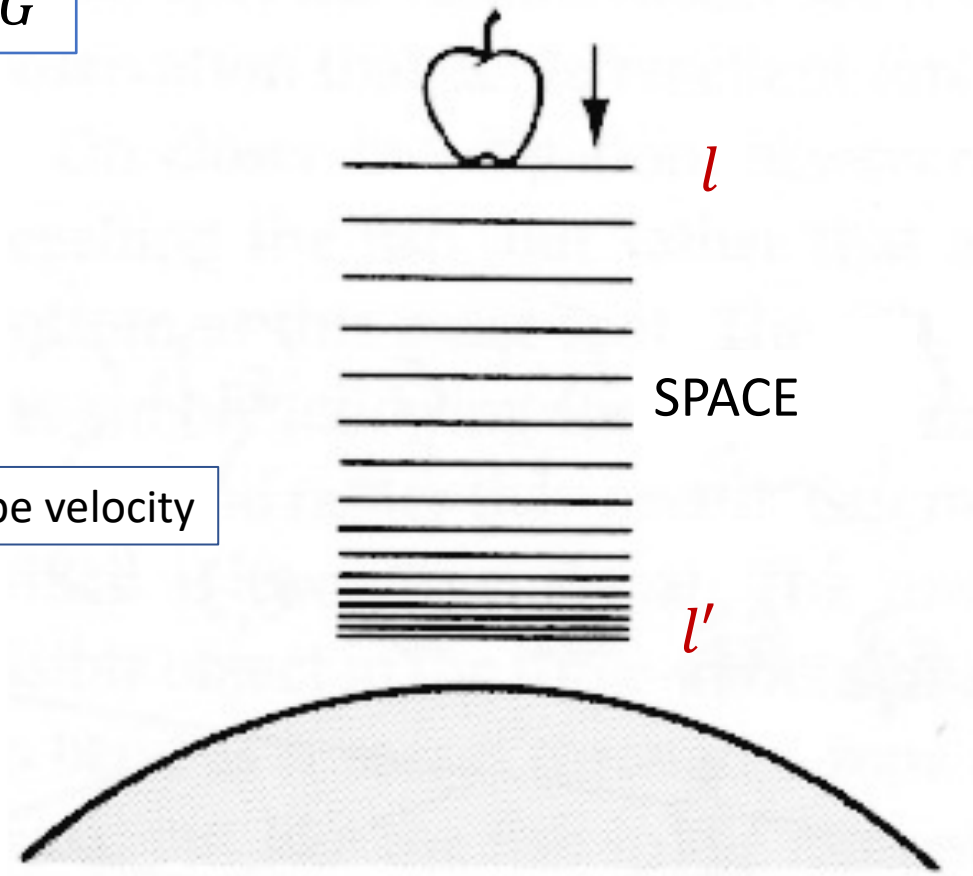
Falling apple:



Compare to accelerating rocket:

Space shrinkage ("curvature"):

$$l' = \sqrt{1-2\frac{GM_\oplus}{Rc^2}} \cdot l$$

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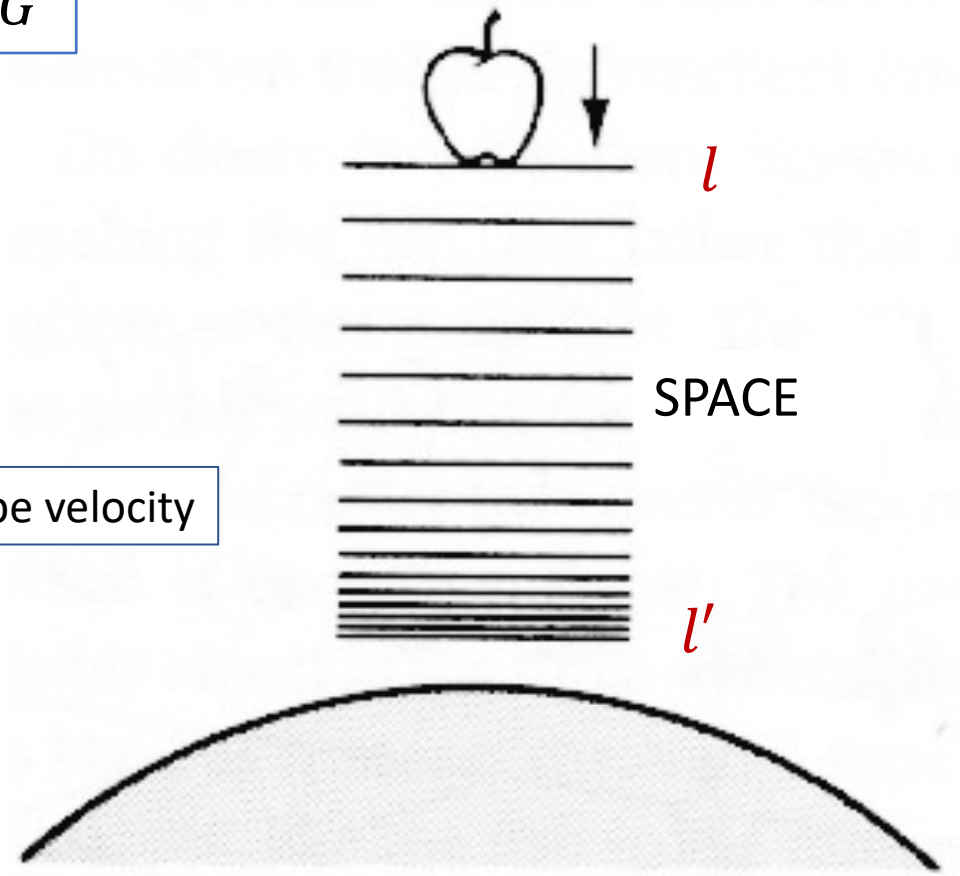
Falling apple:



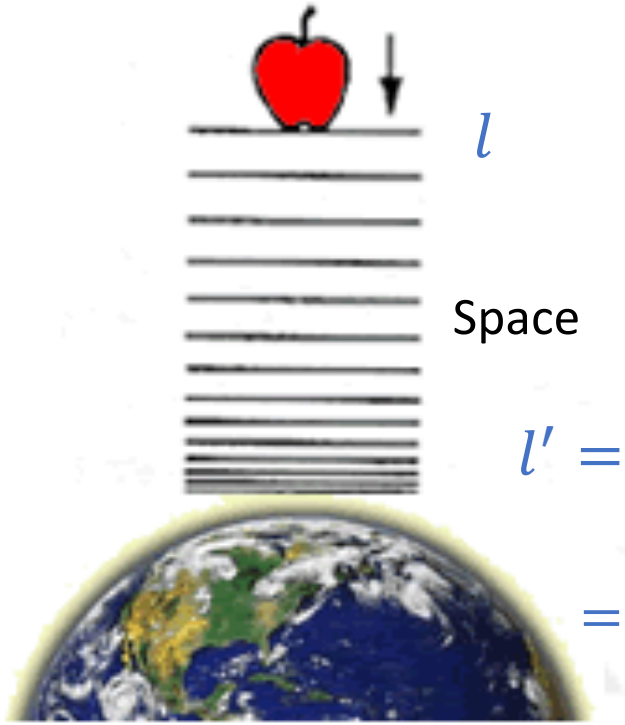
Compare to accelerating rocket:

Space shrinkage ("curvature"):

$$l' = \sqrt{1-2\frac{GM_\oplus}{Rc^2}} \cdot l = \frac{1}{\gamma}L$$

Space-Time curvature

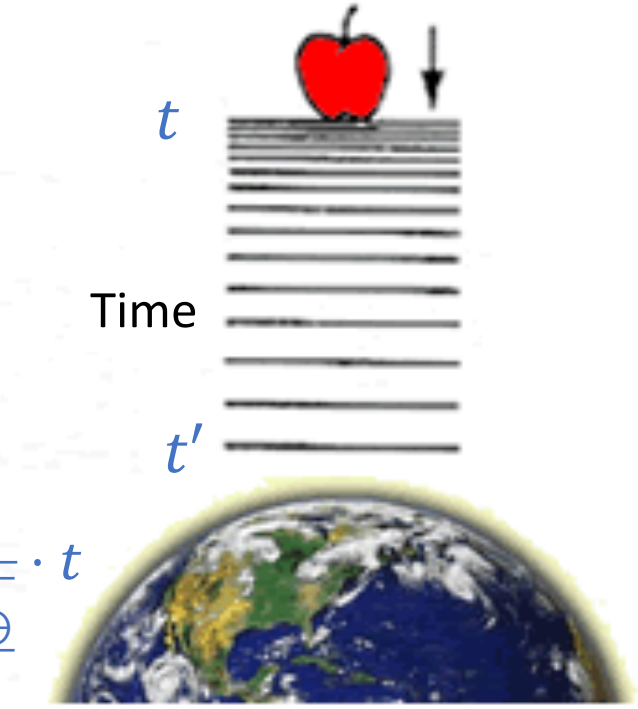
A falling apple accelerates and units of space get more and more contracted:



$$l' = 1/\gamma \cdot l = \sqrt{1 - 2\frac{GM_\oplus}{Rc^2}} \cdot l$$

Space contracts near mass and dilates away from it.

An apple falls into the gravitational field and time runs slower and slower:

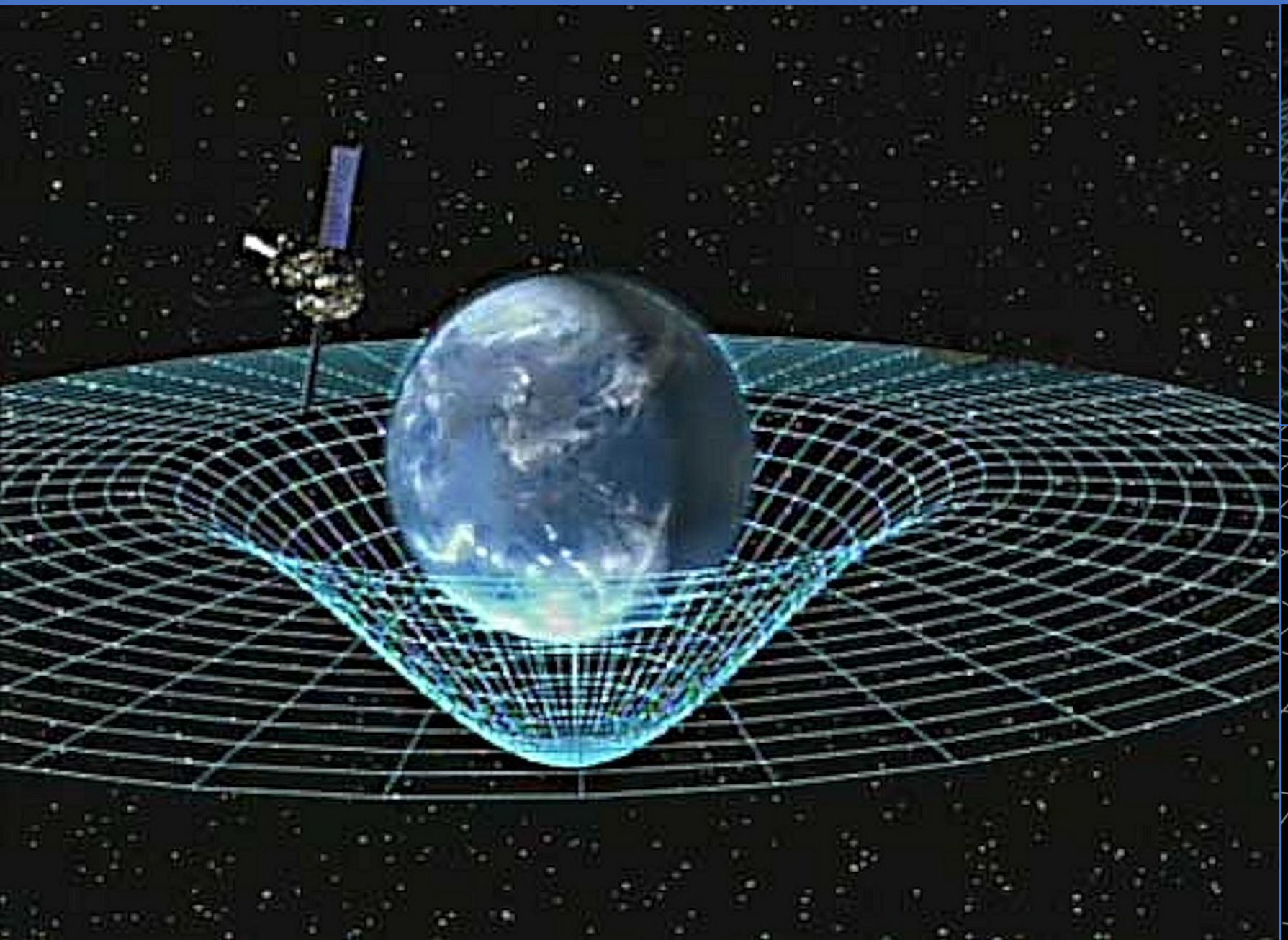


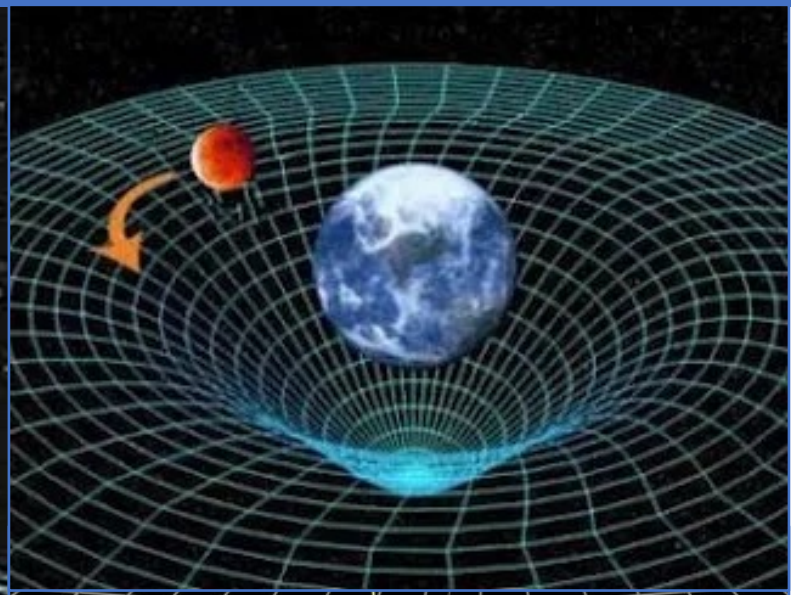
$$t' = \gamma \cdot t = \frac{1}{\sqrt{1 - 2\frac{GM_\oplus}{Rc^2}}} \cdot t$$

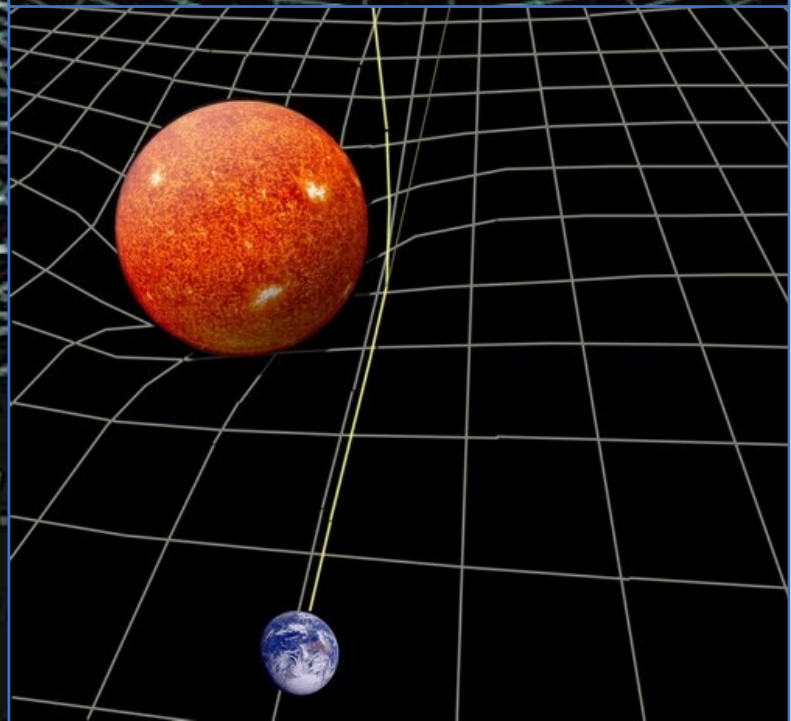
Time slows near mass and speeds up away from it.

