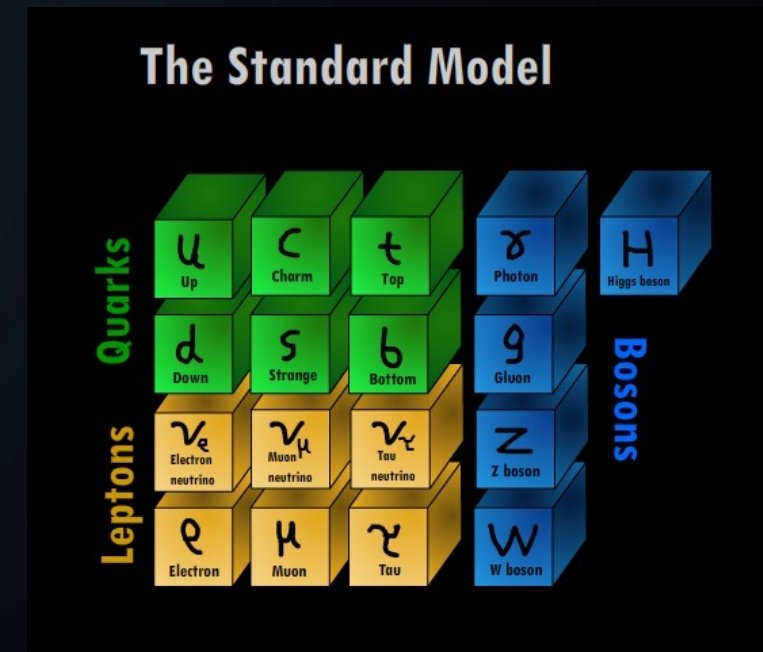
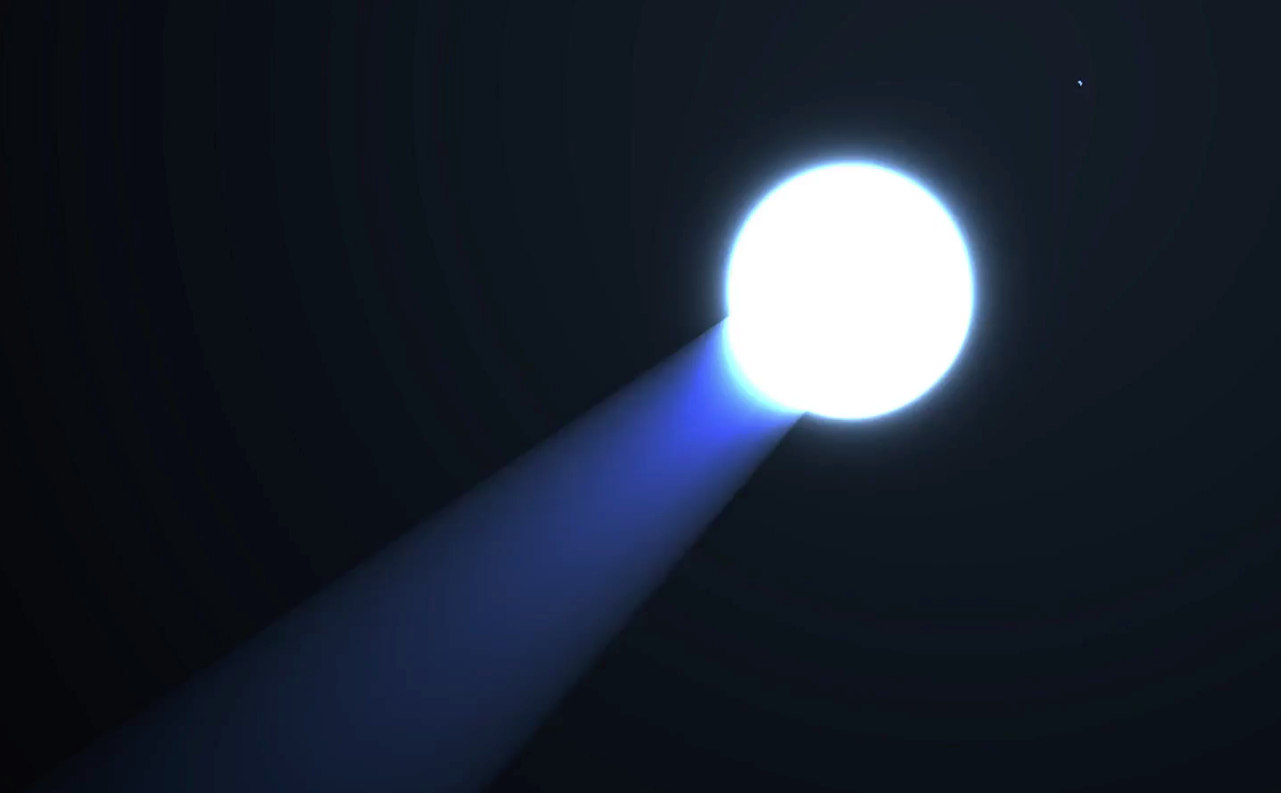




PHY3004: Nuclear and Particle Physics
Marcel Merk, Jacco de Vries



Recap: “Seeing the wood for the trees”

• Lecture 1: “Particles”

- Zooming into constituents of matter
- **Skills: distinguish particle types, Spin**

• Lecture 2: “Forces”

- Exchange of quanta: EM, Weak, QCD
- **Skills: 4-vectors, Feynman diagrams**

• Lecture 3: “Waves”

- Quantum fields and gauge invariance
- **Skills: Dirac algebra, co- & contra variant**

• Lecture 4: “Symmetries”

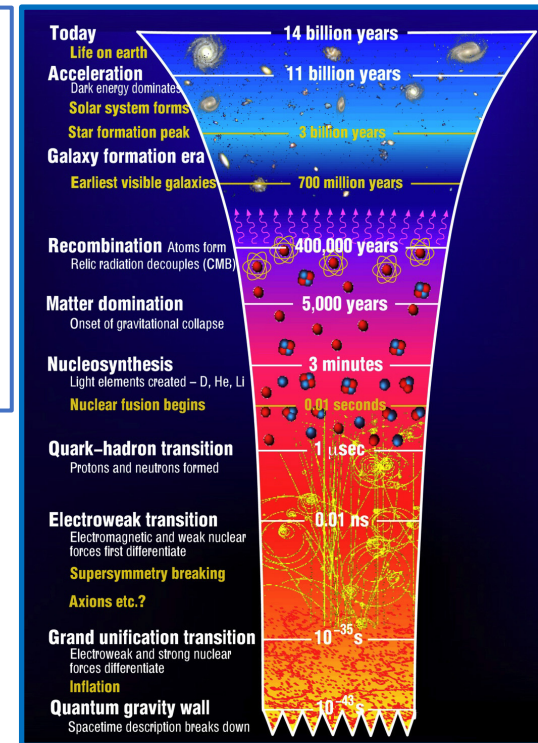
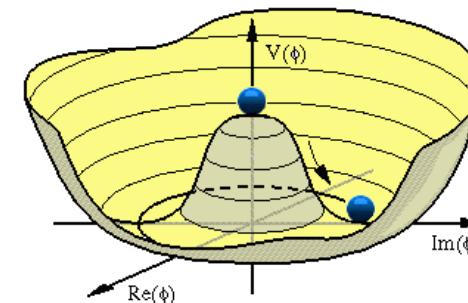
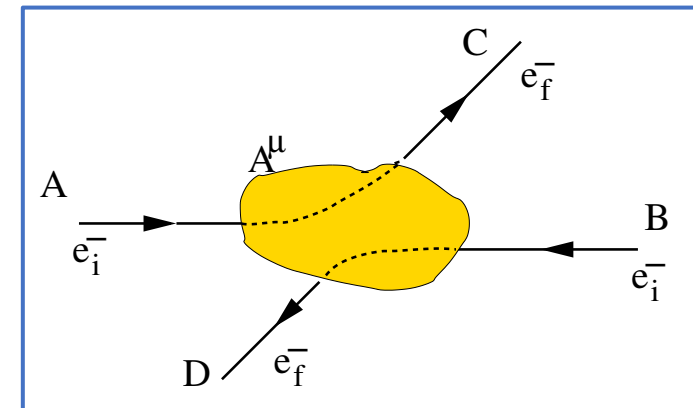
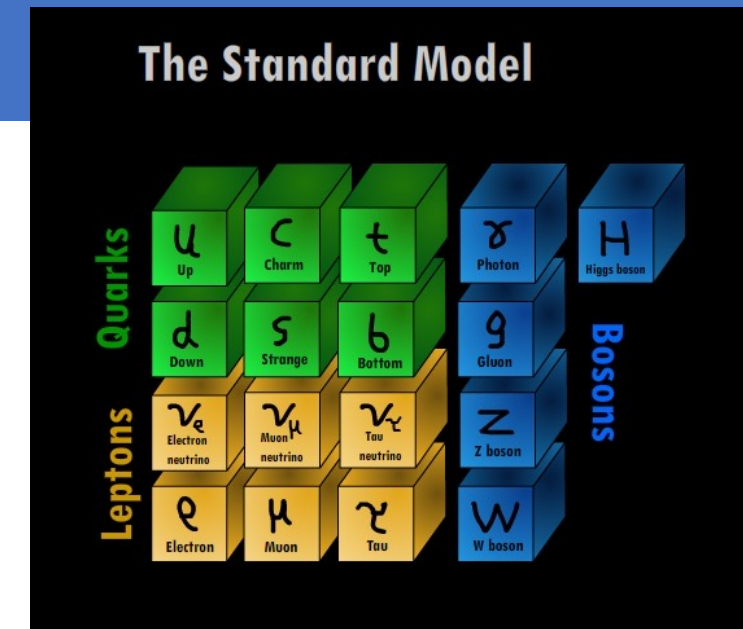
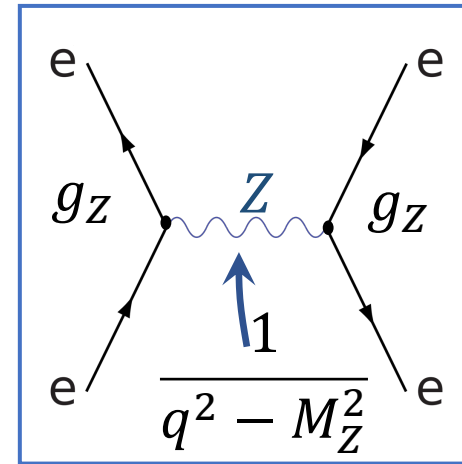
- Standard Model, Higgs, Discrete Symmetries
- **Skills: Lagrangians, Chirality & Helicity**

