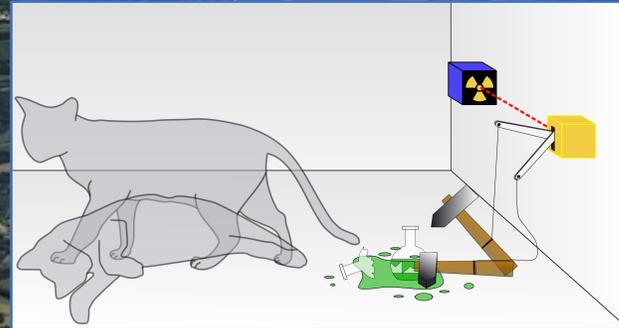
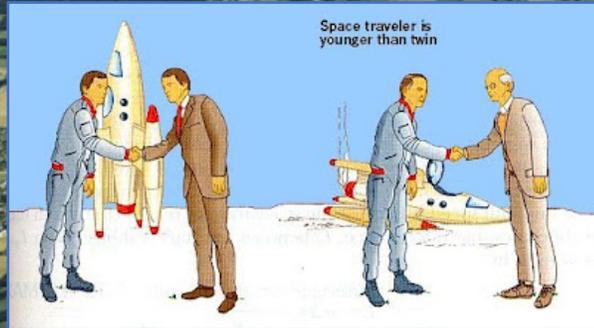
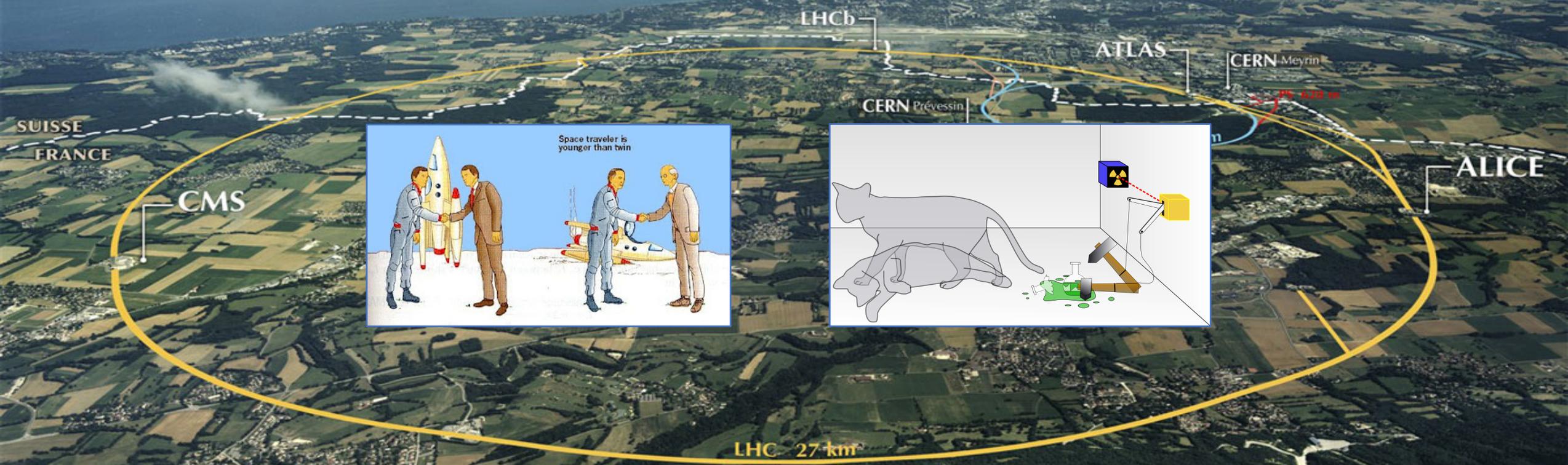


The Relativistic Quantum World

A lecture series on Relativity Theory and Quantum Mechanics

Marcel Merk



University of Maastricht, Sept 16 – Oct 14, 2020

Relativity

Sept. 16:

Lecture 1: The Principle of Relativity and the Speed of Light
Lecture 2: Time Dilation and Lorentz Contraction

Sept. 23:

Lecture 3: The Lorentz Transformation and Paradoxes
Lecture 4: General Relativity and Gravitational Waves

Quantum Mechanics

Sept. 30:

Lecture 5: The Early Quantum Theory
Lecture 6: Feynman's Double Slit Experiment

Oct. 7:

Lecture 7: Wheeler's Delayed Choice and Schrodinger's Cat
Lecture 8: Quantum Reality and the EPR Paradox

Standard Model

Oct. 14:

Lecture 9: The Standard Model and Antimatter
Lecture 10: The Large Hadron Collider

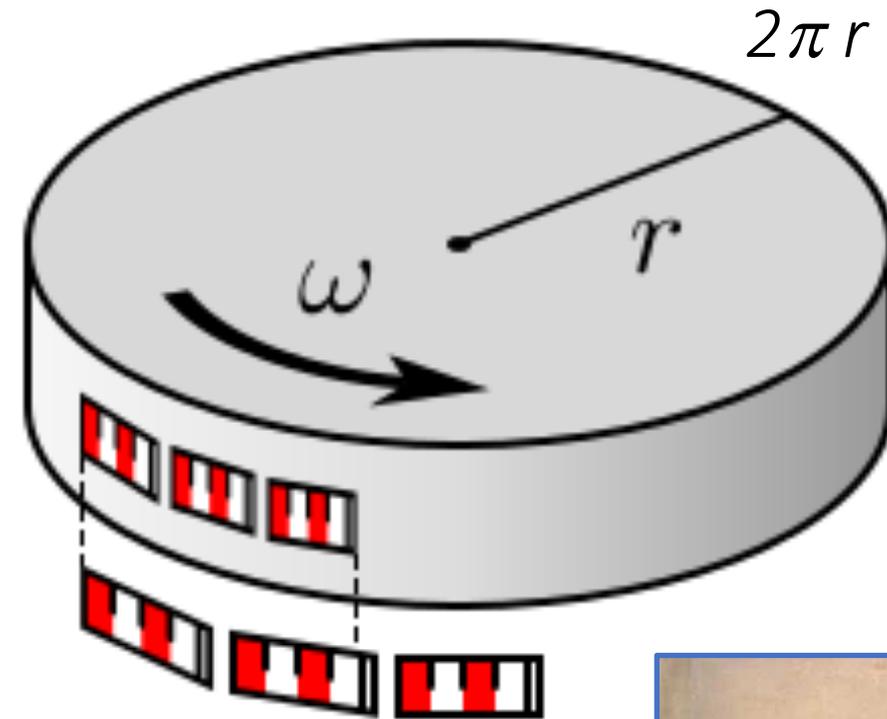
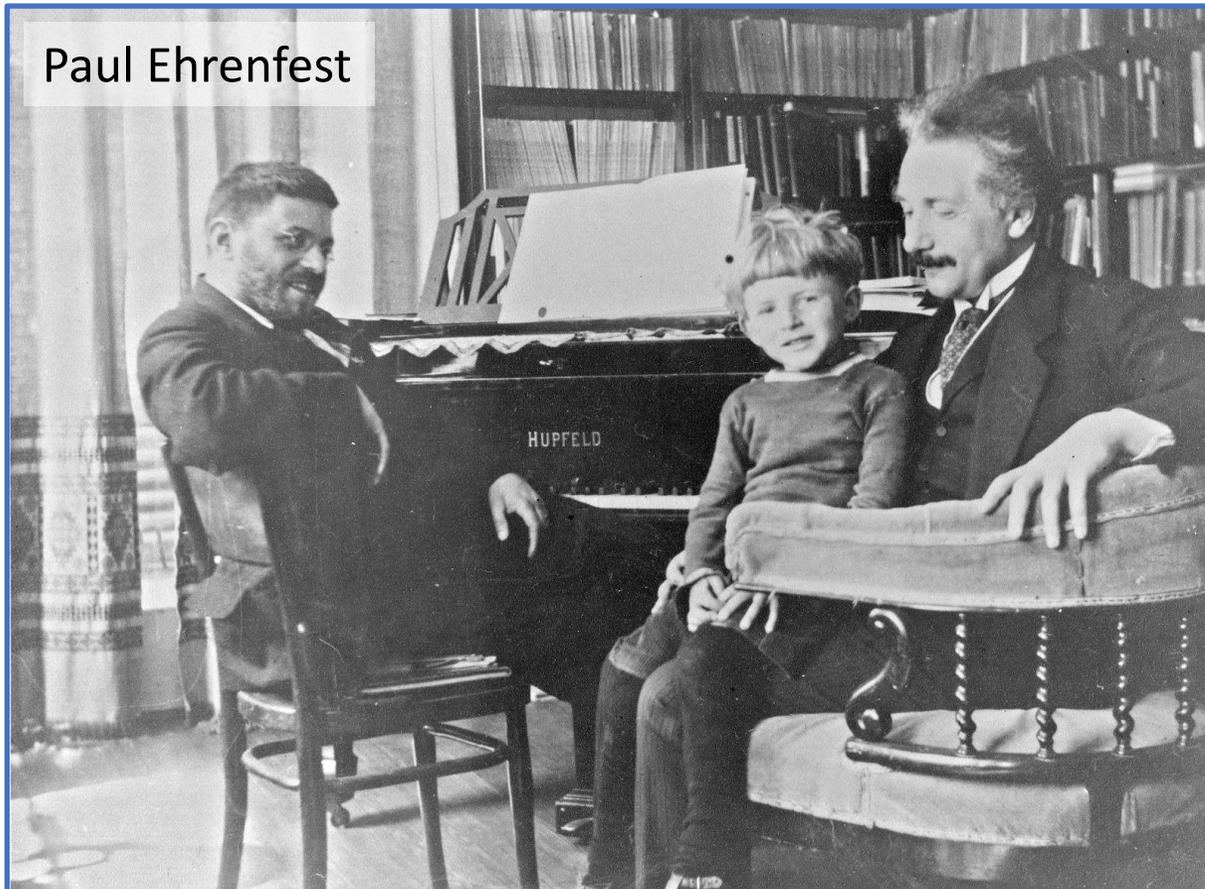
Lecture notes, written for this course, are available: www.nikhef.nl/~i93/Teaching/
Prerequisite for the course: High school level physics & mathematics.

Lecture 4

General Relativity and Gravitational Waves

*“Do not worry about your difficulties in mathematics.
I can assure you mine are still greater.”*
- Albert Einstein

Rotating disk with ruler on the edge:
Circumference: $C = 2 \pi r$



Rotating disk with ruler on the edge:
Circumference: $C = 2 \pi r$

Alice stands next to the disk and sees
rulers on disk Lorentz contracted:

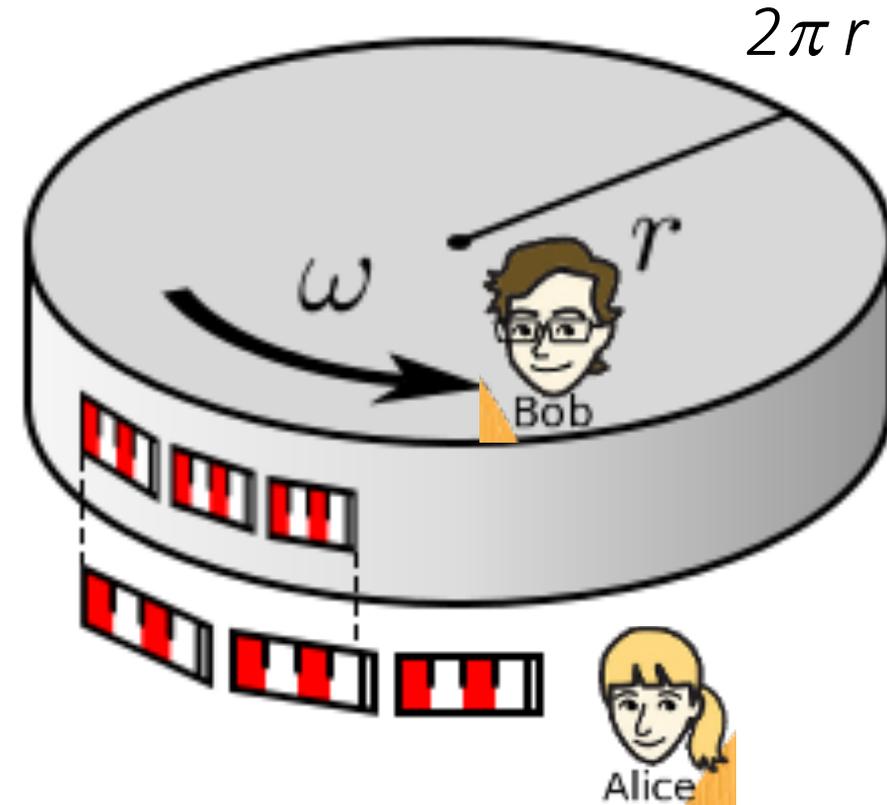
$$C = 2 \pi r / \gamma$$

→ Circumference is smaller!

Bob moves on the disk and sees
rulers next to disk contracted:

$$C = 2 \pi r \cdot \gamma$$

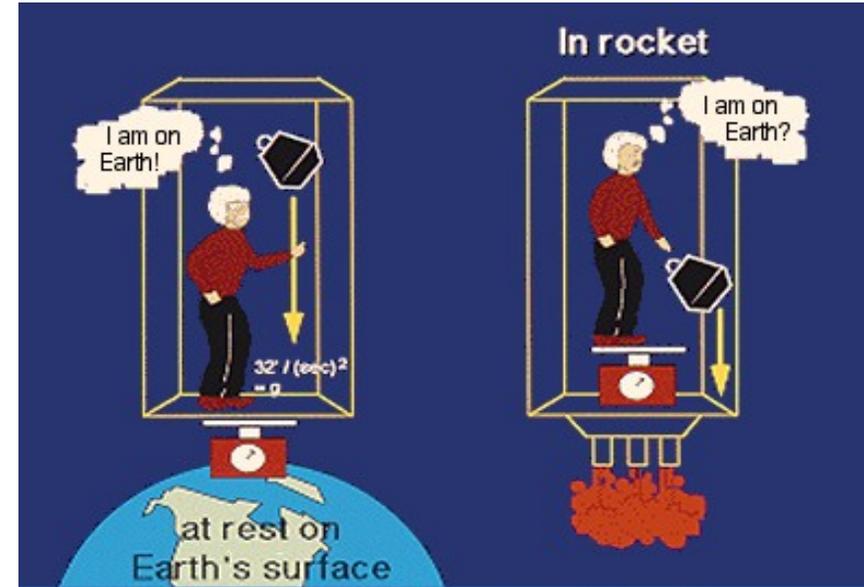
→ Circumference is larger!



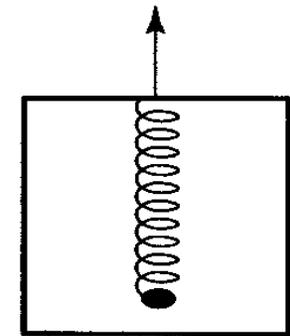
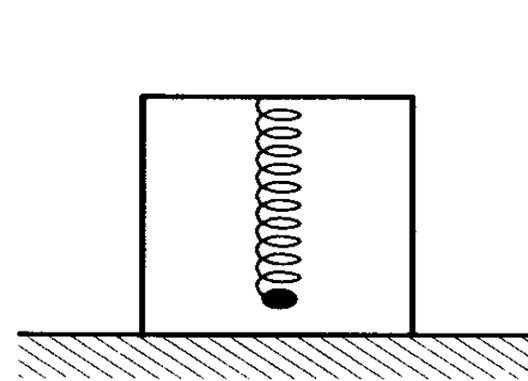
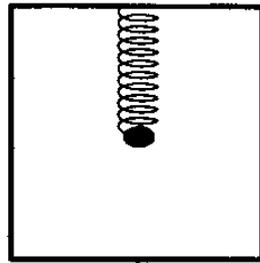
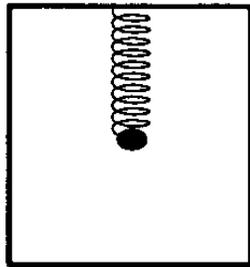
A rotating object is ***not an inertial*** frame:

- Postulate of relativity only worked for ***inertial frames***
- Need to adapt the postulates: special relativity → ***general relativity***

The Equivalence Principle

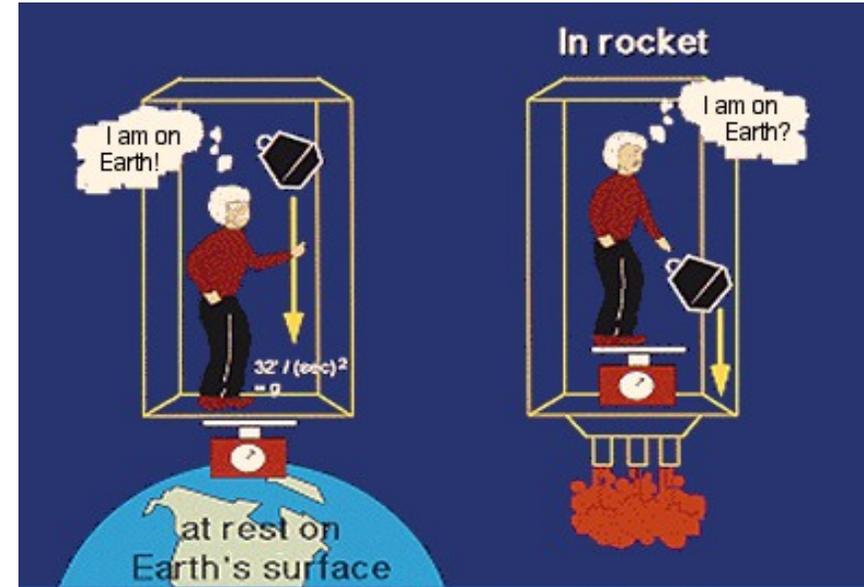


(Inertial Frame)

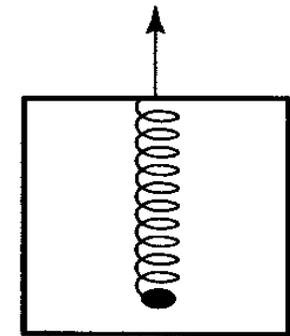
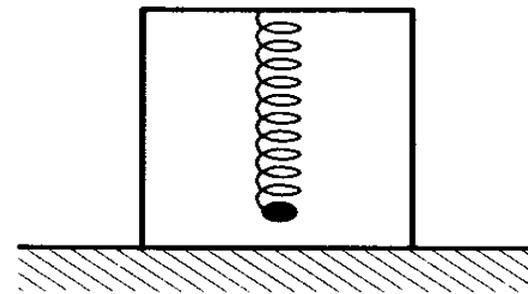
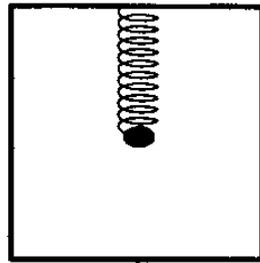
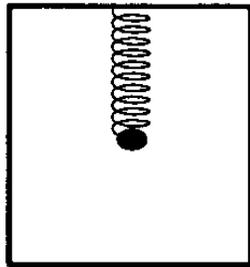


Einstein "happiest thought": there is no way to determine whether you are standing on the earth or accelerating upwards in a rocket in space!

The Equivalence Principle



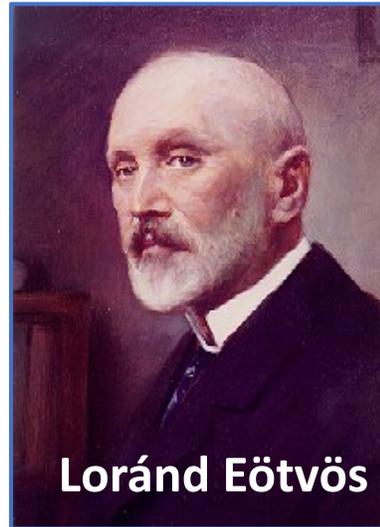
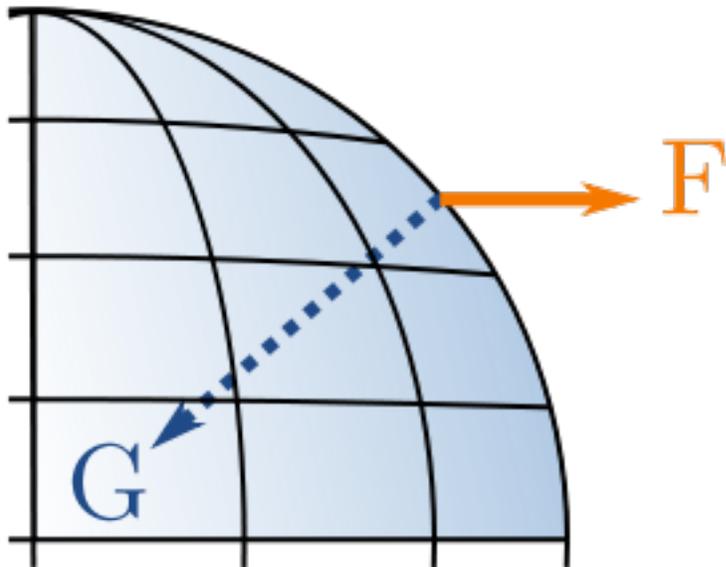
(Inertial Frame)



There is no difference between acceleration force and gravitation.
“Gravitational mass” = “Inertial mass”

The Eötvös Experiment

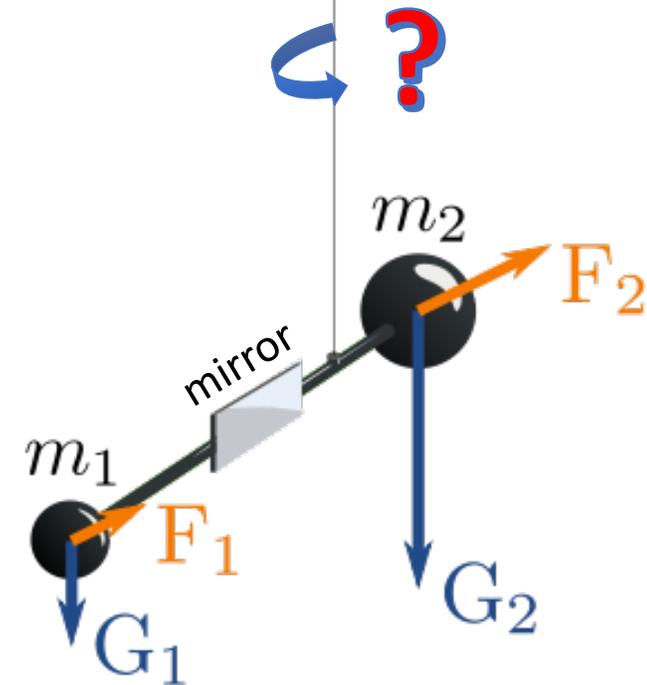
Direction of **gravity** and **centrifugal force** on earth



$$F = m_i a \text{ with } a = \frac{v^2}{R}$$
$$G = m_g g \text{ with } g = \frac{GM_{\oplus}}{R^2}$$

