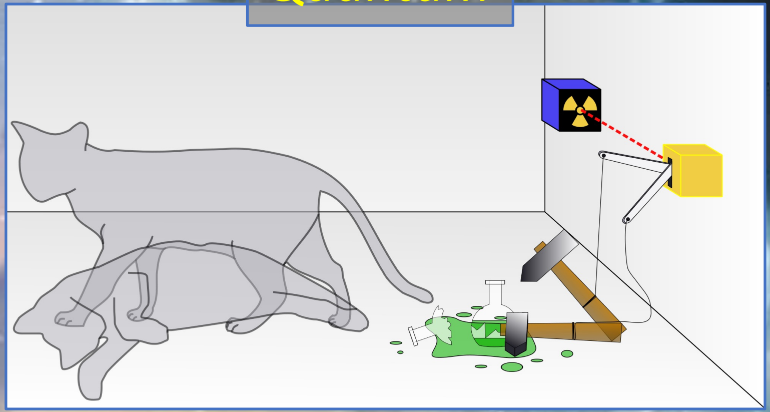
The Relativistic Quantum World

A lecture series on
Relativity Theory and Quantum Mechanics

Relativity

Quantum



Marcel Merk
Studium Generale Maastricht
Nov 1 – Nov 29, 2023

The Relativistic Quantum World

Relativity	Nov. 1:	Lecture 1: The Principle of Relativity and the Speed of Light Lecture 2: Time Dilation and Lorentz Contraction
	Nov. 8:	Lecture 3: The Lorentz Transformation and Paradoxes Lecture 4: General Relativity and Gravitational Waves
Quantum Mechanics	Nov. 15:	Lecture 5: The Early Quantum Theory Lecture 6: Feynman's Double Slit Experiment
	Nov 22:	Lecture 7: Wheeler's Delayed Choice and Schrodinger's Cat Lecture 8: Quantum Reality and the EPR Paradox
Standard Model	Nov. 29:	Lecture 9: The Standard Model and Antimatter Lecture 10: Why is there something rather than nothing?

Lecture notes, written for this course, are available: www.nikhef.nl/~i93/Teaching/
Prerequisite for the course: High school level physics & mathematics.

Lecture 2

Time Dilation and Lorentz Contraction

“I have no special talent. I am only passionately curious.”
- Albert Einstein

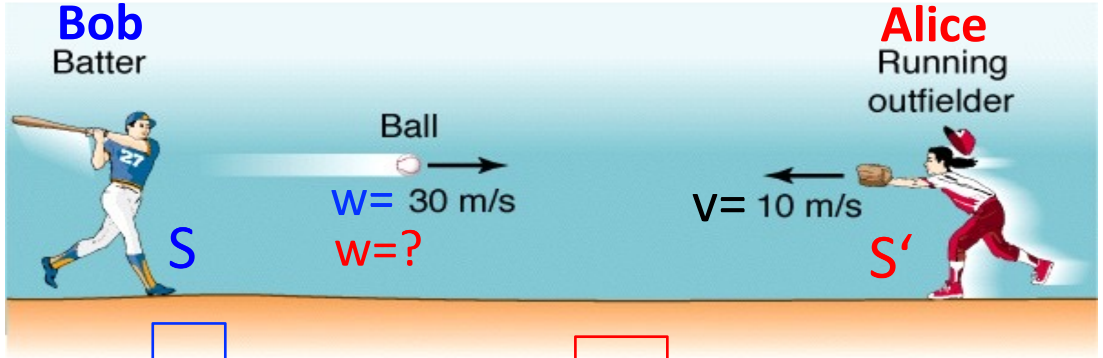
Coordinate Systems

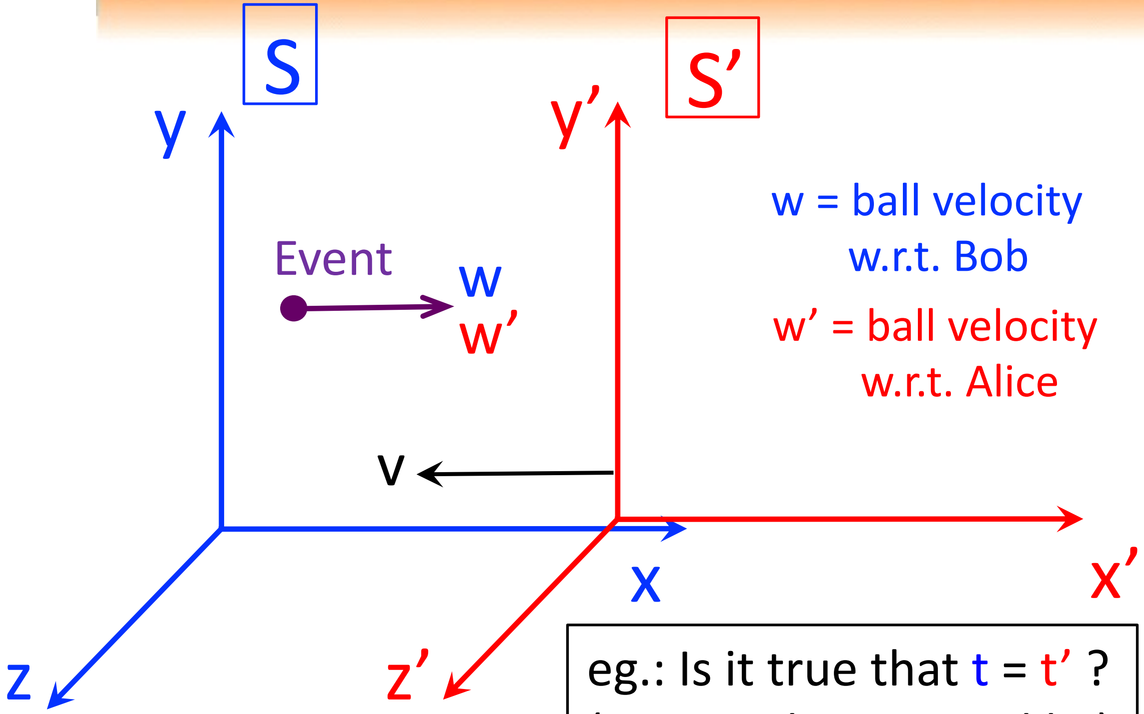
A ***reference system or coordinate system*** is used to determine the time and position of an event.

Reference system S is linked to observer Bob at position (x,y,z) = (0,0,0)
An *event* (batter hits the ball) is fully specified by giving its coordinates in time and space: (t, x, y, z)

Reference system S' is linked to observer Alice who moves with velocity "v" with respect to S of Bob.

How are the coordinates of the event of Bob (batter hits the ball) expressed in coordinates for Alice (t', x', y', z') (running outfielder) ?

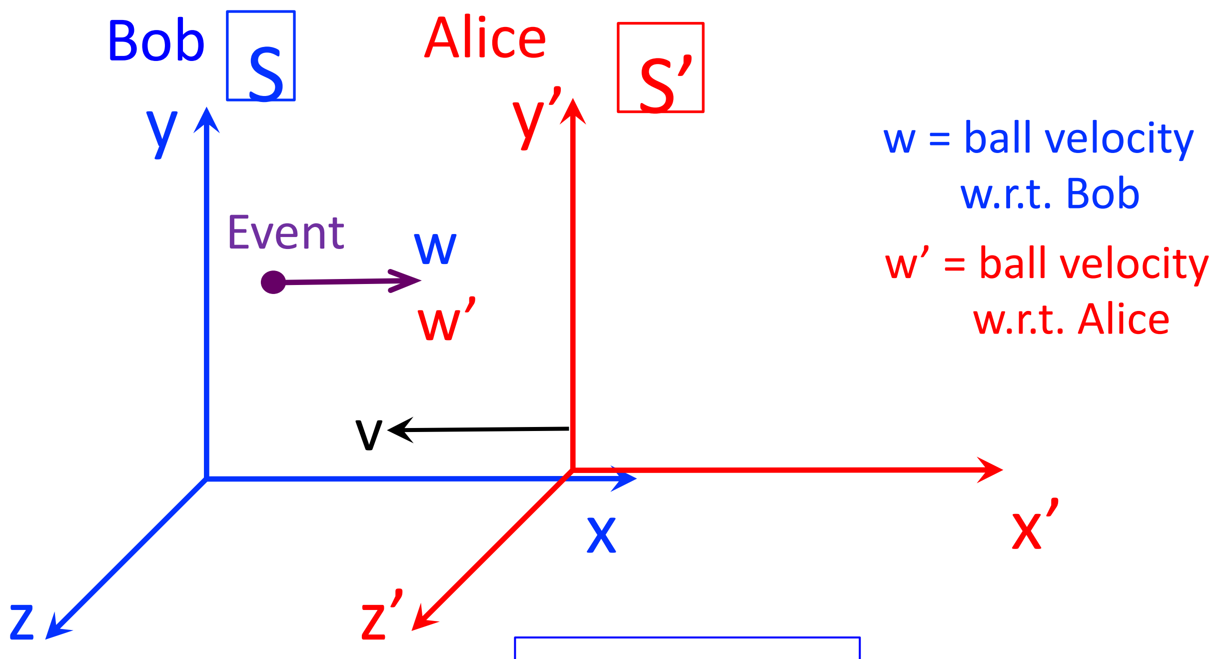




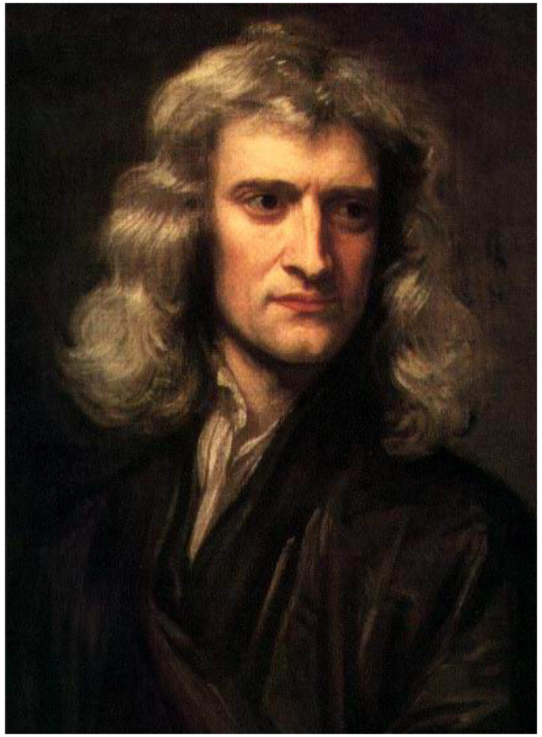
eg.: Is it true that t = t' ?
(universal time – Galilei)

How is the trajectory of the ball for Alice related to that for Bob?

Universality of Time



$$w' = w + v$$



Isaac Newton (1689)

Galileo Galilei (1636)

