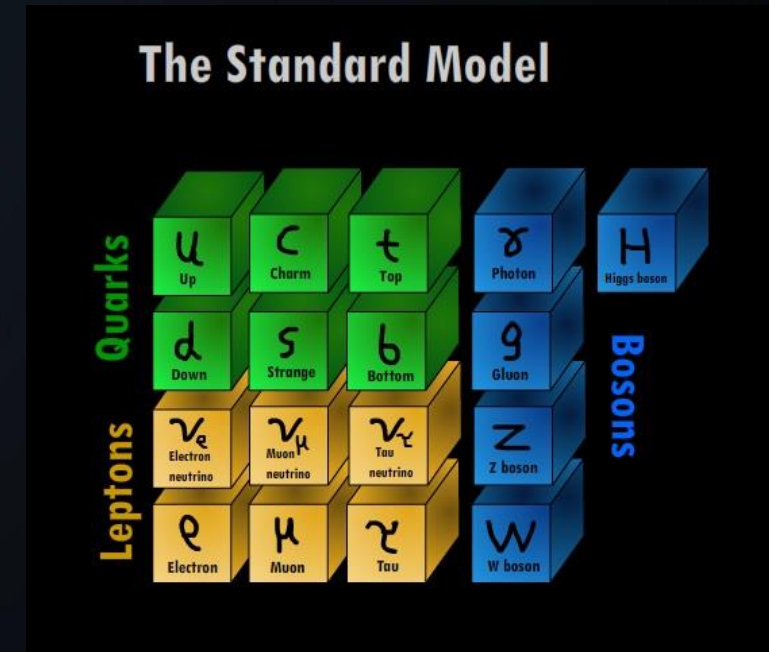
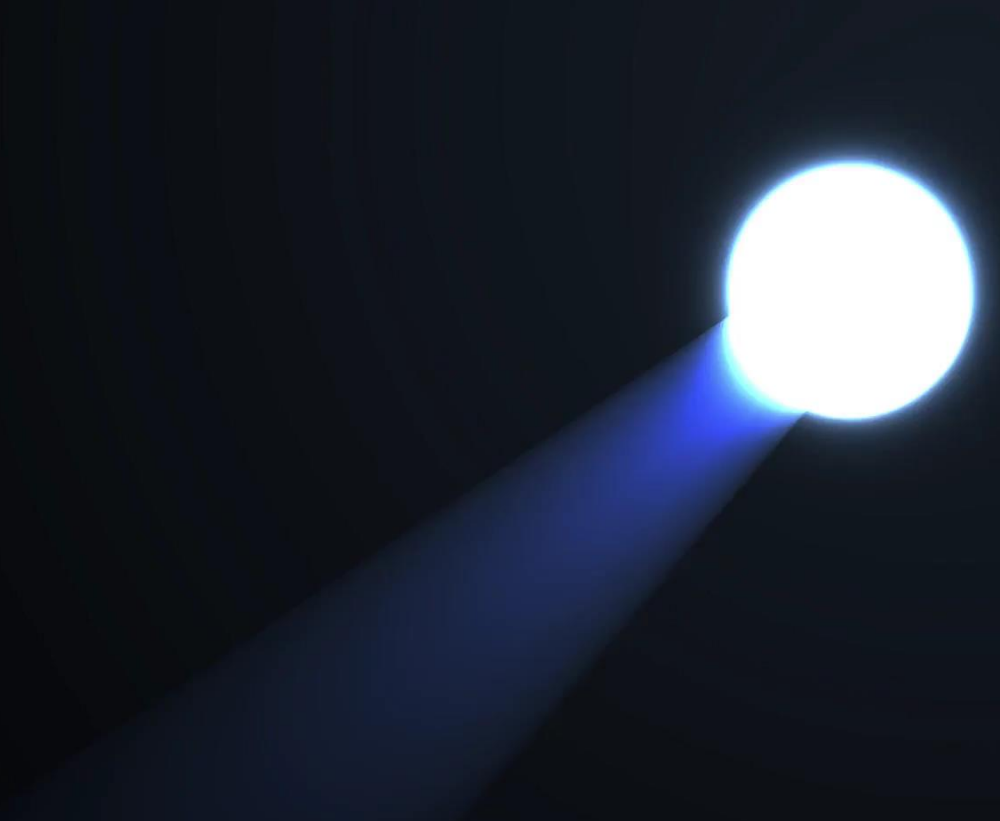




PHY3004: Nuclear and Particle Physics

Marcel Merk, Jacco de Vries



Introducing the lecturers

Lecturers:

Marcel Merk
Jacco de Vries



Tutors:

Evridiki
Chatzianagnostou
Xenofon Chiotopoulos

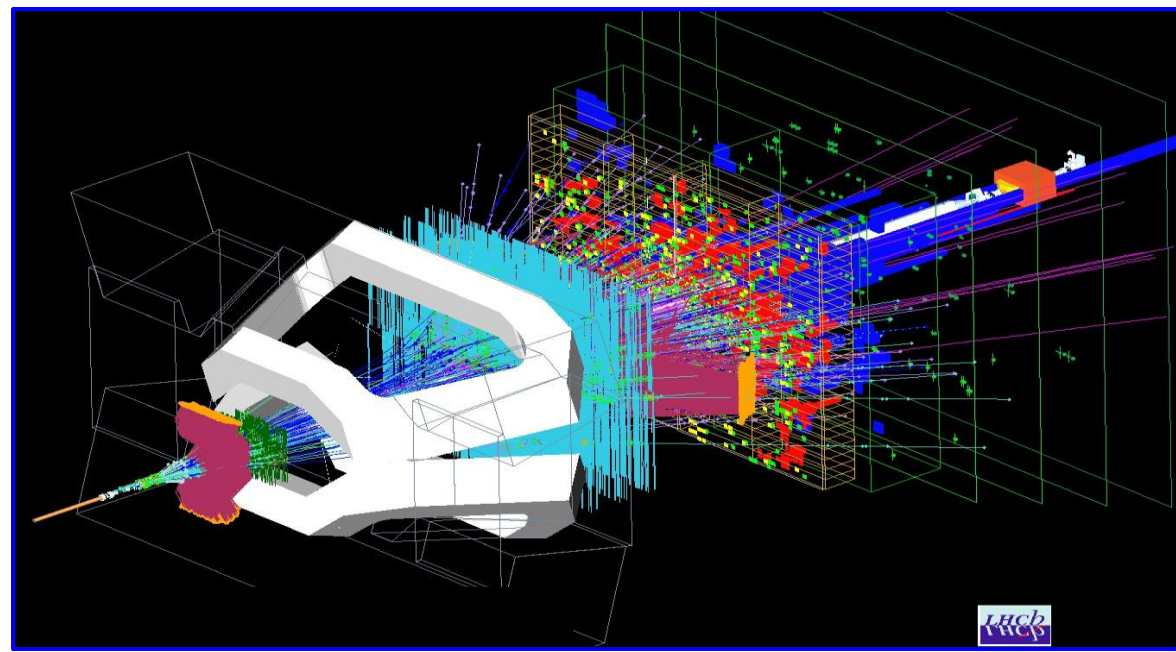
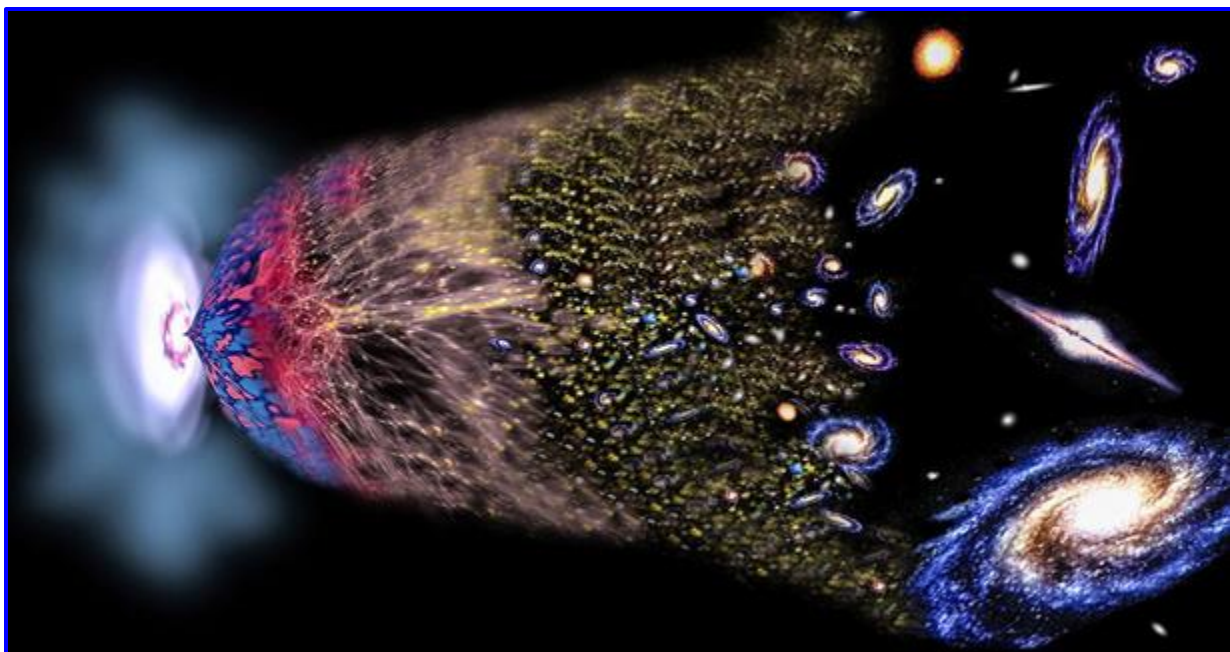


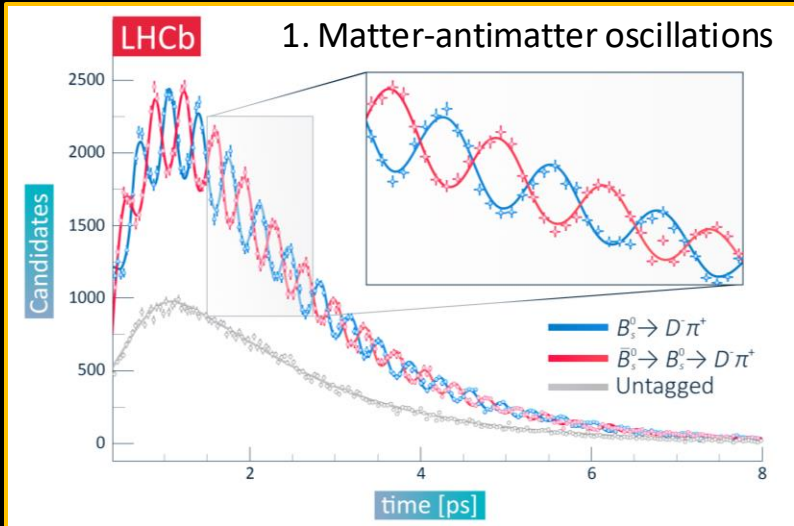
Research (theoretical):

- Why a *matter-vs-antimatter asymmetry* in nature?
- Why do we have *three generations* of particles?