→ $m_i = m_g$

Small (m_1) and big (m_2) mass on a rod suspended by a thin fiber



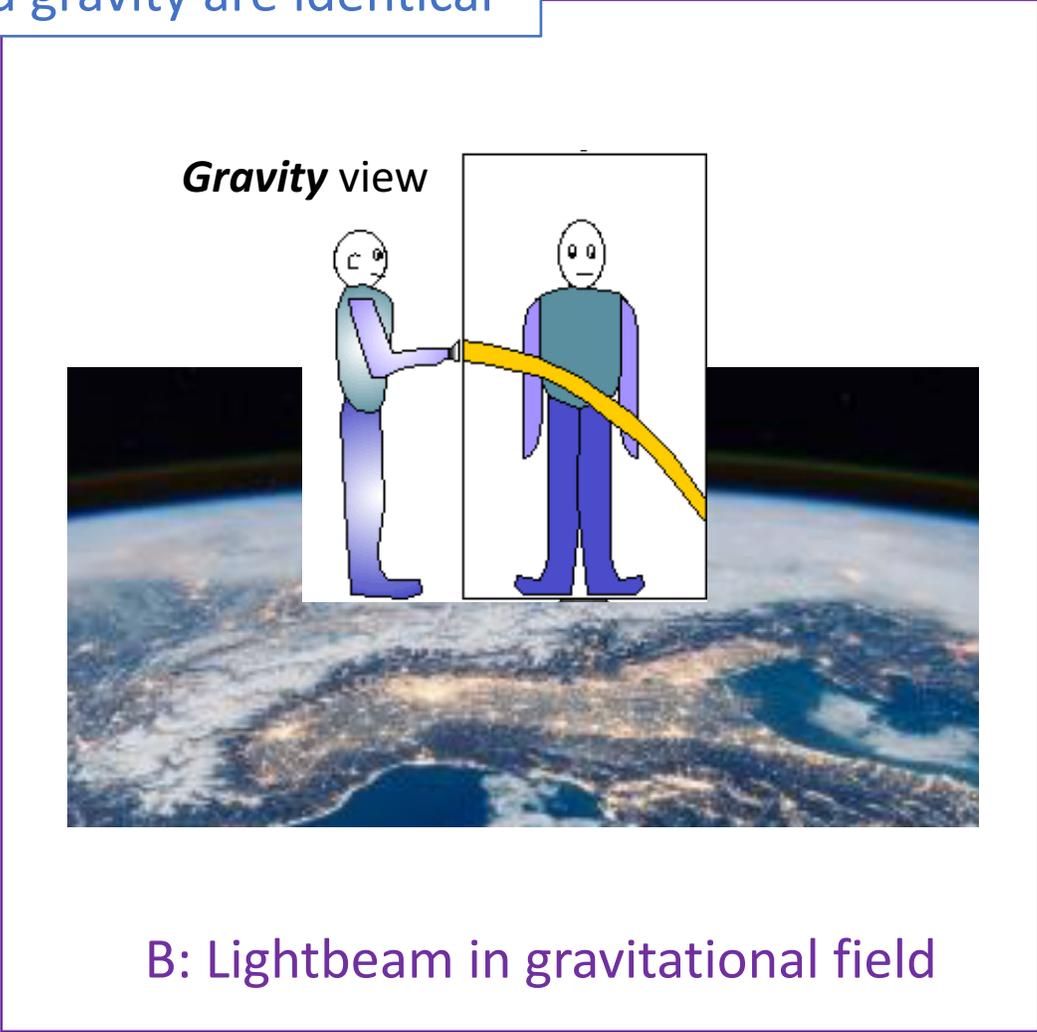
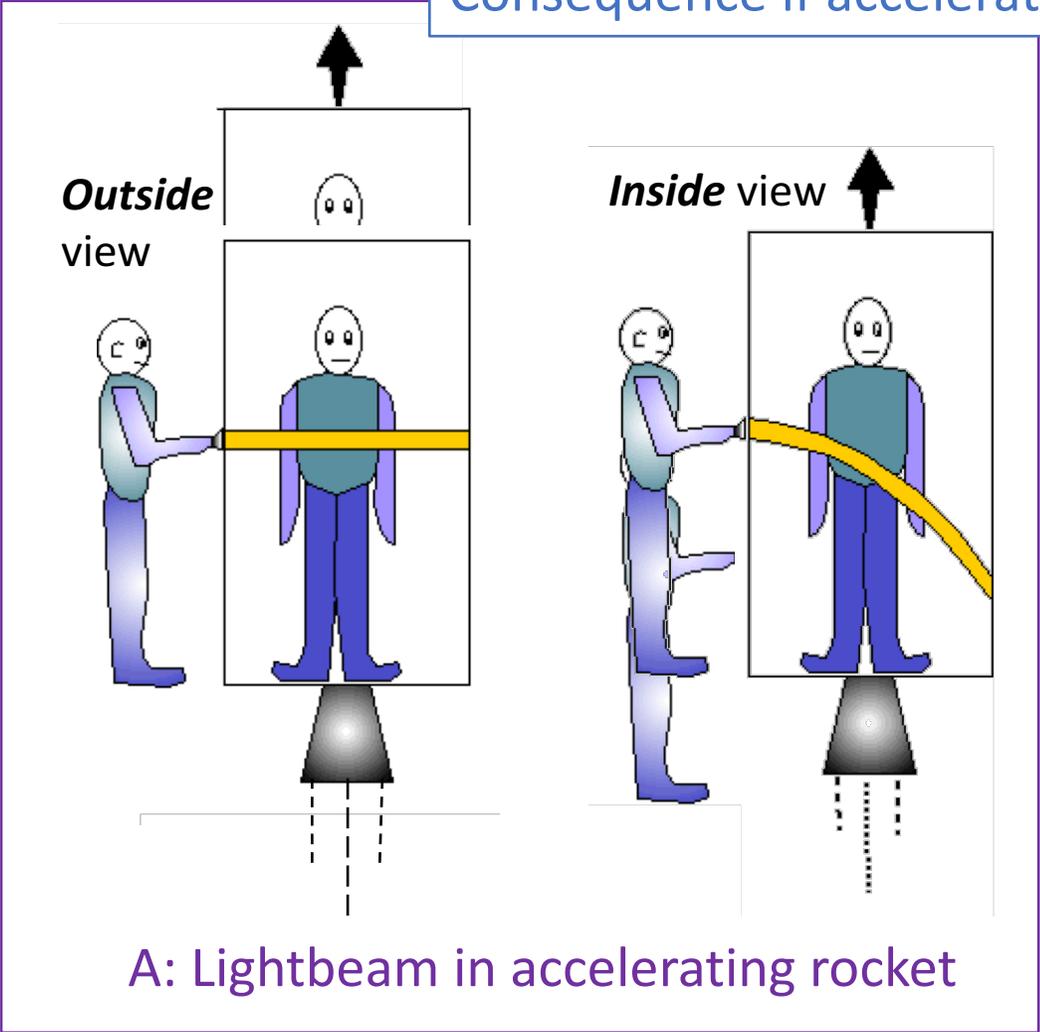
Gravity force **G** depends on Newton's law of gravity: **gravitational mass**

Centrifugal force **F** depends on Newton's law of motion inertial mass: **inertial mass**

The system did **not** rotate. → $F_1/F_2 = G_1/G_2$

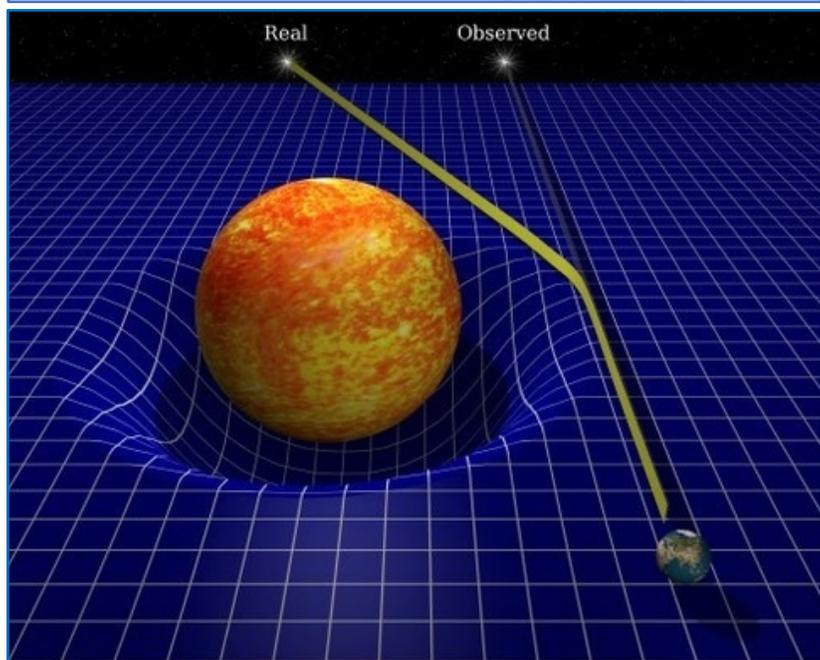
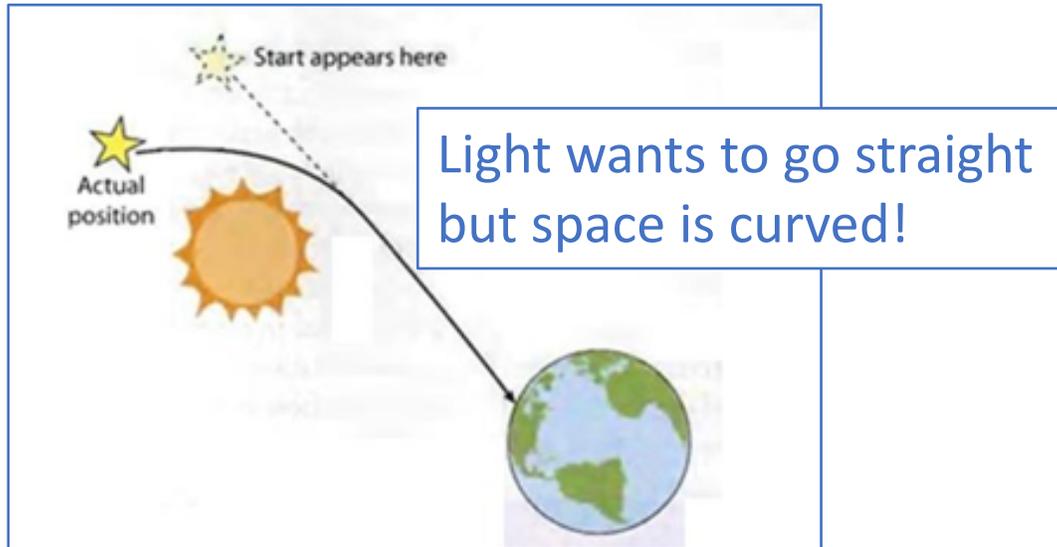
→ Experimental proof that indeed gravitational mass is equivalent to inertial mass.

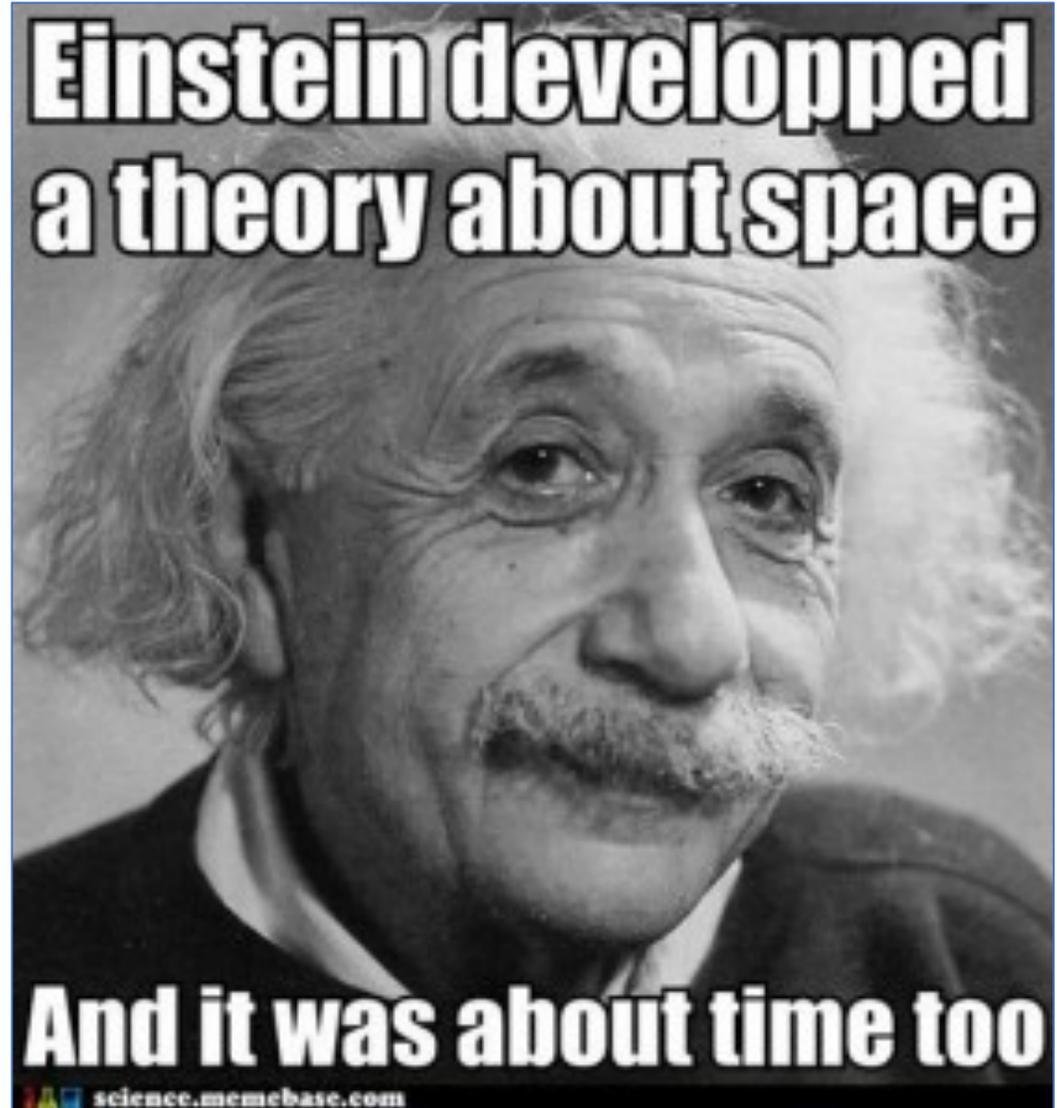
Consequence if acceleration and gravity are identical



Prediction of Einstein: light beam bends under gravity!

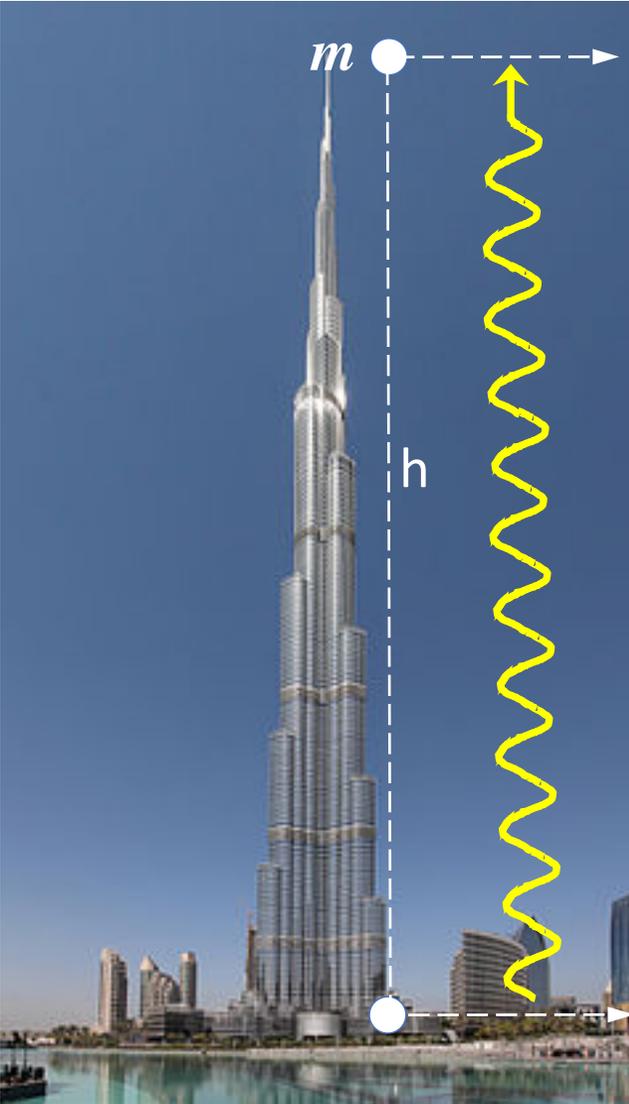
Bending of light in gravitation field of the Sun





Einstein's thought experiment

Particle with mass m falling from tower:



$$E = mc^2 \quad \Rightarrow \quad E = hf$$

From quantum mechanics we know that the energy of light is related to frequency (and wavelength): $E = hf = hc/\lambda$

Perpetuum mobile? \rightarrow No!

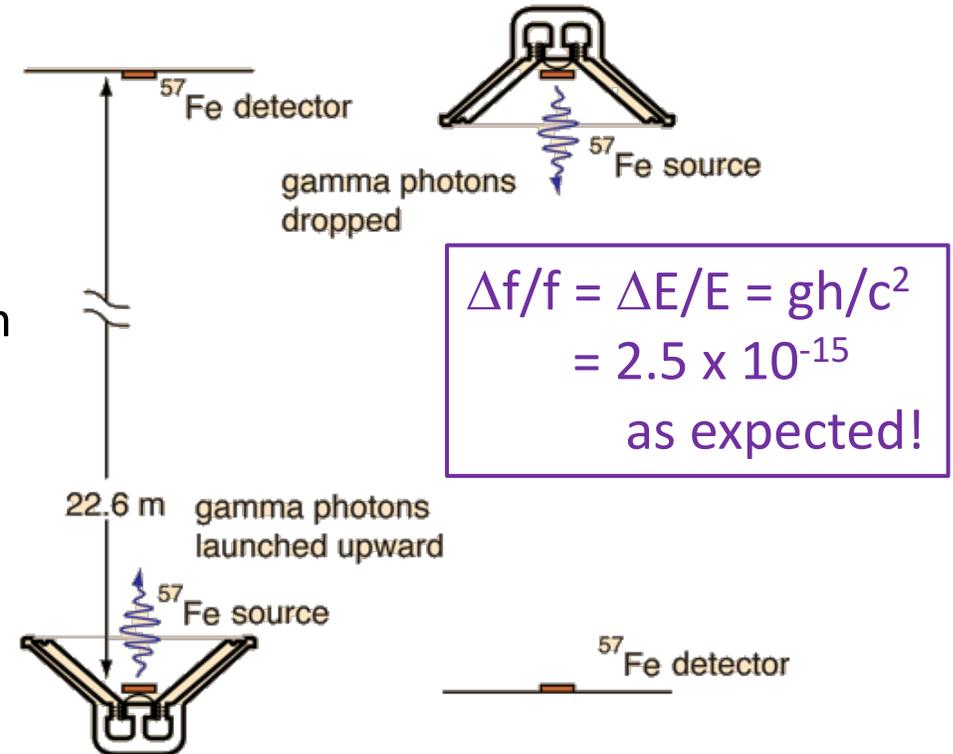
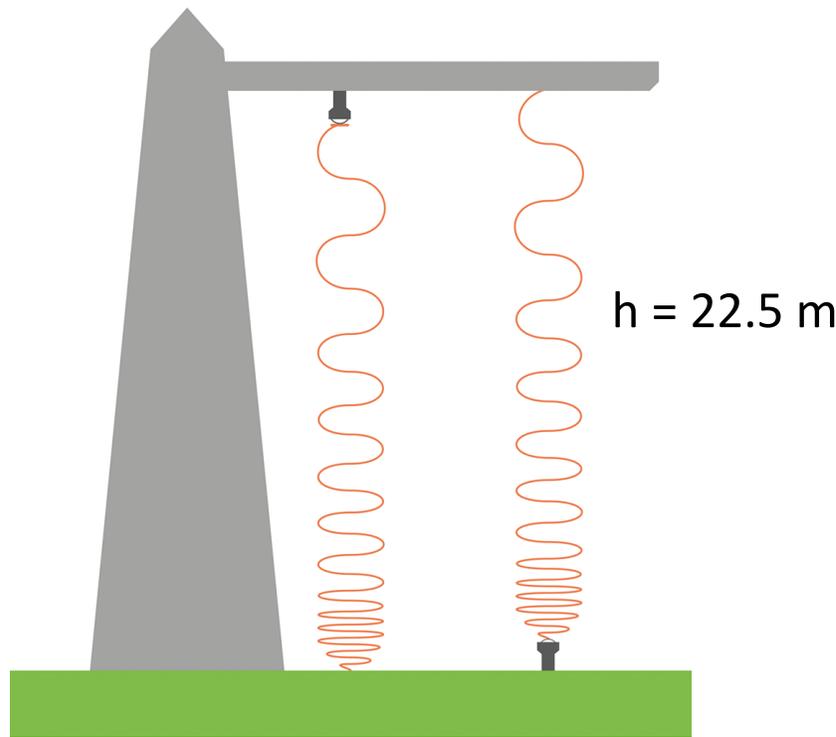
$$hf' > hf$$

Photon loses energy gh/c^2 as it travels up the gravitational field!

$$E' = mc^2 + \frac{1}{2}mv^2 = mc^2 + mgh \quad (E_{kin} = E_{pot})$$
$$= mc^2 (1 + gh/c^2) \quad \Rightarrow \quad E' = hf'$$

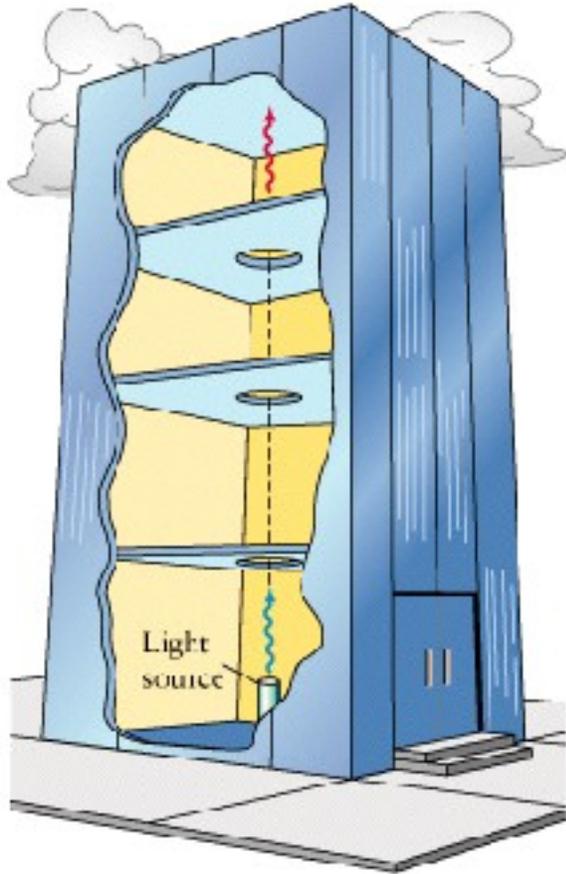
The Harvard Tower Experiment

Harvard Tower Experiment (Pound-Rebka)
at Jefferson lab in Harvard:
Measure red-shift of photons in earth
gravitational field.



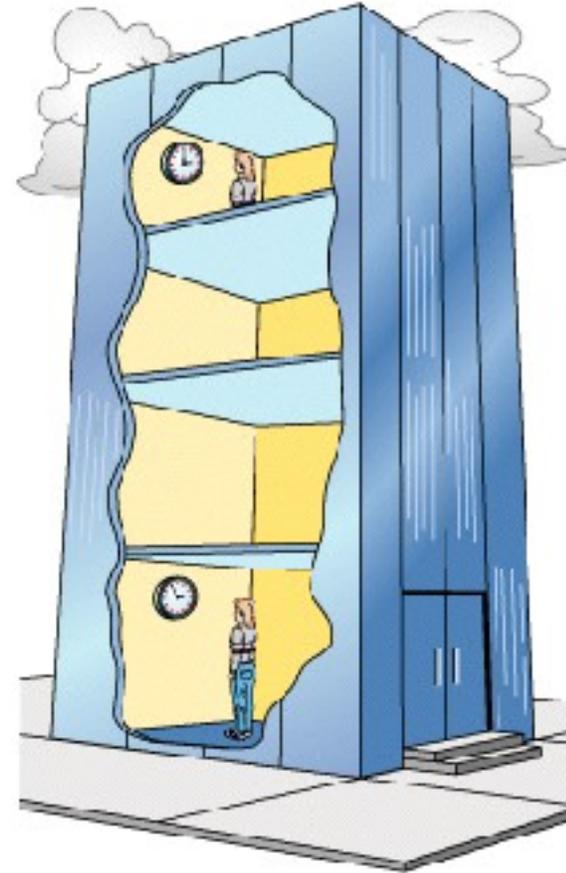
The photon loses energy as it climbs the gravitational field.

Longer wavelength



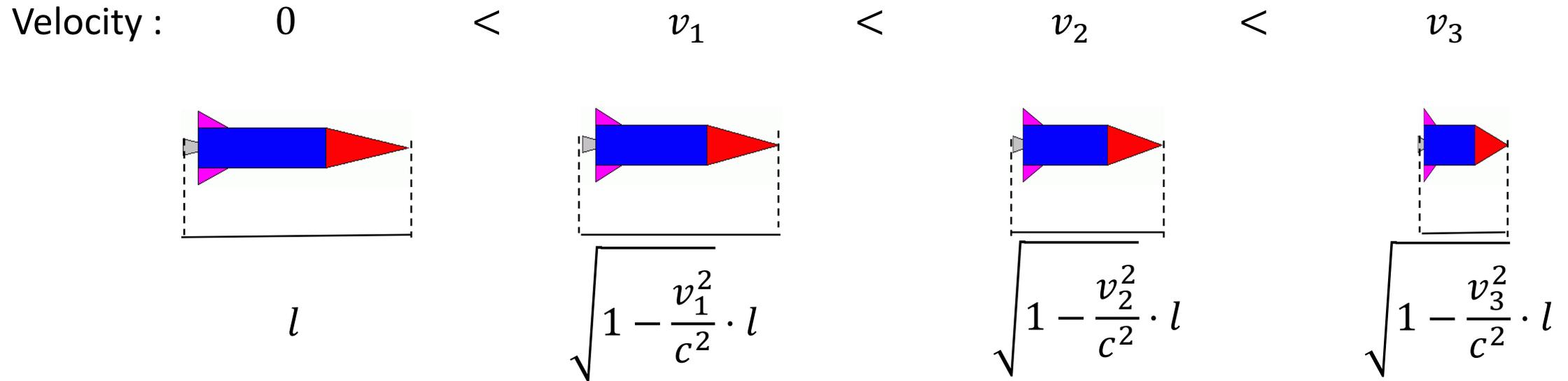
$$c = \lambda f$$

Lower frequency



→ Time ticks faster at higher altitude.