• Lecture 5: “Scattering”

- Cross section, decay, perturbation theory
- **Skills: Dirac-delta function, Feynman Calculus**

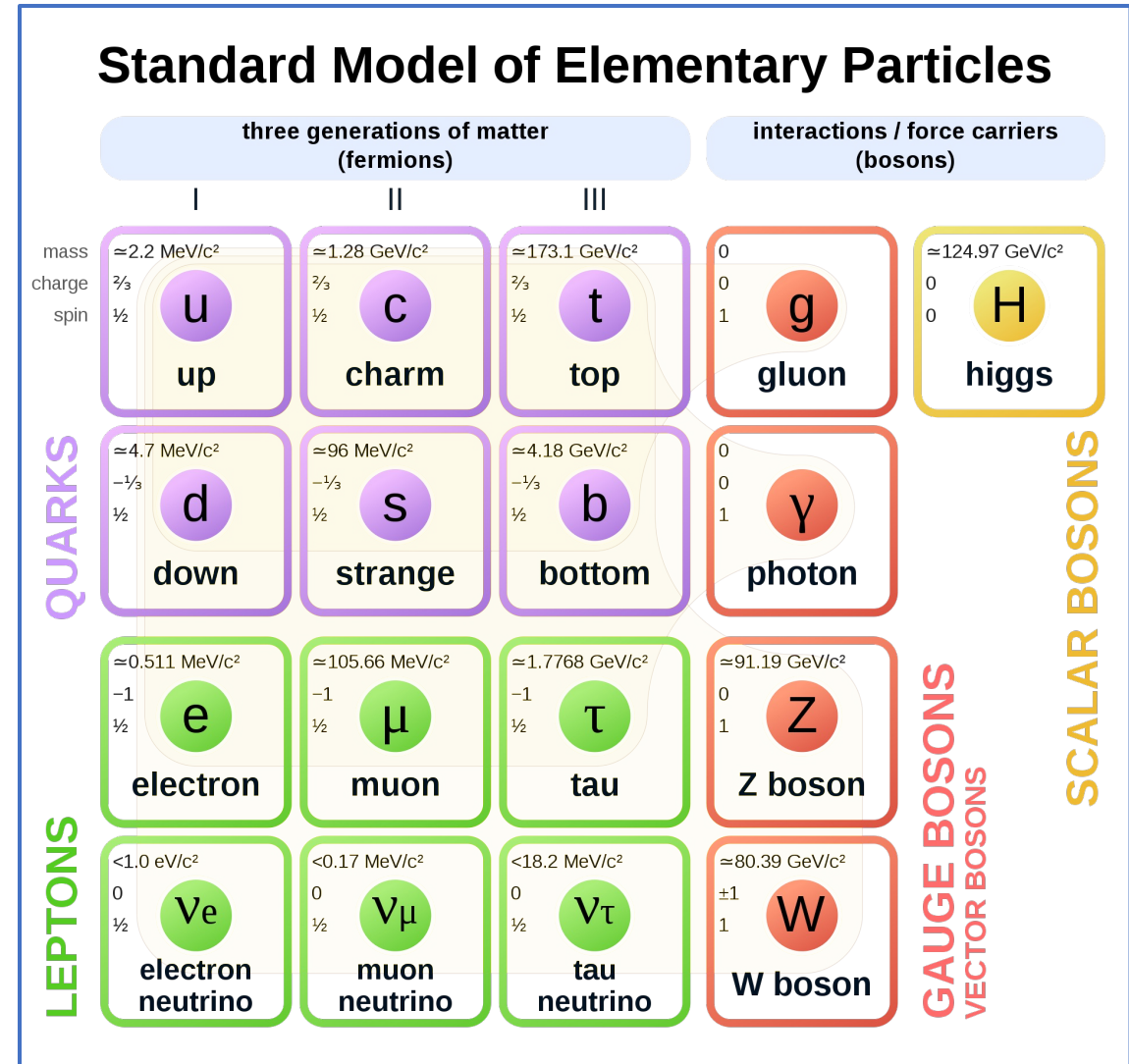
• Lecture 6: “Detectors”

- Energy loss mechanisms, detection technologies
- **Skills: which technologies measure what?**



Classification of particles

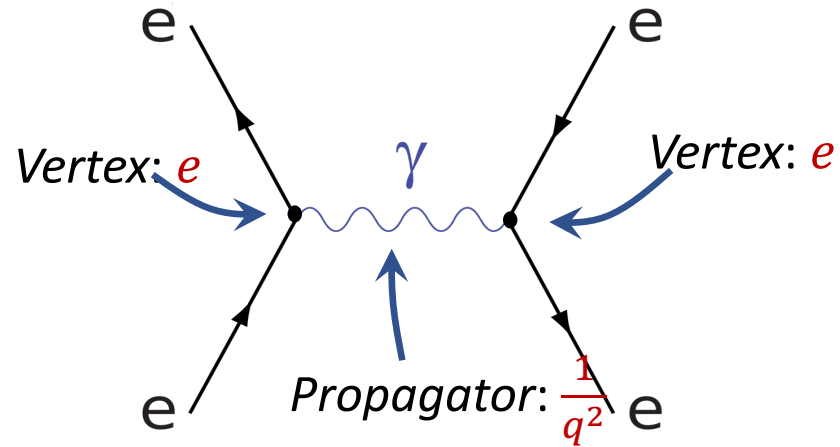
- **Lepton**: fundamental particle
- **Hadron**: consist of **quarks**
 - **Meson**: 1 quark + 1 antiquark (π^+ , B_s^0 , ...)
 - **Baryon**: 3 quarks (p , n , Λ , ...)
 - **Anti-baryon**: 3 anti-quarks
- **Fermion**: particle with half-integer spin.
 - Antisymmetric wave function: obeys Pauli-exclusion principle and Pauli-Dirac statistics
 - All fundamental quarks and leptons are spin- $\frac{1}{2}$
 - Baryons ($S=\frac{1}{2}$, $\frac{3}{2}$)
- **Boson**: particle with integer spin
 - Symmetric wave function: Bose-Einstein statistics
 - **Mesons**: ($S=0$, 1), **Higgs** ($S=0$)
 - **Force carriers**: γ , W , Z , g ($S=1$); graviton ($S=2$)