Space-time is curved in the presence of mass

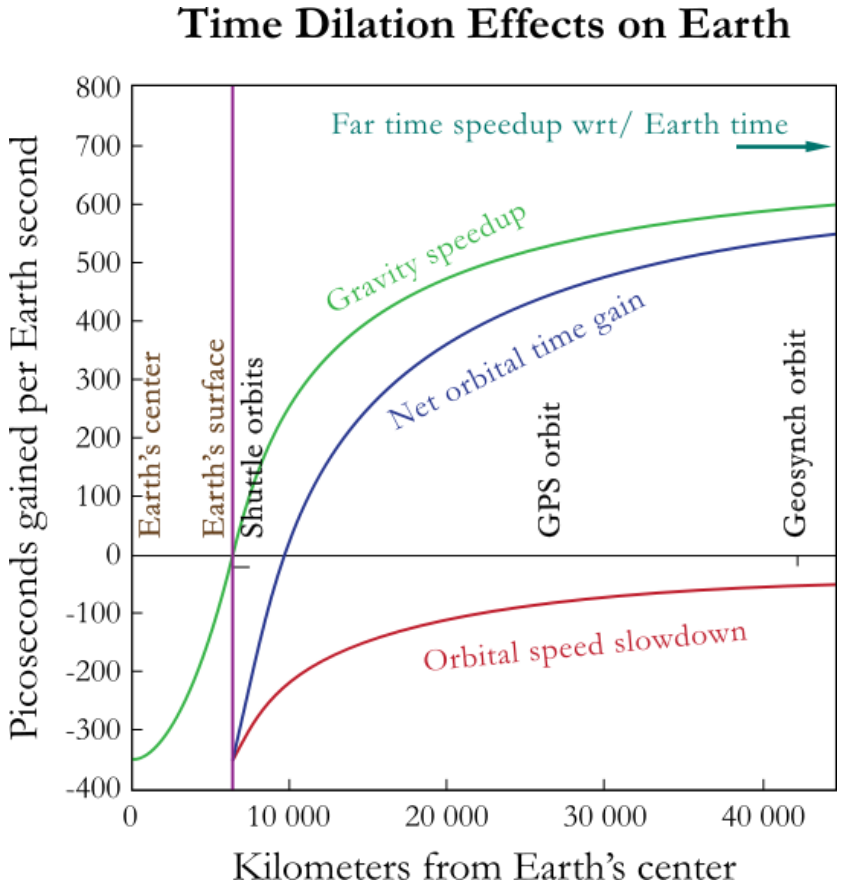
Mass causes curvature in space-time

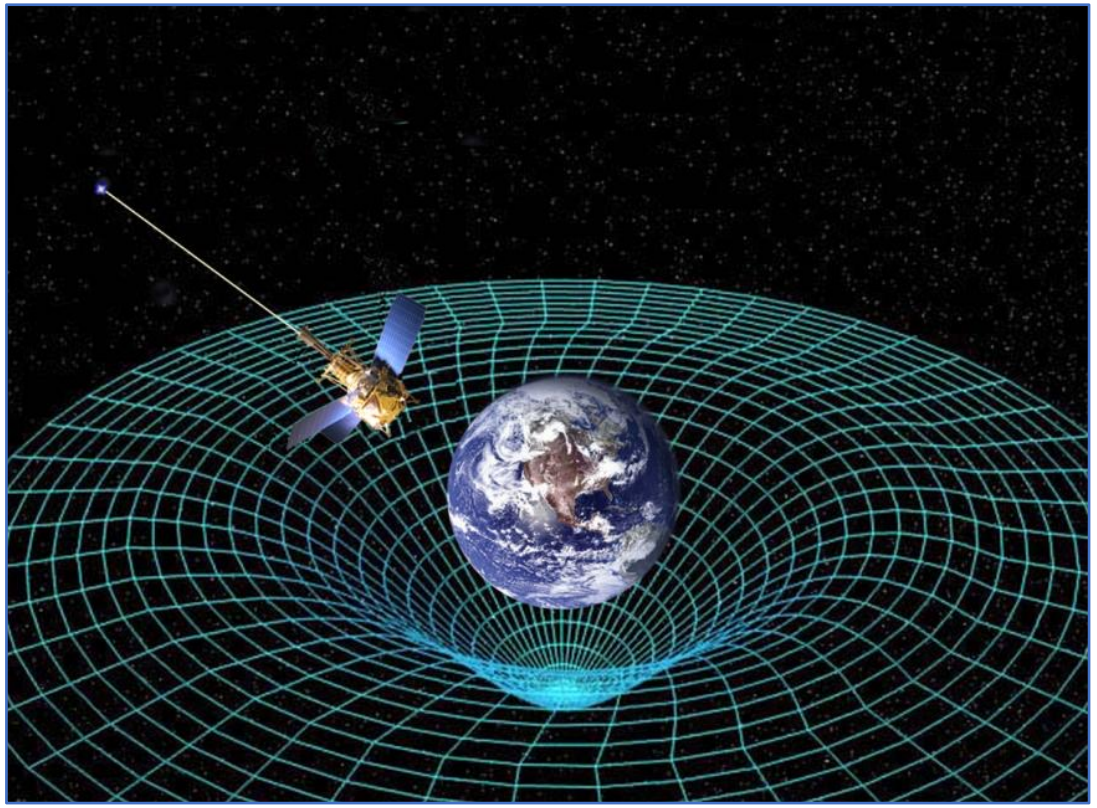






Relativity and GPS



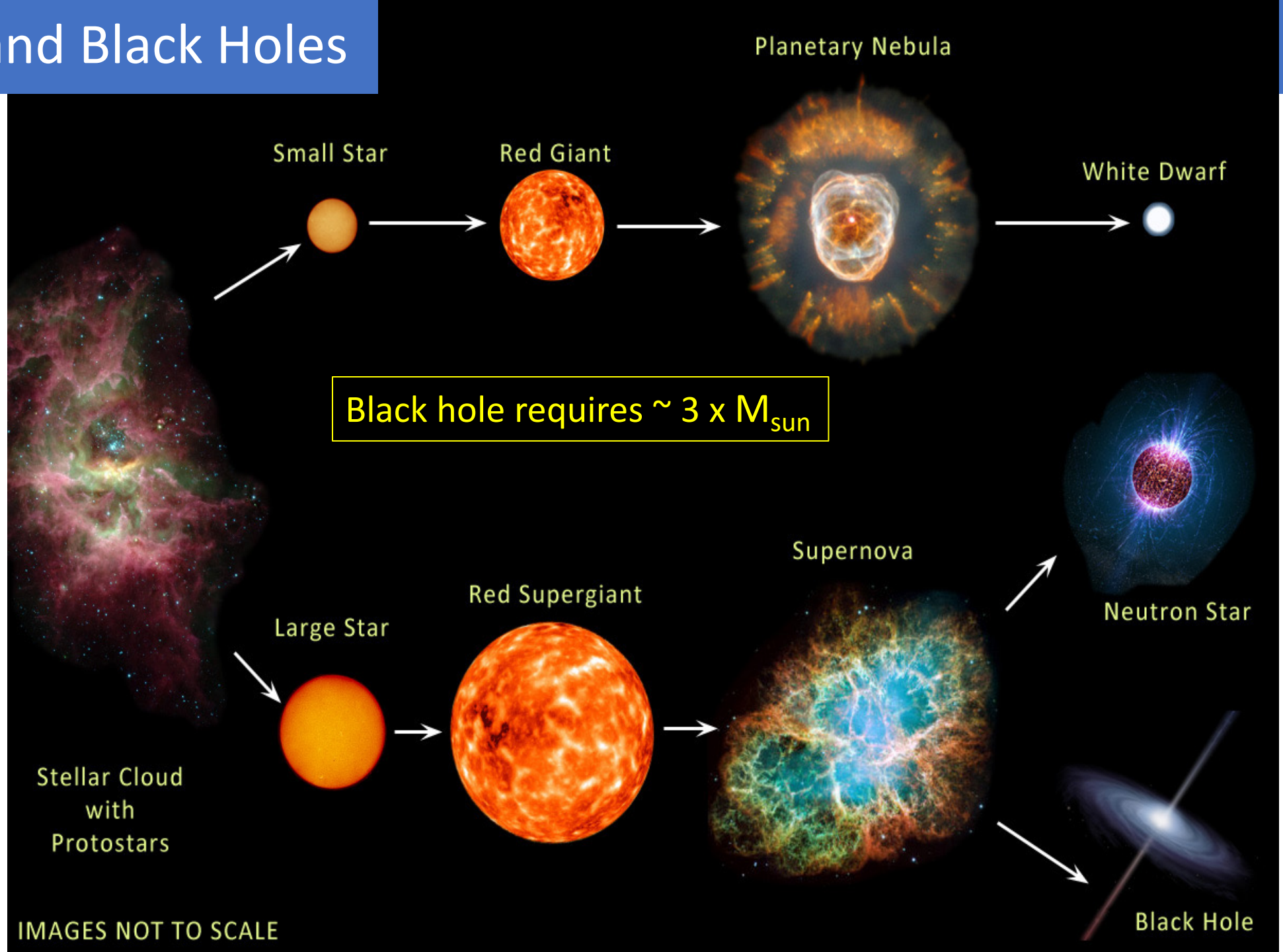


Two effects:

- **Time speeds up** at the satellite in comparison to earth surface **due to gravity**
- **Time slows down** at the satellite **due to high velocity** compared to person on earth

➔ Clocks in satellite and on earth de-synchronize with ~ 40 msec per day!

Stars and Black Holes



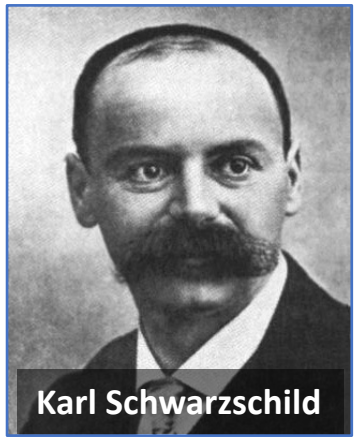
Stars and Black Holes

Gravitational time slowdown near a star with mass M:

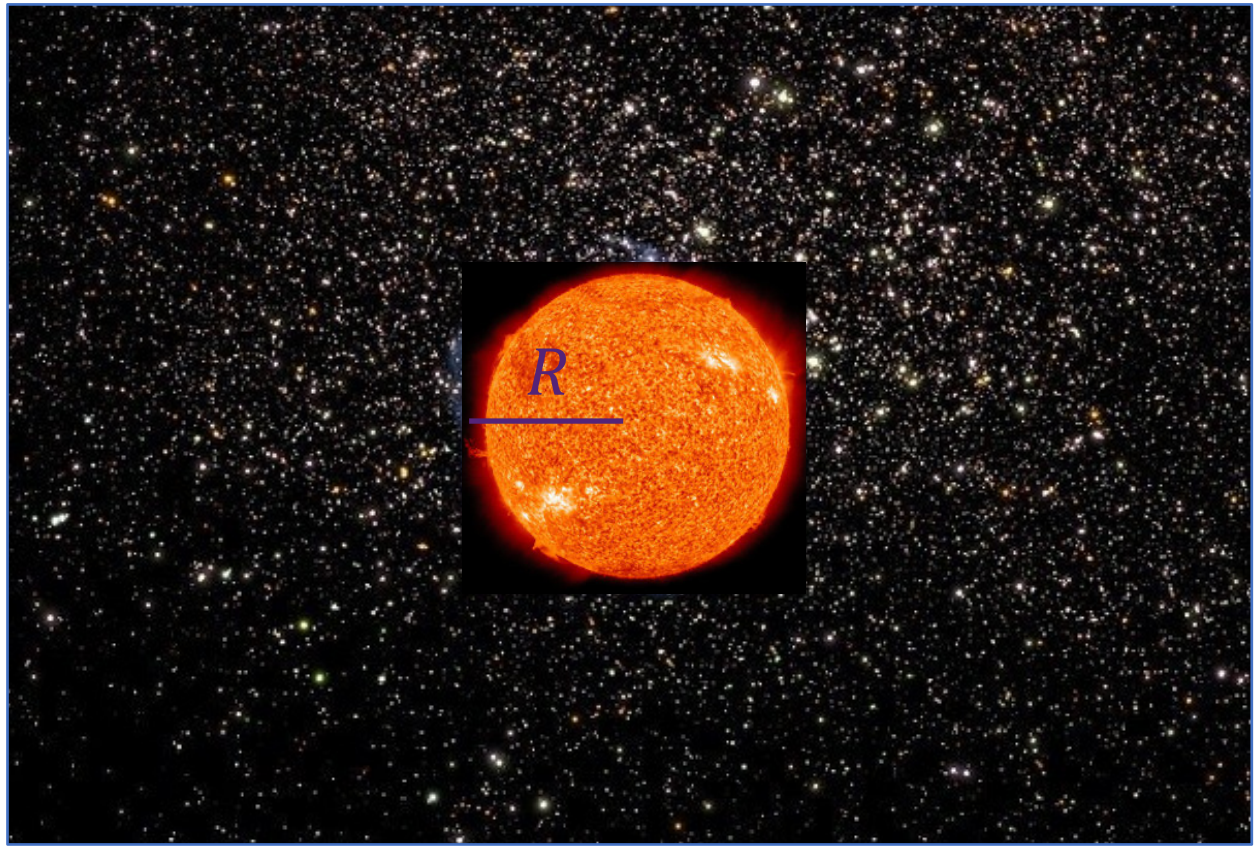
$$\Delta t' = \Delta t \sqrt{1 - \frac{2GM}{Rc^2}}$$

Schwartzschild radius: $R_s = \frac{2GM}{c^2}$

$$\Delta t' = \Delta t \sqrt{1 - \frac{R_s}{R}}$$



Karl Schwarzschild



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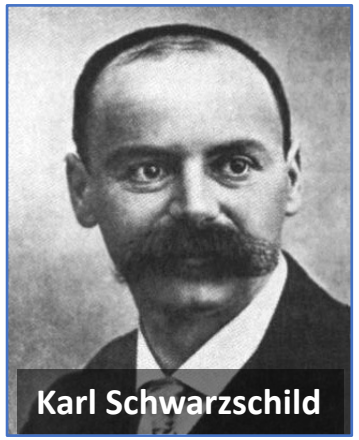
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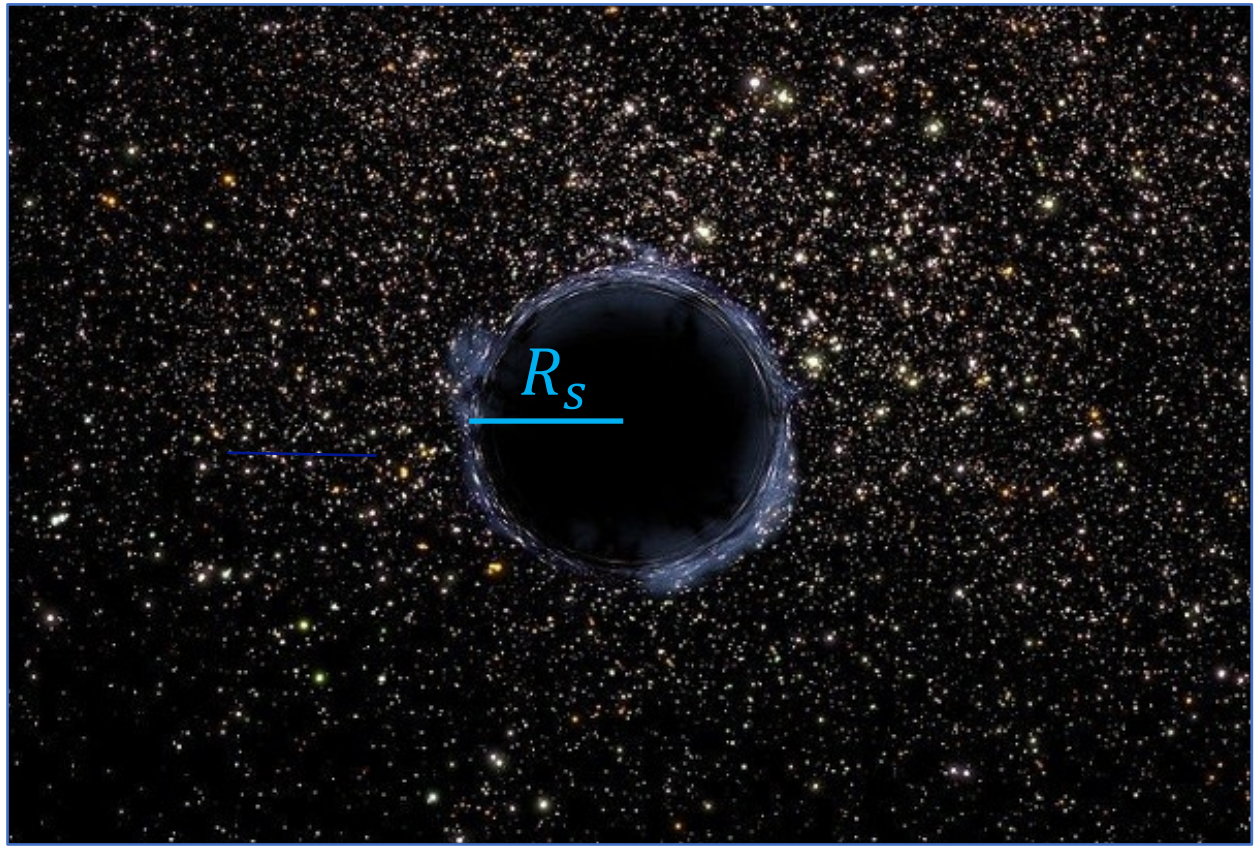

Time stand-still:

If $R = R_s$ then $\Delta t = 0$

(Time stands still at the horizon of a black-hole)

Example our sun: $G = 6.67 \times 10^{-11}\ m^3/kg\ s^2$ (Newton's gravitation constant)

$M_{sun} = 2 \times 10^{30}\ kg$

➔ $R_S = 3\ km$ for a black hole

Stars and Black Holes

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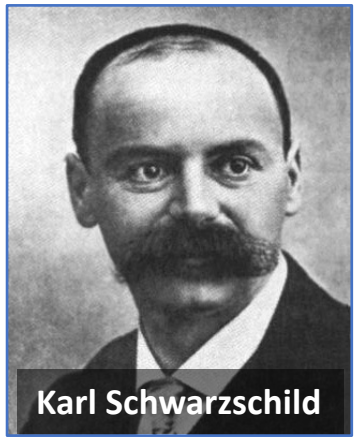
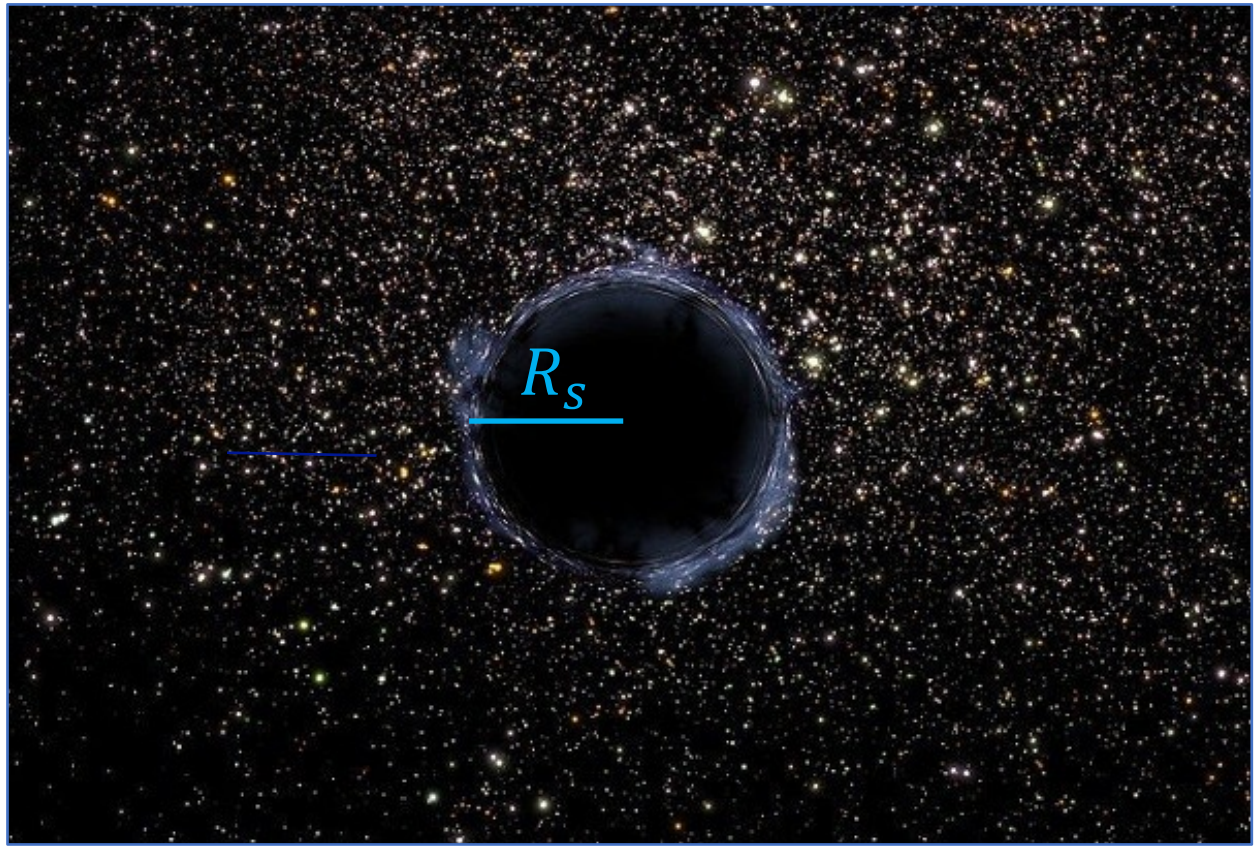

Karl Schwarzschild



Time stand-still:

If $R = R_s$ then $\Delta t = 0$

(Time stands still at the horizon of a black-hole)

Example our earth: $G = 6.67 \times 10^{-11}\ m^3/kg\ s^2$ (Newton's gravitation constant)