From special relativity we know that space contracts at high velocity



Space is seen to shrink further and further with increasing velocity!

$$1/\gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

$$E_{pot} = G \frac{mM_{\oplus}}{R}$$

↓

$$E_{kin} = E_{pot}$$

$$\frac{1}{2}mv^2 = G \frac{mM_{\oplus}}{R}$$

↓

$$v^2 = 2 \frac{GM_{\oplus}}{R}$$

↓

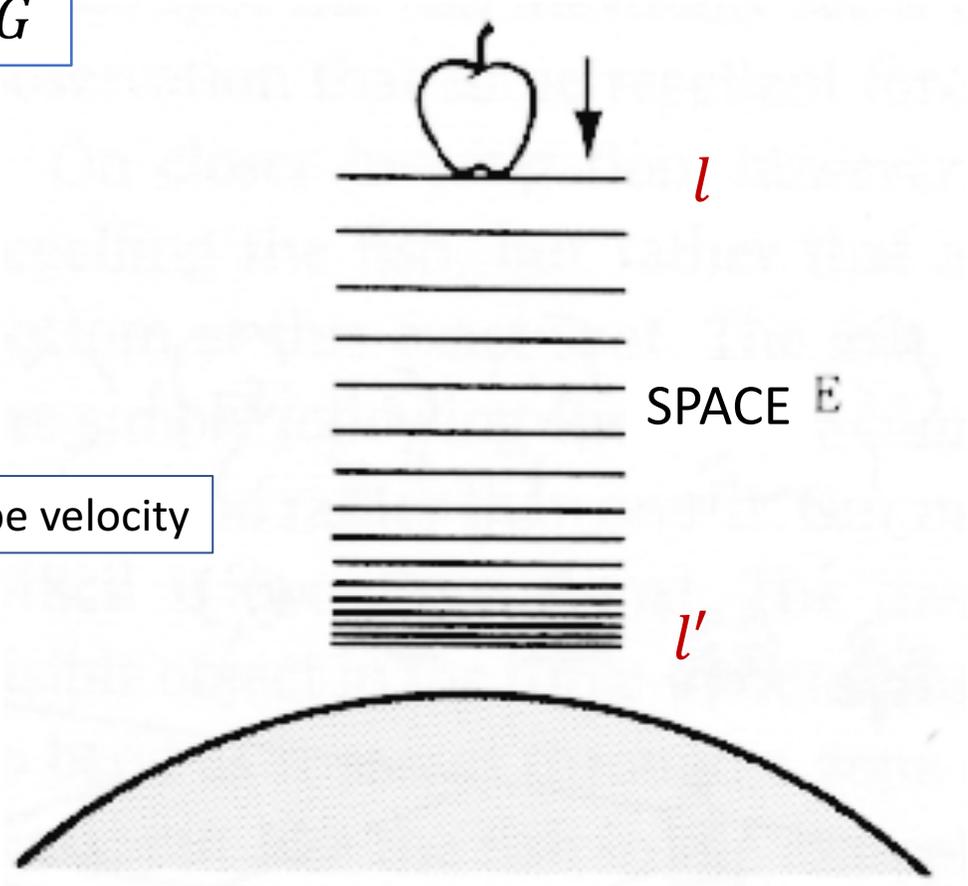
$$\sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - 2 \frac{GM_{\oplus}}{Rc^2}}$$

Newton's
Constant G

v = Escape velocity

Lorentz factor

Falling apple:



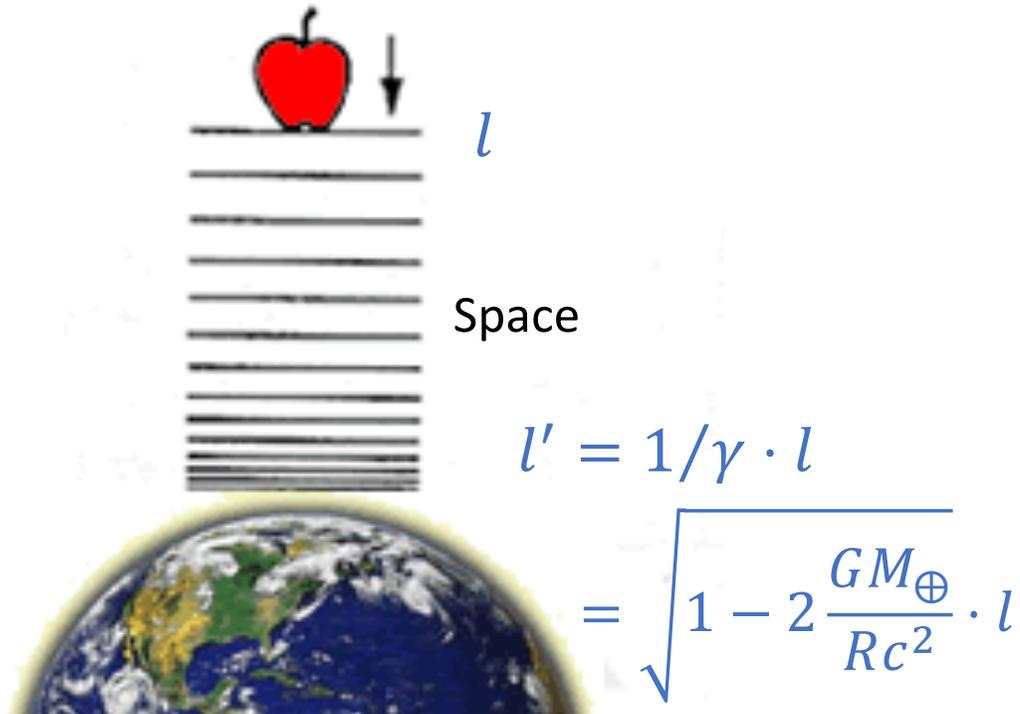
Compare to accelerating rocket:



Space shrinkage ("curvature"):

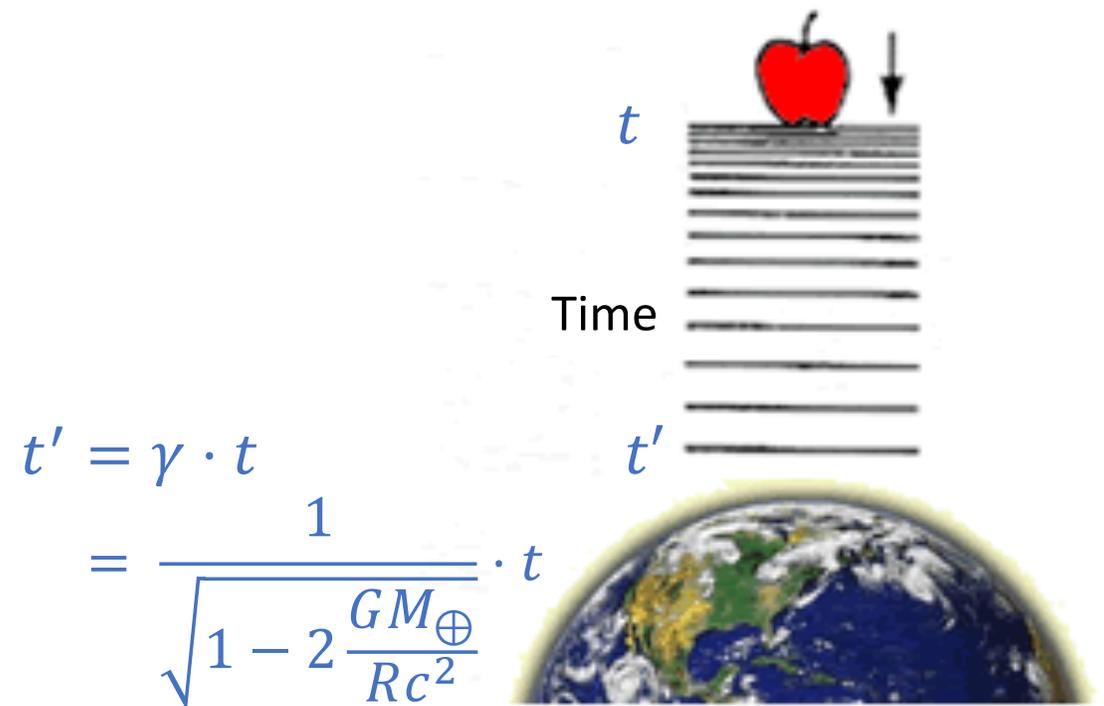
$$l' = \sqrt{1 - 2 \frac{GM_{\oplus}}{Rc^2}} \cdot l$$

A falling apple accelerates and units of space get more and more contracted:



Space contracts near mass and dilates away from it.

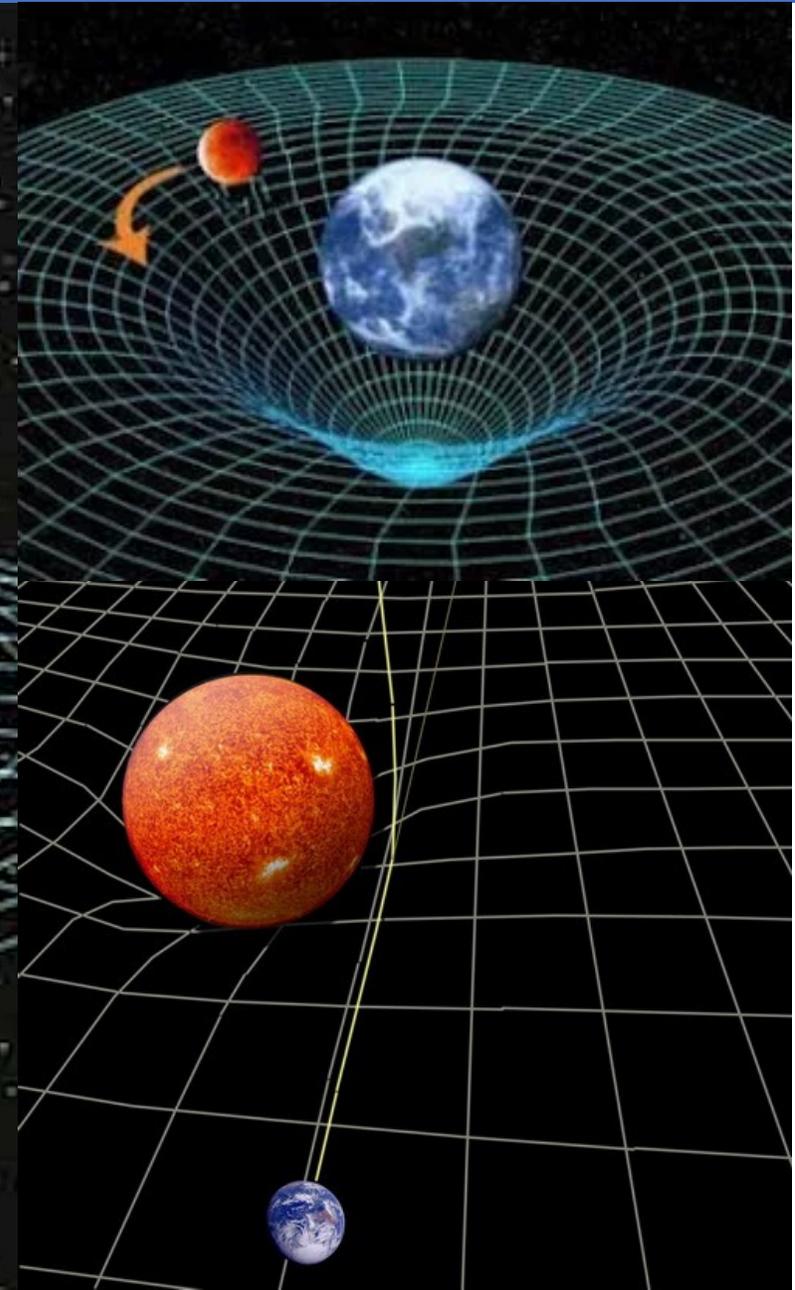
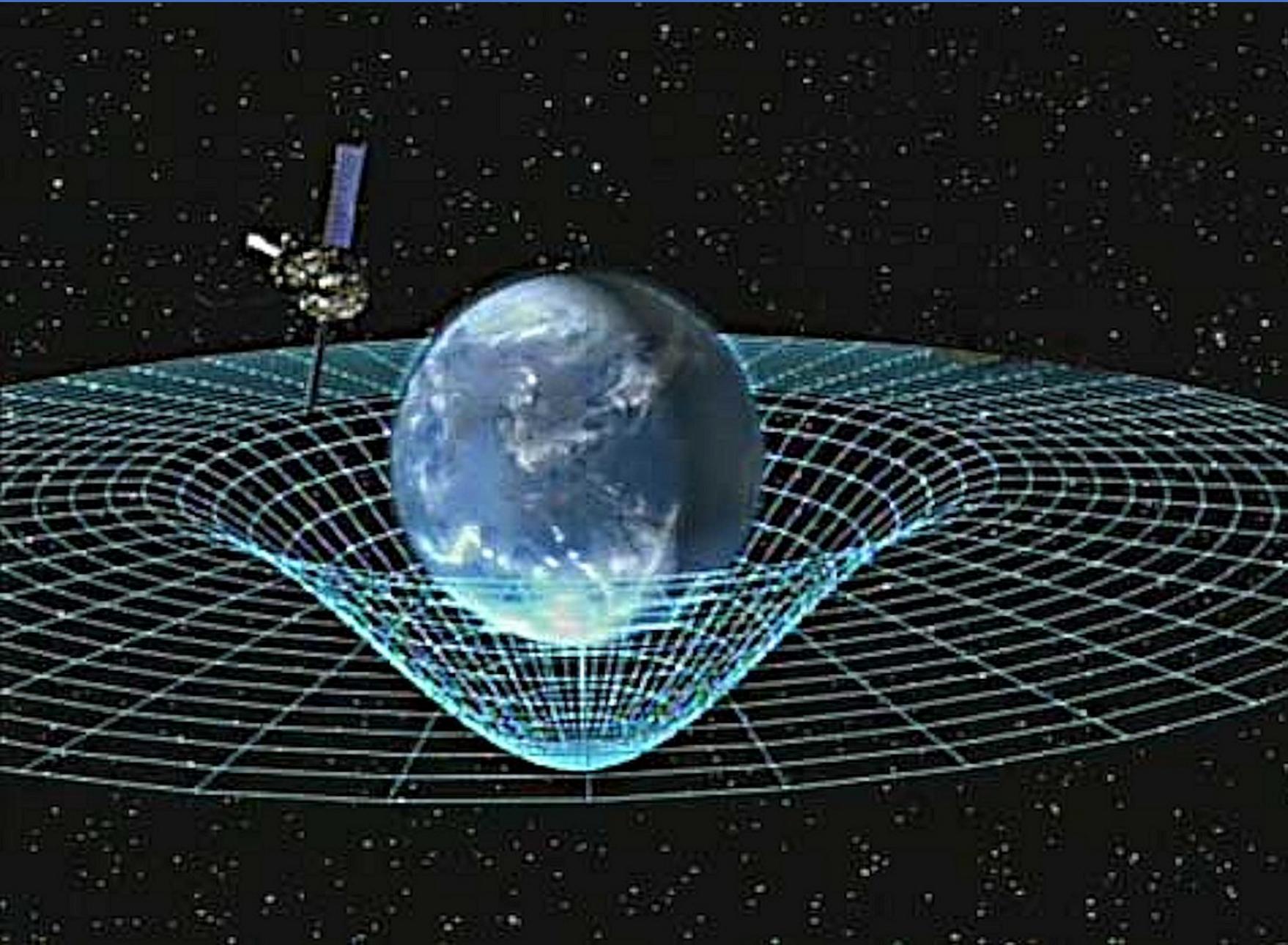
An apple falls into the gravitational field and time runs slower and slower:



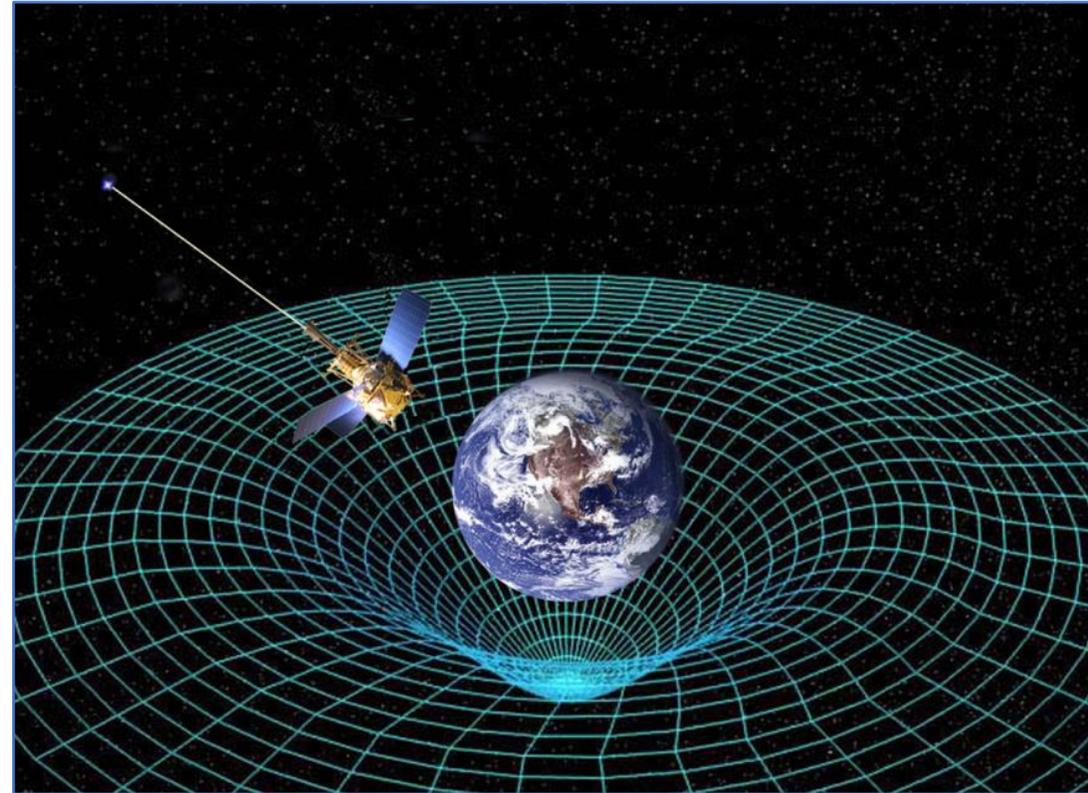
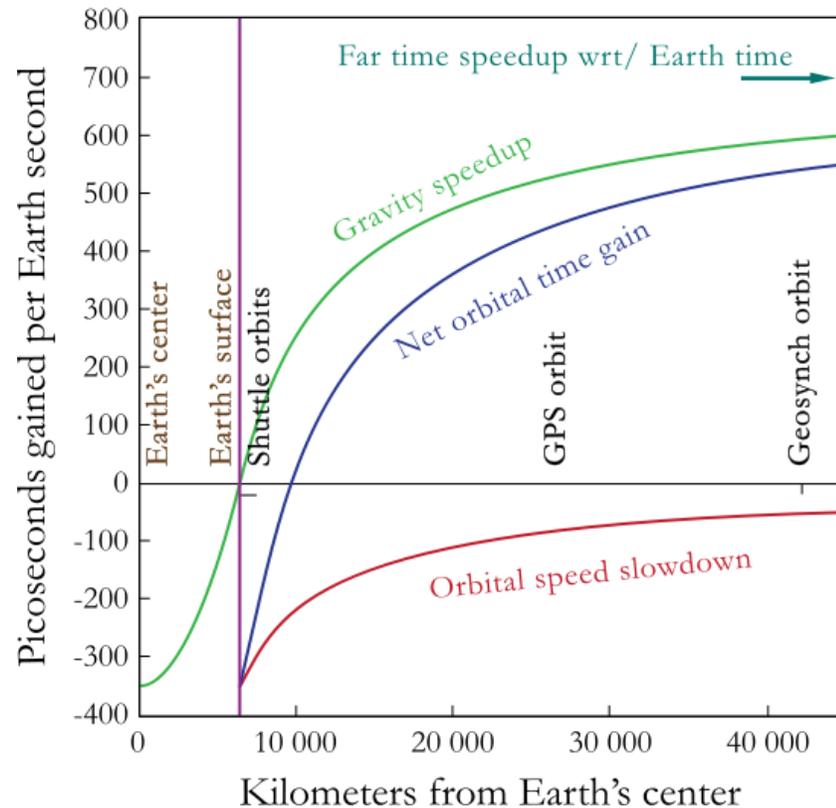
Time slows near mass and speeds up away from it.

Space-time is curved in the presence of mass

Mass causes curvature in space-time



Time Dilation Effects on Earth



Two effects:

- Time speeds up at the satellite in comparison to earth surface due to gravity
- Time slows down at the satellite due to high velocity compared to person on earth

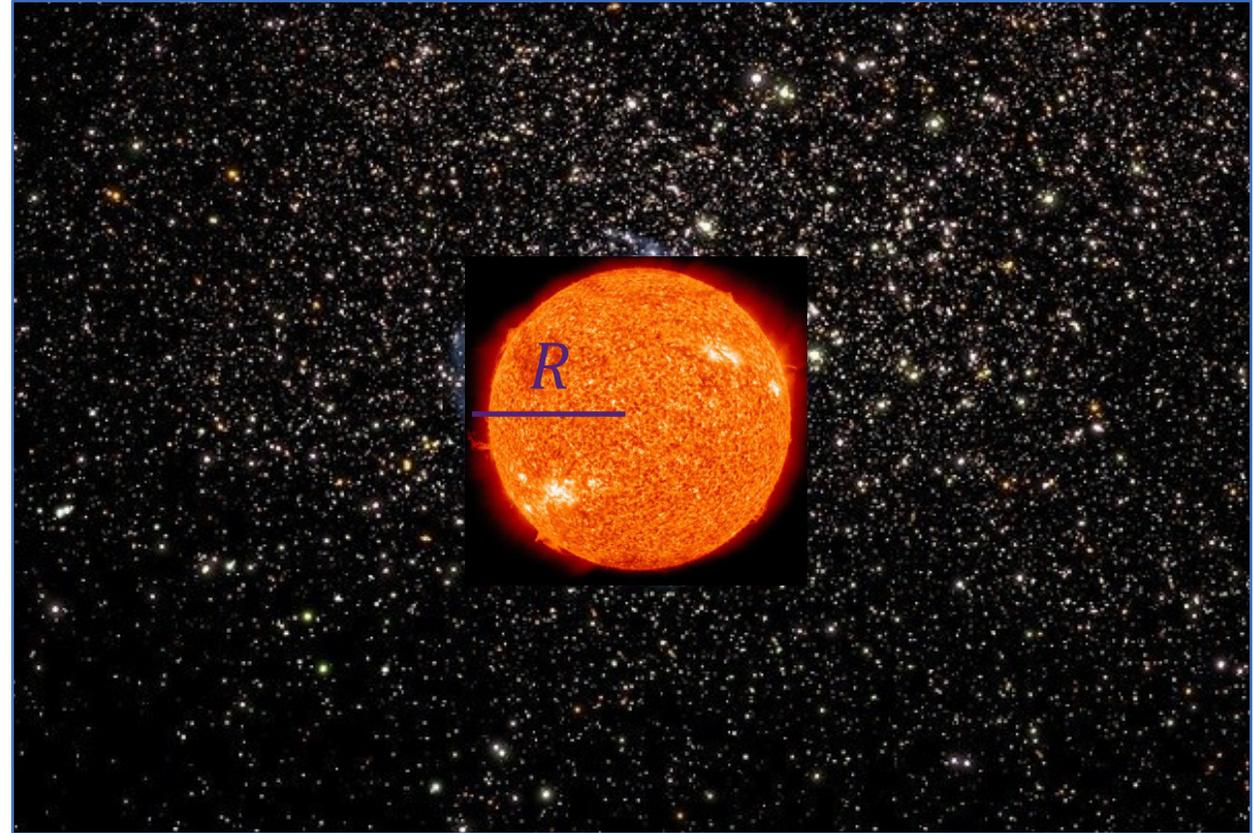
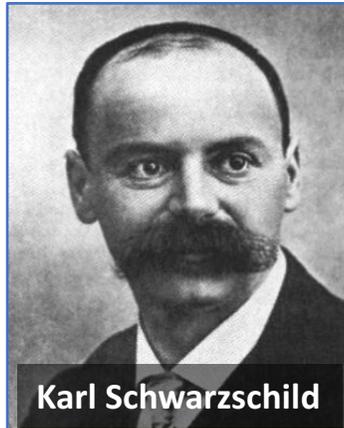
➔ Clocks in satellite and on earth de-synchronize with ~ 40 msec per day!