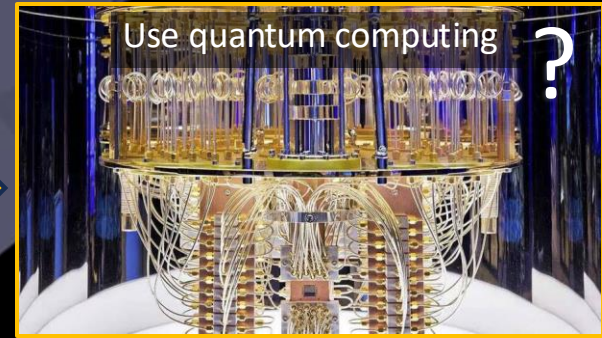
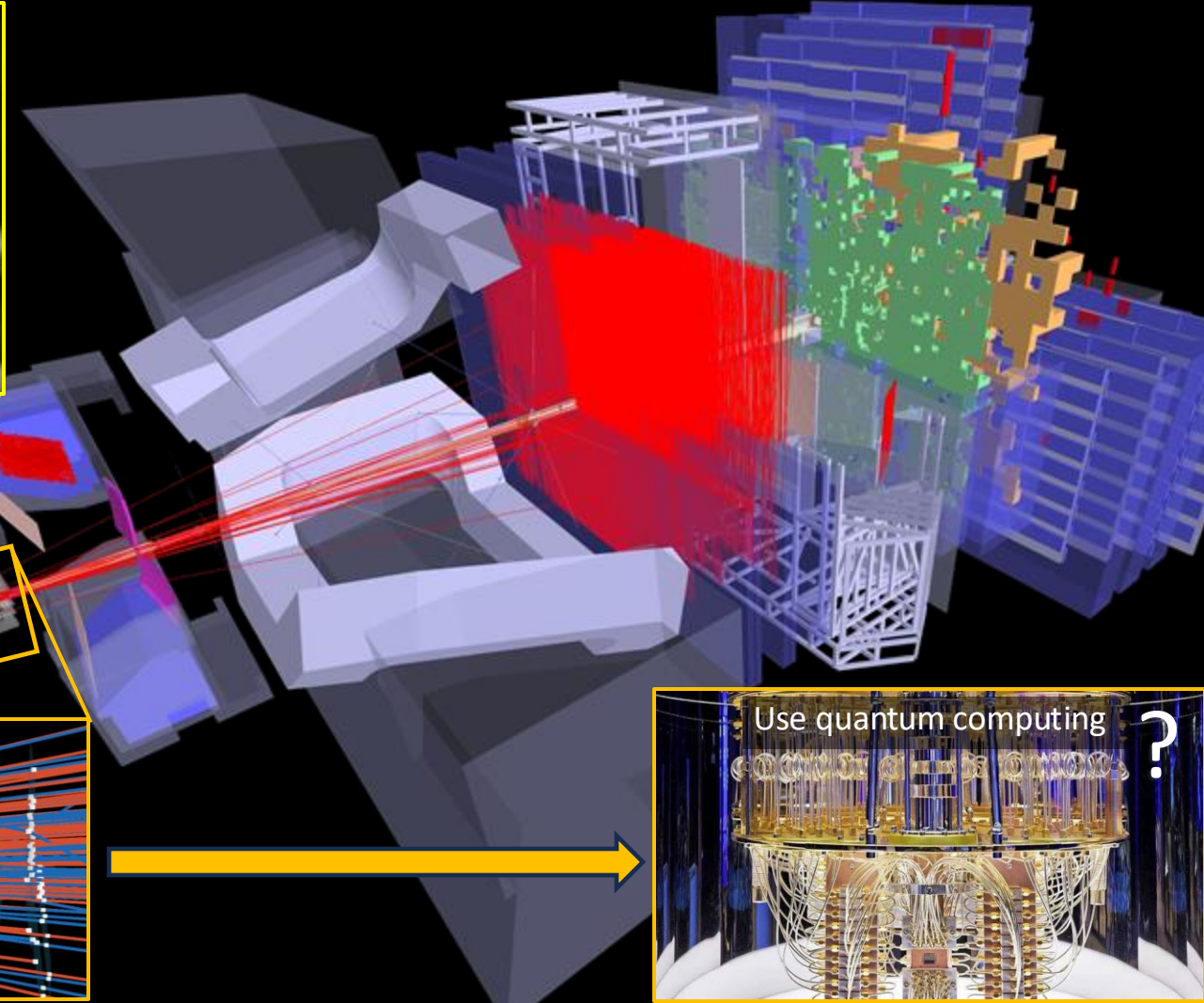
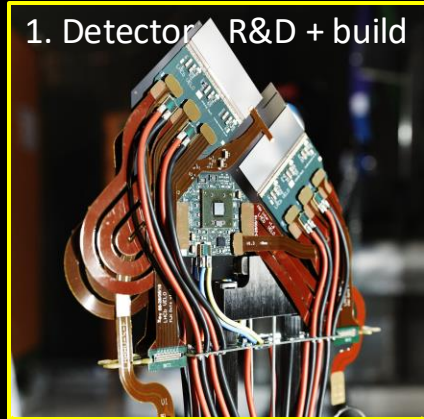
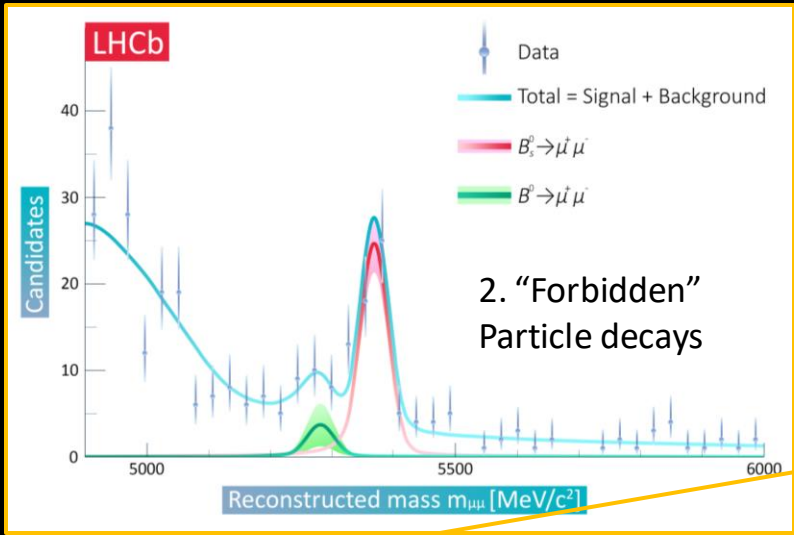
Research (experimental):

- Detector technology at the *Large Hadron Collider*.
- *Measurements of CP violation rare decays*





Theoretical and Experimental Research with the LHCb Detector at the Large Hadron Collider at CERN.



The Goal of these Lectures

- A first course in particle physics theory and experiment
 - Solid preparation for a master on particle physics
 - Pre-reqs are special relativity (“4-vectors”), Lagrange-Hamilton mechanics, Quantum Mechanics
 - Please give these topics attention in tutorials if you need to refresh them
- Various topics introduced
 - Cannot go to “too deep” in each topic
 - Aim is to give you a “feel” for the subject
 - Quite some mathematics later on, but focus on the concepts if you find the math difficult!
- Follow-up possibilities in master education:
 - Theoretical Particle Physics: Field theory courses
 - Theoretical and Experimental Particle Physics: Advanced particle physics courses
 - Experimental Particle Physics: Detection techniques for particles

Particle Physics – contents of the course

- Week 1: “Particles”
 - Nuclear Physics and Particle Physics
- Week 2: “Forces”
 - Electromagnetism, Weak force, Strong force
- Week 3: “Waves”
 - Wave Equations
 - Schrodinger, Klein – Gordon, Dirac, Photon-field
 - Lagrange Hamilton mechanics and Gauge Invariance
- Week 4: “Symmetries”
 - The Standard Model gauge symmetry
 - Discrete symmetries
 - Symmetry breaking: the Higgs Mechanism
- Week 5: “Scattering”
 - Scattering Theory
 - Feynman Calculus
- Week 6: “Experiments”
 - Experimental techniques, LHC detectors

- Week 1 – 5:
 - Homework exercises 20%
 - Tutorial discussions 15%
- Week 6: Research Project
 - Presentation 15%
- Week 7: Open book Exam 50%

Videos: see <https://www.nikhef.nl/~i93/Teaching>

Literature belonging to the lectures:

- Griffiths: “Introduction to Elementary Particles”

Other good books (more advanced):

- Mark Thomson: “Modern Particle Physics”
- Halzen and Martin: “Quarks & Leptons: An introductory Course in Modern Particle Physics”
- Aitchison and Hey: “Gauge Theories in Particle Physics” – 2 volumes
- Lecture notes PP1 master course from Nikhef:
<https://www.nikhef.nl/~i93/Master/PP1/2017/Lectures/Lecture2017.pdf>

Particle Physics – contents of the course

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 - Presentation 15%
 - 50%

All information available in:

- Canvas: **Course Manual** + Lectures + Exercises
- <https://www.nikhef.nl/~i93/Teaching#MSP>

[i93/Teaching](https://www.nikhef.nl/~i93/Teaching)

Primary Particles”

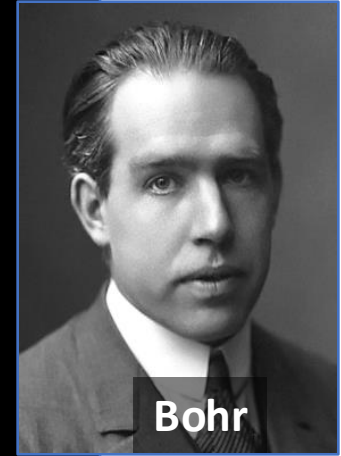
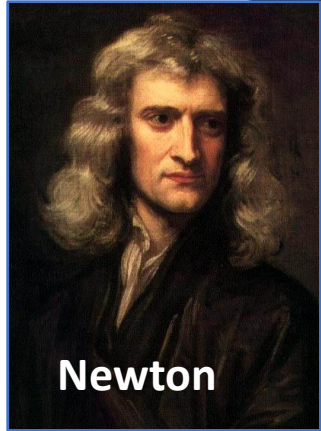
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Intro: Relativity and Quantum Mechanics

Classical Mechanics

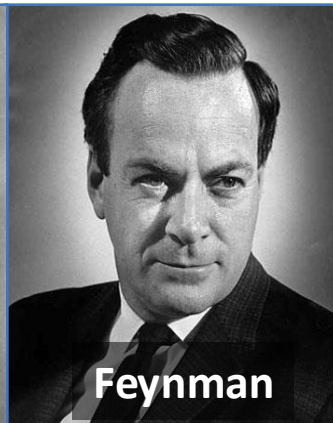
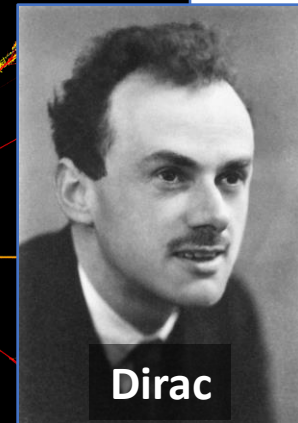
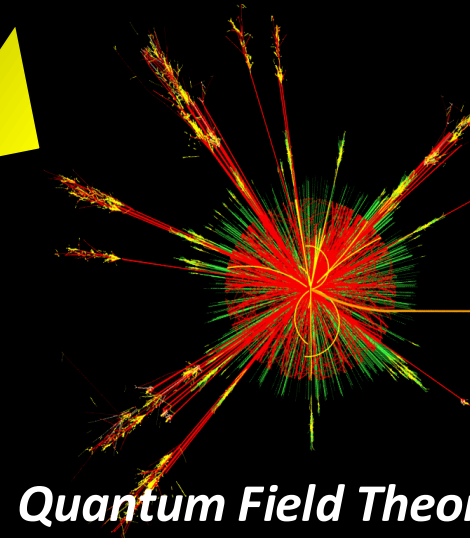
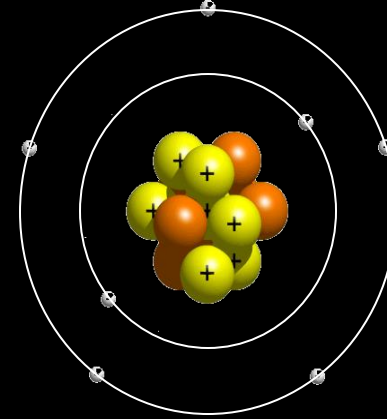
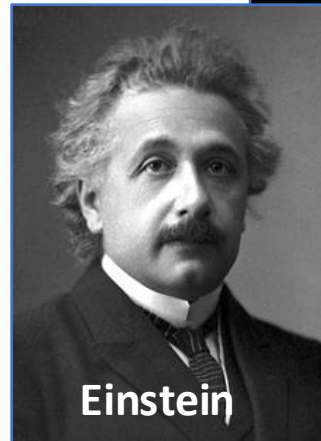
Quantum Mechanics