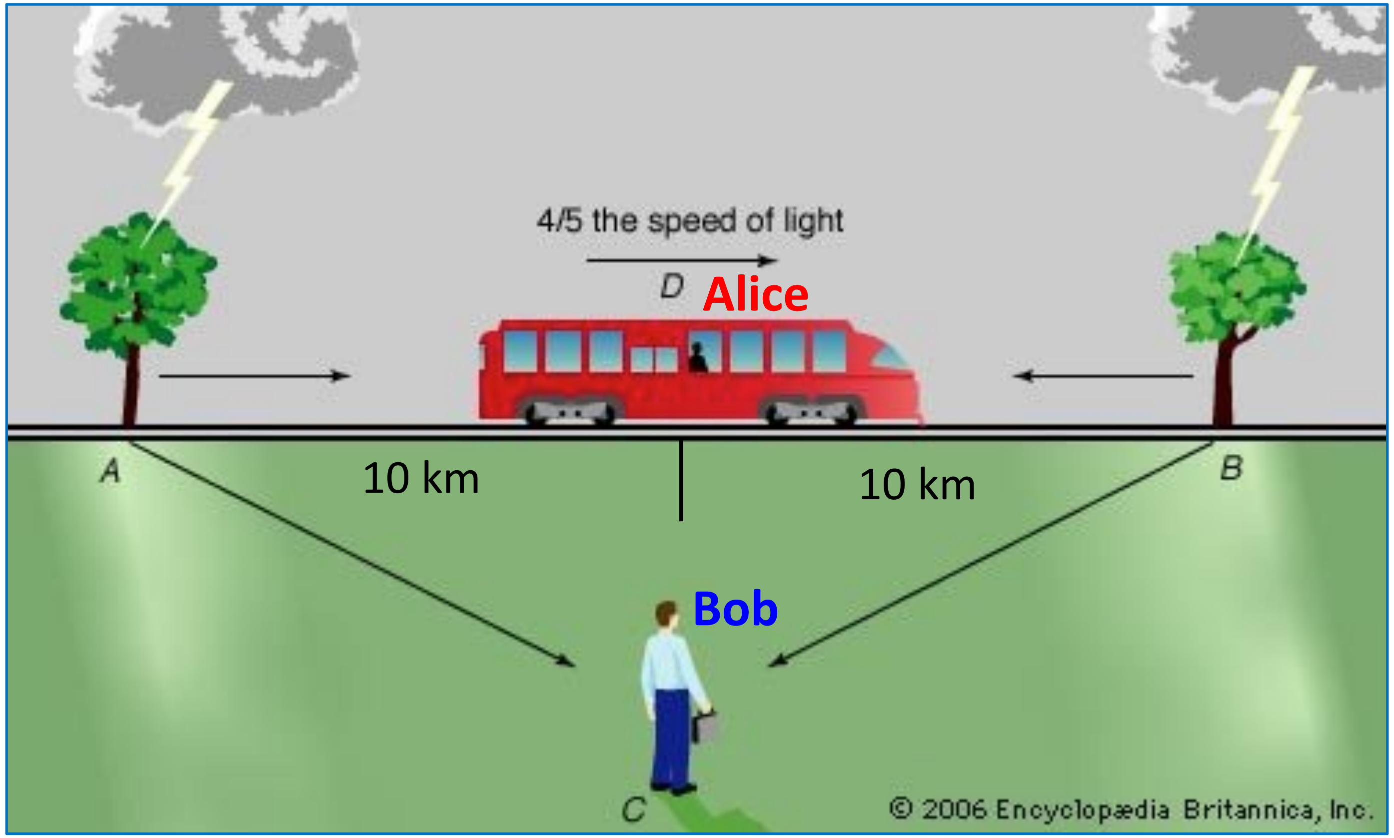
velocity = distance / time , but are *distance* and *time* the same for Bob and Alice?

"Classical" law of adding velocities assumes *time* is universal for all observers.

Let us first look at the concept of "simultaneity" in the eyes of Einstein.

Simultaneity of moving observers (“Gedankenexperiment”)

Bob sees two lightning strokes ***at the same time***. $AC = BC = 10$ km.
At the time of the lightning strike Alice passes Bob at position D. Also: $AD=BD=10$ km.



Alice sees the same events from the speeding train.

By the time the light has travelled 10 km, Alice moved a bit towards B and the light of B reaches her before A.

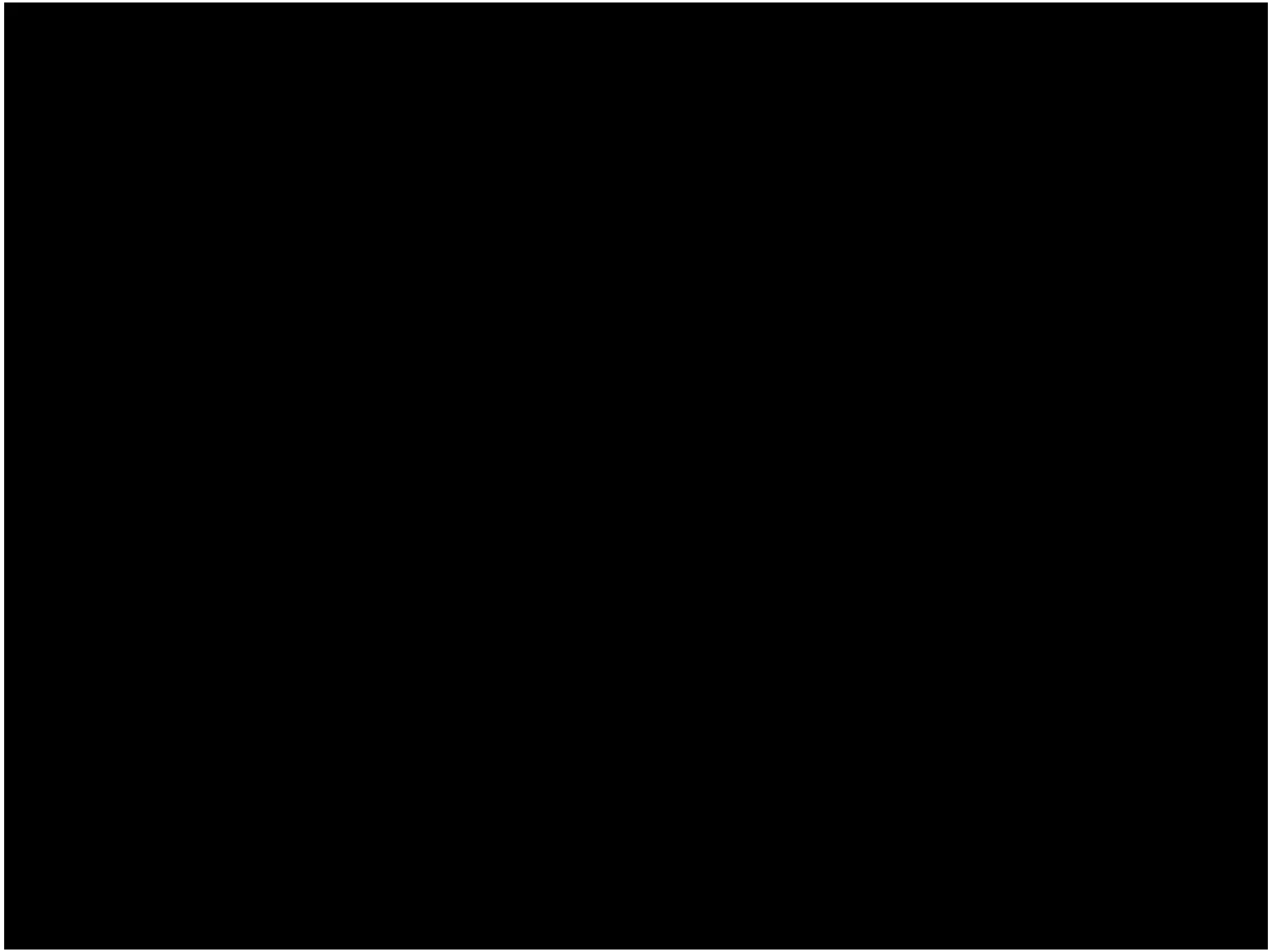
Since also for Alice, the speed of light from AD is the same as that of BD she will conclude that strike B happened ***before*** strike A.

Bob says two lightnings are **simultaneous**, Alice claims they are **not**. **Who is right?**

In case Bob and Alice travelling in empty space: who is moving and who is not?

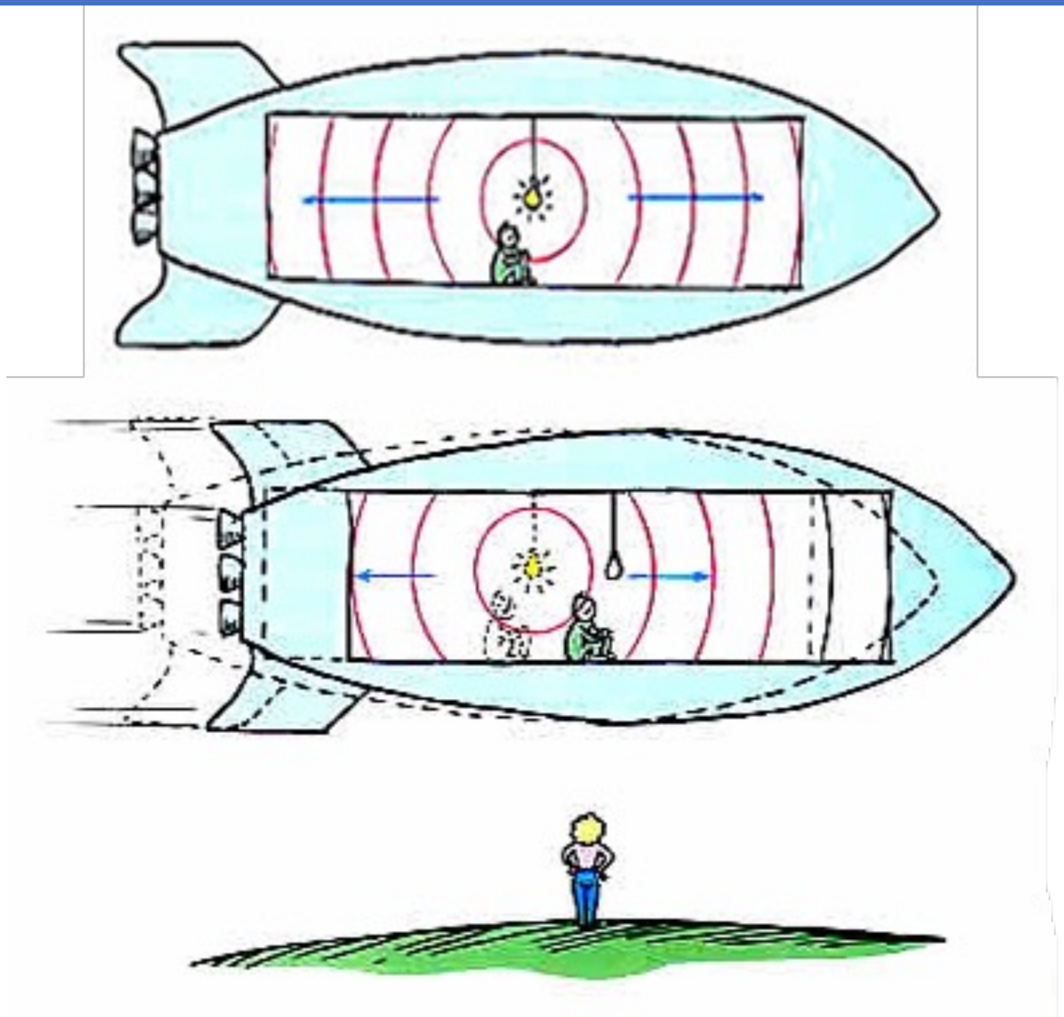
➔ Simultaneity of events depends on the speed of the observer!

Simultaneity of moving observers



Alternative Illustration

Alternative Illustration



View from **inside**

View from **outside**

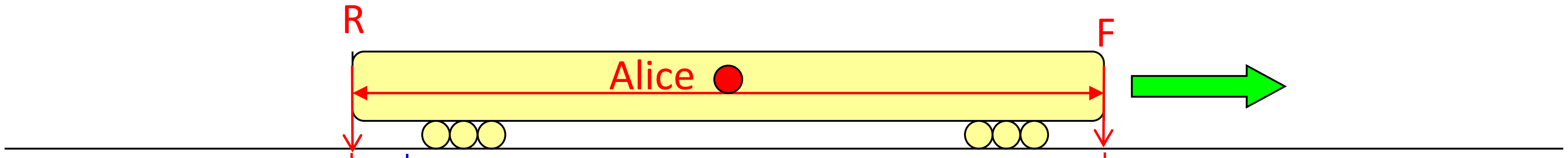
Inside the rocket the light reaches the front and back *simultaneous* independent of the rocket speed.
Seen from the ***outside*** this is different.
But, what is different if we let the rocket "stand still" and the earth move in the opposite direction?

Simultaneity depends on the velocity of the observer.
Time is not universal!

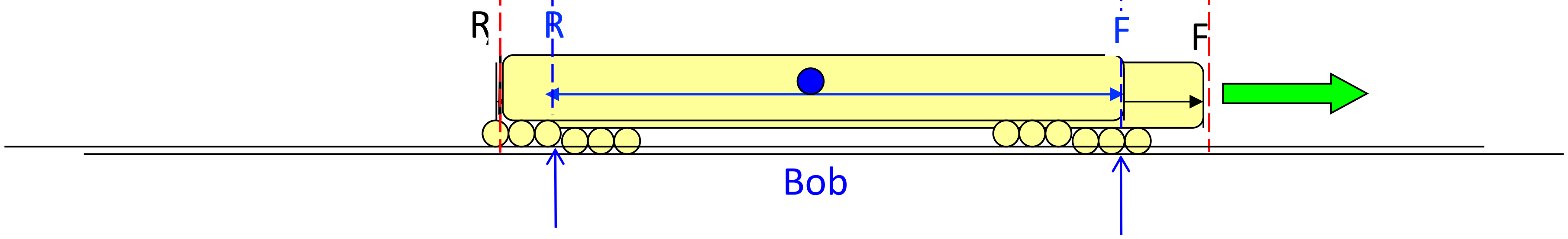
Alternative Explanation

Relativity of Length ("Gedankenexperiment")

Alice: measure the length of the train by setting *simultaneously* two tick marks at the track at position F (front) and R (rear)



Bob: measure the length of the train by setting *simultaneously* two tick marks at the track corresponding to the positions F and R.