Gravitational time slowdown near a star with mass M :

$$\Delta t' = \Delta t \sqrt{1 - \frac{2GM}{Rc^2}}$$

Schwartzschild radius: $R_s = \frac{2GM}{c^2}$

$$\Delta t' = \Delta t \sqrt{1 - \frac{R_s}{R}}$$

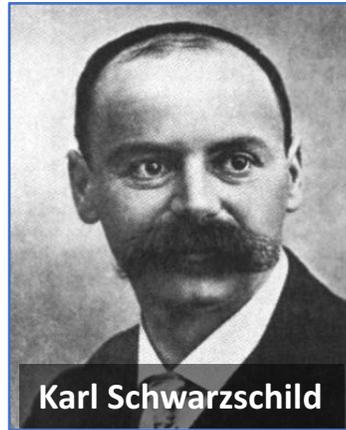


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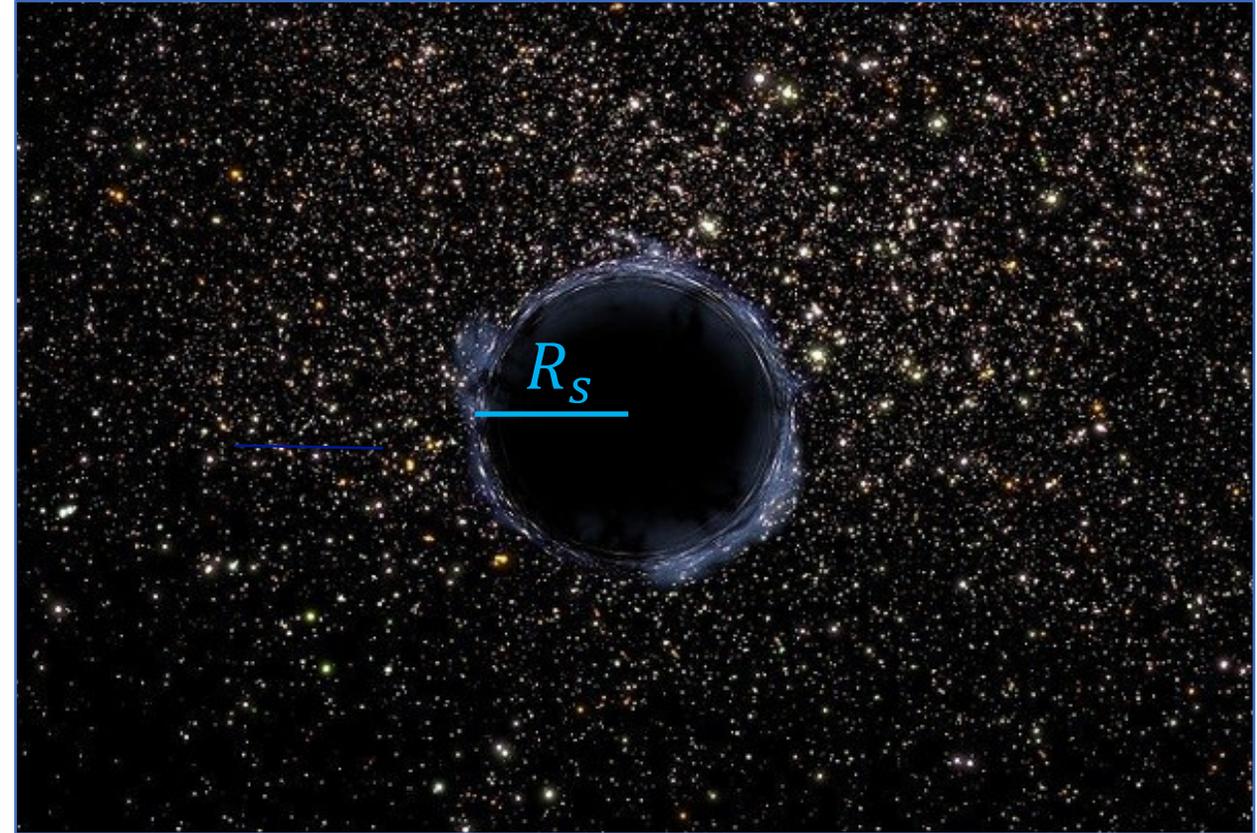
Time stand-still:

If $R = R_s$ then $\Delta t = 0$

Example our sun: $G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2$ (Newton's gravitation constant)

$$M_{\text{sun}} = 2 \times 10^{30} \text{ kg}$$

→ $R_s = 3 \text{ km}$ for a black hole



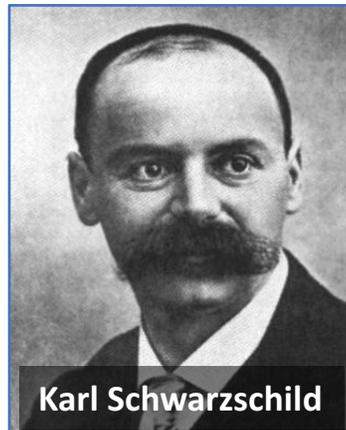
(Time stands still at the horizon of a black-hole)

Gravitational time slowdown near a star with mass M :

$$\Delta t' = \Delta t \sqrt{1 - \frac{2GM}{Rc^2}}$$

Schwartzschild radius: $R_s = \frac{2GM}{c^2}$

$$\Delta t' = \Delta t \sqrt{1 - \frac{R_s}{R}}$$



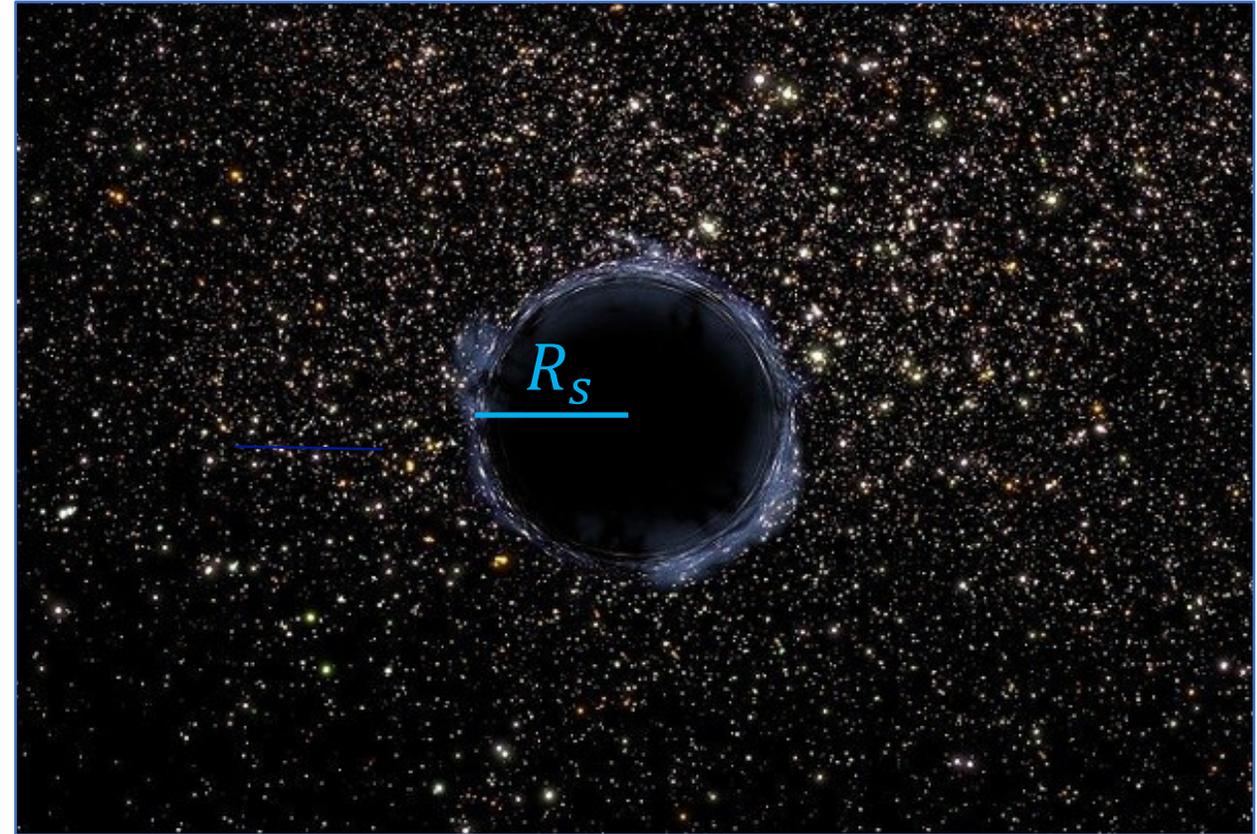
Time stand-still:

If $R = R_s$ then $\Delta t = 0$

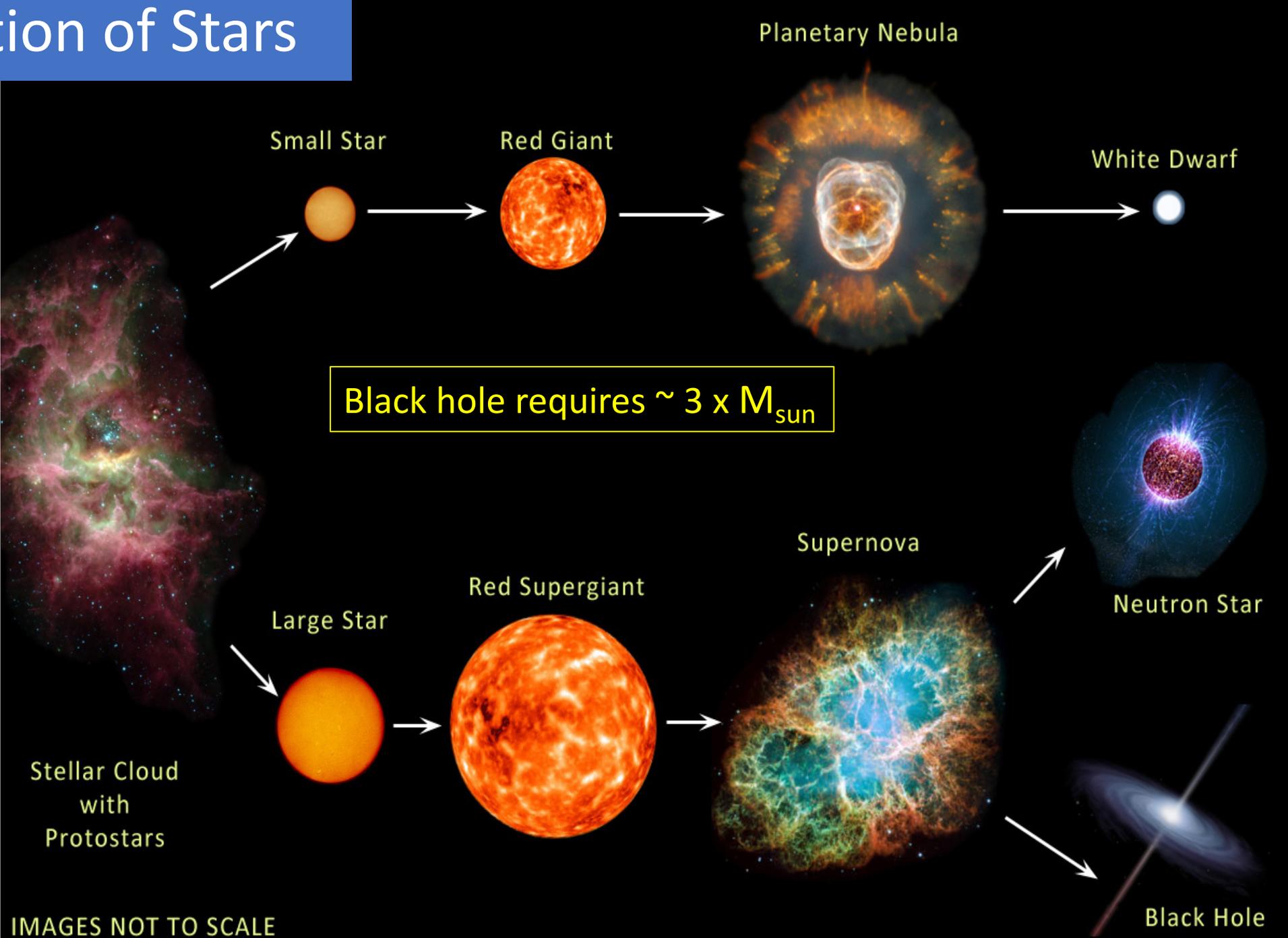
Example our earth: $G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2$ (Newton's gravitation constant)

$$M_{\text{earth}} = 6 \times 10^{24} \text{ kg}$$

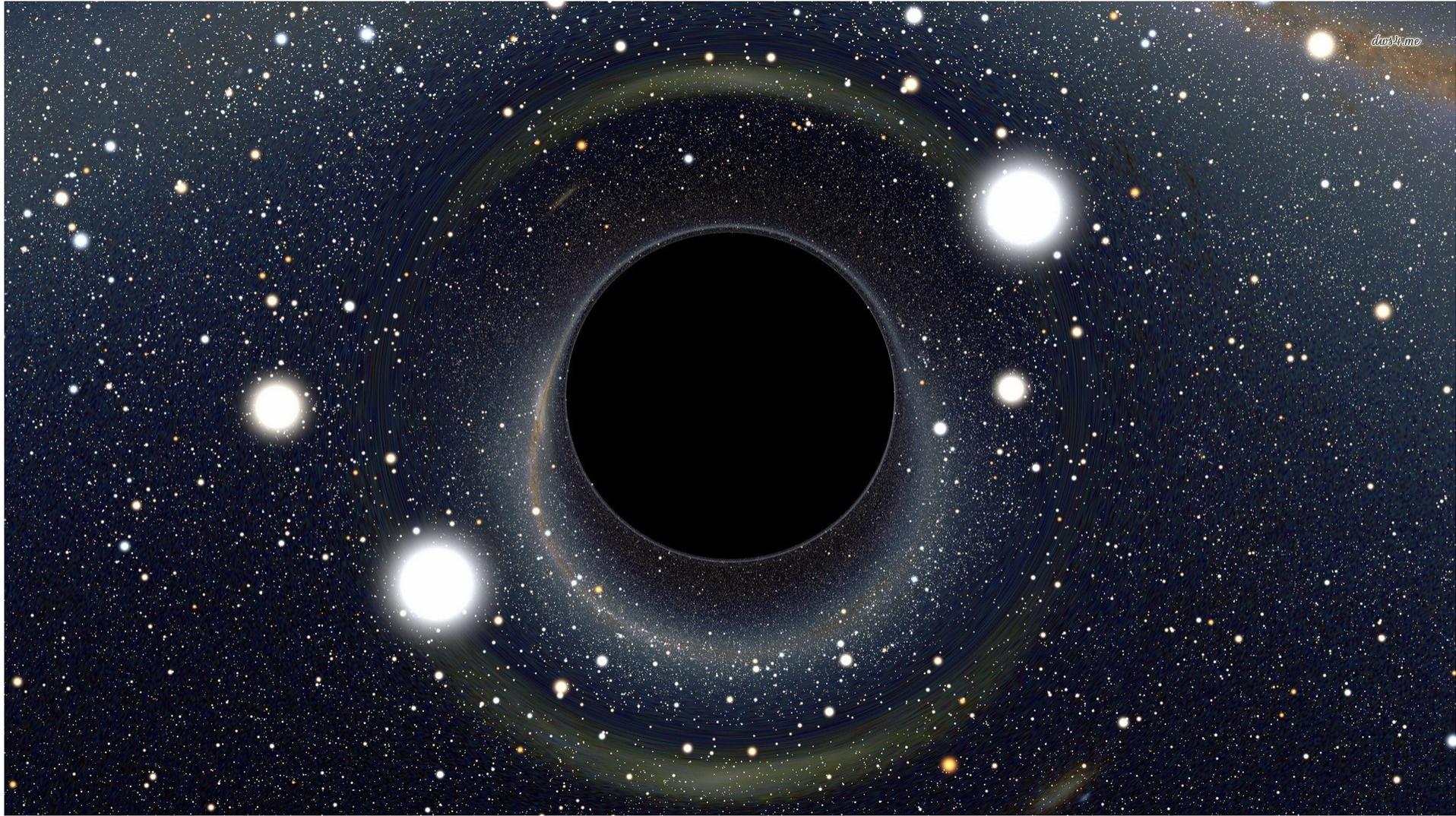
→ $R_s = 9 \text{ mm}$ for a black hole



(Time stands still at the horizon of a black-hole)

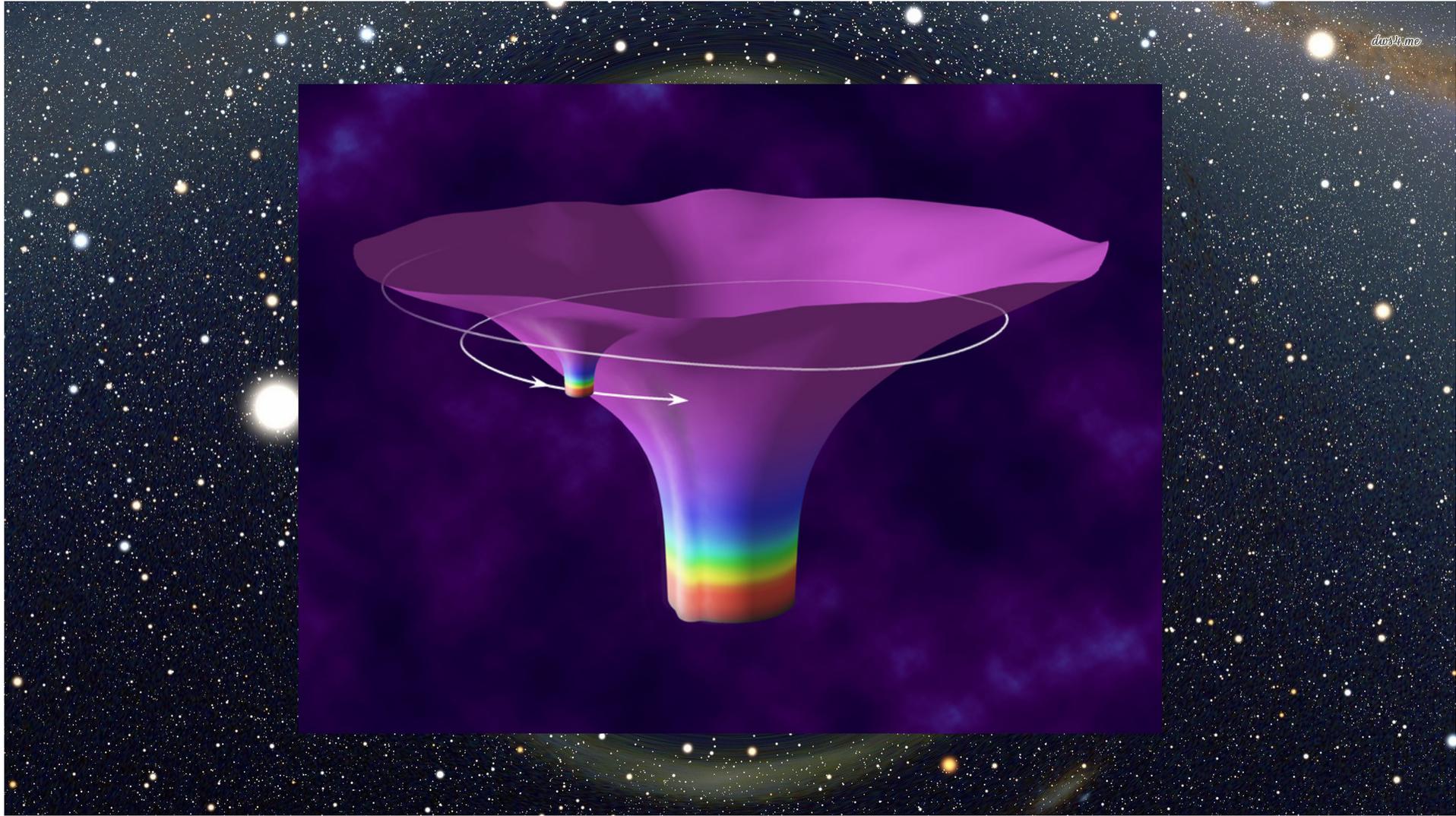


IMAGES NOT TO SCALE



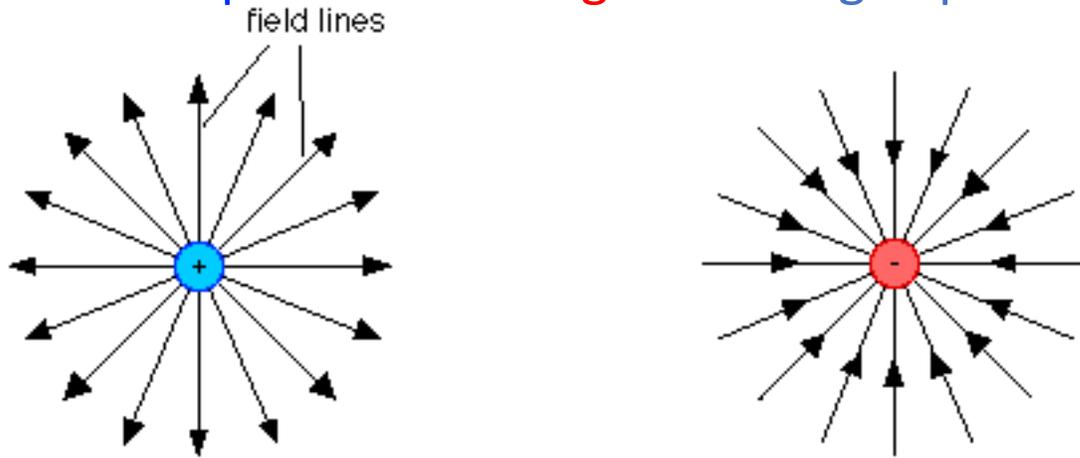
Purely curved space-time!

What is a black hole?



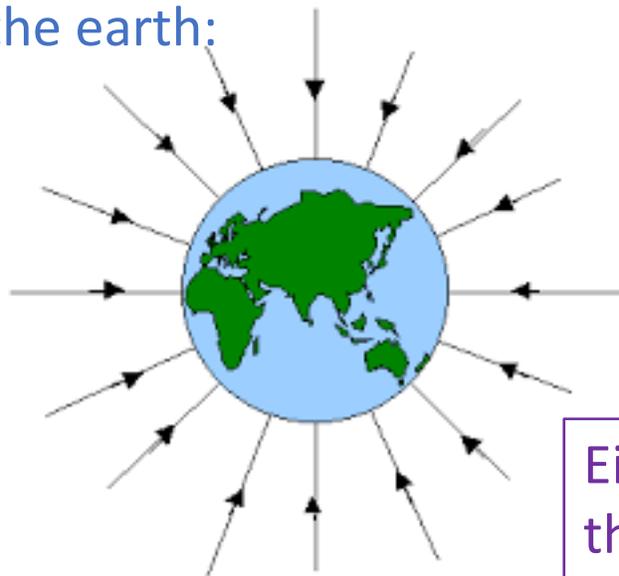
What happens when two black holes meet?