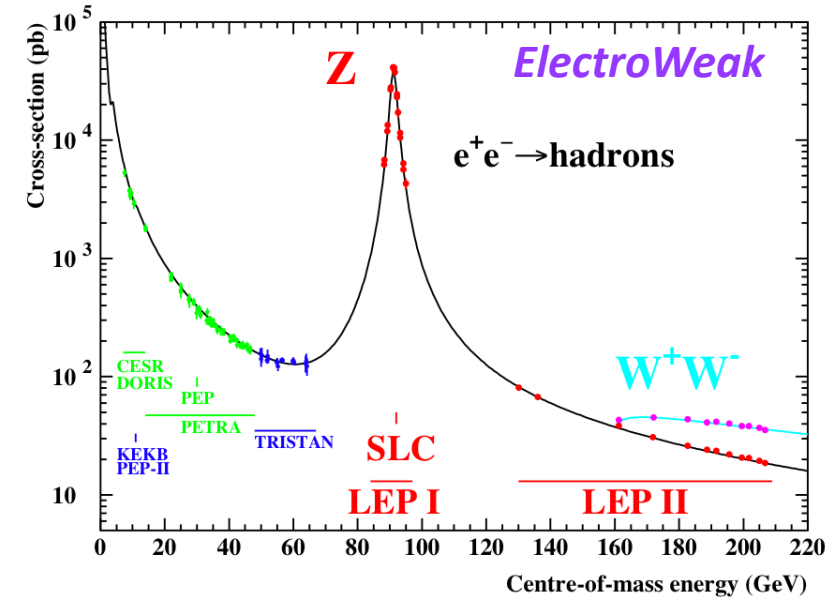
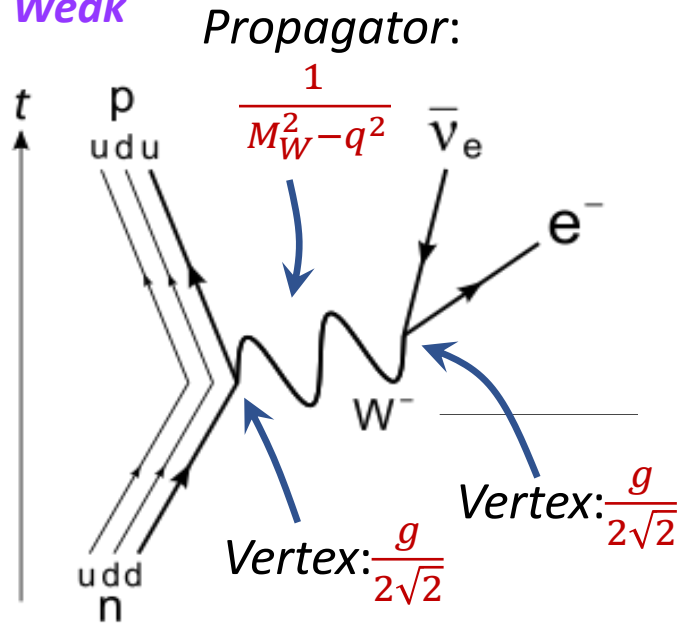
Lecture 2: "Forces"

Griffiths chapter 2

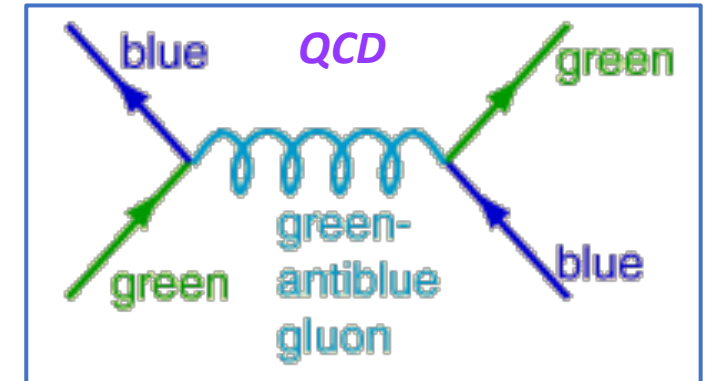
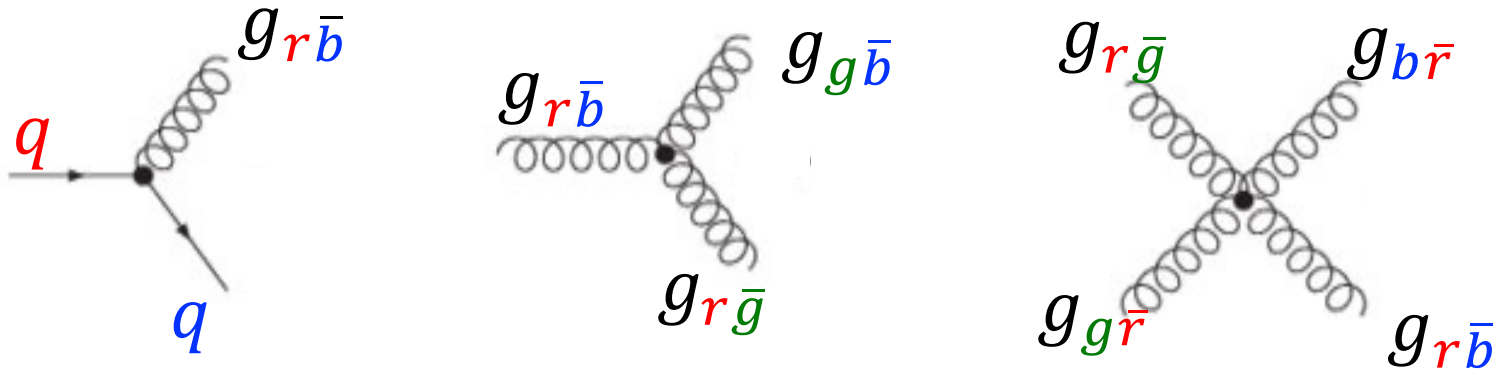
EM (QED)



Weak



Strong (QCD)



Lecture 3: "Waves" – wave equations

Probability interpretation
(Continuity equation)

Quantum Mechanics: $E \rightarrow \hat{E} = i\hbar \frac{\partial}{\partial t}$; $p \rightarrow \hat{p} = -i\hbar \vec{\nabla}$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

Non-relativistic spin 0:

Schrödinger:

$$E = \frac{\vec{p}^2}{2m}$$

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi$$

$$\psi = N e^{i(\vec{p}\vec{x} - Et)}$$

$$\rho \equiv \psi^* \psi = |N|^2$$

$$\vec{j} \equiv \frac{i\hbar}{2m} (\psi \vec{\nabla} \psi^* - \psi^* \vec{\nabla} \psi) = \frac{|N|^2}{m} \vec{p}$$

Relativistic spin 0:

Klein-Gordon:

$$E^2 = p^2 c^2 + m^2 c^4$$

$$-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \phi = -\nabla^2 \phi + \frac{m^2 c^2}{\hbar^2} \phi$$

Example: pions

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

$$\phi = N e^{i(\vec{p}\vec{x} - Et)}$$

$$j^\mu(\rho, \vec{j}) = i[\phi^*(\partial^\mu \phi) - \phi(\partial^\mu \phi^*)]$$

$$\rho = 2|N|^2 E$$

$$\vec{j} = 2|N|^2 \vec{p}$$

$$j^\mu = 2|N|^2 p^\mu$$

Relativistic spin- 1/2:

Dirac: $\gamma^\mu \equiv (\beta, \beta \vec{\alpha})$

$$H = (\vec{\alpha} \cdot \vec{p} + \beta m)$$

Fundamental
quarks and leptons

$$i \frac{\partial}{\partial t} \psi = (-i \vec{\alpha} \cdot \vec{\nabla} + \beta m) \psi$$

$$(i\gamma^\mu \partial_\mu - m) \psi = 0$$

$$\psi = u(p) e^{i(\vec{p}\vec{x} - Et)}$$

$$u(p) = \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$$

$$j^\mu = \bar{\psi} \gamma^\mu \psi$$

$$j^0 = \bar{\psi} \gamma^0 \psi = \psi^\dagger \psi = \sum_{i=1}^4 |\psi_i|^2$$

Relativistic spin-1:

Fundamental
force carriers

Proca:

$$\partial_\mu \partial^\mu A^\nu + m^2 A^\nu = j^\nu$$

EM: $A^\mu = \gamma \rightarrow m = 0$

QCD: $A^\mu = g \rightarrow m = 0$

Weak: $A^\mu = W, Z \rightarrow m \neq 0$

EM: Maxwell equations
for $\vec{E}$ and $\vec{B}$ fields

Lecture 3: “Waves” – gauge invariance

$$S = \int d^4x \mathcal{L}(\phi(x), \partial\phi(x))$$

$$\delta S = 0$$

Lagrangians: Spin 0 Scalar field: $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2$

Spin ½ Dirac fermion $\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$

Spin 1 gauge boson (photon) : $\mathcal{L} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) - j^\mu A_\mu$

Euler Lagrange lead to the wave equations: $\frac{\partial\mathcal{L}}{\partial\phi(x)} = \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi(x))}$ (stationary action in field theory)

All forces result from requiring a symmetry principle: Lagrangian should stay invariant under transformations

1) QED = U(1) symmetry

$$\psi(x) \rightarrow \psi'(x) = e^{iq\alpha(x)}\psi(x)$$

$$A^\mu(x) \rightarrow A'^\mu(x) = A^\mu(x) - \partial^\mu\alpha(x)$$

→ 1 E.M. photon field: $A^\mu(x)$

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi \longrightarrow \mathcal{L} = i\bar{\psi}\gamma_\mu D^\mu\psi - m\bar{\psi}\psi$$

Covariant derivative: $\partial^\mu \rightarrow D^\mu \equiv \partial^\mu + iqA^\mu$

$$\mathcal{L} = \underbrace{i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi}_{\text{“free”}} - \underbrace{q\bar{\psi}\gamma_\mu\psi A^\mu}_{\text{“interaction”}}$$

