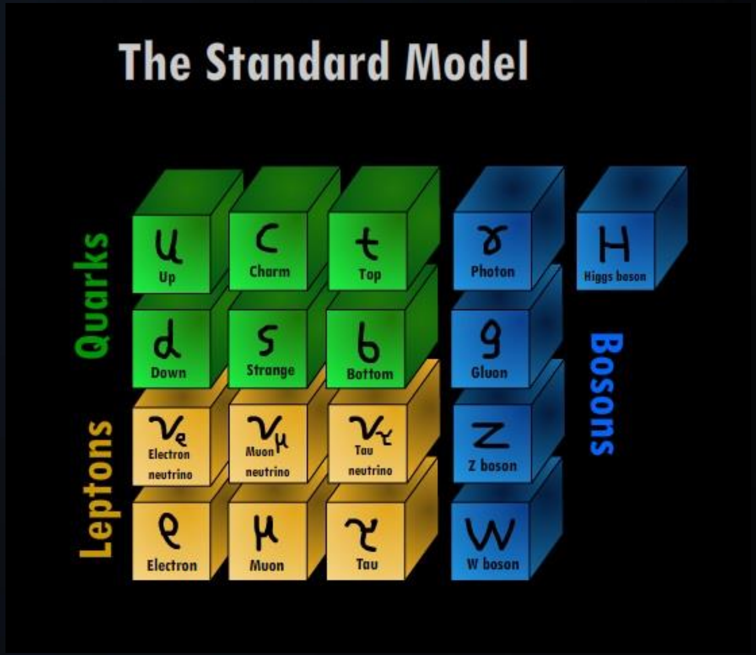
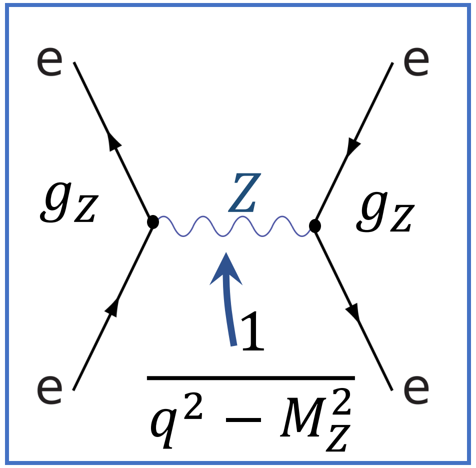
PHY3004: Nuclear and Particle Physics

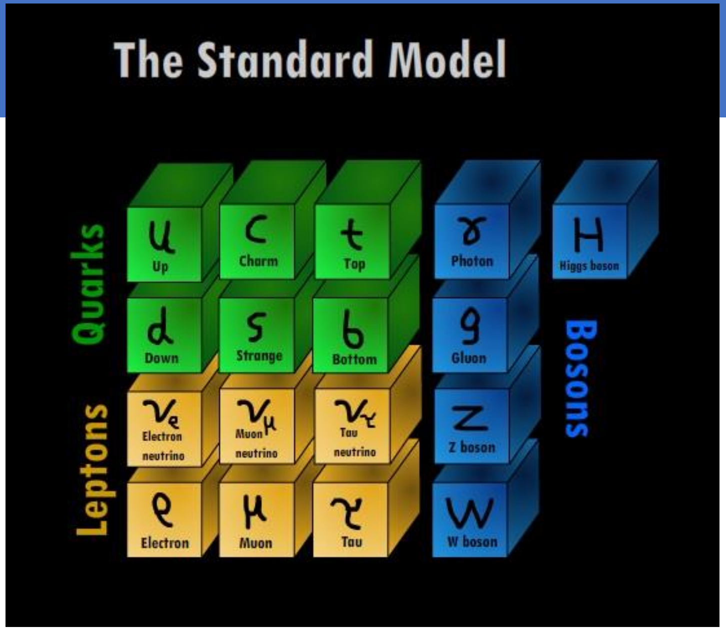
Marcel Merk, Jacco de Vries

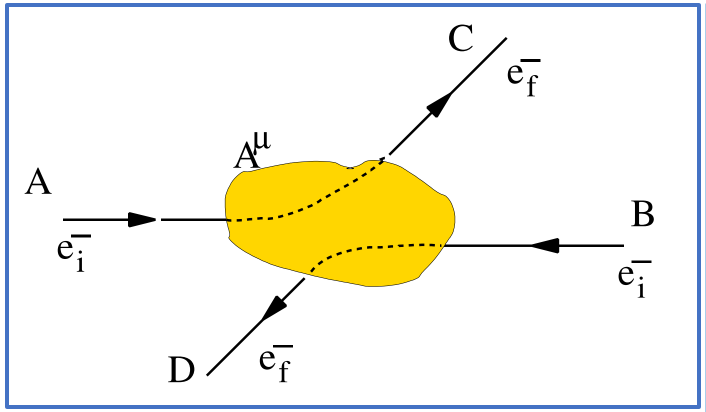


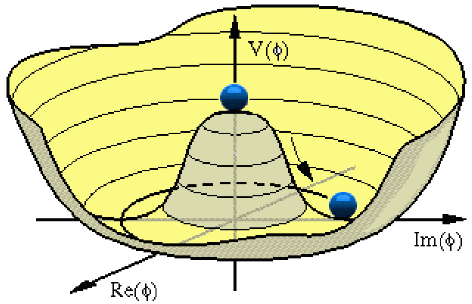
Recap: “Seeing the wood for the trees”

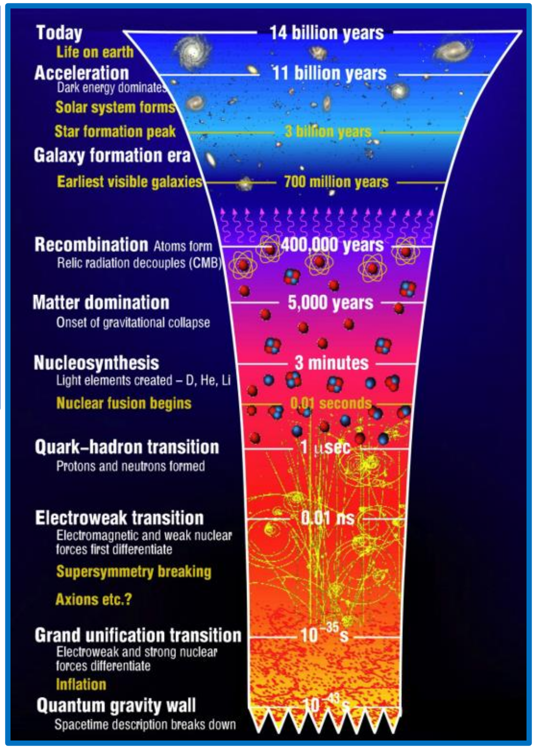
- Lecture 1: “Particles”
  - Zooming into constituents of matter
  - Skills: distinguish particle types, 4-vectors
- Lecture 2: “Forces”
  - Exchange of quanta: EM, Weak, QCD
  - Skills: Spin, Feynman diagrams
- Lecture 3: “Waves”
  - Quantum fields and gauge invariance
  - Dirac algebra, Lagrangian, co- & contra variant
- Lecture 4: “Symmetries”
  - Standard Model, Higgs, Discrete Symmetries
  - Skills: Lagrangians, Chirality & Helicity
- Lecture 5: “Scattering”
  - Cross section, decay, perturbation theory
  - Skills: Dirac-delta function, Feynman Calculus
- Lecture 6: “Detectors”
  - Energy loss mechanisms, detection technologies



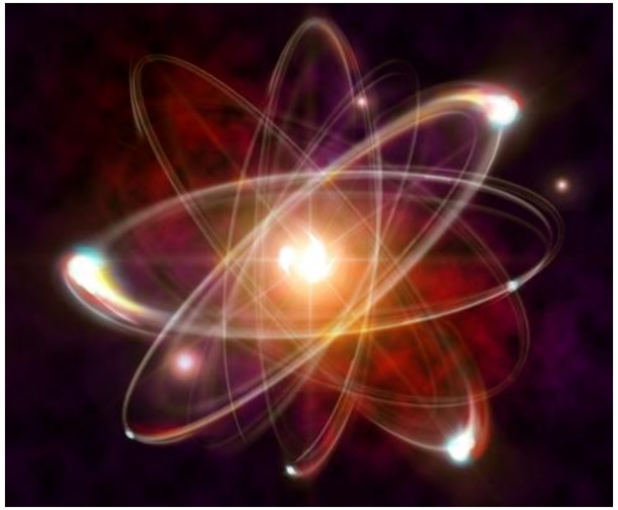


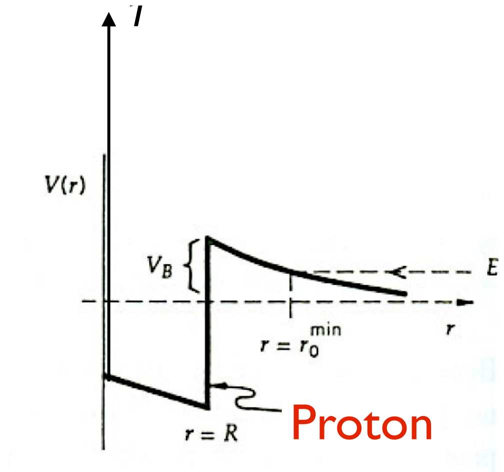


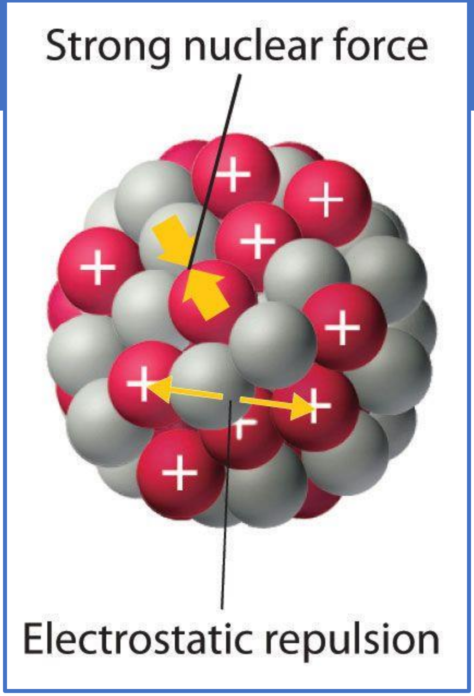


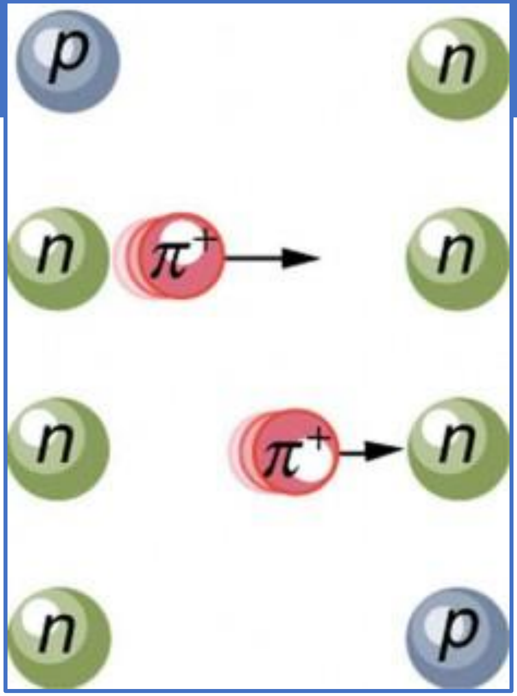


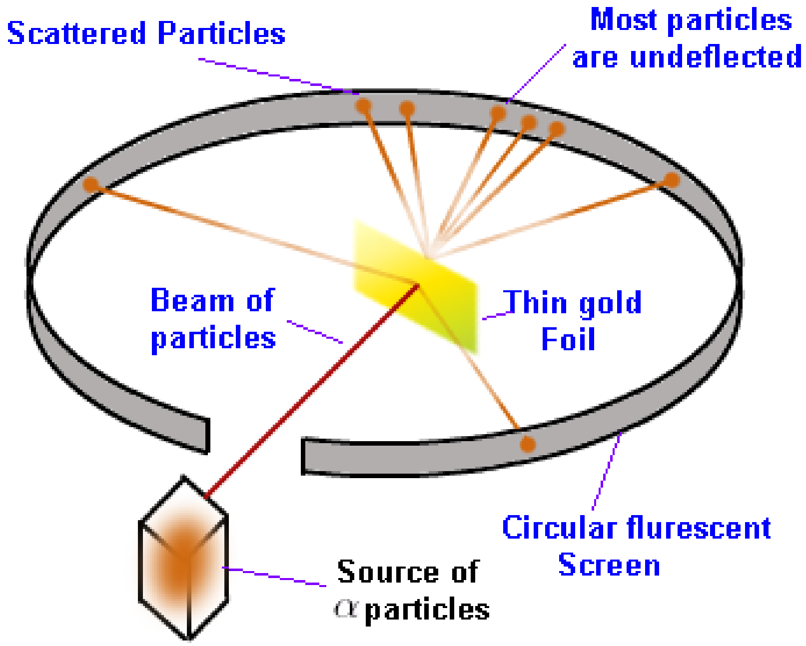
Lecture 1: “Particles” - Nuclear

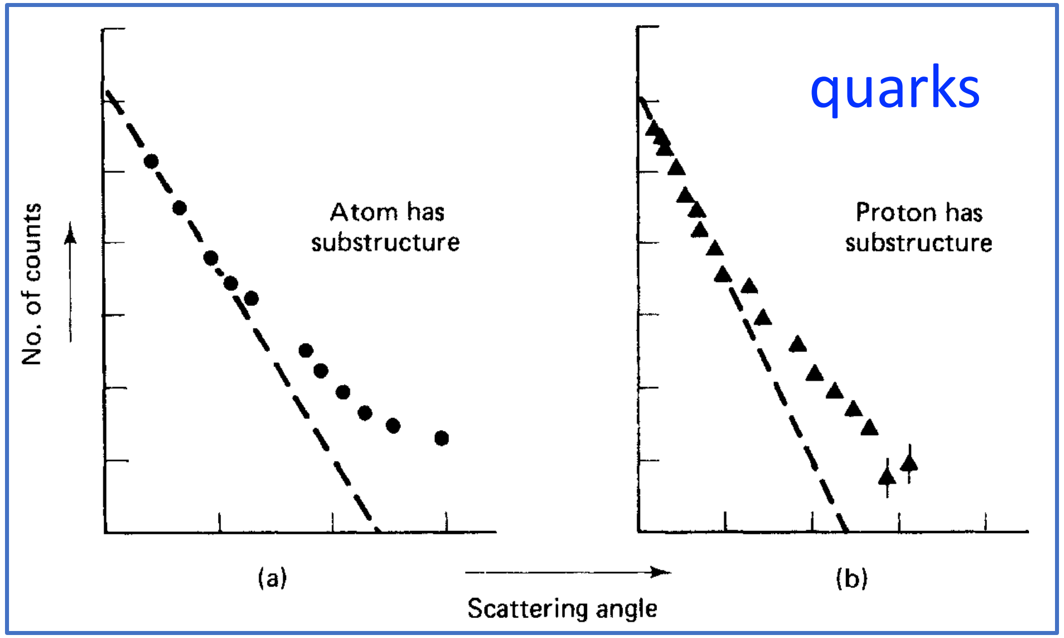




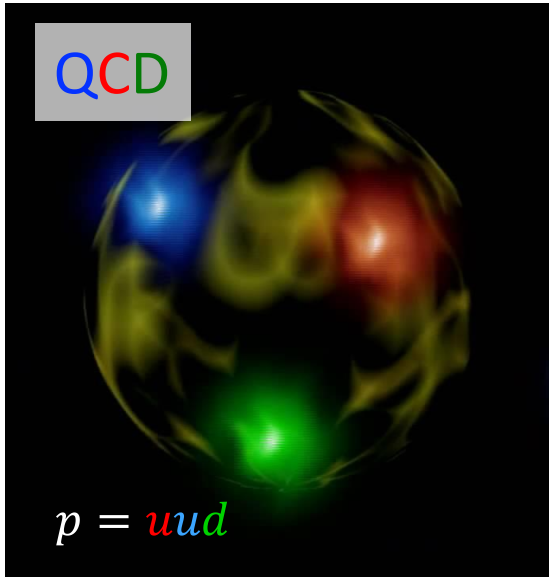


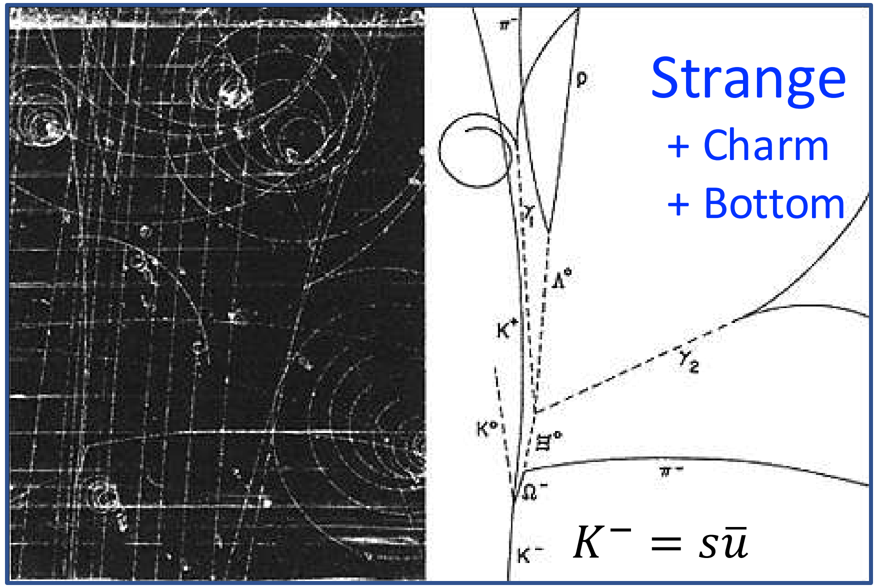


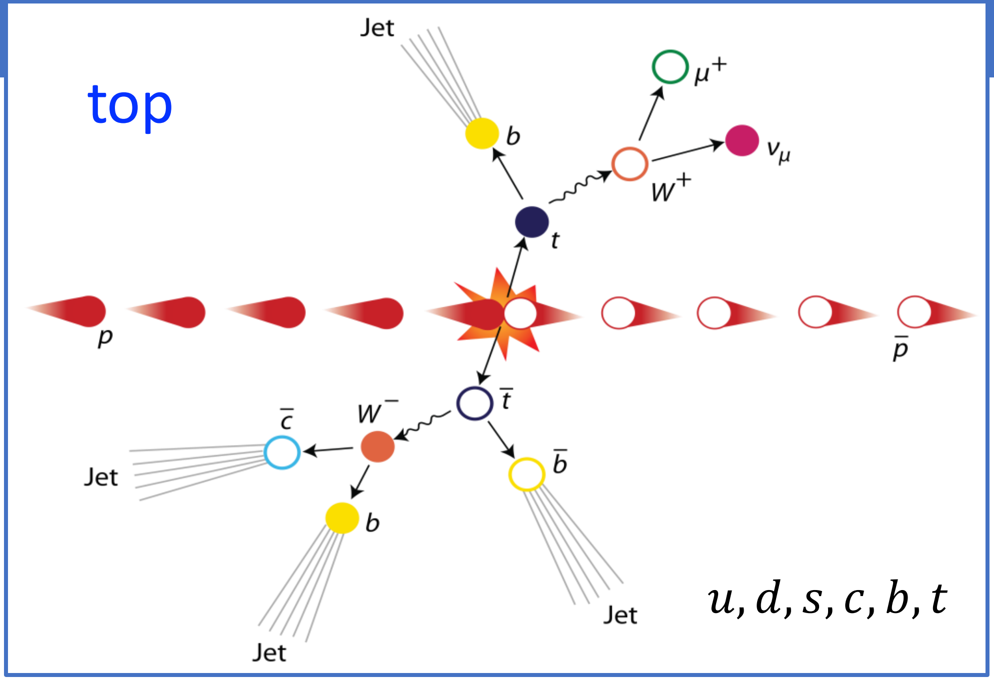


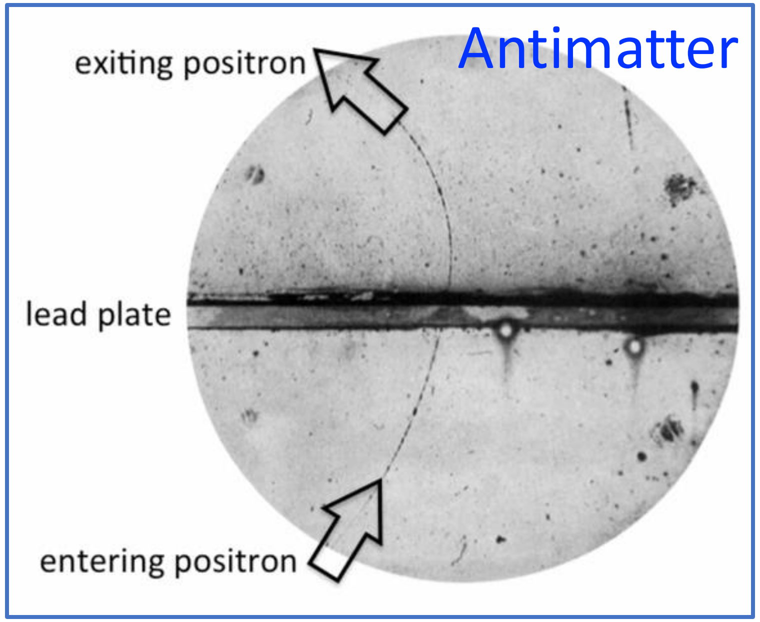


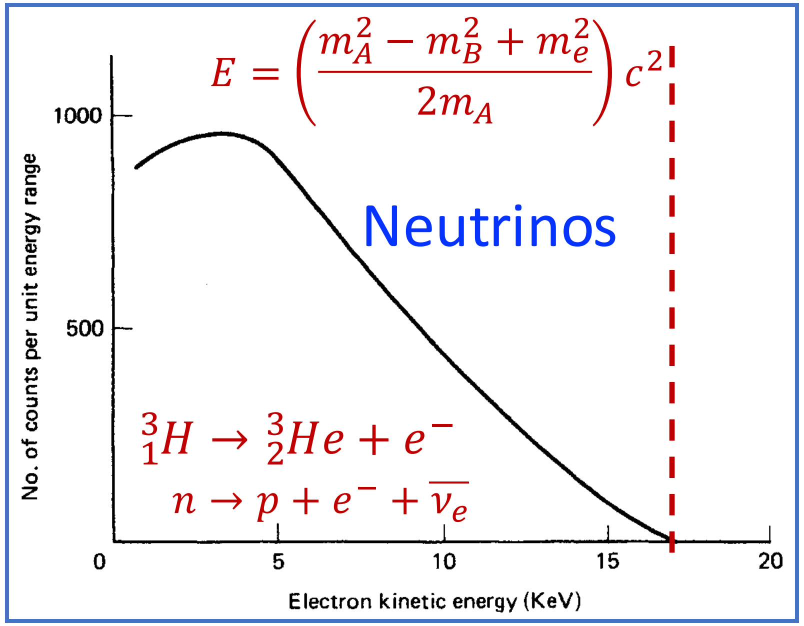
Lecture 1: “Particles” - subatomic

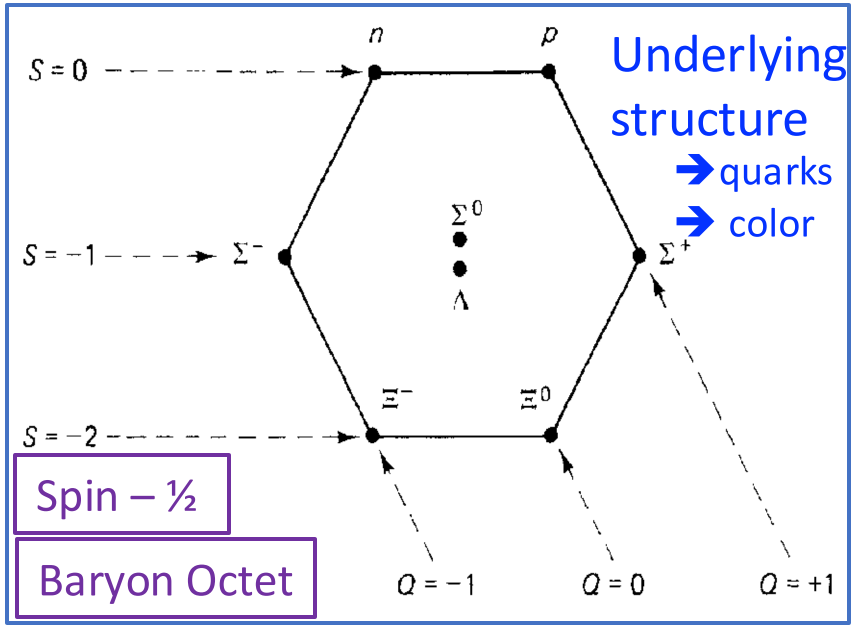








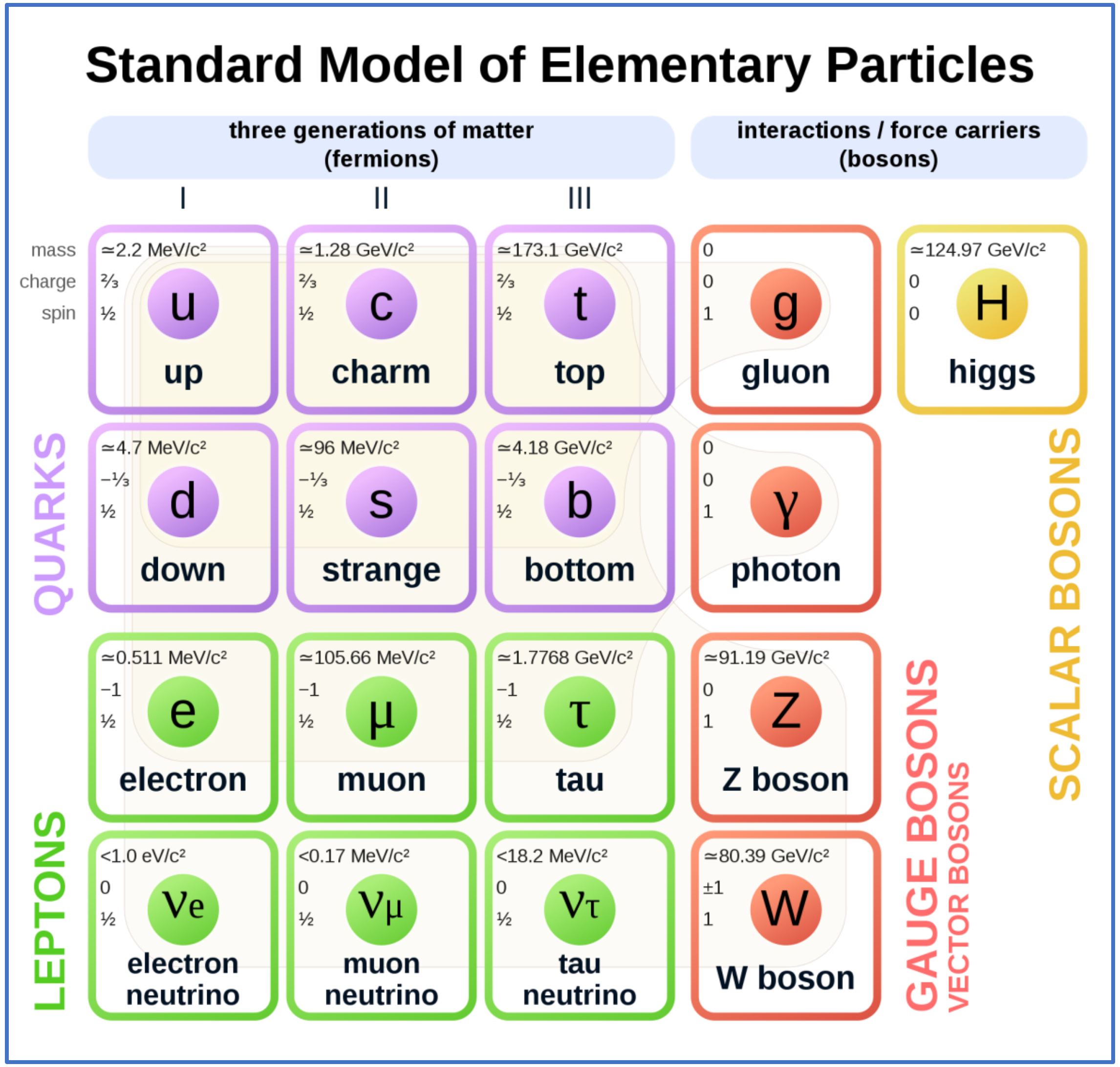




Lecture 1: "Particles"

Classification of particles

- Lepton: fundamental particle
- Hadron: consist of quarks
  - Meson: 1 quark + 1 antiquark (π^+, K^-, B_s^0, …)
  - Baryon: 3 quarks (p, n, Λ, …)
    - Anti-baryon: 3 anti-quarks
- ***Fermion***: particle with half-integer spin.
  - Antisymmetric wave function: obeys Pauli-exclusion principle and Pauli-Dirac statistics
  - All fundamental quarks and leptons are spin-½
  - Baryons ($S={}^1/_2, {}^3/_2$)
- ***Boson***: particle with integer spin
  - Symmetric wave function: Bose-Einstein statistics
  - Mesons: (S=0, 1), Higgs (S=0)
  - Force carriers: γ, W, Z, g (S=1); graviton(S=2)



Lecture 2: "Forces"

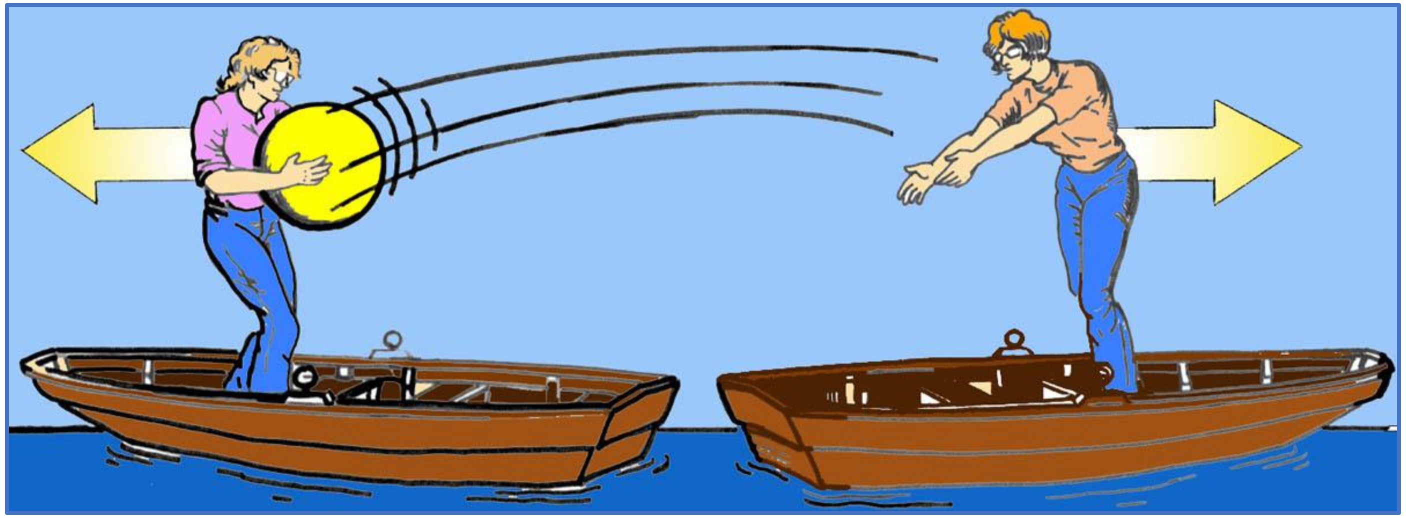
Griffiths chapter 2

- Electromagnetism
- Weak Interaction
- Strong Interaction
- No Gravitation

Lecture : "Forces"

	Gravity	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong
Carried By	Graviton (not yet observed)	W^+ W^- Z^0	Photon	Gluon
Acts on	All	Quarks and Leptons	Quarks and Charged Leptons and W^+ W^-	Quarks and Gluons
Strength	0.000000000000000 0000000000000000 0000000001	0.001	1	100

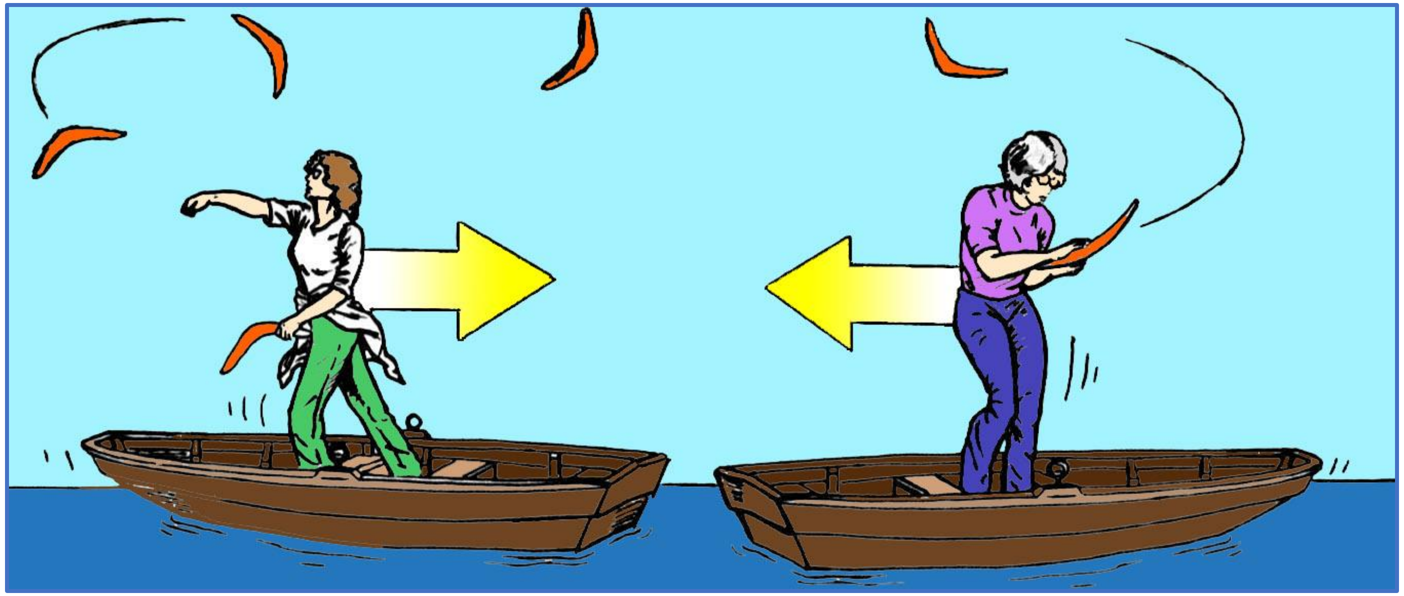
Attractive and Repulsive forces and the quantum exchange



“Repulsive force”

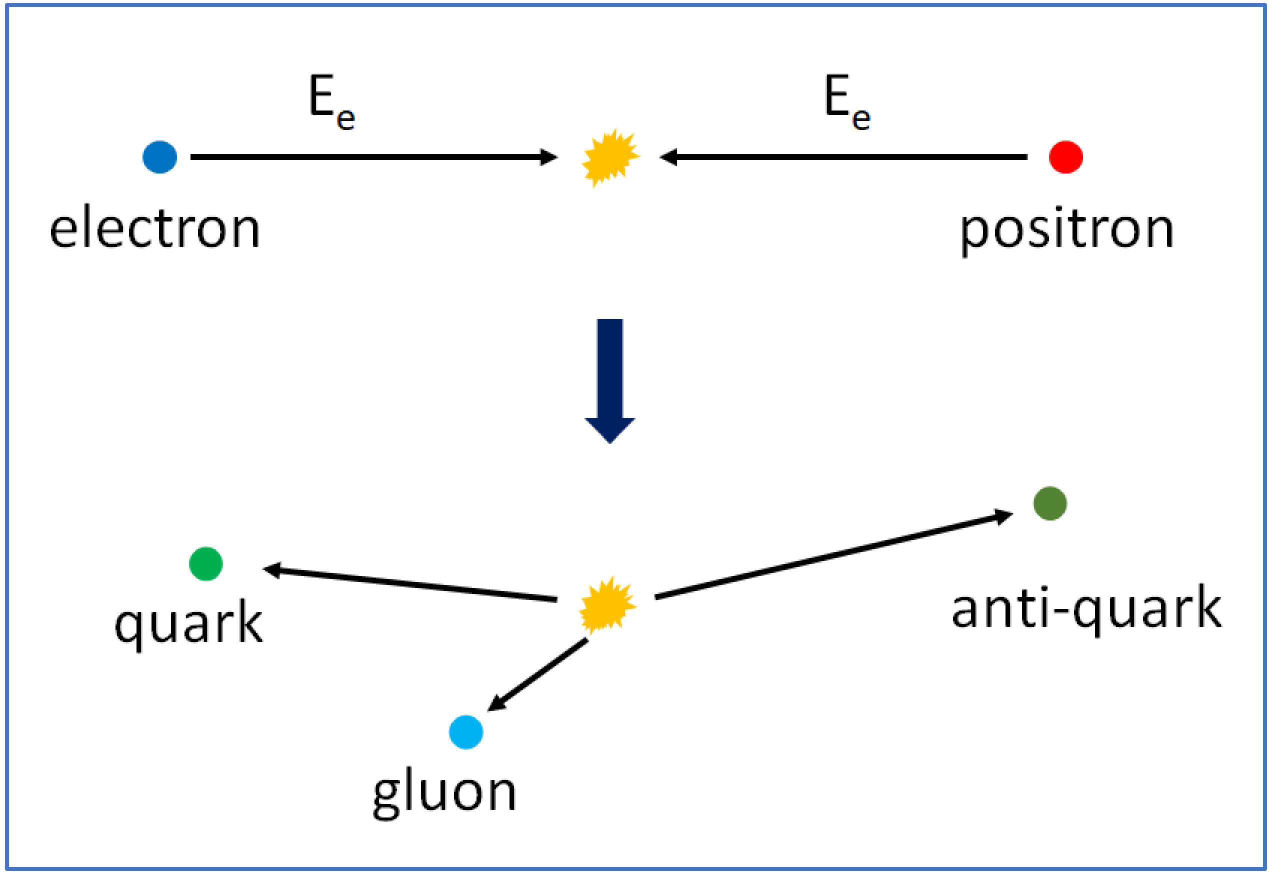
There is no
“action at a distance”

EM: photon
Weak: W, Z bosons
Strong: gluons
Gravitation: graviton(?)