Electric field of **positive** and **negative** charged particle:



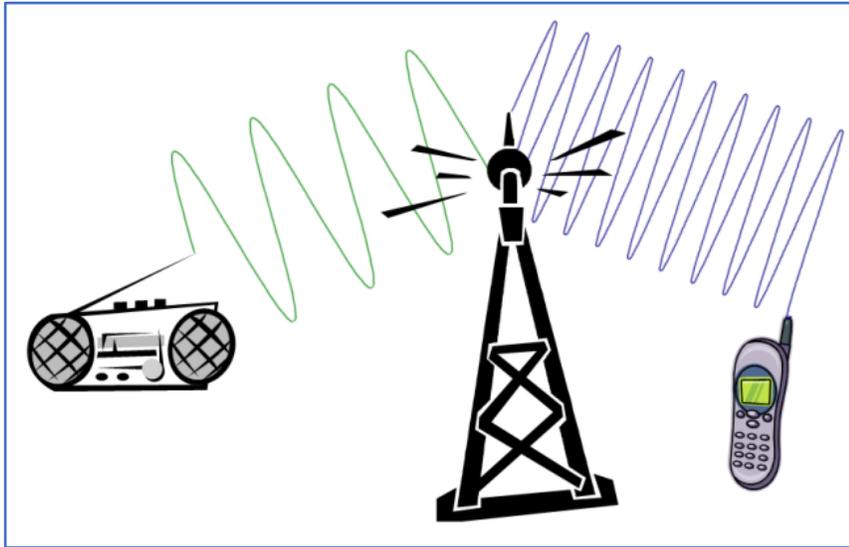
$$F_e = q E = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2}$$

Gravitational field of the earth:



$$F_g = m g = -G \frac{mM}{R^2}$$

Einstein spent most of his life looking for a unified theory of electromagnetism and general relativity.



Electromagnetic waves:

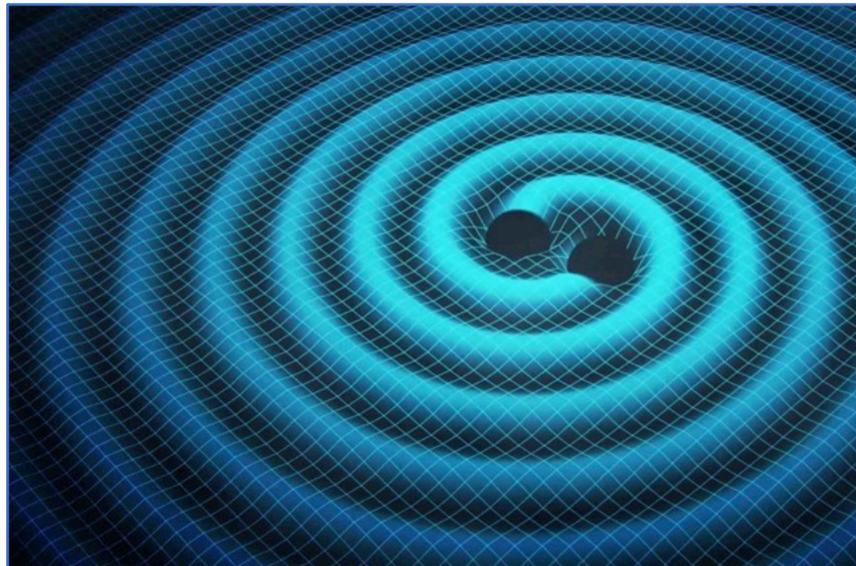
Caused by accelerating electric particles (electrons)
eg.: radio-emission

Maxwell equations:

$$\partial_{\mu} F^{\mu\nu} = \frac{4\pi}{c} J^{\nu}$$

E and B fields

Electric charge
and currents



Gravitational Waves:

Caused by moving masses.

Requires very heavy masses → **black holes.**

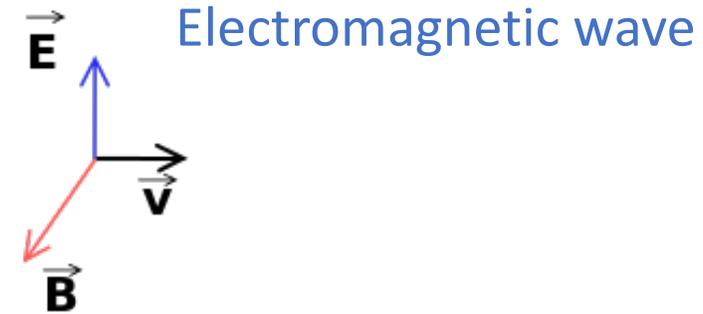
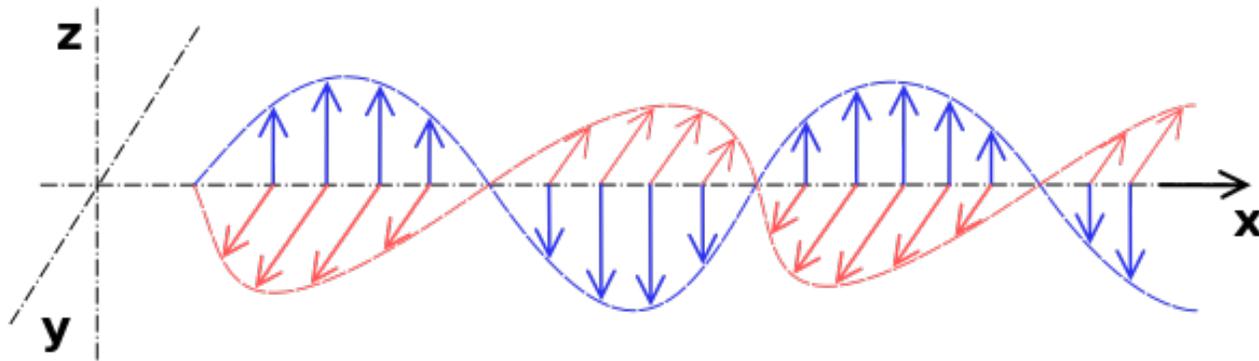
(Einstein thought these couldn't be observed)

Einstein equations:

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

Space-time fields

Mass and mass-flow



Electromagnetic wave:

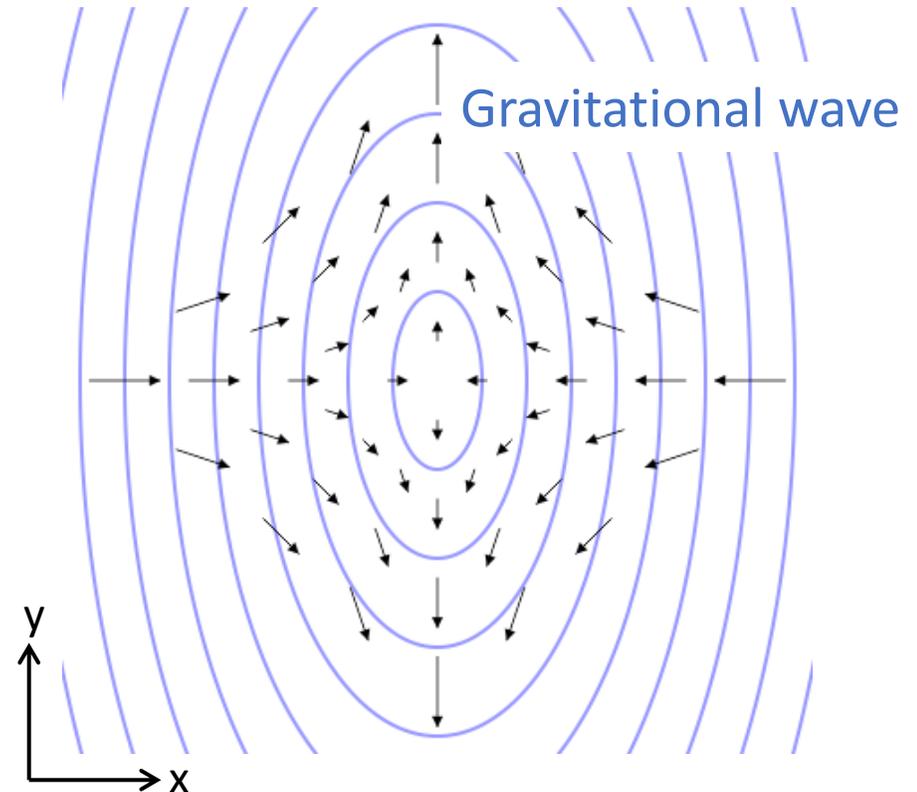
Changing electric and magnetic field propagating through space.

Caused by moving (accelerating!) electric charges.

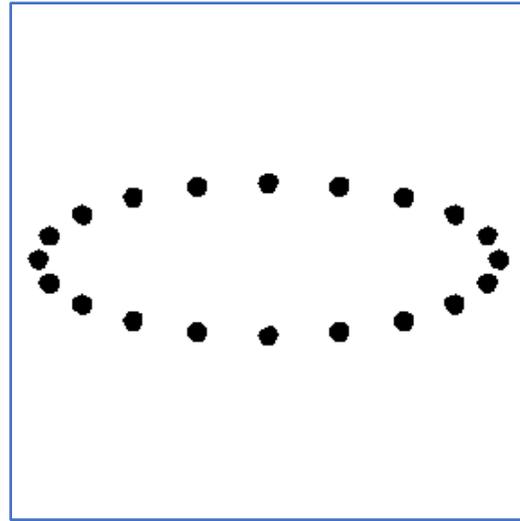
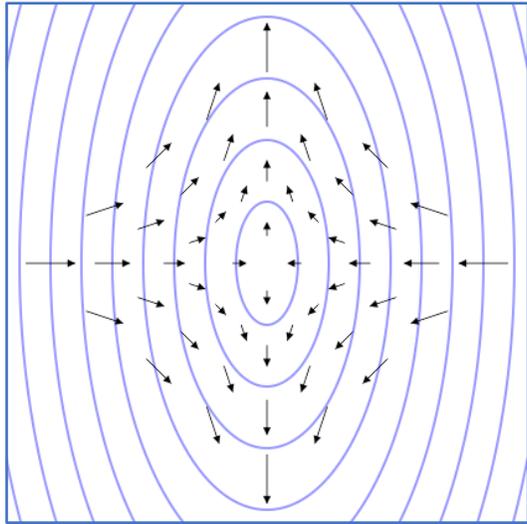
Gravitational wave:

Changing space-time field.

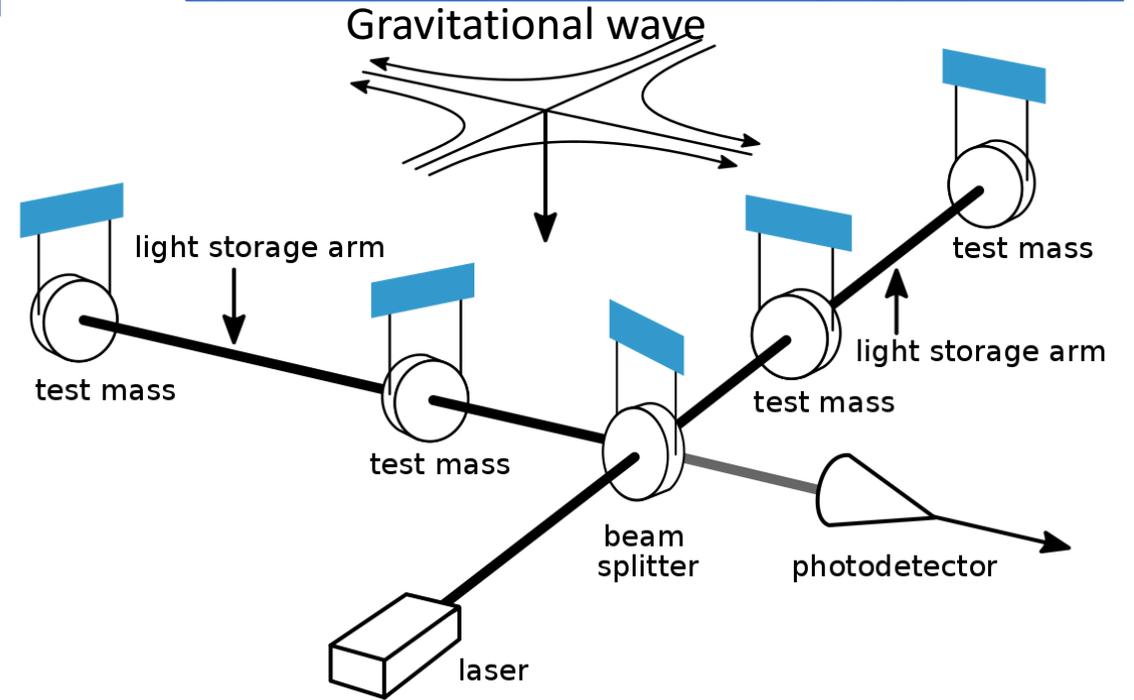
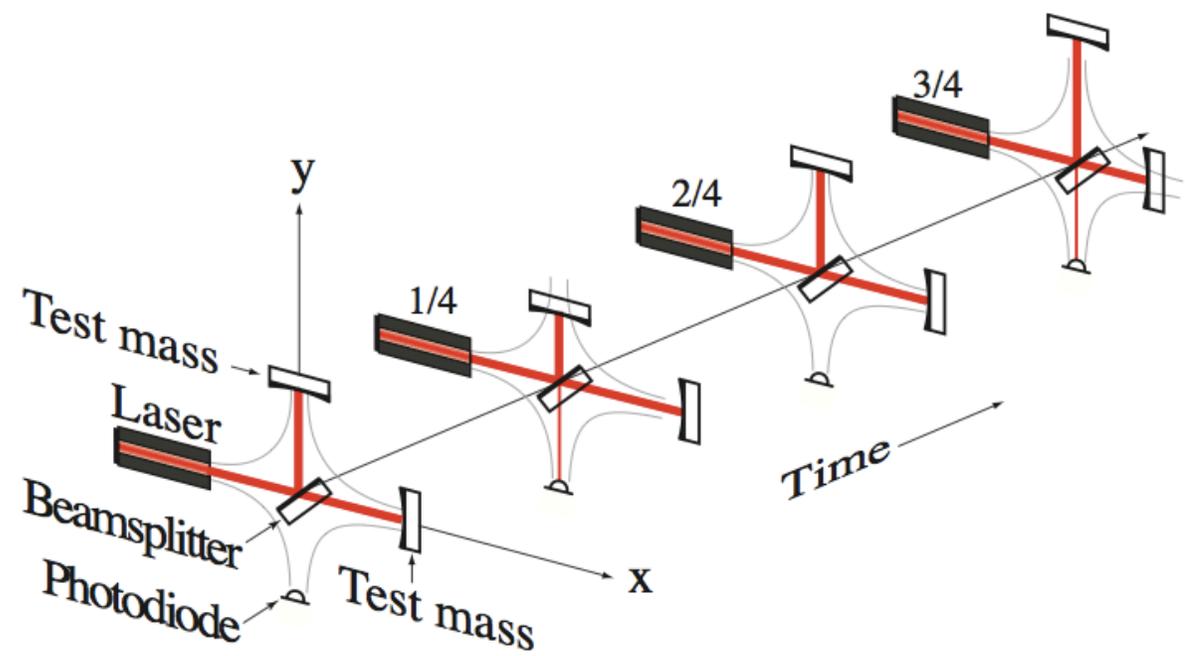
Caused by moving (accelerating!) masses.



Remember the interferometer!

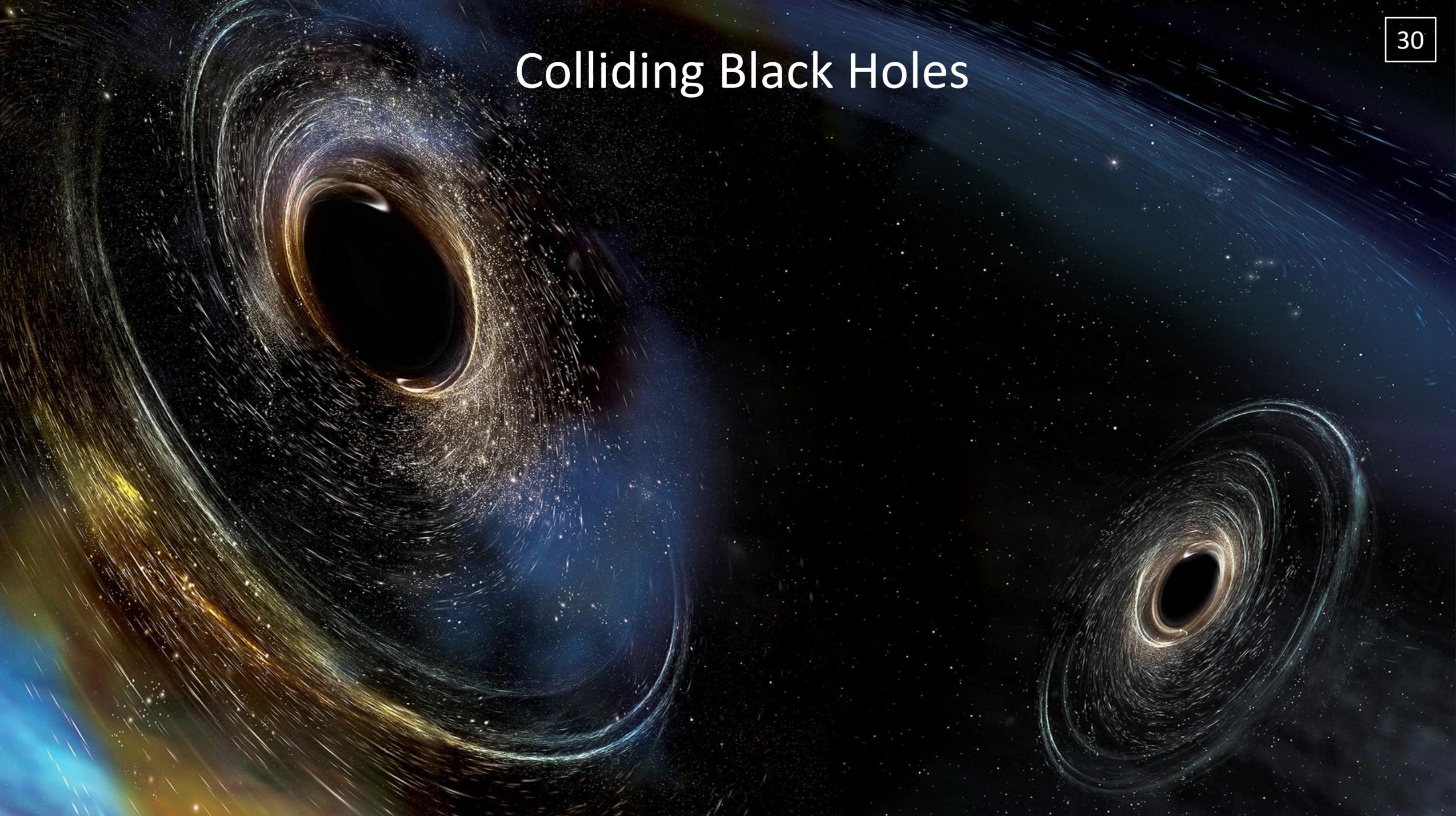


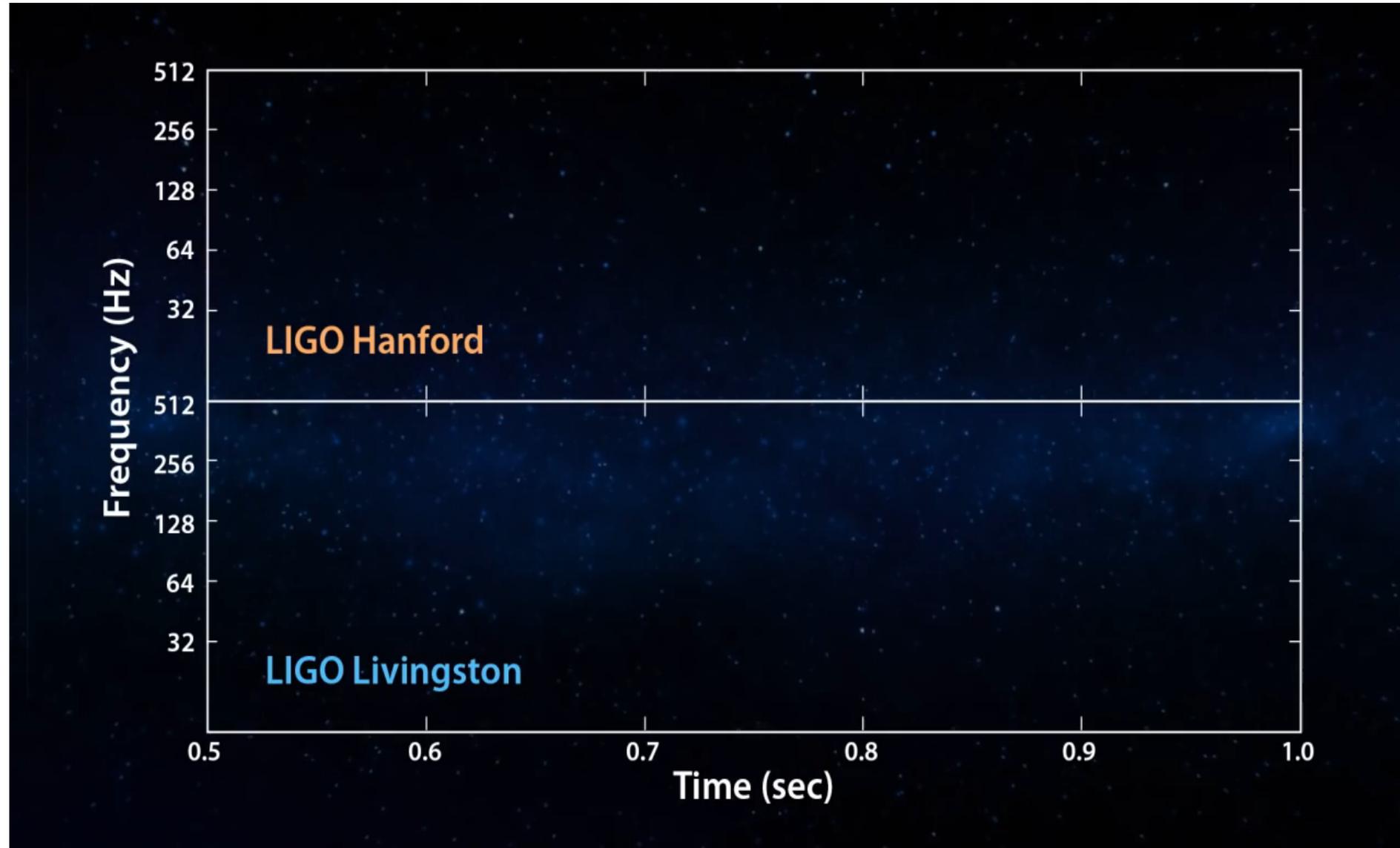
Virgo experiment near Pisa





Colliding Black Holes





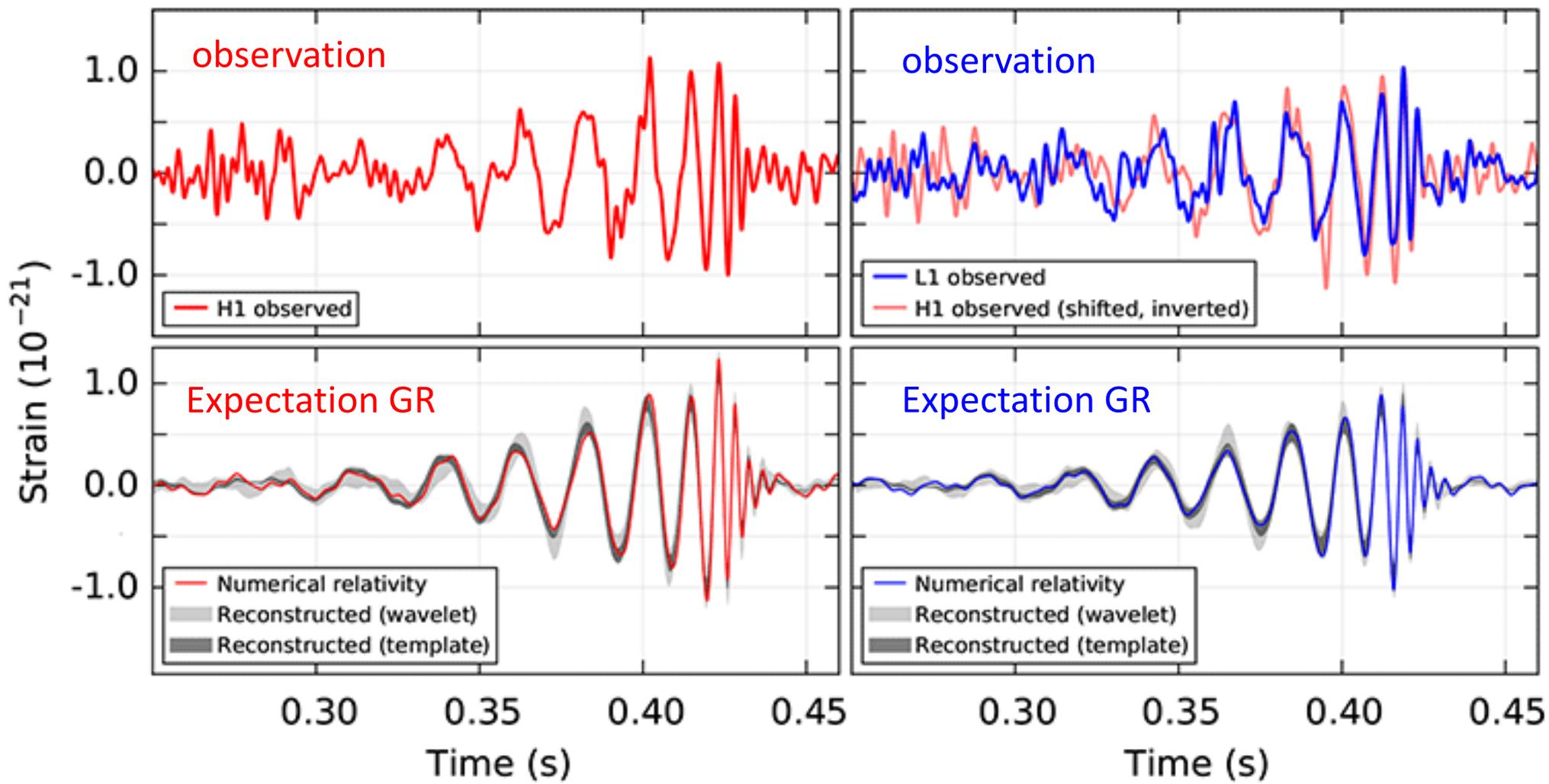
“Chirp” of colliding black holes at 1.3 billion lightyears distance