Think: $\mathcal{L} = T - V$

2) Weak = SU(2) symmetry $\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$

$$\psi(x) \rightarrow \psi'(x) = \exp\left(\frac{i}{2}g\vec{\tau} \cdot \vec{\alpha}(x)\right) \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

→ 3 weak fields: $W^{\mu+}(x), W^{\mu-}(x), Z^\mu(x)$

3) QCD = SU(3) symmetry $\psi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$

$$\psi(x) \rightarrow \psi'(x) = \exp\left(\frac{i}{2}g_s\vec{\lambda} \cdot \vec{\alpha}(x)\right) \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$$

→ 8 colored gluon fields: $g^\mu(x)$

Lecture 4: “Symmetries” – Standard Model – $SU(3)_C \times SU(2)_L \times U(1)_Y$

- The Lagrangian of the Standard Model includes electromagnetic, weak and strong interactions according to the gauge field principle

- Construction of the Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{free}} - \mathcal{L}_{\text{interaction}} = \mathcal{L}_{\text{Dirac}} - gJ^\mu A_\mu$

- With g a coupling constant, J^μ a current ($\bar{\psi}\gamma^\mu\psi$) and A_μ a force field

- A. Local $U(1)$ gauge invariance: symmetry under complex phase rotations

- Conserved quantum number: (hyper-) charge $(\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + iqA_\mu)$

- Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - q \bar{\psi}\gamma^\mu\psi A_\mu$

$SU(2) \times U(1)$: Hypercharge = Y ; Charge = Q ; $Q = T_3 + \frac{1}{2}Y$ $\underbrace{J^\mu}_{EM}$

Note Spinor: $\psi = \begin{pmatrix} \psi_\alpha \\ \psi_\beta \\ \psi_\gamma \\ \psi_\delta \end{pmatrix}$

- B. Local $SU(2)$ gauge invariance: symmetry under transformations in isospin doublet space.

- Conserved quantum number: weak isospin T (LH) $(I\partial_\mu \rightarrow D_\mu = I\partial_\mu + igB_\mu)$

- Lagrangian: $\mathcal{L} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi - \frac{g}{2} \bar{\Psi}\gamma^\mu \vec{\tau}\Psi \vec{b}_\mu$

$$B_\mu = \frac{1}{2} \vec{\tau} \cdot \vec{b}_\mu = \frac{1}{2} \tau_1^a b_\mu^a = \frac{1}{2} \begin{pmatrix} b_3 & b_1 - ib_2 \\ b_1 + ib_2 & -b_3 \end{pmatrix} \quad \underbrace{J^\mu}_{Weak}$$

Note doublet spinors: $\Psi_L = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}_L$

- C. Local $SU(3)$ gauge invariance: symmetry under transformations in colour triplet space

- Conserved quantum number: color $(I\partial_\mu \rightarrow D_\mu = I\partial_\mu + ig_s C_\mu)$

- Lagrangian: $\mathcal{L} = \bar{\Phi}(i\gamma^\mu D_\mu - m)\Phi = \bar{\Phi}(i\gamma^\mu \partial_\mu - m)\Phi - \frac{g_s}{2} \bar{\Phi}\gamma^\mu \vec{\lambda}\Phi \vec{c}_\mu$

C_μ are 3x3 matrices $\rightarrow$ gluon fields

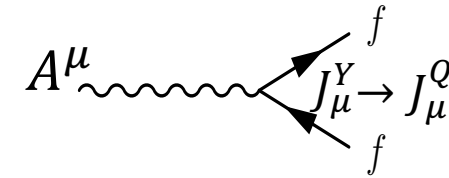
$$\underbrace{J^\mu}_{QCD}$$

Note triplet spinors: $\Phi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$

Lecture 4: “Symmetries” – Standard Model – $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - qJ_{EM}^\mu A_\mu - \frac{g}{2}\vec{J}_{\text{Weak}}^\mu \vec{b}_\mu - \frac{g_s}{2}\vec{J}_{QCD}^\mu \vec{c}_\mu$$

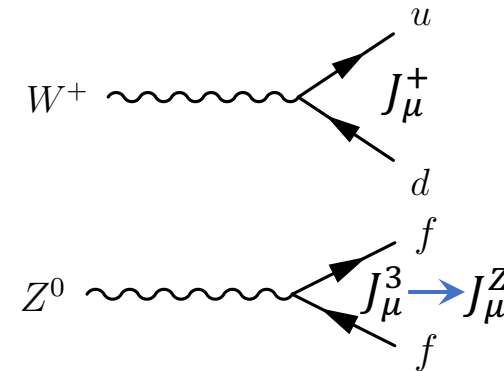
QED U(1) $\mathcal{L}_{\text{int}} = -J_\mu A^\mu$ with $J_\mu = q\bar{\psi}\gamma_\mu\psi$



Electromagnetic interaction

Weak SU(2) : $\mathcal{L}_{\text{int}} = -\vec{J}_\mu \vec{b}^\mu$ with $\vec{J}_\mu = \frac{g}{2} \bar{\Psi} \gamma_\mu \vec{\tau} \Psi$

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(b_\mu^1 \mp ib_\mu^2) \quad J_\mu^\pm = \frac{1}{\sqrt{2}} \bar{\Psi} \gamma_\mu \tau^\pm \Psi \quad \text{with } \tau^\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$$



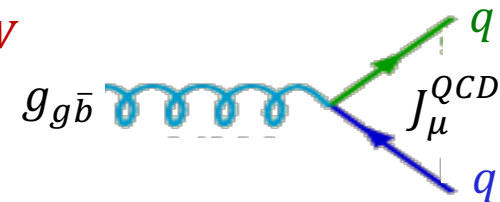
Weak interaction

$$Z_\mu \sim b_\mu^3$$



Electroweak mixing SU(2)xU(1):

$$\begin{aligned} \gamma_\mu &= A_\mu \cos \theta_W + b_\mu^3 \sin \theta_W \\ Z_\mu &= -A_\mu \sin \theta_W + b_\mu^3 \cos \theta_W \end{aligned}$$



Strong interaction

Standard Model: $SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$

Lecture 4: Electroweak Quantum Numbers

For weak isospin some people write T_3 while others write I_3 With : $Q = T_3 + \frac{1}{2}Y$
Or : $Q = I_3 + \frac{1}{2}Y$

Generation				Or : $Q = I_3$		
	I	II	III	I_3	Y	Q
Leptons	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	+1/2	-1	0
	e_R	μ_R	τ_R	-1/2	-1	-1
				0	-2	-1
Quarks	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	+1/2	+1/3	+2/3
	u_R	c_R	t_R	-1/2	+1/3	-1/3
				0	+4/3	+2/3
	d_R	s_R	b_R	0	-2/3	-1/3

Symmetry breaking with a *real* field ϕ

Griffiths §10.9

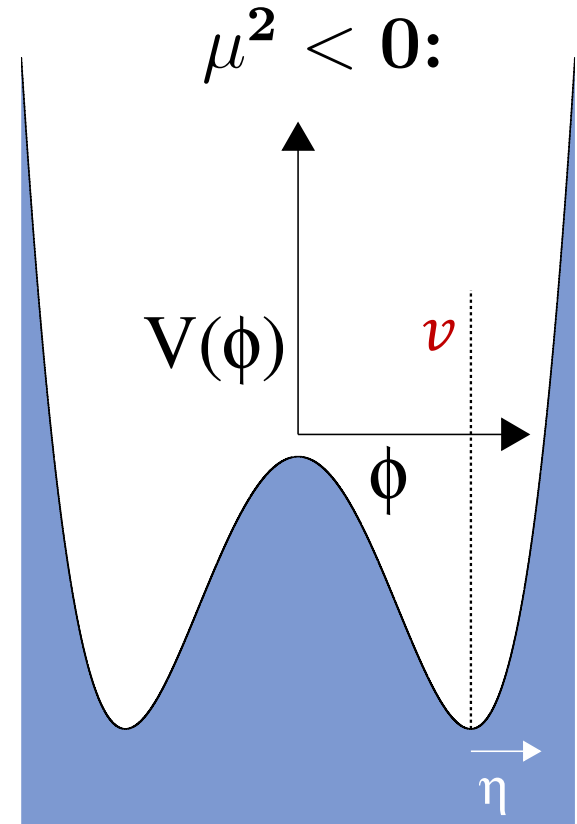
- Explicit mass terms violate the symmetry: $m^2 A_\mu A^\mu \rightarrow m^2 \left(A_\mu + \frac{1}{e} \partial_\mu \alpha \right) \left(A^\mu + \frac{1}{e} \partial^\mu \alpha \right) \neq m^2 A_\mu A^\mu$
- Add a new field to the Lagrangian:

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) = \underbrace{\frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} \mu^2 \phi^2}_{\text{Massive Klein-Gordon term (Spin 0, mass } = \mu)} - \underbrace{\frac{1}{4} \lambda \phi^4}_{\text{Interaction term}}$$

The Lagrangian has a minimum for $\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} = v$ or $\mu^2 = -\lambda v^2$

Conclusion:

- The symmetry of the Lagrangian by adding a symmetric potential ϕ *has not been destroyed*
- The *vacuum is no longer* in a symmetric position