“Atractive force”

Example of a quantum process: $e^+ + e^- \rightarrow q\,\bar{q}\,g$



- Electron-positron annihilation
- Produce a “virtual” photon
- Photon produces a quark + antiquark
- Quark can radiate a gluon

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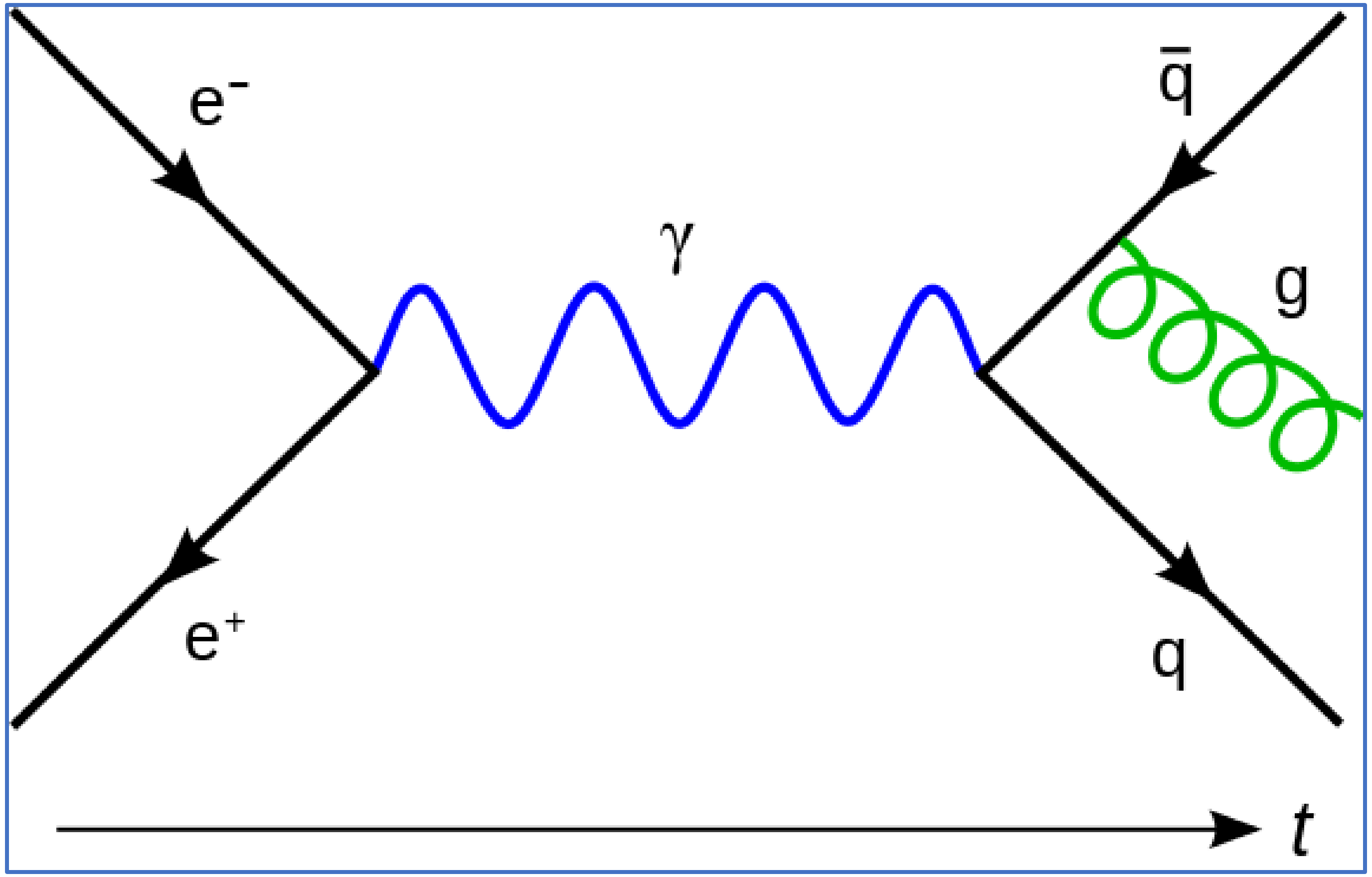
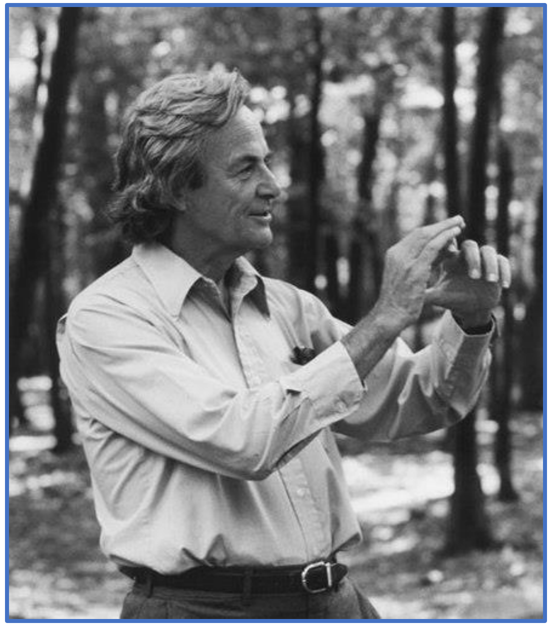




Feynman diagram

- Electron-positron annihilation
- Produce a "virtual" photon
- Photon produces a quark + antiquark
- Quark can radiate a gluon

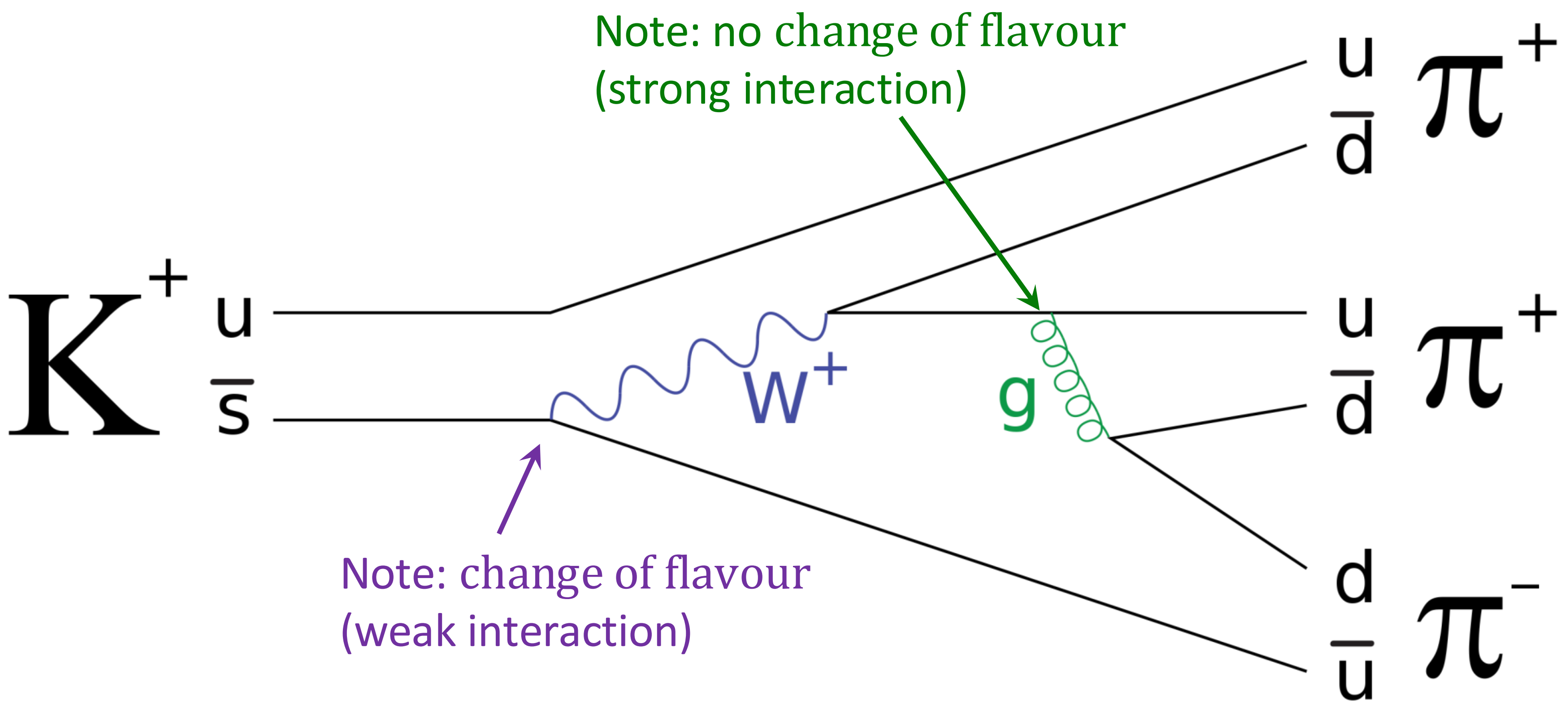
Note:

This is a ***graphic*** representation of a ***calculation*** that represents a quantum event. We use it to ***talk*** about it.

Do ***not*** take it ***literally*** as what happens

Another example

- Kaon decay with *weak* interaction mediated by a W-boson
- Quark anti-quark produced by the *strong* interaction mediated by a gluon



Part 1

The Electromagnetic Interaction

Quantum Electrodynamics (QED)

Quantum Electrodynamics (QED)

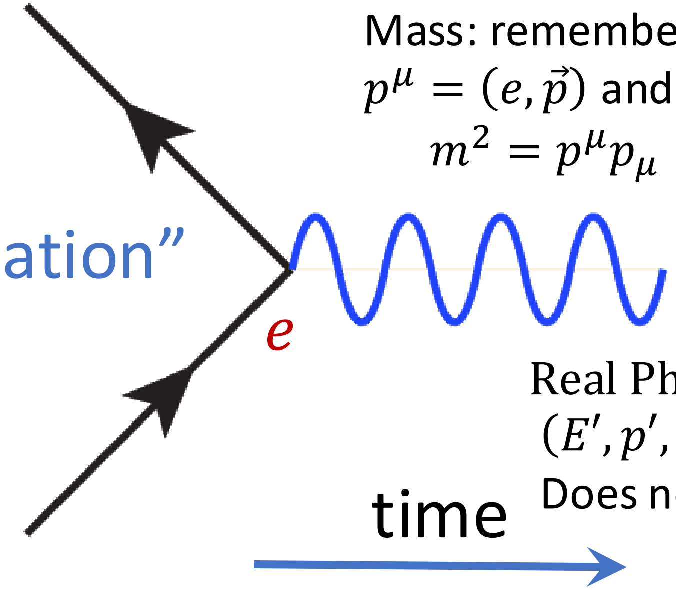
- There is only one fundamental quantum interaction process: ie one “vertex”

Mass: remember $(c = 1)$

$p^{\mu} = (e, \vec{p})$ and $p_{\mu} = (e, -\vec{p})$

$$m^2 = p^{\mu} p_{\mu} = E^2 - \vec{p}^2$$

“annihilation”

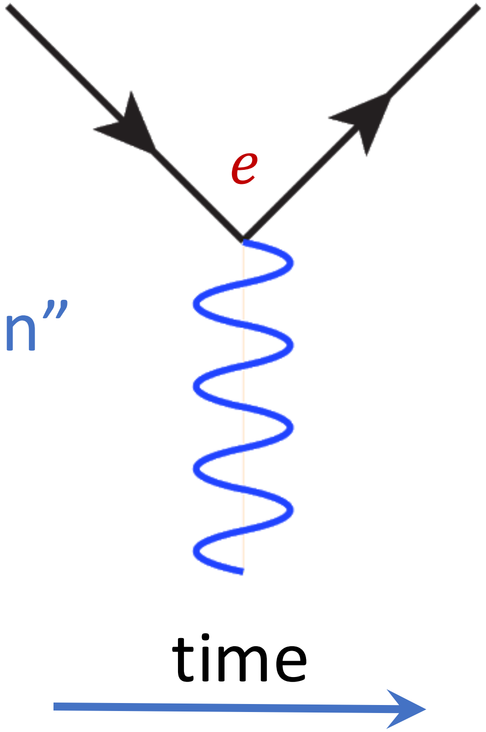


Real Photon: $E = |\vec{p}| \equiv p: \;\; p^{\mu} = (p, p, 0{,}0)$

$(E', p', 0{,}0) + (E', -p', 0{,}0) = (2E', 0{,}0{,}0)$

Does not work! See Griffiths page 64, 65

“radiation”

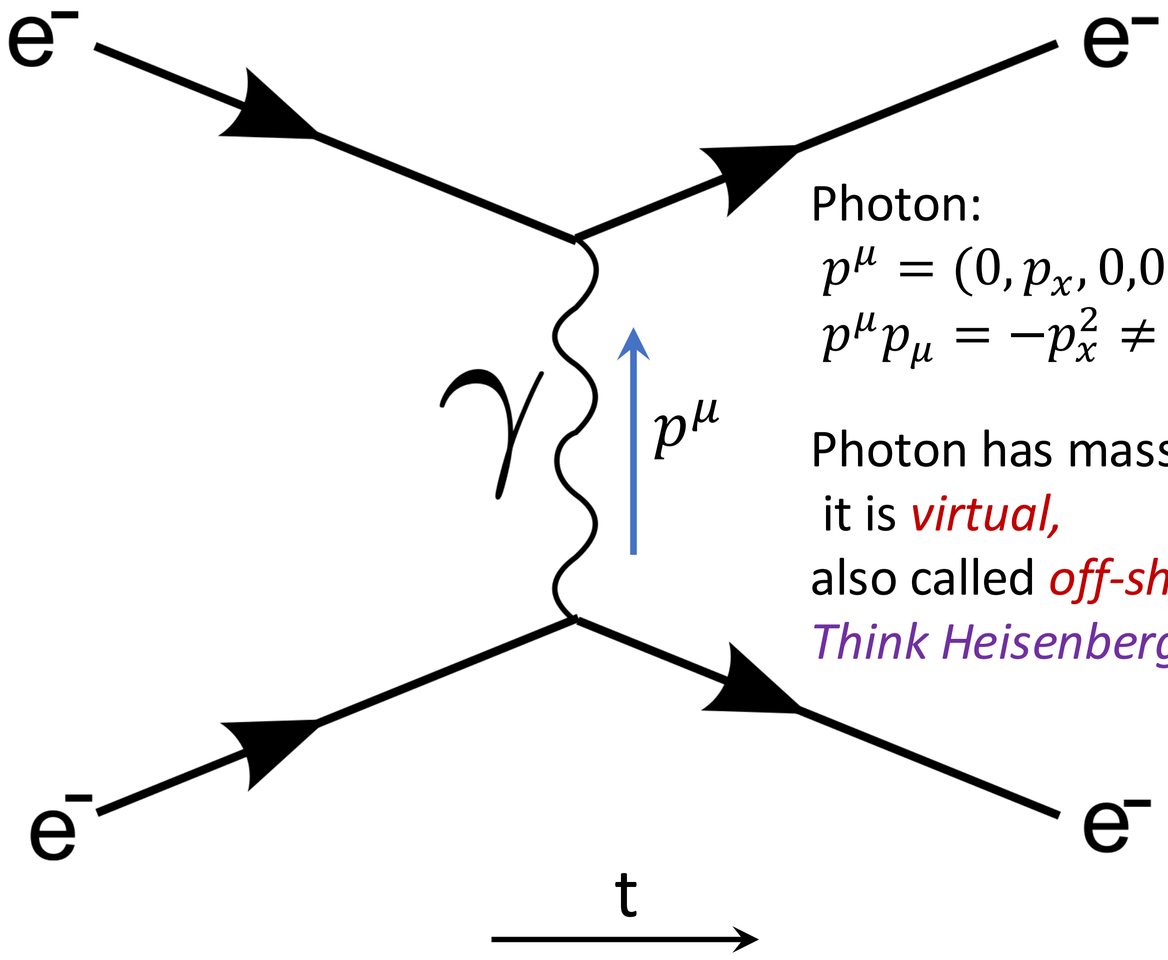


- “strength” of the “vertex coupling” is equal to the elementary charge e
- Note: this vertex can only be part of a process since energy and momentum cannot be conserved at the same time in this “2-to-1” process → check with C.M.
  - This is an off-shell or virtual photon

A Real QED Process: Möller scattering

- "Coulomb" repulsion of elastic colliding electrons

- Convention: the direction of an arrow w.r.t. time determines whether it is a colliding particle or antiparticle
  - Should leave out the charge to be unambiguous



Photon:

$p^\mu = (0, p_x, 0,0)$

$p^\mu p_\mu = -p_x^2 \neq 0$??

Photon has mass;

it is *virtual,*

also called *off-shell*

Think Heisenberg!

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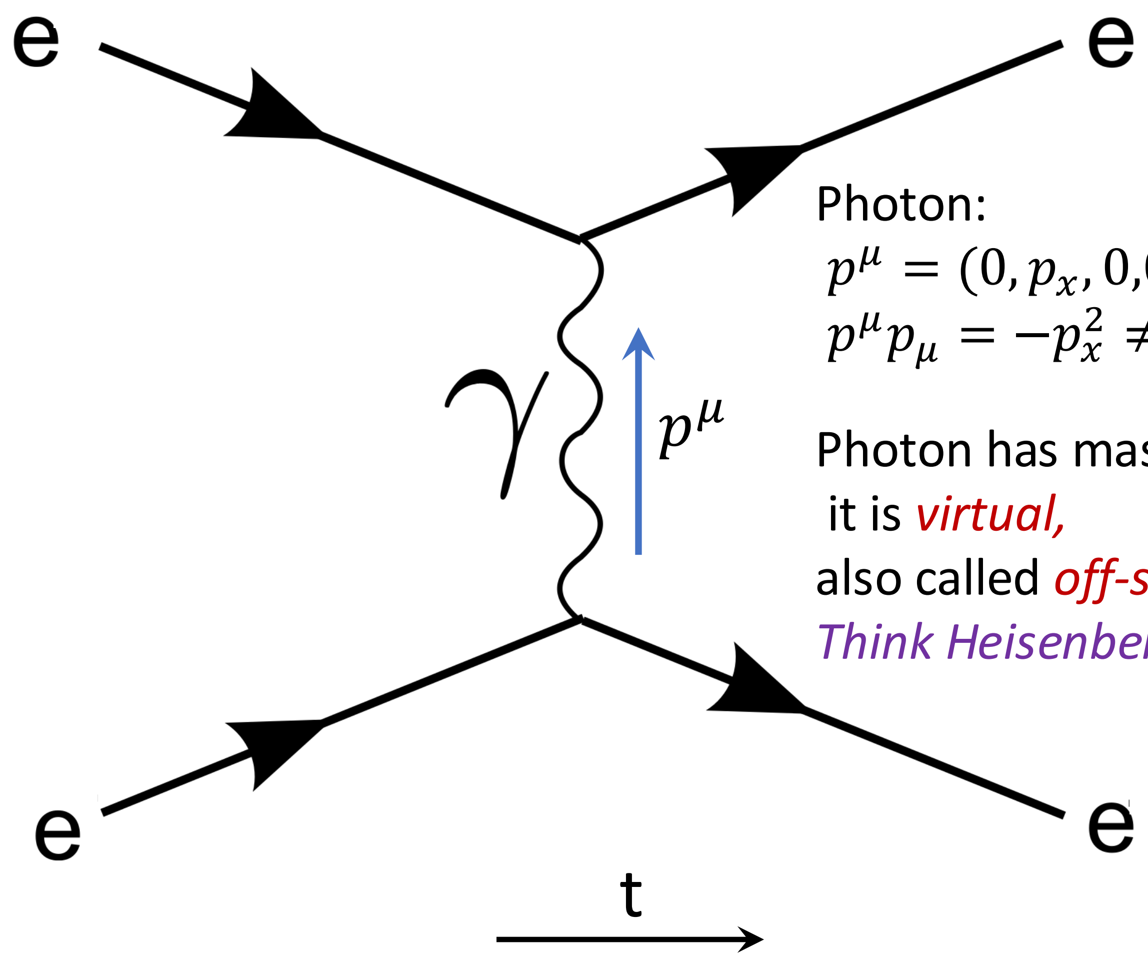

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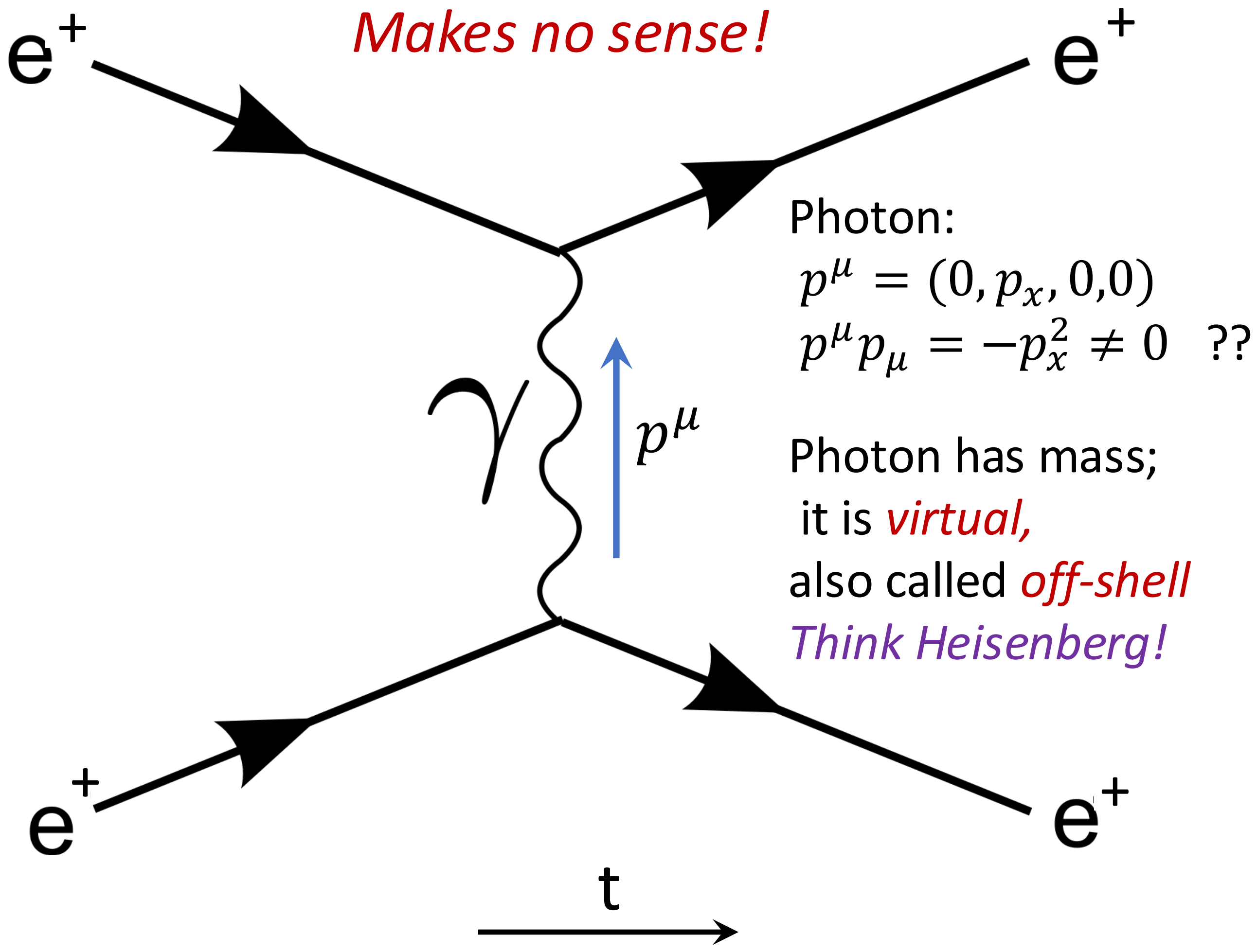
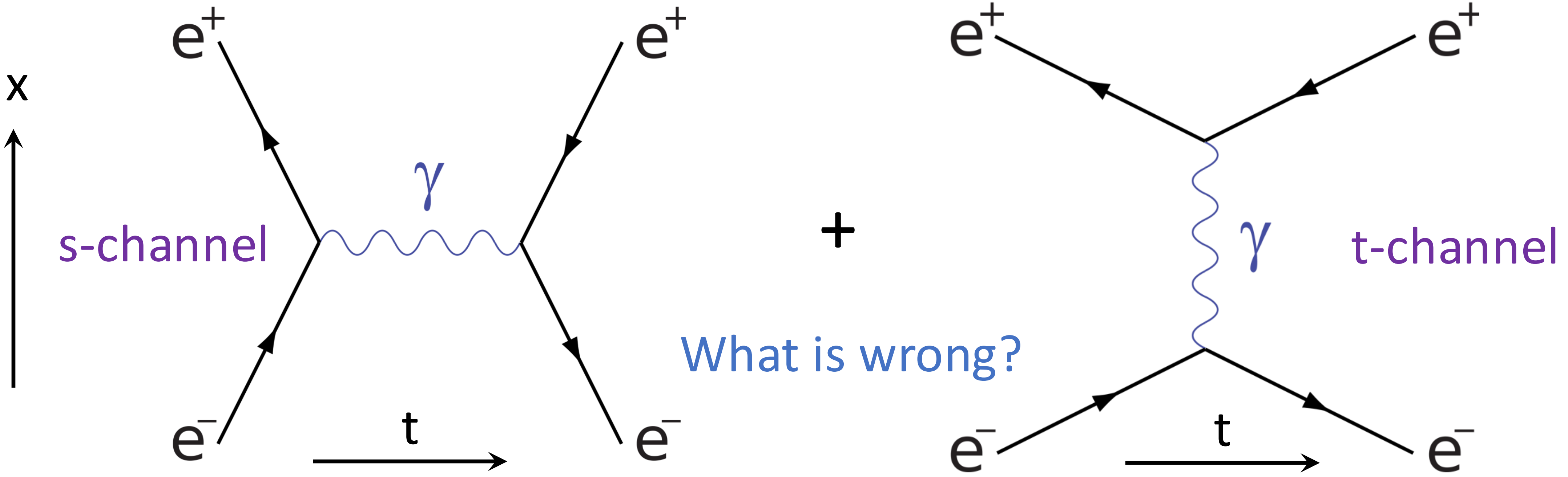
A Real QED Process: Möller scattering

- “Coulomb” repulsion of elastic colliding electrons
- Convention: the direction of an arrow w.r.t. time determines whether it is a colliding particle or antiparticle
  - Should leave out the charge to be unambiguous
- Feynman diagrams are always drawn with *particles*
  - *Anti-particles* are represented with an arrow pointing against the direction of time.



A Real Process: Bhabha Scattering

- Scattering of electron and positron, two quantum processes (“Feynman diagrams”) in one real process:
  - Timelike (!) “s-channel” exchange (name comes from Mandelstam variable s)
    - e^- and e^+ annihilate into a photon, which converts back to a e^- and e^+ pair
  - Spacelike (!) ”t-channel” exchange (name comes from Mandelstam variable t)
    - e^- and e^+ scatter in each others EM field



What is wrong?

A Real Process: Bhabha Scattering

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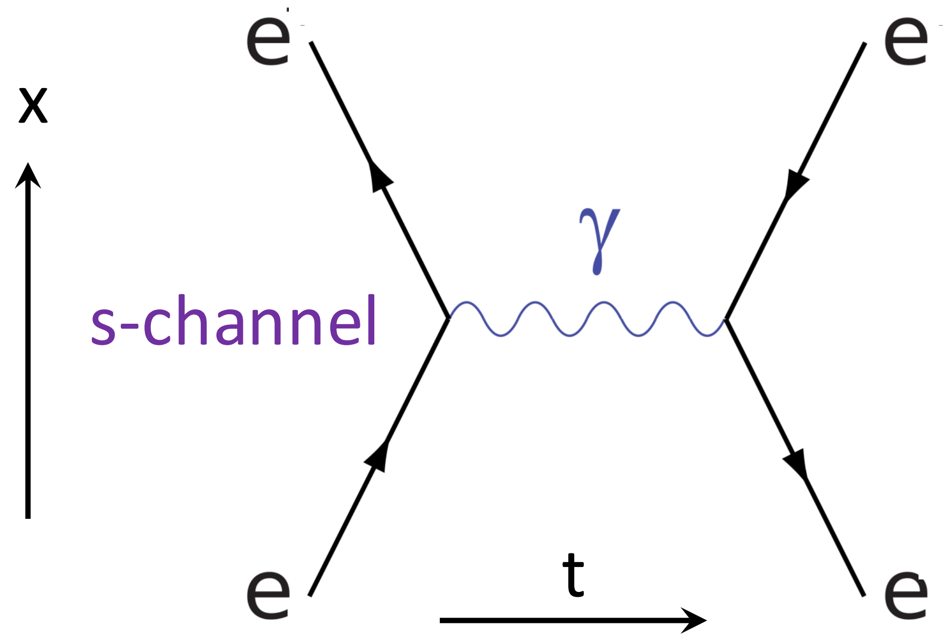

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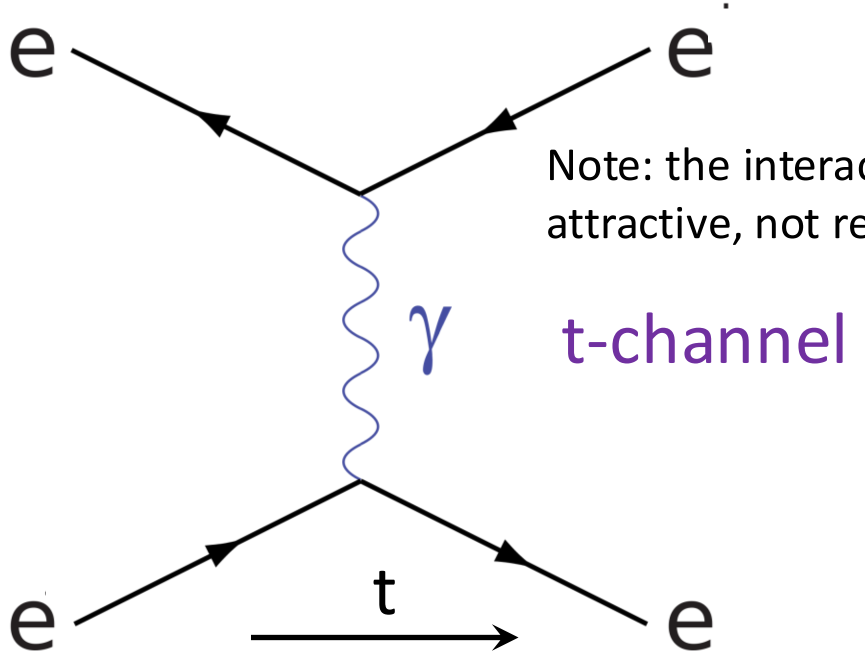
Now its OK!