Consistent signals seen in Washington and Louisiana

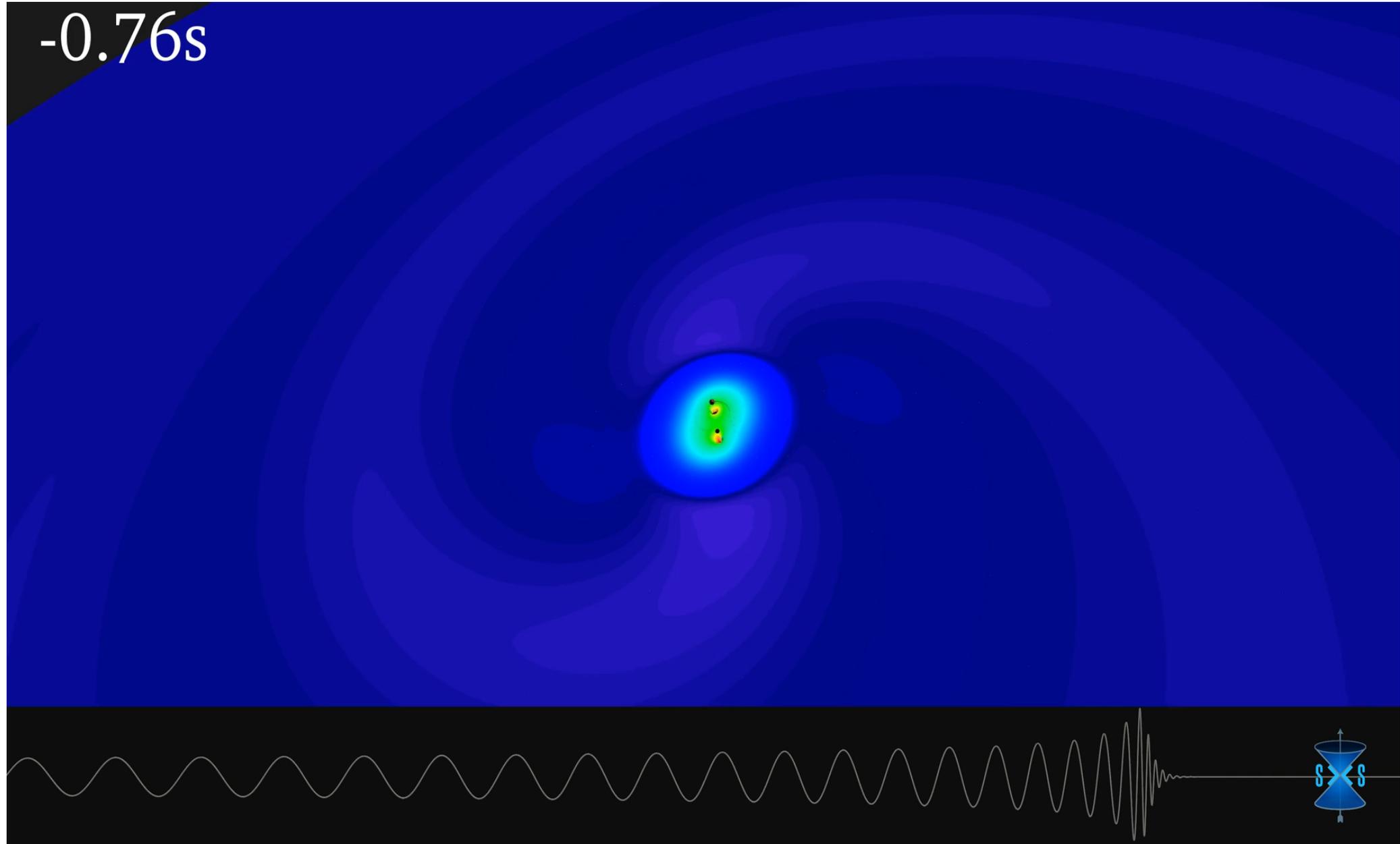
(GW150914)

Hanford, Washington

Livingston, Louisiana

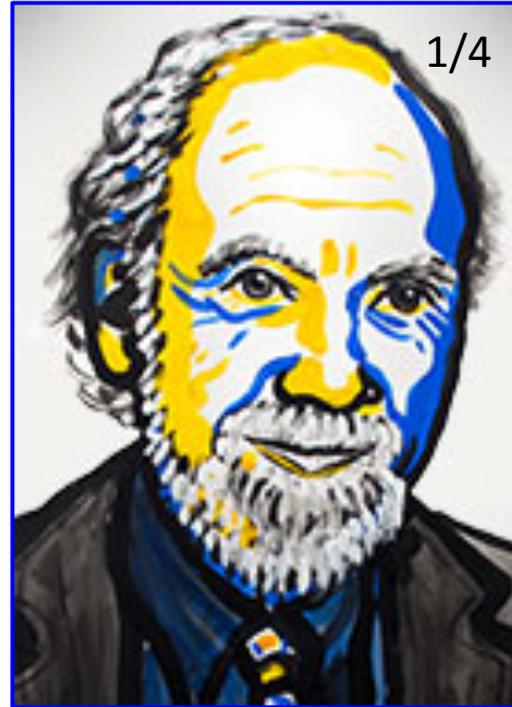








Rainer Weiss

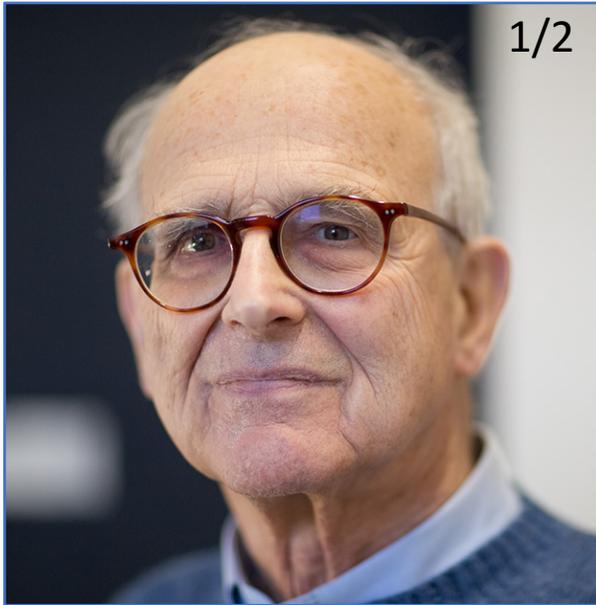


Barry C. Barish

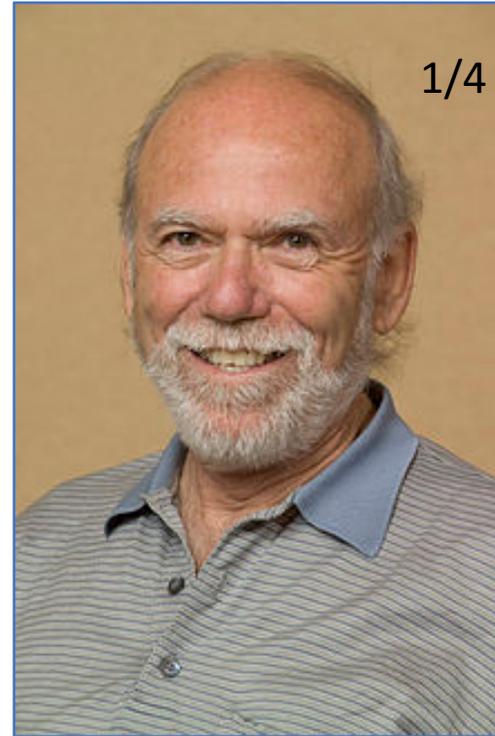


Kip S. Thorne

“For decisive contributions to the LIGO detector and the observation of gravitational waves”



Rainer Weiss



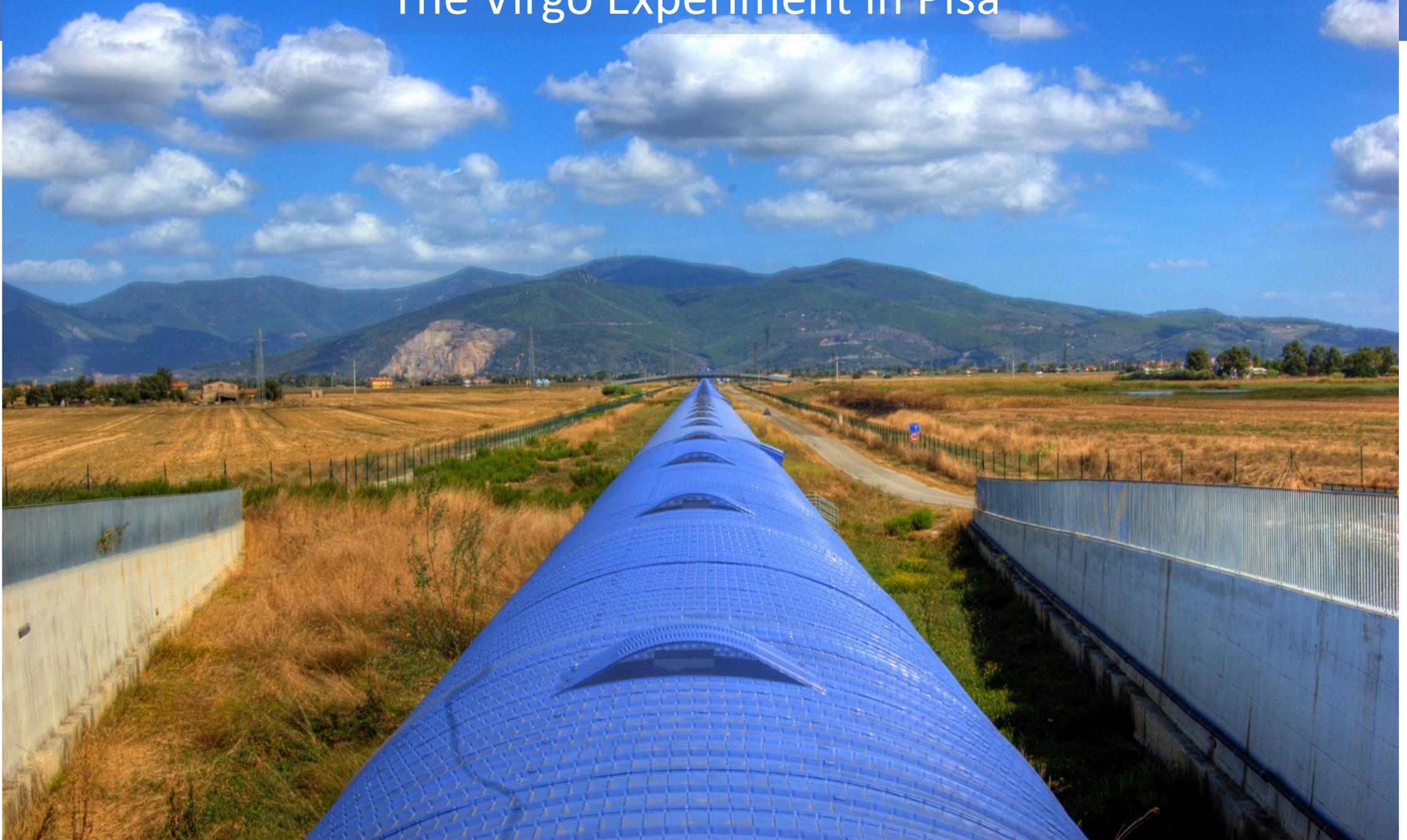
Barry C. Barish



Kip S. Thorne

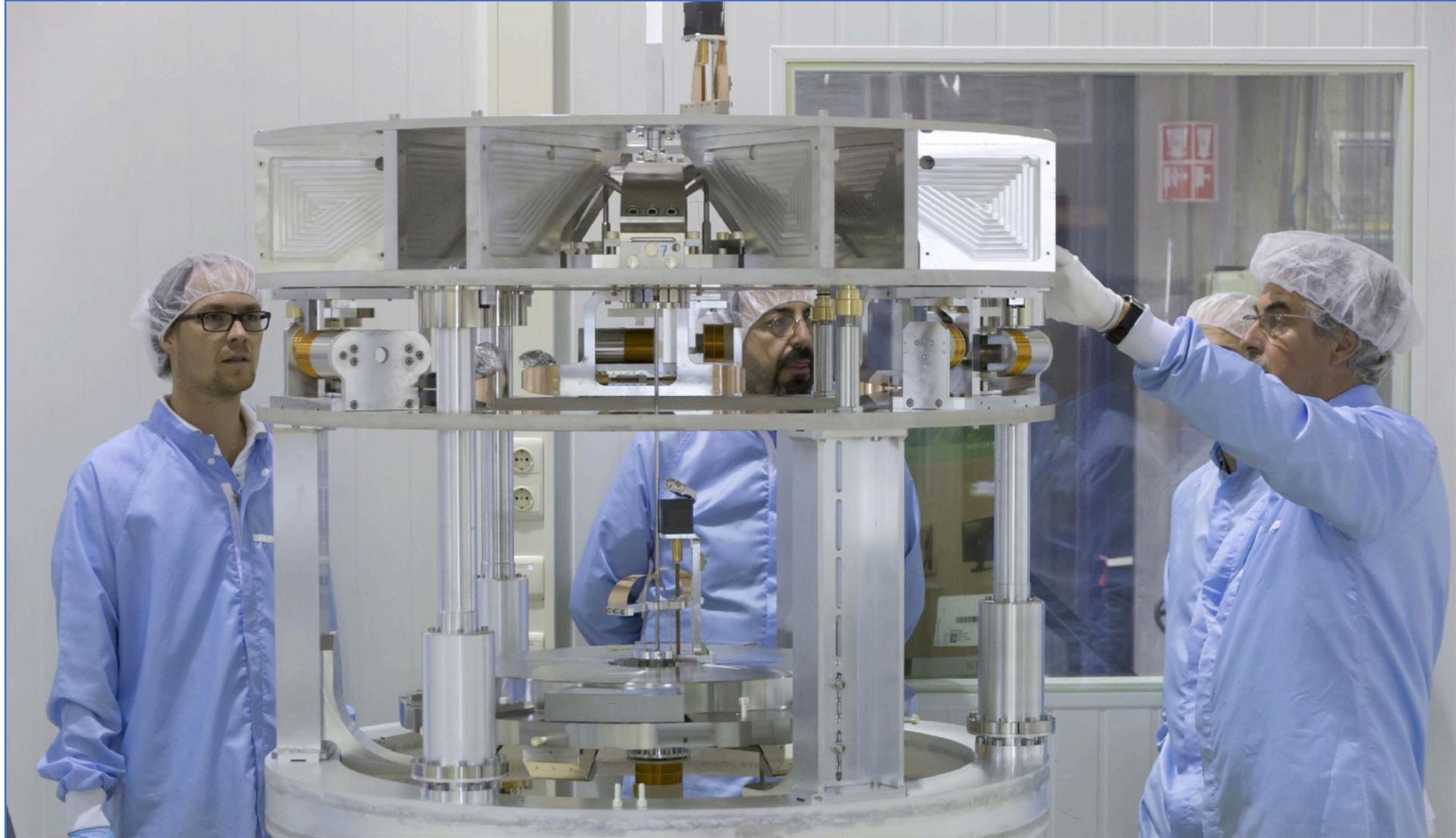
“For decisive contributions to the LIGO detector and the observation of gravitational waves”

The Virgo Experiment in Pisa

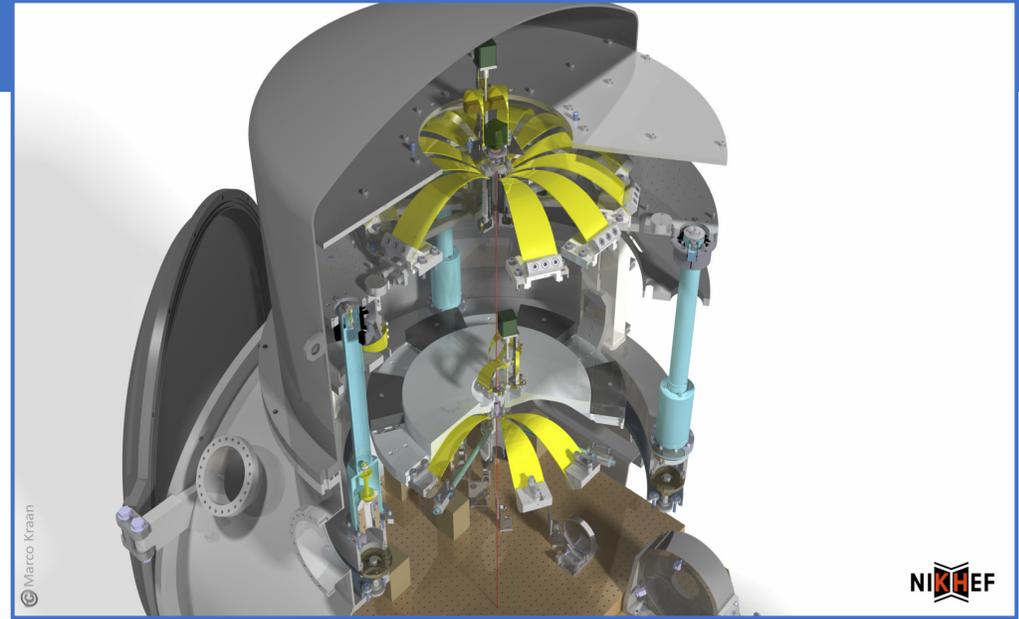




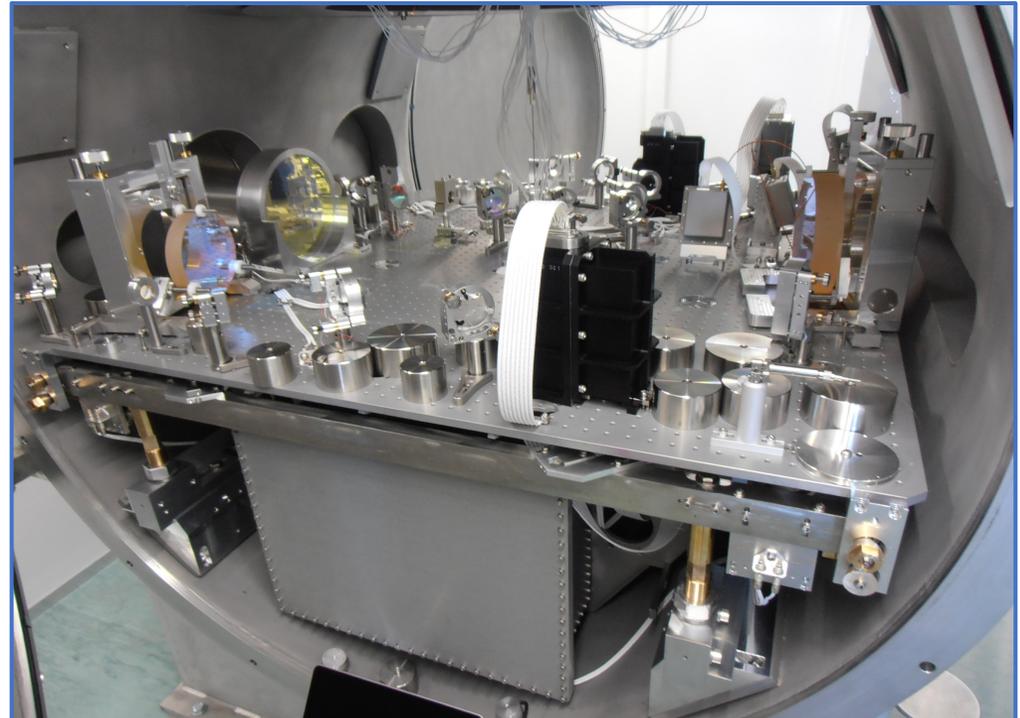
Largest vacuüm vessel in Europe: Pressure $\sim 10^{-10}$ mbar