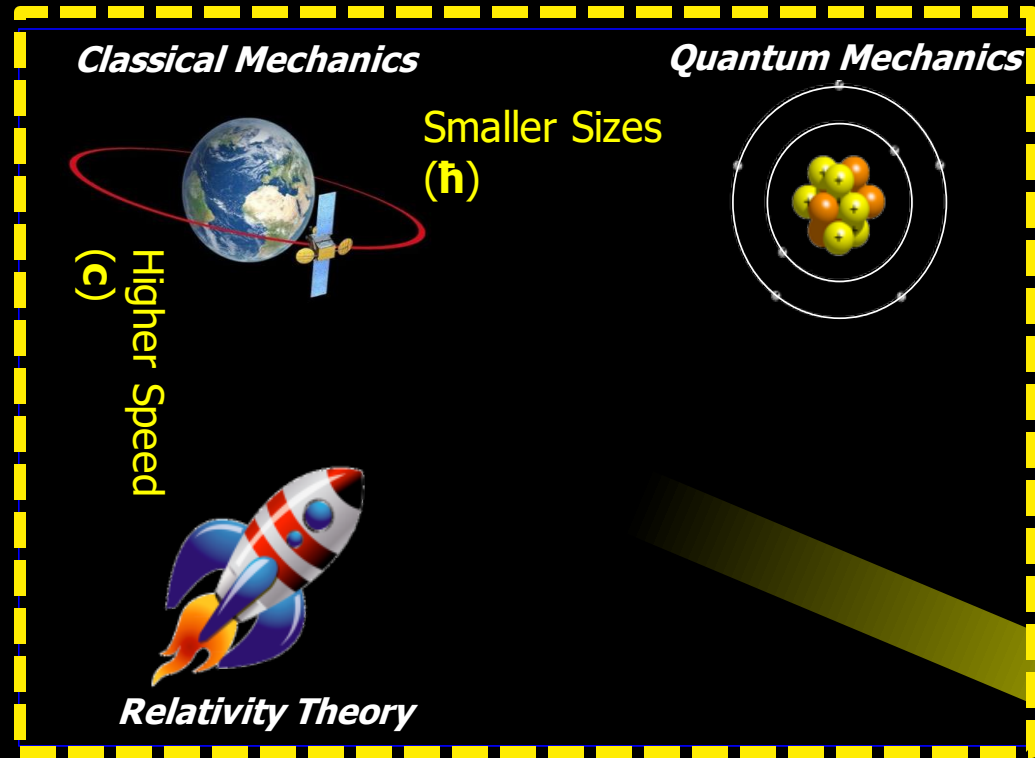
Smaller Sizes ($\hbar$)

Higher Speed (c)

From our experience we are not used to relativistic and quantum mechanical phenomena. That's why we find it counterintuitive.



Intro: How does nature behave in extreme conditions?

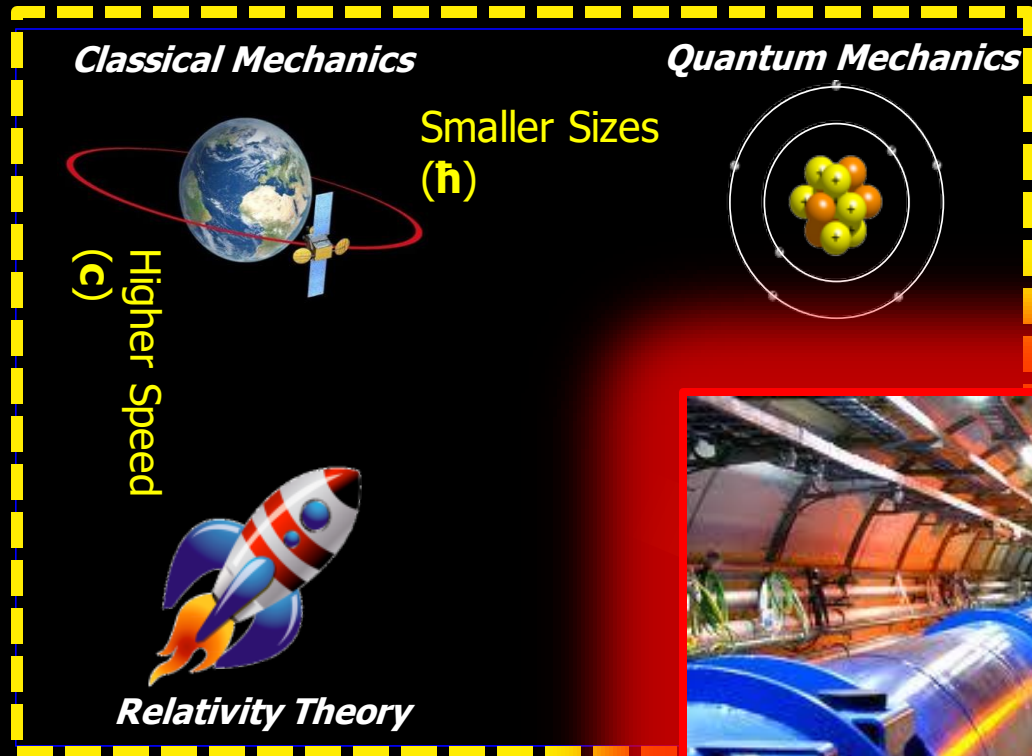


← Experimentally tested domain of the Standard Model

Big Bang

↑ ↑
← Do we understand the physics also here? →

Intro: How does nature behave in extreme conditions?



← Experimentally tested domain of the Standard Model

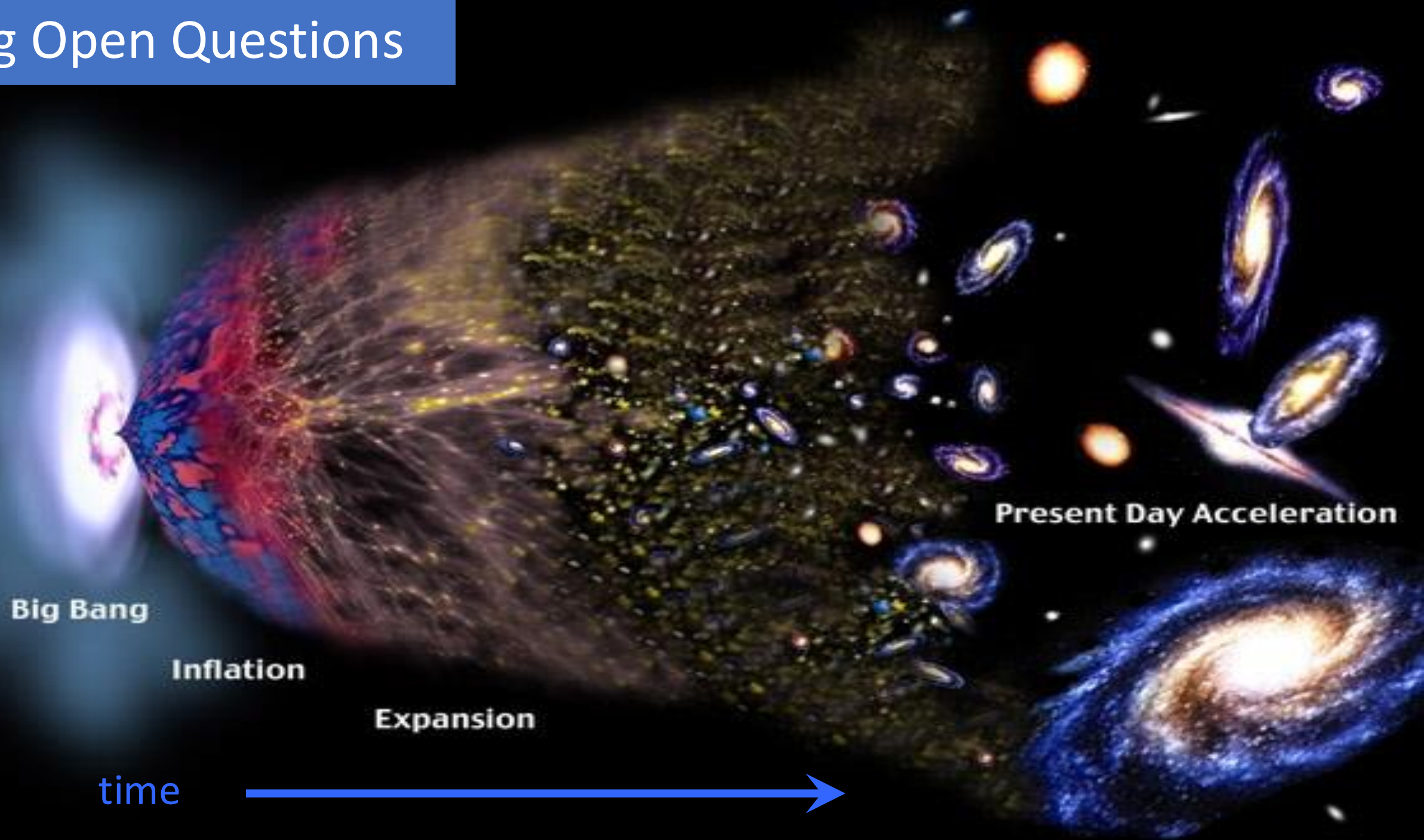


The Large Hadron Collider

Big Bang

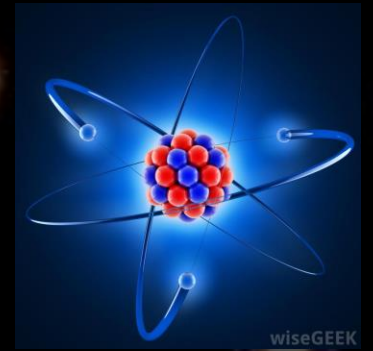


Big Open Questions



Big Open Questions

1. Which are the elementary **building blocks** of matter?



2. Which **forces** exist in nature and what are their essential differences?



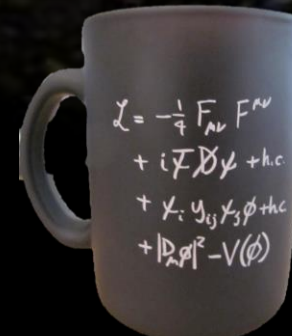
3. Is **empty space** ('the vacuum') truly empty?

Present Day Acceleration

4. Can we explain the existence of **our universe** from the *big bang* using the known laws of nature?

Inflation

- *Wanted: a consistent theory that can answer these questions*



Lecture 1: Particles

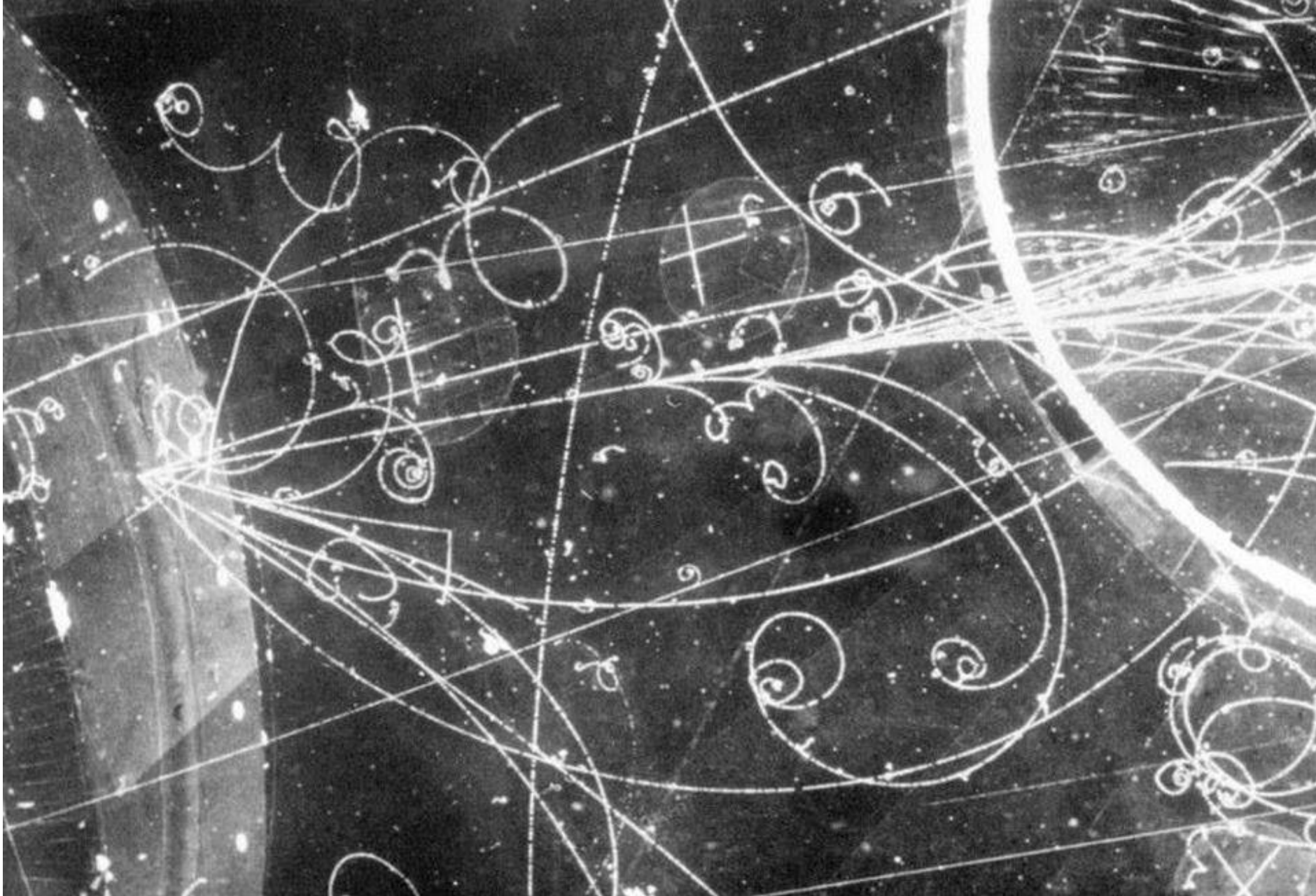
Part 1

The Structure of Matter (Nuclear Physics)