Note: the interaction is attractive, not repulsive!

Feynman Diagram

- Calculation using Feynman rules:

$$\mathcal{M} = \frac{e^2}{q^2}$$

- Fine structure constant:

$$\alpha = \frac{e^2}{4\pi}$$

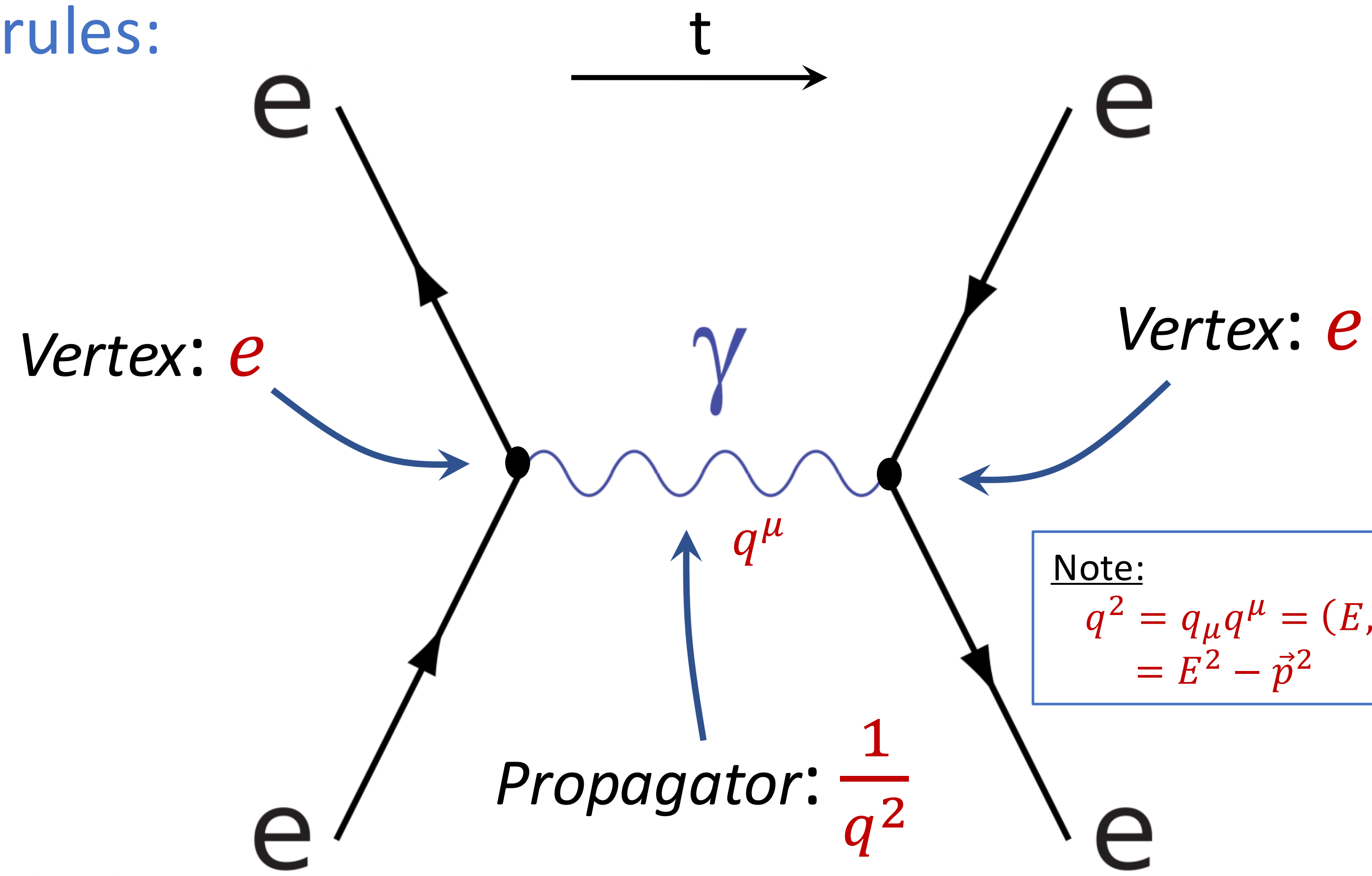
Hence: $\mathcal{M} \propto \frac{\alpha}{q^2}$

q^2 is "virtuality" of the photon

- Strength of the interaction
  - Determined by the coupling constant e:

$$\alpha = \frac{e^2}{4\pi} = 1/137$$

why???



q^μ = Energy, momentum 4-vector

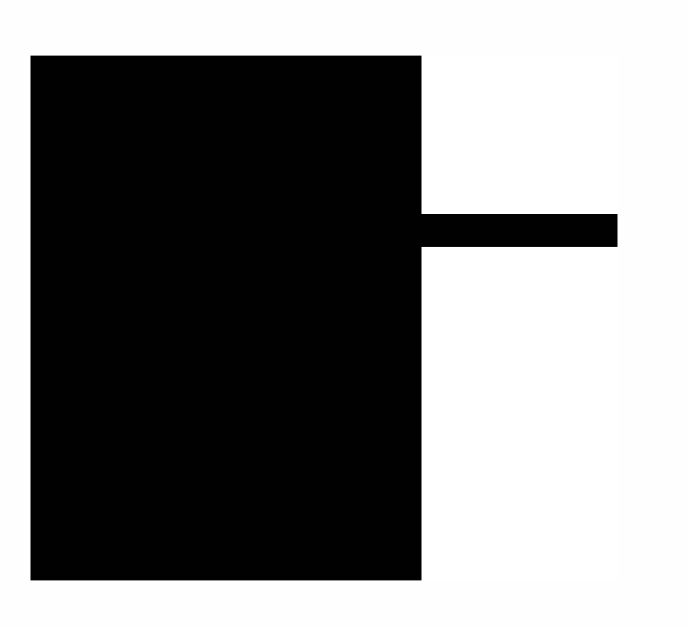
Note:
$q^2 = q_\mu q^\mu = (E, \vec{p})^2$
$= E^2 - \vec{p}^2$

Note: a real photon should have $q^2 = 0$!

Scattering with photons

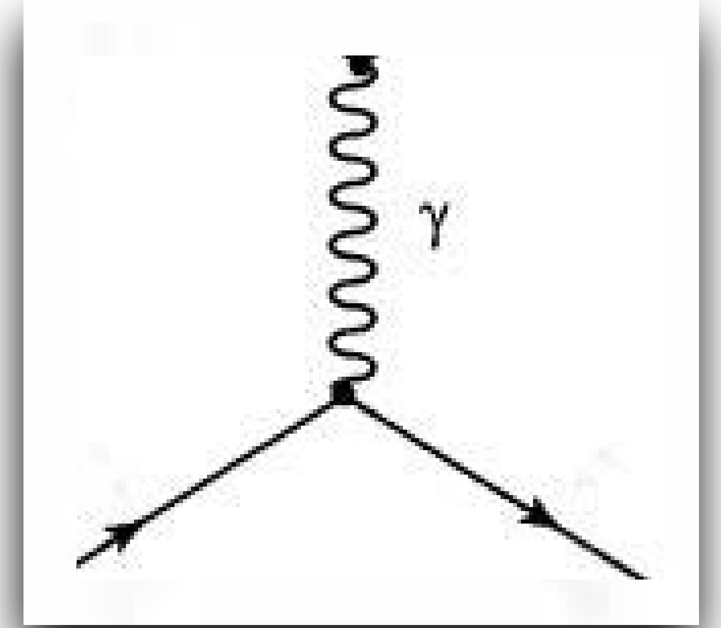
Real processes with external photon lines

t →



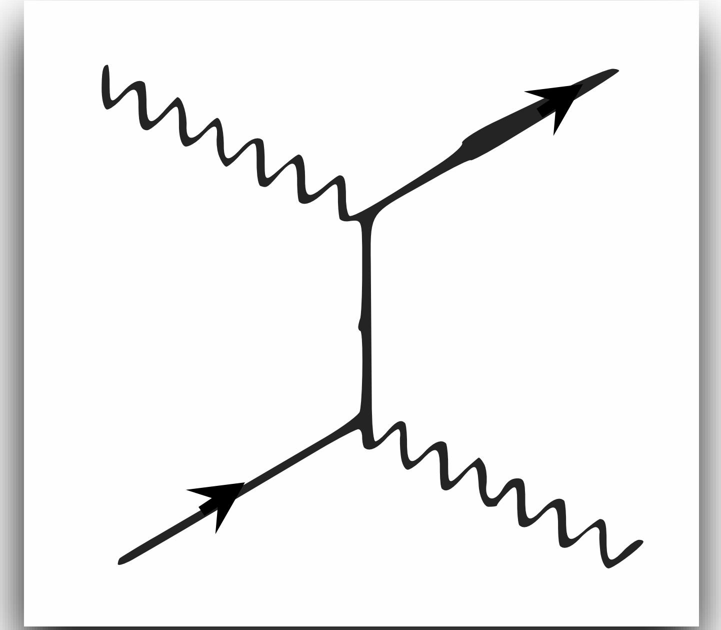
Pair annihilation

$q + \overline{q} \rightarrow \gamma + \gamma$



Pair production

$\gamma + \gamma \rightarrow q + \overline{q}$



Compton scattering

$q + \gamma \rightarrow q + \gamma$

Remember “crossing”

Perturbation Theory

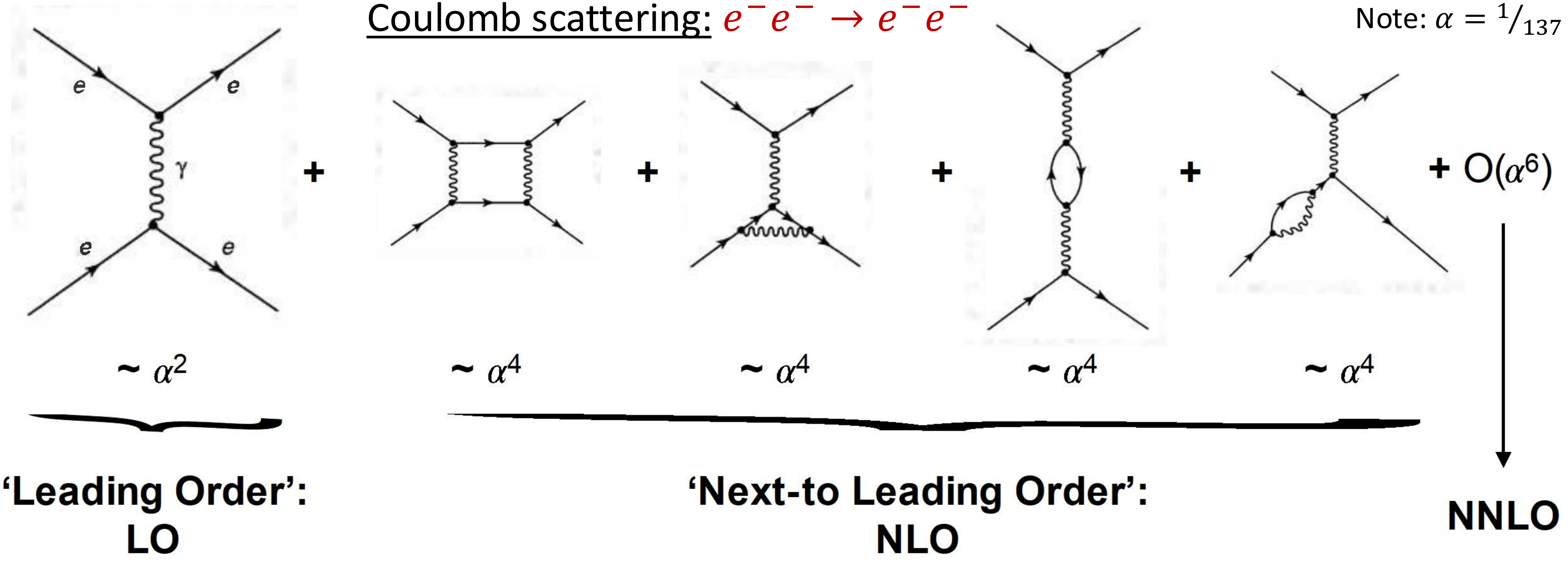
See lecture 5

Feynman diagrams sofar describe the lowest order "Perturbation Theory".
Calculations with Feynman diagrams are never exact, always an approximation.

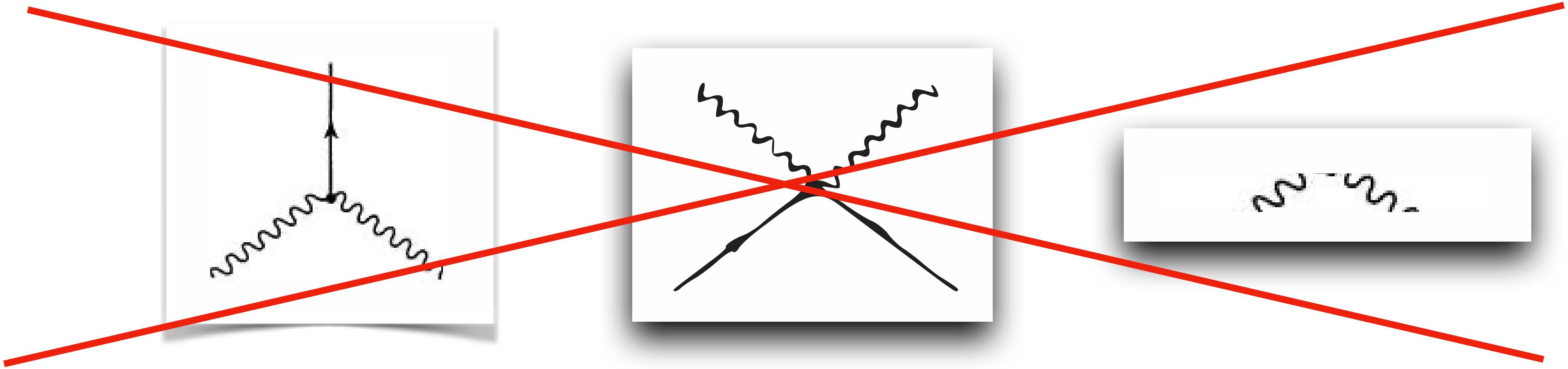
- We simply do not know how to do *exact* calculations in particle physics
- QED is extremely precise because many complicated diagrams have been calculated

<u>Coulomb scattering:</u> $e^- e^- \to e^- e^-$

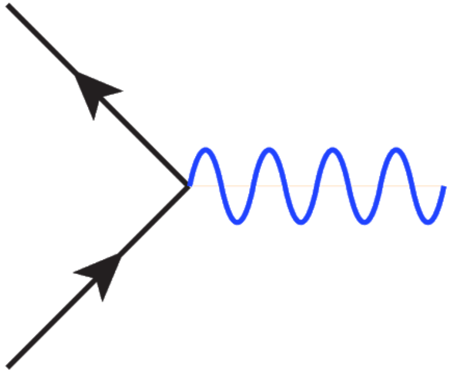
Note: $\alpha = {}^{1}/_{137}$



Not all diagrams exist



- You can only use combinations of the fundamental QED vertex:



The background of Feynman Diagrams

- More about scattering and Feynman diagrams in Lecture 5

- For an intuitive approach to Feynman diagrams, see wikipedia: https://en.wikipedia.org/wiki/Quantum_electrodynamics and references therein, or the booklet of Feynman:
  - “The strange theory of light and matter”

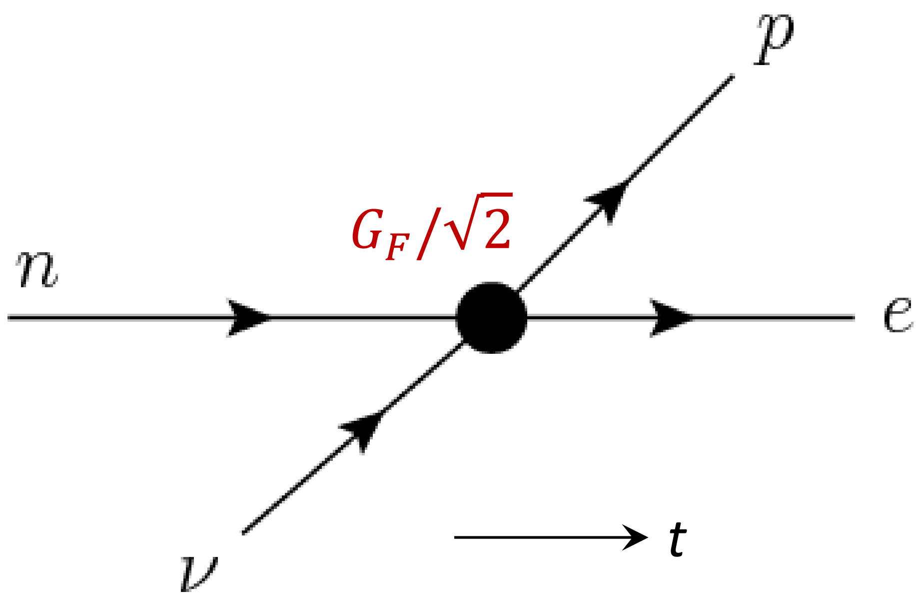
Griffiths section 2.4

Part 2
The Weak Interaction

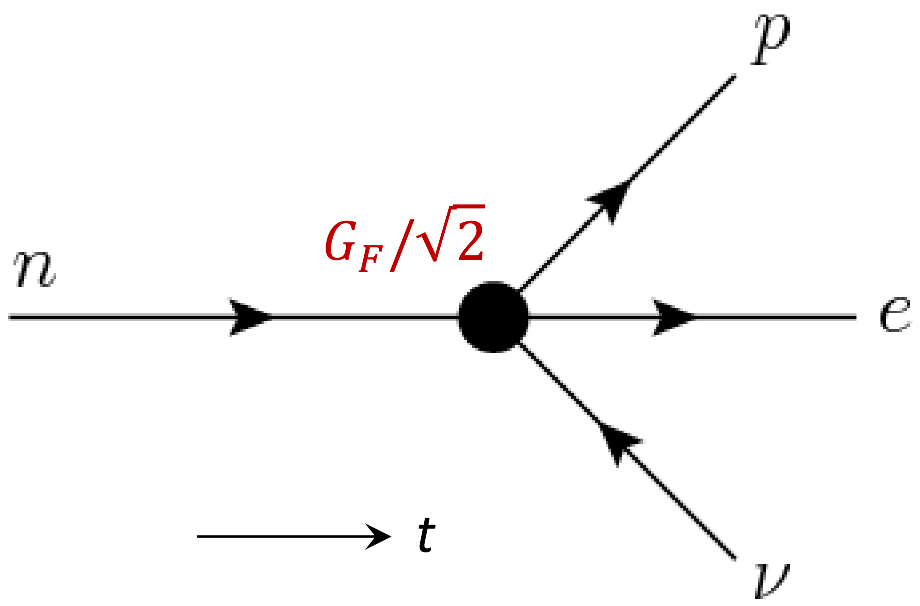
Historical: Fermi 4-point interaction

- Becquerel discovered radioactivity based on the weak force
  - $n \rightarrow p + e + \overline{\nu}$
- The original model for the weak interaction is from Fermi
  - For obvious reasons it is called the 4-point 'contact' interaction (1933).
  - The strength of the interaction was given by Fermi's constant $G_F = 10^{-5}$

$$n + \nu \rightarrow p + e$$

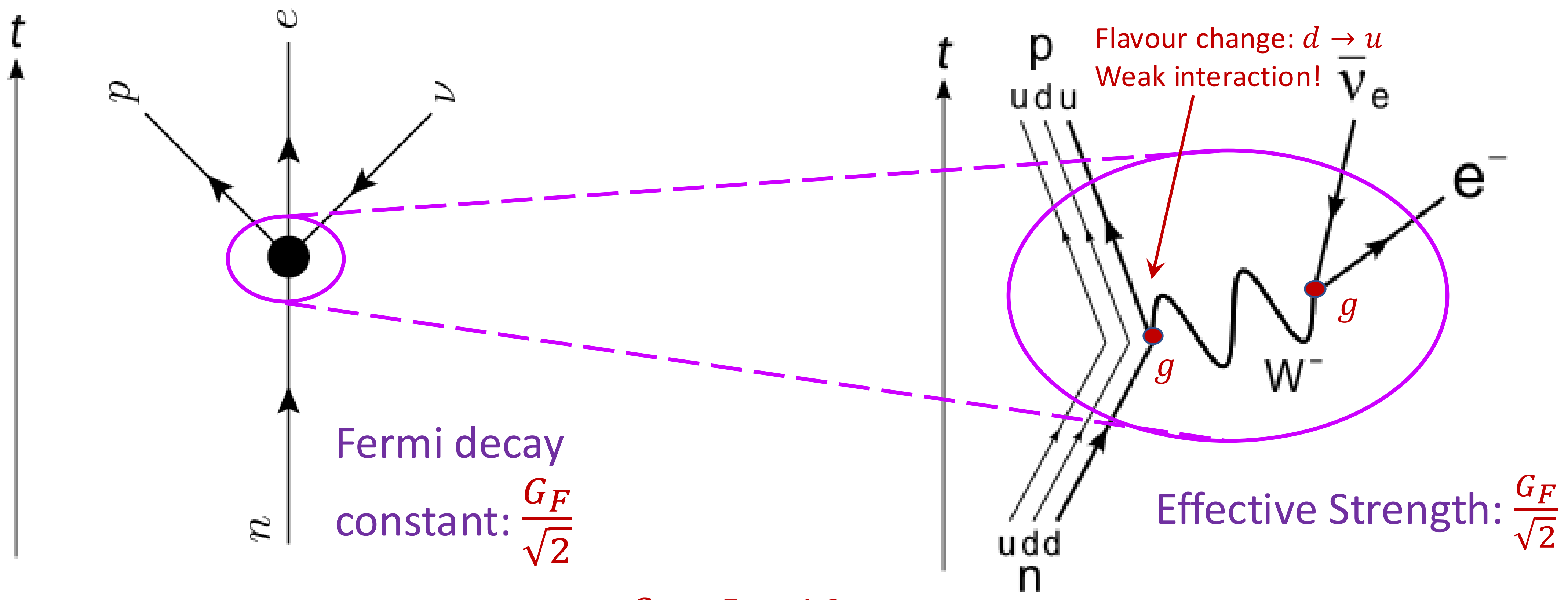


$$n \rightarrow p + e + \overline{\nu}$$



Quantum zoom-in on Fermi 4-point interaction

- Fermi 4-point interaction
- Very short range W exchange

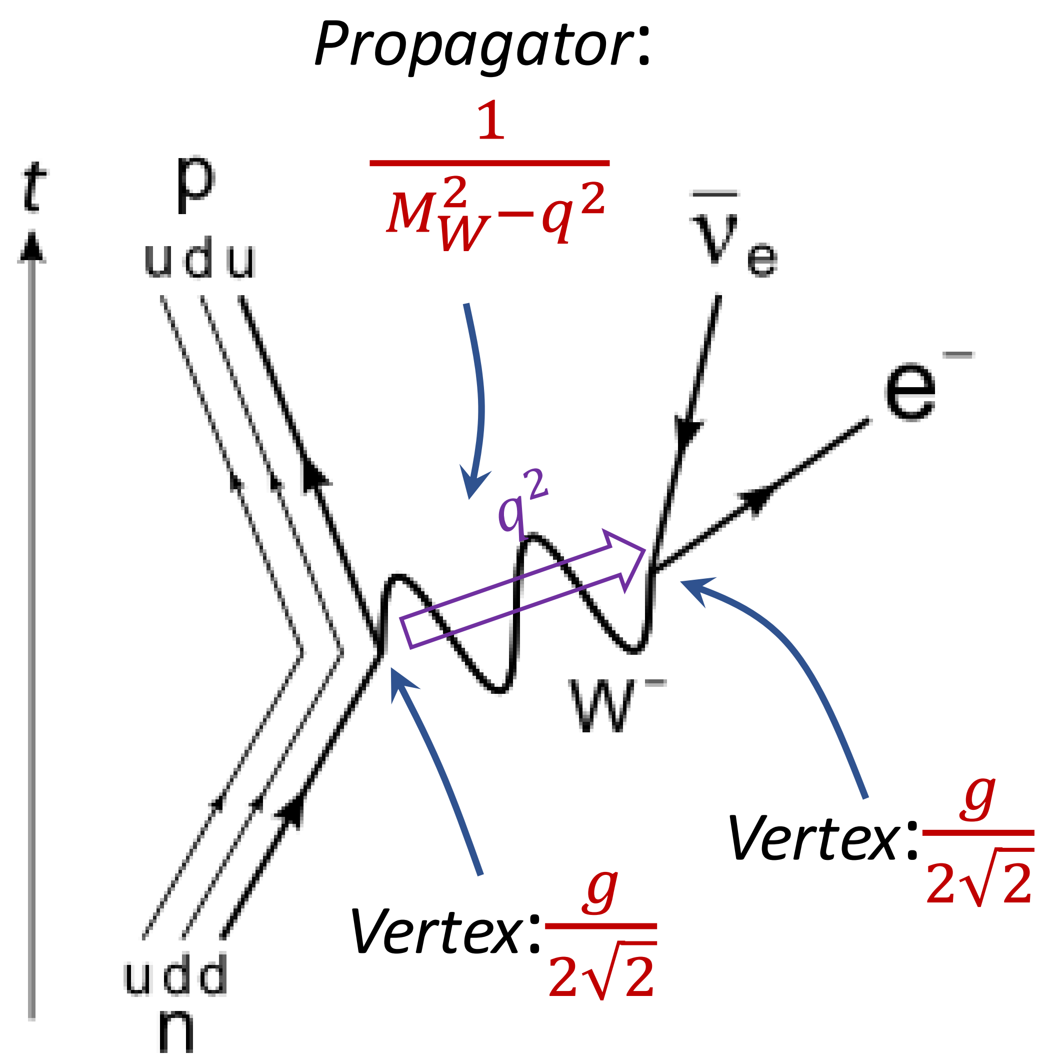


G_F = Fermi Constant

- Similar to electrodynamics the weak interaction has a propagator
  - W spin-1 boson
- At quark level it involves a $d \to u$ transition: $d \to u\, e^-\, \bar{\nu}_e$

The Weak Interaction

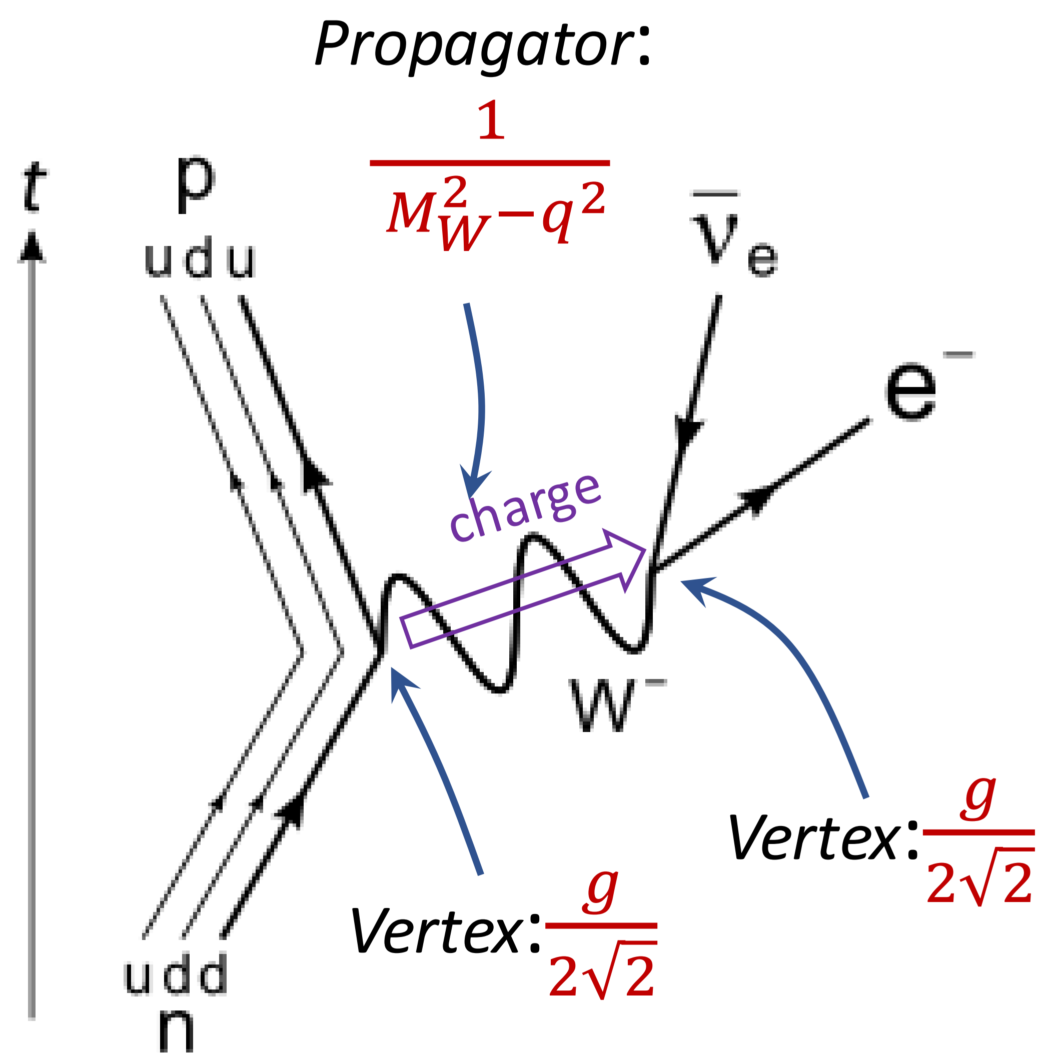
- Feynman rules



- W-boson has electric charge
  - The process is called a *charged current*
- q^2 is the momentum transfer from the hadronic to leptonic vertex
  - Here due to mass difference $m_n - m_p$

The Weak Interaction

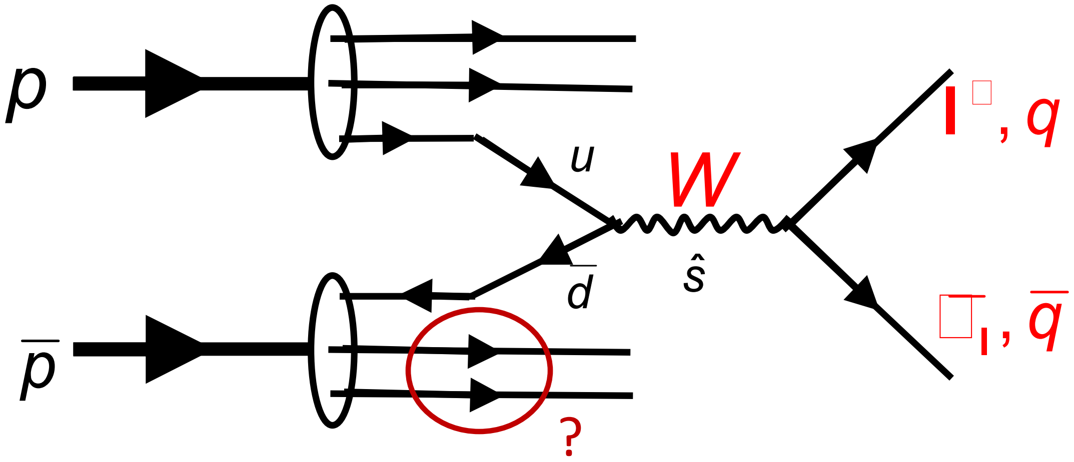
- For $q^2 \ll M_W^2$: $G_F = \frac{g}{2\sqrt{2}} \frac{1}{M_W^2} \frac{g}{2\sqrt{2}} = \frac{g^2}{8M_W^2}$



- W-boson has electric charge
  - The process is called a *charged current*
- q^2 is the momentum transfer from the hadronic to leptonic vertex
  - Here due to mass difference $m_n - m_p$
- Effect of propagator is only noticeable when $q^2 \sim M_W^2$ (M_W=80.4 GeV)
  - If q^2 is large: ***resonance***
    - In that case weak force is strong
  - If q^2 is small: then effectively a 4-point Fermi coupling
    - Propagator $\sim 1/M_W^2$ ➔ weak force
    - $G_F = \frac{g^2}{8M_W^2}$; $\alpha_w = \frac{g^2}{2\pi} = \frac{1}{30} > \alpha_{QED}$
- The weak coupling constant is large!
  - Interaction is weak due to large W mass