Seismic Damping Table

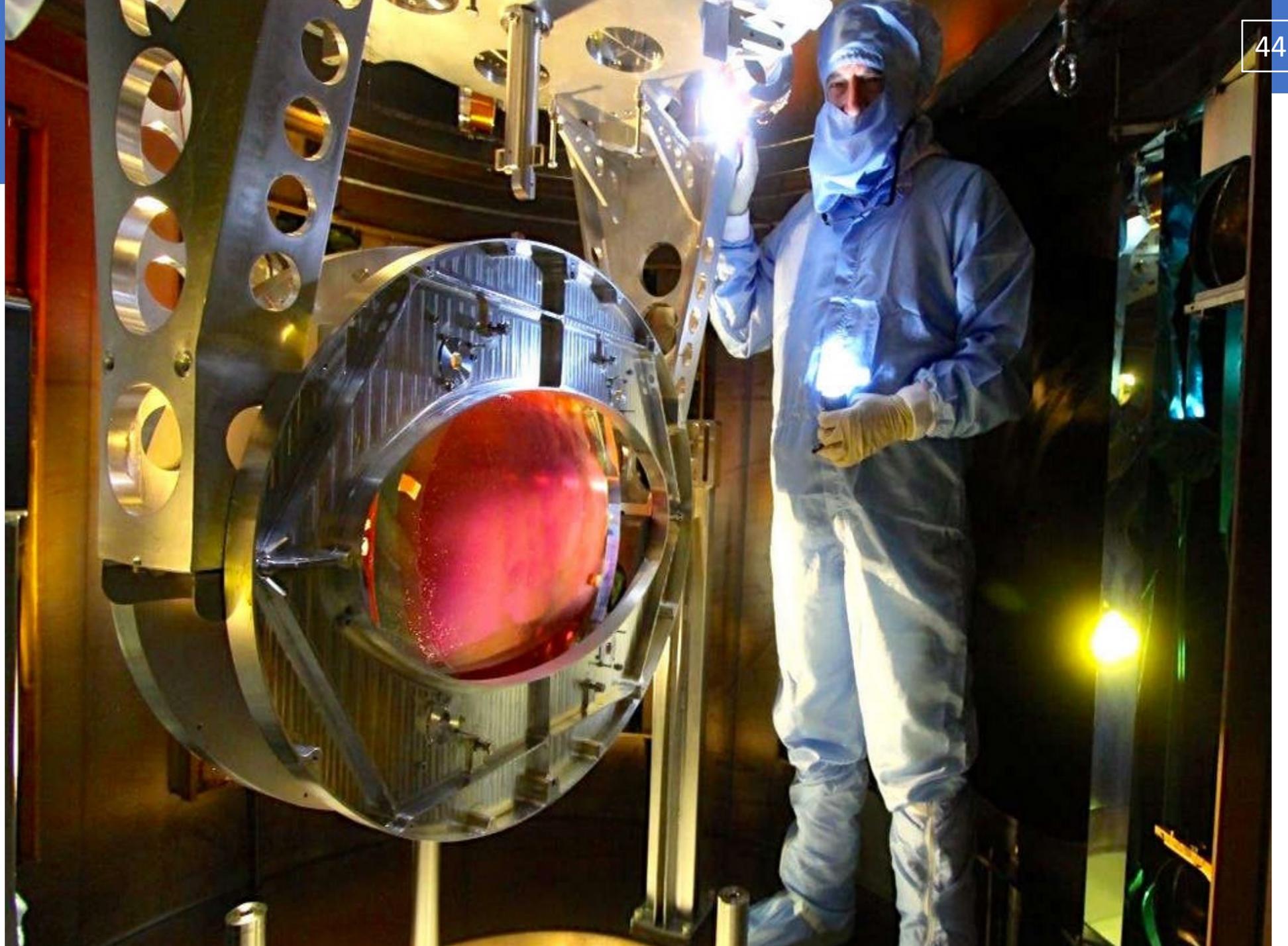


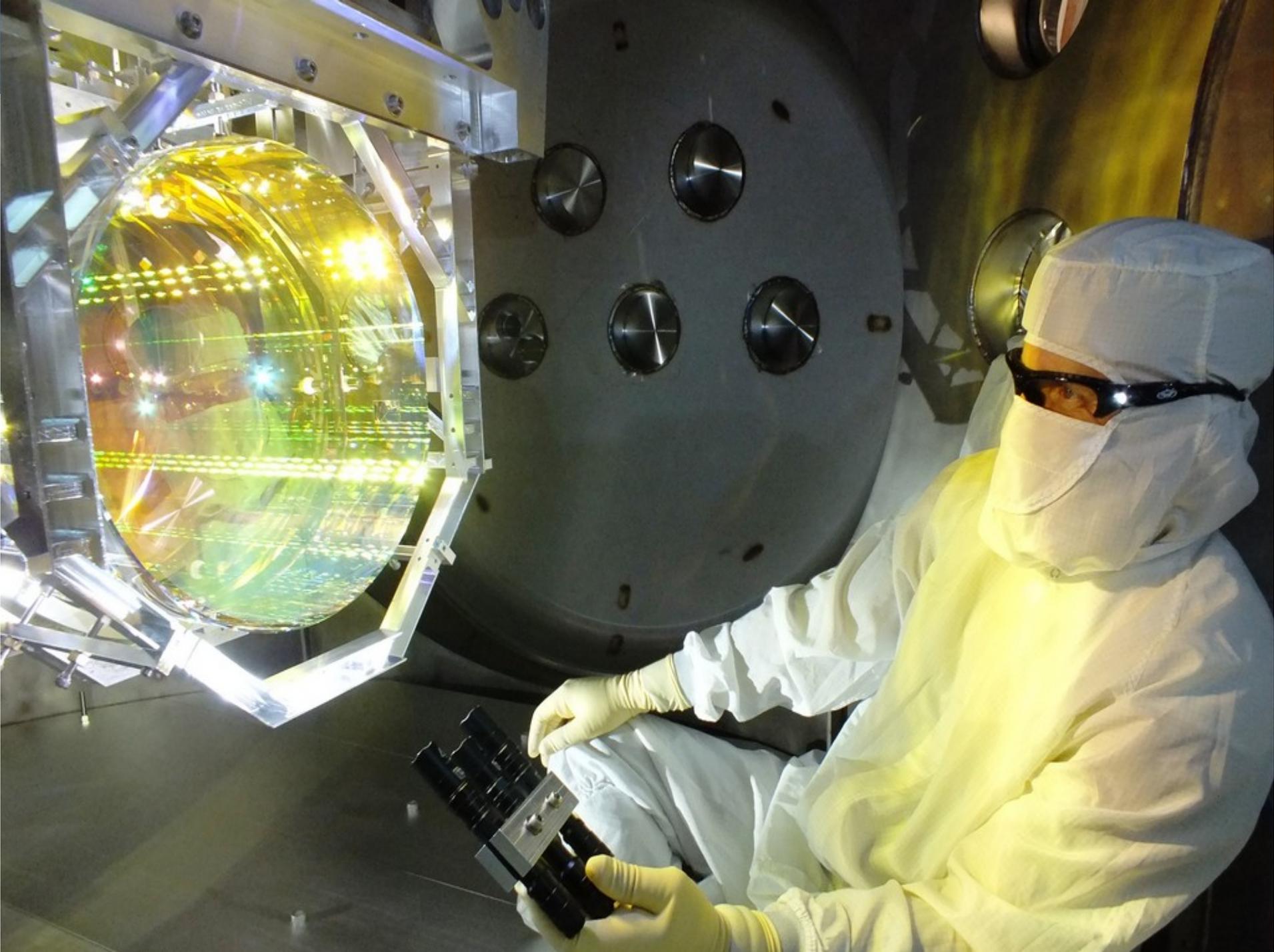
© Marco Kraan

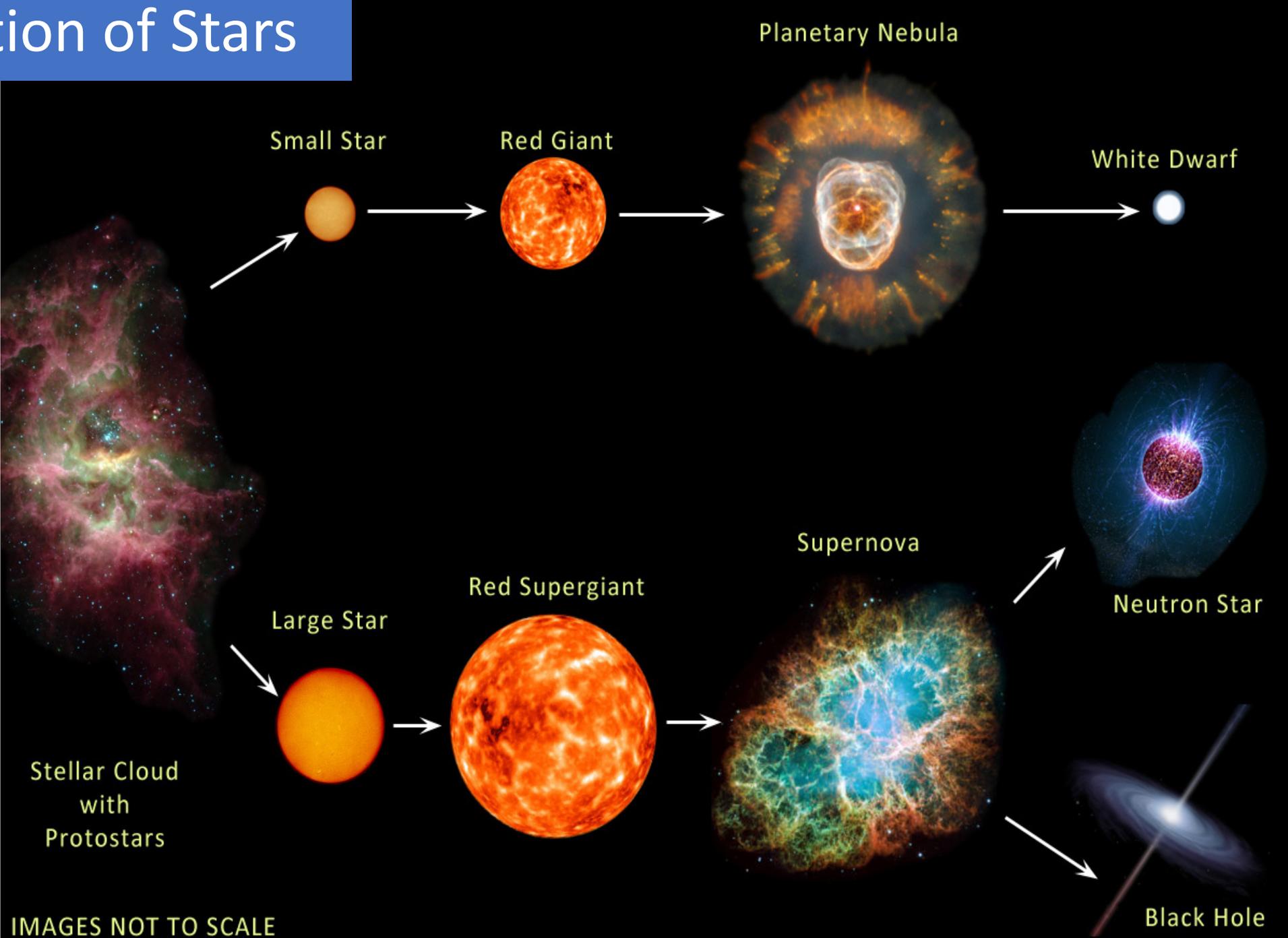




Beam Splitter



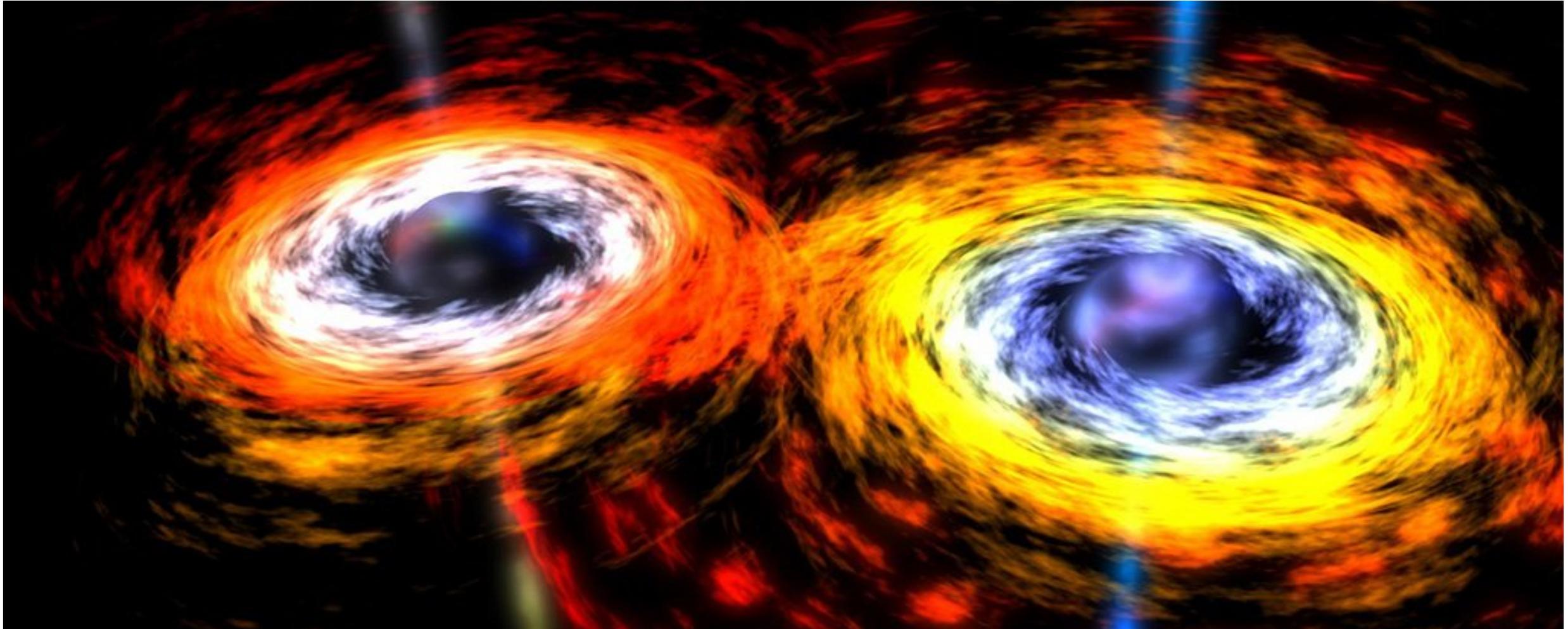


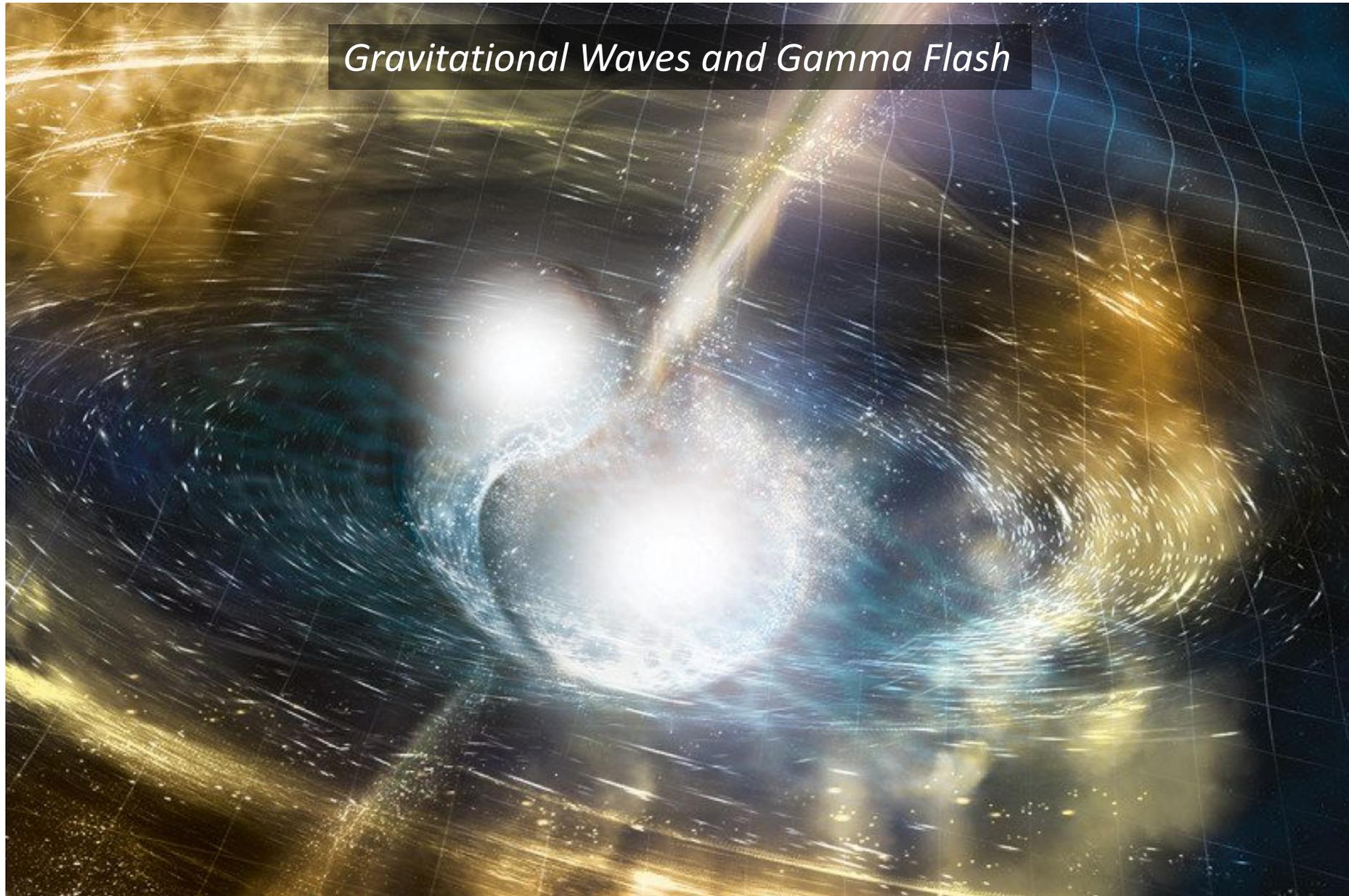


IMAGES NOT TO SCALE



Gravitational Waves and ...



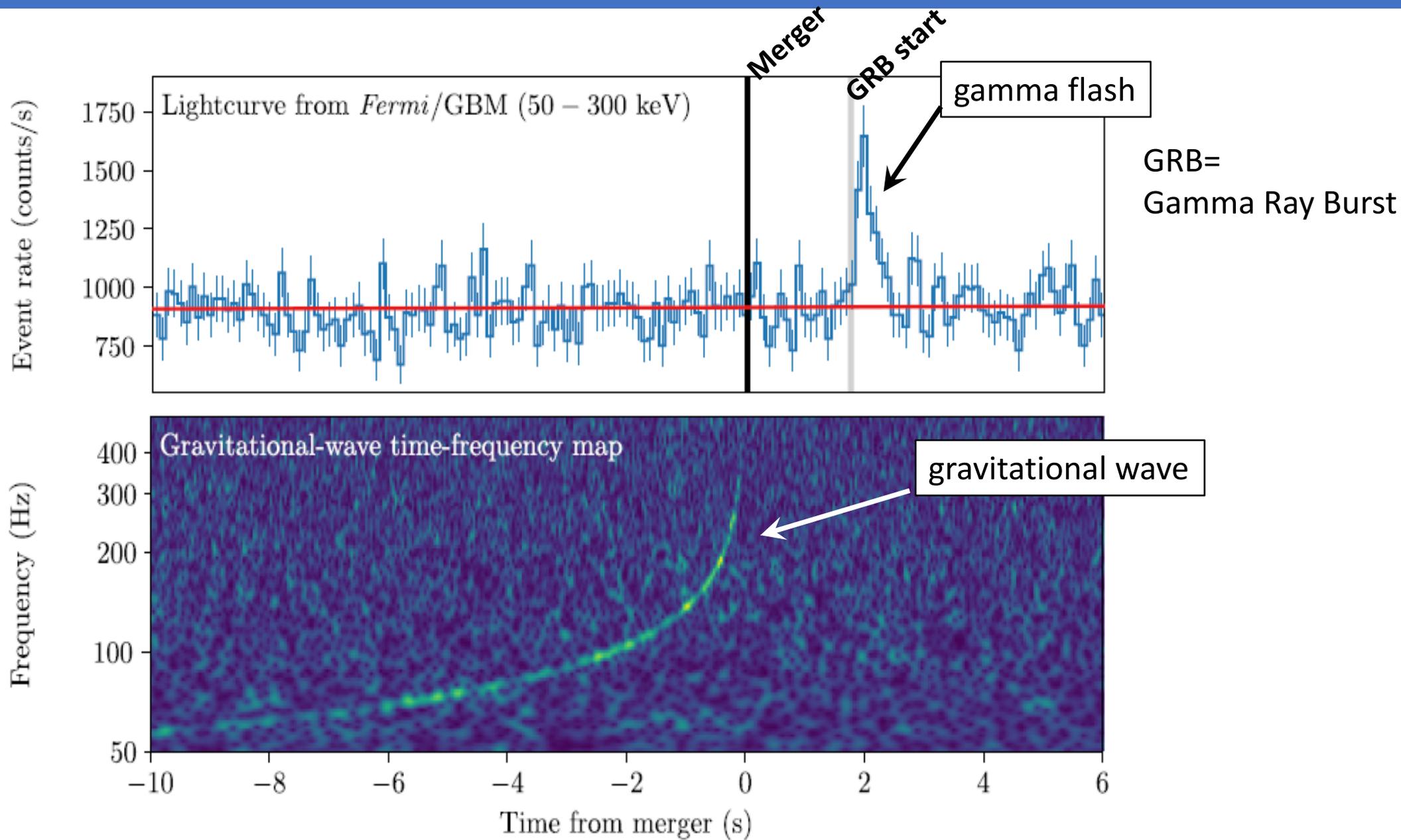


Gamma flash 1.7 sec later...



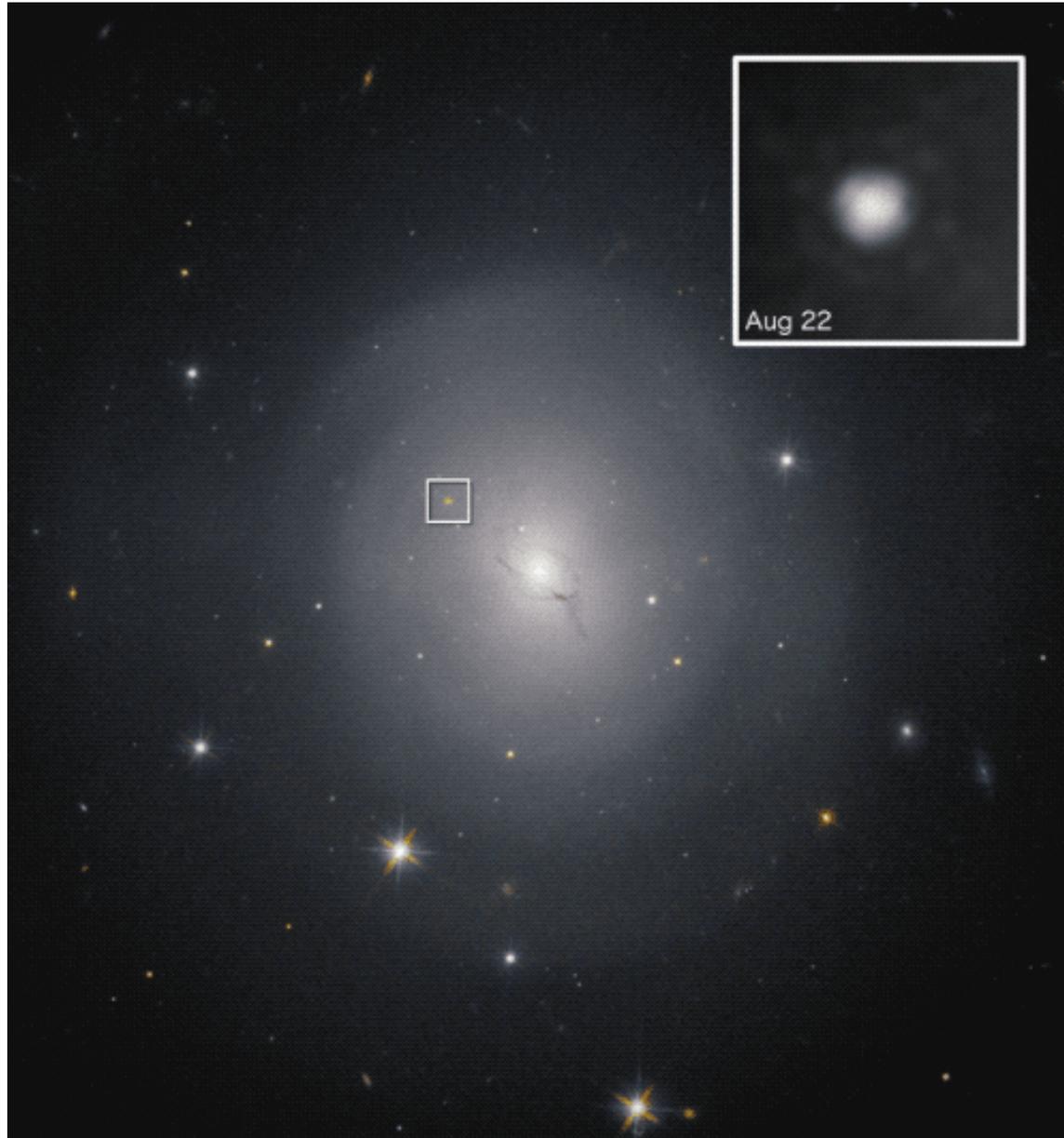
INTEGRAL

Fermi Space Telescope



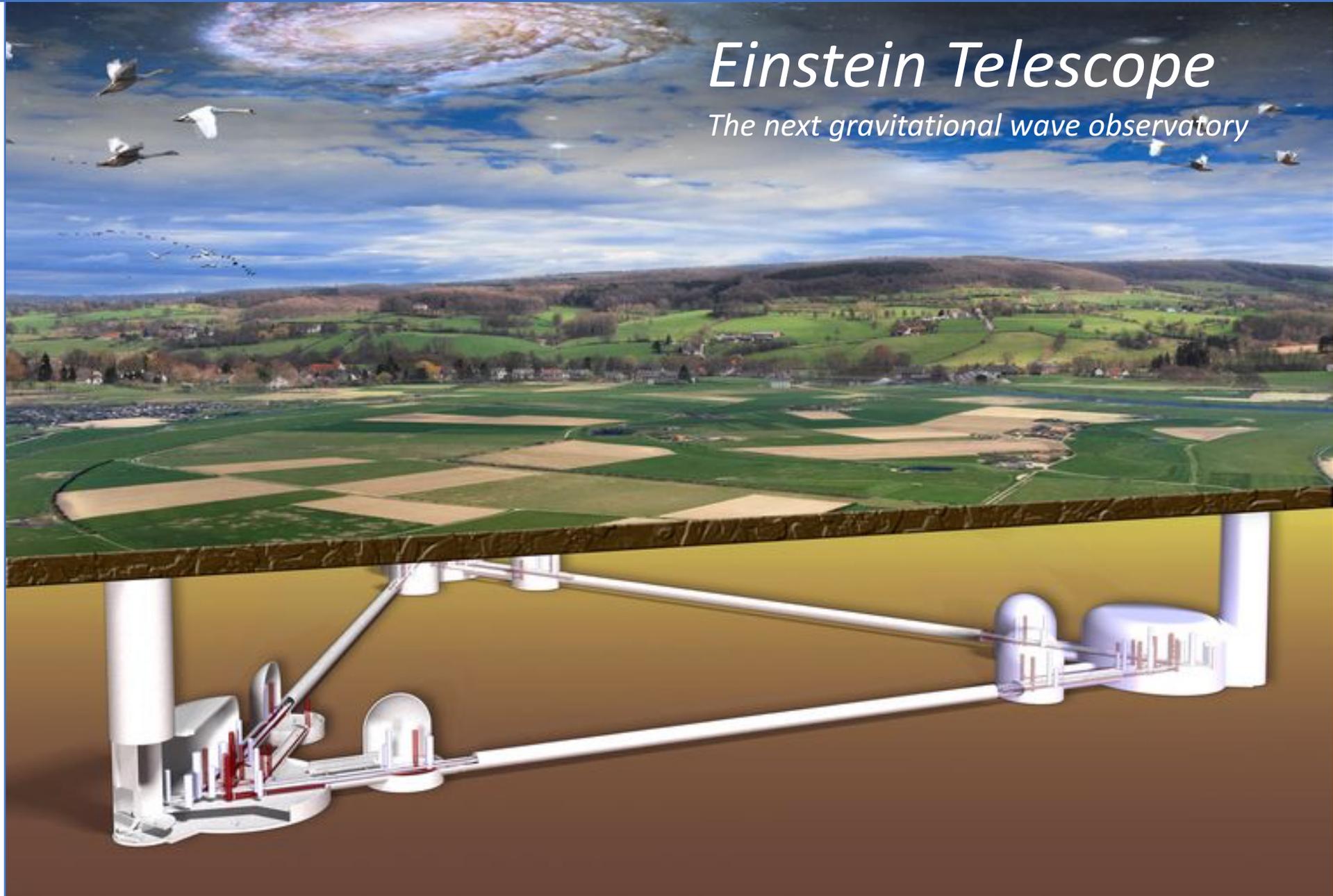


Kilonova: production heavy elements: gold, platinum etc.