$M_{earth} = 6 \times 10^{24}\ kg$

➔ $R_S = 9\ mm$ for a black hole

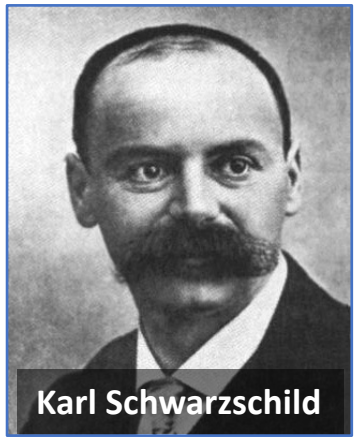
Stars and Black Holes

Gravitational time slowdown near a star with mass M:

$$\Delta t' = \Delta t \sqrt{1 - \frac{2GM}{Rc^2}}$$

Schwartzschild radius: $R_s = \frac{2GM}{c^2}$

$$\Delta t' = \Delta t \sqrt{1 - \frac{R_s}{R}}$$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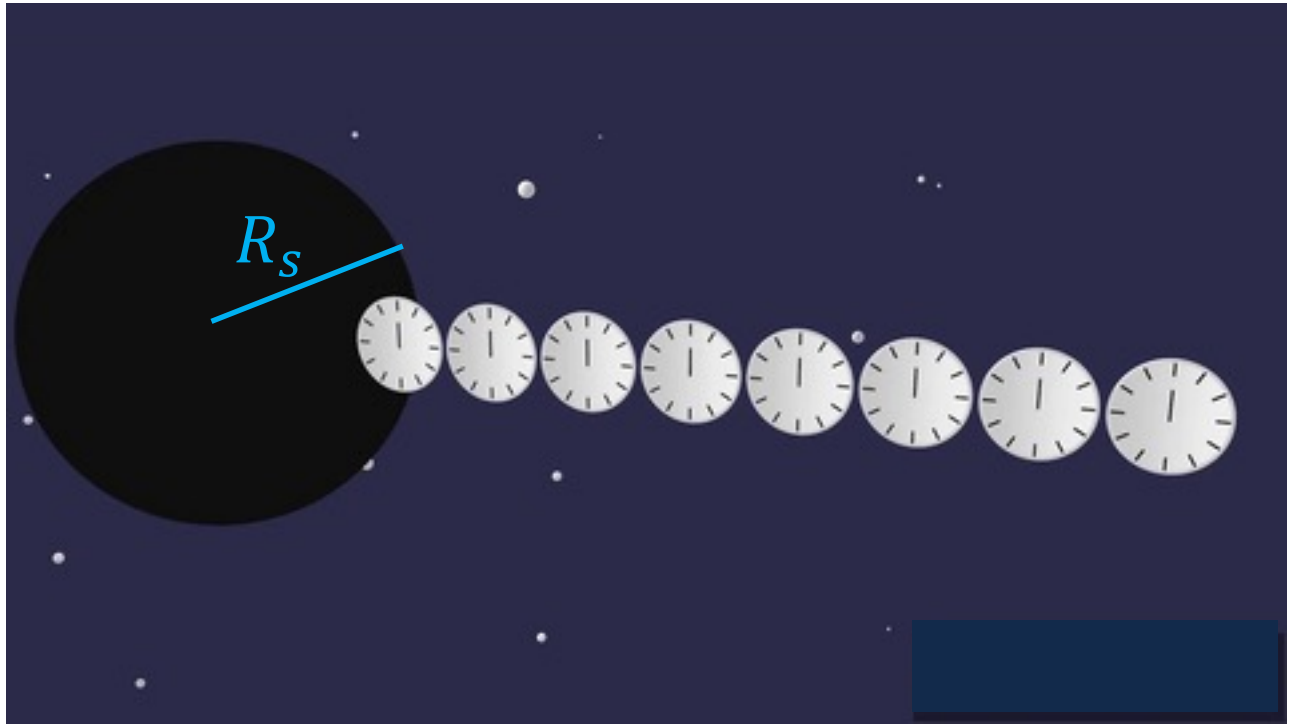

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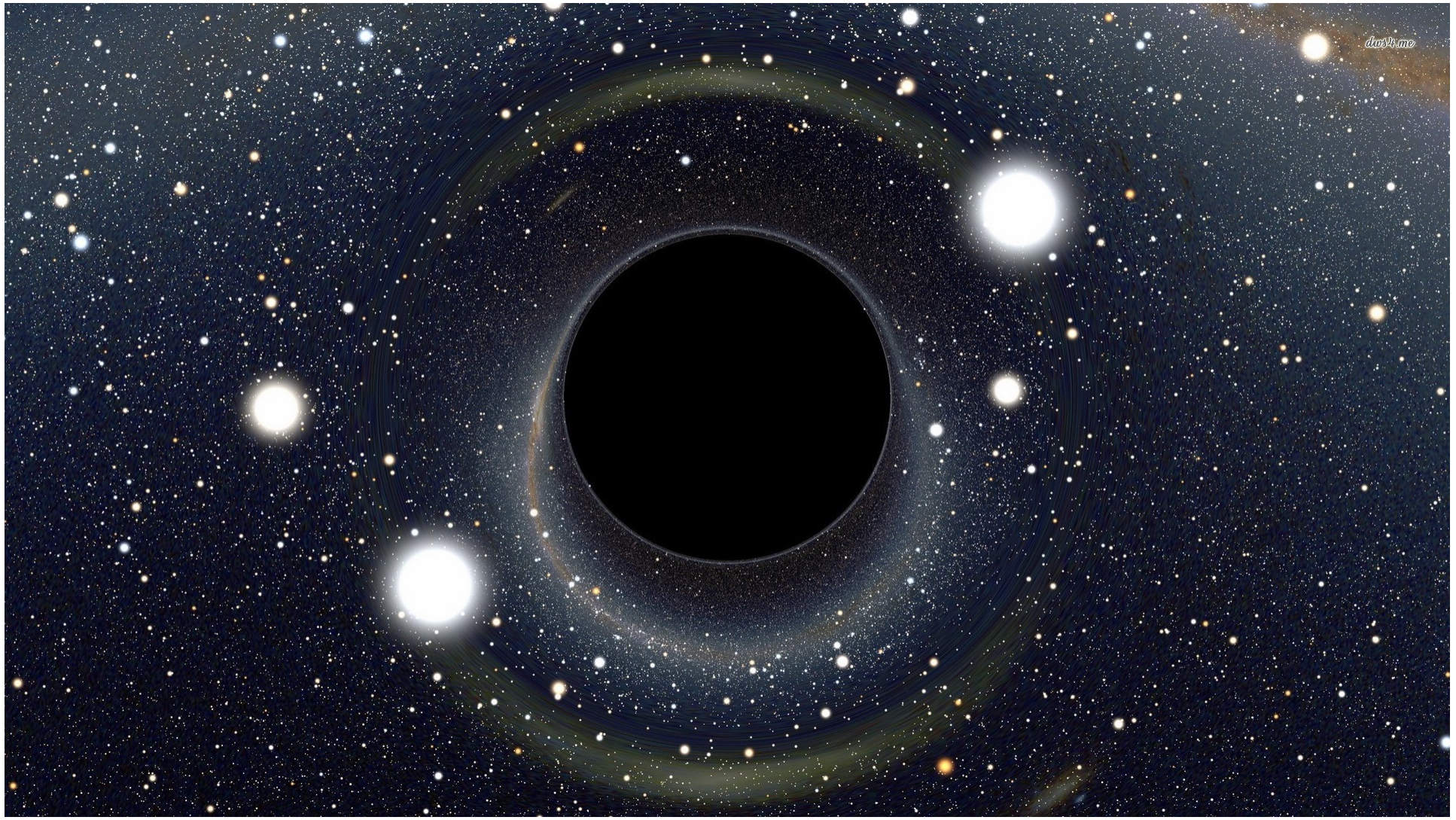
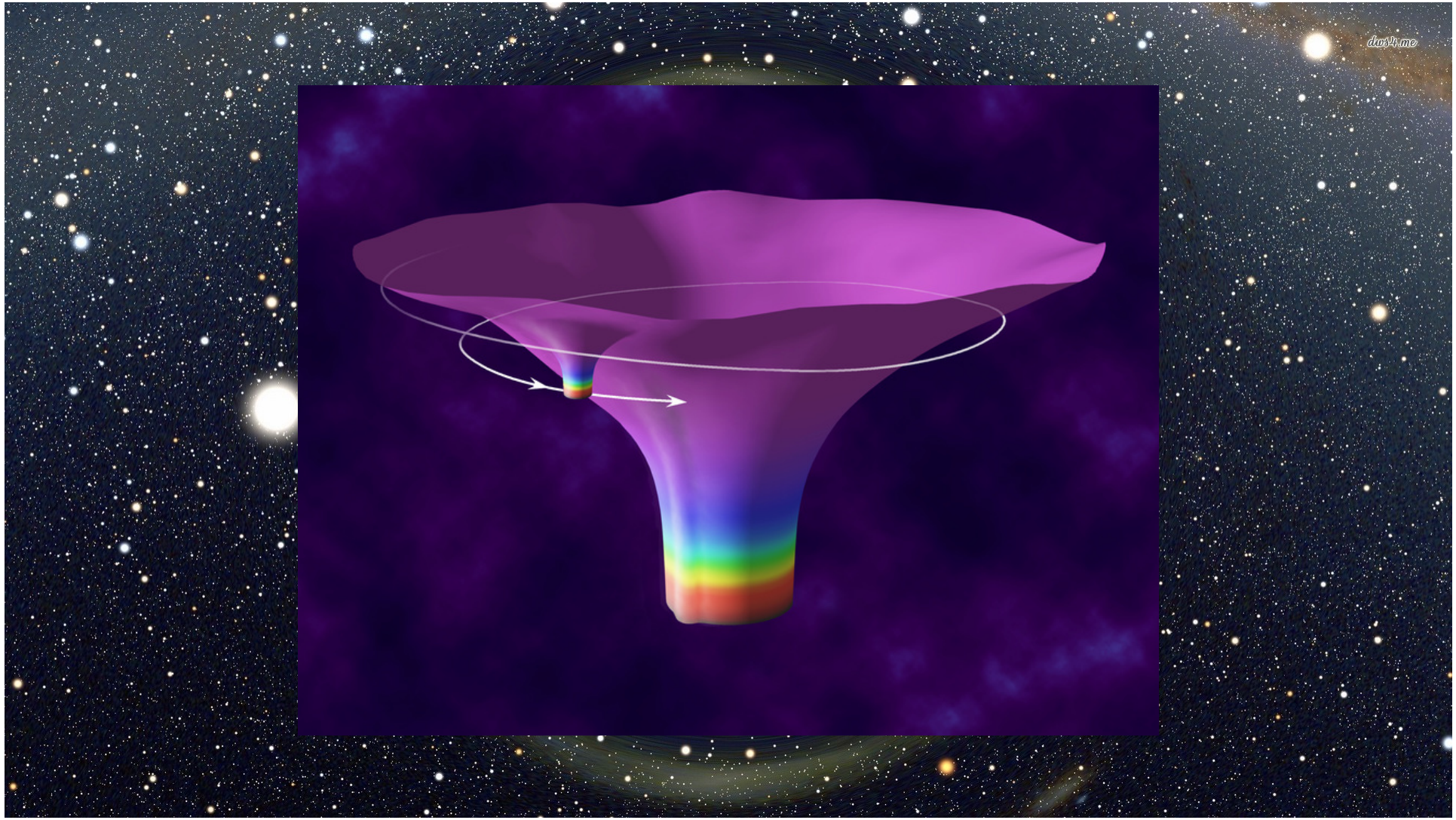
What *is* a black hole?



Purely curved space-time!

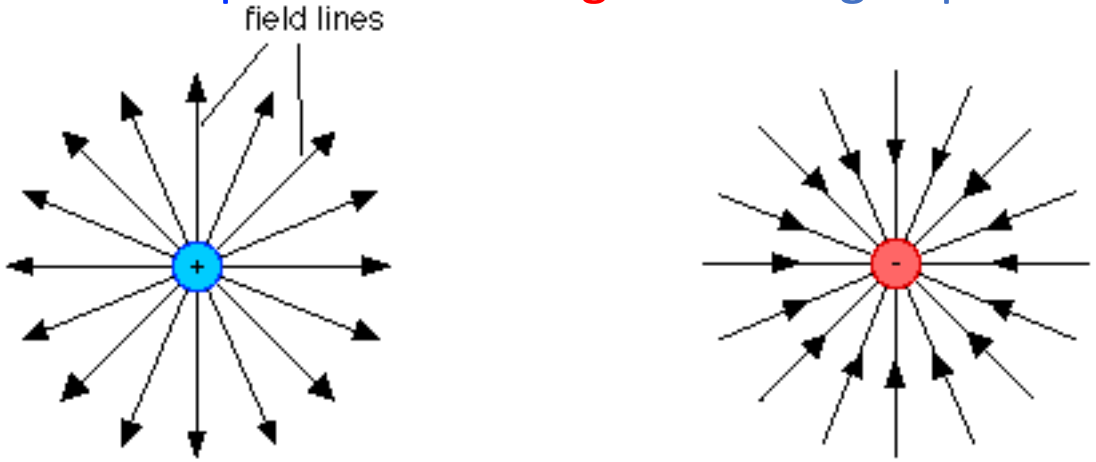
What *is* a black hole?



What happens when two black holes meet?

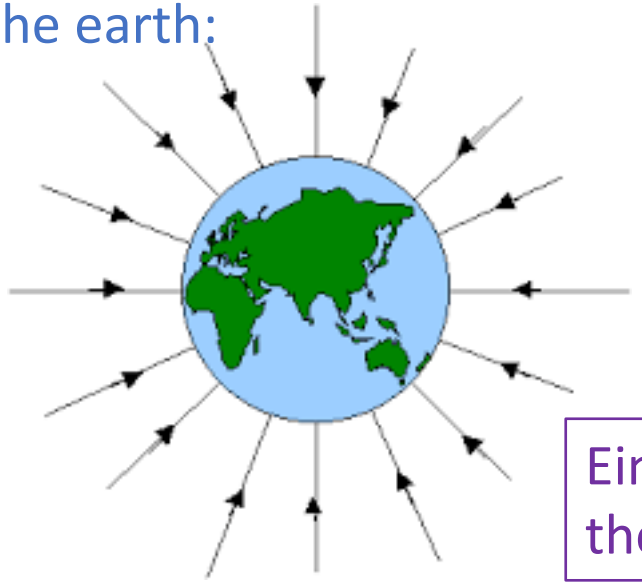
Intermezzo: Electric vs Gravitational Fields

Electric field of positive and negative charged particle:



$$F_e = q\,E = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$

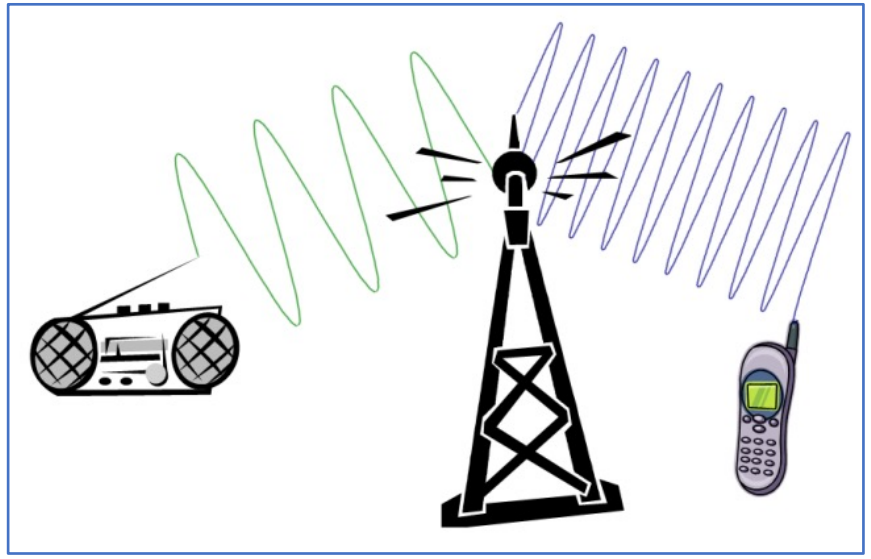
Gravitational field of the earth:



$$F_g = m\,g = -G\,\frac{mM}{R^2}$$

Einstein spent most of his life looking for a unified theory of electromagnetism and general relativity.

Waves and Radiation



Electromagnetic waves:

Caused by accelerating electric particles (electrons)

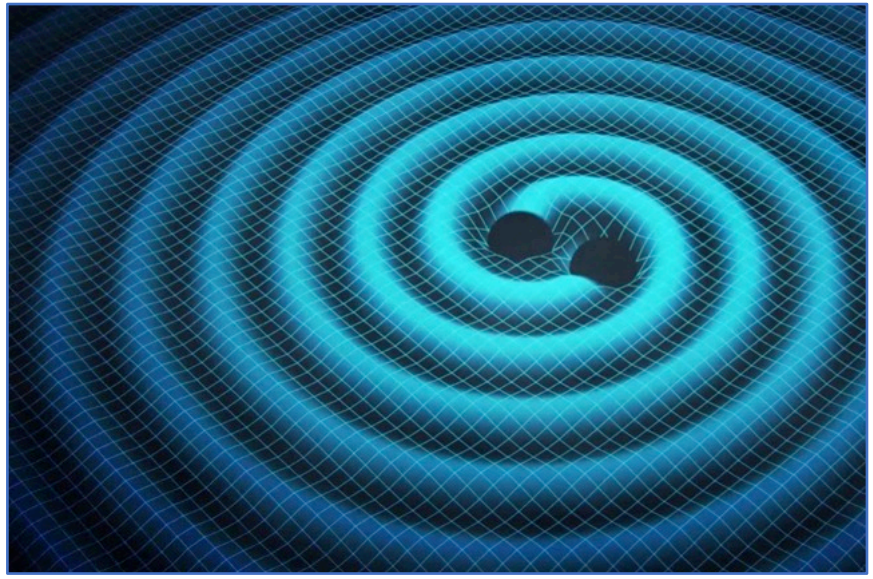
eg.: radio-emission

Maxwell equations:

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu$$

E and B fields

Electric charge and currents



Gravitational Waves:

Caused by moving masses.

Requires very heavy masses →**black holes.**

(Einstein thought these couldn't be observed)

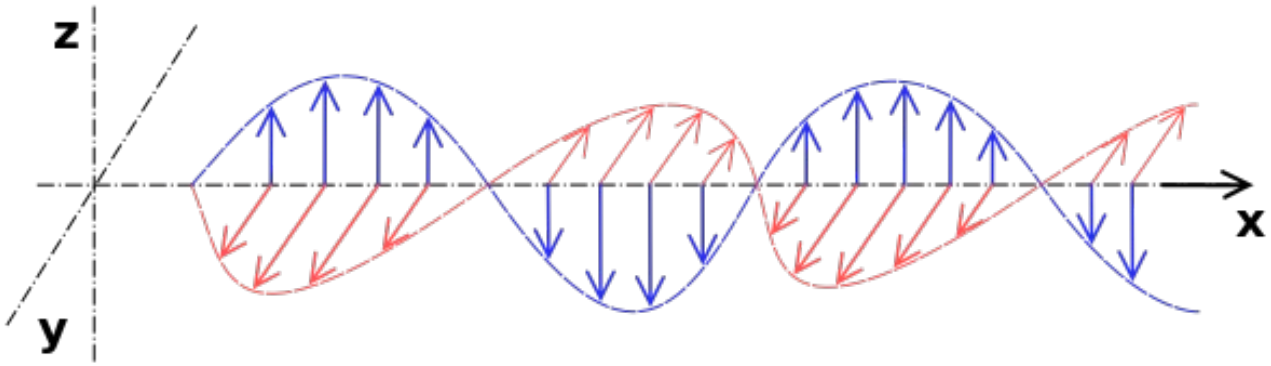
Einstein equations:

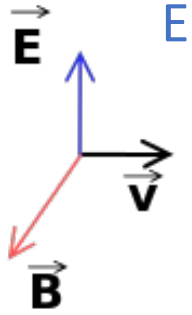
$$G^{\mu\nu} + \Lambda g^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

Space-time fields

Mass and mass-flow

Electromagnetic and Gravitational Waves

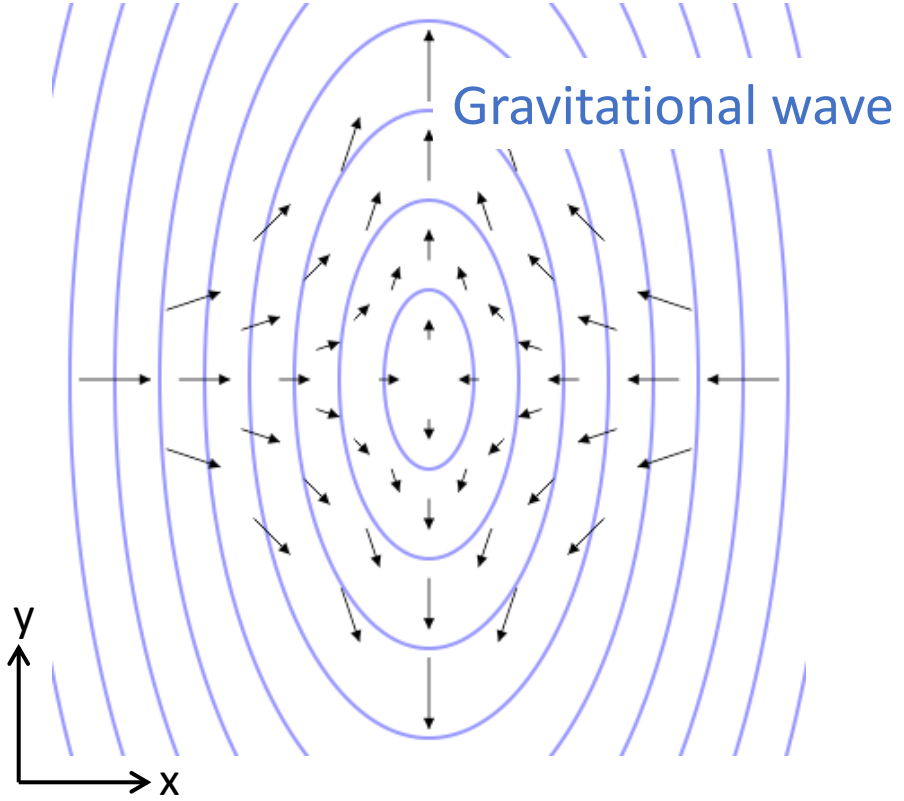




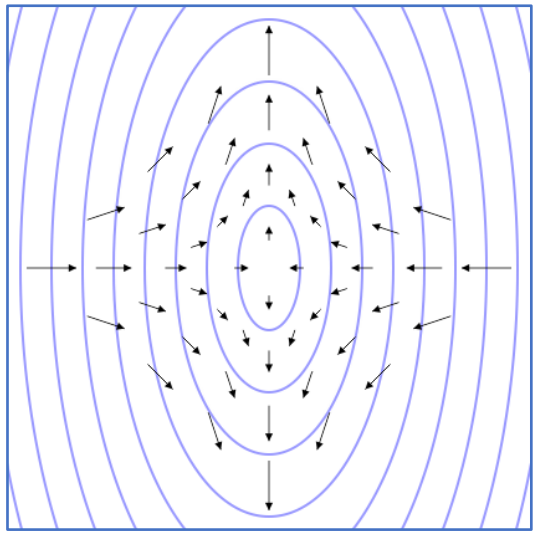
Electromagnetic wave

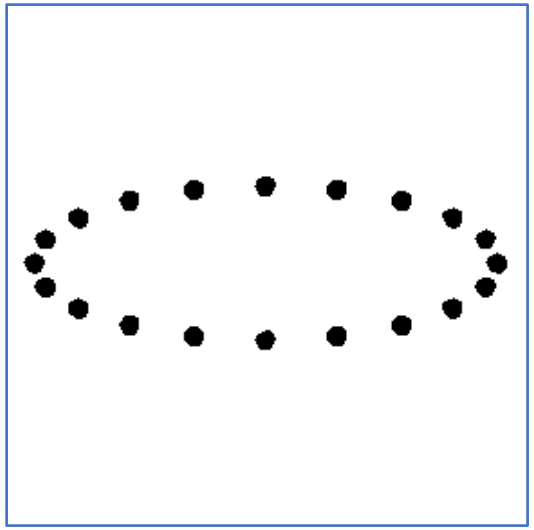
Electromagnetic wave:
Changing electric and magnetic field propagating through space.
Caused by moving (accelerating!) electric charges.