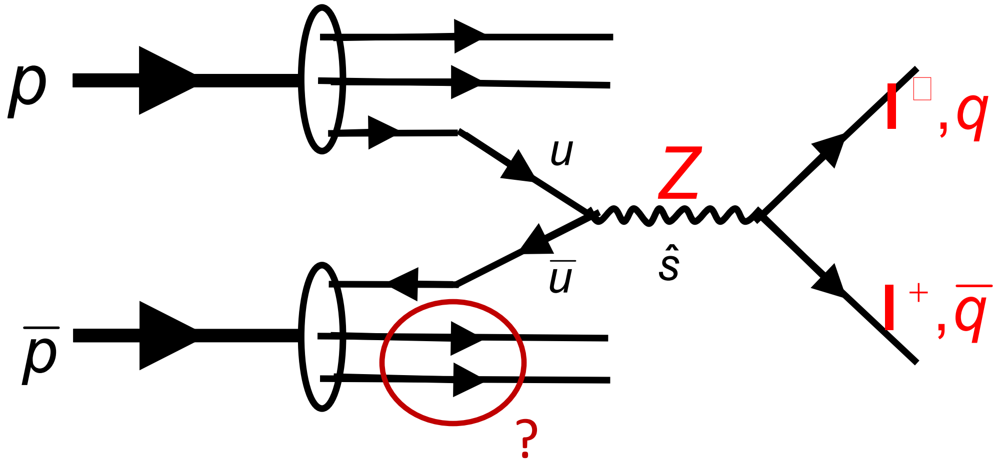
W and Z bosons

- Glashow, Salam, Weinberg (GSW model, 1968) predict three weak mediators
  - W^+ , W^- , Z - bosons
- Discovery of the W and Z bosons at CERN, 1983, UA1 and UA2 experiments
  - Look at high energy proton - antiproton collisions
  - $M_W = 80\ GeV/c^2$, $M_Z = 91\ GeV/c^2$ as predicted by GSW model



$p\bar{p} \to W \to l\bar{\nu}_l + X$

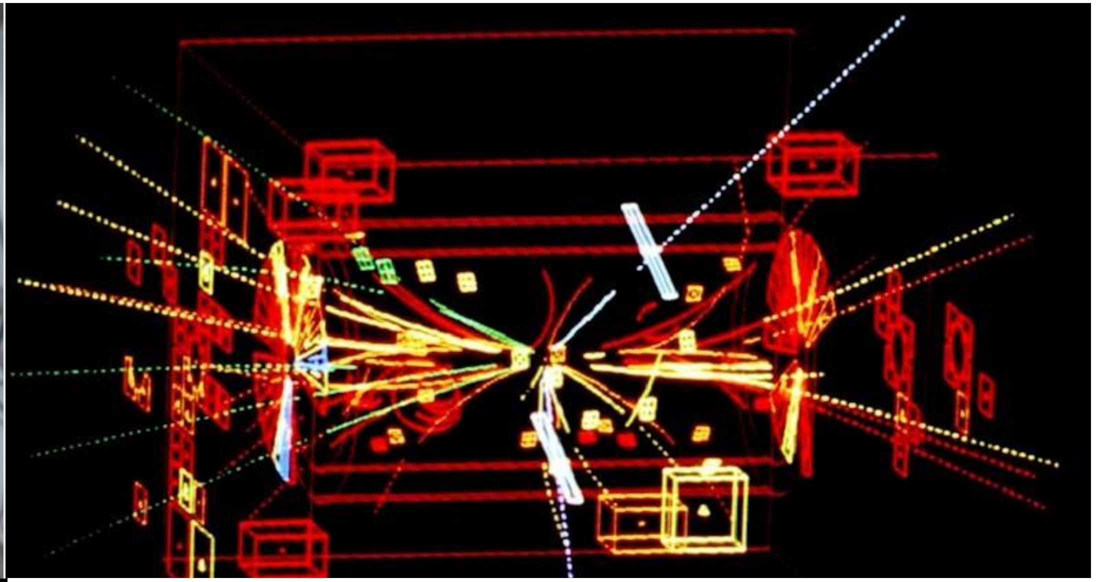
Flavour *changing charged* currents: $u\bar{d}$

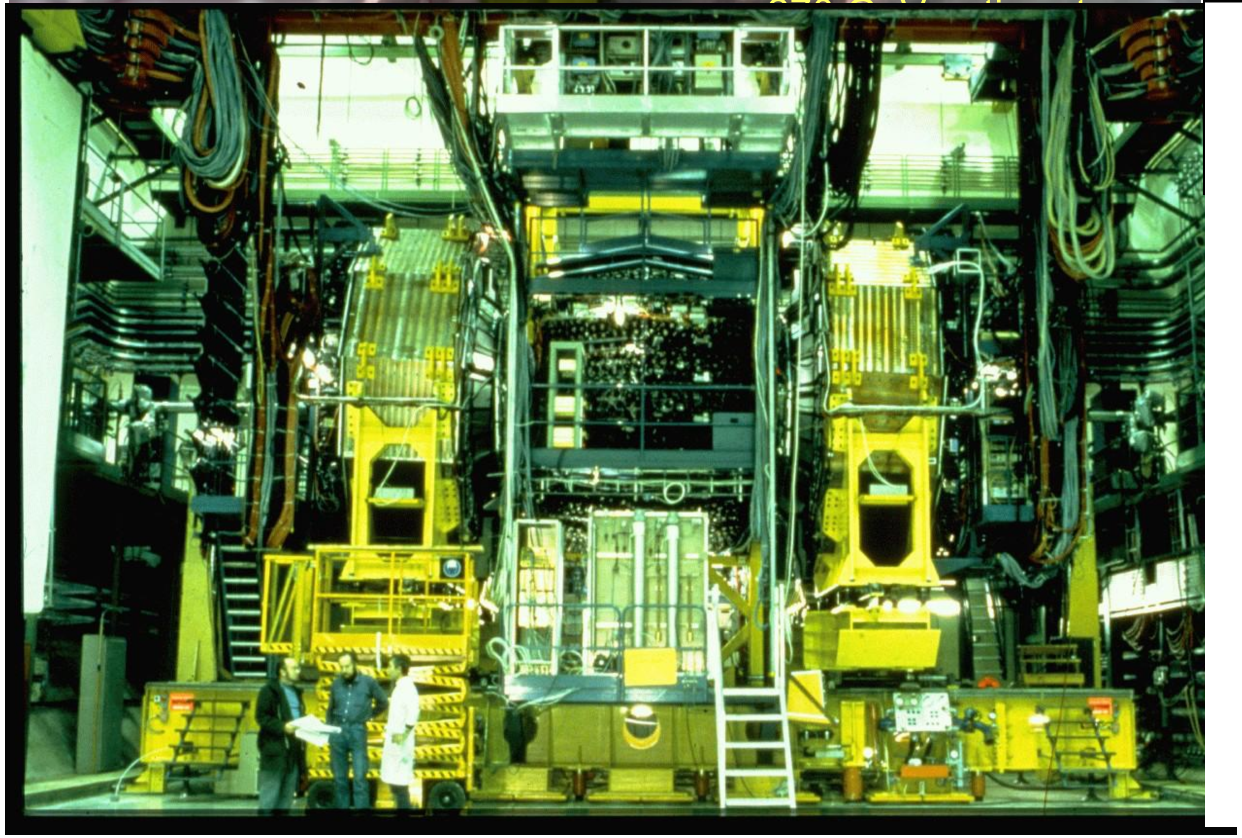


$p\bar{p} \to Z \to f\bar{f} + X$

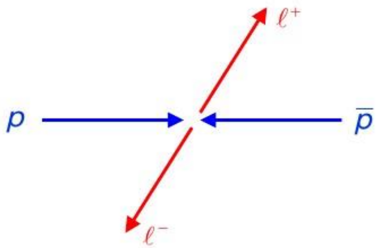
Flavour *conserving neutral* currents: $u\bar{u}$

Discovery of the Z boson at CERN in 1983



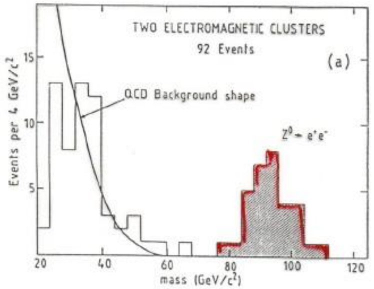


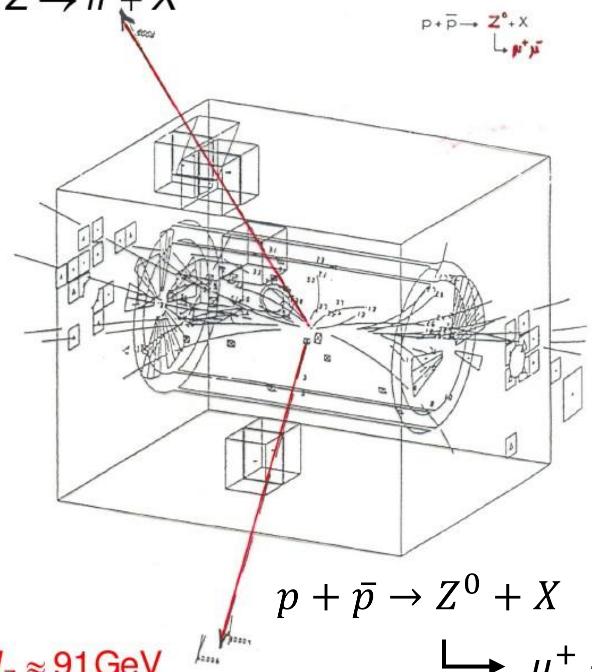
$p\bar{p} \to Z \to f\bar{f} + X$



High-energy lepton pair:

$m_{\ell\ell}^2 = (p_{\ell^+} + p_{\ell^-})^2 = M_Z^2$





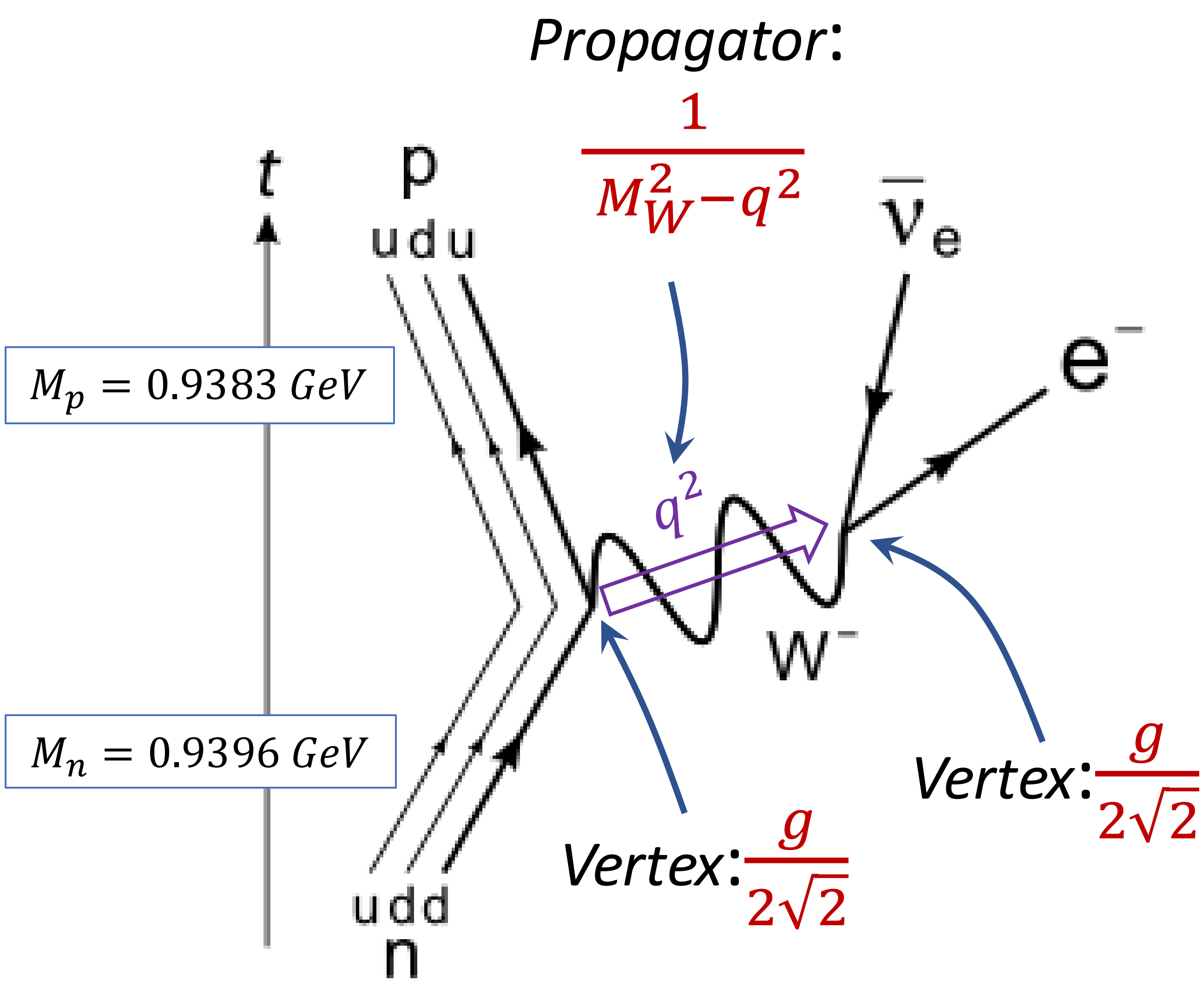
$M_Z \approx 91\,\text{GeV}$

$p + \bar{p} \to Z^0 + X$

$\hookrightarrow \mu^+ + \mu^-$

Virtuality

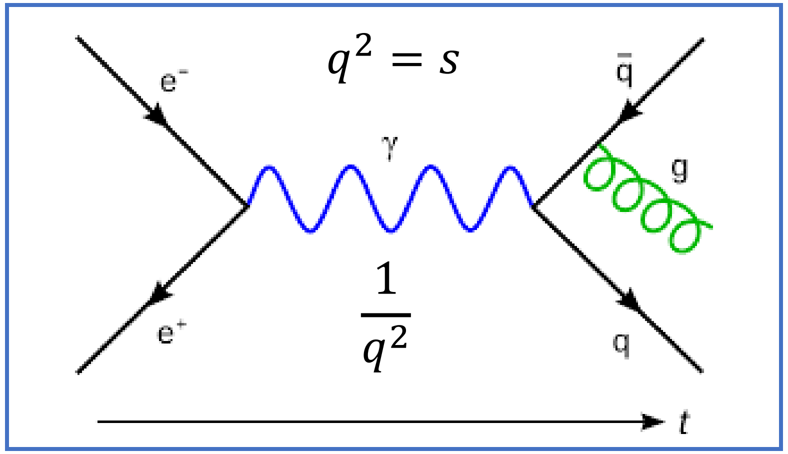
- How can beta decay $n \rightarrow p + e + \bar{\nu}$ work, if the W has a mass of 80 GeV??

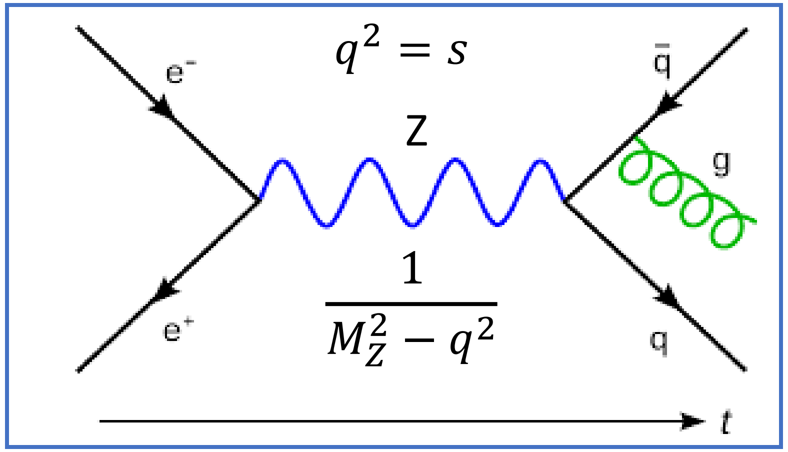


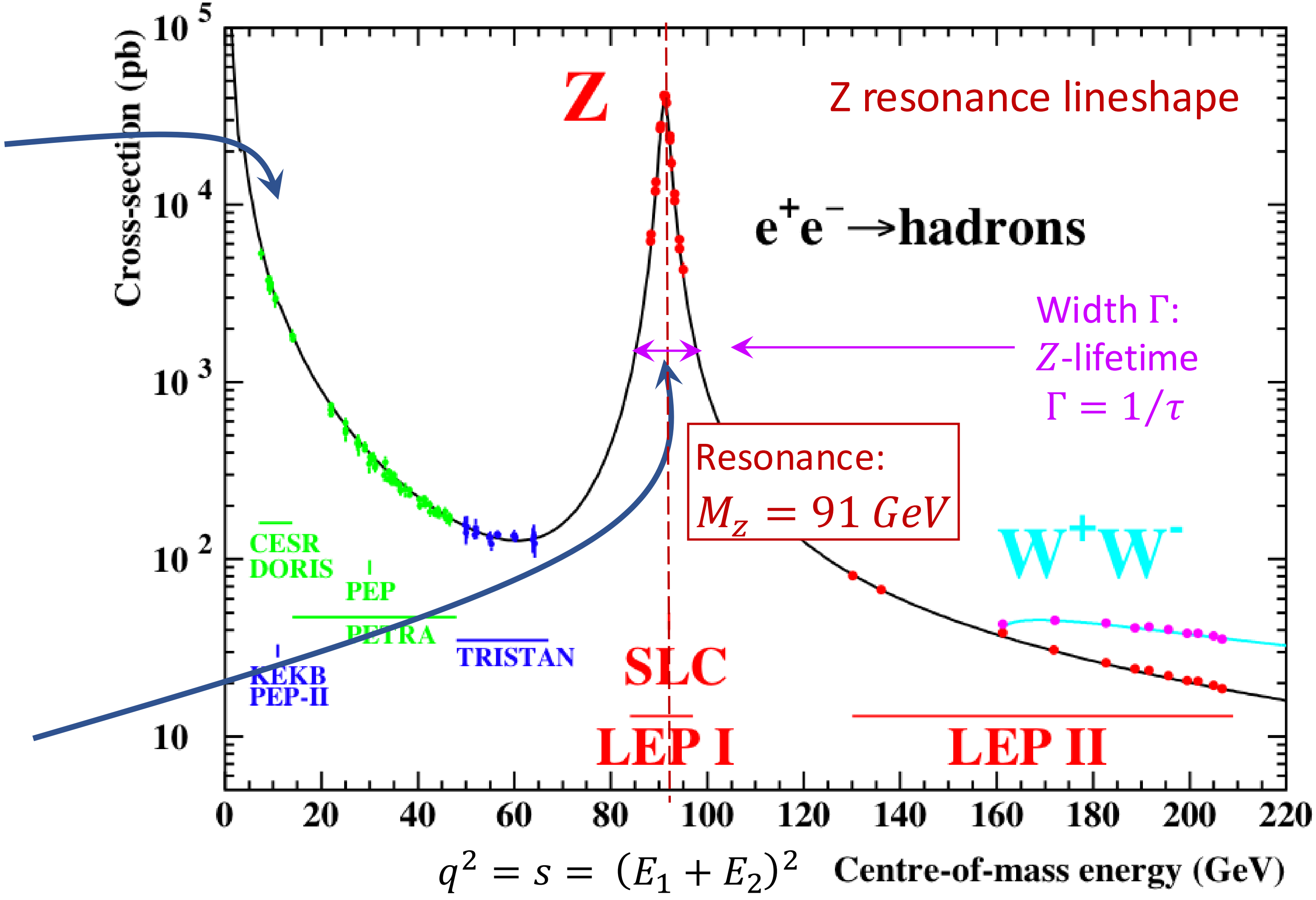
- Exchange of “virtual” particles, also called “off-shell”: $q^2 = E^2 - \vec{p}^2 \neq m^2$
- Heisenberg:
  E is undetermined as $\Delta E \Delta t \geq \hbar/2$ with
  $\Delta E = \sqrt{\vec{q}^2 + m^2}$
  For small q^2 $(\Delta E = mc^2)$ → range force R :
  $R \sim c\Delta t \approx \frac{\hbar}{2mc} \sim 1.2 \times 10^{-18}\ m$
- Notice the resonance behaviour for $q^2 \approx M_W^2$

Precision studies with Z bosons: "Electroweak Force"

- The LEP collider did many precision studies of the weak interaction with e^+e^- annihilation collisions at collision energy around 91 GeV/c^2.



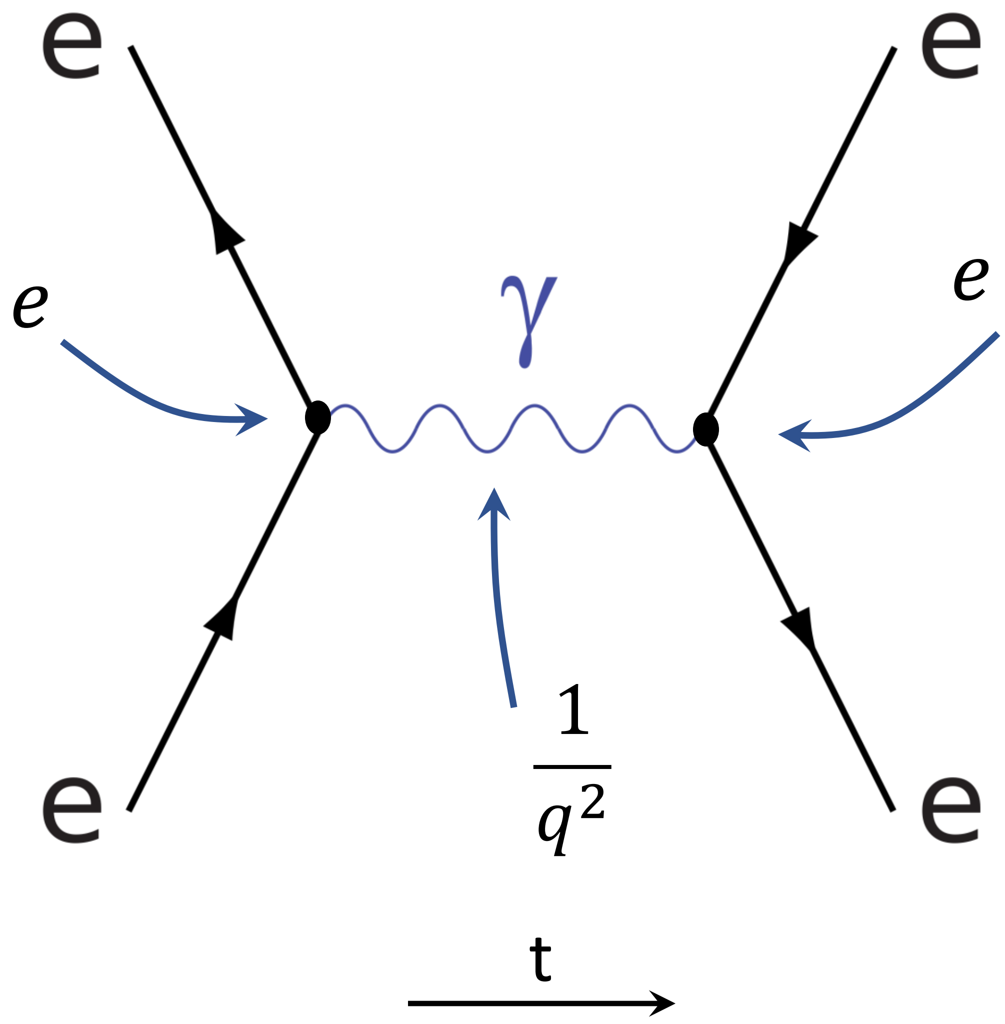




Similarity of Electromagnetic (γ) and Weak (Z) force

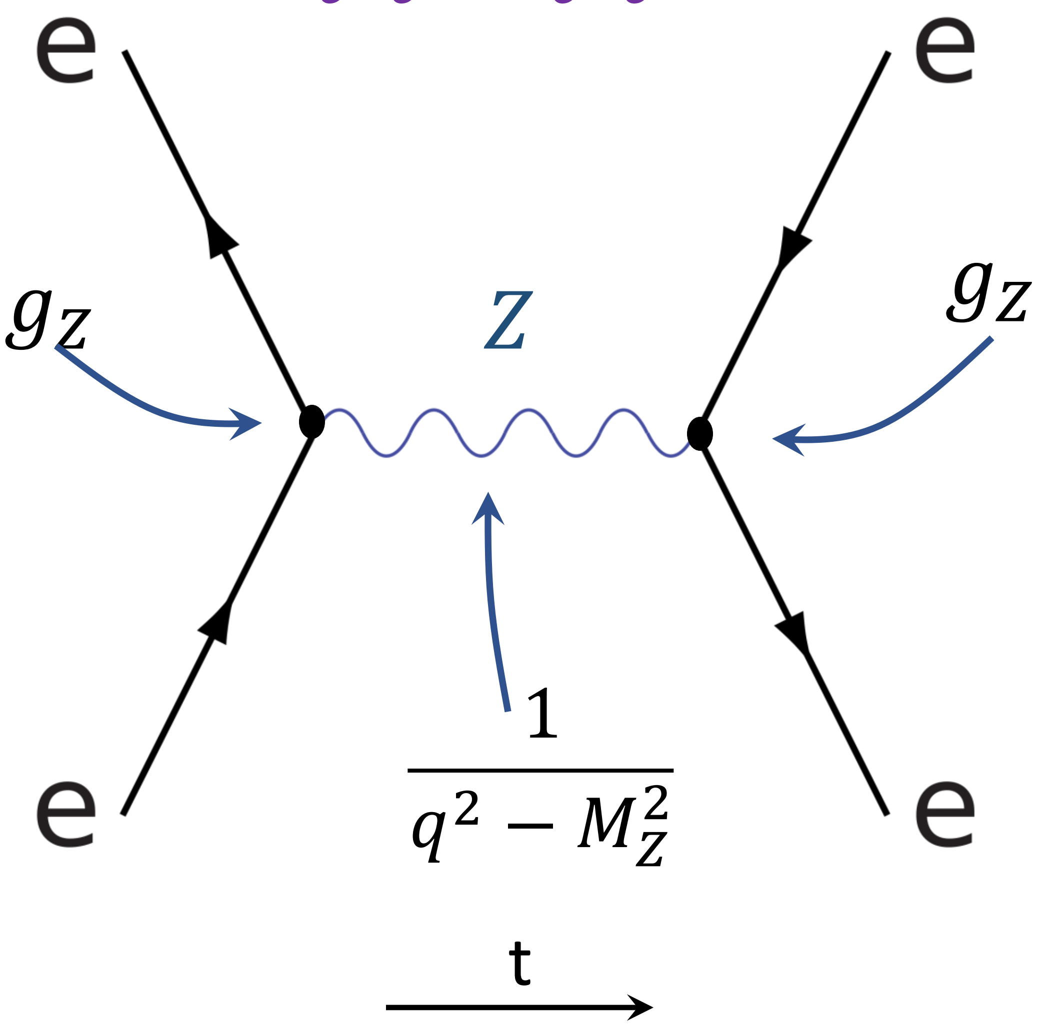
- Electromagnetic:

$e^+e^- \to e^+e^-$



- Weak neutral current:

$e^+e^- \to e^+e^-$



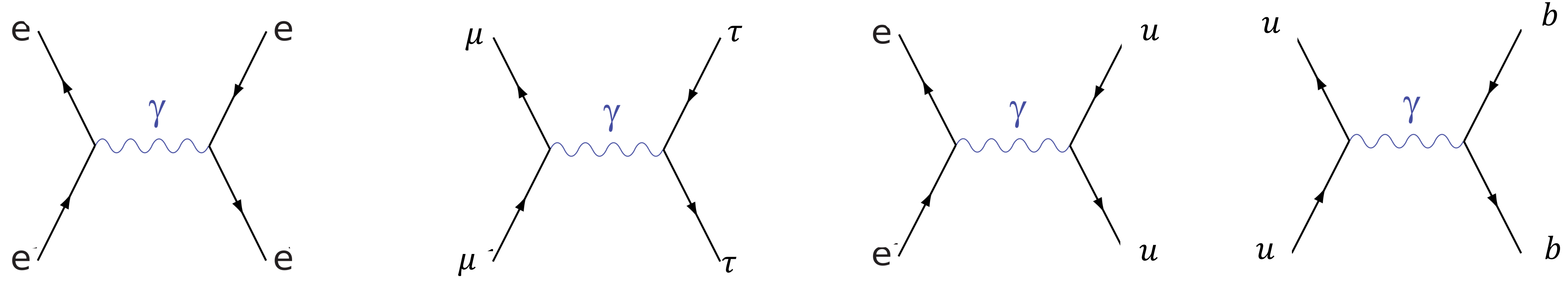
What is the weak "charge"? → Weak isospin

- Compare weak isospin quantum number T or I to spin quantum number S
  - A doublet includes $I_3 = -1/2$ and $I_3 = +1/2$, the charges for the Z similar to Q for E.M.
  - W^+ and W^- are Isospin raising and Isospin lowering interactions, Z does not change I_3

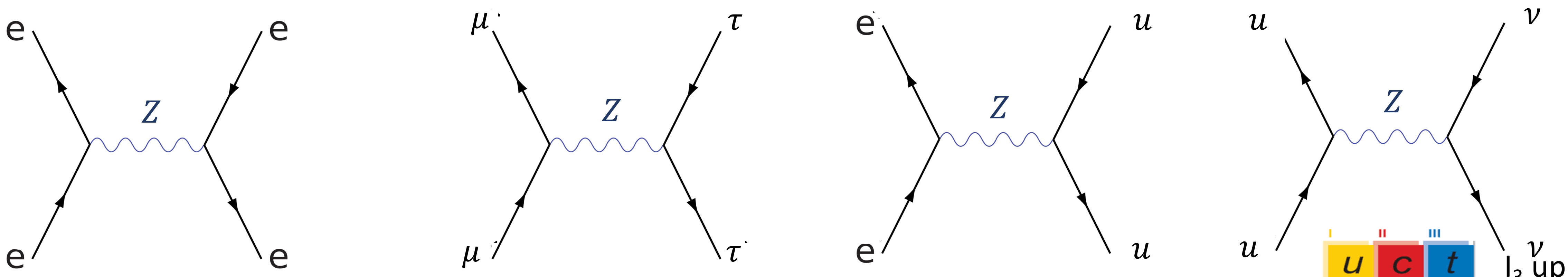
	Generation I	Generation II	Generation III	I_3	Y	Q
Leptons	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$+1/2$ $-1/2$	-1 -1	0 -1
	e_R	μ_R	τ_R	0	-2	-1
Quarks	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$+1/2$ $-1/2$	$+1/3$ $+1/3$	$+2/3$ $-1/3$
	u_R	c_R	t_R	0	$+4/3$	$+2/3$
	d_R	s_R	b_R	0	$-2/3$	$-1/3$

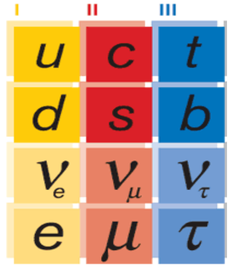
Examples of processes: “neural currents”

- Electromagnetic: couples to electric charge: ‘+ and –’



- Weak “Neutral Currents”: couples to weak charge: isospin ‘up’ and ‘down’



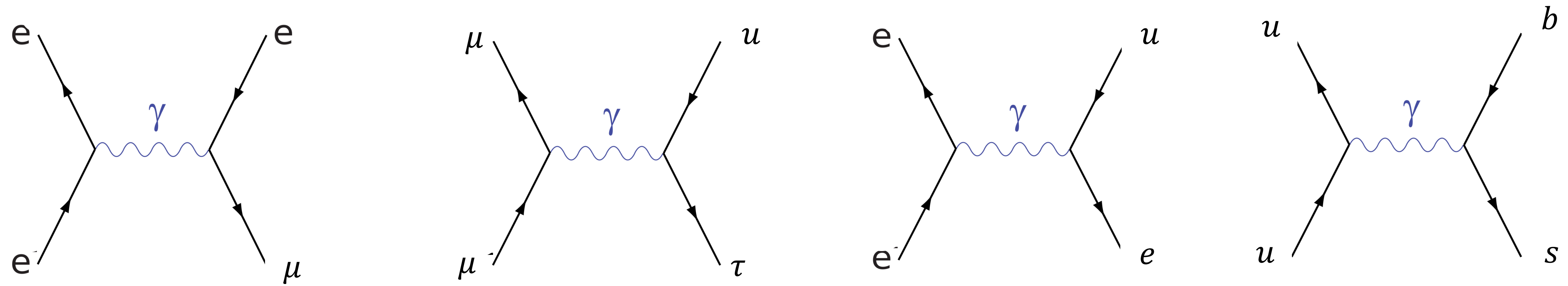


I_3 up
I_3 down
I_3 up
I_3 down

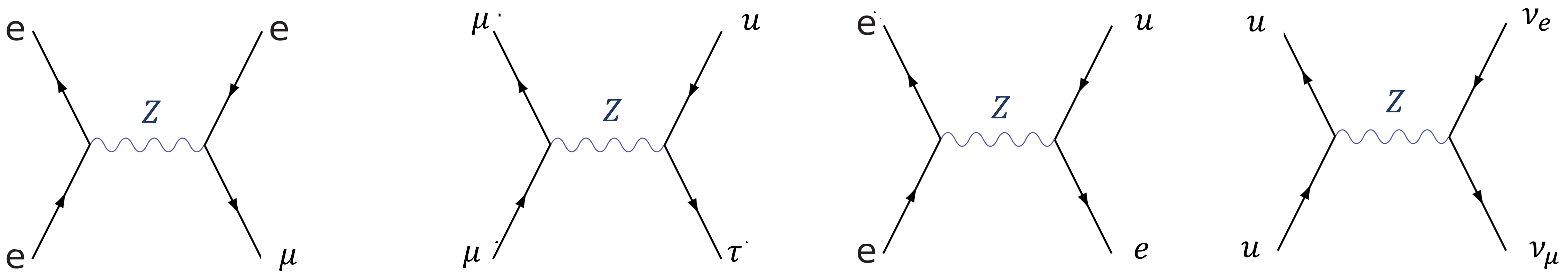
Z boson also couples to neutrinos: neutrinos have weak charge