Since Alice and Bob don't agree on the simultaneity of making the tick marks they will observe a different length. Alice will claim Bob puts tick mark at Front *too early* and rear *too late* such that he sees a *shorter* train: ***Lorentz contraction***.

Relativity of Length (“Gedankenexperiment”)

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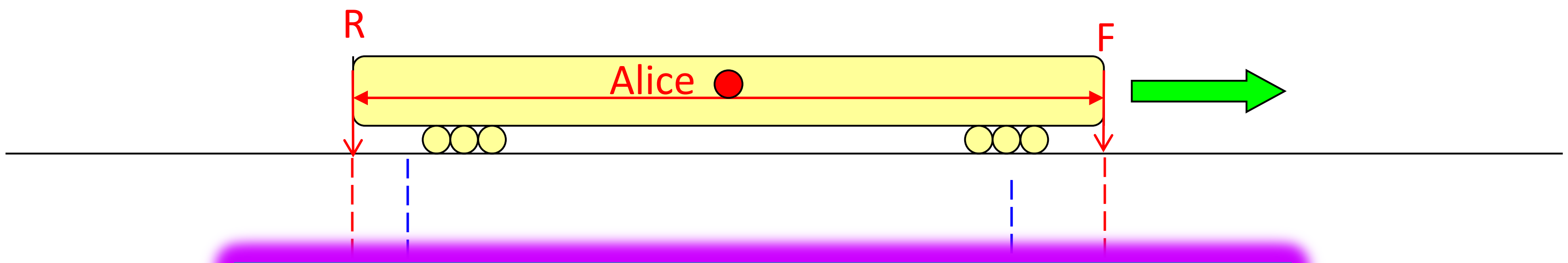
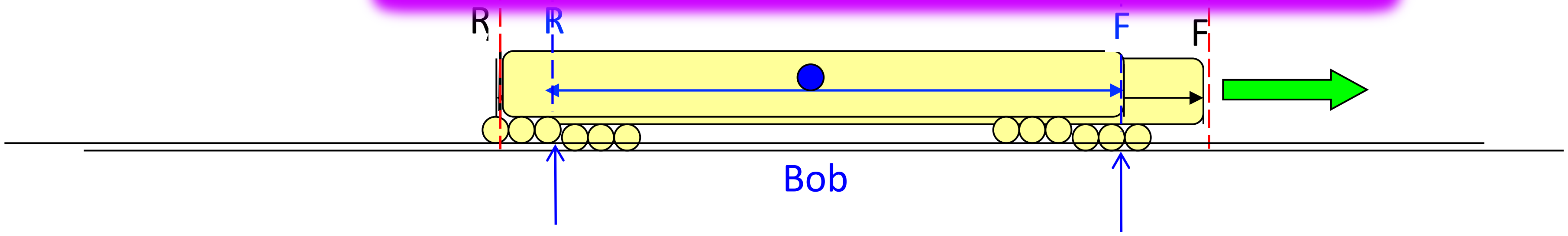


Bob: me … two
tick mar …

This is not a measurement artifact, but Due to the nature of space and time itself as it is experienced by different observers!



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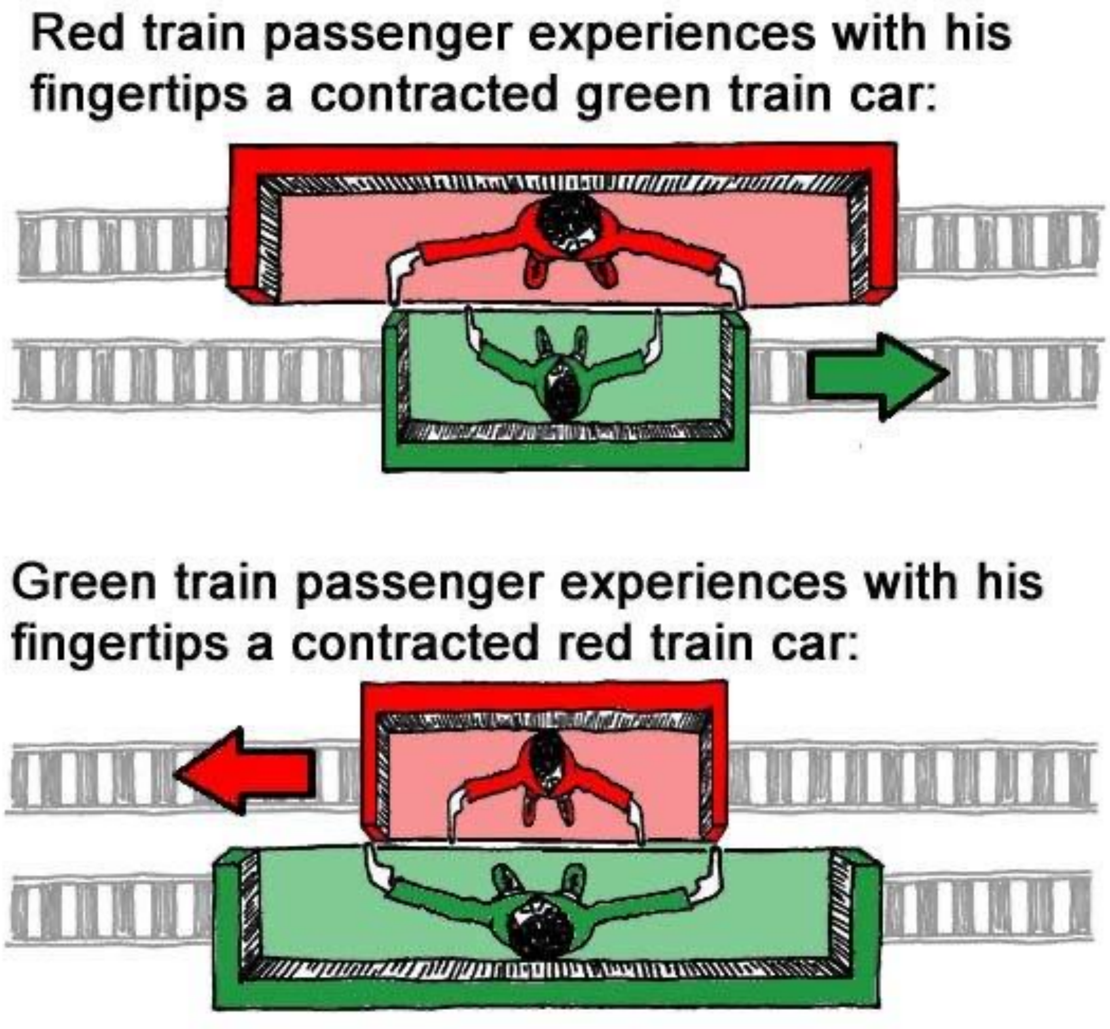
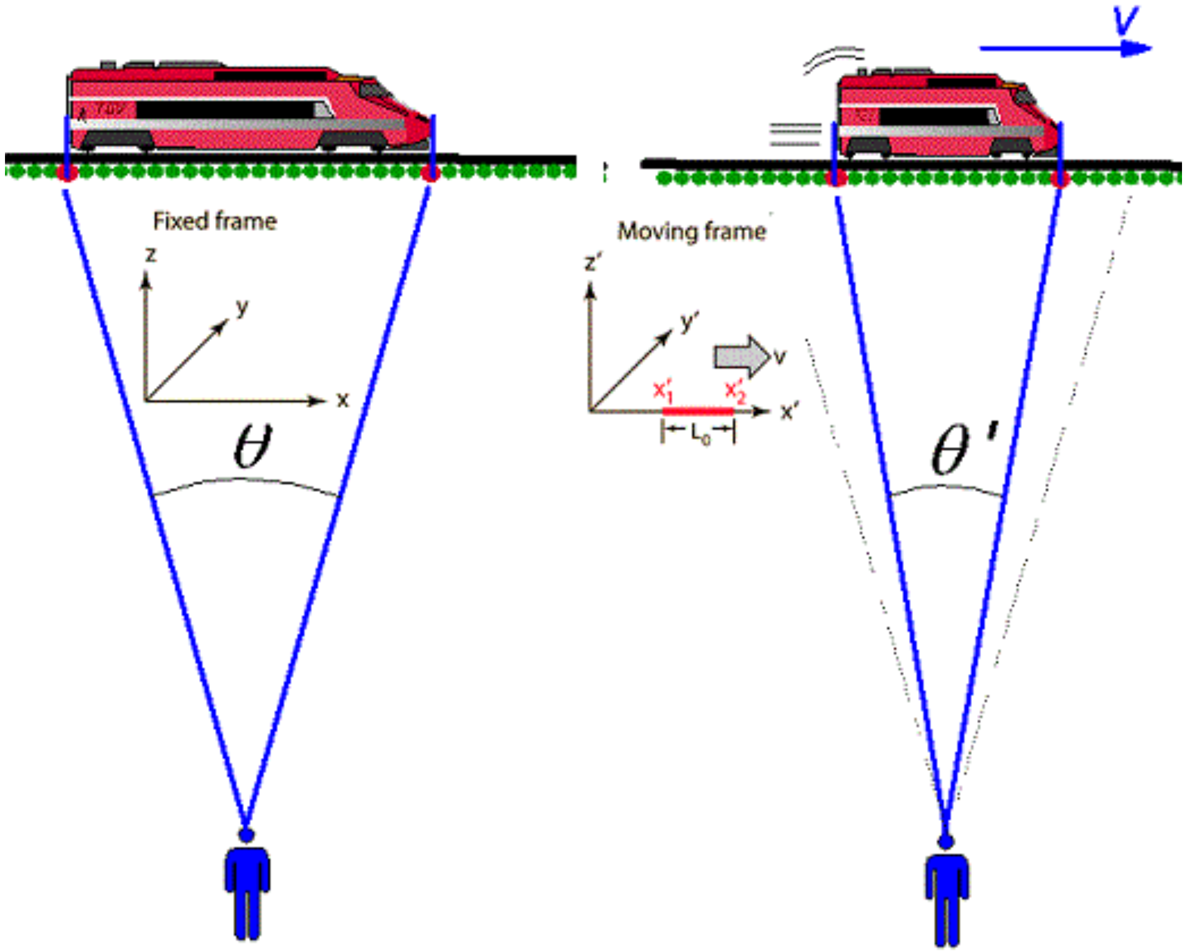
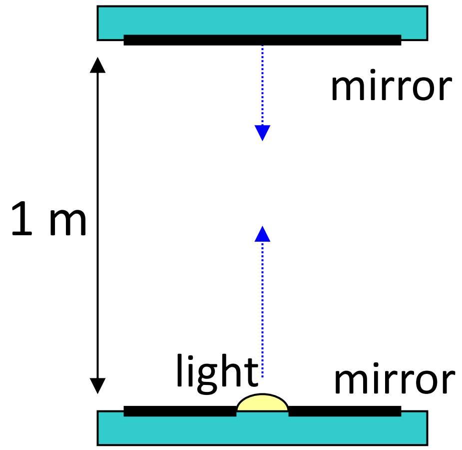
Lorentz Contaction & Relativity