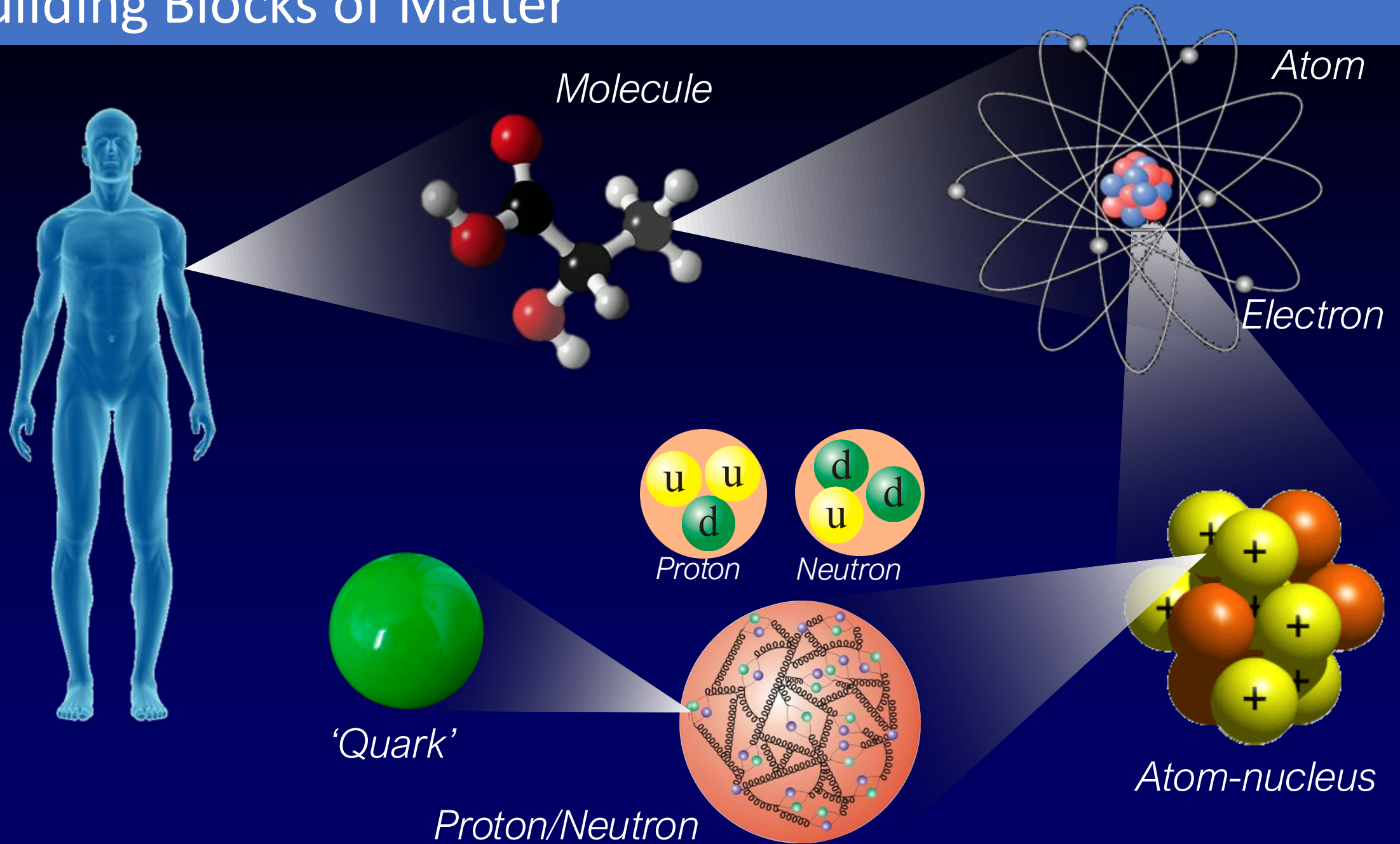
See the online slides

Not part of the exam material

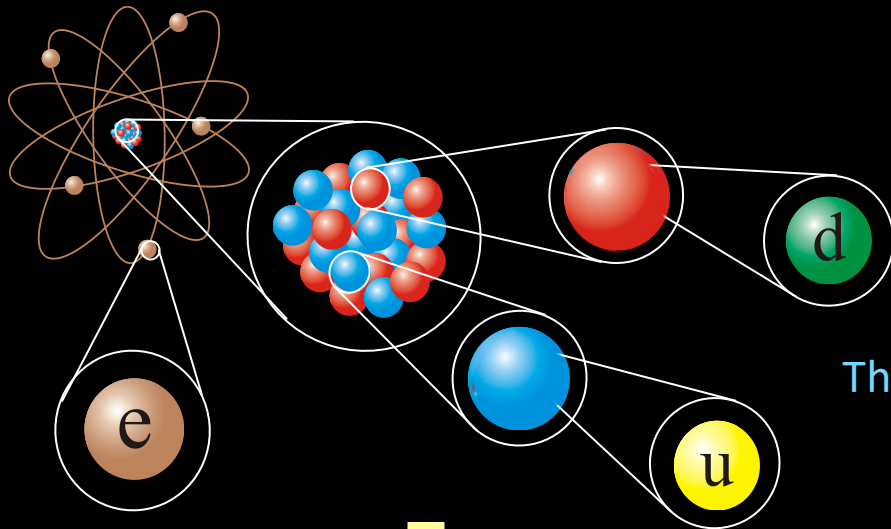
Lecture 1: “Particles”



Building Blocks of Matter



Stable Matter on Earth



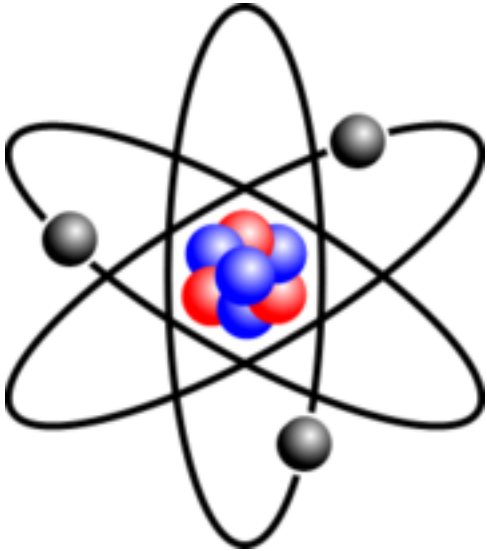
The Lego blocks of nature

	1 Ia	2 IIa											13 IIIa	14 IVa	15 Va	16 VIa	17 VIIa	18 0 He
1	1 H												5 B	6 C	7 N	8 O	9 F	10 Ne
2	3 Li	4 Be																
3	11 Na	12 Mg	3 IIIb	4 IVb	5 Vb	6 VIb	7 VIIb	8 VIIIb	9 VIIIb	10 VIIIb	11 Ib	12 IIb	13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	(43) Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	(104) Rf	(105) Db	(106) Sg	(107) Bh	(108) Hs	(109) Mt	(110) Ds	(111) Rg	(112) Cn	(113) Uut	(114) Fl	(115) Uup	(116) Lv	(117) Uus	(118) Uuo
Lanthaniden			57 La	58 Ce	59 Pr	60 Nd	(61) Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
Actiniden			89 Ac	90 Th	91 Pa	92 U	(93) Np	(94) Pu	(95) Am	(96) Cm	(97) Bk	(98) Cf	(99) Es	(100) Fm	(101) Md	(102) No	(103) Lr	



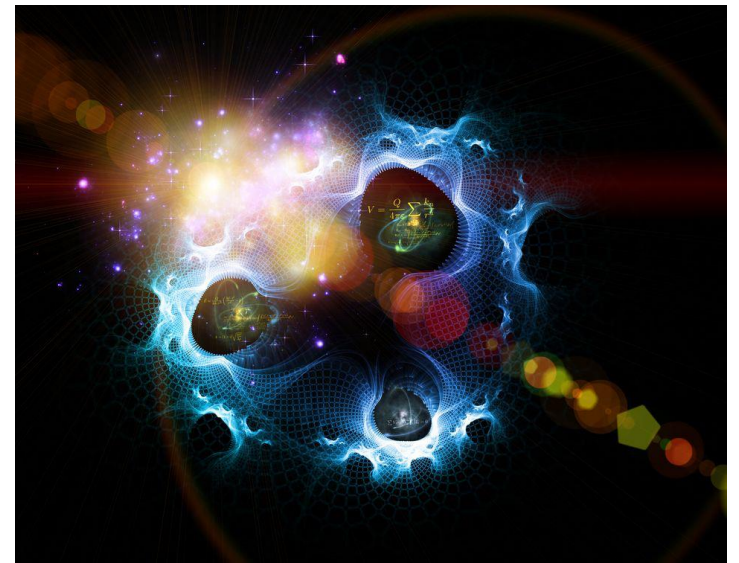
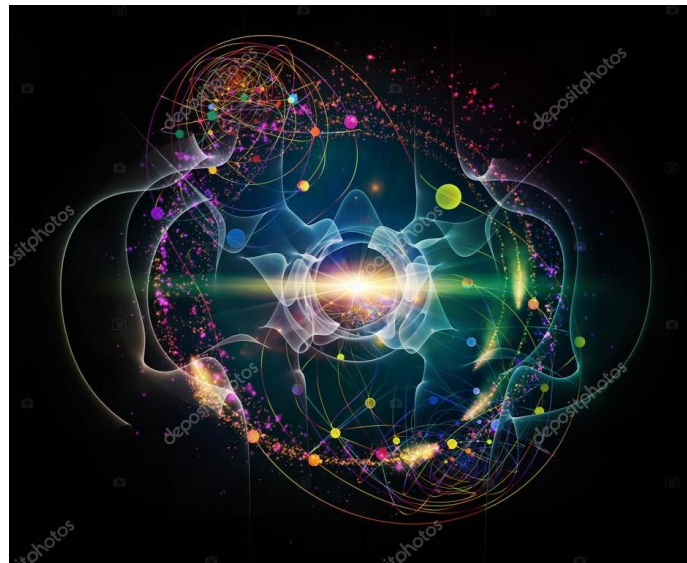
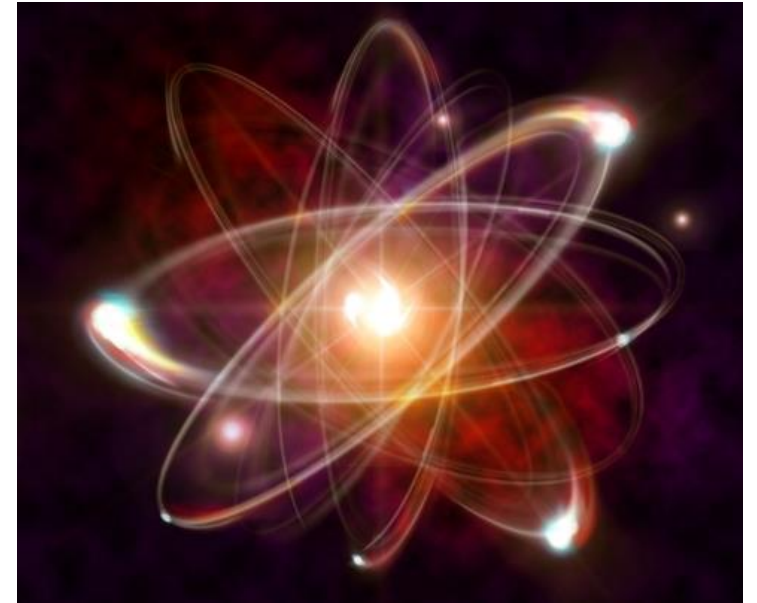
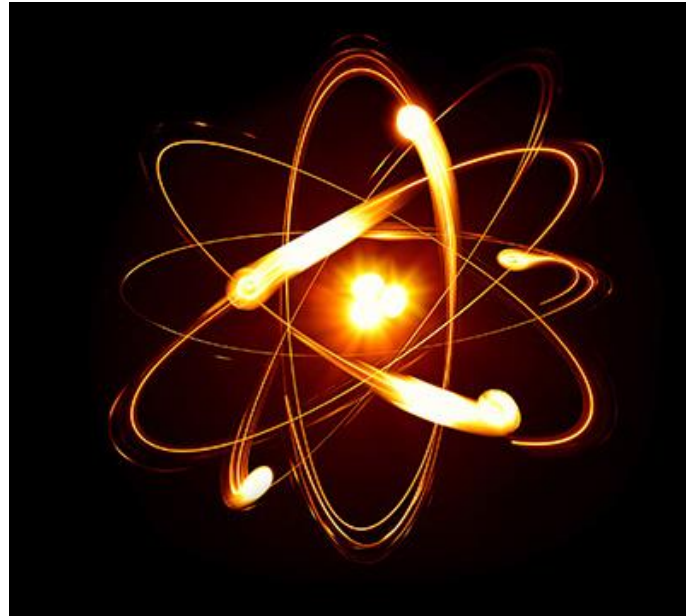
Which mental picture do you make of an atom?