Examples of processes that ***do not exist***

- Electromagnetic:



- Weak "Neutral Currents":

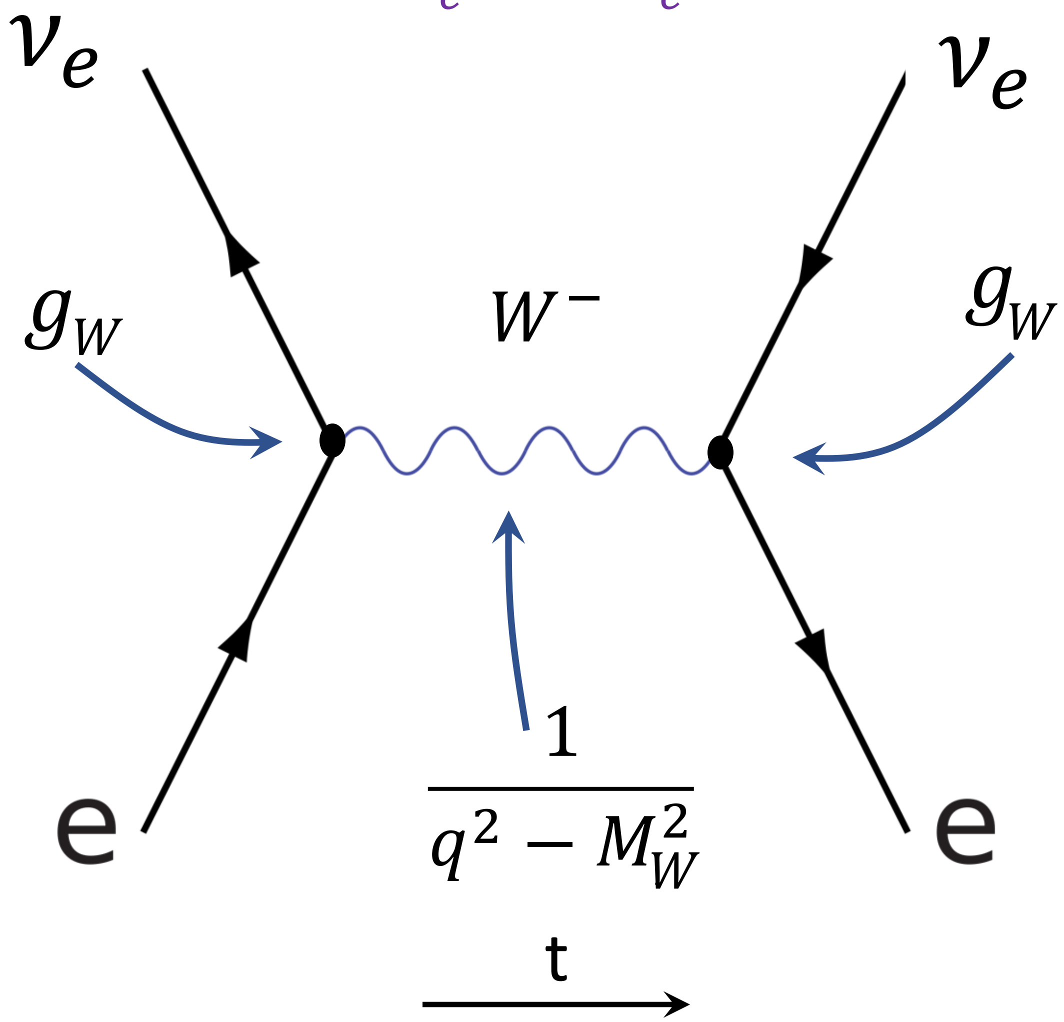


γ and Z only couple to particles *within one generation* and *only to quark or lepton pairs*

Weak charged current ($W^{\pm}$) and weak neutral current (Z)

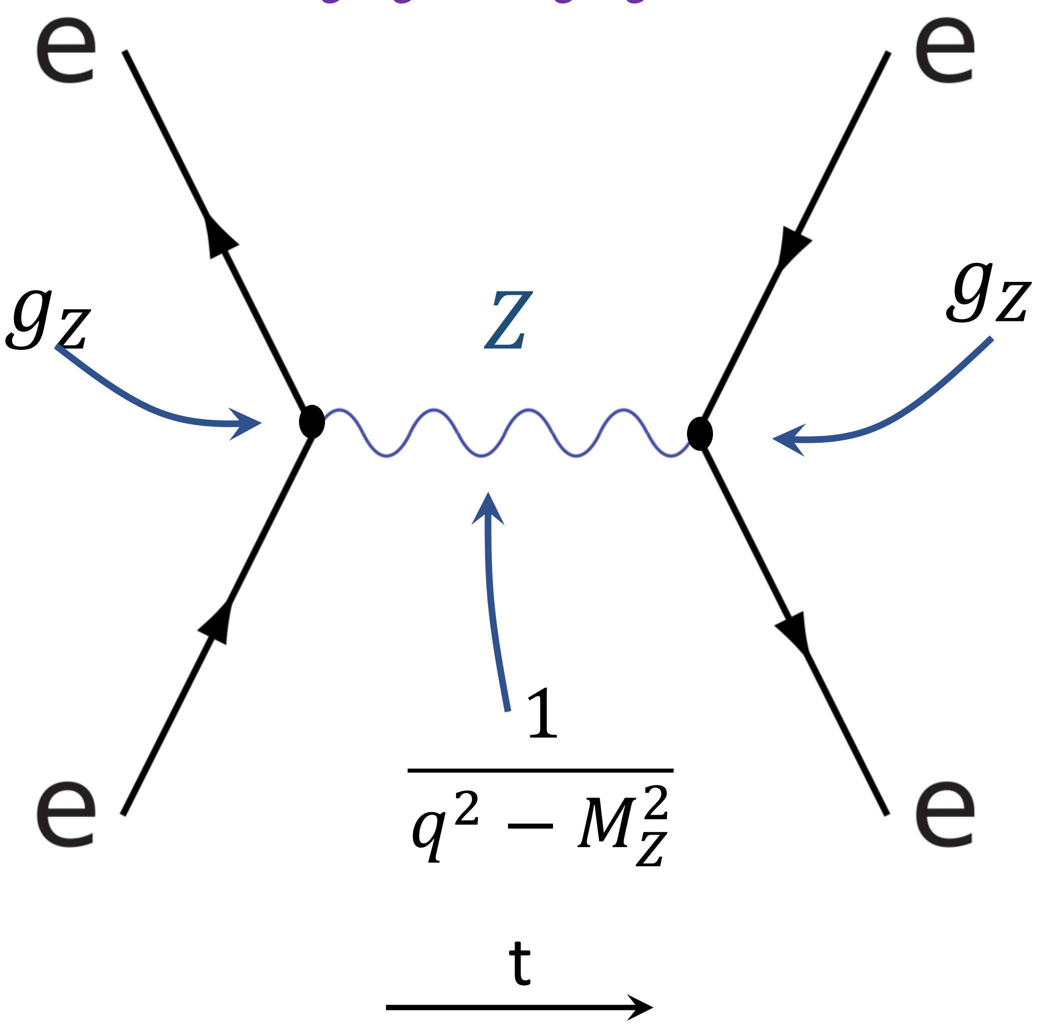
- Weak charged current:

$e^- \nu_e \to e^- \nu_e$



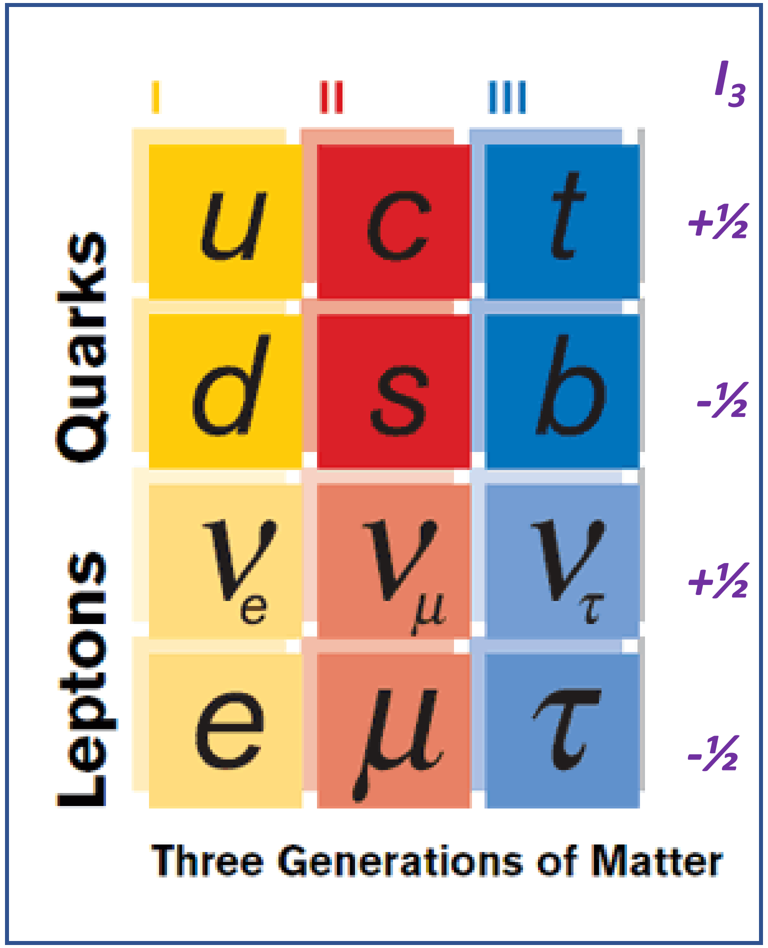
- Weak neutral current:

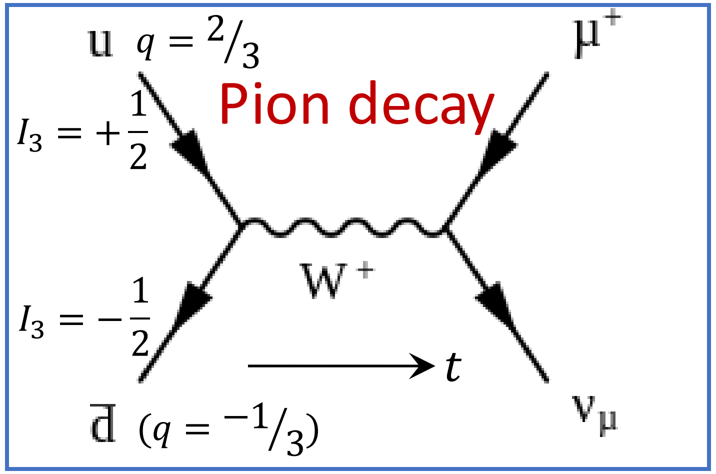
$e^+ e^- \to e^+ e^-$

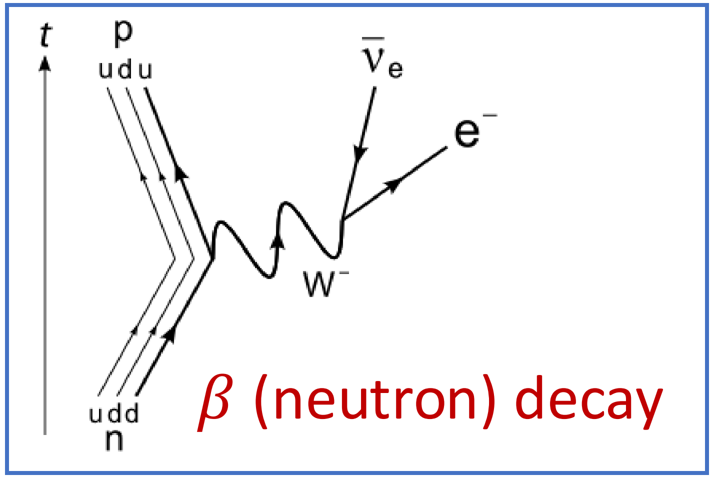


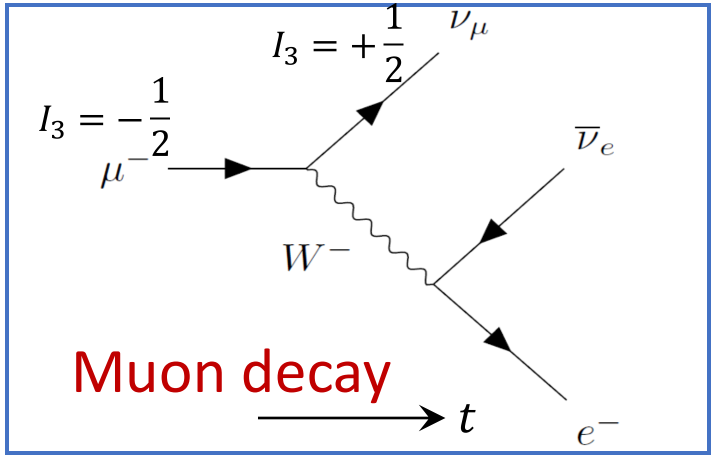
The W weak force: "charged currents"

- The W-boson carries electric charge
- The W connects the weak ***isospin*** (I_3) 'up' and 'down' type particles of a generation
  - W^+ and W^- do the opposite
- 2nd and 3rd generation muon and tau can decay because neutrino's are light!







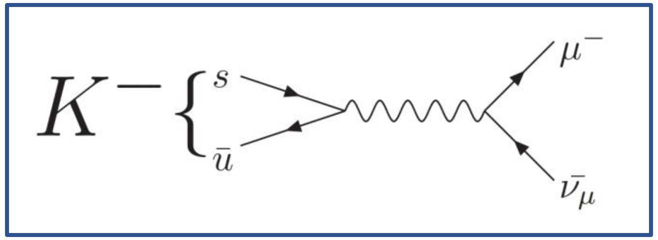


- How does a kaon decay?
  - $K^- = s\overline{d}$
- The c-quark is heavier than the s quark:
  - $s \rightarrow c\ e\ \overline{\nu_e}$ is not possible

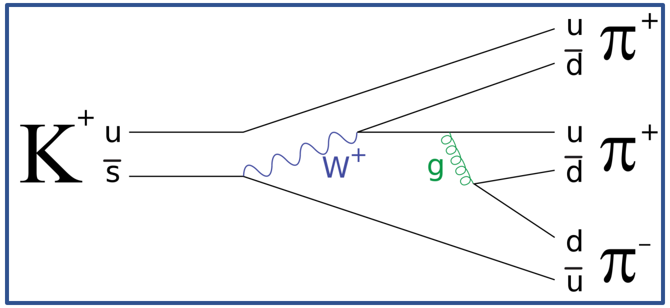
Generations

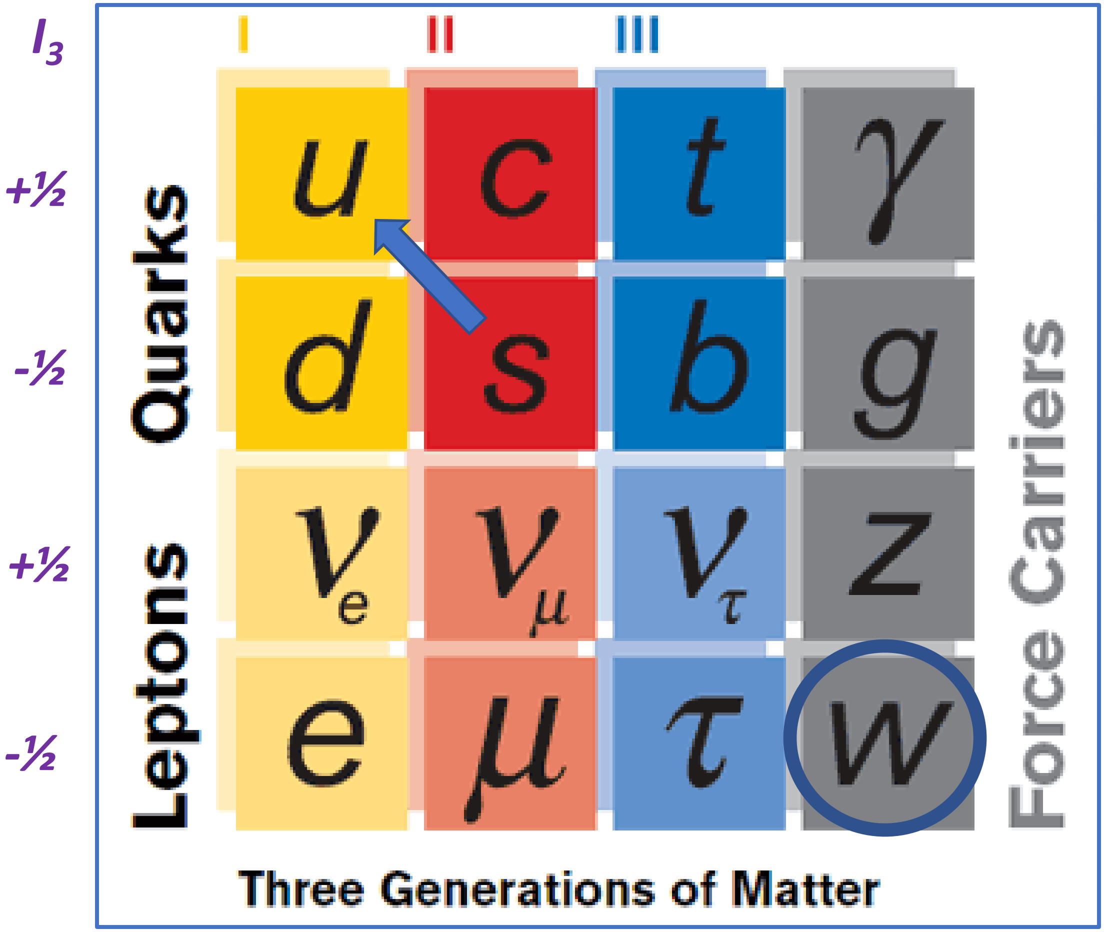
- Does the W ***only*** couple between particles of one generation?
  - Leptons: yes
  - Quarks: no!
    - Couplings between generations are possible for the W!

- Kaon decays
  - "Leptonic": $K^+ \to \mu^+ \nu_\mu$



  - "Hadronic": $K^+ \to \pi^+ \pi^+ \pi^-$





Generations

- Does the W ***only*** couple between particles of one generation?
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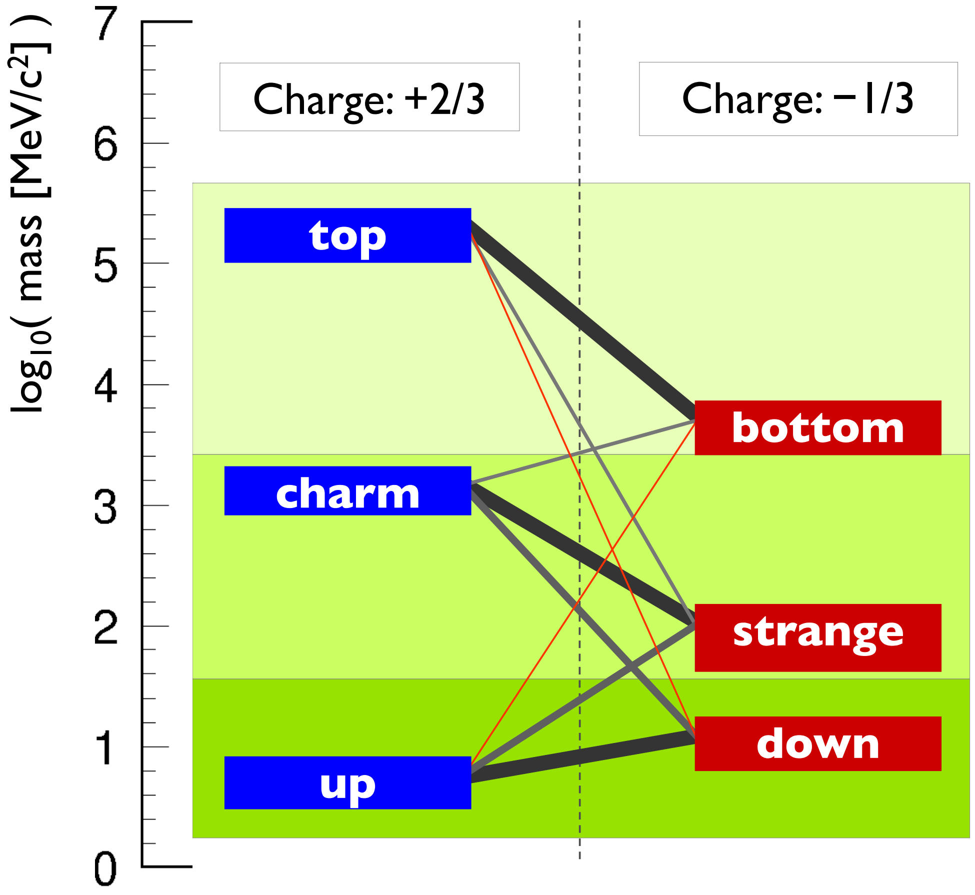
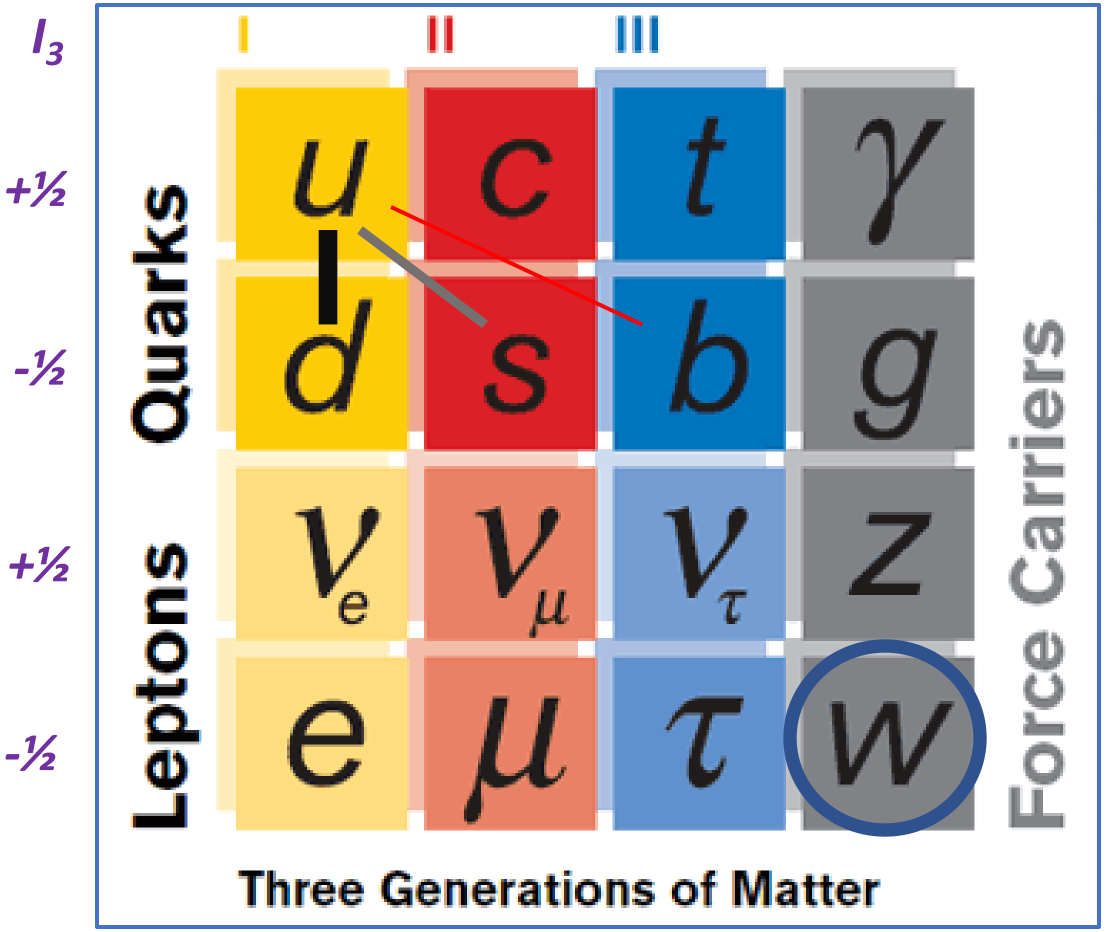
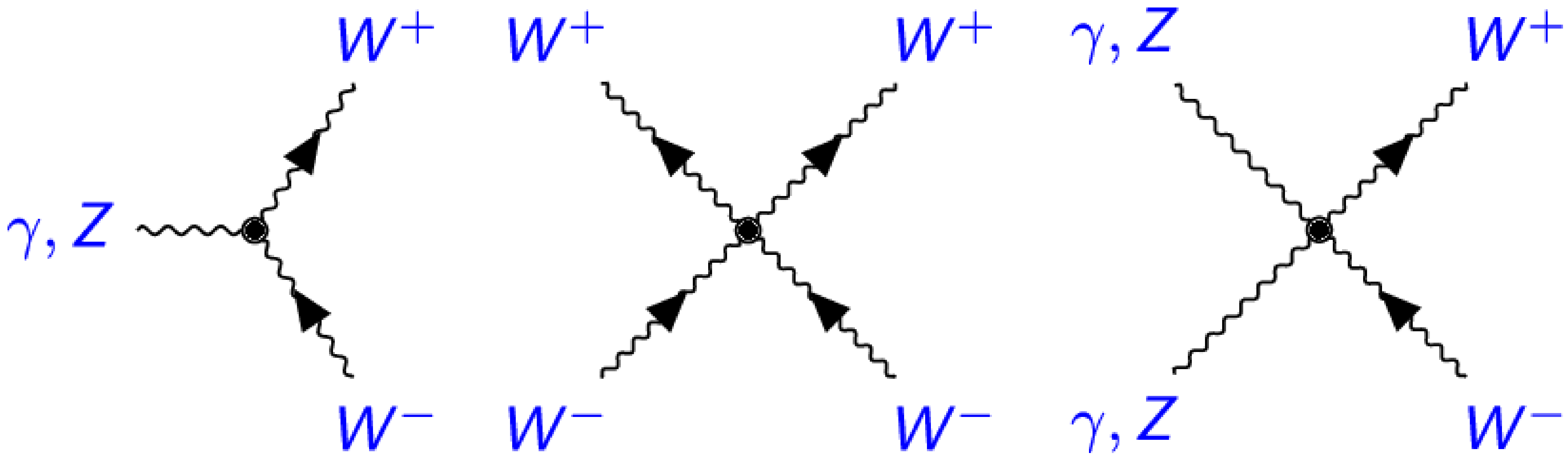




➔ Flavour Puzzle": See the discussion topic on the rich structure of the weak interaction

Self coupling

- In electromagnetism a photon has no electric charge
- In the weak interaction the force carriers have weak charge themselves
  - The weak charge is called "weak isospin": $W^-: I_3 = -1 \quad ; \quad Z^0 : I_3 = 0 \quad ; \quad W^+: I_3 = +1$
  - The following diagrams are possible

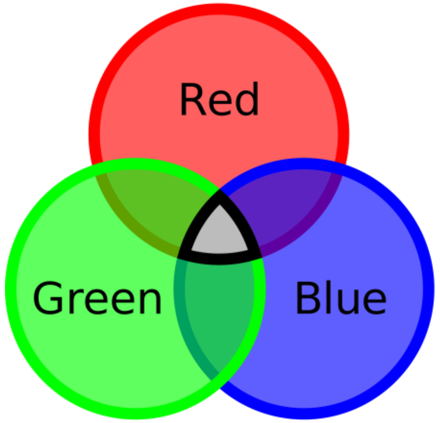


Griffiths section 2.3

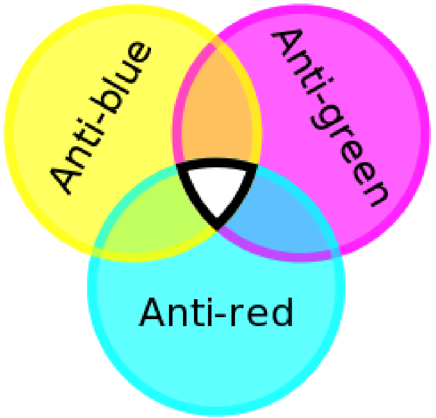
The Strong Interaction
Quantum Chromodynamics (QCD)

Color singlets

- Quarks are "locked-up" in hadrons
  - Technical term: confinement
- Quarks carry color charge: r , g , b
- All physical objects are color neutral or color *singlets: confinement!*
  - Baryons: rgb or $\bar{r}\,\bar{g}\,\bar{b}$
  - Mesons: $r\,\bar{r}$ or $b\bar{b}$ or $g\,\bar{g}$
- The color force, transmitted by gluons, is very strong
  - Requires a lot of energy to separate a color charge from a color neutral object
  - Trying to do so will produce another color neutral object

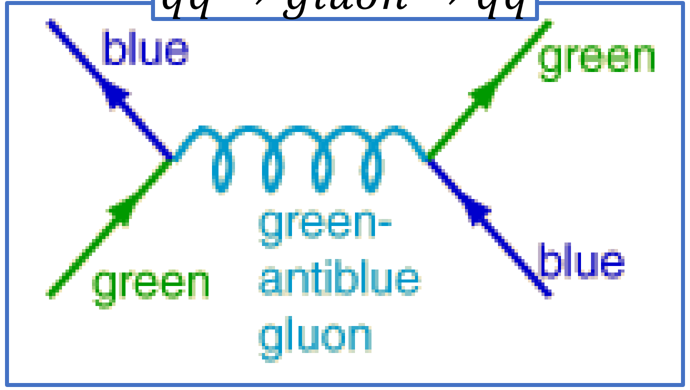


baryon



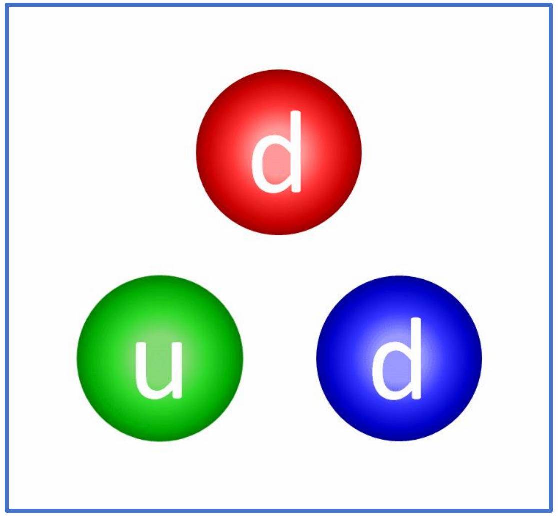
anti-baryon

$q\bar{q} \rightarrow gluon \rightarrow q\bar{q}$



Gluon color: $g\,\bar{b}$
There are 8 gluons
+ 1 colorless singlet
$3 \otimes 3 = 8 \oplus 1$

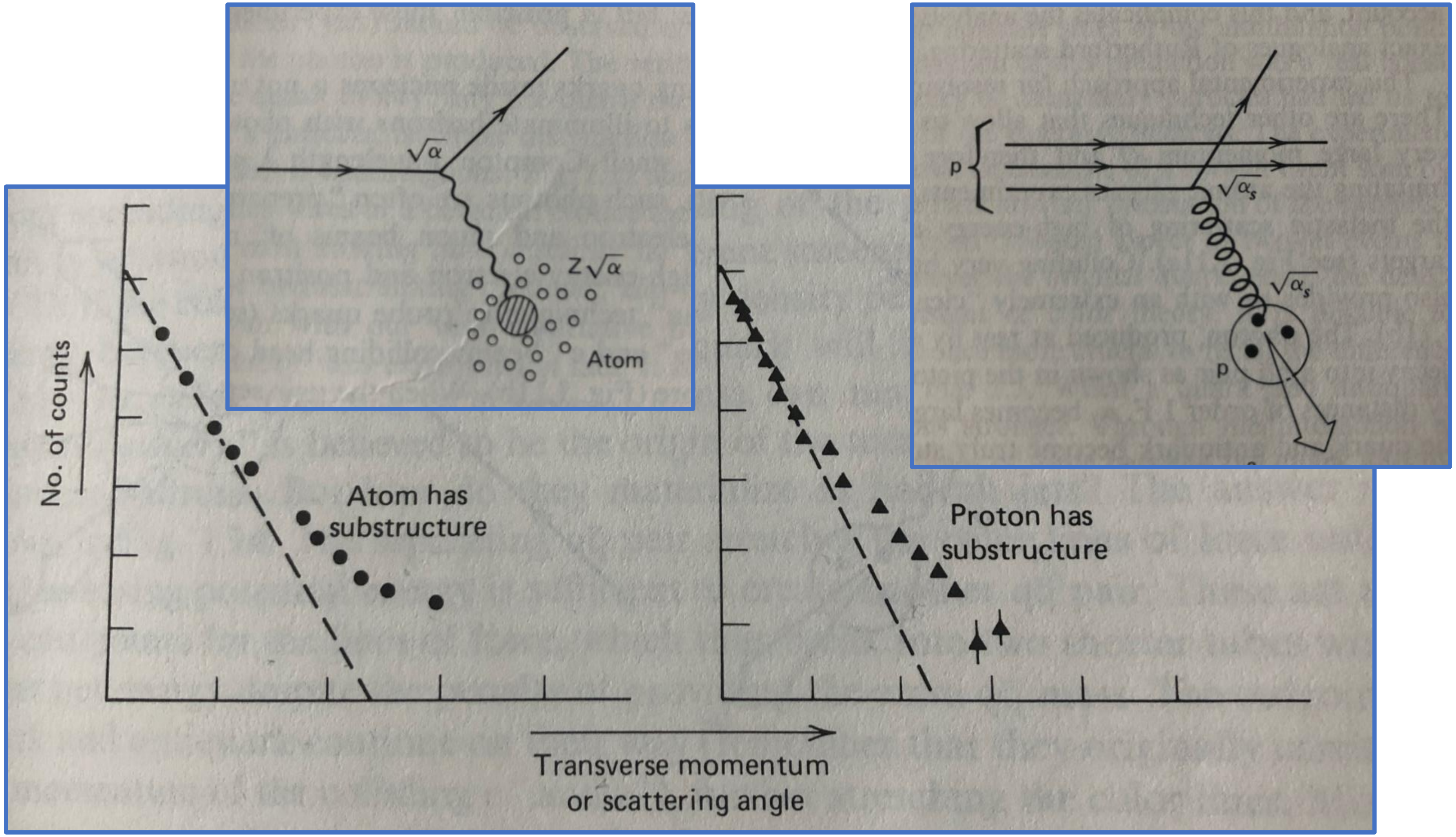
- Animation of a color neutral neutron:



- Are quarks and gluons real or a bookkeeping devise?