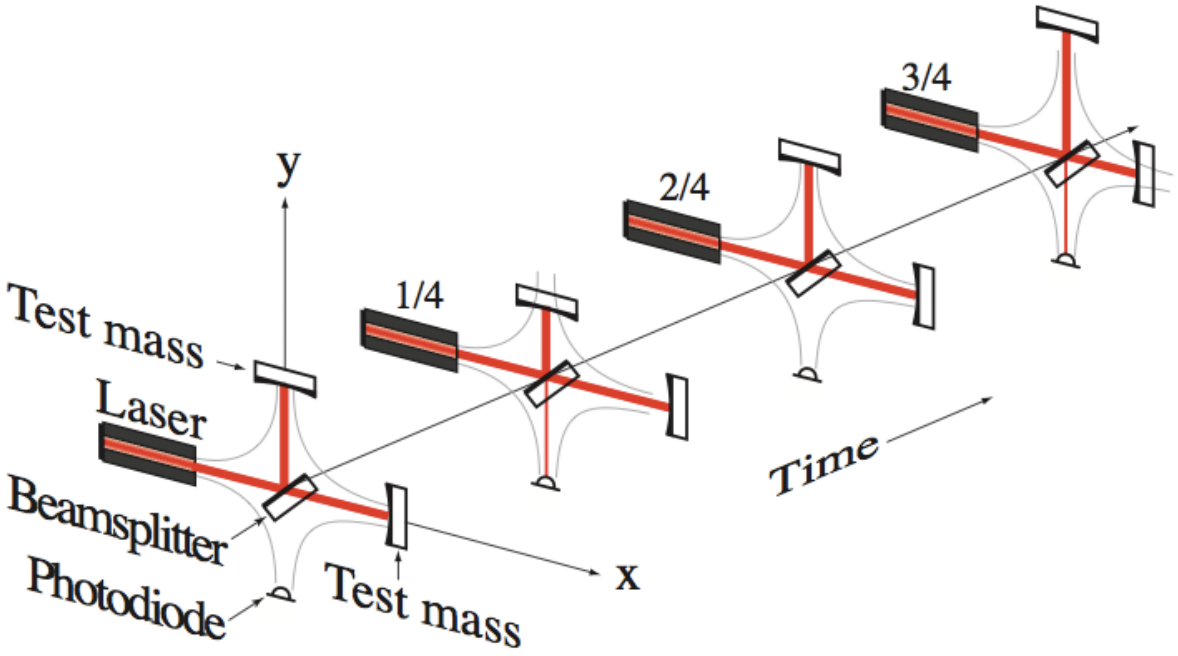
Gravitational wave:
Changing space-time field.
Caused by moving (accelerating!) masses.

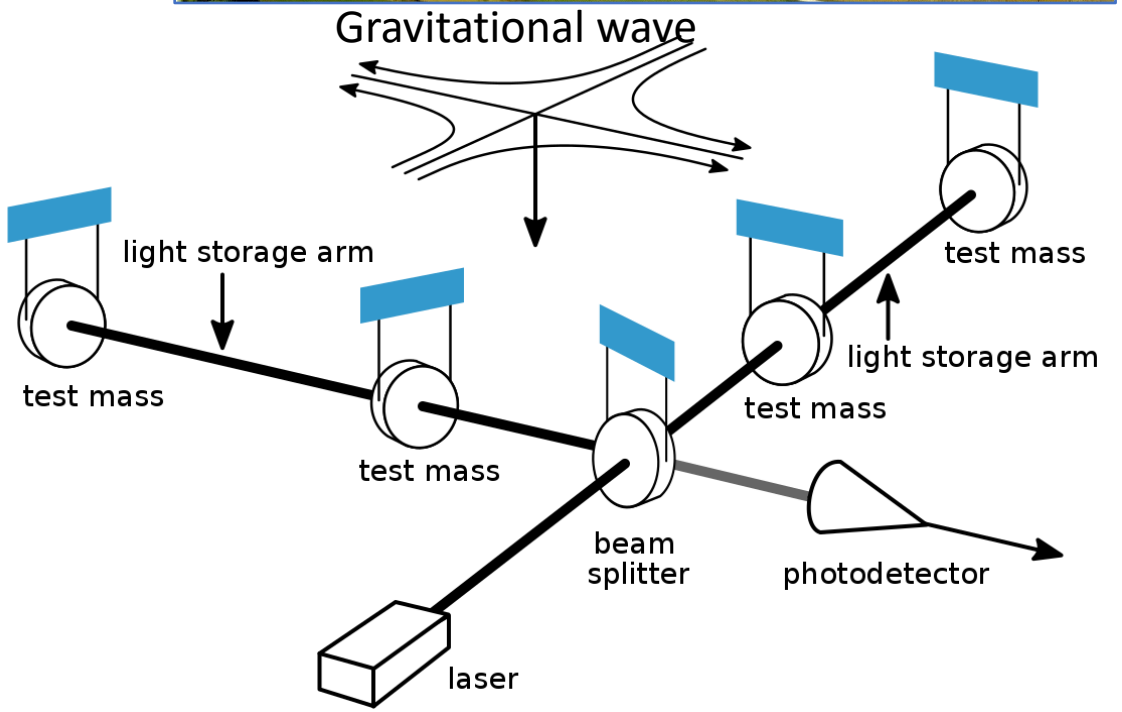


Remember the interferometer!

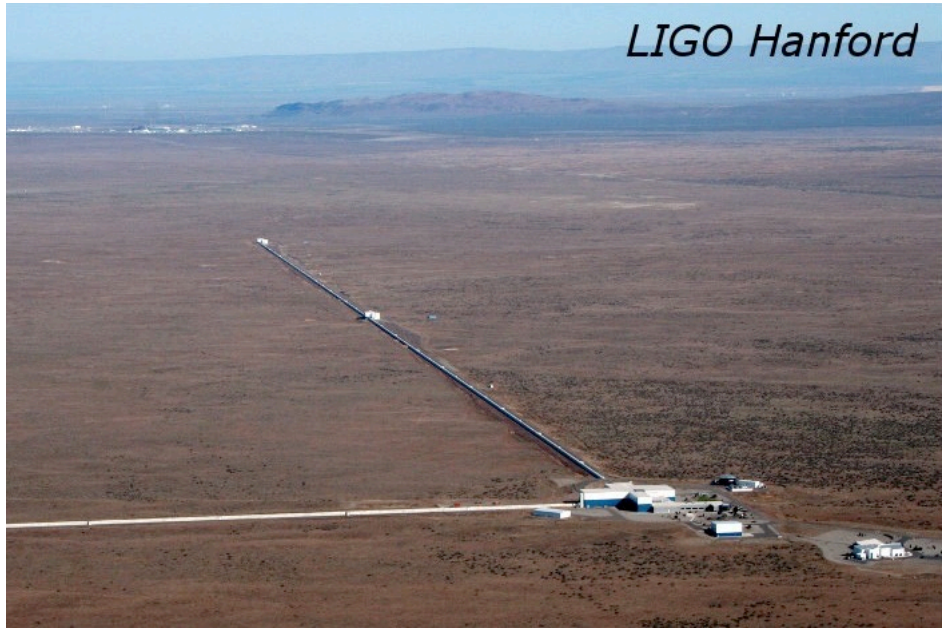




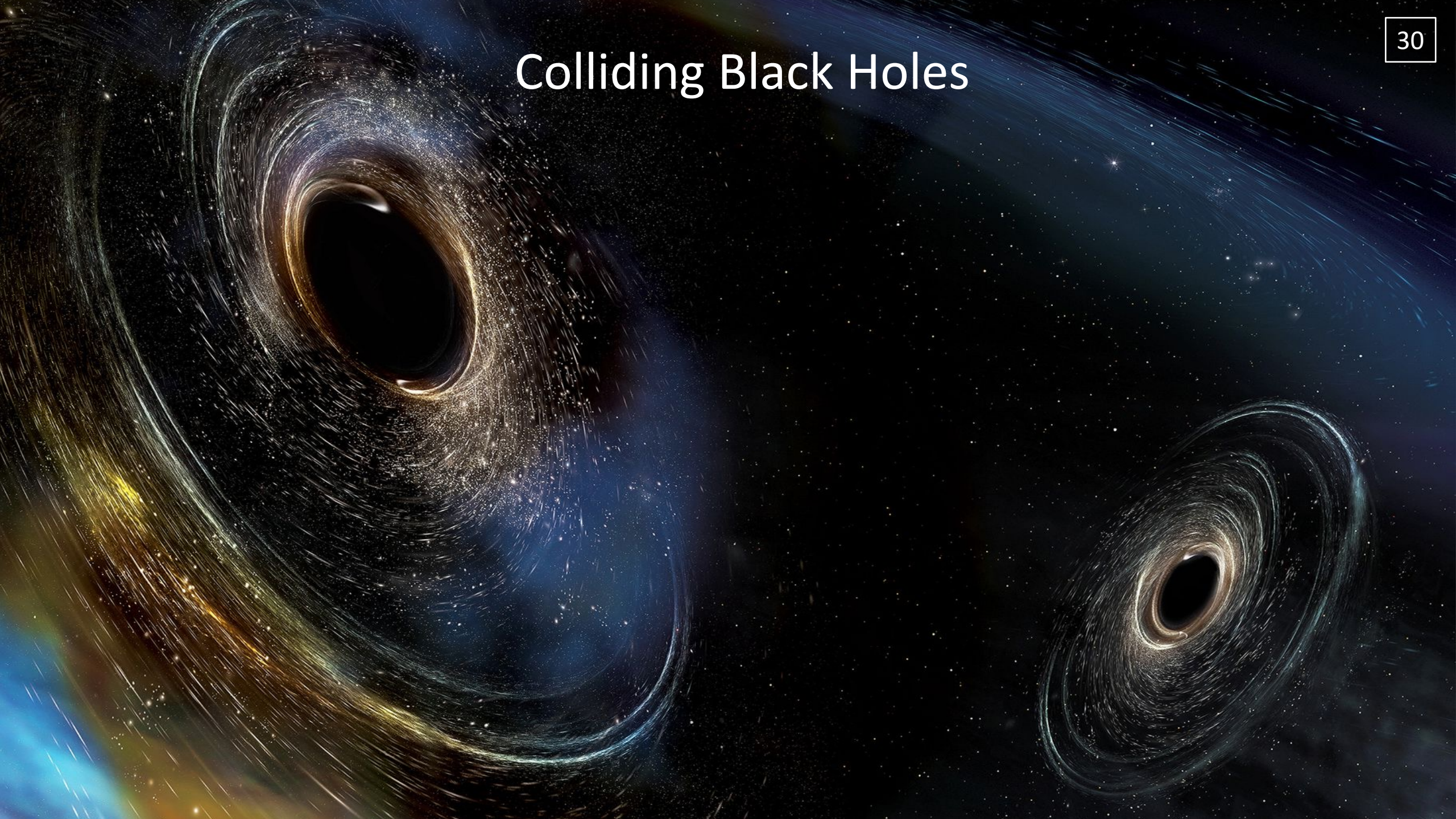




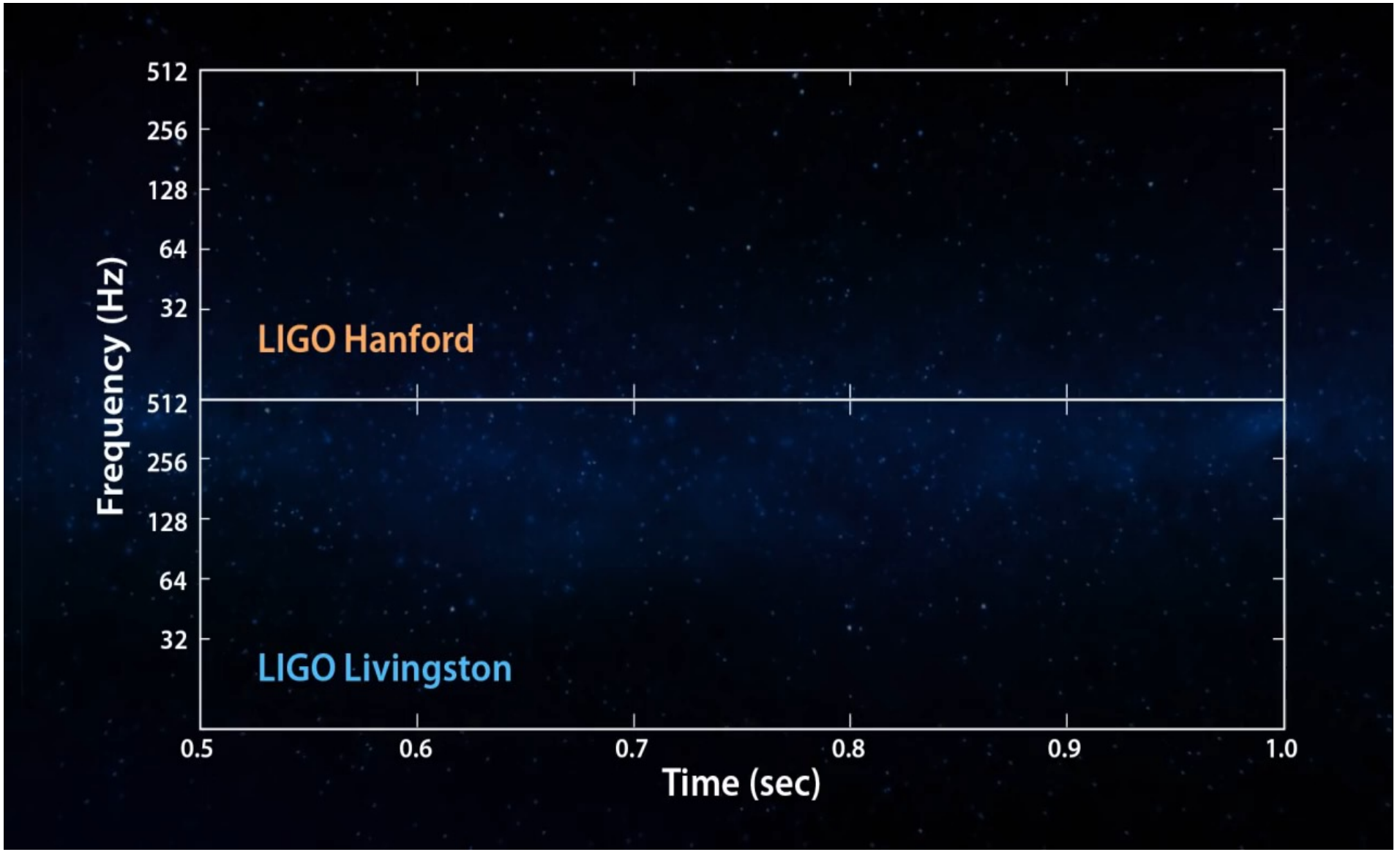
Current Facilities



Colliding Black Holes



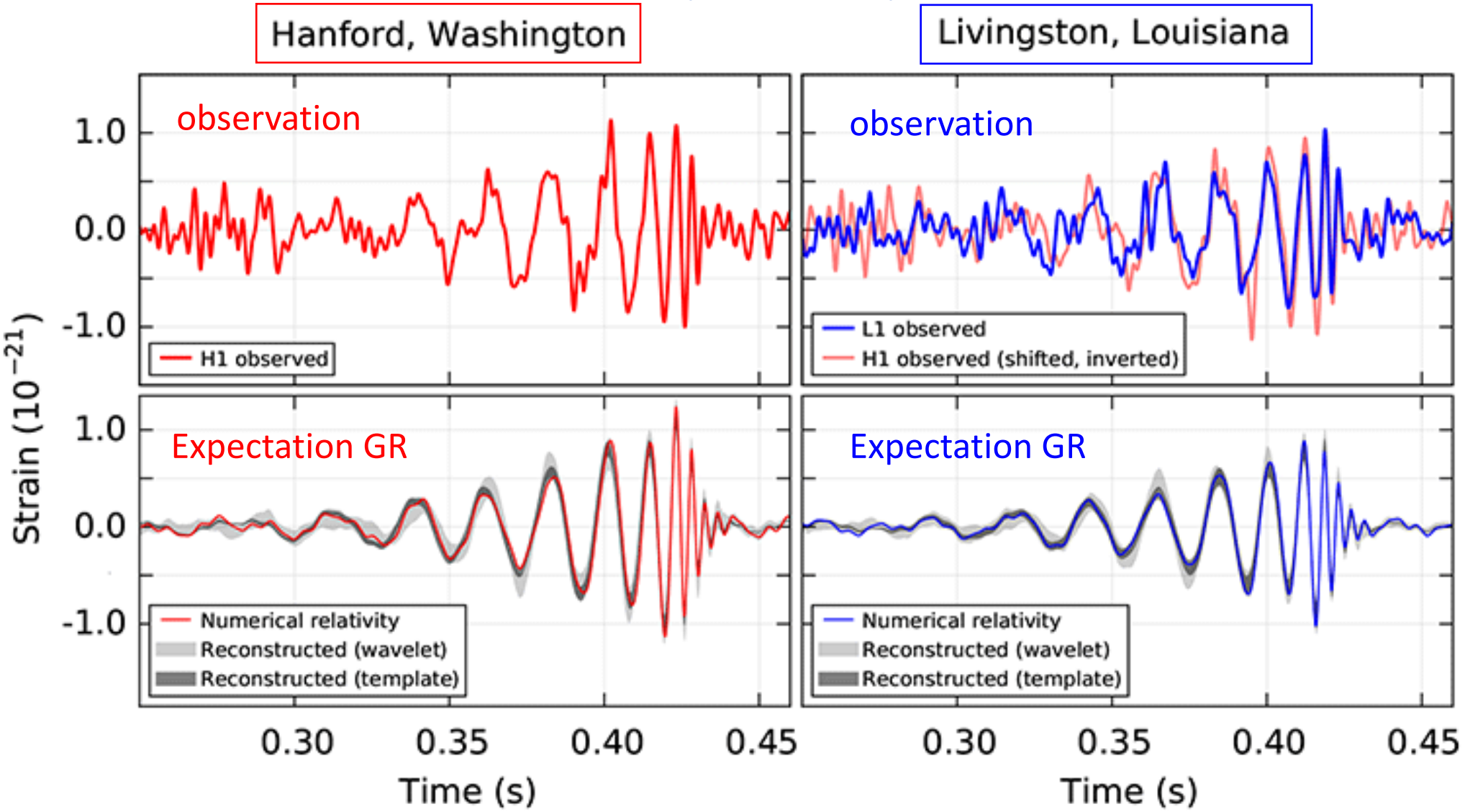
First detection of gravitational waves: GW150914