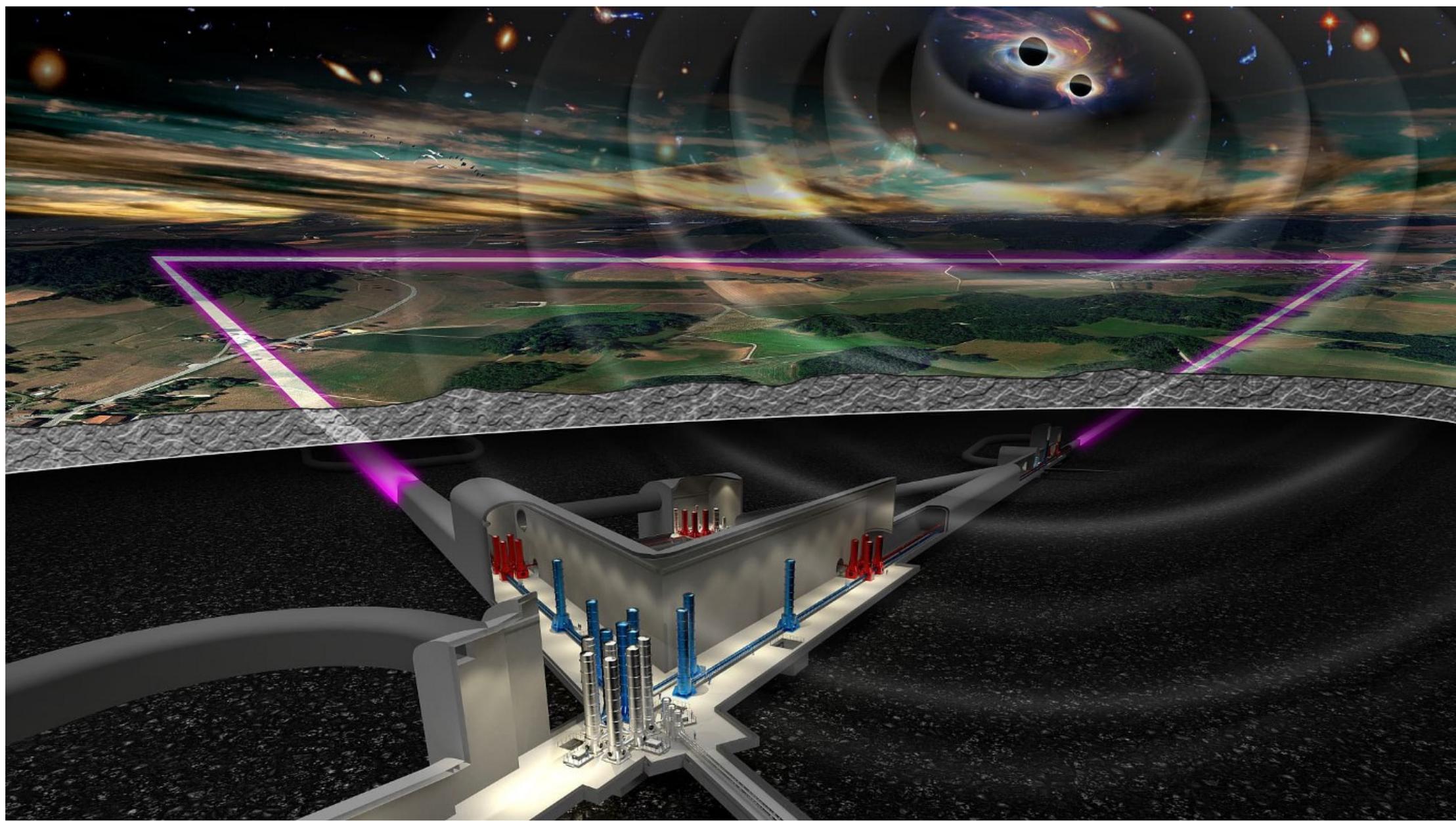


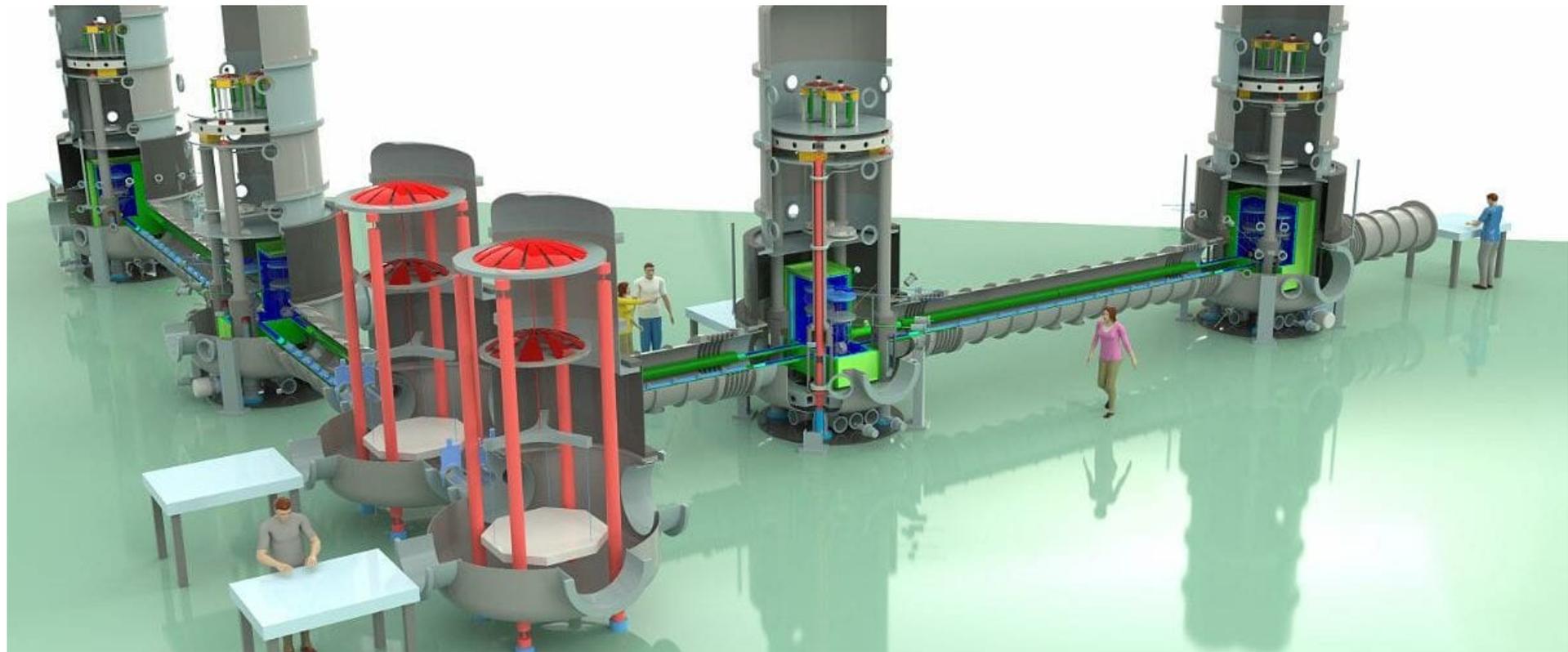
Possible Future Facility...





Possible Future Facility...





RWTH Aachen, UCL Louvain, Hasselt, Ghent, Antwerp, VUB Brussels, ULB Brussels, Liege, Radboud University Nijmegen, TU Eindhoven and Hamburg

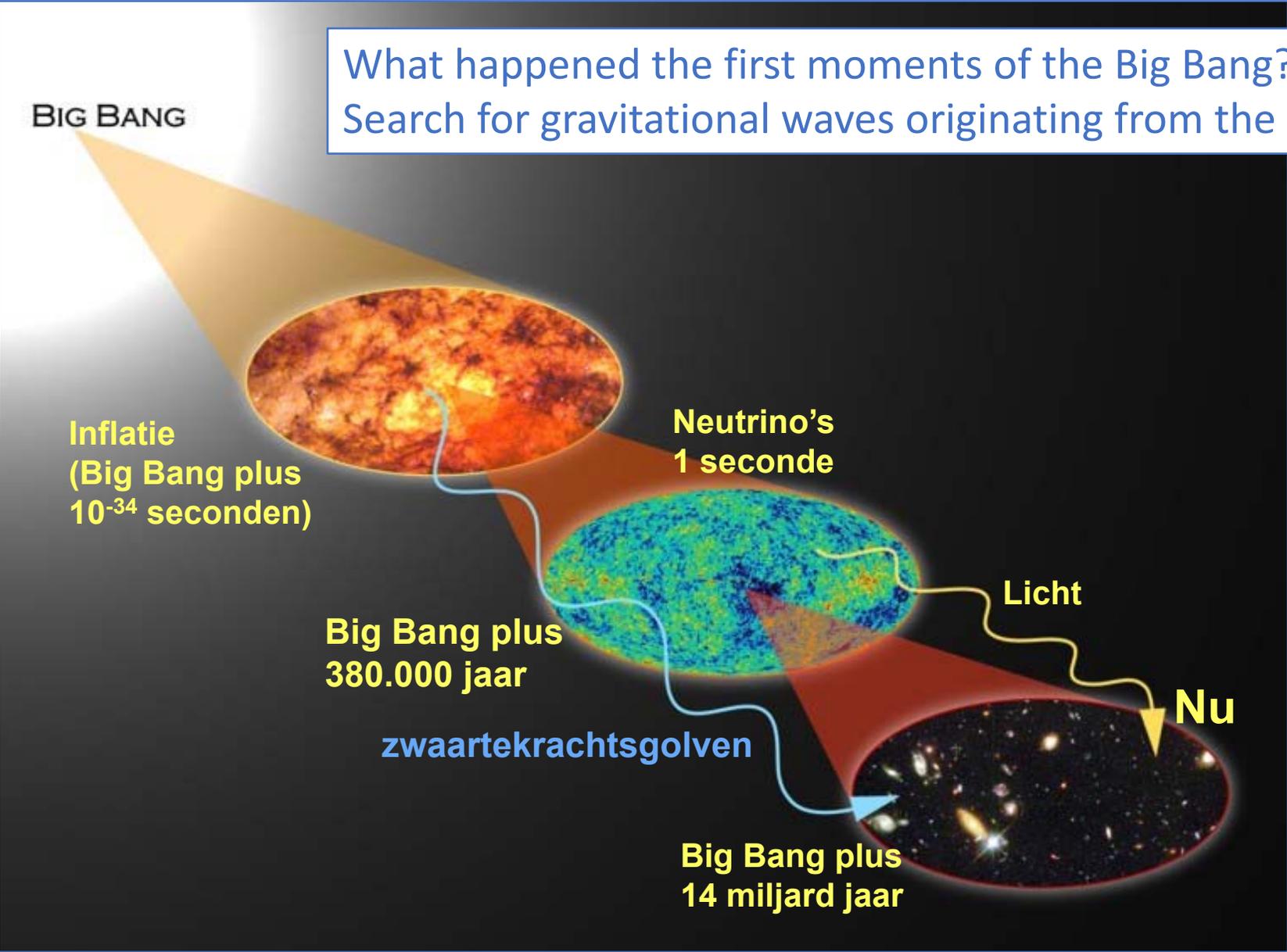


Wat happens at the edge of a black hole (quantum effects)?

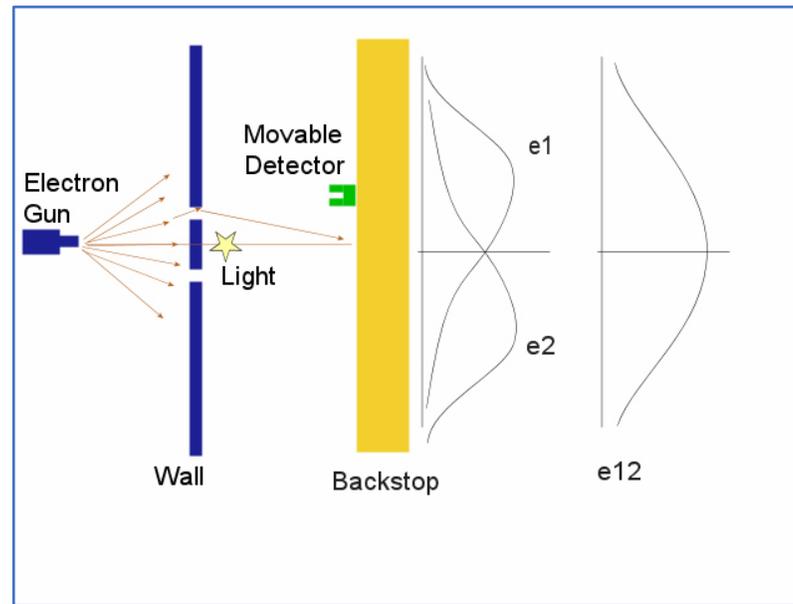
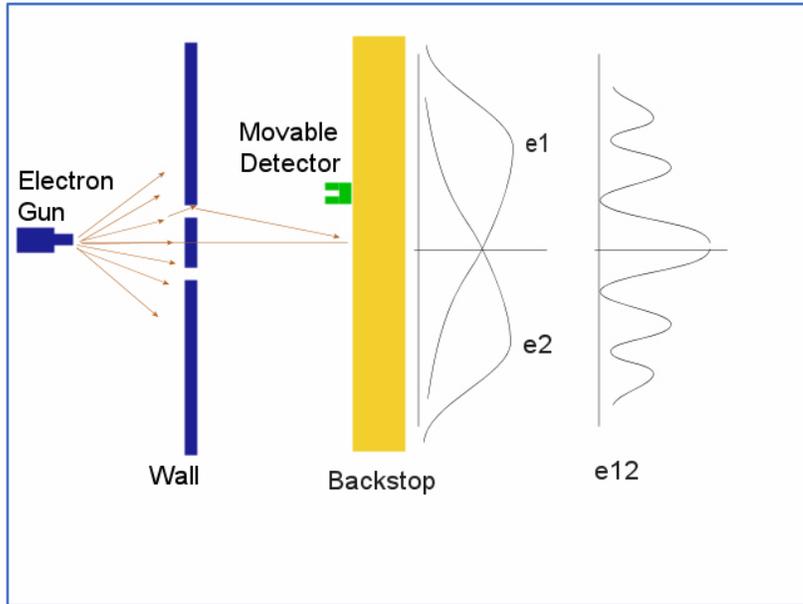


Is Einstein's theory still valid under these extreme conditions?

What happened the first moments of the Big Bang?
Search for gravitational waves originating from the very first moment!

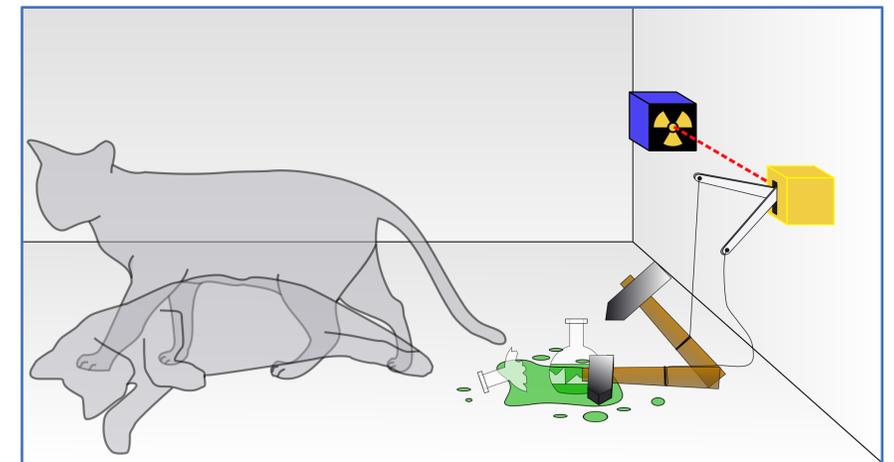


Next week: Quantum Mechanics



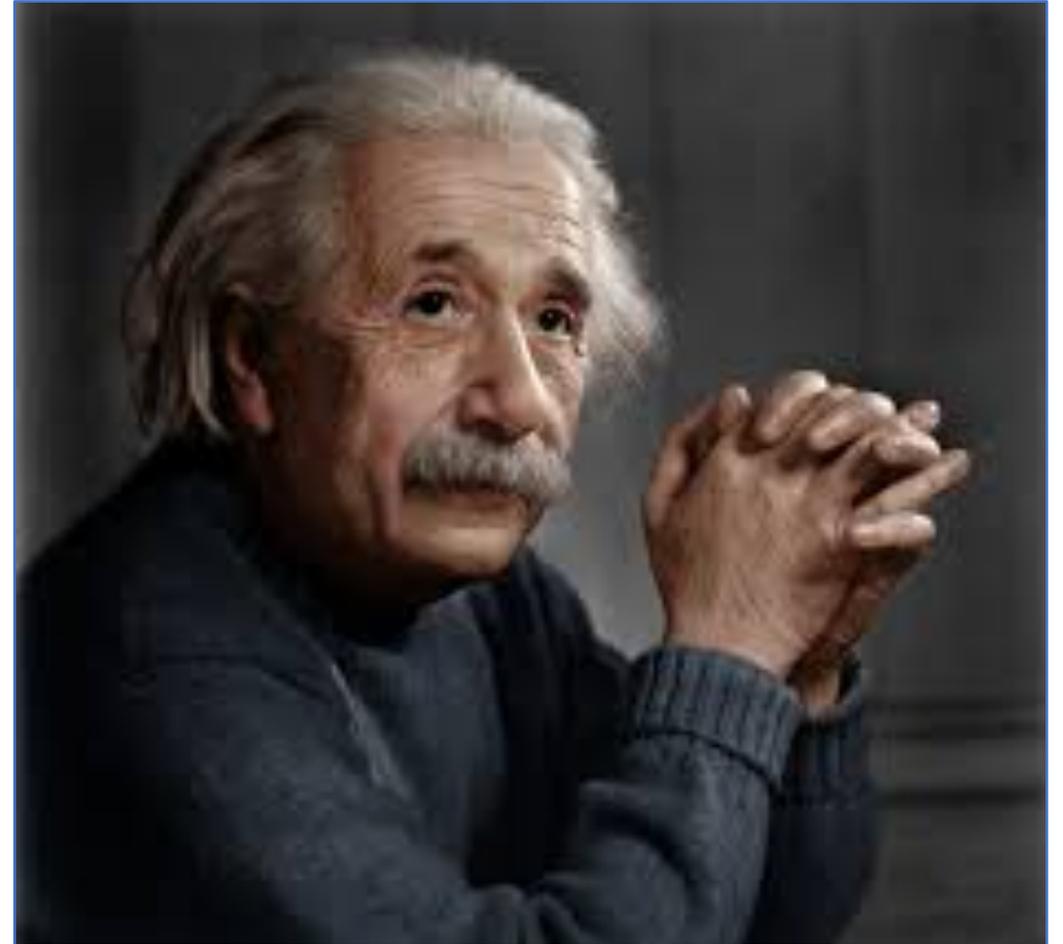
Quantum mechanics developed by Bohr and Heisenberg leads to "absurd" thought experiments of Feynman and Wheeler. Einstein and Schrödinger did not like it.

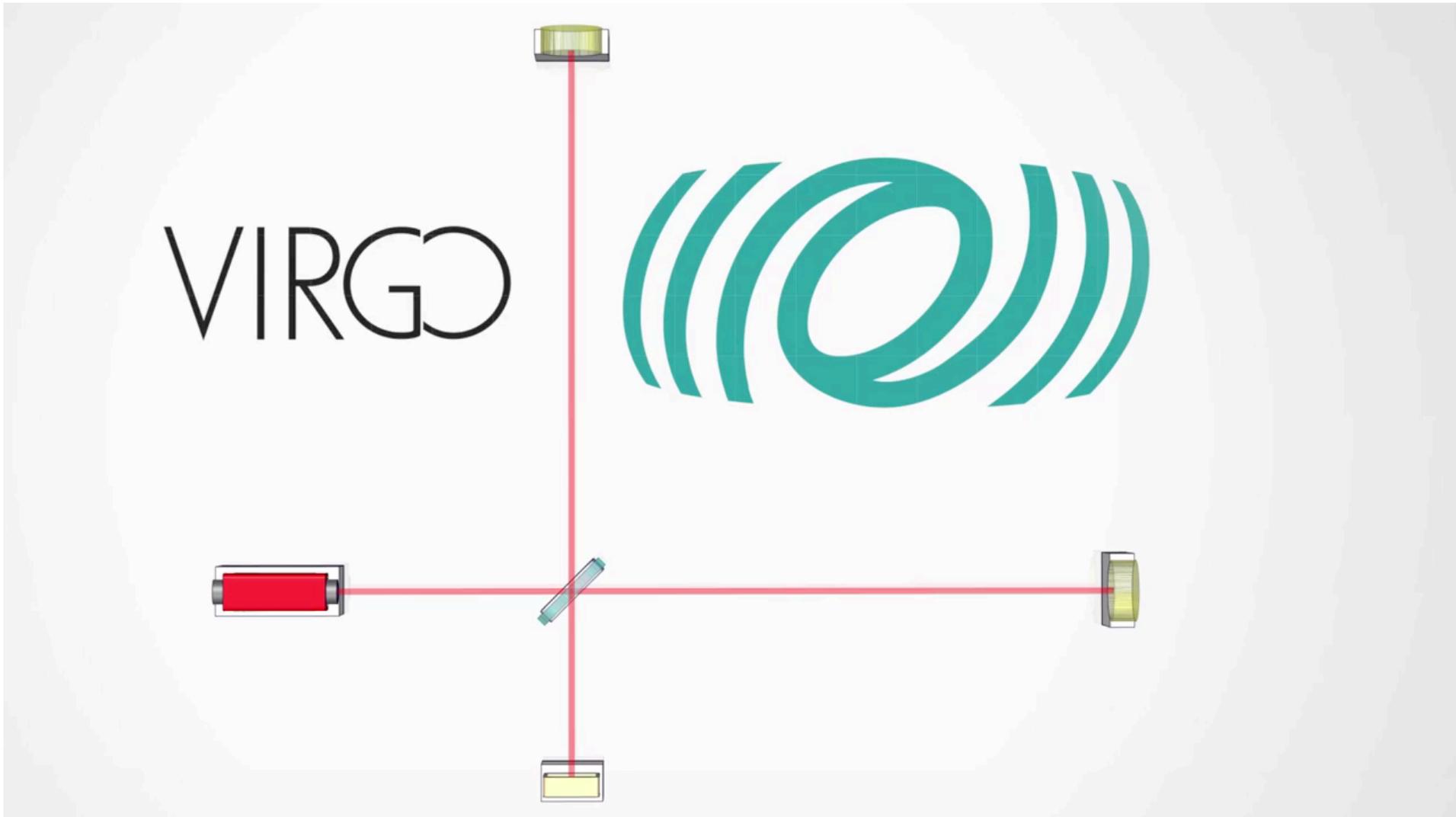
Even today people are debating its interpretation....



Extra Slides

- “Imagination is more important than knowledge”
- “Education is what remains after one has forgotten what one has learned at school.”
- “I fear the day that technology will surpass our human interaction. The world will have a generation of idiots.”
- “A person who never made a mistake never tried anything new.”





The Gravitational Wave Spectrum