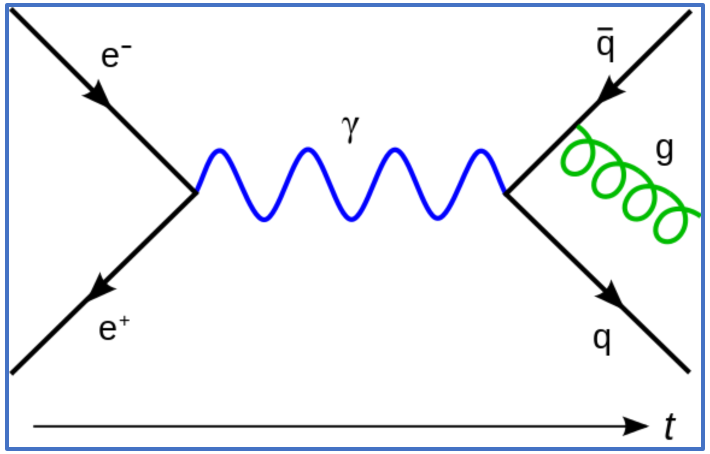
Proton Substructure: discovery of "partons" or "quarks"(1968)

- Similarly to Rutherford scattering ("substructure of the nucleus") deep inelastic scattering of electrons on protons show the proton substructure

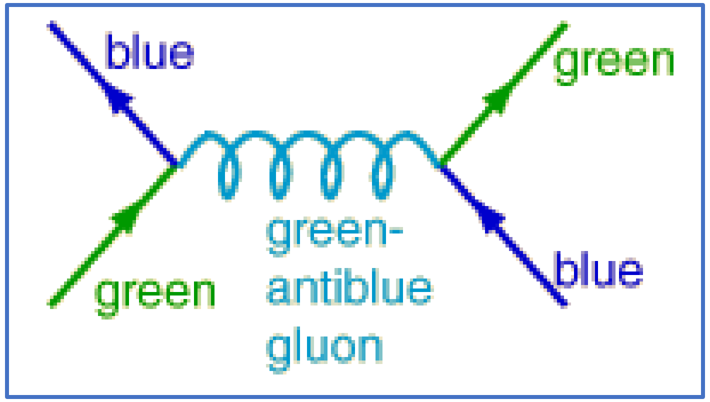


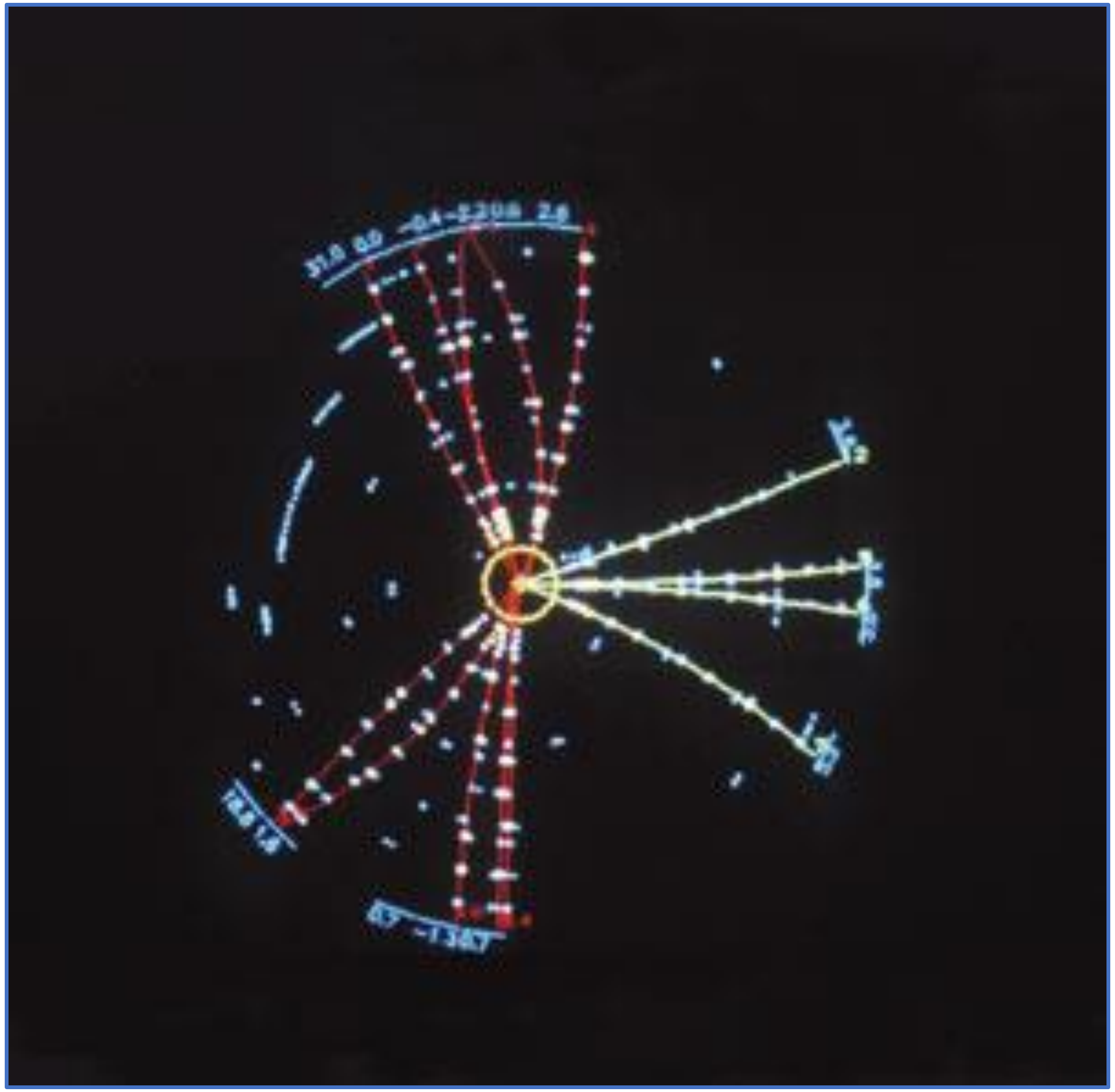
Discovery of the Gluon (1979)

- The gluon was discovered in e^+e^- collisions at DESY in Hamburg
  - "3-jet events"

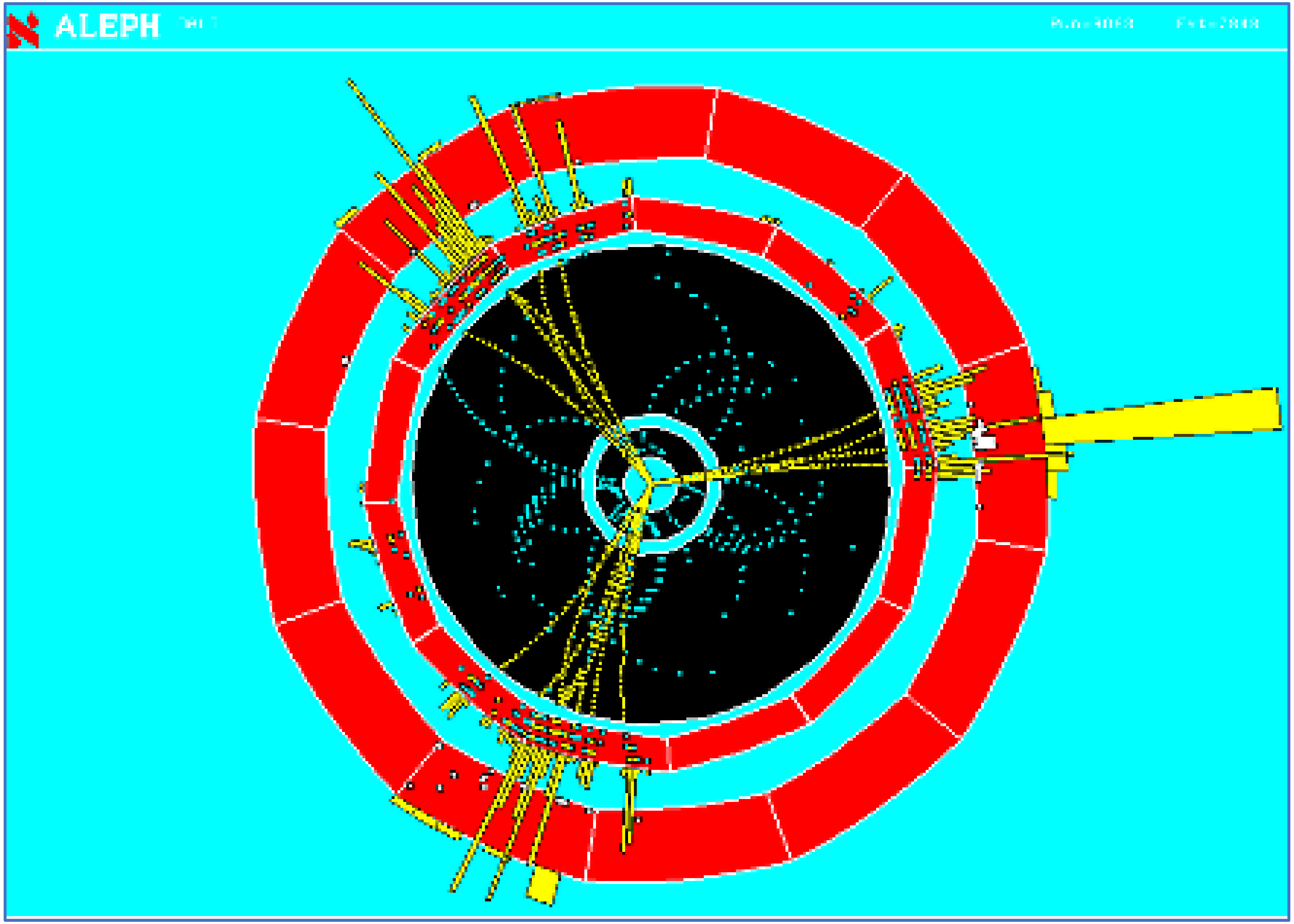


- Feynman diagram for quark-quark interaction
  - At LHC



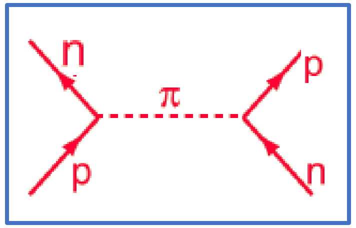


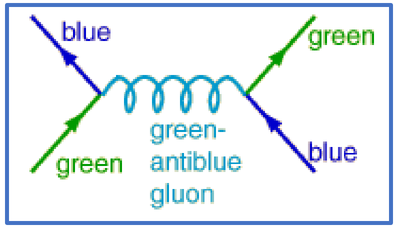
3-Jet event at LEP - Delphi (1989 – 2000)

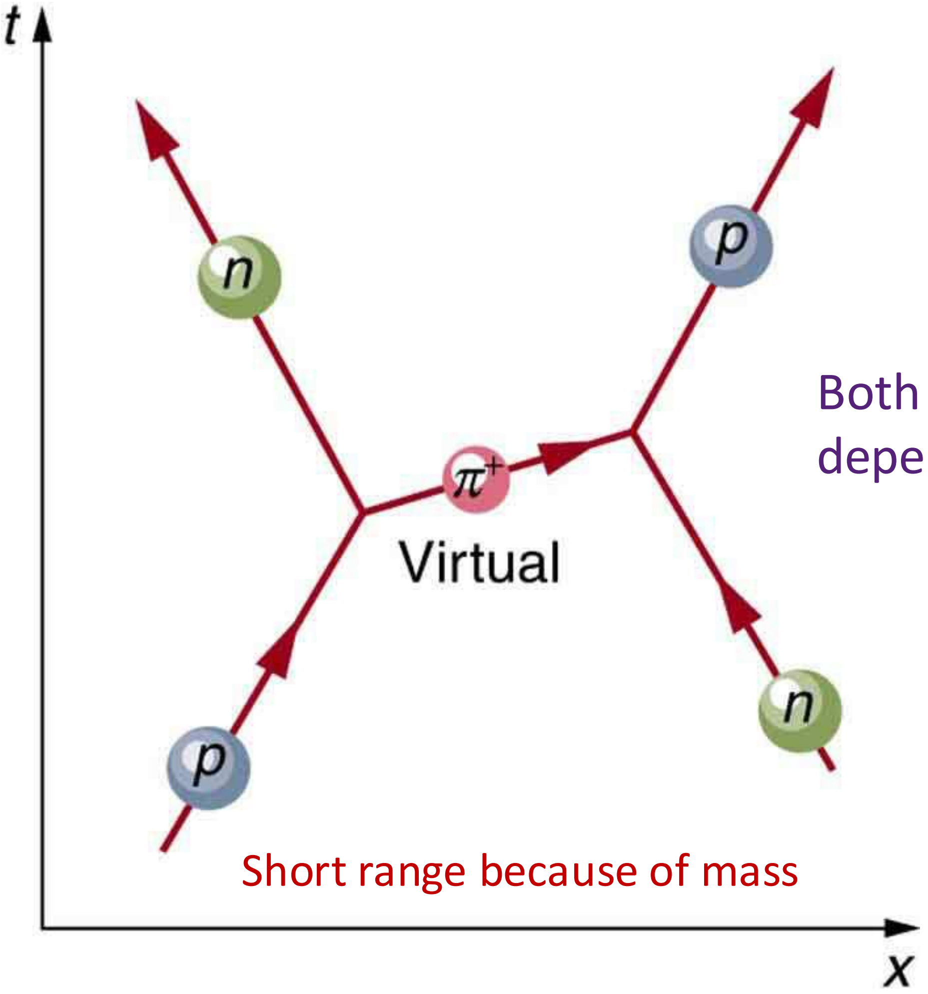


Yukawa's pion exchange vs gluon exchange

- Yukawa: Nuclear force is carried by *massive* pion
- QCD: Strong force is mediated by *massless* gluon

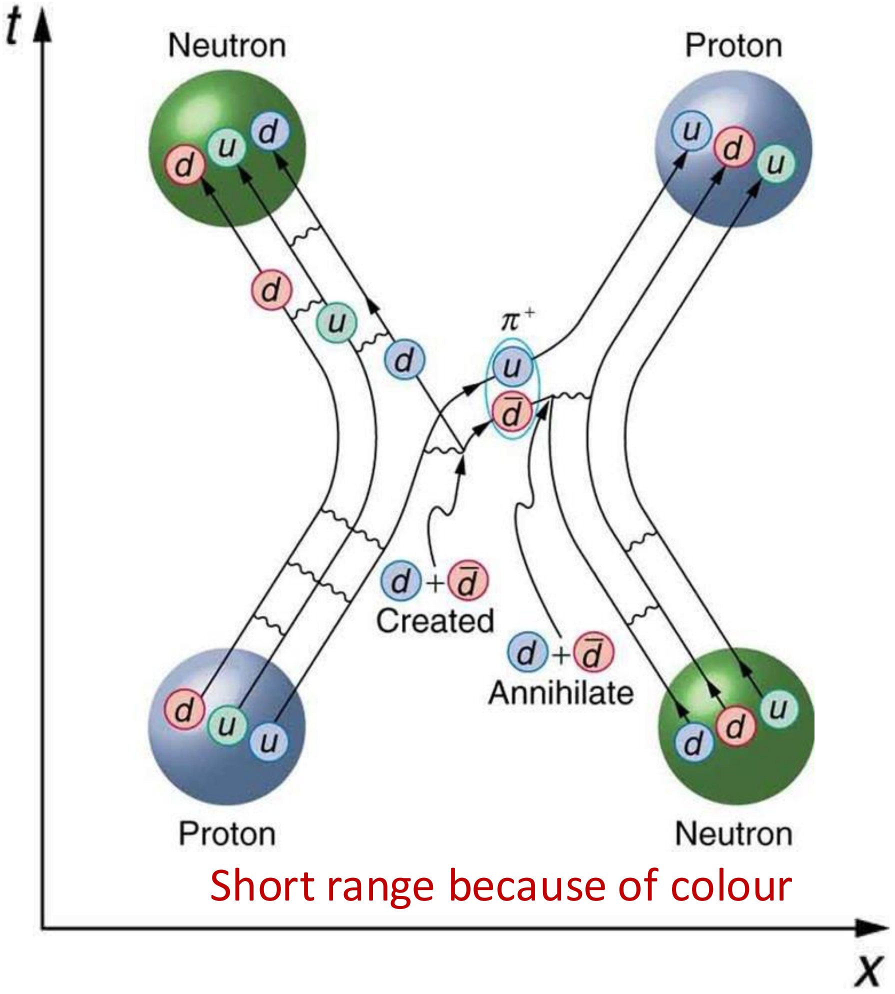






Short range because of mass

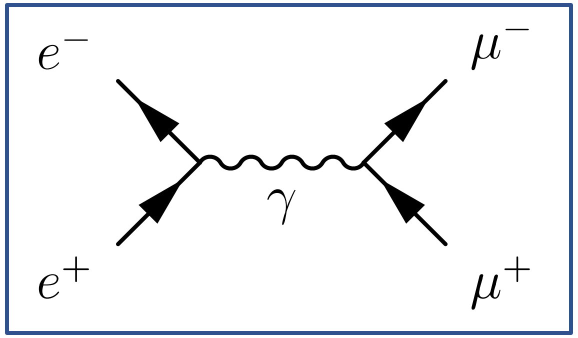
Both are "correct" depending on "zoom"

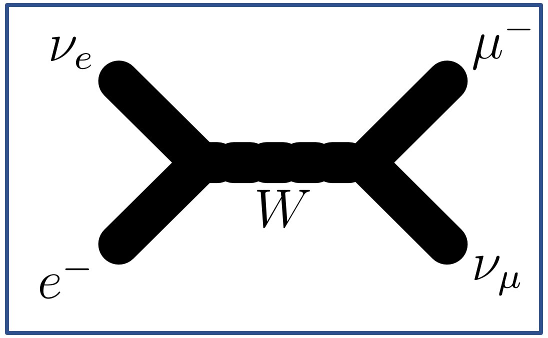


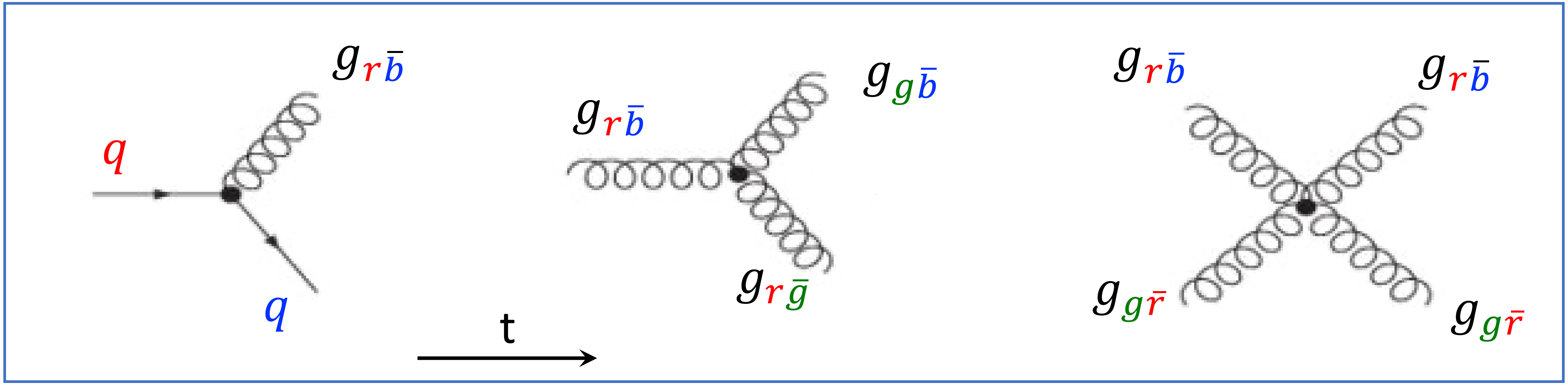
Short range because of colour

Fundamental Vertices of the strong interaction

- QED: photon couples to charge
- Weak: W couples to weak isospin
- Strong gluon couples to color
  - 3 colors and anti-colors
  - Gluon carries charge itself!
    - 8 different gluons (9 – 1 symmetric)





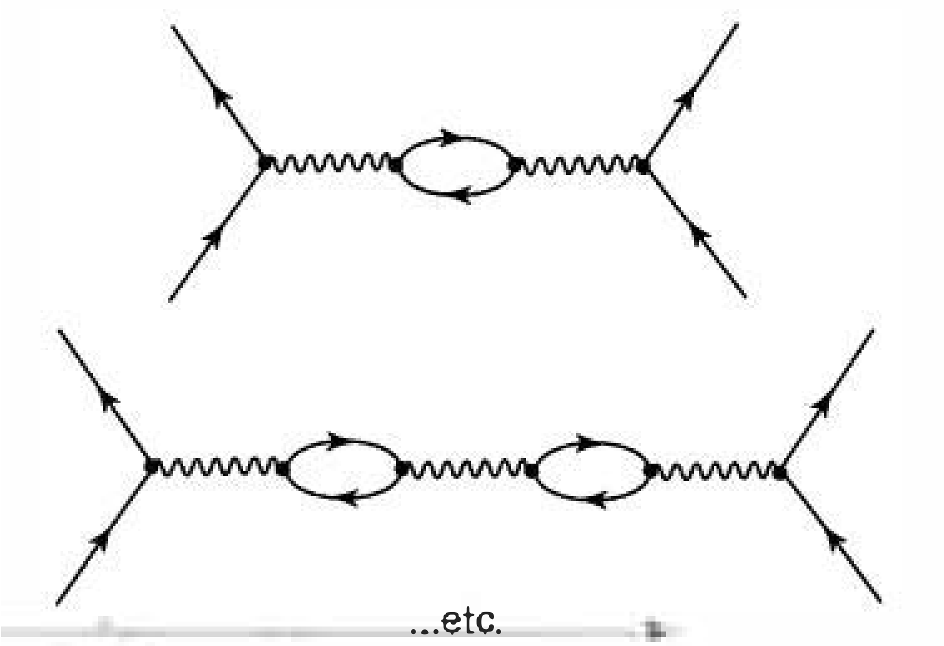


- Leptons carry no color: leptons do not feel strong (nuclear) interaction

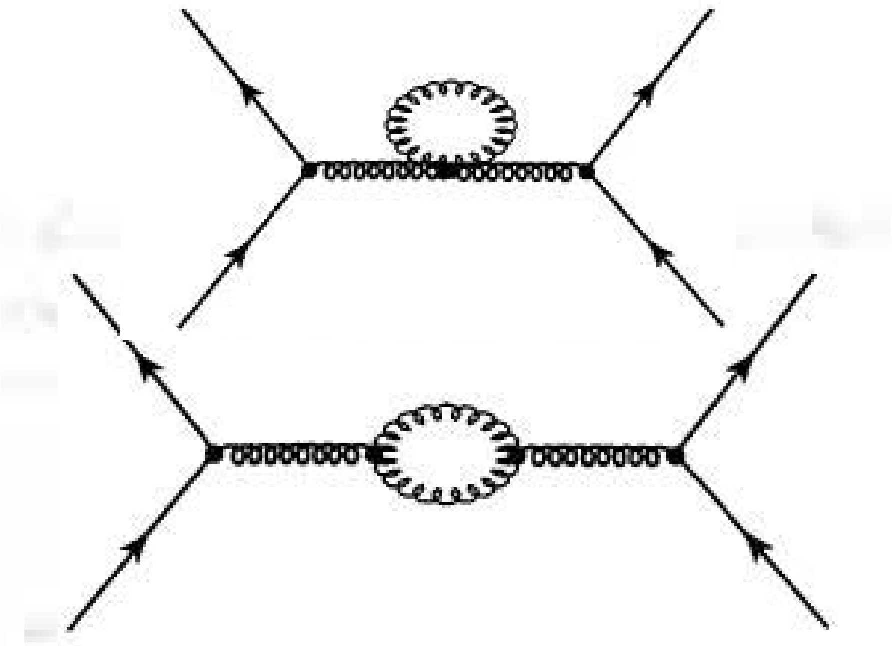
Abelian and non-Abelian

- Photons have no electric charge:
  - Photons *do not* interact with each other ("Abelian" in group theory)
- Gluons carry color charge:
  - Gluons *do* interact with each other ("non-Abelian" in group theory)

QED, QCD



QCD



- Vacuum behaves like dielectric: by QED and even more so by QCD…

Screening

- Charge screening effect:
  - Around a charge $q(+)$ dipoles align
  - Halo of negative charge
  - Effective charge (at distance) reduced

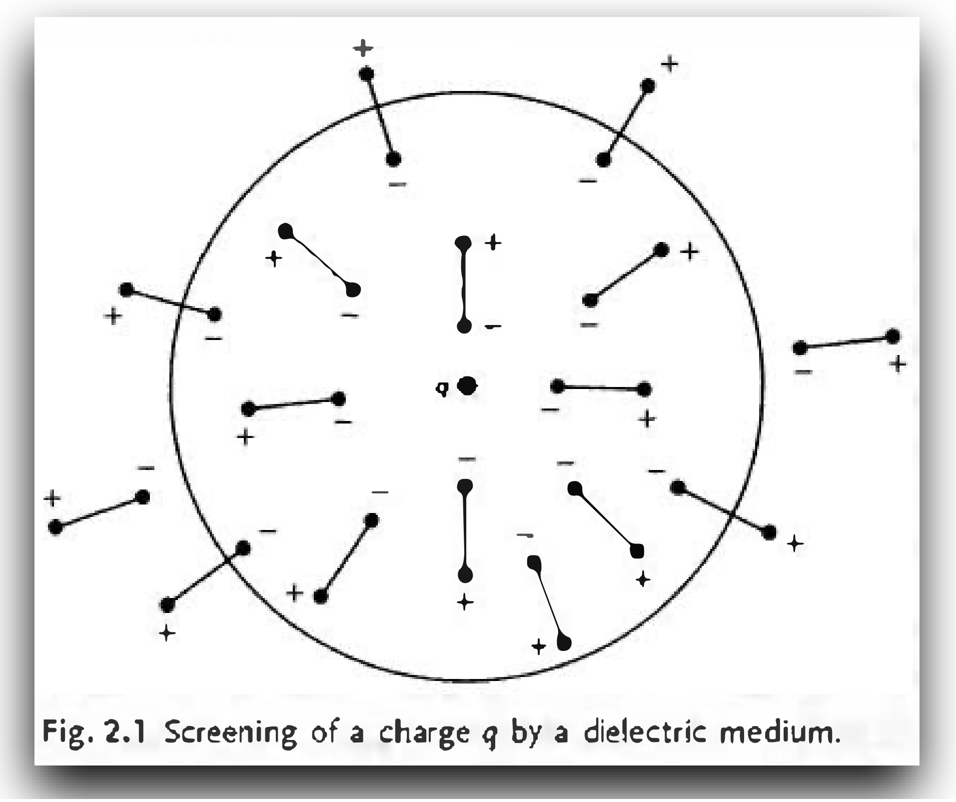
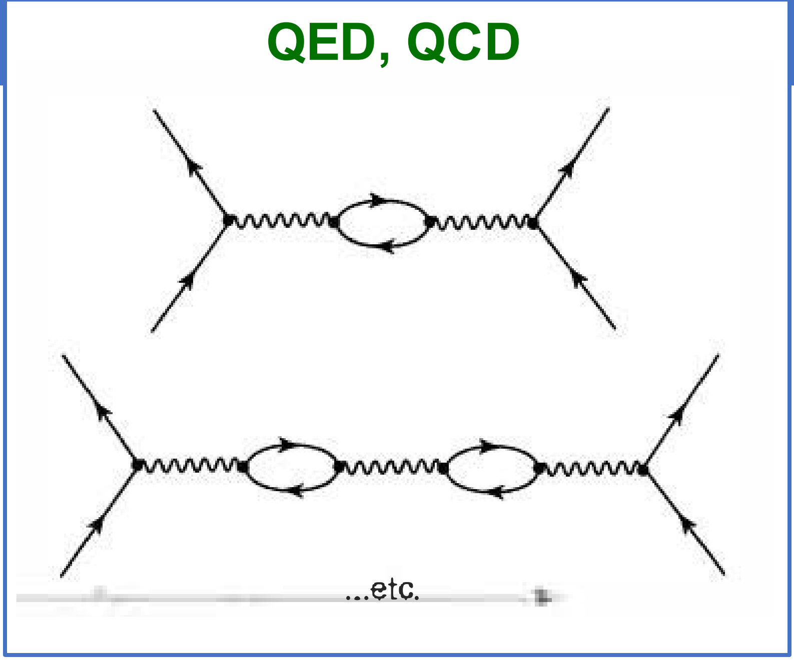
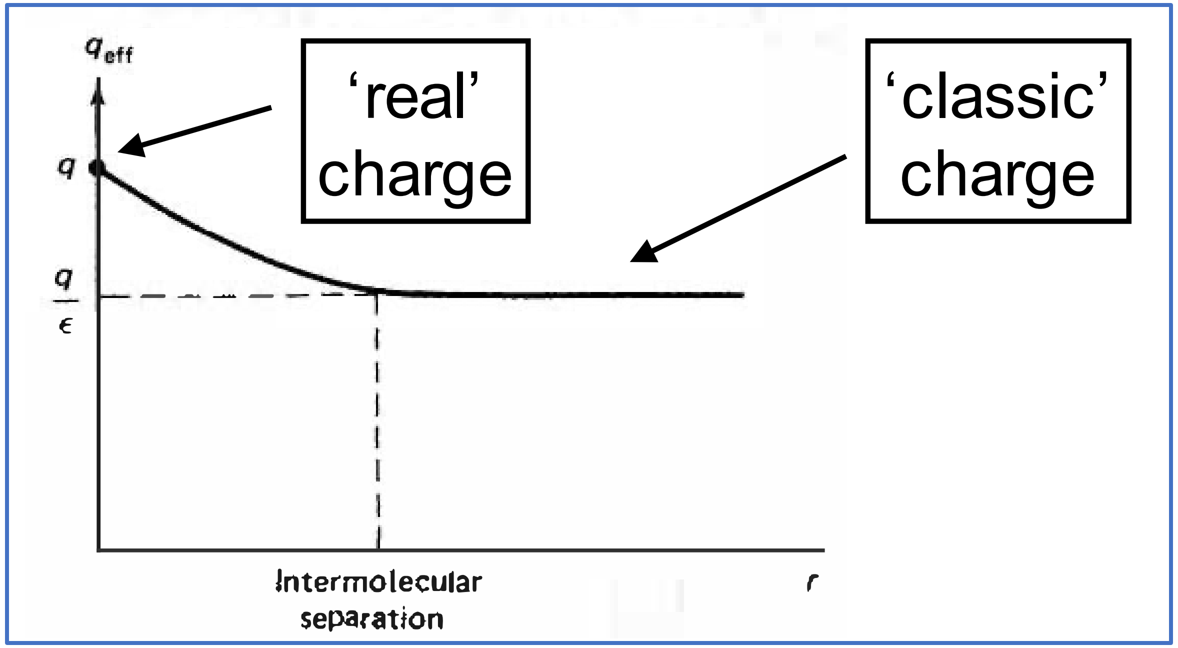


Fig. 2.1 Screening of a charge q by a dielectric medium.