Symmetry breaking with a *complex* field ϕ

Griffiths §10.9

- Introduce a complex scalar field: $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$
- The Lagrangian term is: $\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - V(\phi)$, with $V(\phi) = \mu^2(\phi^* \phi) + \lambda (\phi^* \phi)^2$
- Lagrangian:

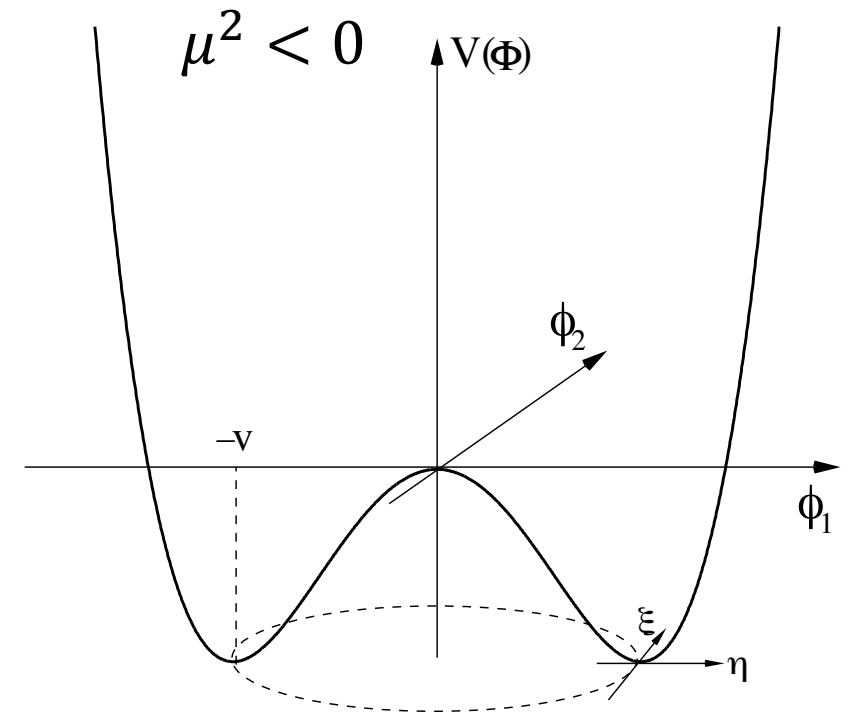
$$\mathcal{L}(\phi_1, \phi_2) = \frac{1}{2} (\partial_\mu \phi_1)^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 - \frac{1}{2} \mu^2 (\phi_1^2 + \phi_2^2) - \frac{1}{4} \lambda (\phi_1^2 + \phi_2^2)^2$$

Conclusion:

- The symmetry of the Lagrangian by adding a symmetric potential ϕ *has not been destroyed*
- The *vacuum is no longer* in a symmetric position

The real case includes a complex (isospin doublet) field ϕ

- ϕ degrees of freedom lead to mass terms for the W^+, W^-, Z^0
- ϕ can also couple to fermions $\rightarrow$ particle masses



- Symmetry breaking:

$$\phi_0 = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$$

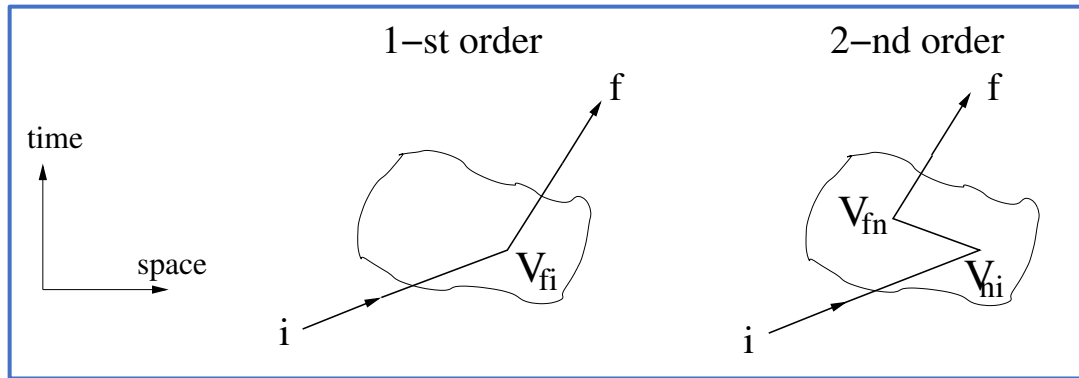
Lecture 5 : “Scattering” – non-Rel.

Perturbation theory

1) $V(x, t)$ is fixed

Solve wave equation
Iteratively... $i \frac{\partial \psi}{\partial t} = (H_0 + V(\vec{x}, t))\psi$

...use plane waves $\psi = \sum_{n=0}^{\infty} a_n(t) \phi_n(\vec{x}) e^{-iE_n t}$



$$\sigma = N/\mathcal{L} \quad d\sigma = \frac{W_{fi}}{\text{flux}} d\Phi \quad W_{fi} \equiv \lim_{T \rightarrow \infty} \frac{|T_{fi}|^2}{T}$$

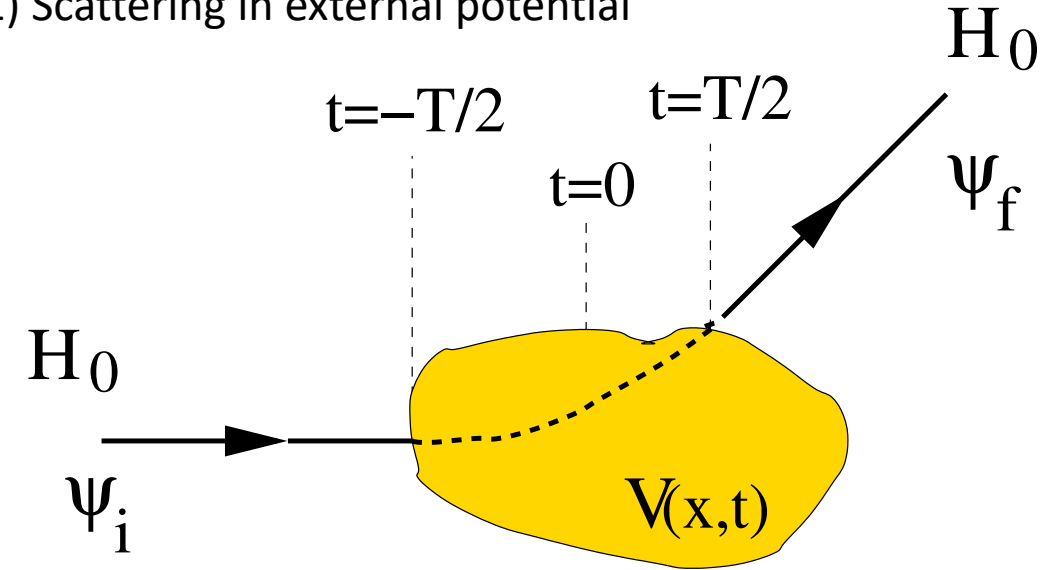
$$T_{fi} = -i \int d^4x \psi_f^*(x) V(x) \psi_i(x) = -2\pi V_{fi} \delta(E_f - E_i)$$

Energy conservation

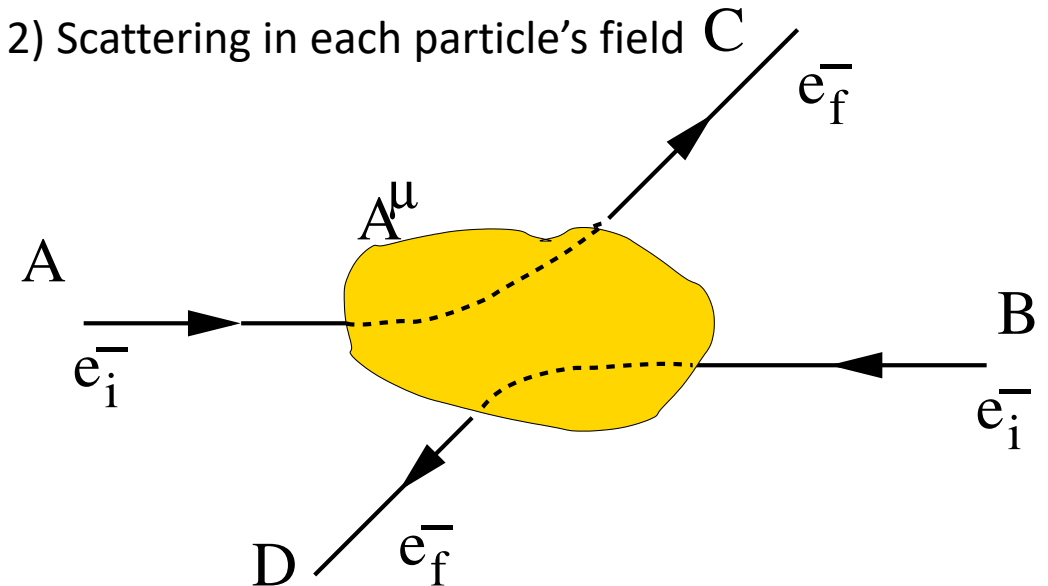
2) Determine V from A field scattering particles
(Solve Maxwell equation)