"Chirp" of colliding black holes at 1.3 billion lightyears distance

Consistent signals seen in Washington and Louisiana

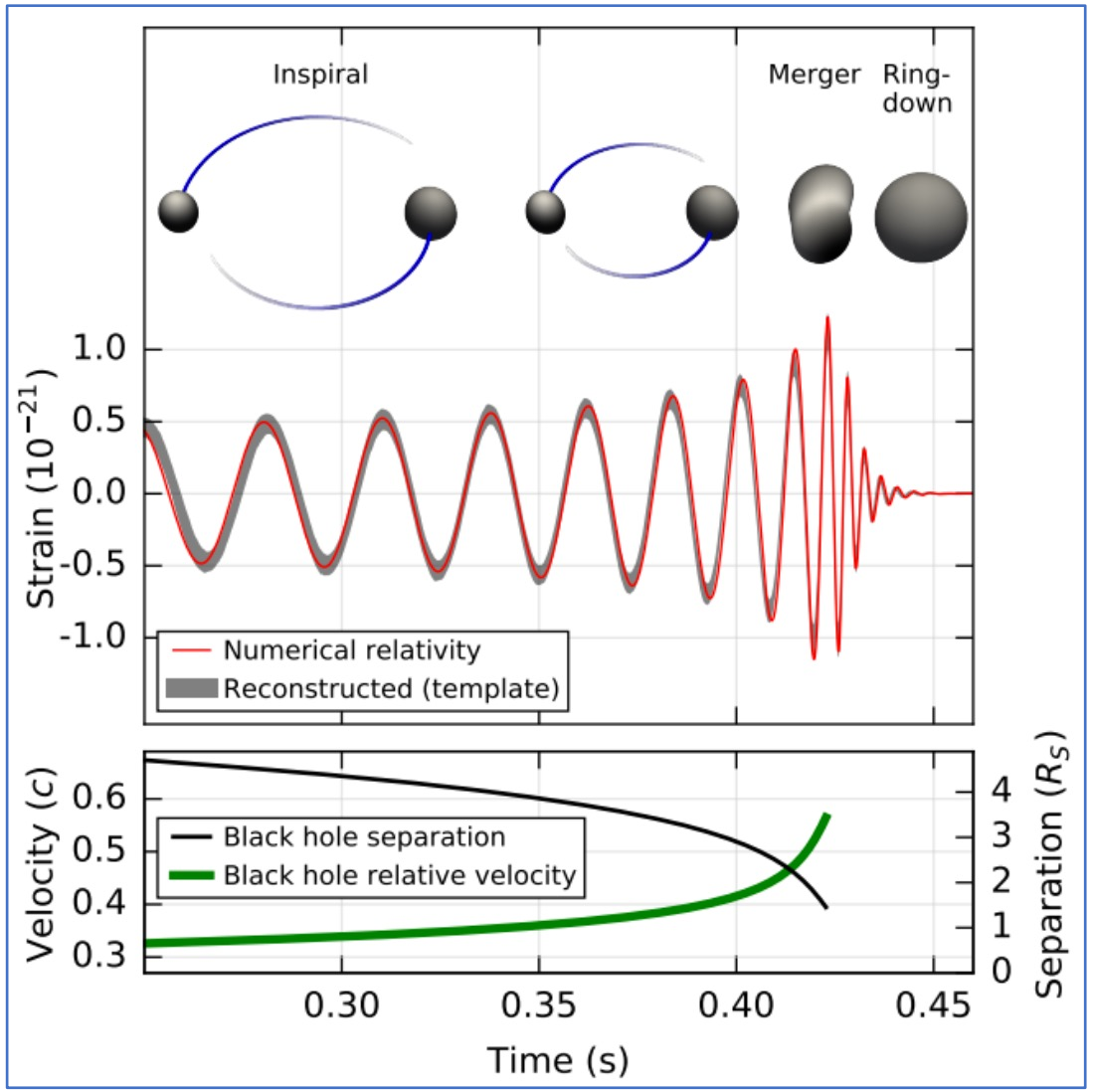
(GW150914)



Detection of Gravitational Waves

Two Merging Black Holes

Two massive colliding/merging black holes:



Distance: 1.3 billion lightyears

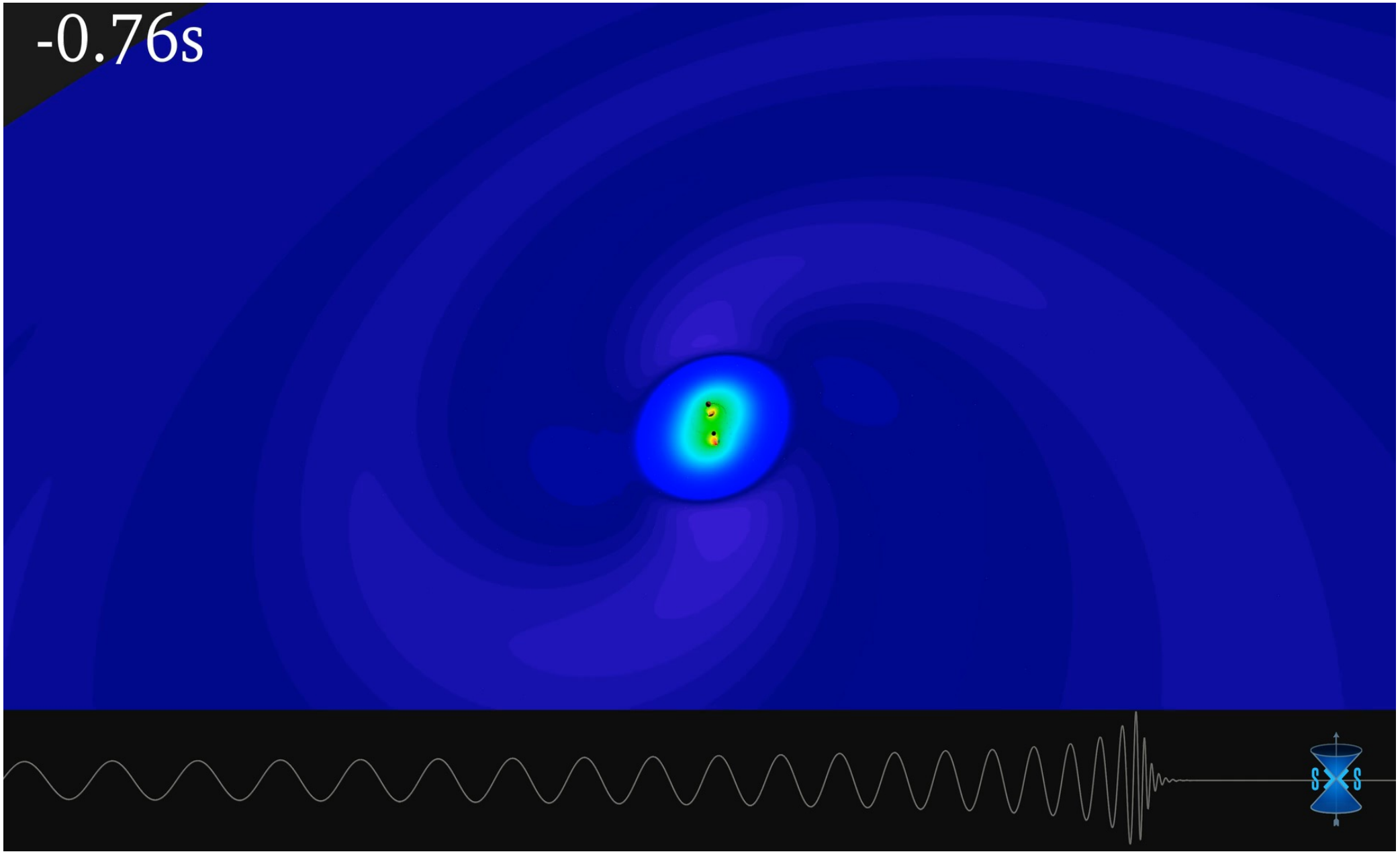
B.H.1 = 36 x mass of the sun
B.H.2 = 29 x mass of the sun
New BH: 62 solar masses
➔3 solar masses of energy ($E=mc^2$) radiated into space

Relative change of space (strain)
0.00000000000000000001%

Rotation speed increasing to half the light speed!

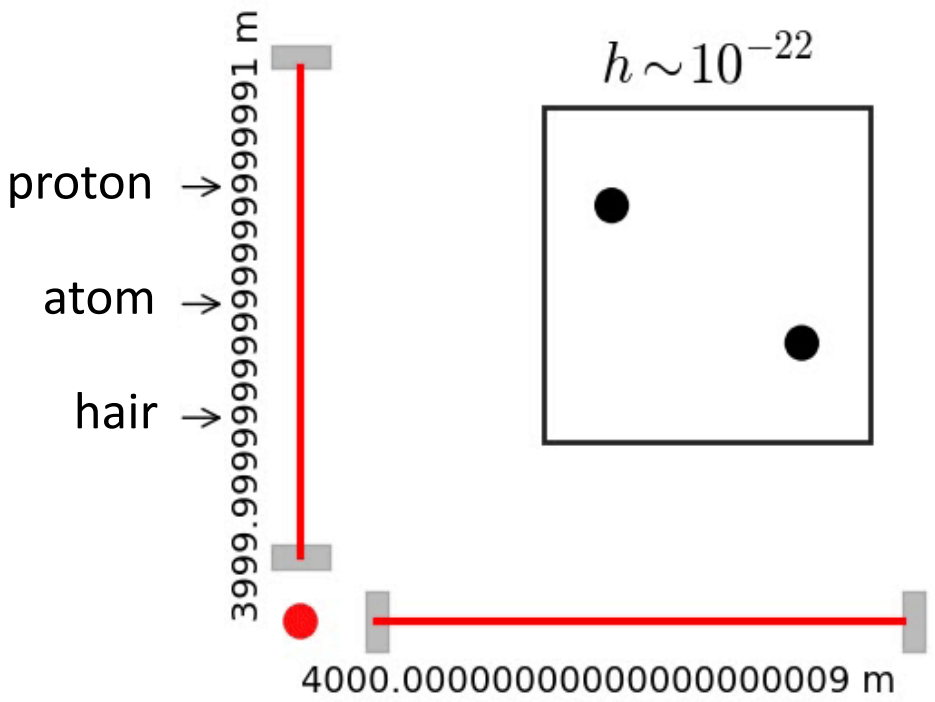
More energy was emitted in gravitational waves than all the visible (EM) energy of all stars in the universe!

Numerical Relativity Simulation for GW150914

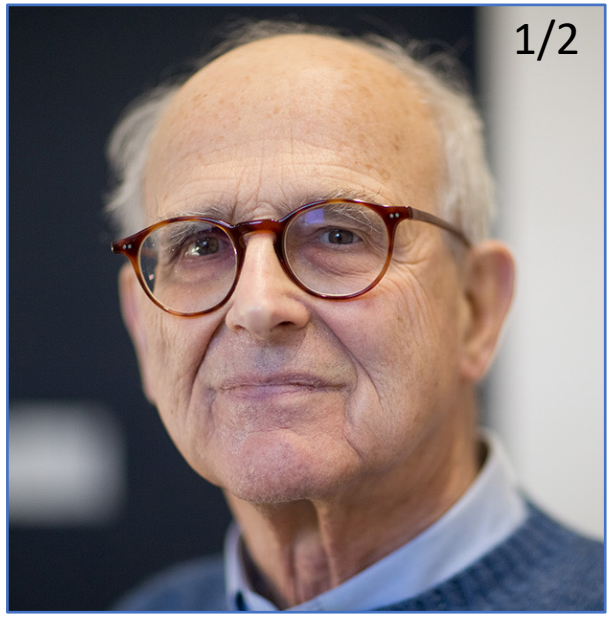


Which precision is needed?

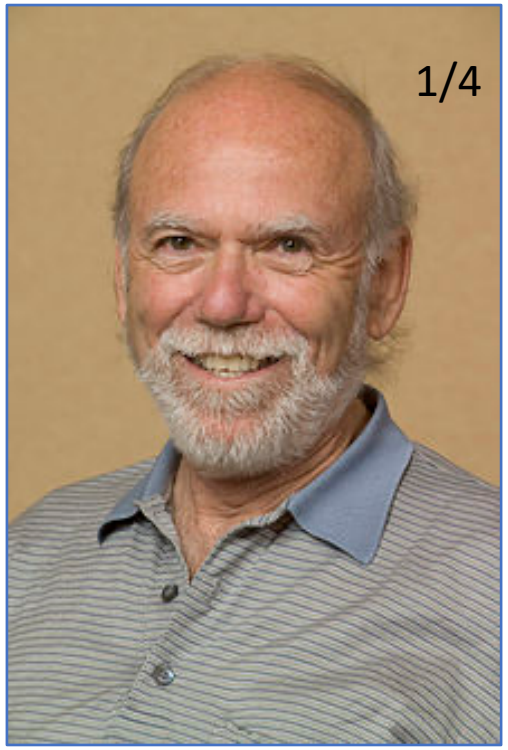
Laser
(f,



Nobel Prize in Physics 2017



Rainer Weiss



Barry C. Barish

Kip S. Thorne

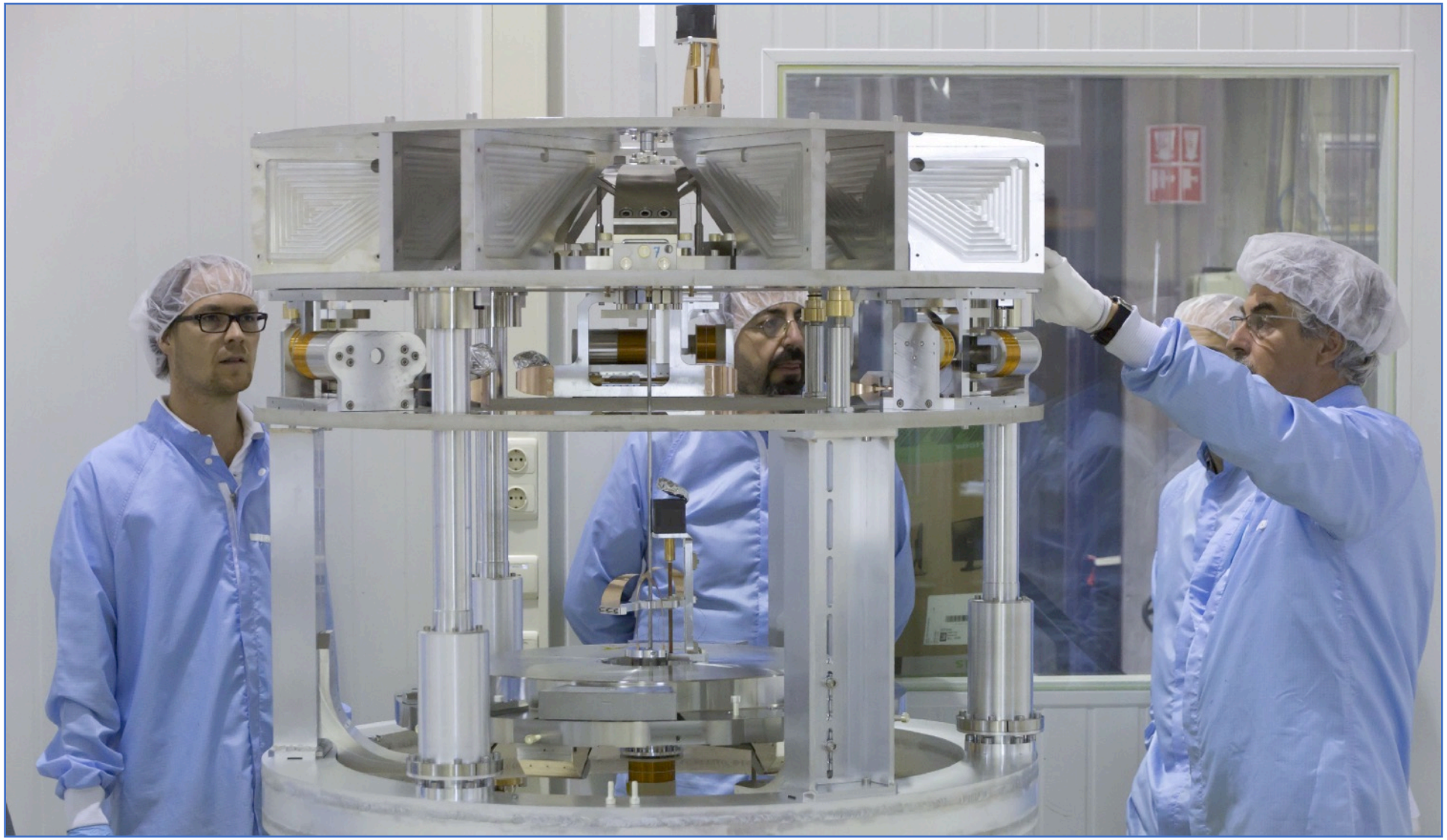
"For decisive contributions to the LIGO detector and the observation of gravitational waves"