Lorentz Contaction & Relativity

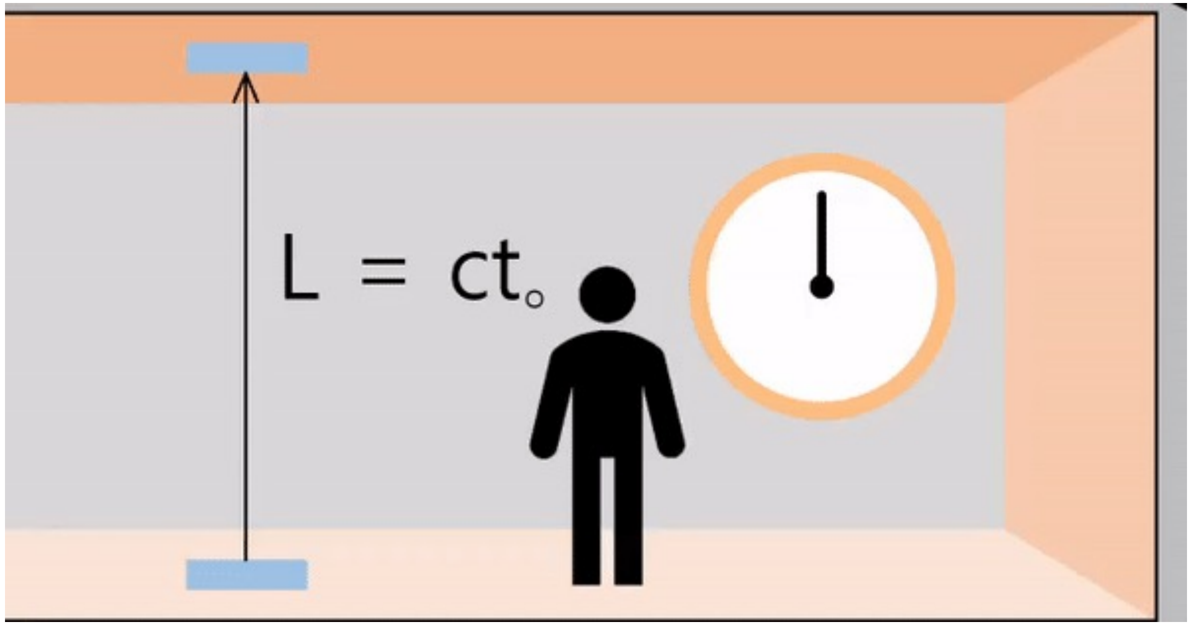
Perfect clock on a Relativistic Train



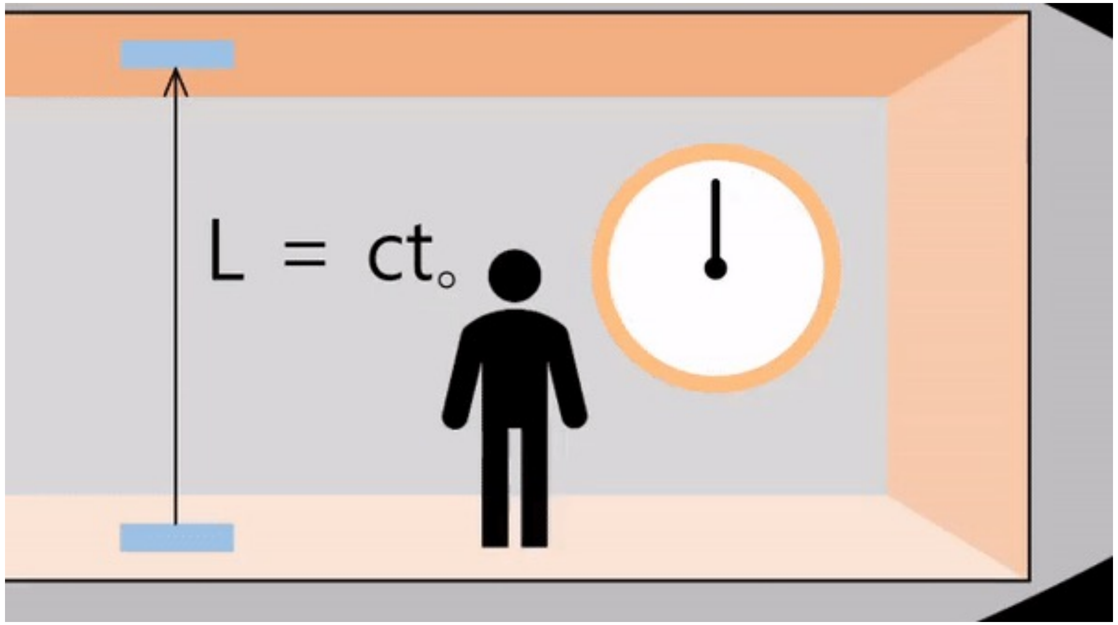
Light-clock: 300 million ticks per second.

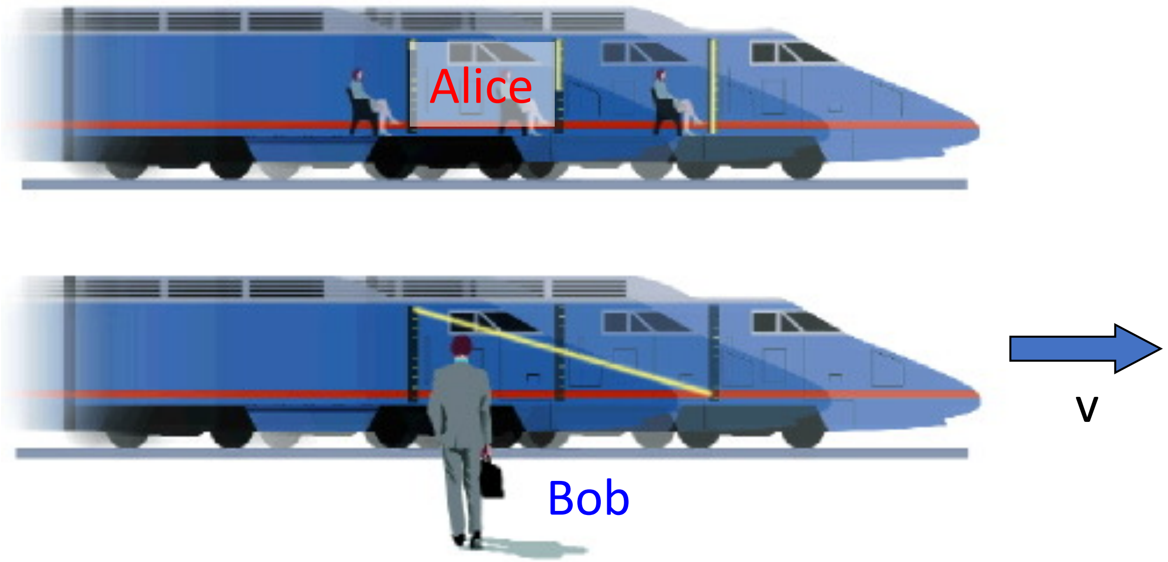
Perfect clock on a Relativistic Train

S: Alice in the train:



S’: Bob at the station:

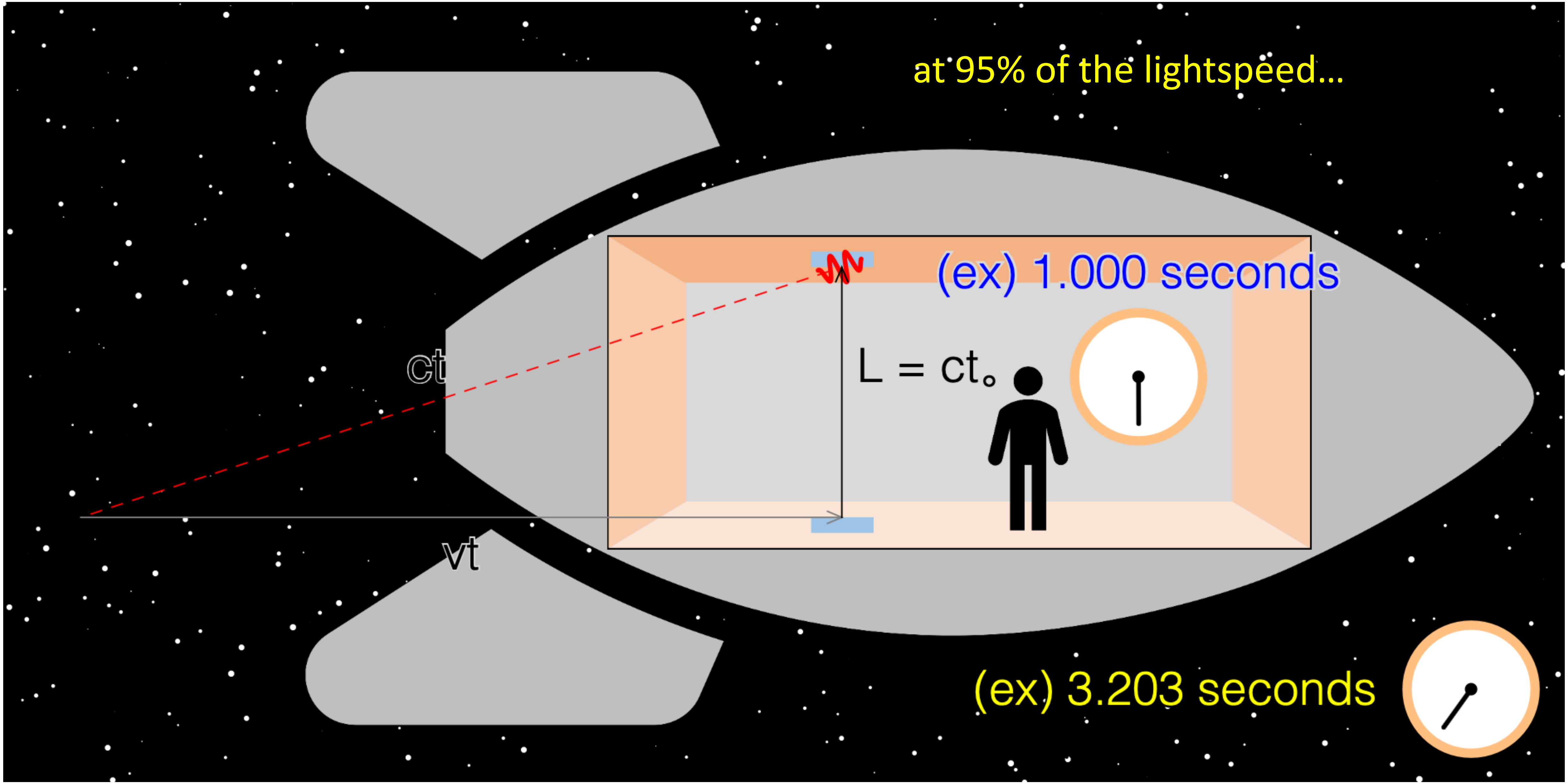




Bob sees that the ticks of Alice’s clock slow down!
Bob concludes that **time runs slower** for Alice than for himself:
Time Dilation!!!

$$\Delta t' = \frac{1}{\sqrt{1-(v^2/c^2)}} \cdot \Delta t \equiv \gamma \cdot \Delta t$$

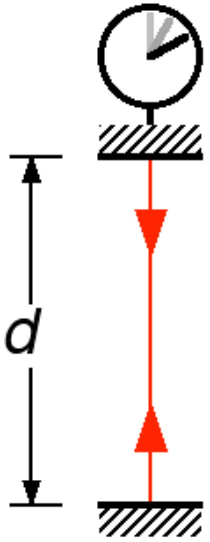
If you go very fast time slow-down is a very large effect

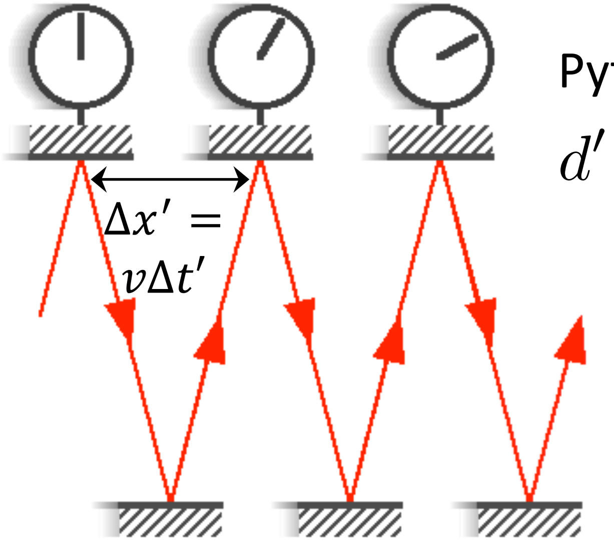


Time Dilation

S: Alice on the train:

S': Bob at the station:





$$\Delta t = d/c$$

$$\Delta t' = d'/c$$

Pythagorean theorem:

$$d' = \sqrt{d^2 + (v\,\Delta t')^2}$$

substitute

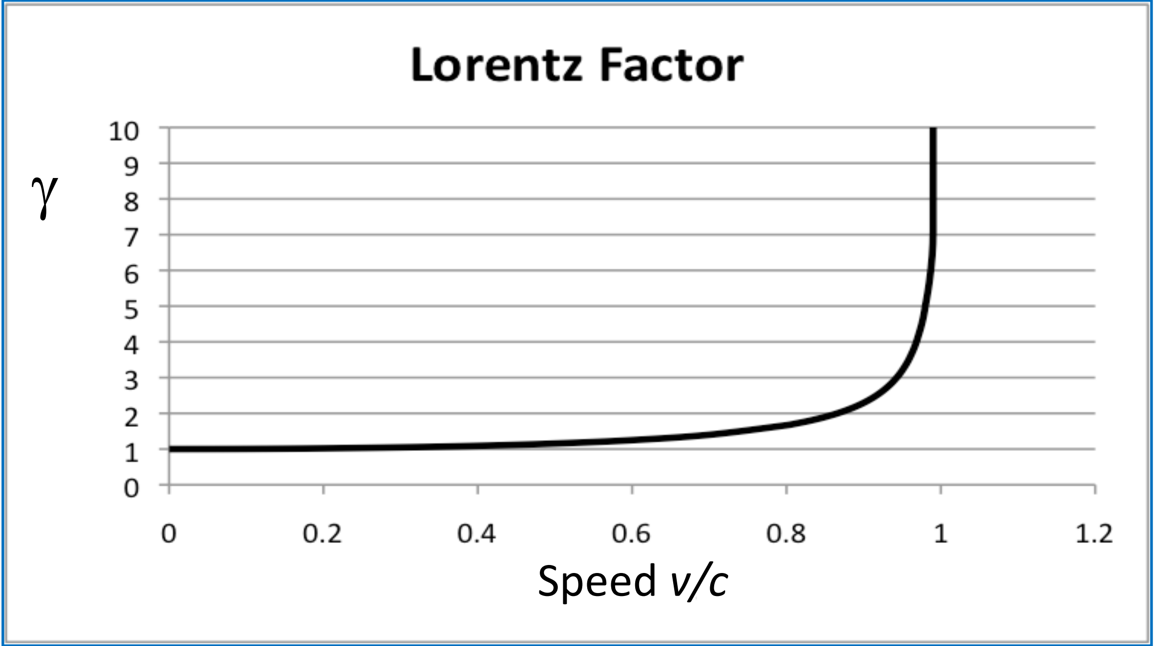
"juggling" with linear algebra

$$d'^2 = d^2 + (v\Delta t')^2$$
$$(c\Delta t'^2) = (c\Delta t)^2 + (v\Delta t')^2$$
$$(c^2 - v^2)\,(\Delta t')^2 = c^2\,(\Delta t)^2$$
$$\Delta t'^2 = \frac{c^2}{c^2 - v^2}(\Delta t)^2 = \frac{1}{1 - v^2/c^2}(\Delta t)^2$$

$$\Delta t' = \frac{1}{\sqrt{1 - (v^2/c^2)}} \cdot \Delta t \equiv \gamma \cdot \Delta t$$

γ is called the **time dilation** factor or Lorentz factor.

Time Dilation



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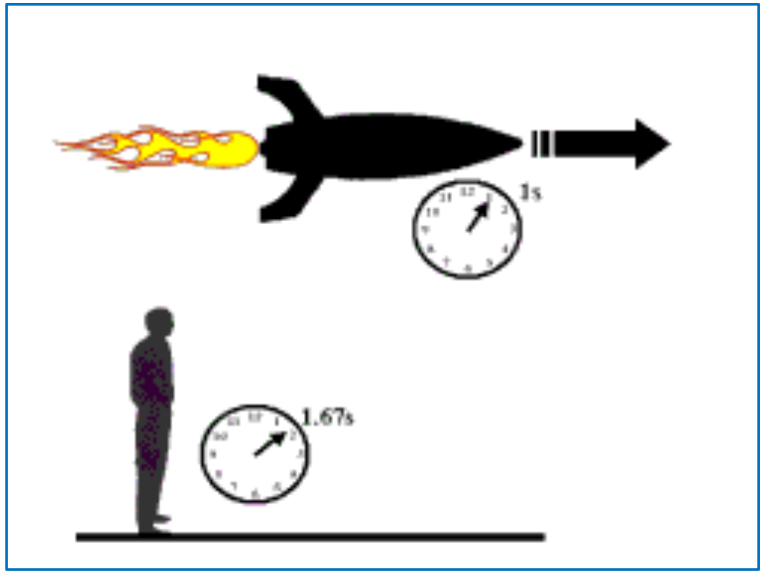


For “low” (***v<<c***) relative speeds: ***no effect***, time stays the same. ***This we know from every day life.***

For “high” (***v→c***) relative speeds: ***large effect***, time runs very slow! ***This we have never really seen in every day life.***

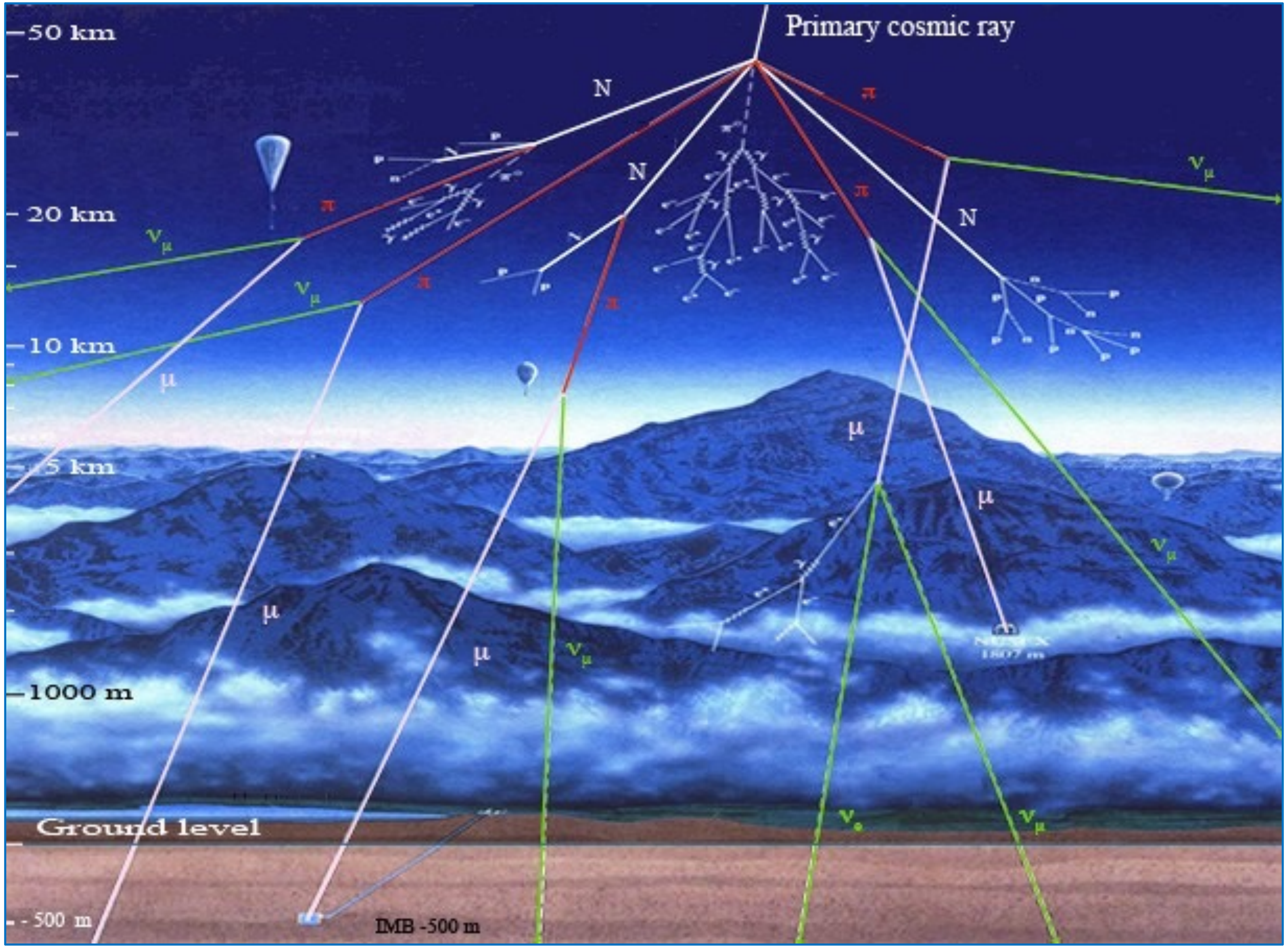
Example:
Rocket goes at
v=0.8 c = 4/5 c : $\gamma = \frac{1}{\sqrt{1-(\frac{4}{5})^2}} = \frac{1}{\sqrt{\frac{9}{25}}} = \frac{5}{3}$

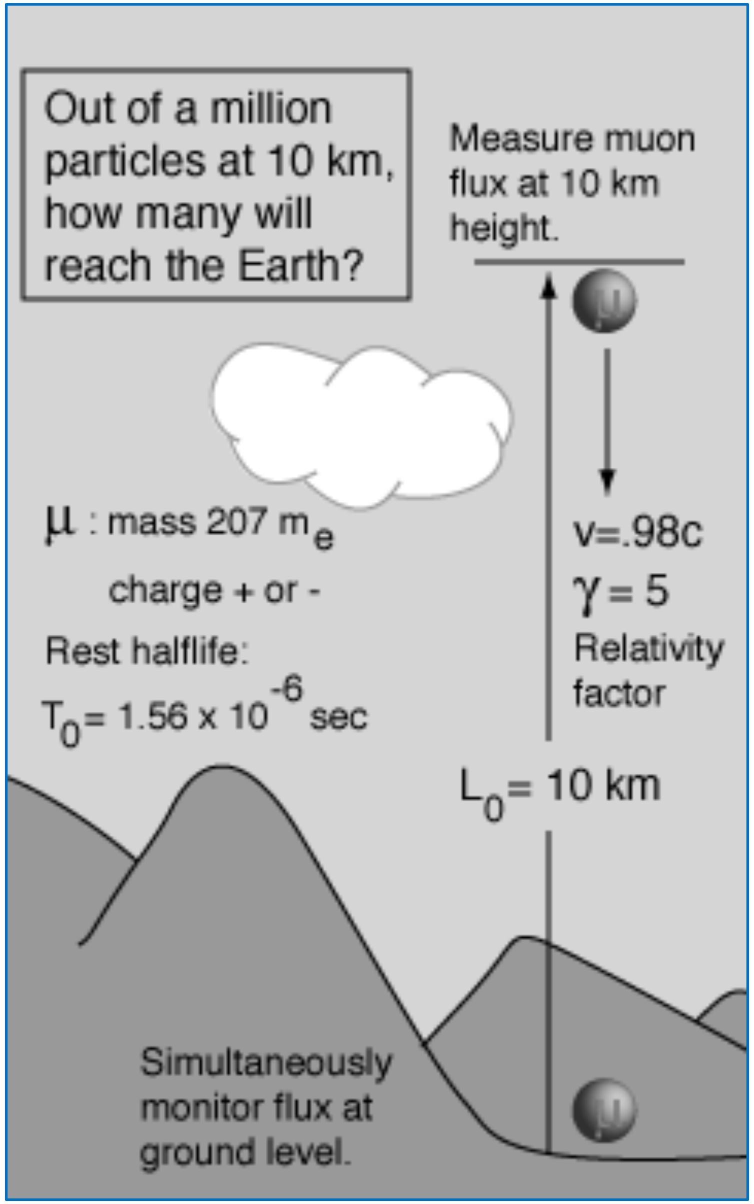
1 second inside the rocket lasts 1.66 seconds on earth.

Einstein and Relativity

“To you it was fast”

Time Dilation: Is it real? Cosmic Muons! (Real Experiment)





Time Dilation: Is it real? Cosmic Muons! (Real Experiment)

Muons: unstable particles with a decay life-time of: t=1.56 μs = 0.00000156 s

(After 1.56 μs 50% survive, after 2x1.56 μs 25%, …etc.: $½^n$)

Muon particles are produced at 10 km height (by cosmic rays) with ~98% light-speed.