Relativistic: $V_{fi} \rightarrow \mathcal{M}$ “matrix element”

1) Scattering in external potential



2) Scattering in each particle's field



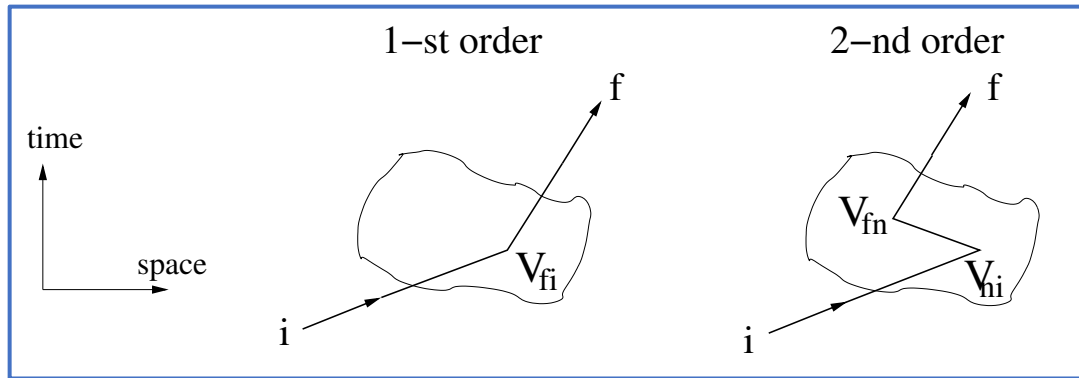
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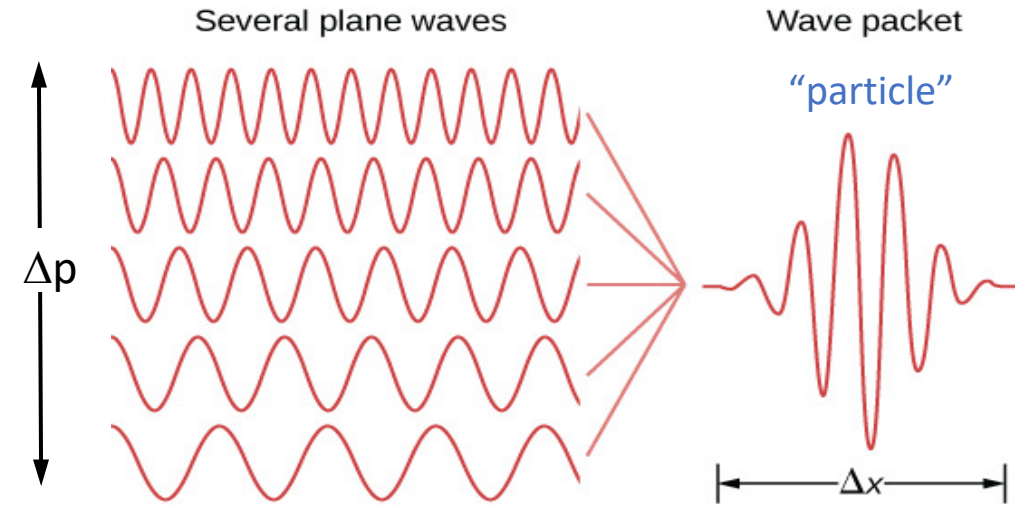
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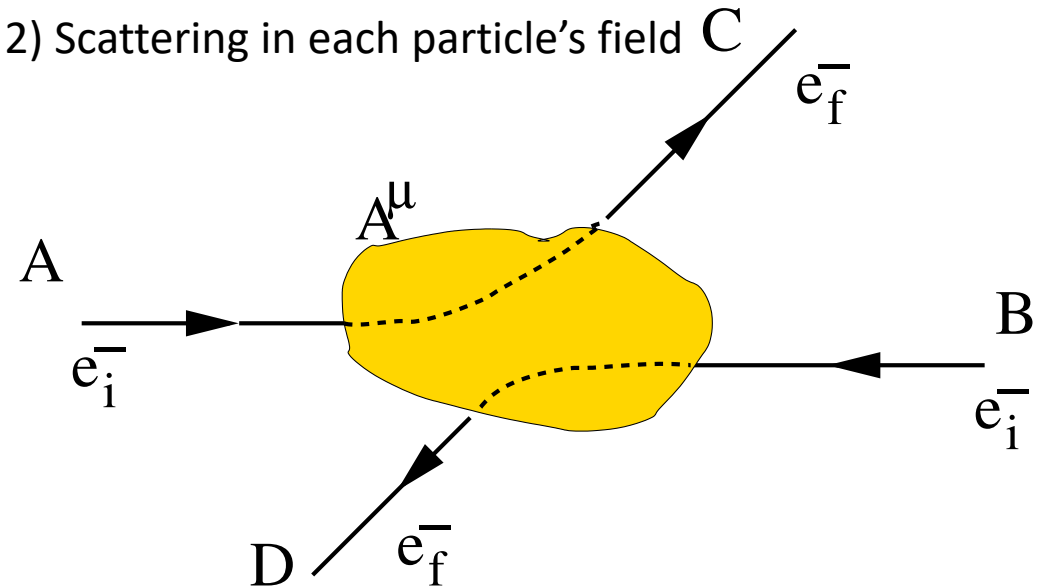
Energy conservation

2) Determine V from A field scattering particles
(Solve Maxwell equation)

Relativistic: $V_{fi} \rightarrow \mathcal{M}$ “matrix element”



2) Scattering in each particle's field C



Lecture 5 : "Scattering" - Relativistic

$$d\sigma = \frac{W_{fi}}{\text{flux}} d\Phi \Rightarrow \sigma = \frac{1}{\text{flux}} \int W_{fi} d\Phi$$

Cross section:

Flux

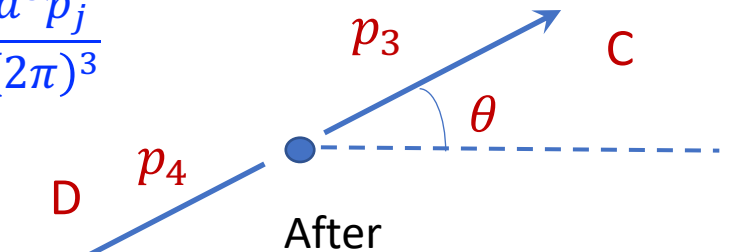
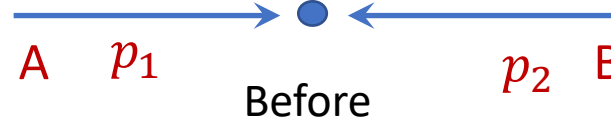
Matrix element

Energy momentum conservation

Phase space

$$\sigma = \frac{S}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 \dots - p_n) \times \prod_{j=3}^n \frac{1}{2E_j} \frac{d^3 \vec{p}_j}{(2\pi)^3}$$

Eg: "2-to-2" scattering:



$$\sigma = \frac{S}{64\pi^2 (E_1 + E_2) |\vec{p}_1|} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \frac{d^3 \vec{p}_3}{E_3} \frac{d^3 \vec{p}_4}{E_4}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S |\mathcal{M}|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

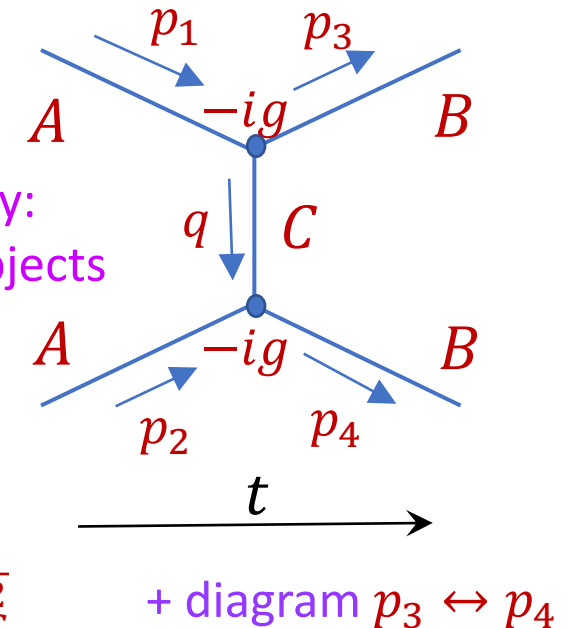
How to determine $\mathcal{M}$? $\rightarrow$ Feynman rules (depend on actual theory/interaction):

Feynman rules (ABC theory):