Classic picture of the atom



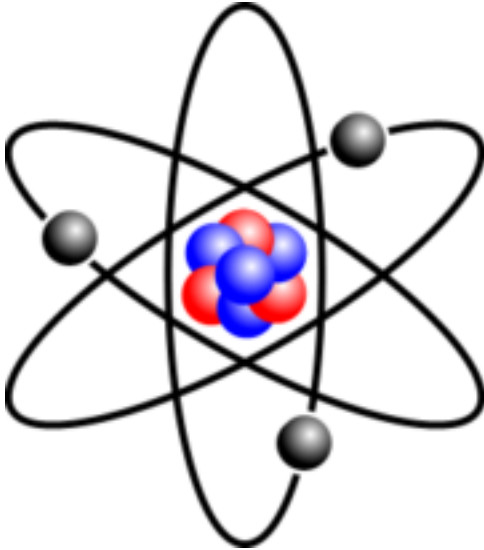
- Does something “move”?
- How empty is it?
- How does an electron “exist”?

➔ Consider QM interpretations



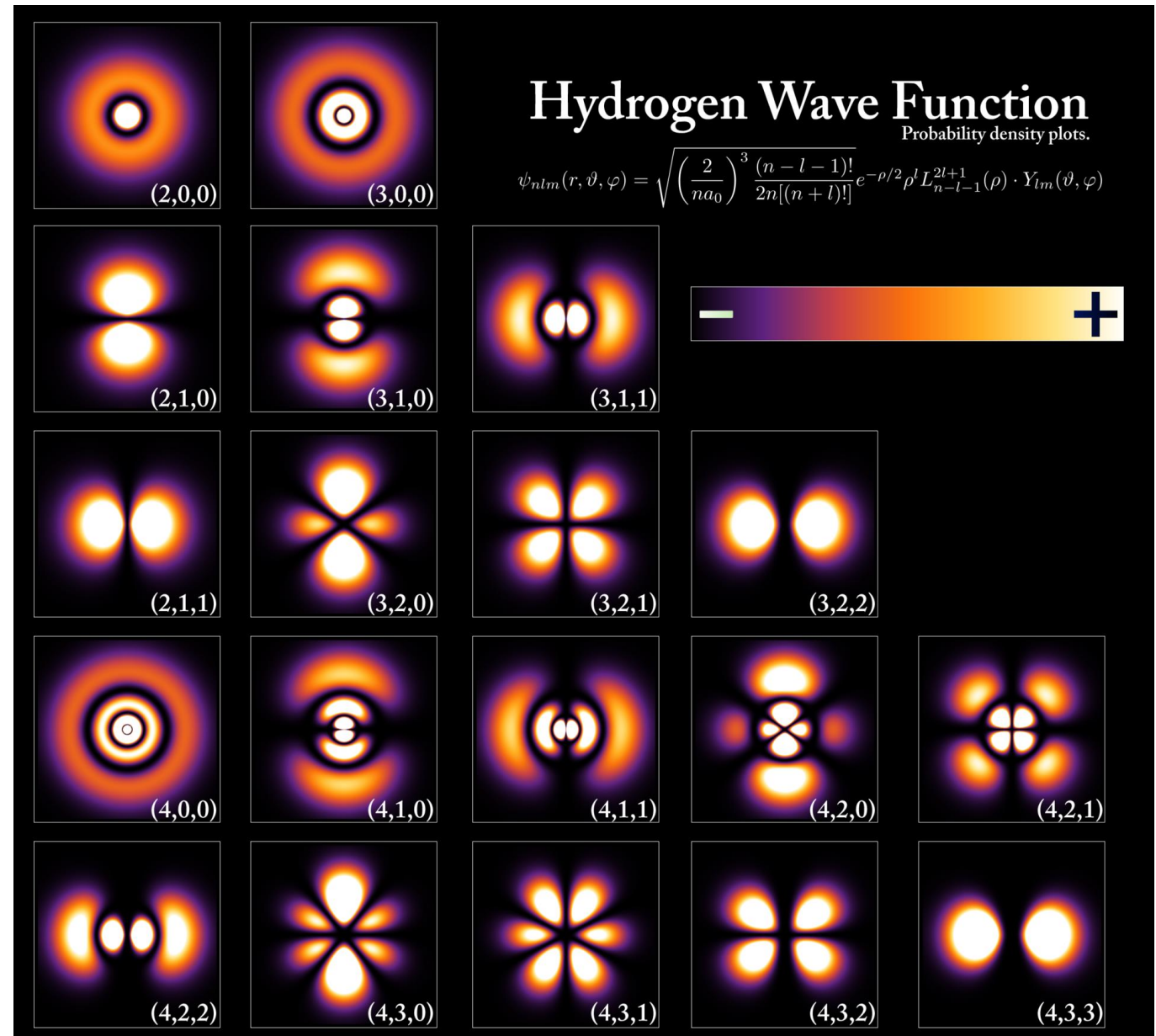
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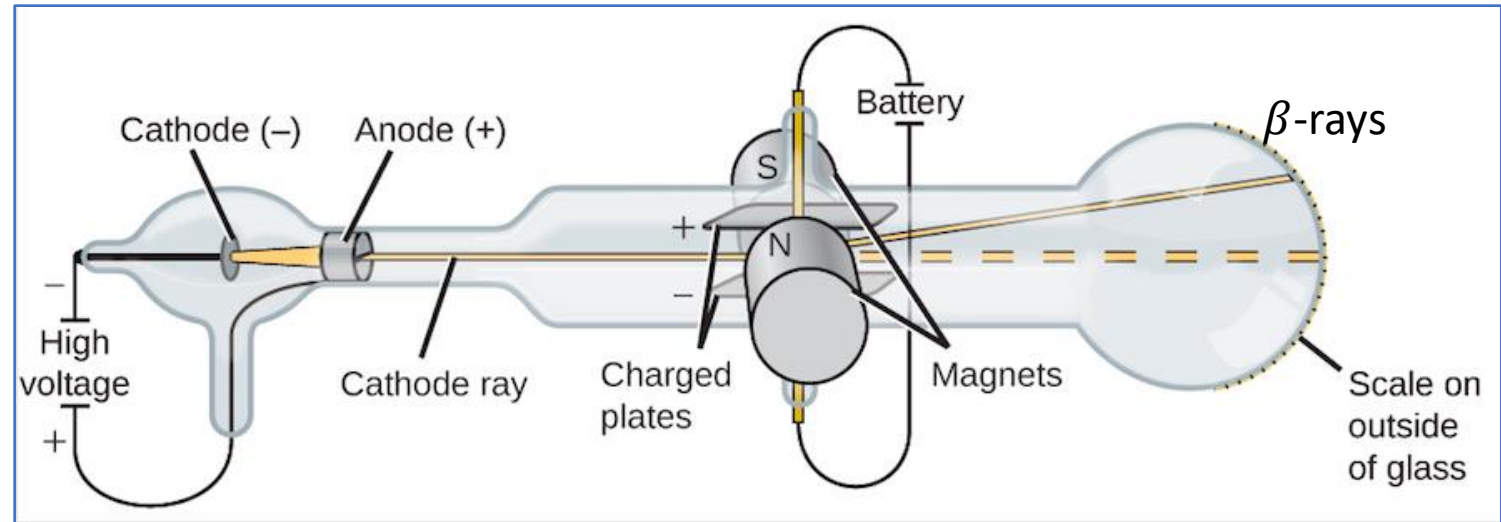
➔ Consider QM interpretations



Discovery of the Electron

Griffiths §1.1

- JJ Thomson (1897):
Cathode rays are bent by electric and magnetic field
 - Charged particles



1. Compensate Electric and Magnetic deflection:

$$F_E = F_B$$

$$qE = qvB ; v = E/B$$

2. Only B-field, cycle orbit:

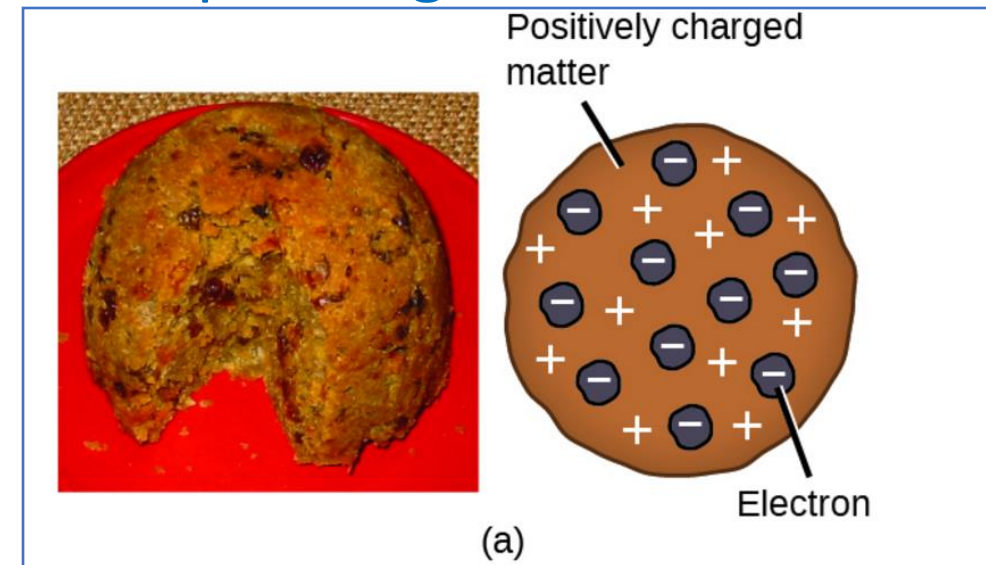
$$F_c = F_B$$

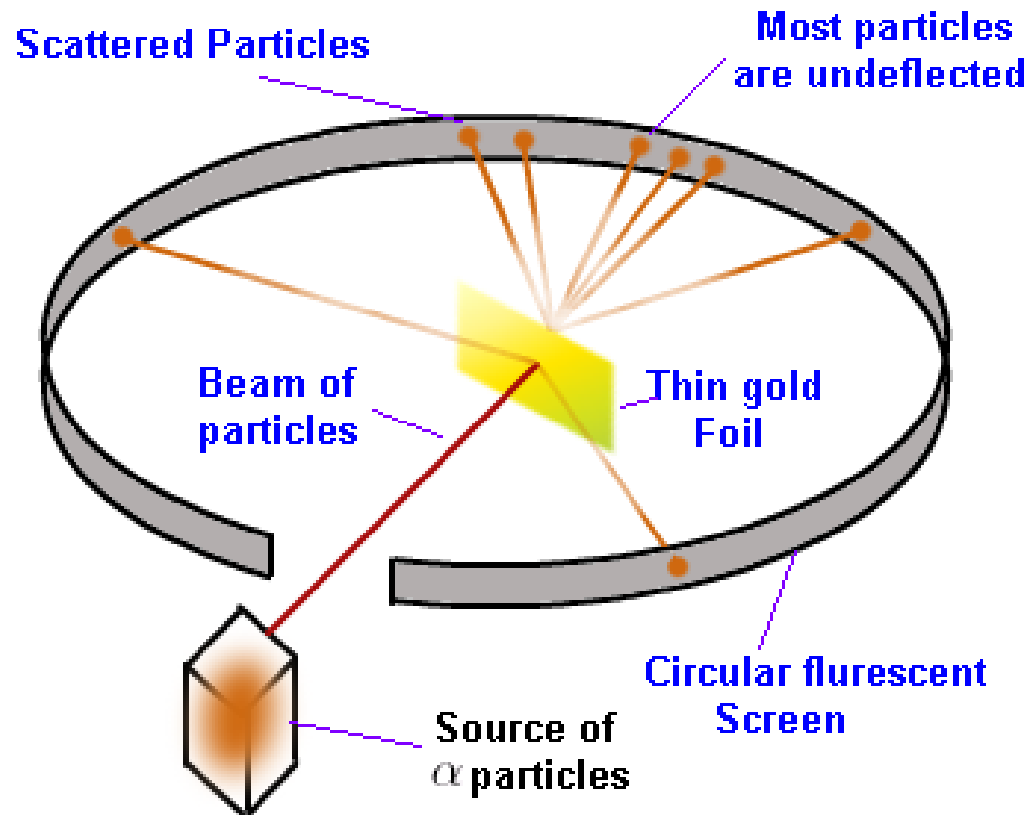
$$\frac{mv^2}{r} = qvB$$

$$\frac{q}{m} = \frac{E}{rB^2}$$

Electrons are much lighter than ions of same charge!