The Virgo Experiment in Pisa

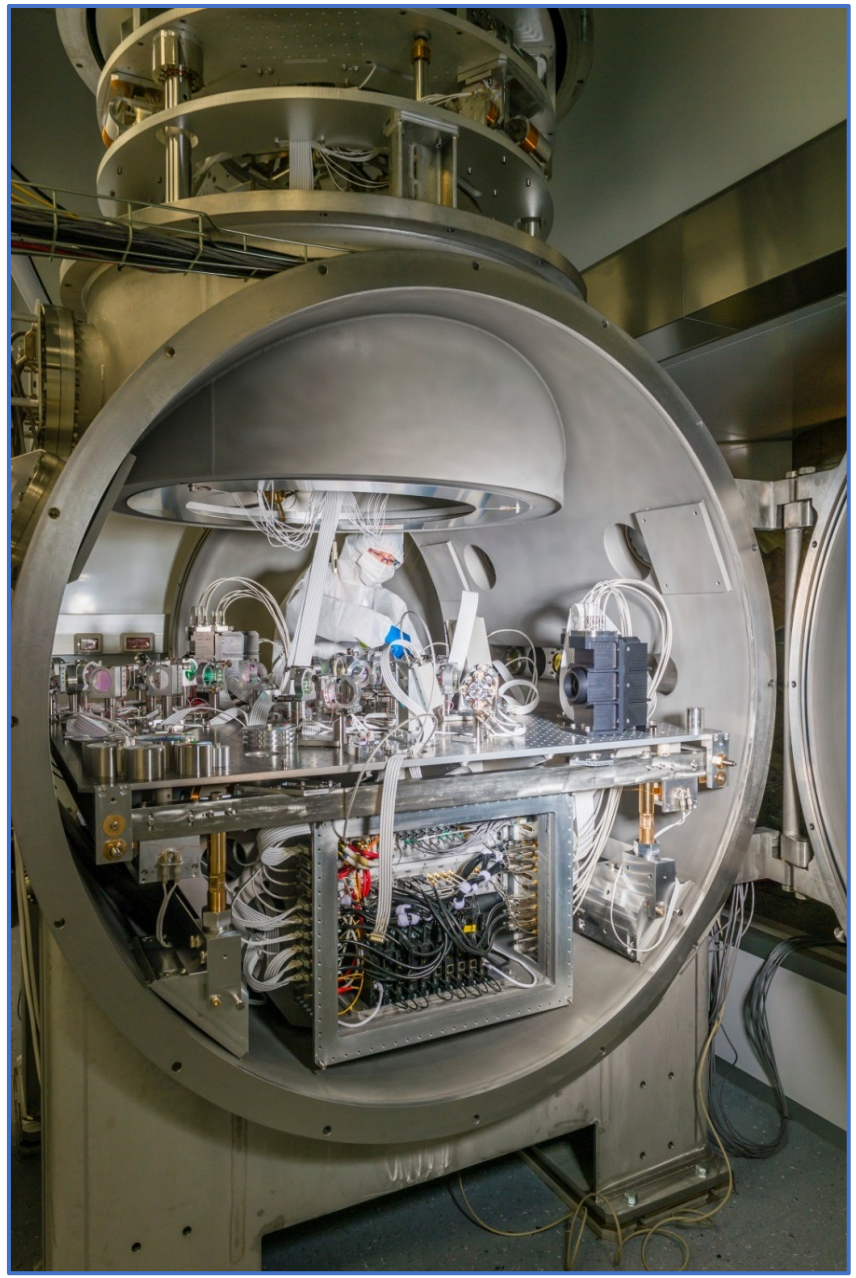
Ultrahigh Vacuüm

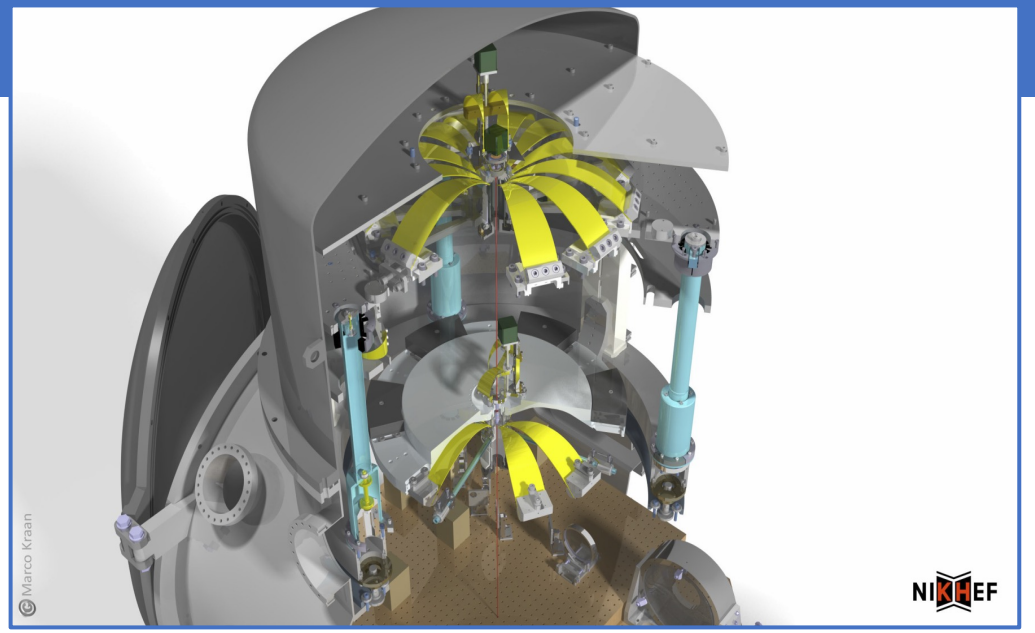
Largest vacuüm vessel in Europe: Pressure ~ 10^{-10} mbar