1. Diagram: see sketch
2. Labels: see sketch
3. Two vertices: $(-ig)^2 = -g^2$
4. Propagators: one internal line: $\frac{i}{q^2 - m_C^2}$
5. Conservation of $E, \vec{p}$ twice: $(2\pi)^4 \delta^4(p_1 - p_3 - q)$ and $(2\pi)^4 \delta^4(p_2 + q - p_4)$
6. Integrate: one integral: $1/(2\pi)^4 d^4 q$
7. Erase delta-function and multiply by i to find:

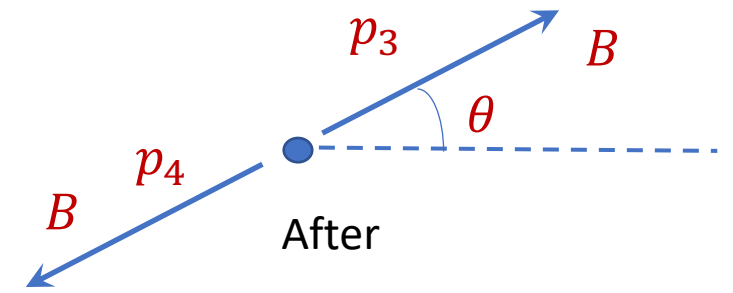
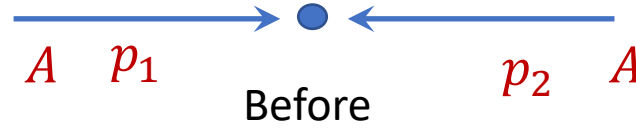
Example diagram:
 $A + A \rightarrow B + B$

From ABC $\rightarrow$ Standard Model theory:
More complicated rules and spin objects
 $\rightarrow$ Master level education



$$\mathcal{M} = \frac{g^2}{(p_4 - p_2)^2 - m_C^2}$$

$A + A \rightarrow B + B$ Scattering: $d\sigma/d\Omega$



- Look at the matrix element and assume that $m_A = m_B = m$ and $m_C = 0$ (eg. a photon):

$$\mathcal{M} = \frac{g^2}{(p_3 - p_2)^2} + \frac{g^2}{(p_4 - p_2)^2}$$

$$\begin{aligned} (p_4 - p_2)^2 - m_C^2 &= p_4^2 + p_2^2 - 2p_2 \cdot p_4 \\ &= m_4^2 + m_2^2 - 2p_2 \cdot p_4 \\ &= 2m^2 - 2E_2E_4 + 2(\vec{p}_2 \cdot \vec{p}_4) \\ &= 2m^2 - 2\left(\sqrt{m^2 + \vec{p}^2}\right)\left(\sqrt{m^2 + \vec{p}^2}\right) + 2\vec{p}^2 \cos \theta \\ &= -2\vec{p}^2(1 - \cos \theta) \end{aligned}$$

$$(p_3 - p_2)^2 - m_C^2 = -2\vec{p}^2(1 + \cos \theta)$$

$$\mathcal{M} = \frac{g^2}{-2\vec{p}^2(1 - \cos \theta)} + \frac{g^2}{-2\vec{p}^2(1 + \cos \theta)} = -\frac{g^2}{2\vec{p}^2 \sin^2 \theta}$$

Note $p_i \equiv p_i^\mu$ and that for 4-vectors:
 $p_i \cdot p_j = p_{i\mu} p_j^\mu = E_i E_j - \vec{p}_i \cdot \vec{p}_j$
 and that $p^2 = p_\mu p^\mu = E^2 - \vec{p}^2 = m^2$
 (Invariant mass)

- Plug in: ($s = 1/2$)

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \left(\frac{g^2}{16\pi E \vec{p}^2 \sin^2 \theta} \right)^2$$

- Consider beam of particles on a target

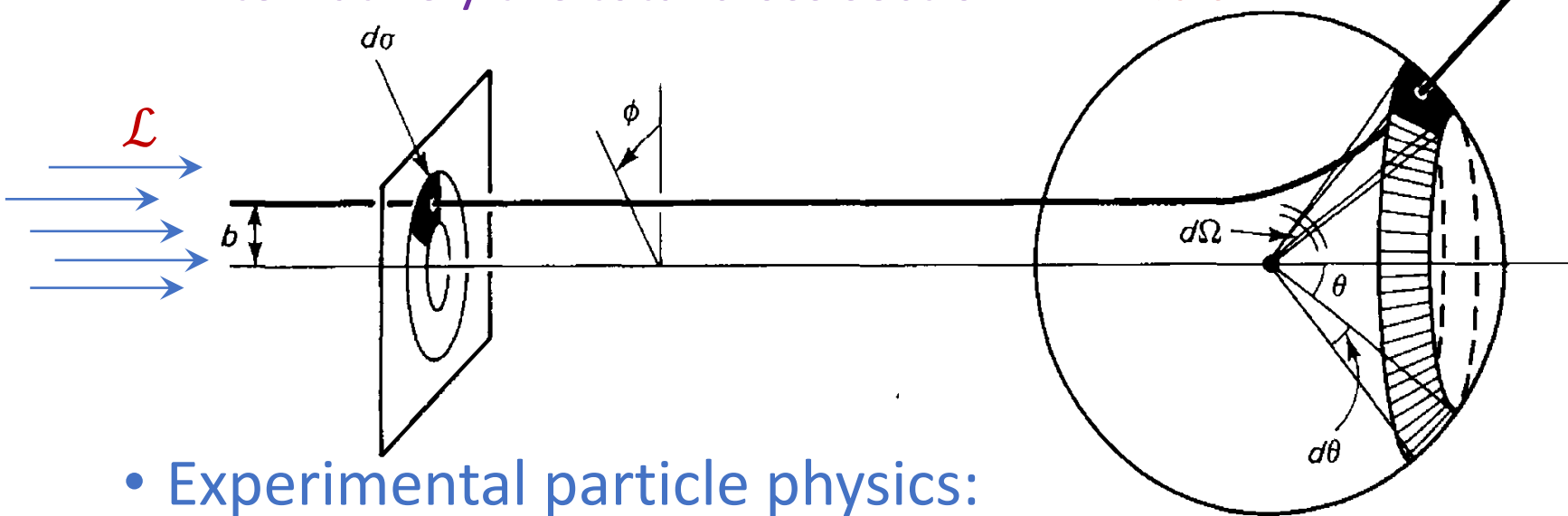
- Luminosity $\mathcal{L}$ is number of particles per unit time, per unit area.

- Number of particles passing through area $d\sigma$: $dN = \mathcal{L} d\sigma$

- Number of particles scattering into solid angle $d\Omega$: $dN = \mathcal{L} d\sigma = \mathcal{L} D(\theta) d\Omega$

- By counting one can measure the *differential cross section*: $\frac{d\sigma}{d\Omega} = D(\theta) = \frac{dN}{\mathcal{L} d\Omega}$

- Alternatively the total cross section: $N = \mathcal{L} \sigma$

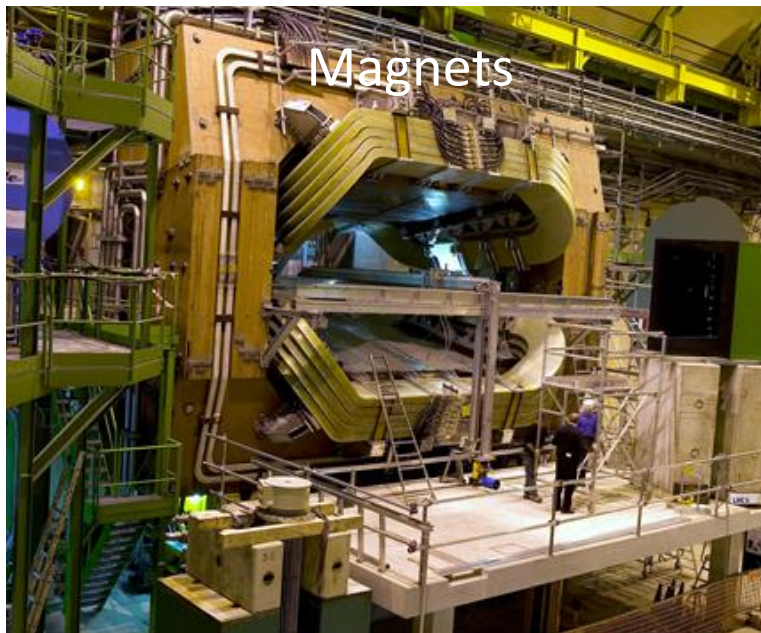
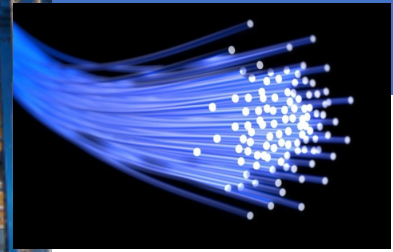


These aspects are needed when you Compare theory with experiments.