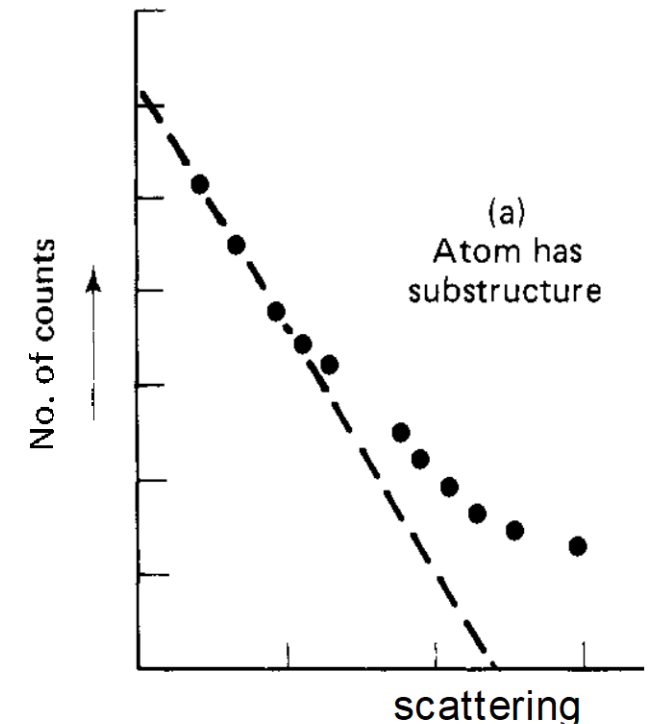
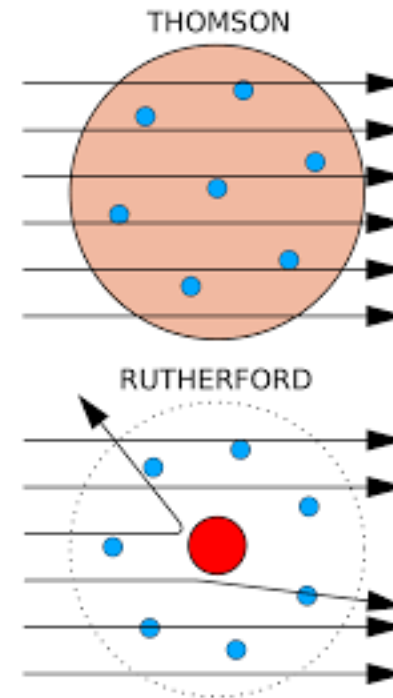
- Plum-pudding model of atoms:





- Atom is basically just vacuum:
 - Atom size: $\sim 10^{-10} m$
 - Nucleus size: $\sim 10^{-15} m$

- Rutherford (1911)
 - α -particles at gold target
 - Most particles pass undisturbed, few have a "hard" collision
 - Atom has substructure: small heavy nucleus



Example Rutherford scattering

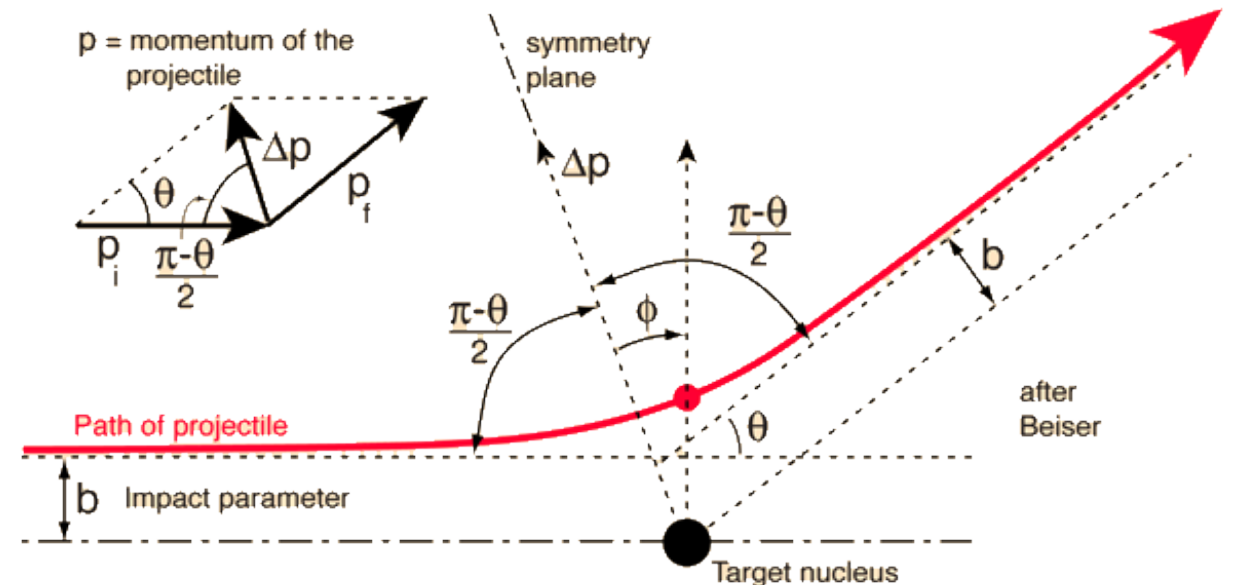
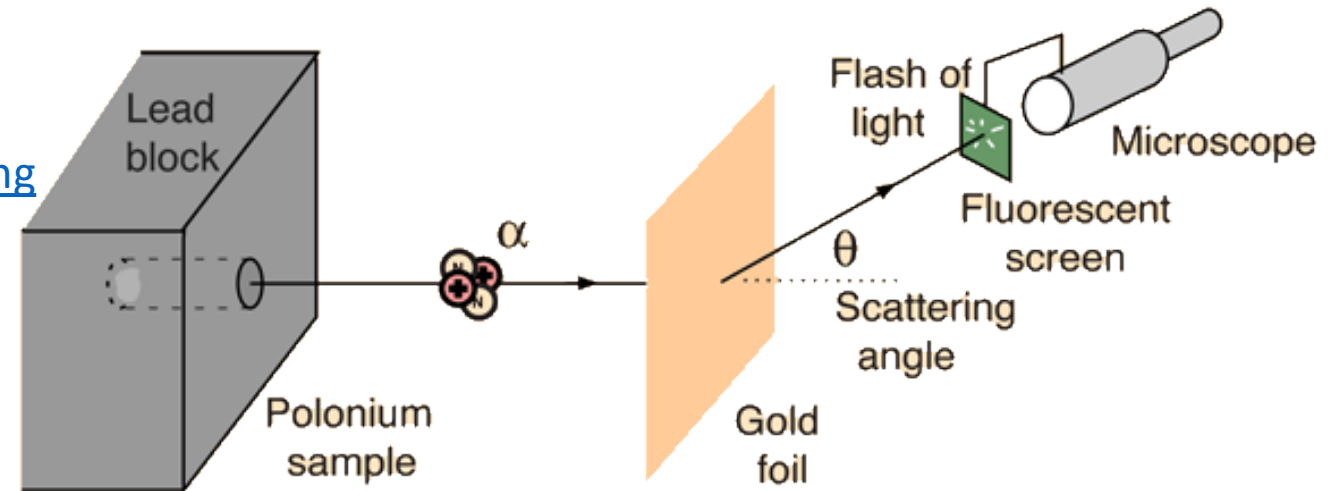
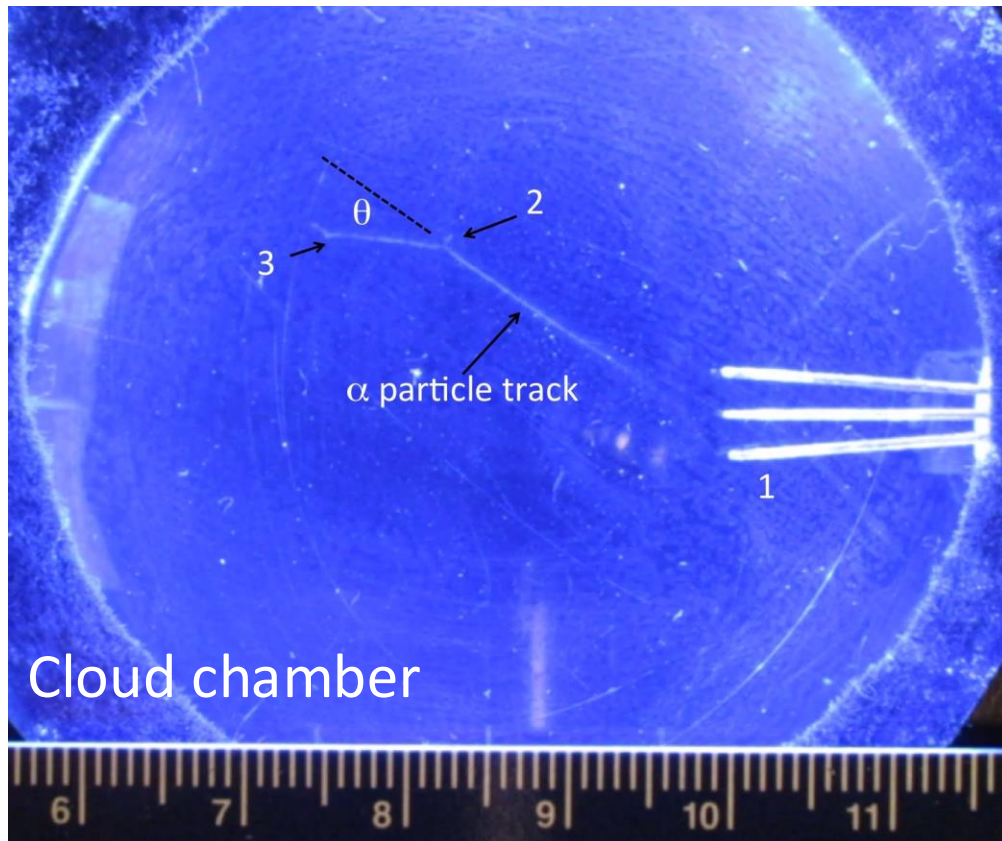
Griffiths §1.1

- Rutherford scattering on nucleus

- Formula derivation:

- https://en.wikipedia.org/wiki/Rutherford_scattering

- → not required



Example Rutherford scattering

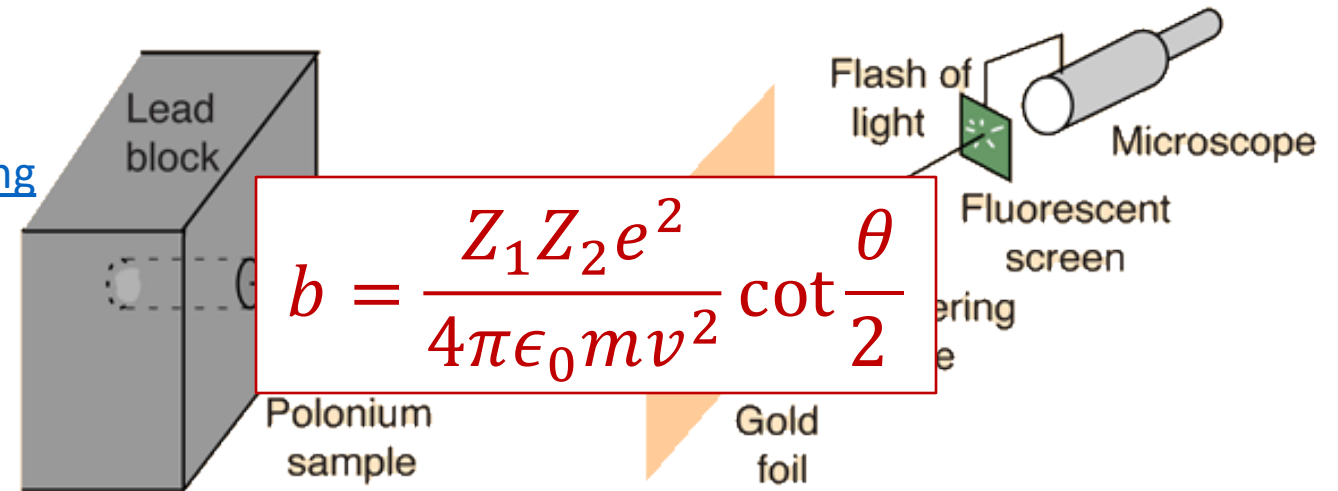
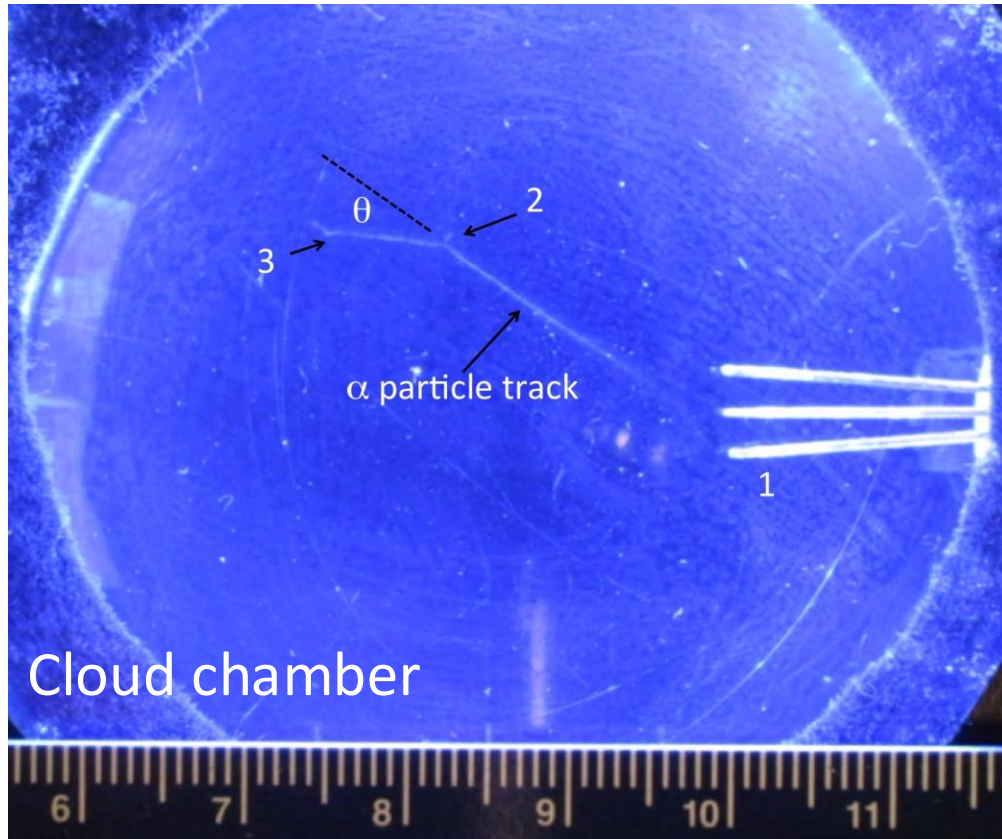
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- Rutherford scattering on nucleus

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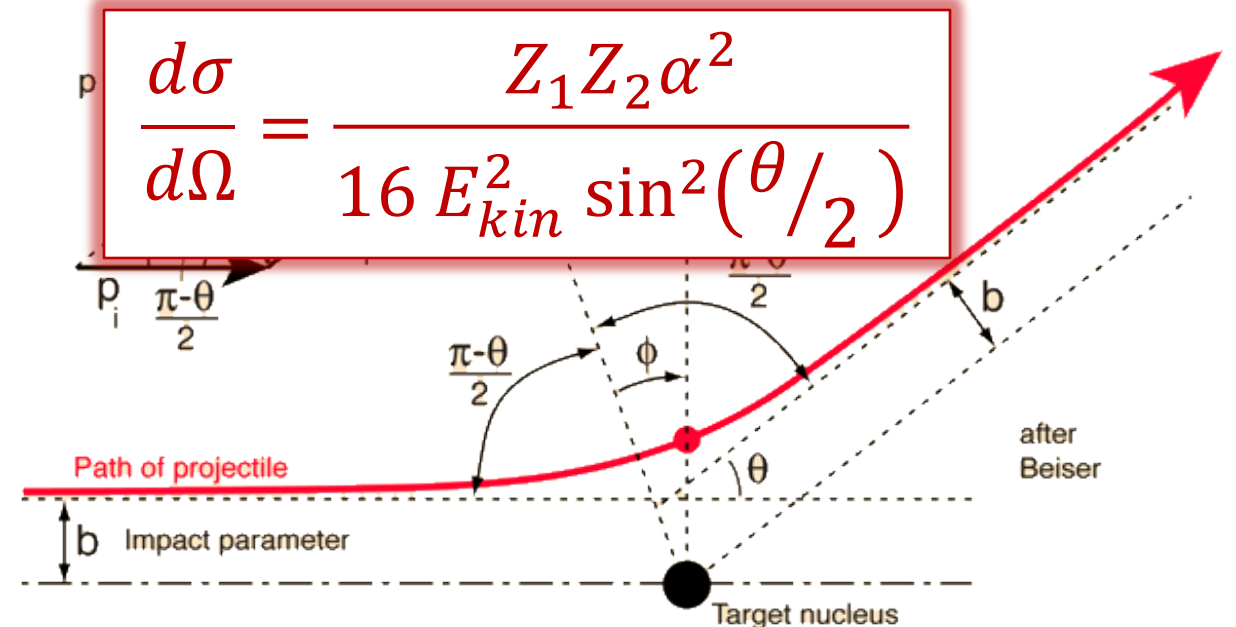
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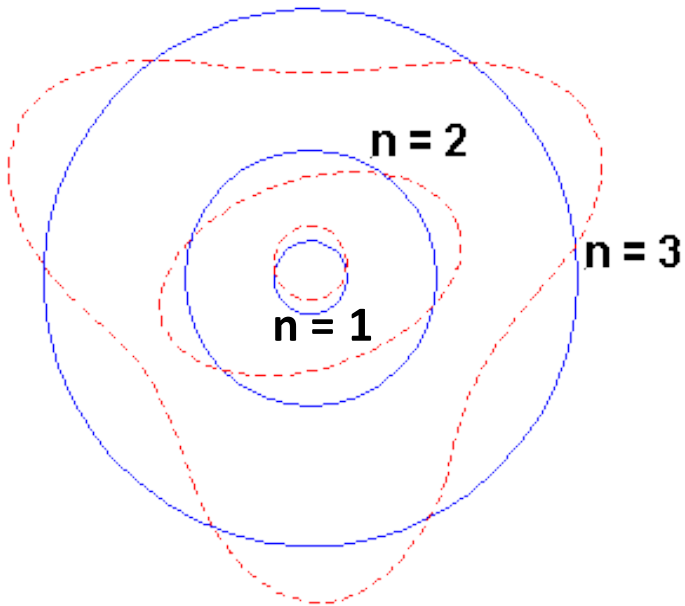


$$b = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 m v^2} \cot \frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{Z_1 Z_2 \alpha^2}{16 E_{kin}^2 \sin^2(\theta/2)}$$

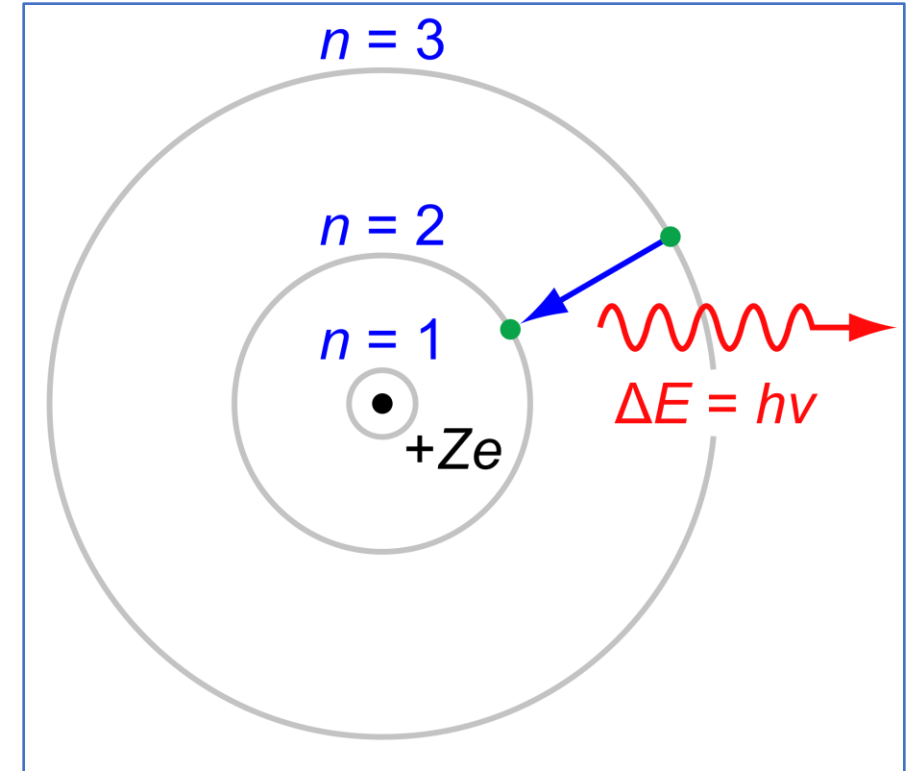


- Niels Bohr, 1914:
 - Calculate atomic energy levels using semi classical method

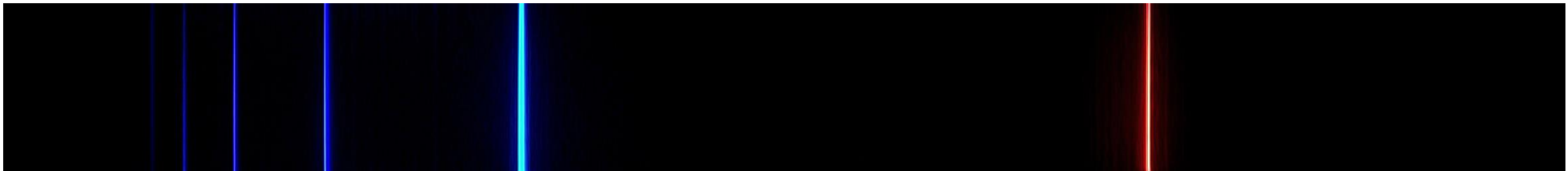


de Broglie: $\lambda = h / p$

$$\begin{aligned} L &= r p \\ L &= r h / \lambda \\ L &= r n h / (2 \pi r) \\ L &= n h / (2 \pi) = n \hbar \end{aligned}$$