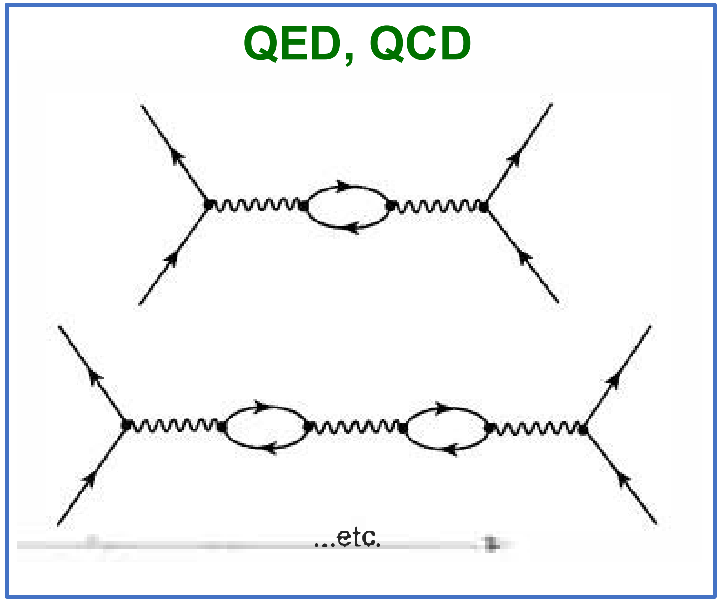
Anti-screening and asymptotic freedom

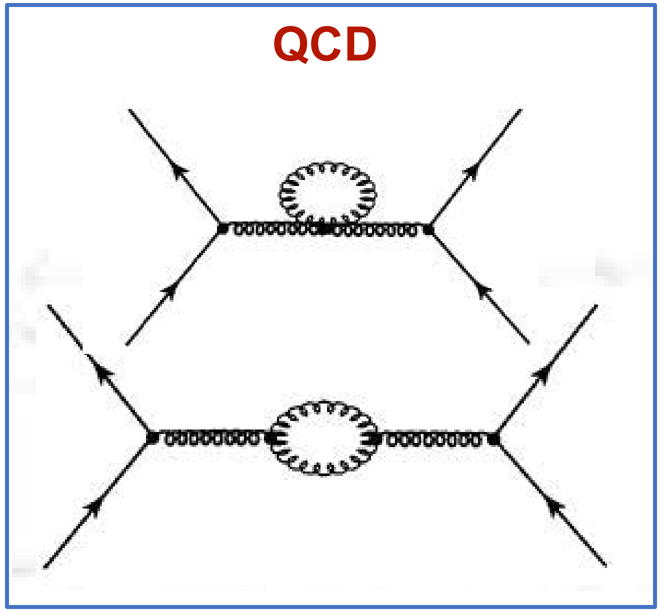
- Anti-screening:
  - gluon self coupling loops have opposite effect
  - Crucial parameter:

    $a \equiv 2f - 11c$

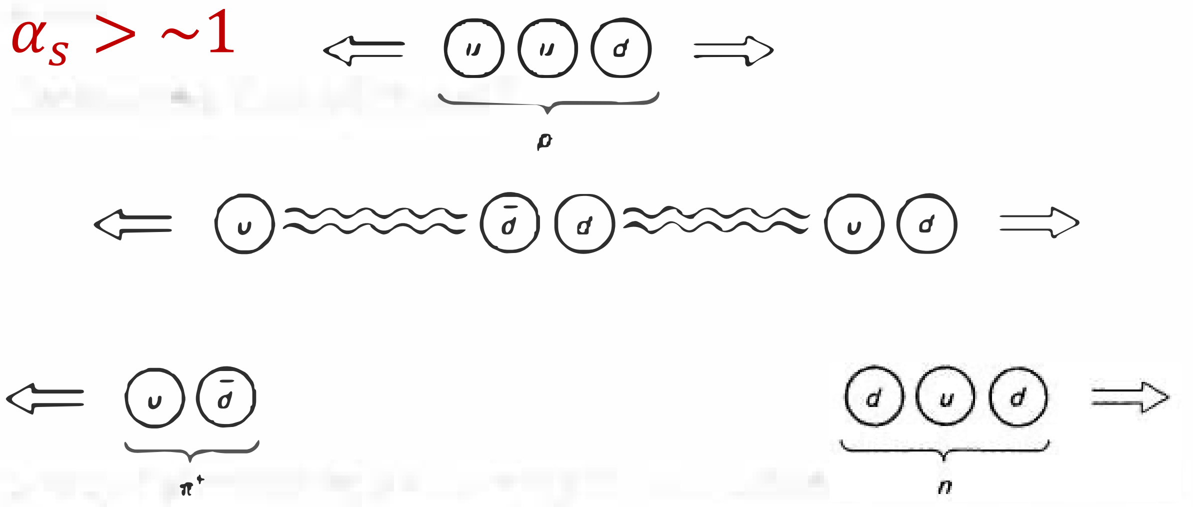
    where: f = number of flavours

    c = number of colors
  - In case $a > 1$ (QED) coupling *increases* for short distance:
  - In case $a < 1$ (QCD) coupling *decreases* for short distance





- *Confinement*: at large distance:
  - $\alpha_s > \sim 1$

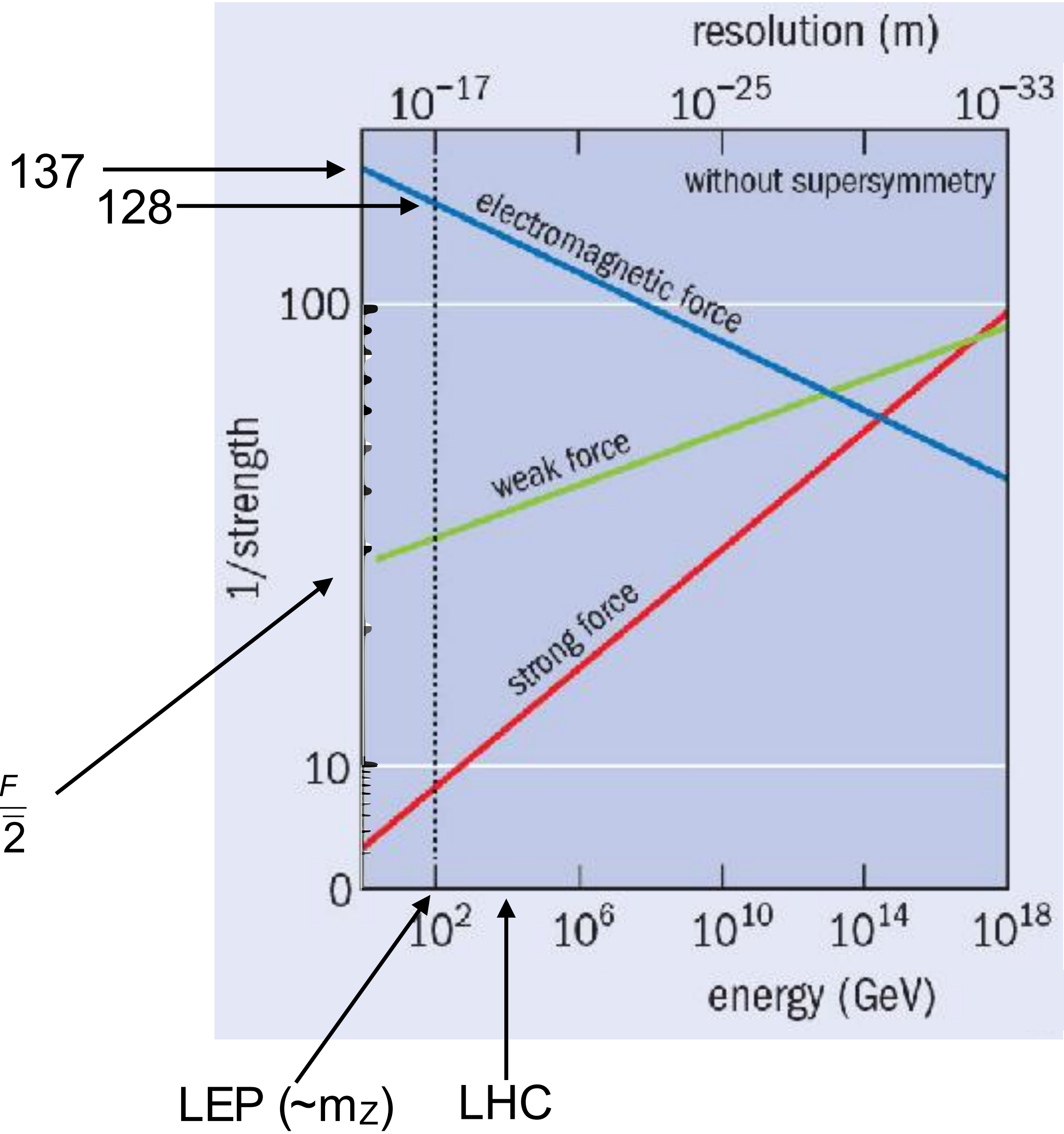


- *Asymptotic freedom:* at short distance:
  - Coupling $\alpha_s \sim 0$ at very small distances
  - Quarks and gluons become "free"

➔Quark gluon plasma

Running coupling constants and unification?

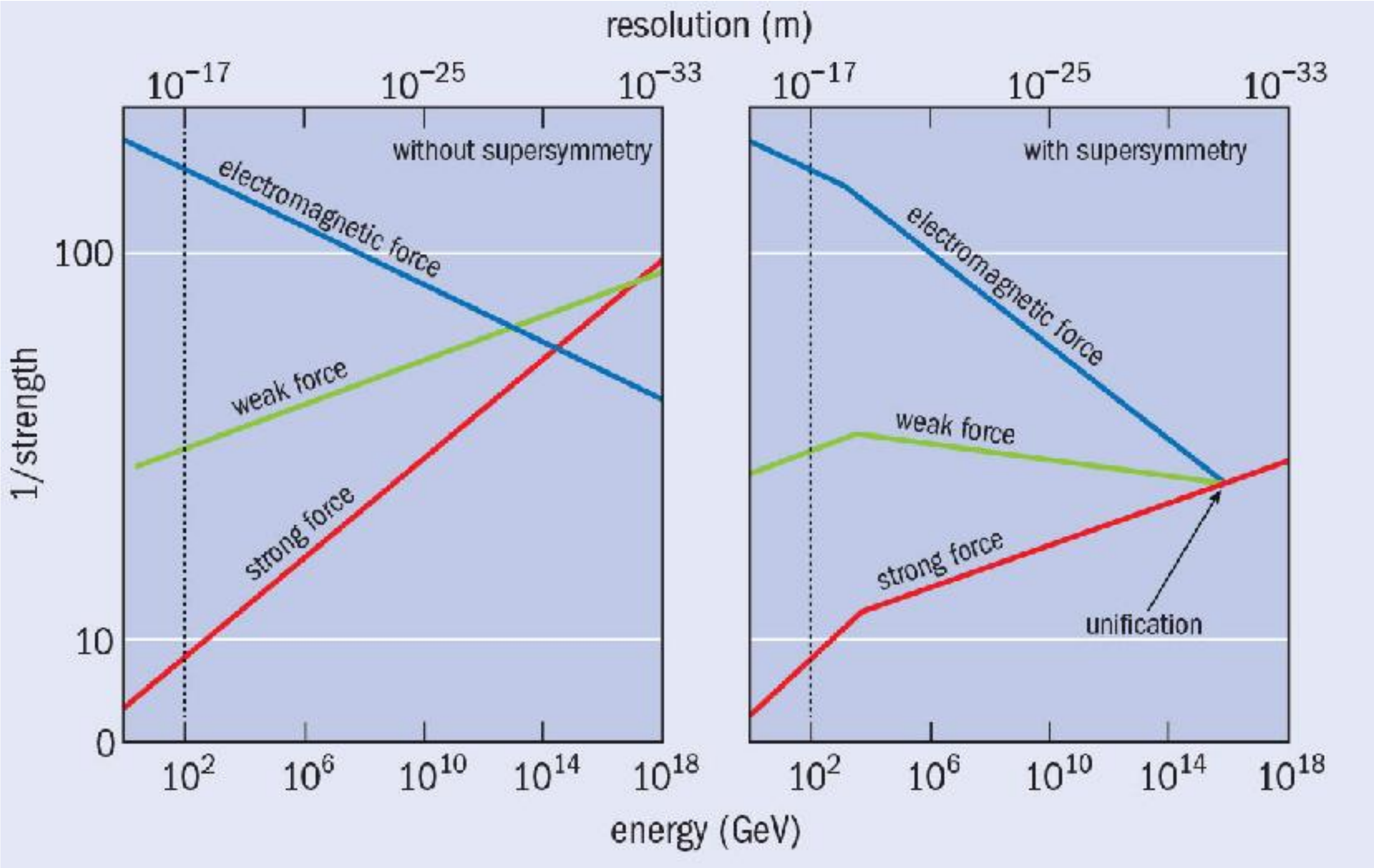
- Running couplings: ‘beta’-function
- Higher energy means shorter distance
- Lines do not cross in one point



$$\frac{g_W^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$$

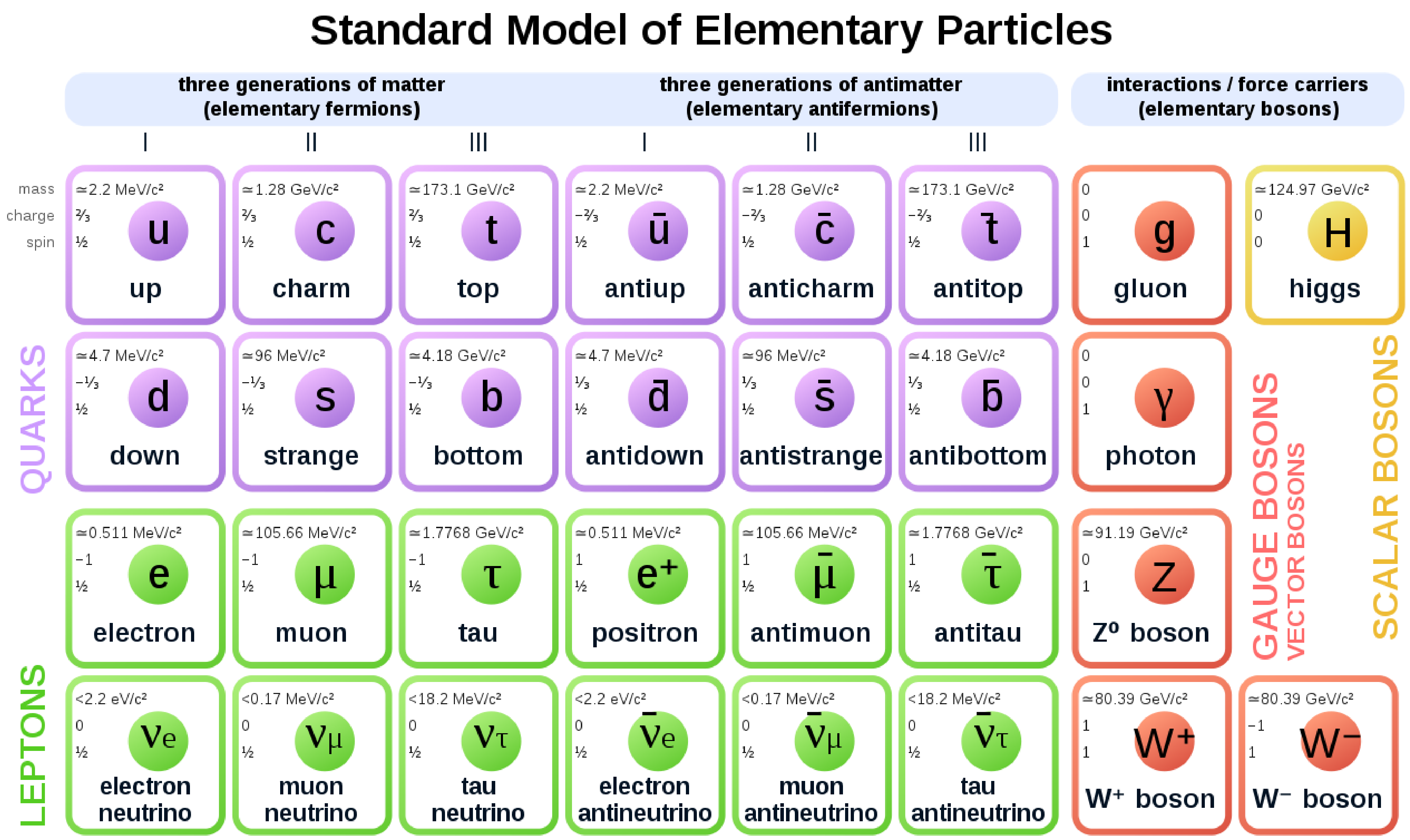
Running coupling constants and unification?

- Unification with SUSY?
- 10^{15} GeV
  - Grand Unified Theory
- 10^{19} GeV:
  - Planck scale → quantum gravity?



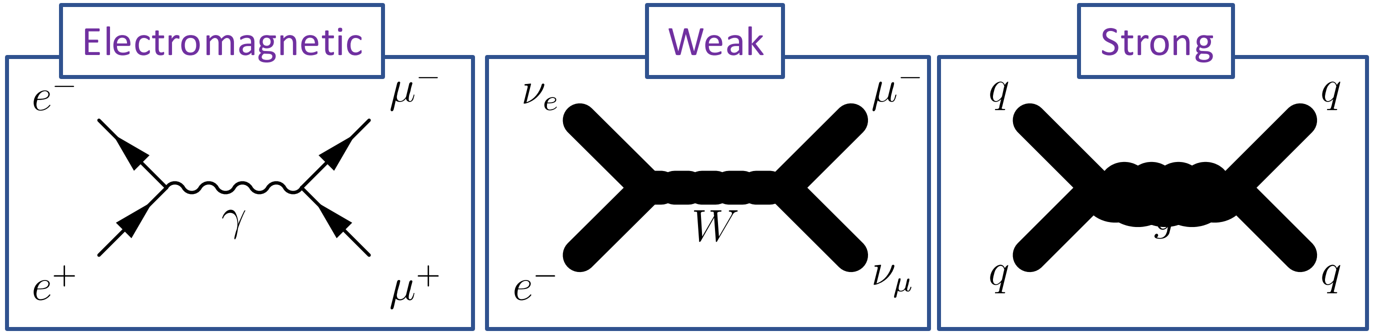
- 10^{15} GeV —> Grand Unified Theory?
- Planck scale (~10^{19} GeV) —> Gravity! —> Theory of Everything?

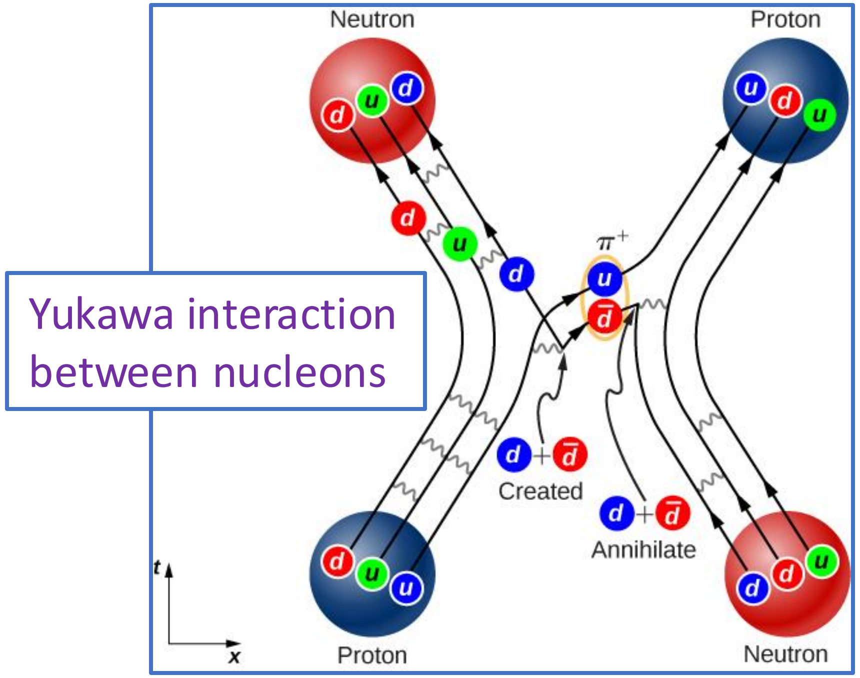
Standard Model of particles and forces



Summary Forces: Intermediate Vector Bosons

- SM: forces are transmitted by vector mesons, particles with spin 1:
  - Electromagnetism:
    - Long range : photon γ , $M_\gamma = 0$
    - Photon carries no E.M. charge
  - Weak interaction:
    - Very short range: W and Z bosons, $M_W = 80\ GeV$, $M_Z = 91\ GeV$
    - Weak isospin charge. W and Z have I-charge.
  - Strong interaction:
    - Very short range: gluon g , $M_g = 0$
      - Gluons carry color → confinement → very short range
    - Pairs of quarks can transmit strong force: Yukawa's mesons
      - Short range





- Gravitation: transmitted by spin-2 particle?
  - Graviton
    - Long range: $M_g = 0$

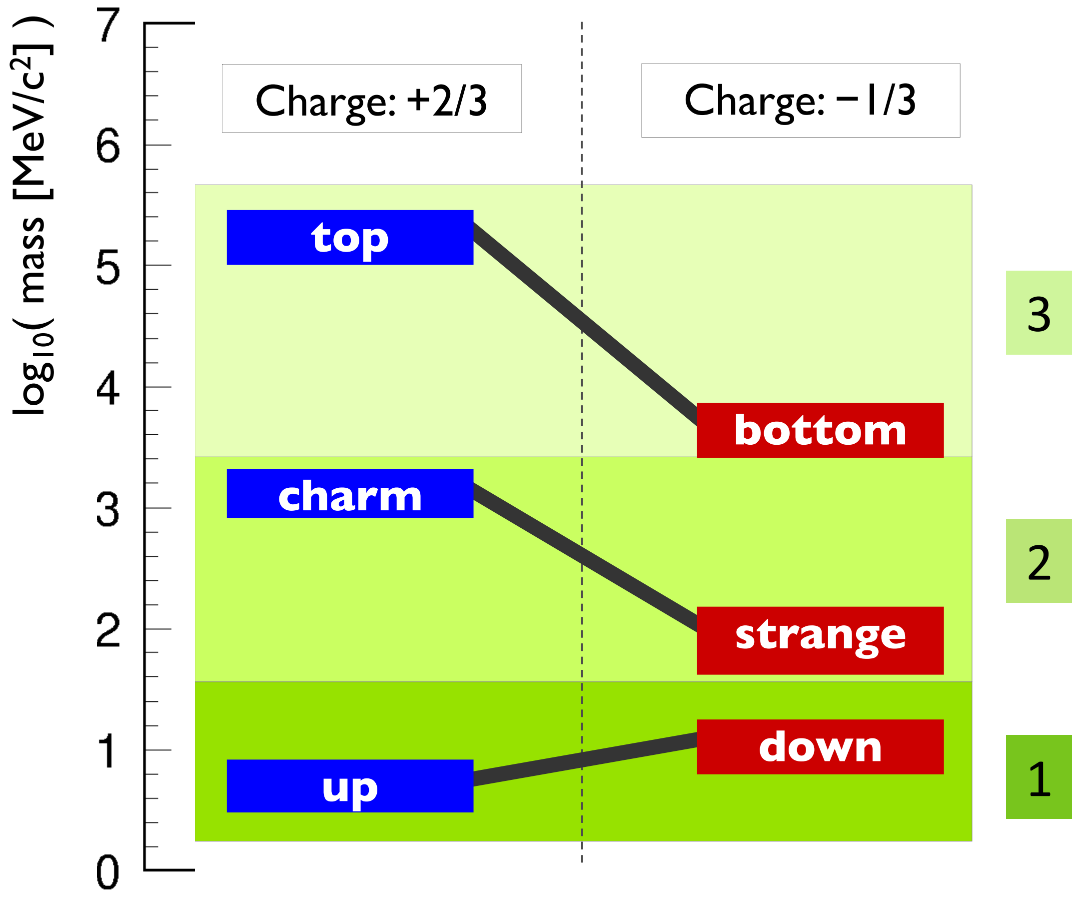
Lecture 2: Discussion Topics

Discussions Topics belonging to
Lecture 2

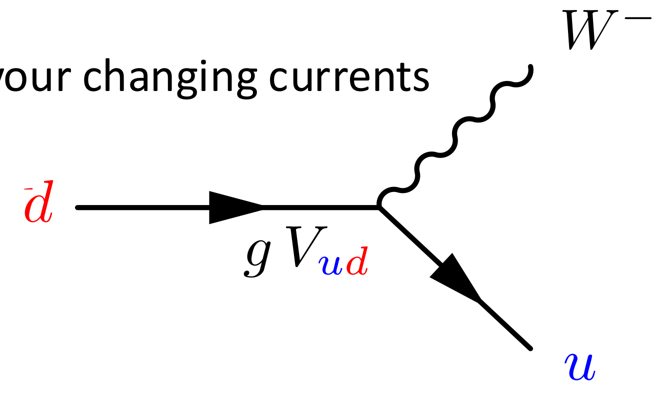
Topic-4: Flavour changing Weak interaction

- What are neutral currents?; what are charged currents?
- Which transitions are allowed by the charged current?
- What is the consequence for stability of 2nd and 3rd generation particles?

Topic-4: Flavour Changing Quark Interactions

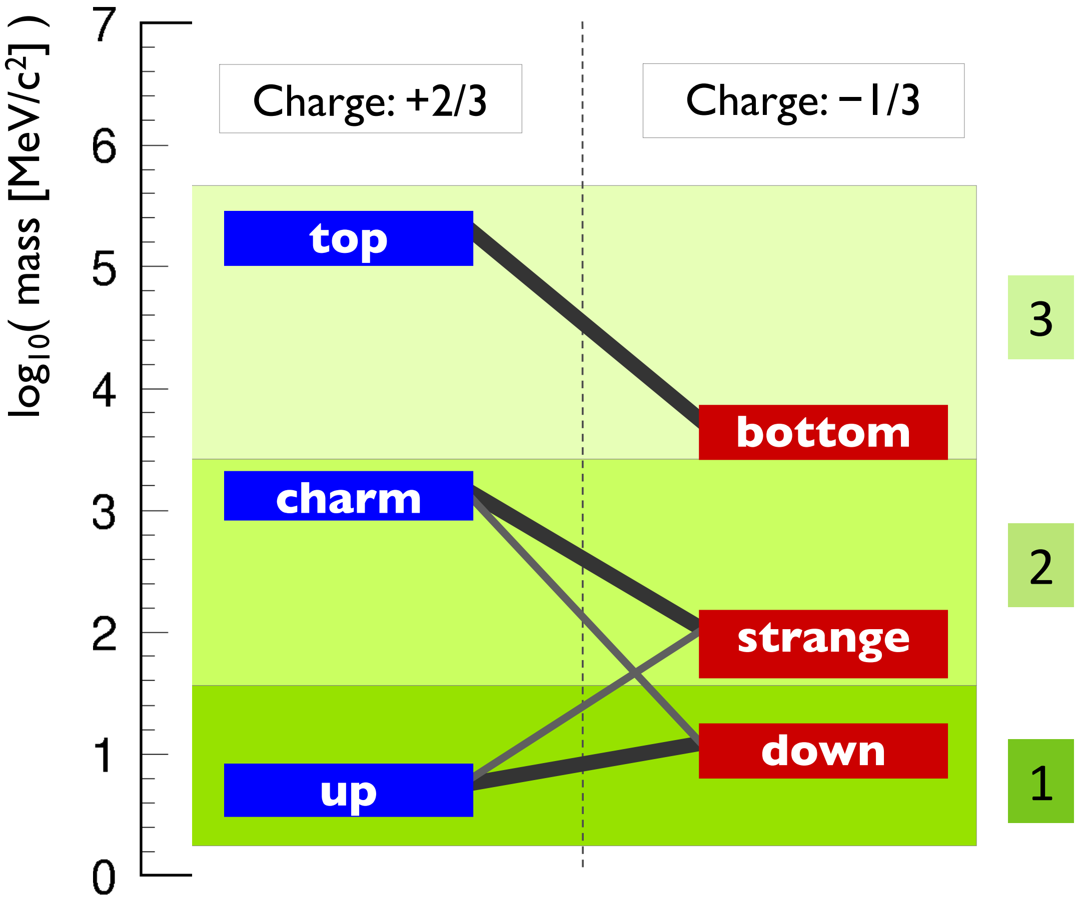


Flavour changing currents

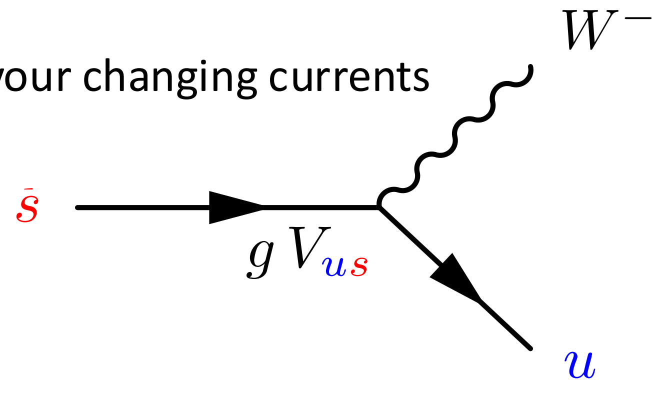


$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & & \\ & V_{cs} & \\ & & V_{tb} \end{pmatrix}$$

Topic-4: Flavour Changing Quark Interactions

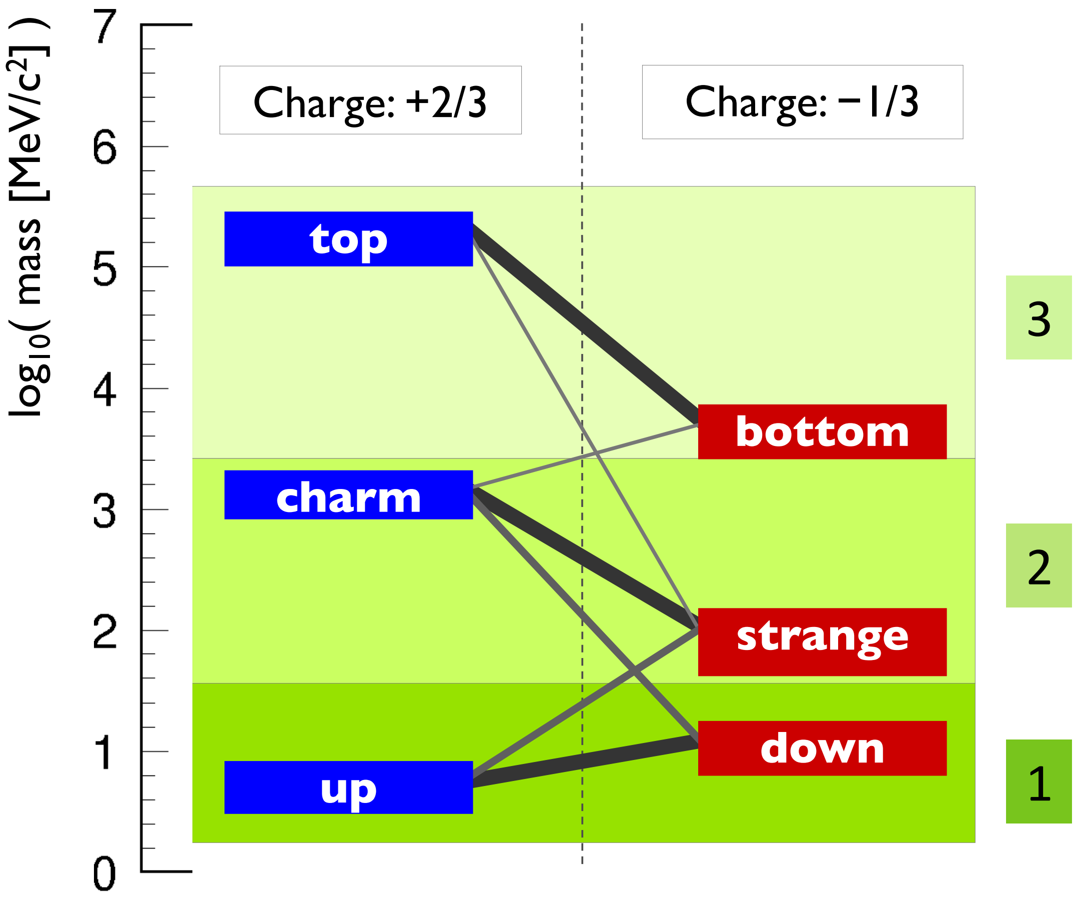


Flavour changing currents

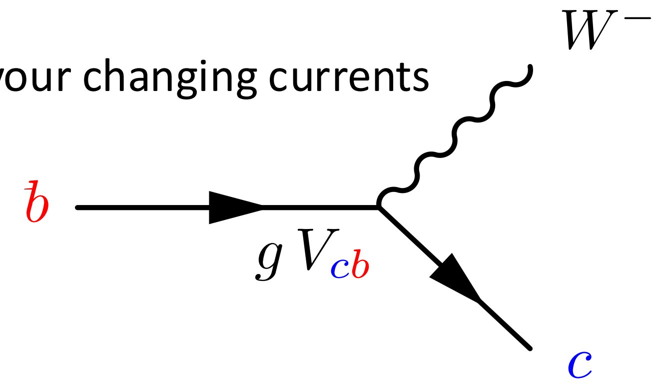


$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & \\ V_{cd} & V_{cs} & \\ & & V_{tb} \end{pmatrix}$$

Topic-4: Flavour Changing Quark Interactions

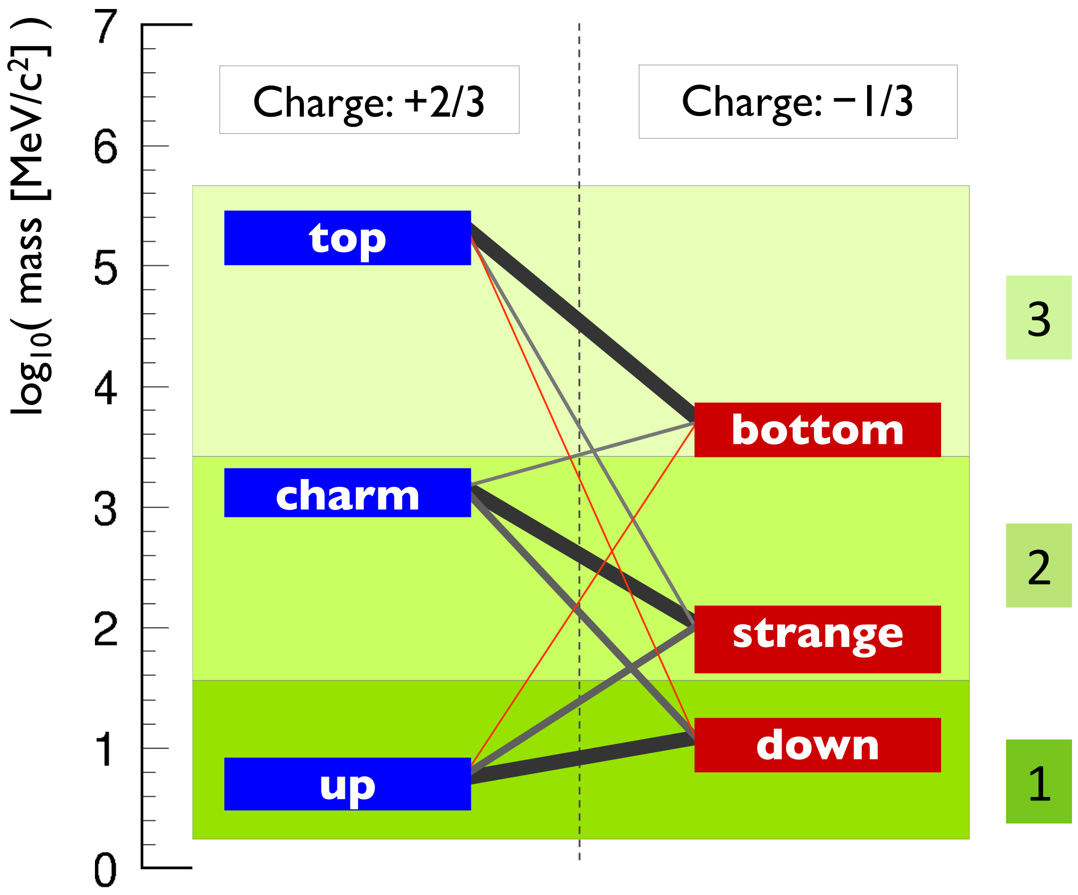


Flavour changing currents

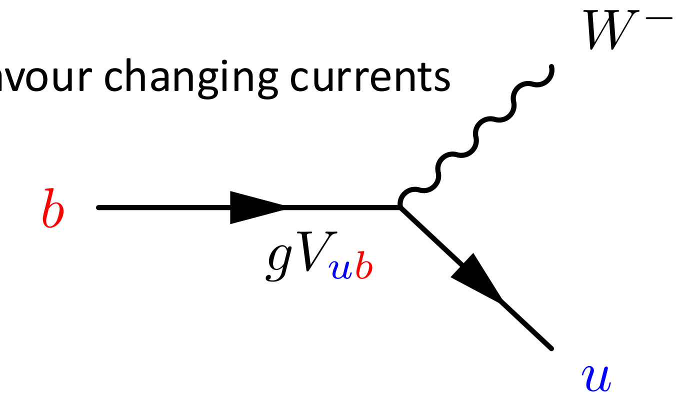


$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & \\ V_{cd} & V_{cs} & V_{cb} \\ & V_{ts} & V_{tb} \end{pmatrix}$$