- Experimental particle physics:

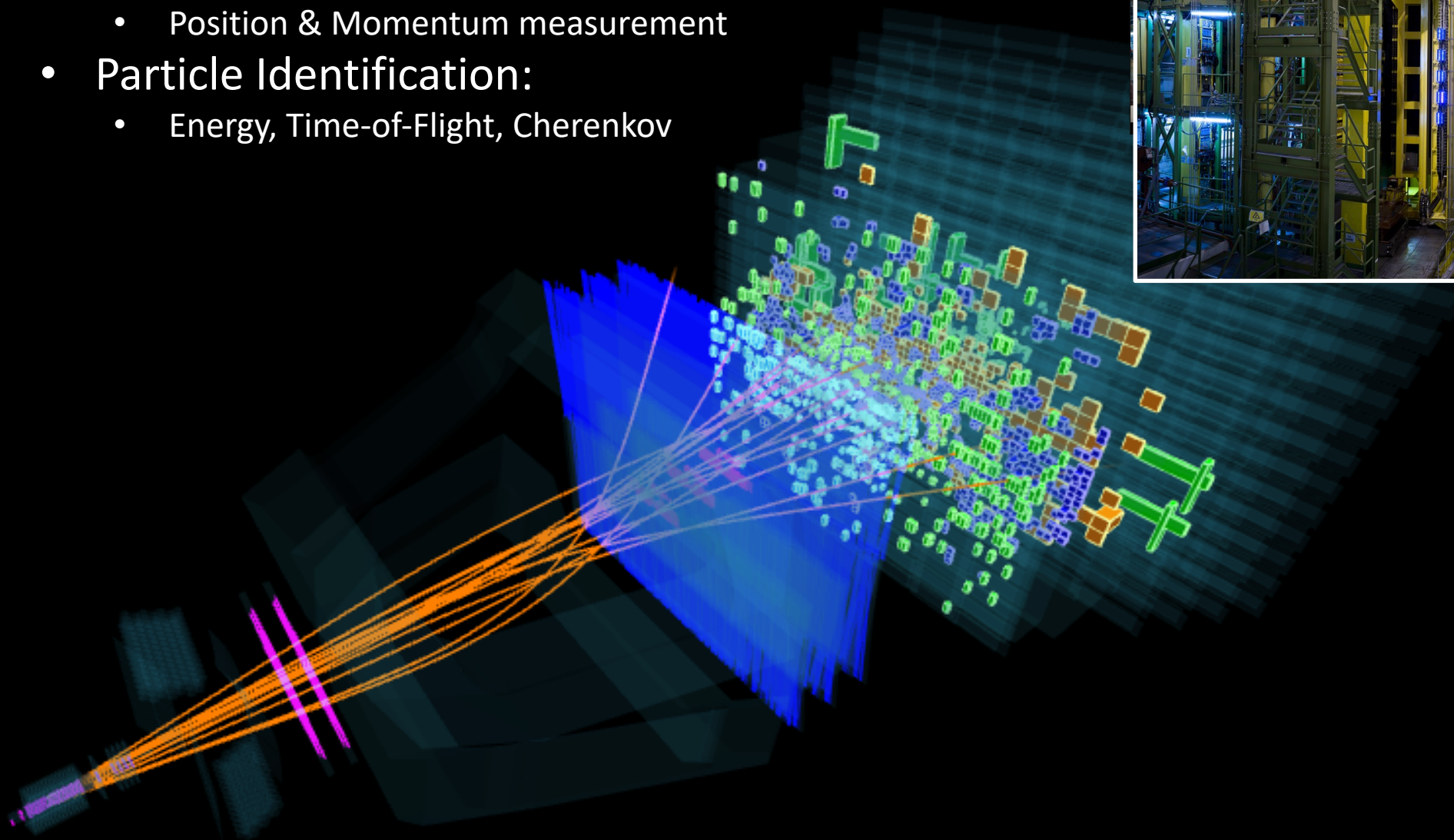
- Measure number of events N and the luminosity $\mathcal{L}$ to find cross section $\sigma = N/\mathcal{L}$
- Compare with theoretical calculation of σ (or $\frac{d\sigma}{d\Omega}$) using e.g. Standard Model

Lecture 6 : “Detectors” - Technologies



Lecture 6 : “Detectors” - Combination

- Charged particle tracking:
 - Position & Momentum measurement
- Particle Identification:
 - Energy, Time-of-Flight, Cherenkov



Skill: Four vectors & co- and contra-variance

Griffiths: chapter 3

- Four vector: $x^\mu = (x^0, x^1, x^2, x^3)$ with $x^0 = ct \Rightarrow x^0 = t$ (since $c \equiv 1$)
- We call this a **contravariant** vector and: $x^\mu = (x^0, \vec{x})$
- Lorentz transformation:

$$x^{\mu'} = \Lambda^\mu_{\nu'} x^\nu ; \Lambda^\mu_{\nu'} = \begin{pmatrix} \gamma & -\beta & 0 & 0 \\ -\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{aligned} x^{0'} &= \gamma(x^0 - \beta x^1) \\ x^{1'} &= \gamma(x^1 - \beta x^0) \\ x^{2'} &= x^2 \\ x^{3'} &= x^3 \end{aligned} \quad g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Lorentz transformation leaves the following invariant: $|x|^2 = x^{0^2} - |\vec{x}|^2$
 - $|x|^2 = x^{0^2} - |\vec{x}|^2 = (ct)^2 - |\vec{x}|^2 = (ct')^2 - |\vec{x}'|^2$
- Introduce **covariant** vectors $x_\mu = \sum_\nu g_{\mu\nu} x^\nu = g_{\mu\nu} x^\nu$ Note the Einstein summation convention
- Inproduct invariants: $I = a_\mu b^\mu = a \cdot b = a'_\mu b'^\mu$ for any Lorentz 4-vectors a^μ and b^μ
 - Example invariant mass: $E^2 = \vec{p}^2 c^2 + m^2 c^4 \Rightarrow p^\mu = (E, \vec{p}) \Rightarrow p_\mu p^\mu = E^2 - \vec{p}^2 = m^2$

Skill: Four vectors & co- and contra-variance

- Contravariant vector:

$$x^\mu = (ct, \vec{x})$$

$$p^\mu = (E, \vec{p})$$

But covariant derivative:

$$\partial^\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, -\vec{\nabla} \right)$$

- Covariant vector:

$$x_\mu = (ct, -\vec{x})$$

$$p_\mu = (E, -\vec{p})$$

But covariant derivative:

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, \vec{\nabla} \right)$$

Note that the minus sign is “opposite” to the case of the coordinate four-vectors.

- Use cases:

$$\partial_\mu A^\mu = \partial^\mu A_\mu = \frac{\partial A^0}{\partial t} + \frac{\partial A^1}{\partial x} + \frac{\partial A^2}{\partial y} + \frac{\partial A^3}{\partial z}$$

$$\partial_\mu \partial^\mu \phi = \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \phi}{\partial z^2}$$

Skill: Dirac Gamma Matrices

- Dirac γ matrices: $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ in 4x4 matrices.

- We will use the Dirac-Pauli representation

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Or: $\gamma^0 = \beta = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix}$ and $\gamma^k = \beta \alpha_k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$ with Pauli matrices σ_k

Define also the chirality matrix: $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

- Note: although the gamma indices are Lorentz-indices (“space-time”, the gamma-matrices are not 4-vectors! (They are simply constants.)

Note the indices:
(confusing!)

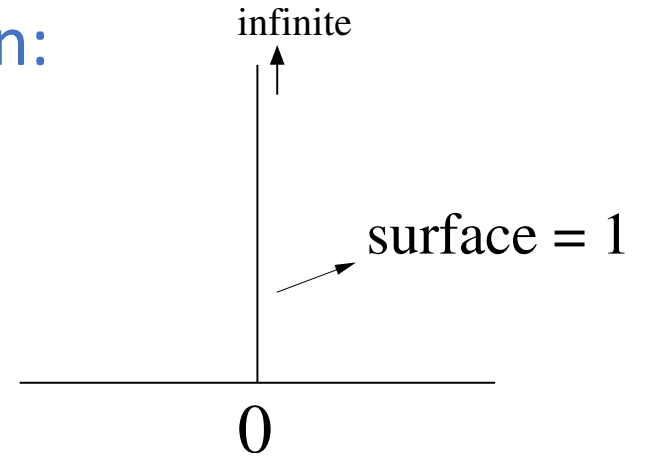
$\mu, \nu = 0, 1, 2, 3$ are the
Lorentz indices in space-time:

Dirac matrix indices: 1, 2, 3, 4
Have to do with the row and
column indices of the matrix
(and spinors)

- Consider a function defined by the following prescription:

$$\delta(x) = \lim_{\Delta \rightarrow 0} \begin{cases} 1/\Delta & \text{for } |x| < \Delta/2 \\ 0 & \text{otherwise} \end{cases}$$

- The integral of this function is normalized: $\int_{-\infty}^{\infty} \delta(x) dx = 1$



- For a function $f(x)$ we have: $f(x)\delta(x) = f(0)\delta(x)$

...and therefore: $\int_{-\infty}^{\infty} f(x)\delta(x) dx = f(0) \int_{-\infty}^{\infty} \delta(x) dx = f(0)$

- An important representation of the Dirac delta function is:

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} dk$$