Expectation: even at light-speed it would take them a time:

t = 10 km / 300 000 km/s = 33 μs

to reach the ground = 21 x half-life time: $½^{21}$ ~ 1 / 1 000 000

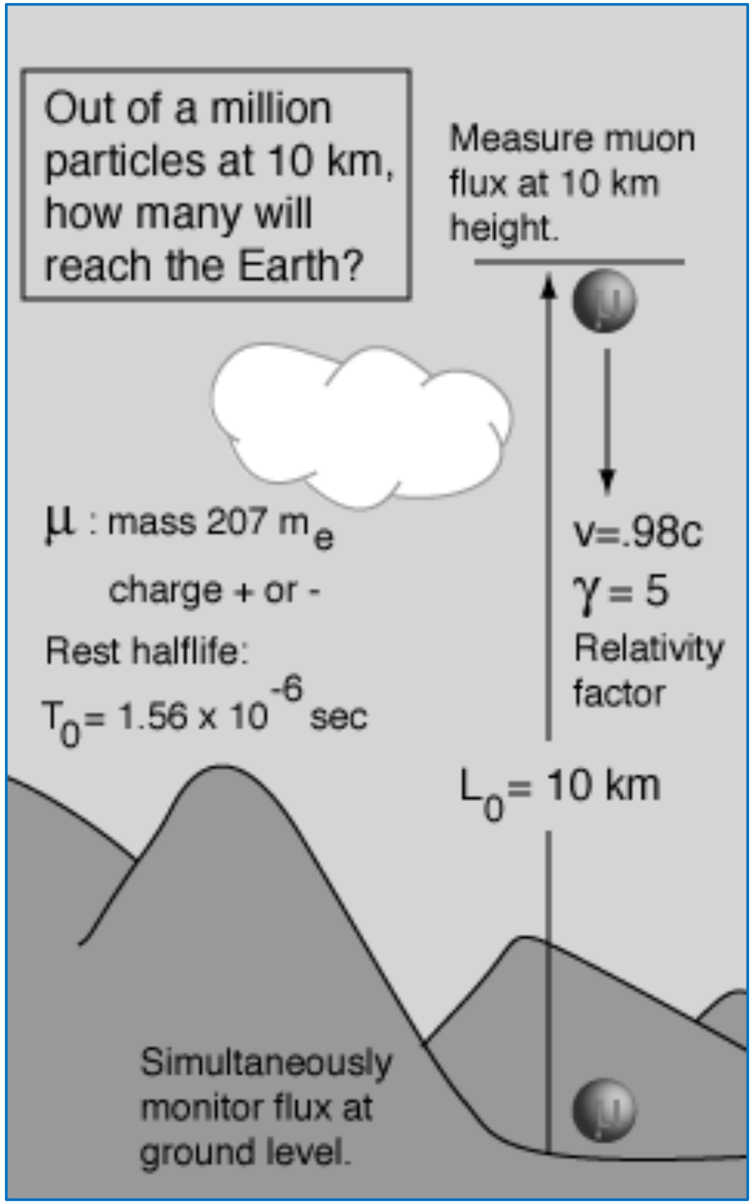
Expect: only 1 in a million muons arrive on the ground!

Measurement: ~ 5% makes it to the ground!

Relativity: $\gamma = 1/\sqrt{1-0.98^2} \approx 5$

→ Lifetime = 5x1.56 μs = 7.8 μs ; Takes 33/7.8 = 4.2 half-lives!

Consistent with observation! Since: $½^{(4.2)}$ = 0.05 →5%



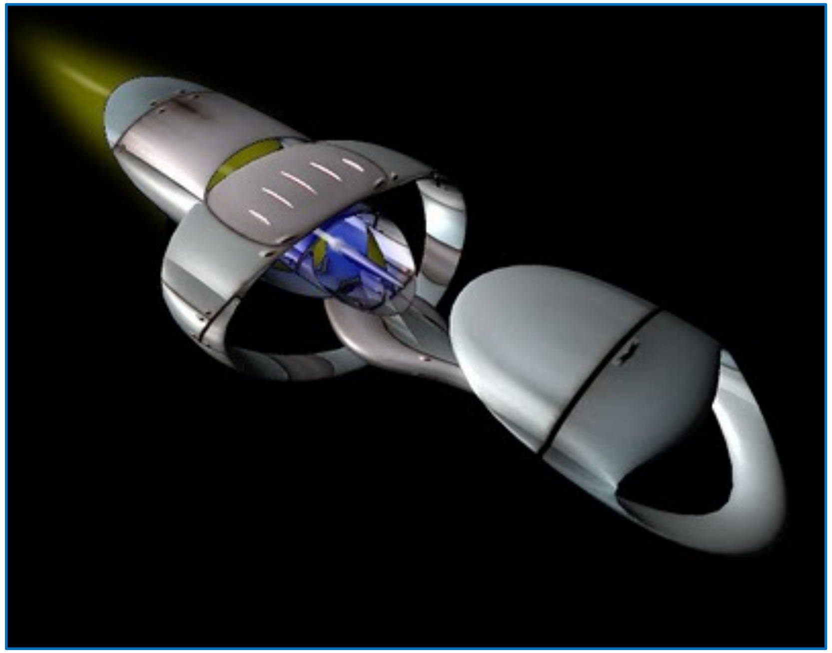
➔ Also in GPS navigation devices relativity is essential!

Lorentz Contraction (Gedankenexperiment)

Alice boards a super spaceship with her clock and travels with v=0.8c to a distant star (L = 8 light-years).

From earth, Bob calculates that the trip takes about ***10 years***, since: L = v t → t = L / v → t = 8 / 0.8 = 10

Bob calculates that since Alice's clock runs slower, for her the trip takes ***6 years***, since $\gamma = 1 / \sqrt{(1-0.8^2)} = 5/3$
and L = v t = v γ t' → t' = t / γ = 6



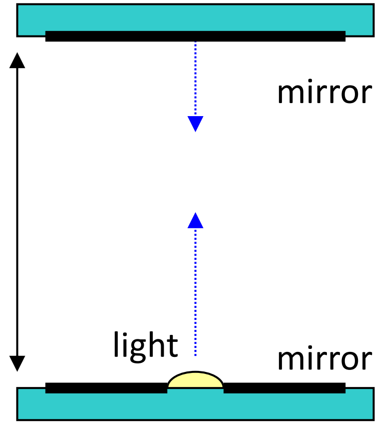
Since:

1) Alice and Bob agree on the velocity v
2) Alice and Bob agree on the number of clock ticks
3) For Alice a clock tick does not change, so the trip takes indeed 6 years

Alice observes: L' = v t' = 0.8 x 6 = 4.8 light-years!

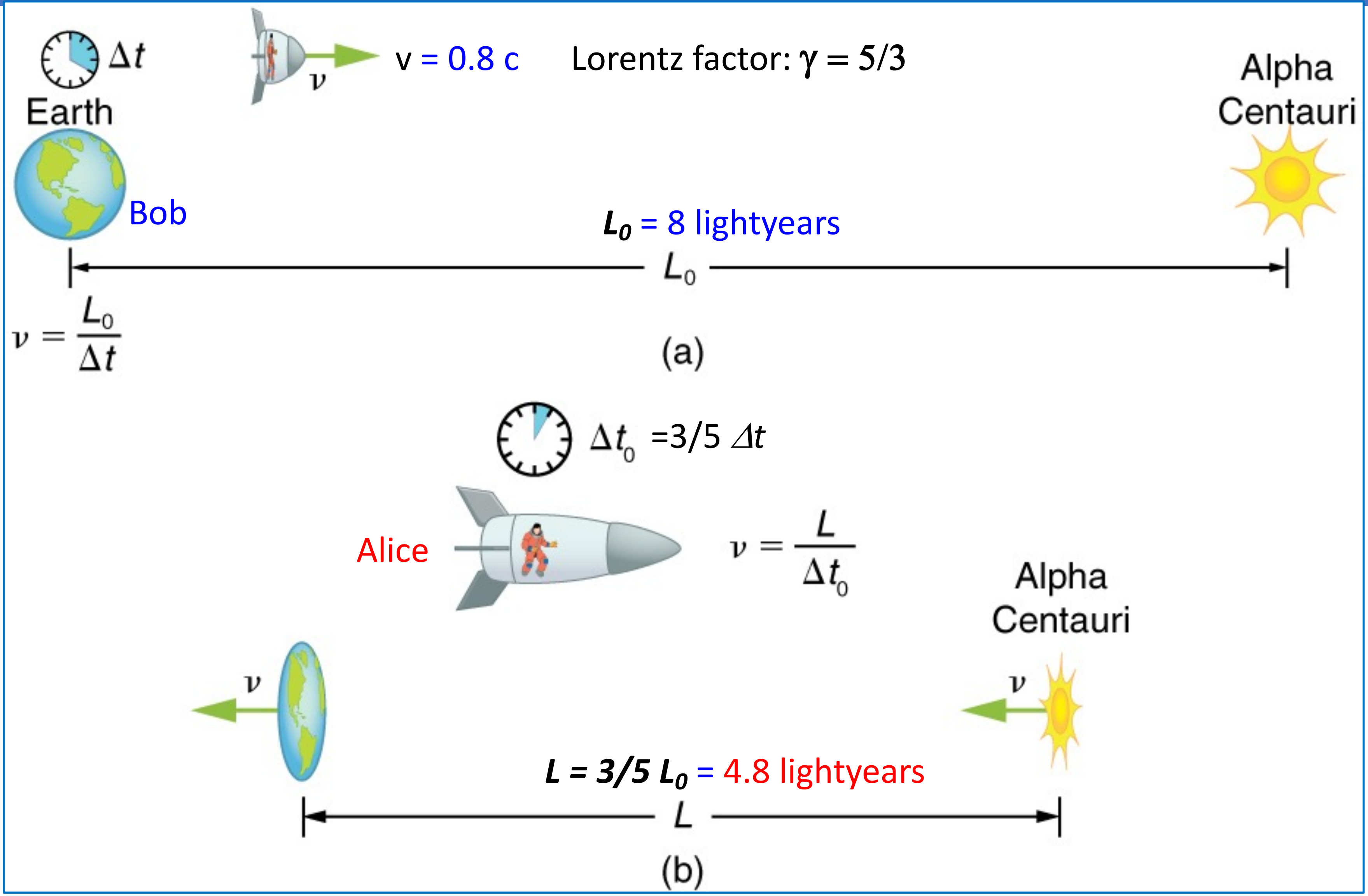
Lorentz contraction: L' = L / γ

→ Distances shrink at high speed!



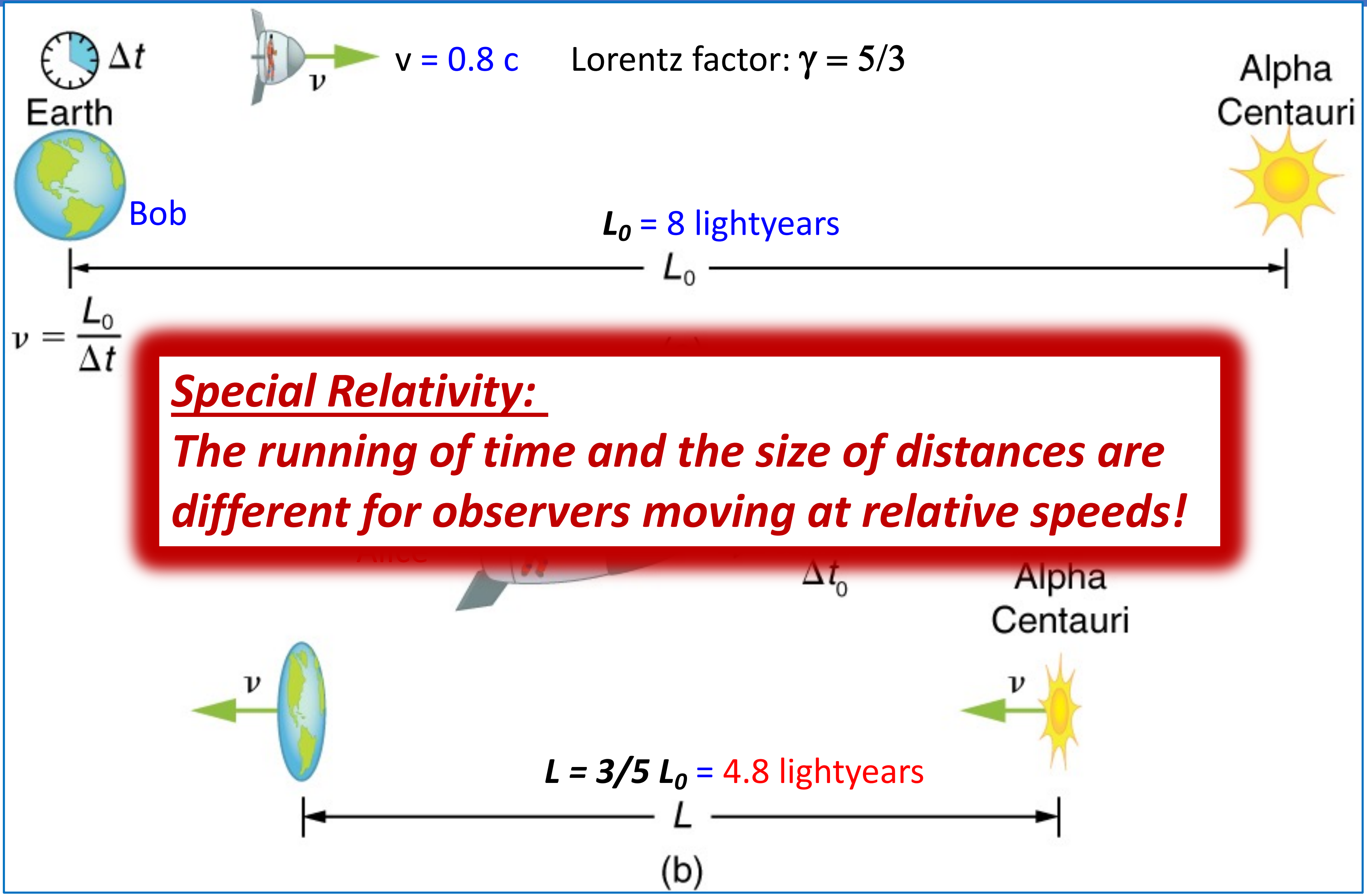
$$L' = L/\gamma = L \cdot \left(\sqrt{1 - v^2/c^2}\right)$$