Topic-4: Flavour Changing Quark Interactions

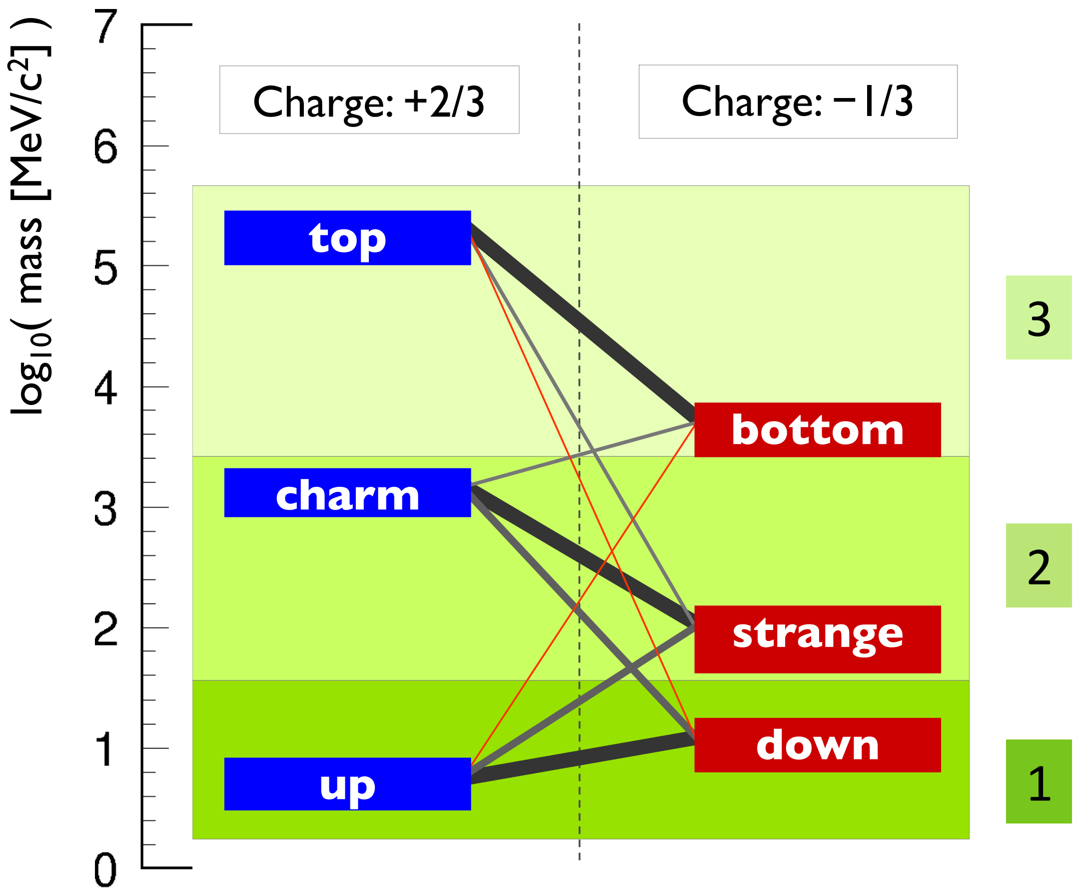


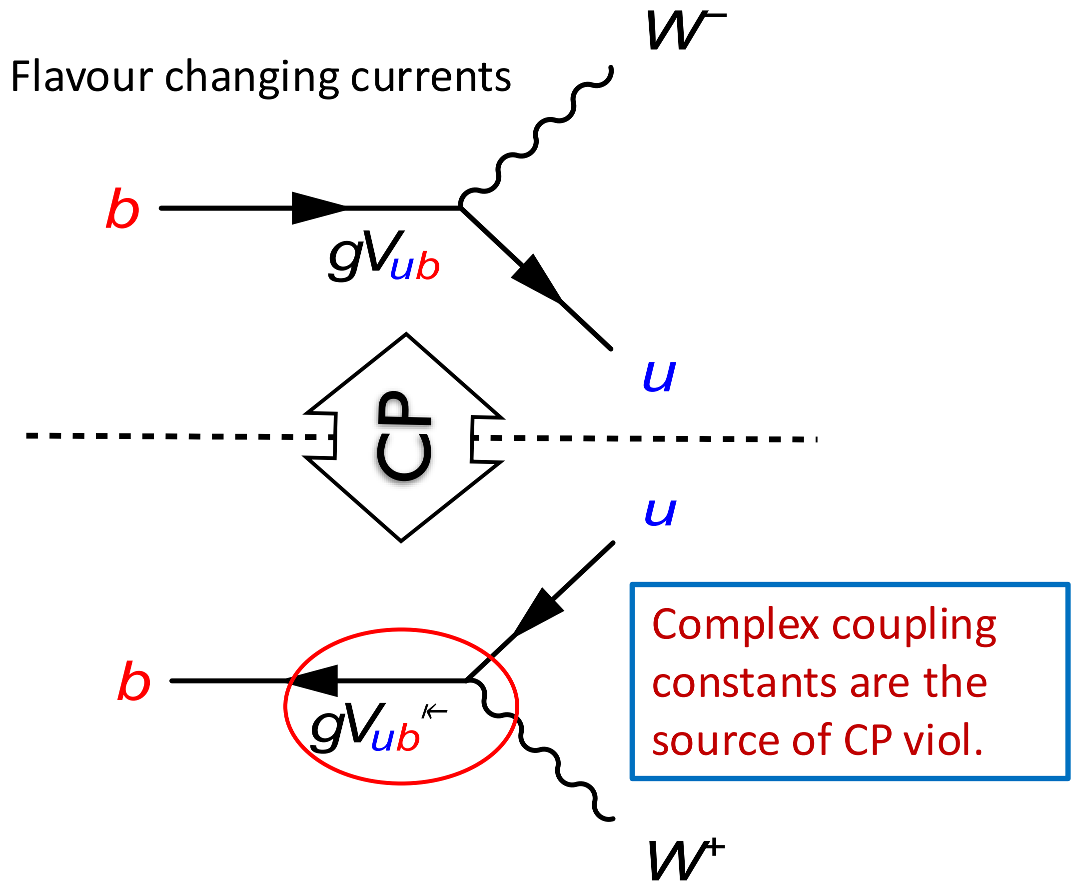
Flavour changing currents



$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Topic-4: Flavour Changing Quark Interactions – CP Violation





- Particles and antiparticles have complex conjugated coupling constants
  - This leads to CP violation
  - Matter dominated universe

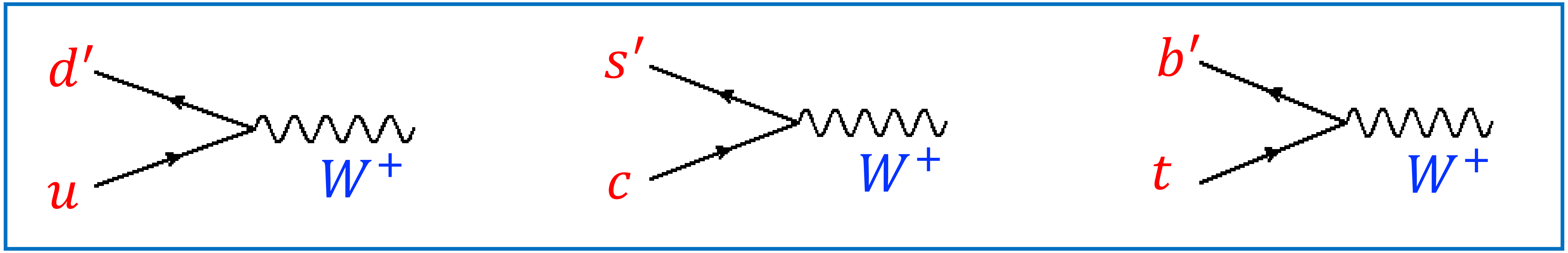
$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Topic-5: The CKM and PMNS matrices

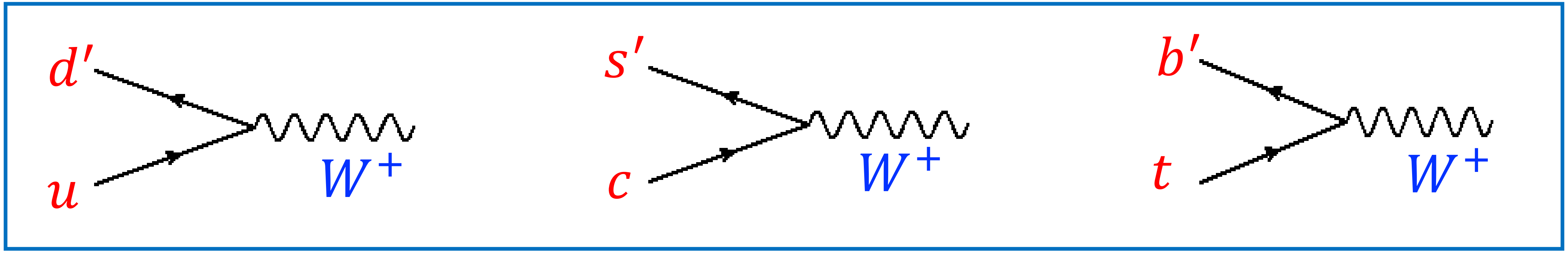
- What is different and what is the same for quarks and lepton in the charged current weak interaction?
- Explain how possibly a matter – antimatter asymmetry can be implemented?
  - Think of complex coupling constants

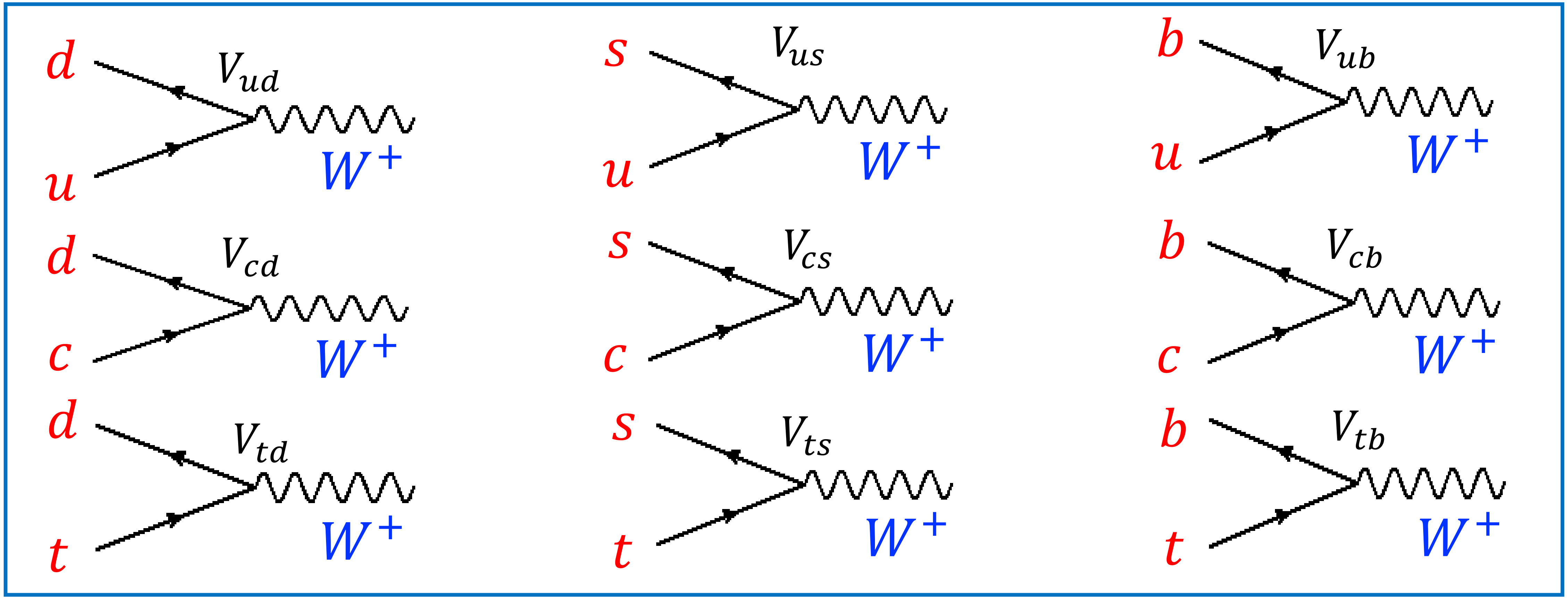
Topic-5: A story of eigenstates

- *Mass* eigenstates $|q\rangle$ are the eigenstate solutions of the free Hamiltonian
- *Weak* or *flavour* eigenstates $|q'\rangle$ are the eigenstate solutions of the weak interaction Hamiltonian.
  - They are unitary linear combination, or "rotation" of mass eigenstates.
- The the weak interaction can be written as:



Topic-5: Flavour eigenstates and Mass eigenstates





Topic-5: Why a matrix notation?

- Model:
  - Charged weak current does ***not*** couple to

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$$

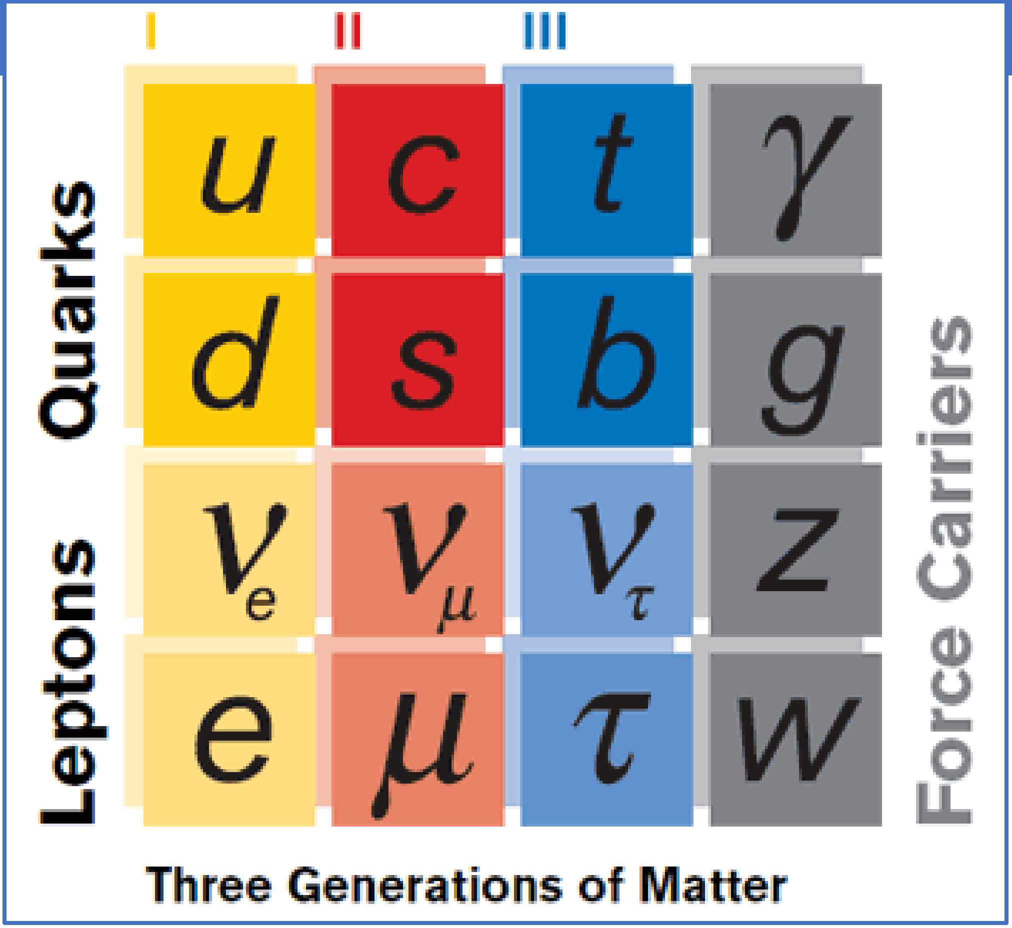
  - but instead to

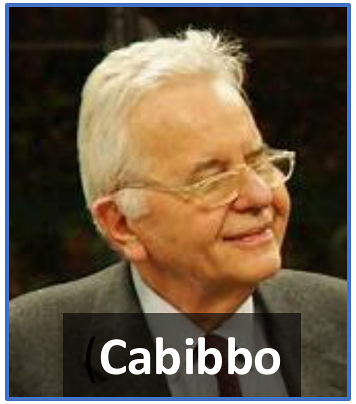
$$\begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}, \begin{pmatrix} t \\ b' \end{pmatrix}$$

  - Where

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM matrix:
Cabibbo,
Kobayashi
Maskawa





Topic-5: The CKM matrix V_{CKM} - 3 vs 2 Generations

$$V_{CKM}: \quad \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \end{array}$$

- Wolfenstein parametrization: V_{CKM} =

$$\begin{pmatrix} 1 - {}^{1}/_{2}\,\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - {}^{1}/_{2}\,\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

➔ 1 CP violating phase

$$V_{CKM}: \quad \begin{array}{c} \\ u \\ c \end{array} \begin{array}{c} \begin{array}{cc} d & s \end{array} \\ \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \end{array}$$

V_{CKM} =

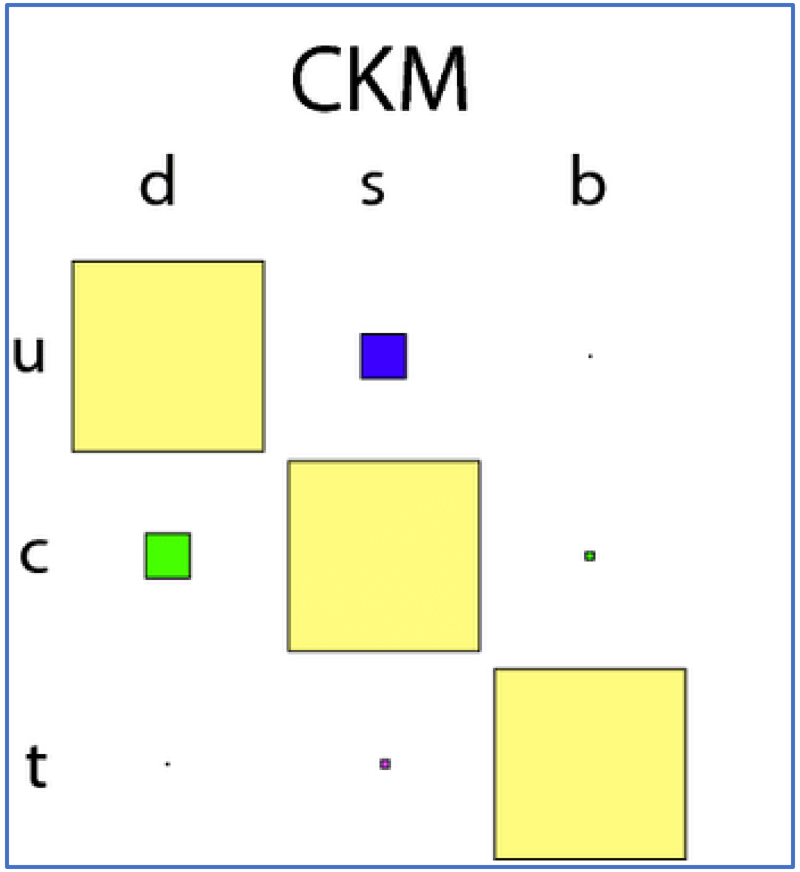
$$\begin{pmatrix} 1 - {}^{1}/_{2}\,\lambda^2 & \lambda \\ -\lambda & 1 - {}^{1}/_{2}\,\lambda^2 \end{pmatrix}$$

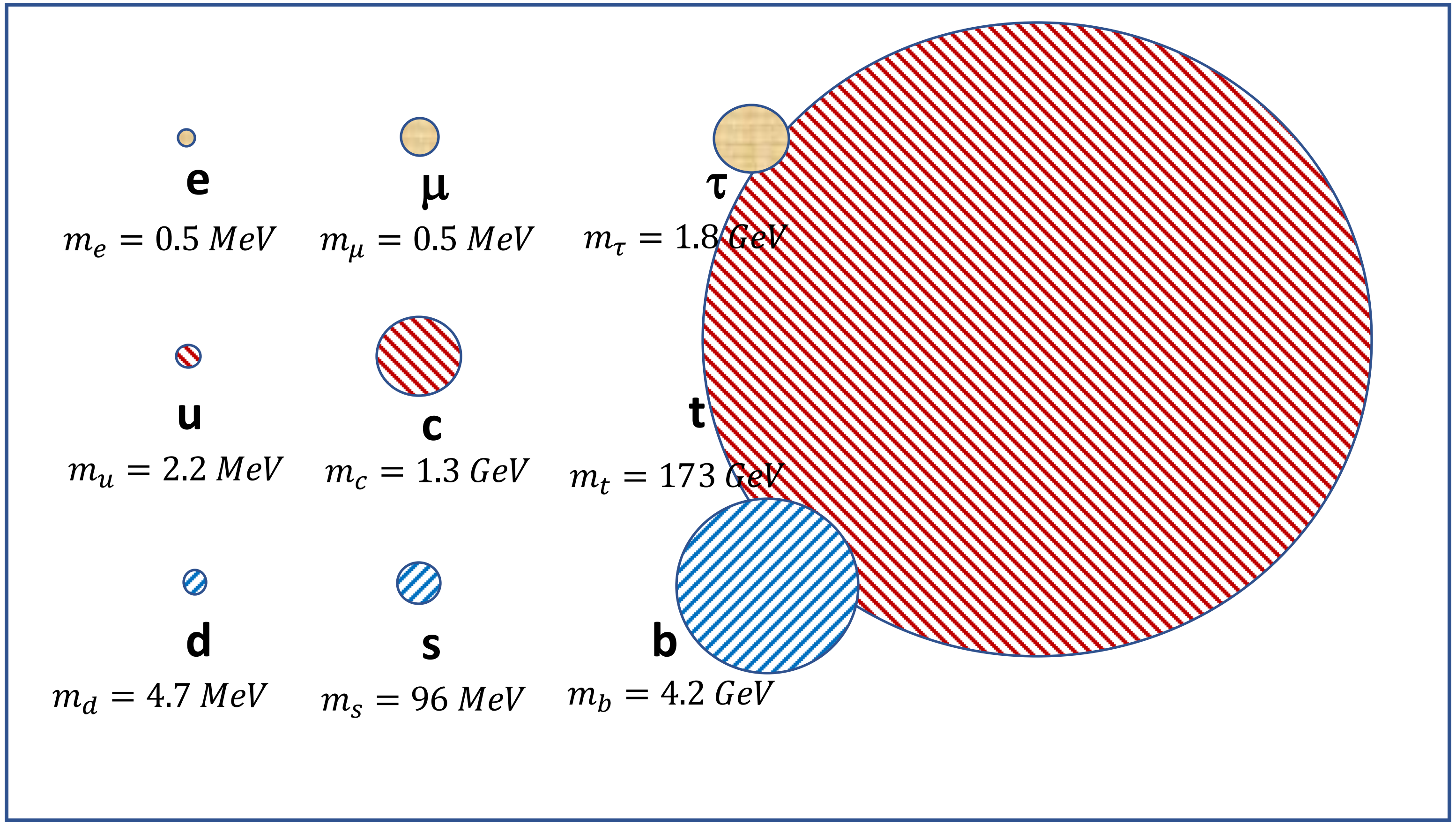
➔ No CP violation

- *3 generations is the minimal particle content to generate CP violation* (In Standard Model).

Topic-5: The Flavour Puzzle

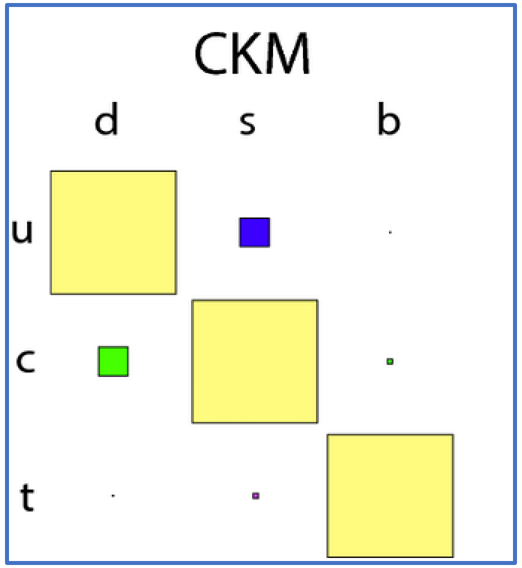
- Why 3?
- Why are the couplings what they are?
- Is there a relation with the masses of the quarks?

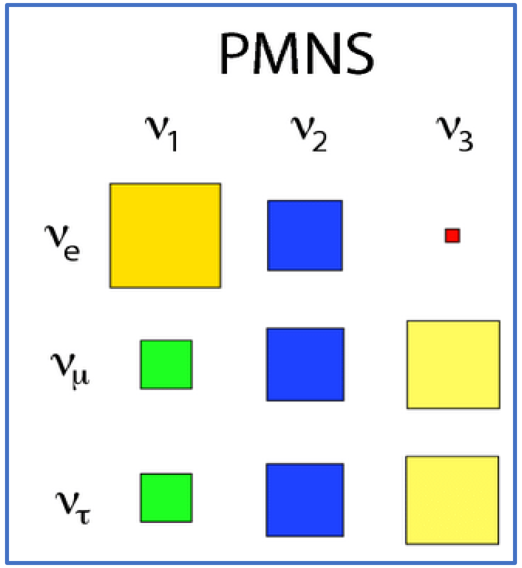




Topic-5: Flavour in the leptons!

- It turns out neutrino's have mass, too!
  - The mass is very tiny
- The generation mixing also occurs for neutrino's
  - Slightly different nomenclature:
  - Quarks (CKM): $\begin{pmatrix} u' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$
  - Leptons (PMNS): $\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} \\ U_{2e} & U_{2\mu} & U_{c\tau} \\ U_{3e} & U_{3\mu} & U_{3\tau} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$
    - U_{PMNS} : Pontecorvo, Maki, Nakagawa, Sakata mixing matrix
- (Difficult) Question: why is lepton mixing not seen in decays?
  - It is only seen in neutrino oscillations





Topic-6: Variational calculus and Lagrangians

- Explain the idea behind variational calculus?
- What is a conservative Force?
- Define a Lagrangian: $\mathcal{L} = T - V = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(x,y,z)$
- Show how Newton's law in one dimension: $F = m\ddot{x}$ leads to the Euler Lagrange equation (in case of a conservative force):

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = \frac{\partial \mathcal{L}}{\partial x}$$

Topic-6: Lagrange Formalism classical

Griffiths §10.1

- Classical Mechanics: The Lagrangian leads to equations of motion
  - $L(q_i, \dot{q}_i) = T - V$ where q_i and $\dot{q}_i$ are the generalized coordinates and velocities.
  - The path of a particle is found from *Hamilton's principle of least action*

$$S = \int_{t_1}^{t_2} dt\, L(q, \dot{q}) \qquad \delta S = 0$$

From this the *Euler Lagrange equations* follow and provide the equations of motion:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}$$

See: https://en.wikipedia.org/wiki/Lagrangian_mechanics

- Example: Ball falls from height $y = h$: $q = y$, $\dot{q} = dy/dt = v_y$
  - $E_{pot} = V = mgq$
  - $E_{kin} = T = \frac{1}{2} m\dot{q}^2$

$$L = T - V = \frac{1}{2} m\dot{q}^2 - mgq$$

- Euler Lagrange: $\partial L/\partial q = mg$; $\partial L/\partial \dot{q} = m\dot{q}$
  - $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}$ gives $m\ddot{q} = mg \rightarrow \dot{q} = gt + v_0 \rightarrow q = y = \frac{1}{2} gt^2 + v_0 t + y_0$

Lecture 2: Exercises

Exercises belonging to Lecture 2

Exercise-4: Variational calculus Lagrange Formalism classical

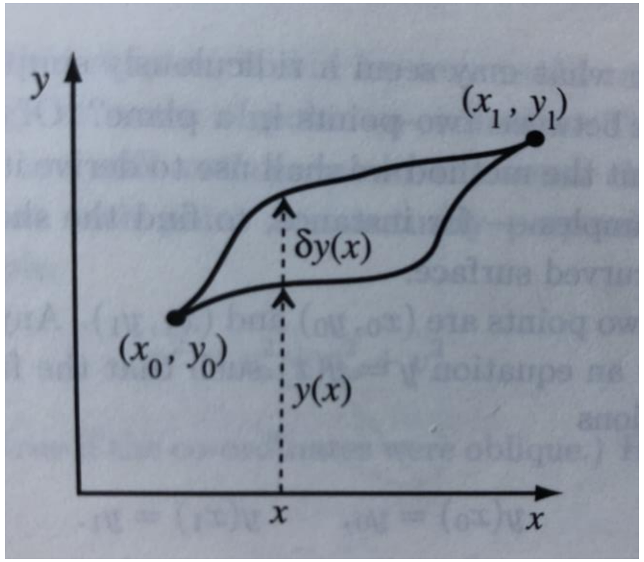
- Example of variational calculus and least action principle: what is the shortest path between two points in space?
  - Distance of two close points:

$$dl = \sqrt{dx^2 + dy^2} = \sqrt{dx^2\left(1 + \left(\frac{dy}{dx}\right)^2\right)} = \sqrt{1 + y'^2}dx \quad \text{with } y' = dy/dx$$

  - Total length from (x_0, y_0) to (x_1, y_1) :

$$l = \int_{x_0}^{x_1} dl = \int_{x_0}^{x_1} \sqrt{1 + y'^2}dx = \int_{x_0}^{x_1} f(y, y')dx$$

  - Task is to find a function $y(x)$ for which l is minimal



- In general assume the path length is given by: $I = \int_{x_0}^{x_1} f(y, y')dx$
- Variational principle: shortest path is stationary: $\delta I = 0$

a) Write $\delta f(y, y') = \frac{\partial f}{\partial y}\delta y + \frac{\partial f}{\partial y'}\delta y'$ where $\delta y' = \delta\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\delta y)$

Show using partial integration that $\delta I = 0$ leads to the Hamilton Lagrange equation $\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0$

b) Here for the shortest path we have $f(y') = l = \sqrt{1 + y'^2}$.

Then $\partial f/\partial y = 0$ and $\partial f/\partial y' = y'/\sqrt{1 + y'^2}$

Show that the variational principle leads to a straight line path: $\frac{d}{dx}\left(\frac{y\prime}{\sqrt{1+y'^2}}\right) = 0$ or that y' is a constant:

$dy/dx = a$; $y = ax + b$

Exercise-5: 4-Vector derivatives

a) Start with the expression for a Lorentz transformation along the x^1 axis. Write down the *inverse* transformation (i.e. express (x^0, x^1) in $(x^{0\prime}, x^{1\prime})$)

b) Use the chain rule to express the derivatives $\partial/\partial x^{0\prime}$ and $\partial/\partial x^{1\prime}$ in $\partial/\partial x^0$ and $\partial/\partial x^1$

c) Use the result to show that $(\partial/\partial x^0, -\partial/\partial x^1)$ transforms in the same way as (x^0, x^1)

d) In other words the derivative four-vectors transform inversely to the coordinate four-vectors:

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}\right) \text{ and } \partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

Note the difference w.r.t. the minus sign!

e) Explicit 4-vectors: (ct, x, y, z) and $\left(E/c, p_x, p_y, p_z\right)$ → use next $c \equiv 1$

Co- and contravariant derivatives

Griffiths: chapter 3

- Contravariant vector:

$$x^\mu = (ct, \vec{x})$$

But contravariant derivative:

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}\right)$$

- Covariant vector:

$$x_\mu = (ct, -\vec{x})$$

But covariant derivative:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

Note that the minus sign is “opposite” to the case of the coordinate four-vectors.

Exercises 6, 7, 8

6. [Griffiths exercise 2.2] "Crossing lightsabers"
   - Draw the lowest-order Feynman diagram representing Delbruck scattering: $\gamma + \gamma \rightarrow \gamma + \gamma$
   - This has no classical analogue. Explain why.
7. [Griffiths exercise 2.4]
   - Determine the invariant mass of the virtual photon in each of the lowest-order Feynman diagrams for Bhabha scattering. Assume electron and positron at rest.
8. [Griffiths exercise 2.7]
   - Examine the processes in ***the left column*** of Griffiths exercise 2.7 and state which one is possible or impossible, and why / with which interaction.

   Hint: draw the corresponding Feynman diagrams if needed.