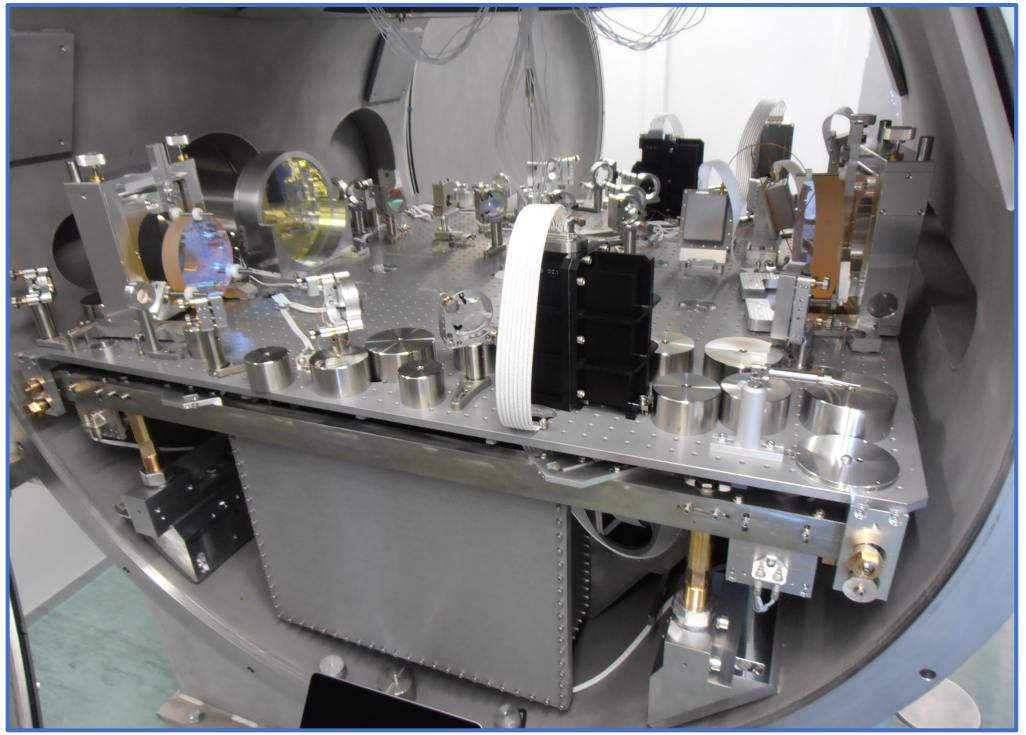
Seismic dampingsystem – made @Nikhef



Seismic Damping Table

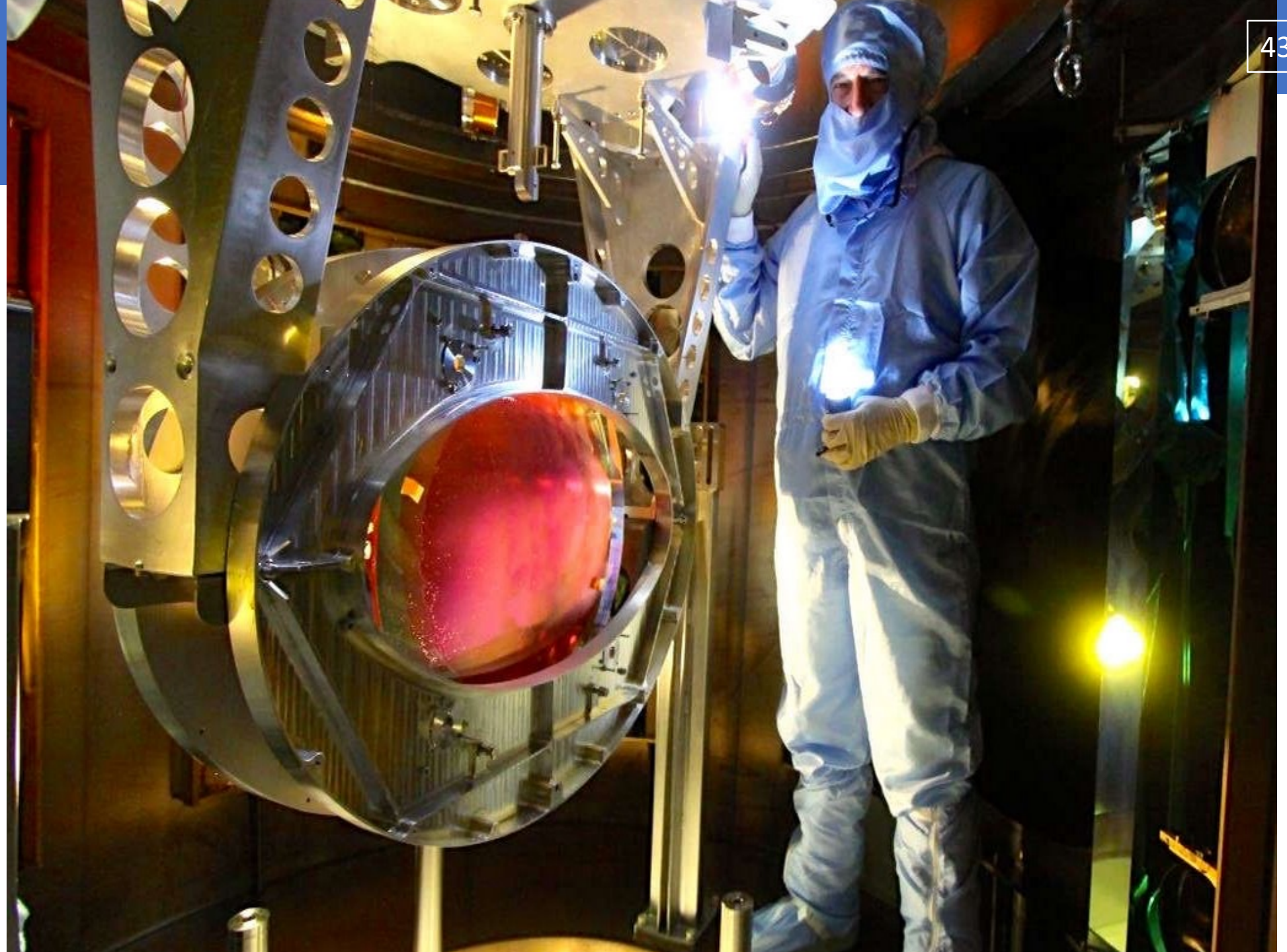


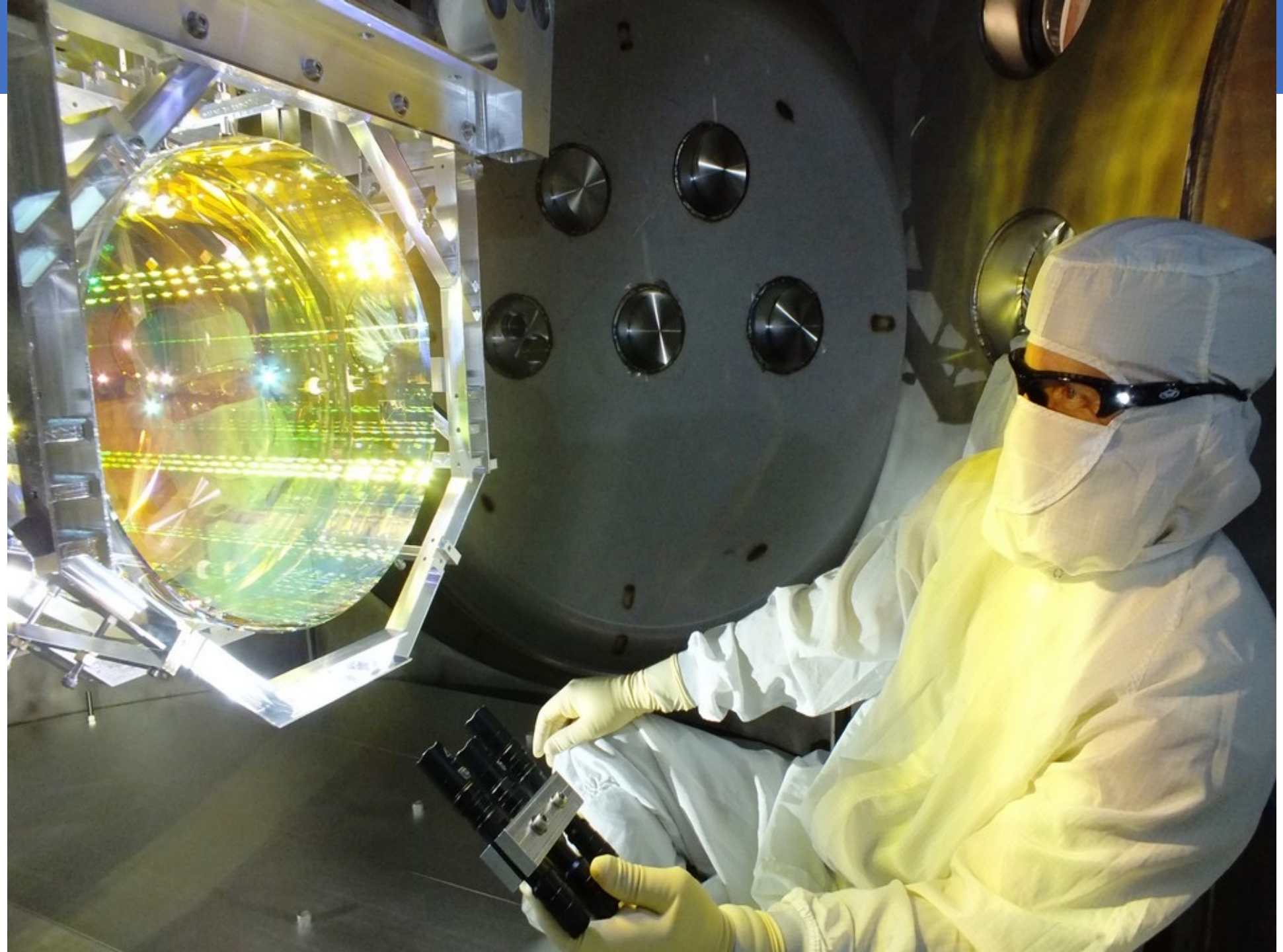




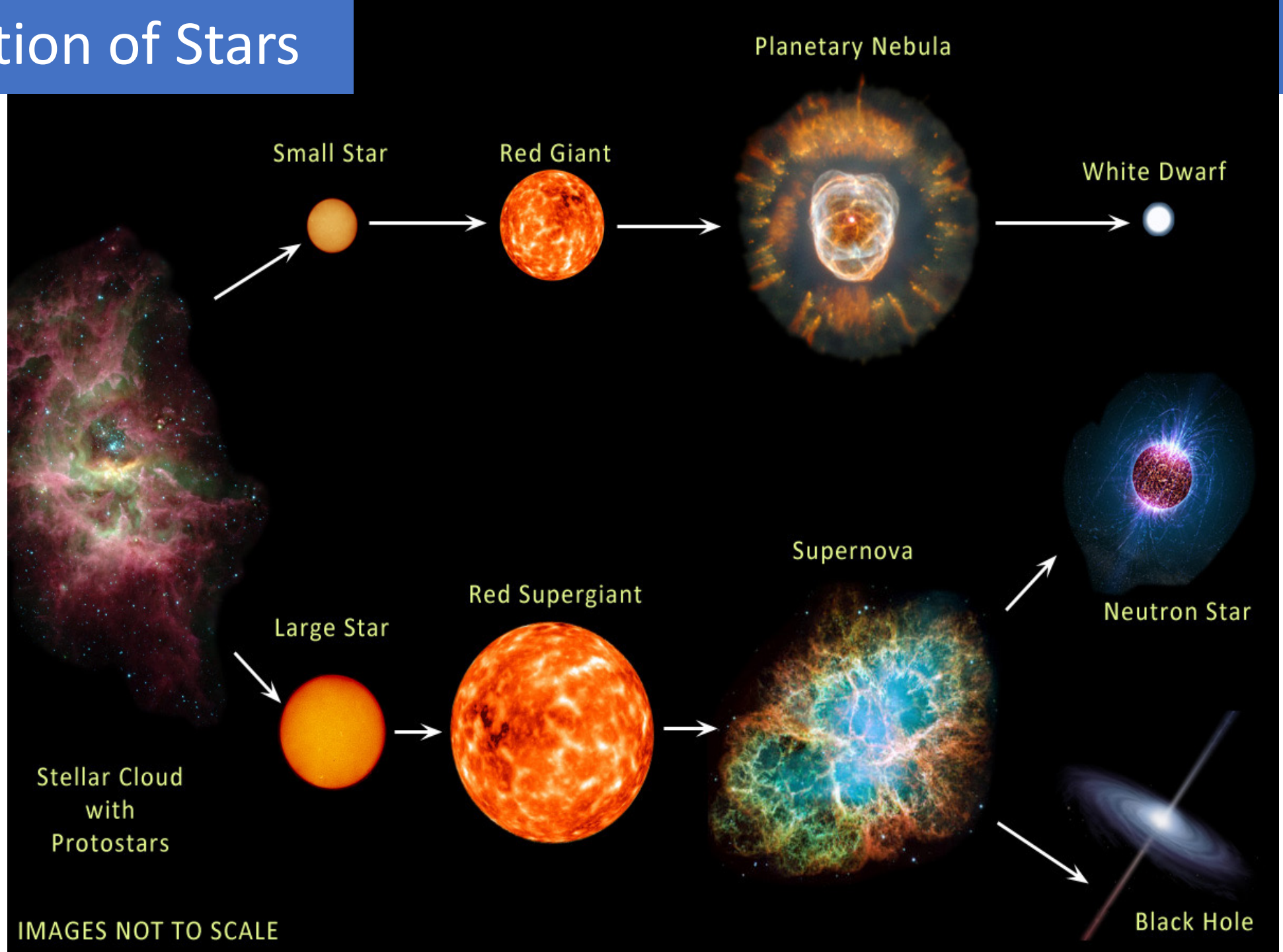
Input mode cleaner end mirror

Beam Splitter





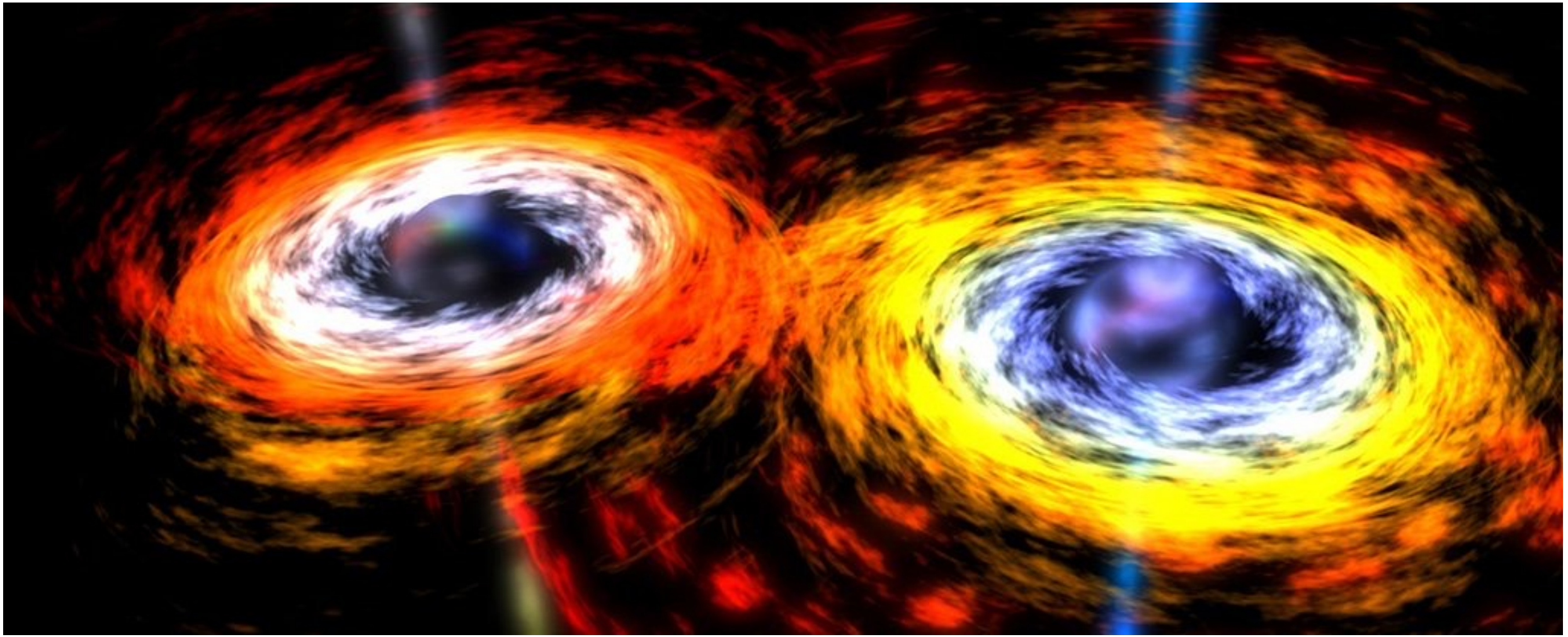
Evolution of Stars



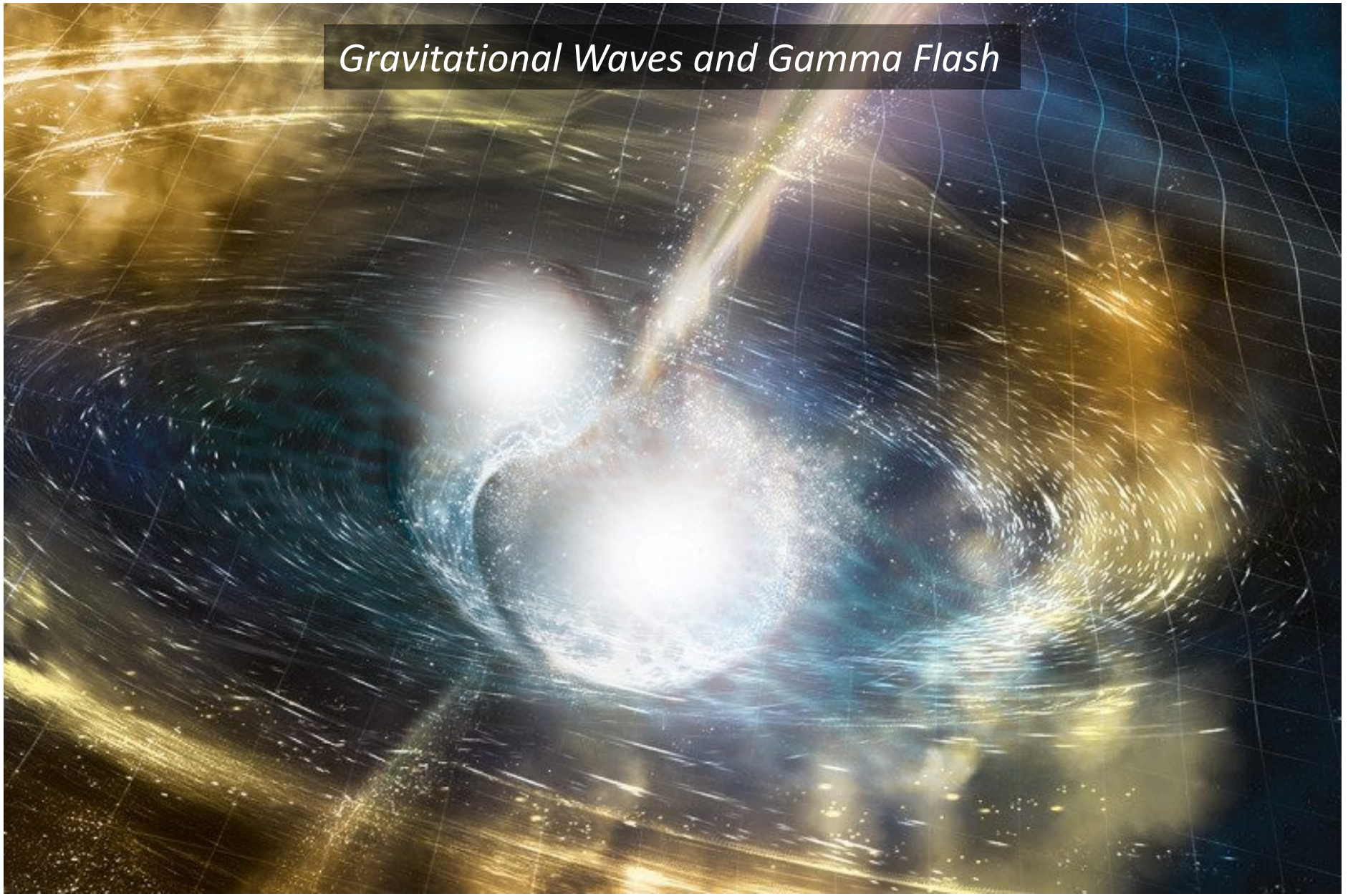
Neutron Star

Coalescing Neutron Stars: “Multimessenger” Astronomy

Gravitational Waves and …

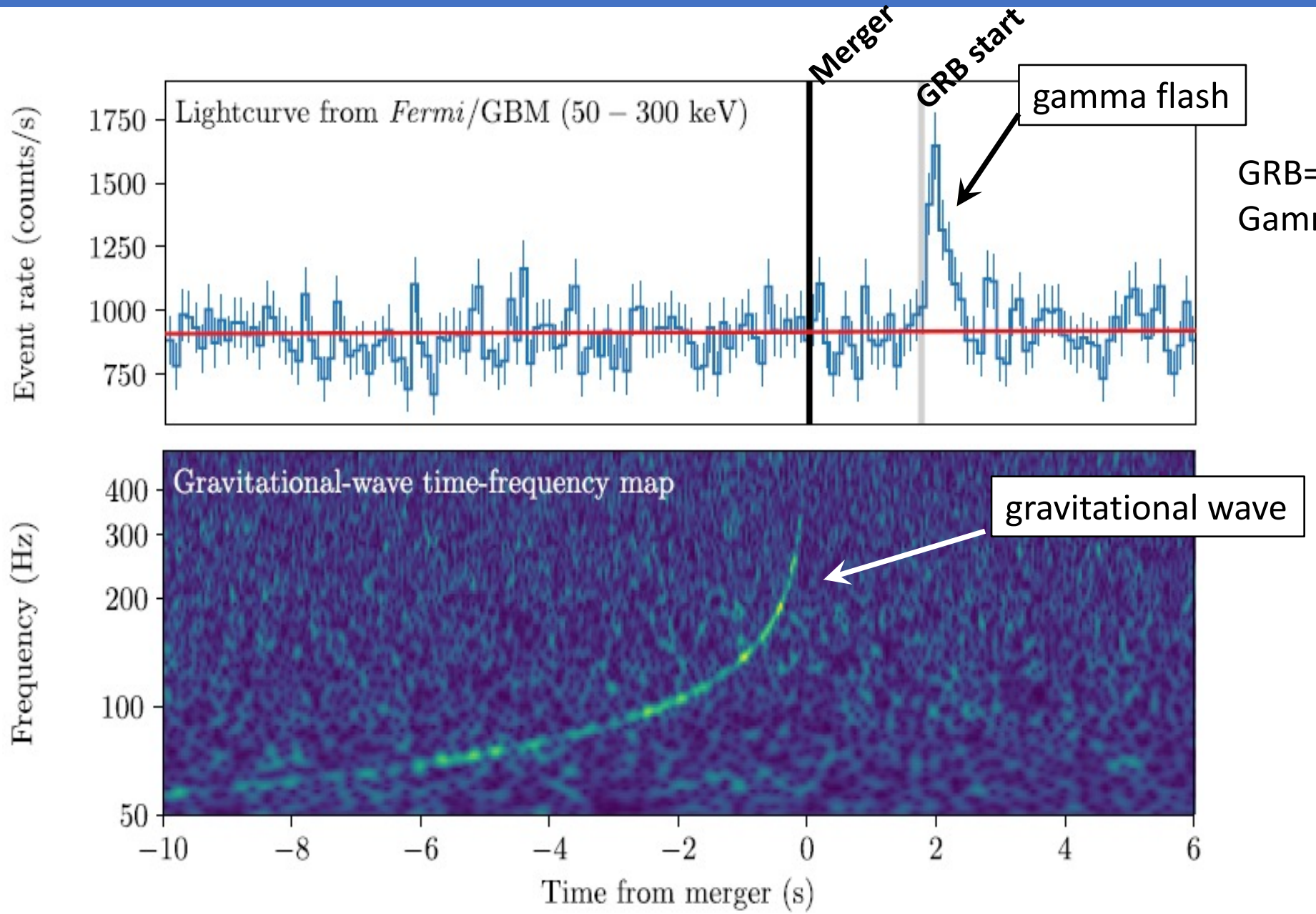


Coalescing Neutron Stars: “Multimessenger” Astronomy




Gamma flash 1.7 sec later…
INTEGRAL
Fermi Space Telescope

“Multimessenger” Astronomy

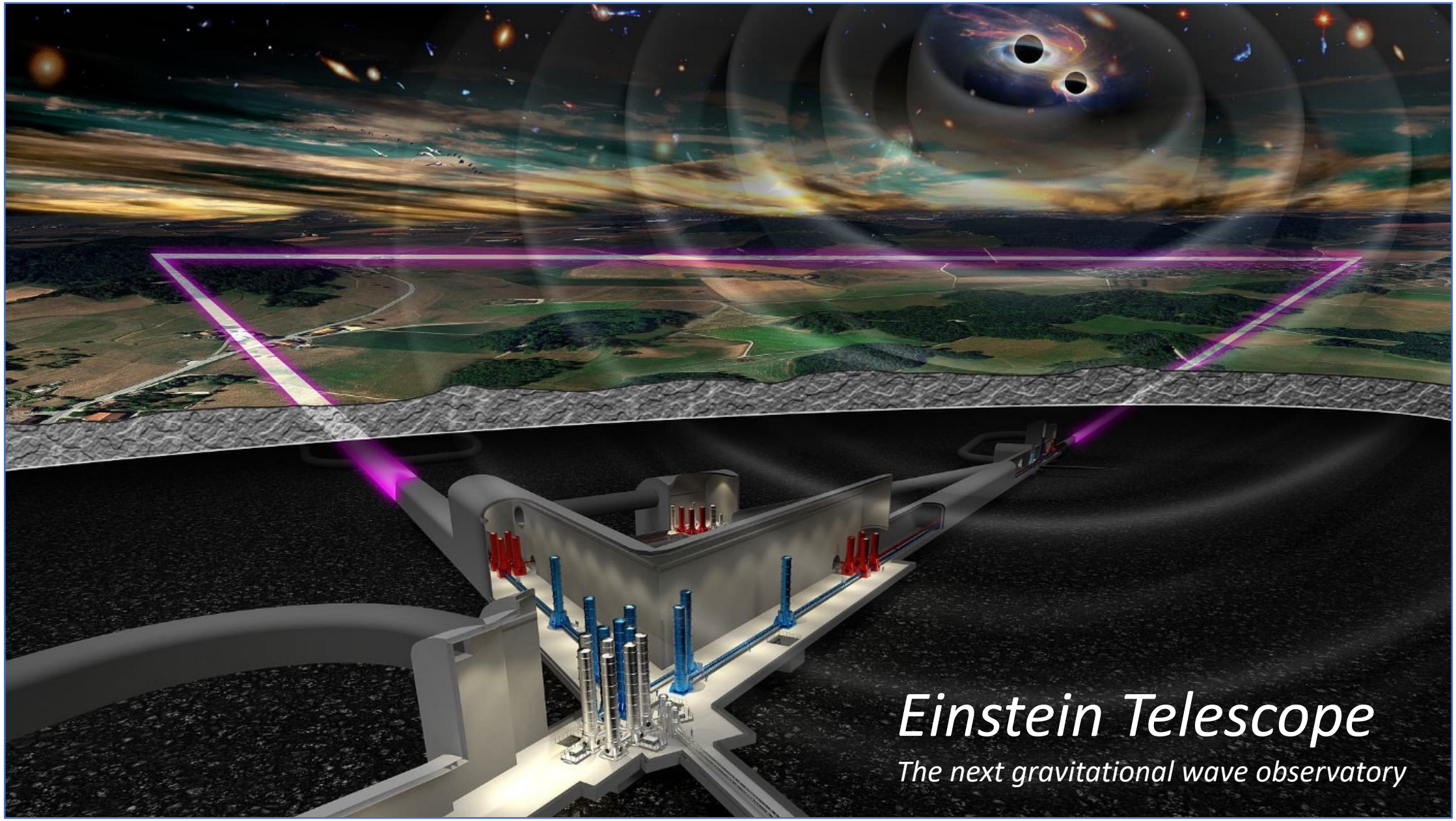


GRB=
Gamma Ray Burst

Neutron star merger: multimessenger

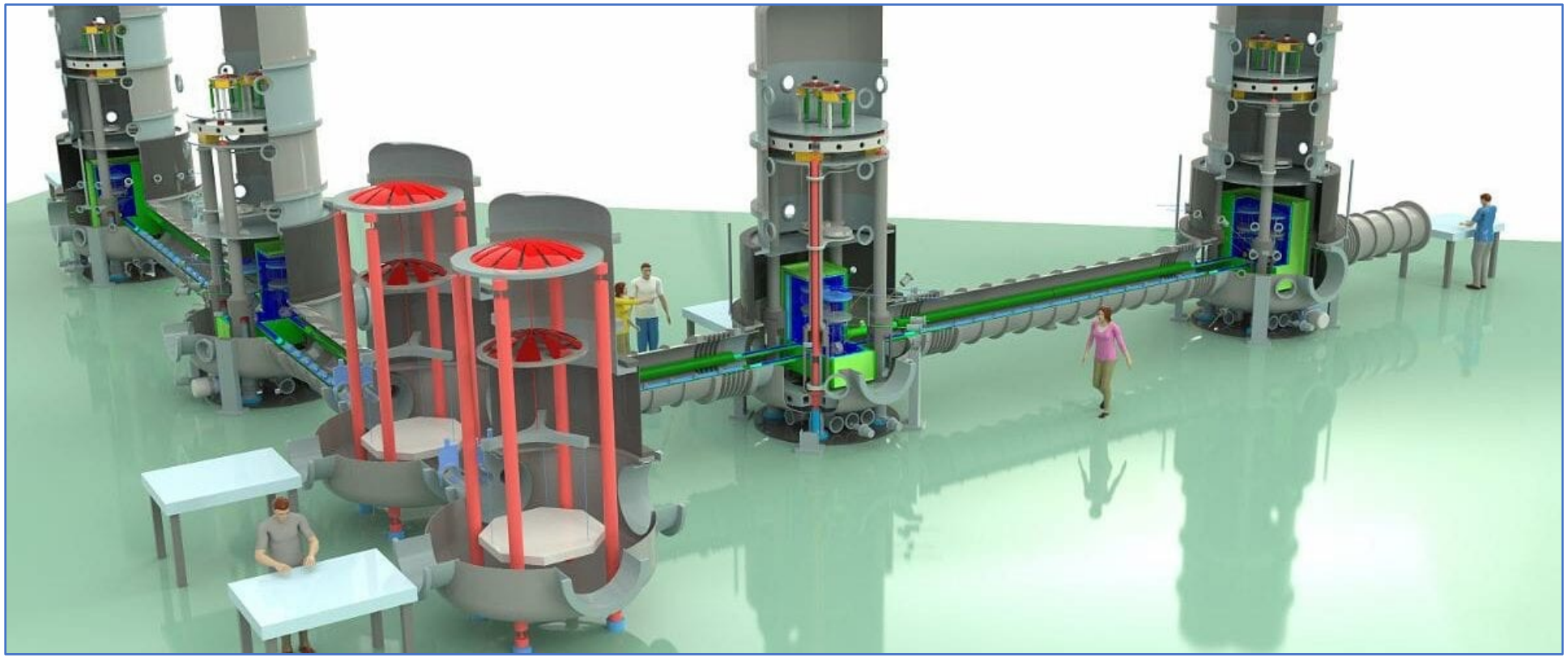
Kilonova: production heavy elements: gold, platinum etc.

Possible Future Facility…



ET Pathfinder in Maastricht

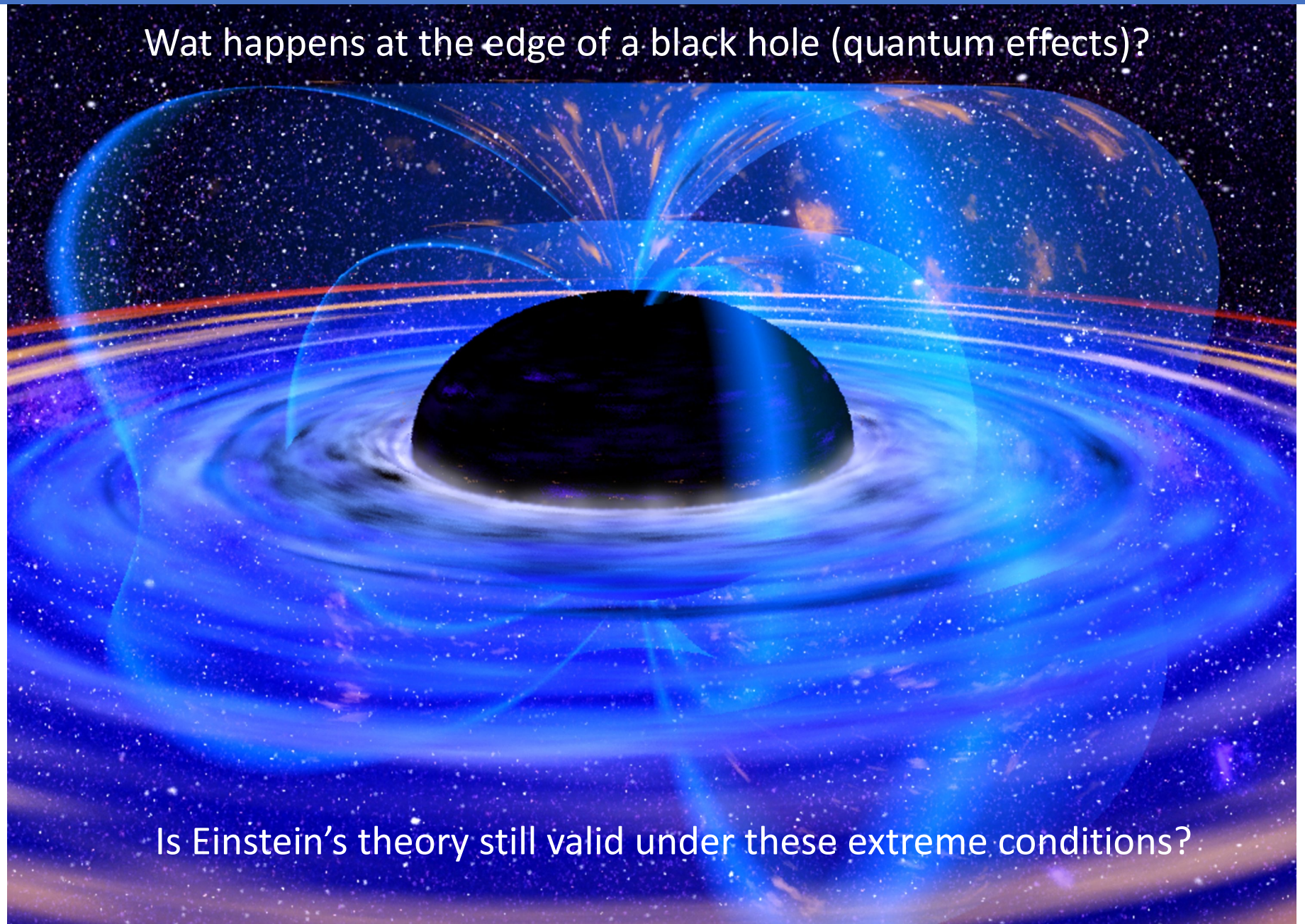
ET Pathfinder



Nikhef, RWTH Aachen, UCL Louvain, Hasselt, Ghent, Antwerp, VUB Brussels, ULB Brussels, Liege, Radboud University Nijmegen, TU Eindhoven and Hamburg