Lorentz Contraction and time dilation



This is called: ***Lorentz contraction:*** $L' = L / \gamma$. **Distances shrink at high velocities!**

Lorentz Contraction

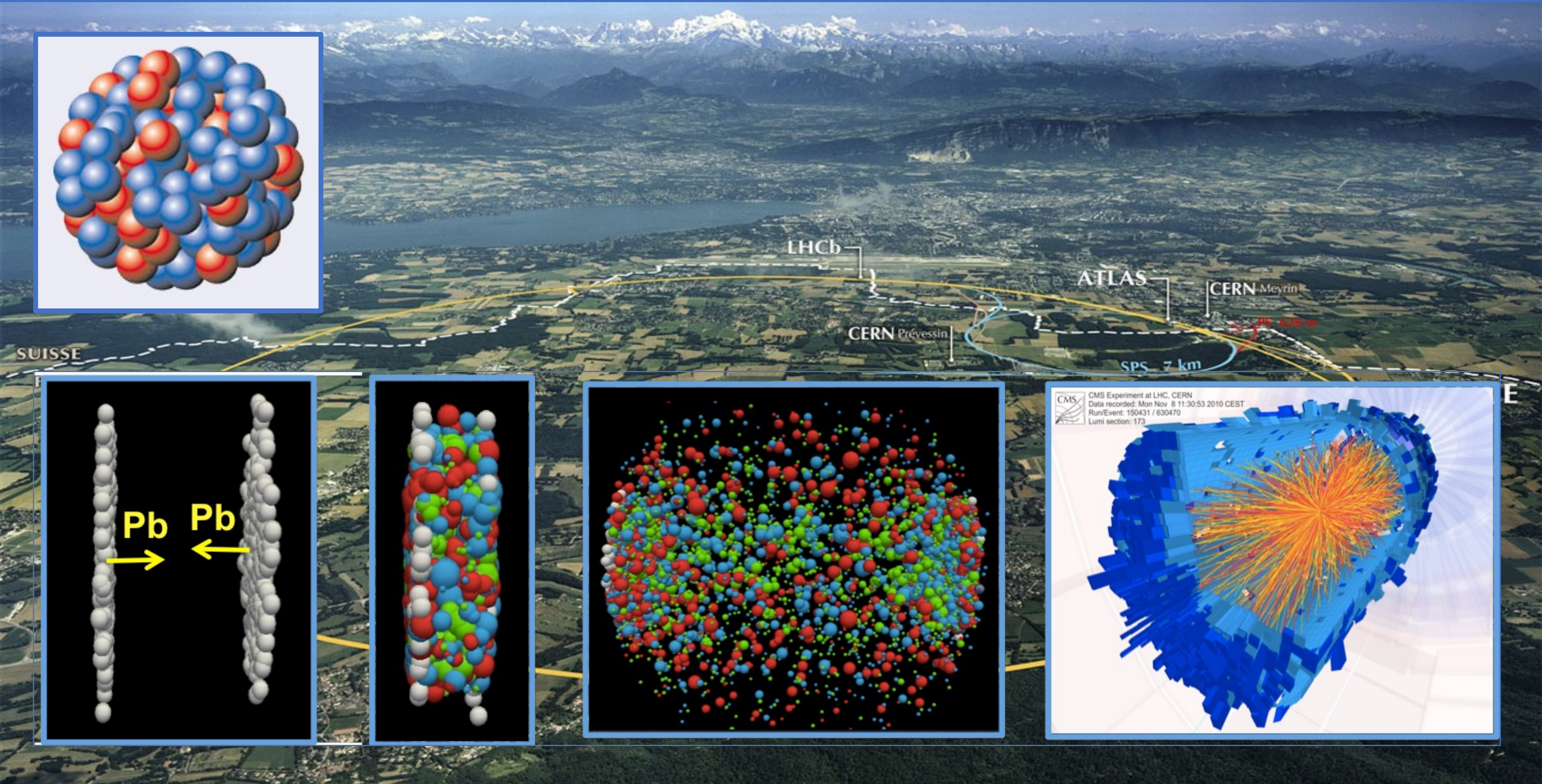


Special Relativity:

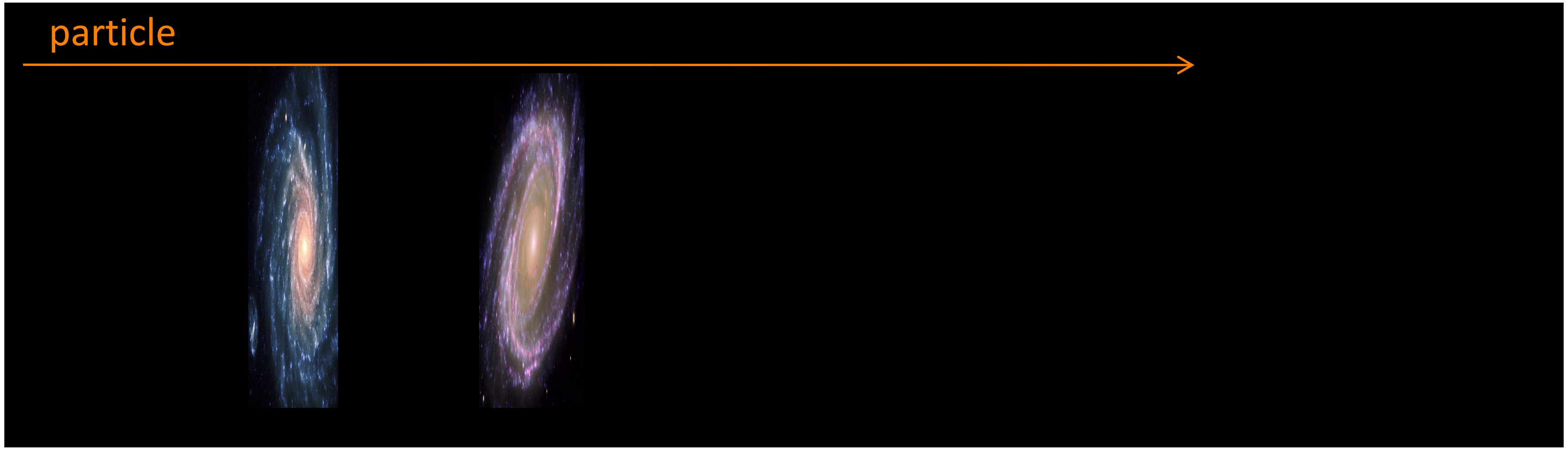
The running of time and the size of distances are different for observers moving at relative speeds!

This is called: ***Lorentz contraction: $L' = L / \gamma$.*** **Distances shrink at high velocities!**

Colliding Lead Nuclei “***Pancakes***” at the LHC

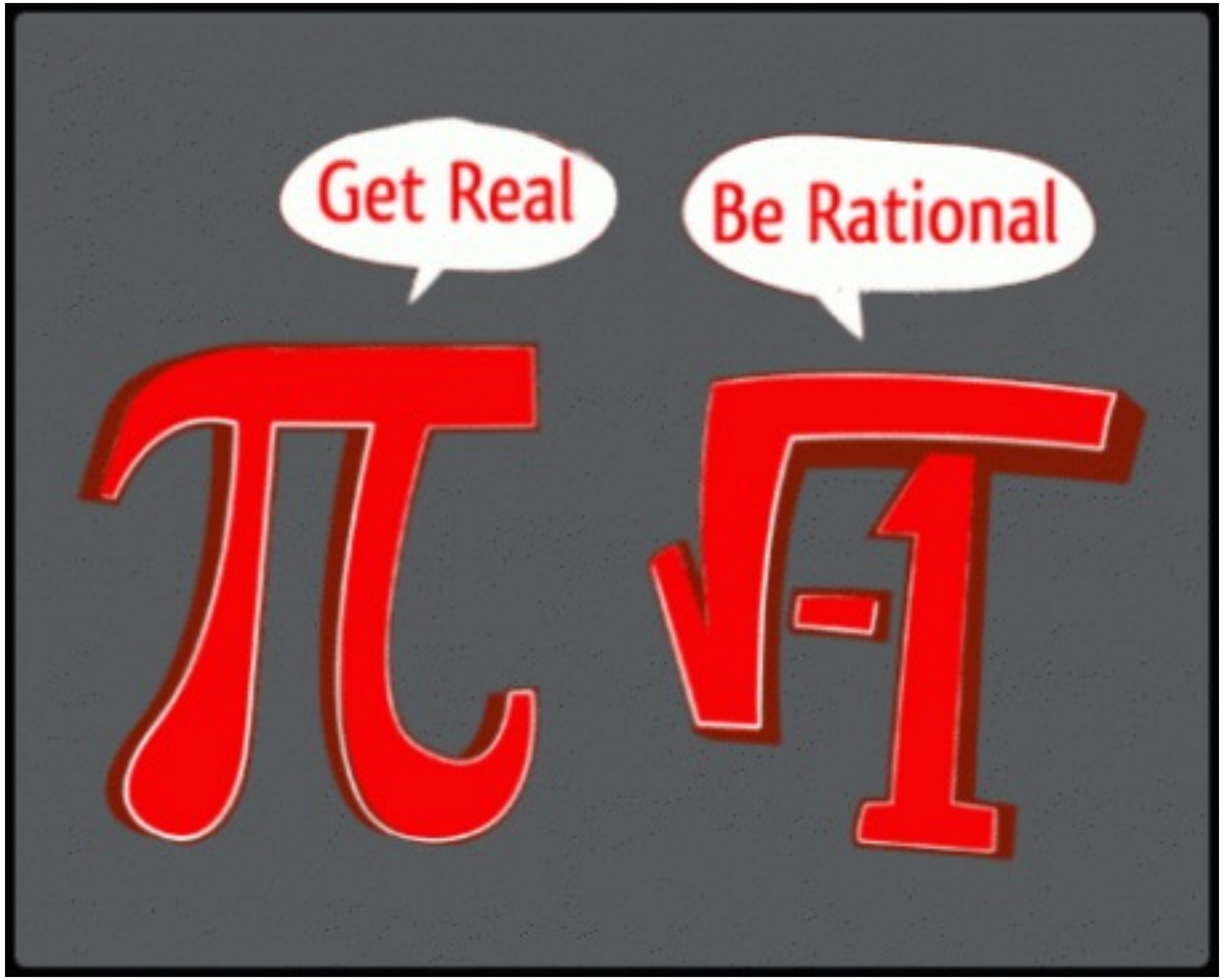


Different perspectives of the universe

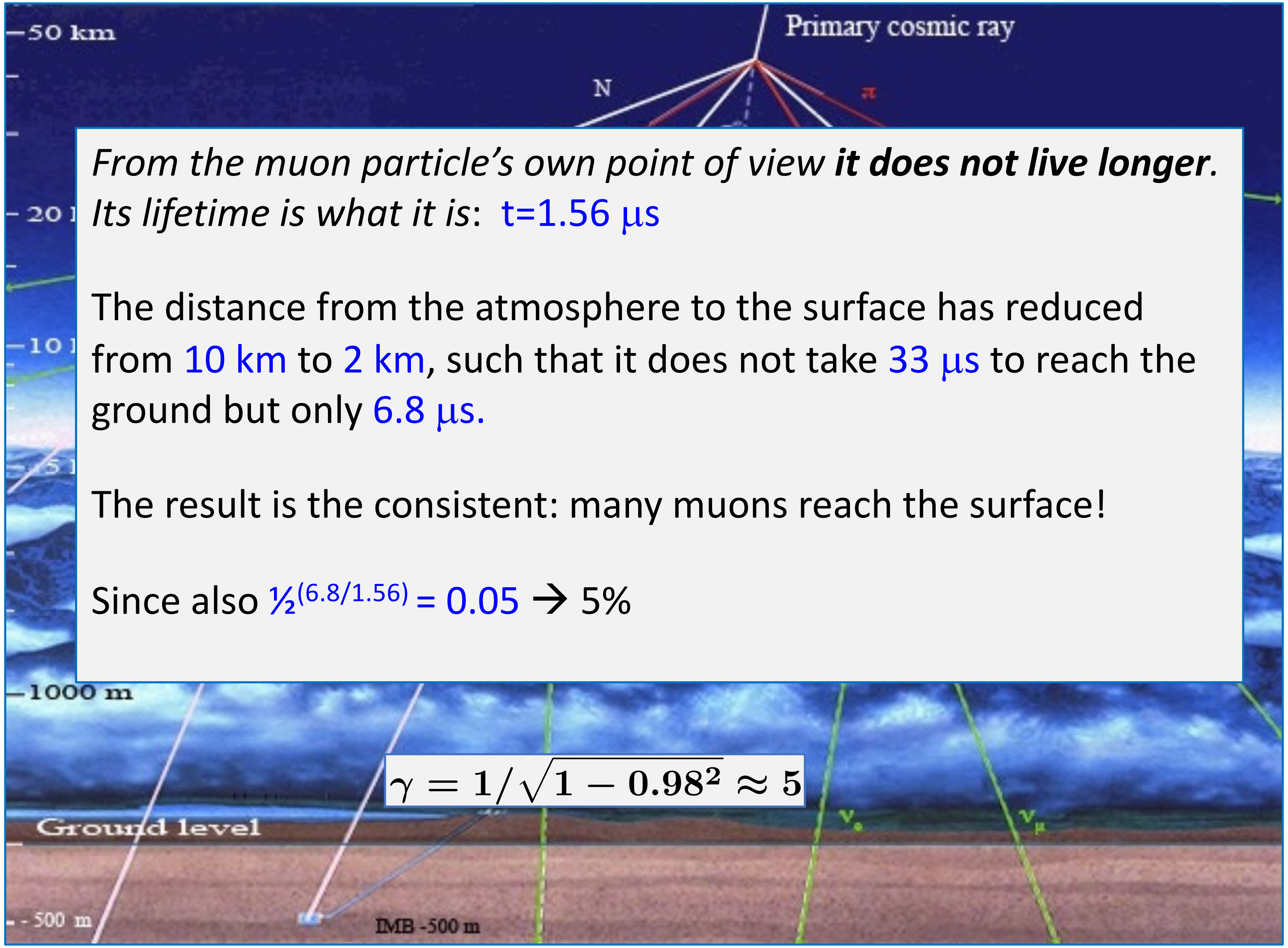


How does a photon see the universe?

For a photon time does not exist!


Get Real
Be Rational

Muon particles revisited



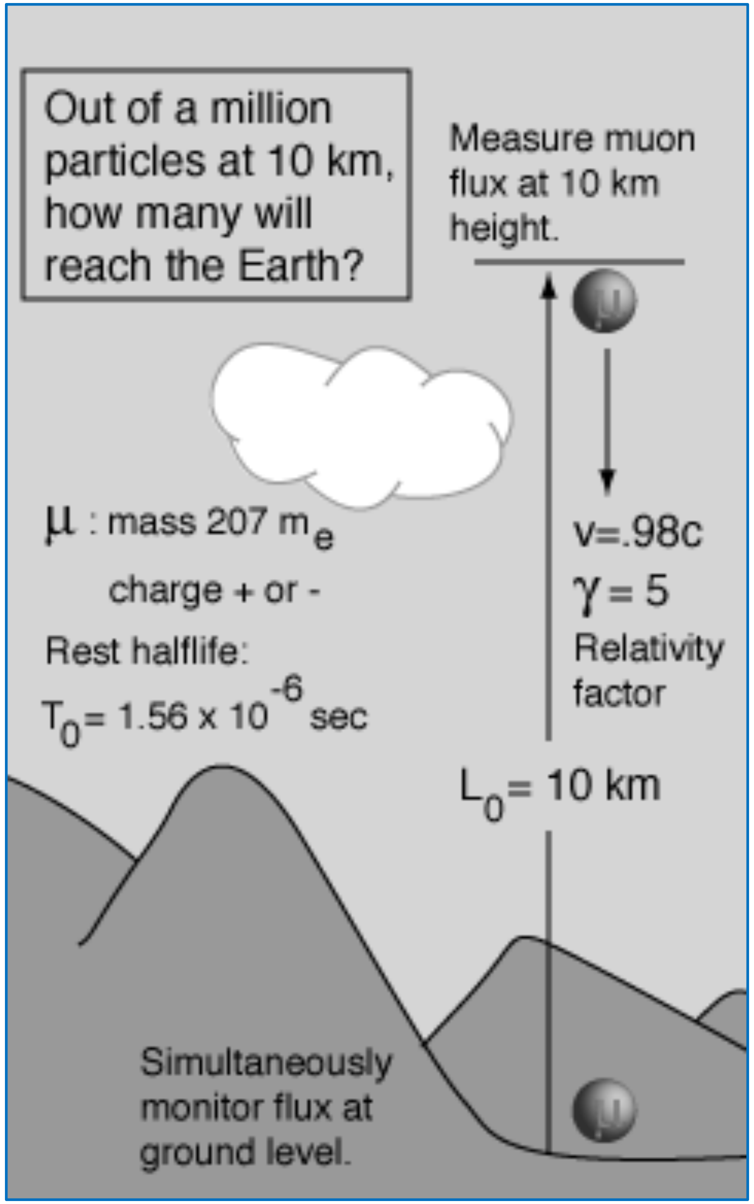
*From the muon particle's own point of view **it does not live longer**.*
Its lifetime is what it is: t=1.56 μs

The distance from the atmosphere to the surface has reduced from 10 km to 2 km, such that it does not take 33 μs to reach the ground but only 6.8 μs.

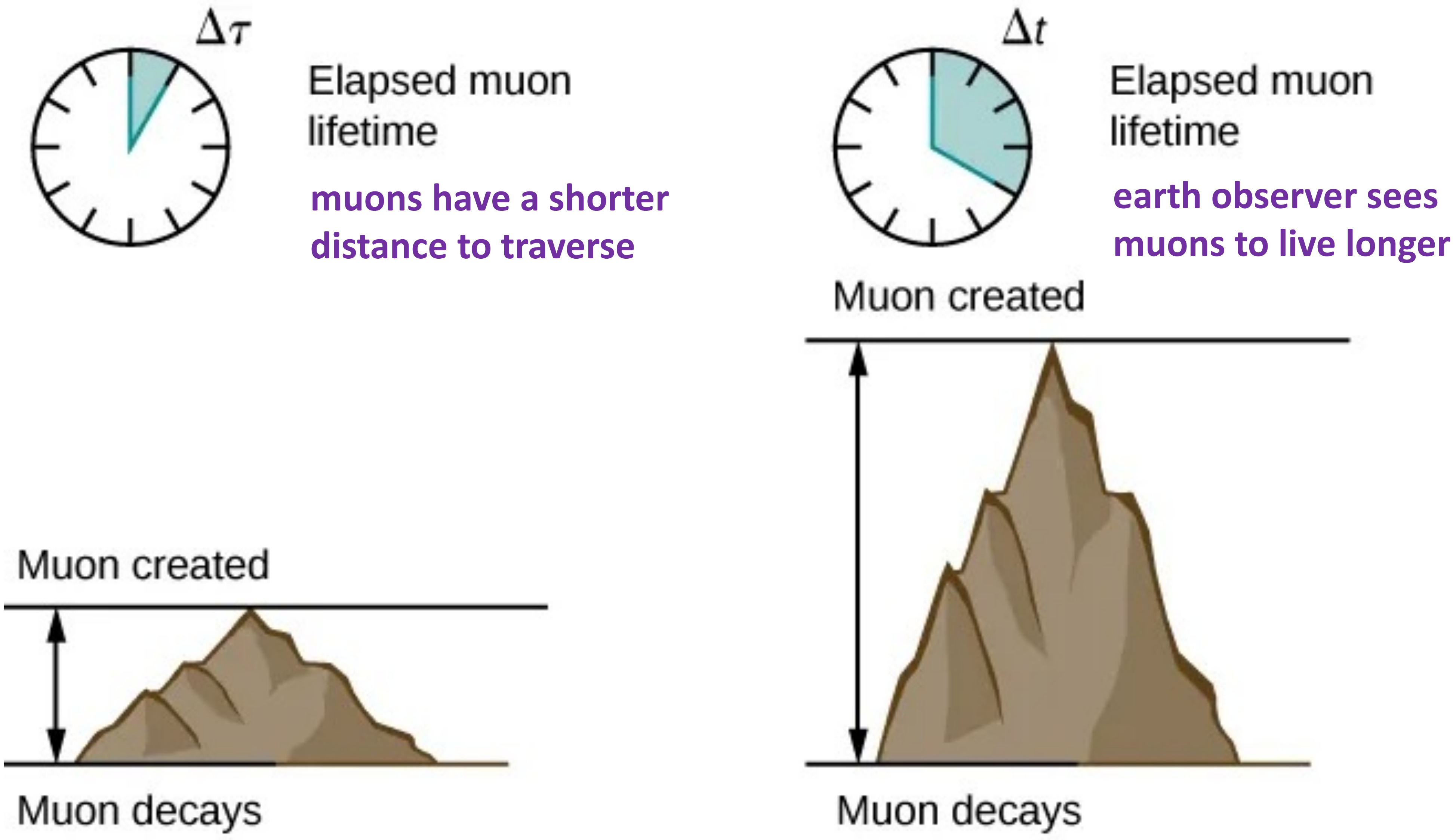
The result is the consistent: many muons reach the surface!

Since also $\frac{1}{2}^{(6.8/1.56)} = 0.05$ → 5%

$$\gamma = 1/\sqrt{1-0.98^2} \approx 5$$



Muon particles revisited



(a) Muon's reference frame

(b) Earth's reference frame

muons have a shorter distance to traverse

earth observer sees muons to live longer

Relativistic Effects

Time dilation:

$$\Delta t' = \gamma\, \Delta t$$

Lorentz contraction:

$$L' = L/\gamma$$

with the relativistic factor:

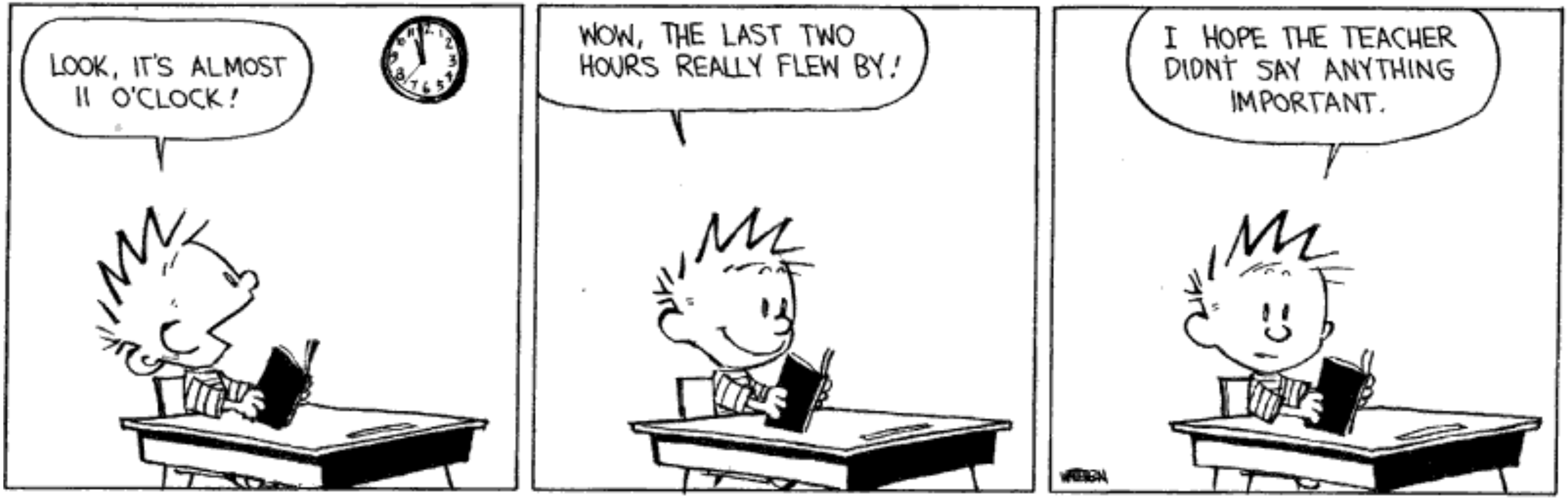
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Two definitions:

Time in rest frame = "eigen-time" or "proper-time"

Length in rest frame = "proper length"

Calvin’s Relativity



Next Lecture… Paradoxes!

Causality…

Travelling to the future…