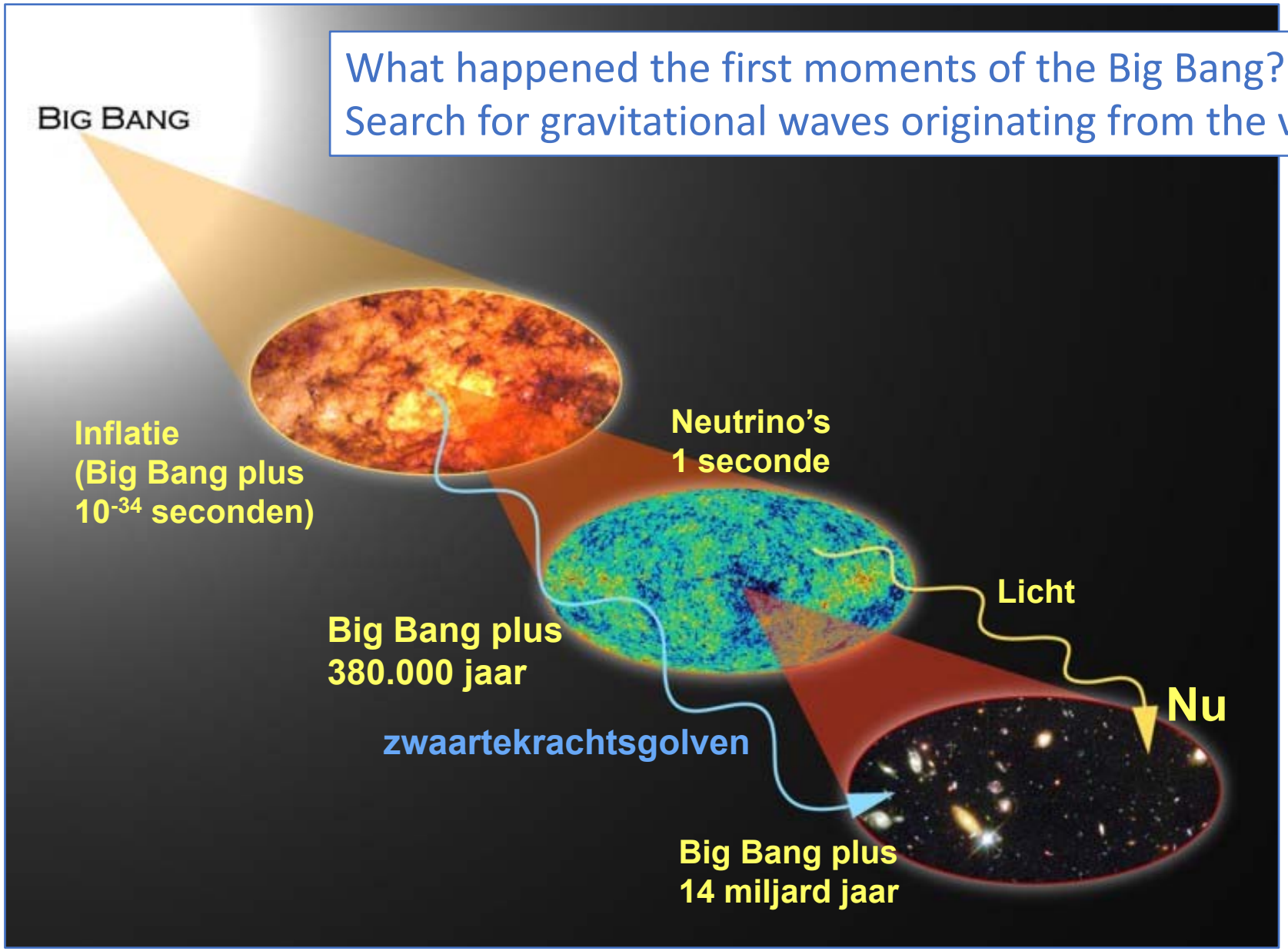
ET Pathfinder

Fundamental Black hole physics

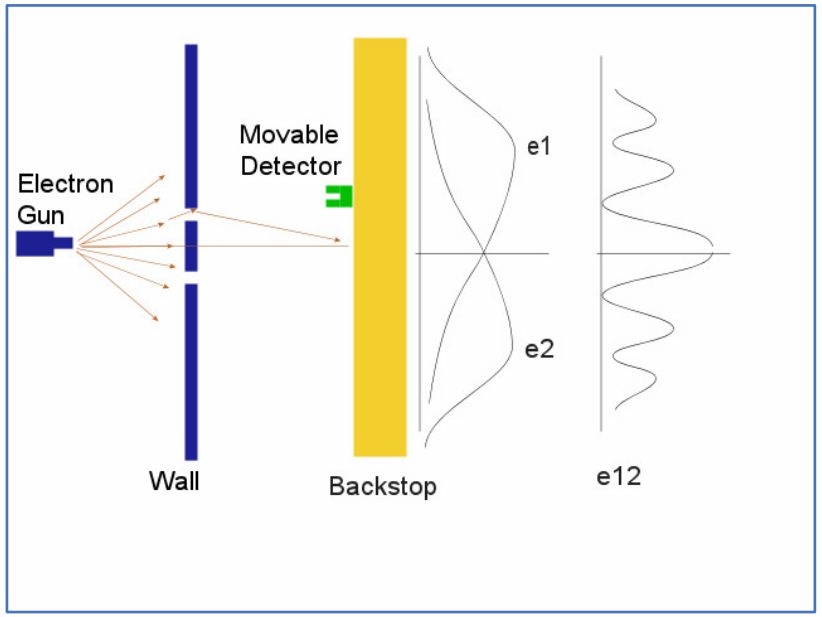


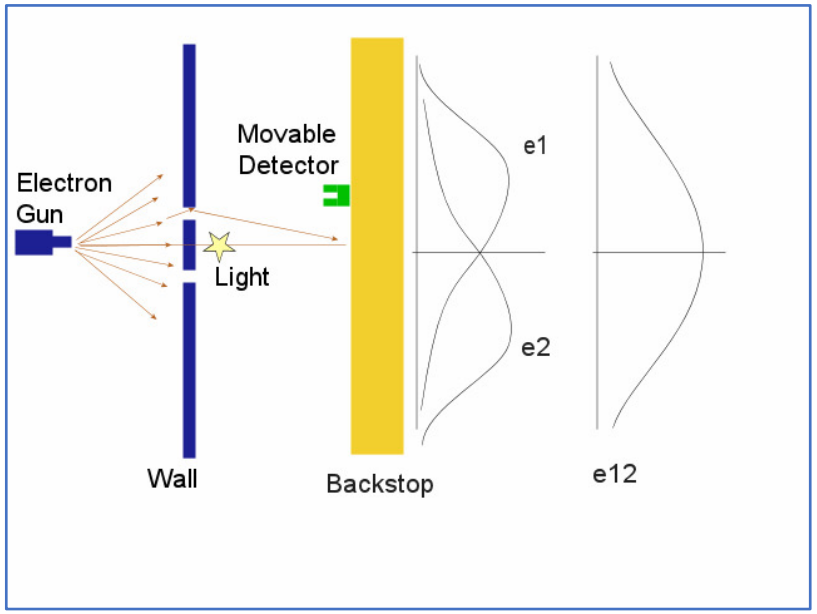
Looking into the Big Bang

What happened the first moments of the Big Bang?
Search for gravitational waves originating from the very first moment!



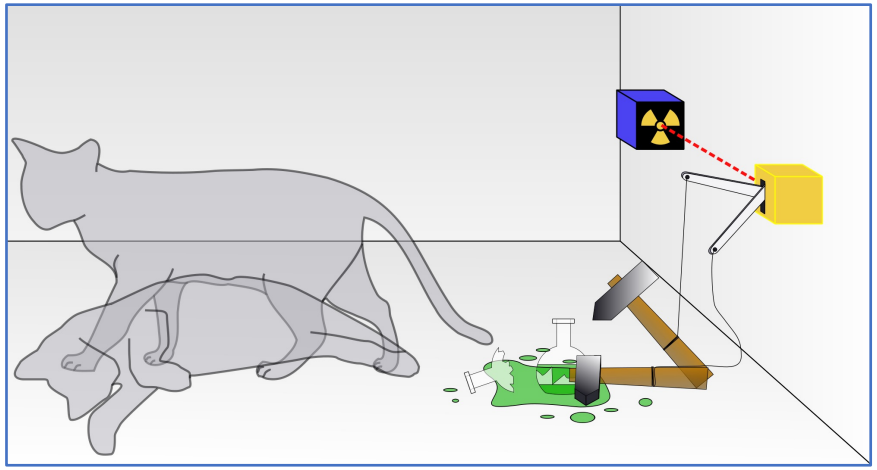
Next week: Quantum Mechanics





Quantum mechanics developed by Bohr and Heisenberg leads to ”absurd” thought experiments of Feynman and Wheeler. Einstein and Schrödinger did not like it.

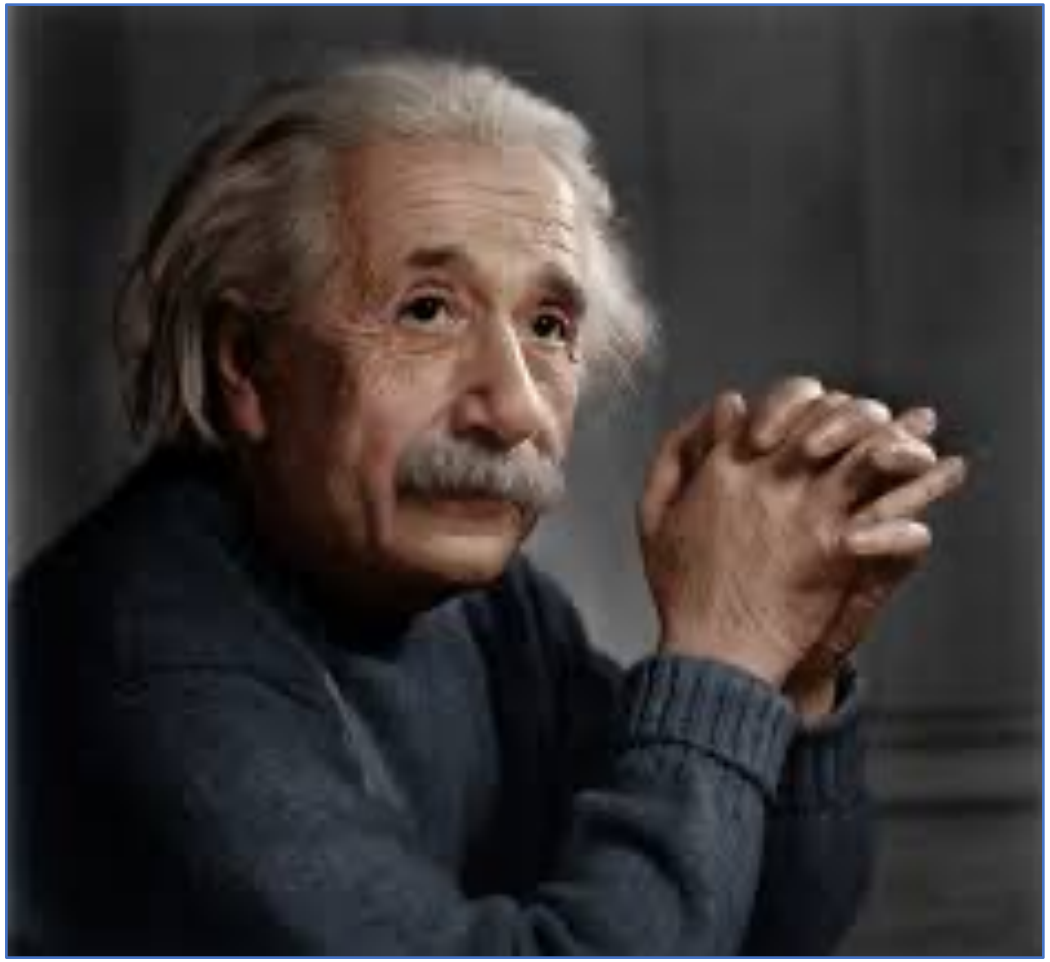
Even today people are debating its interpretation….



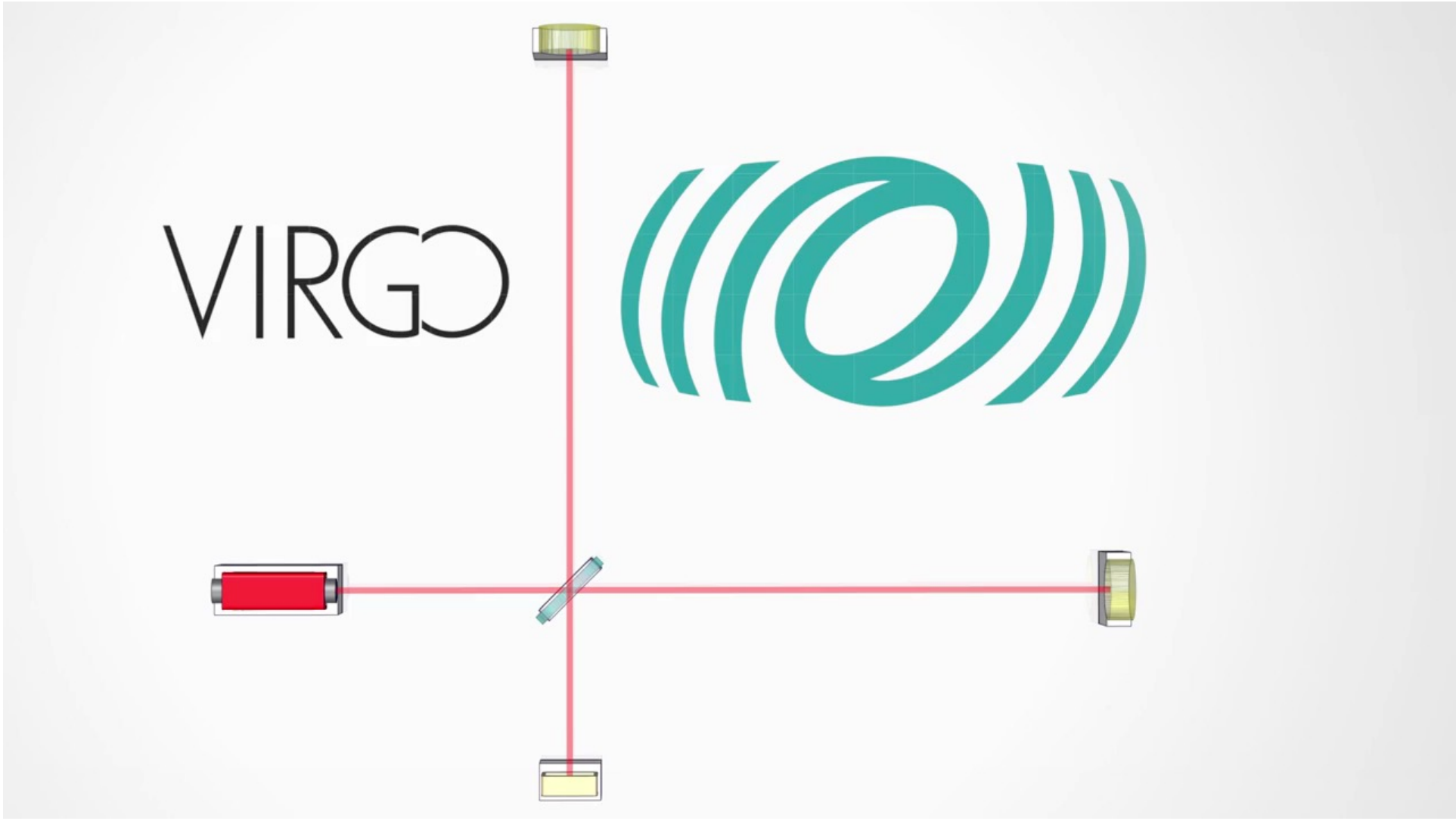
Extra Slides

Einstein Quotes

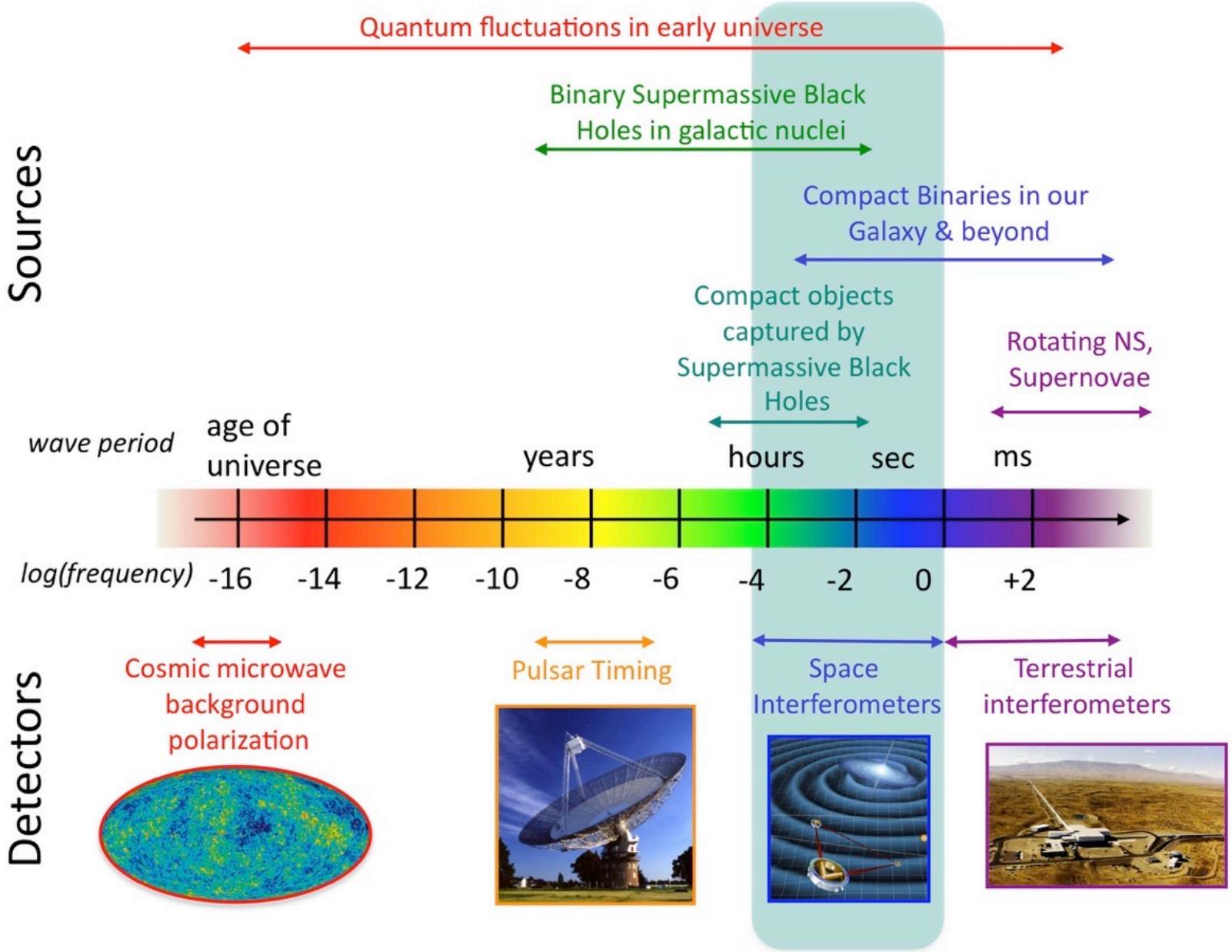
- “Imagination is more important than knowledge”
- “Education is what remains after one has forgotten what one has learned at school.”
- “I fear the day that technology will surpass our human interaction. The world will have a generation of idiots.”
- “A person who never made a mistake never tried anything new.”



Gravitation Wave Detection Method (in Dutch)



The Gravitational Wave Spectrum



Nobel Prize in Physics (Oct 6, 2020) – Black Holes

½ : Roger Penrose

For the discovery that black hole formation is a robust prediction of the general theory of relativity.

¼ : Reinhard Genzel

¼ : Andrea Ghez

For the discovery of a supermassive compact object at the centre of our galaxy.

Center of our Milky Way Galaxy

Supermassive Black Hole in the center of our Galaxy