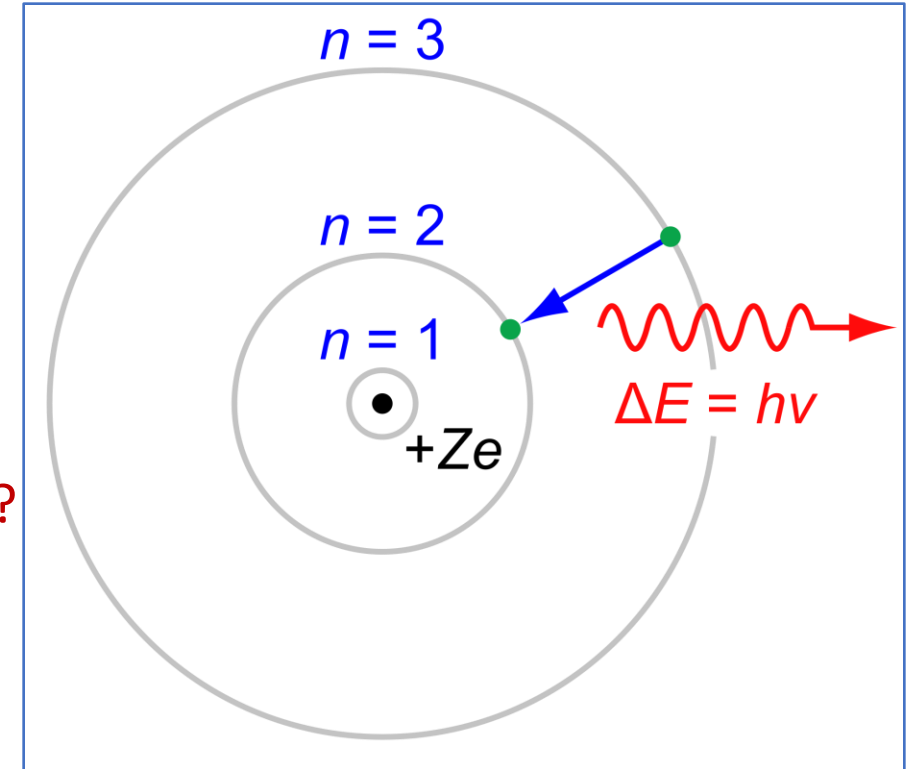
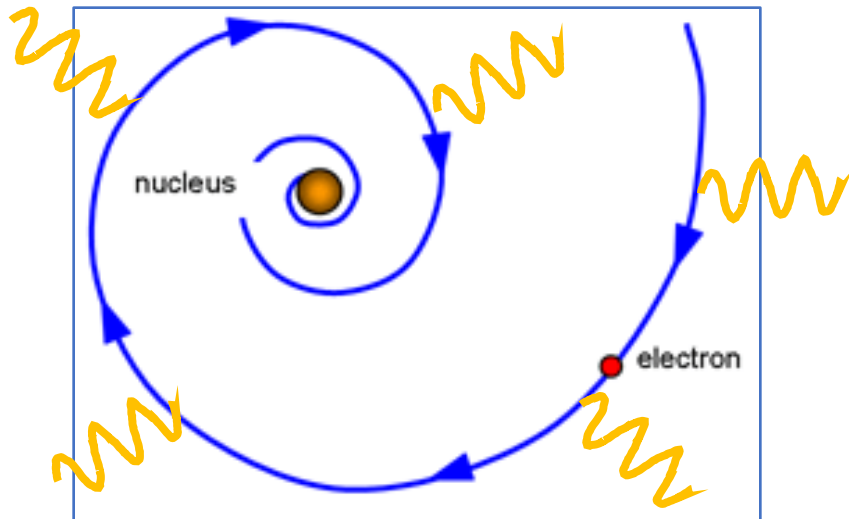
- Balmer spectrum of visible wavelengths in hydrogen:



- Niels Bohr, 1914:
 - Calculate atomic energy levels using semi classical method

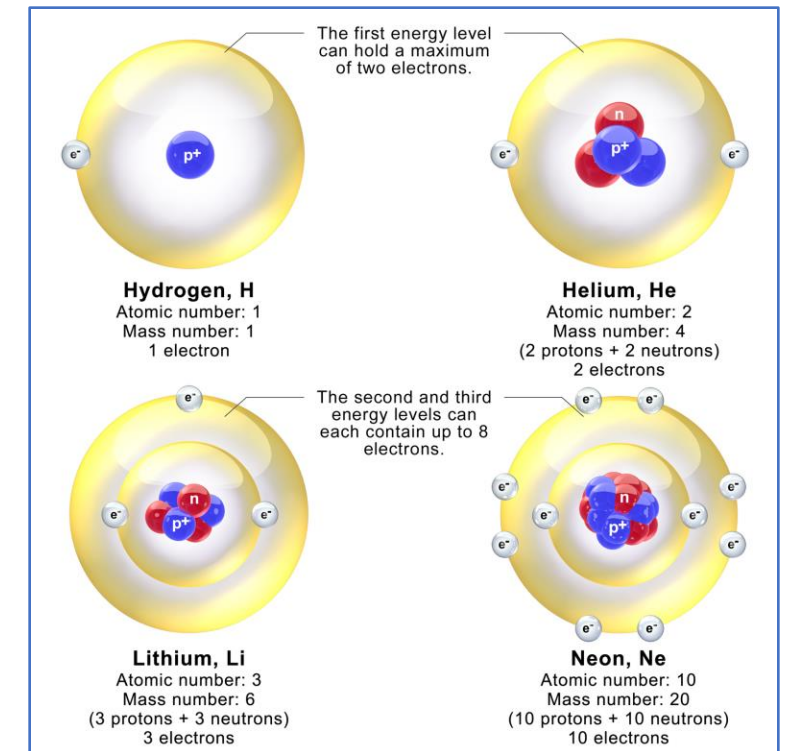
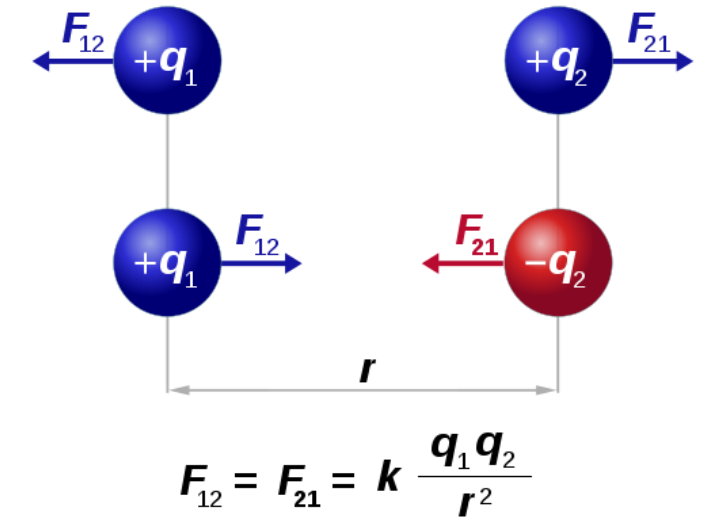
Exercise-2 “Pub Quizz”:

- 1: How come an atom is stable?
- 2: What hits each other when you clap your hands?
- 3: Why does a hydrogen electron not interact with the proton?



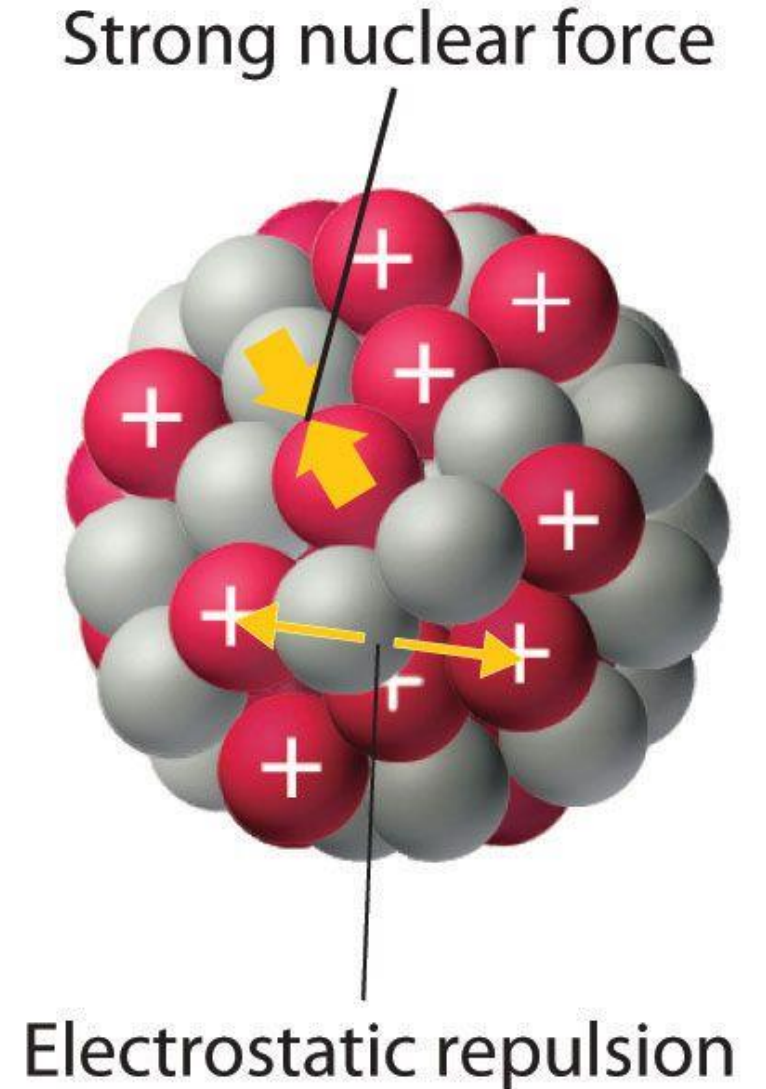
Nuclear Physics

- Nuclei include protons, p
 - Masses of nuclei studied in chemistry (Avogadro)
 - Hydrogen = 1 proton: $M_H = 1 \text{ GeV}/c^2 = 200 M_e$
- Study q/m of nuclei:
 - Masses of nuclei do not scale with charge, but with $A \approx 2 \times Z$
 - A = mass number, Z = atomic number = nucleus charge
 - What keeps heavy nuclei together?
- Chadwick 1932: Discovery of neutron, n
 - Neutron has almost same mass as proton
 - Neutrons act as a glue to nucleons in the nucleus
 - Later we will see it is due to the gluon color force
- Notation: ${}_Z^A\text{MyAtom}$, with $Z = \#p$, $A = \#(p + n)$
 - Isotopes: same $\#p$, different $\#n$



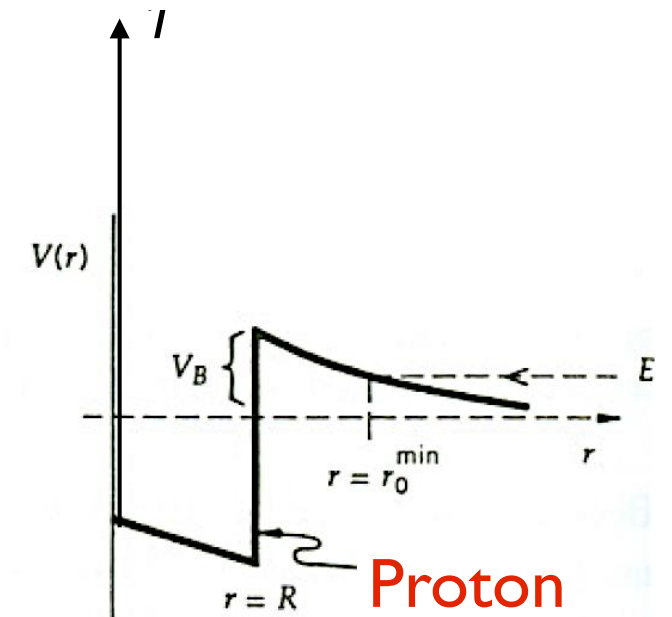
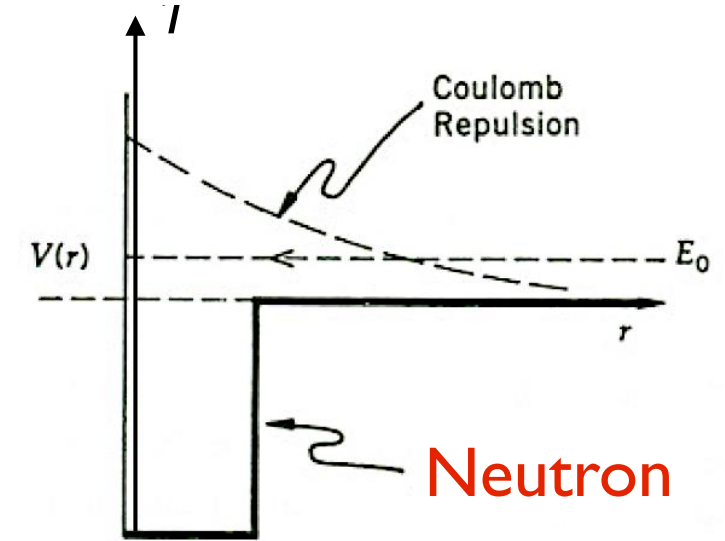
Electric Force vs Nuclear Force

- Electric Force: inverse square law
 - Generated by exchange of massless photon quanta
- Nuclear Force
 - must be (much) stronger than electric force in the nucleus
 - must be (much) weaker than electric force outside the nucleus



Electric Force vs Nuclear Force

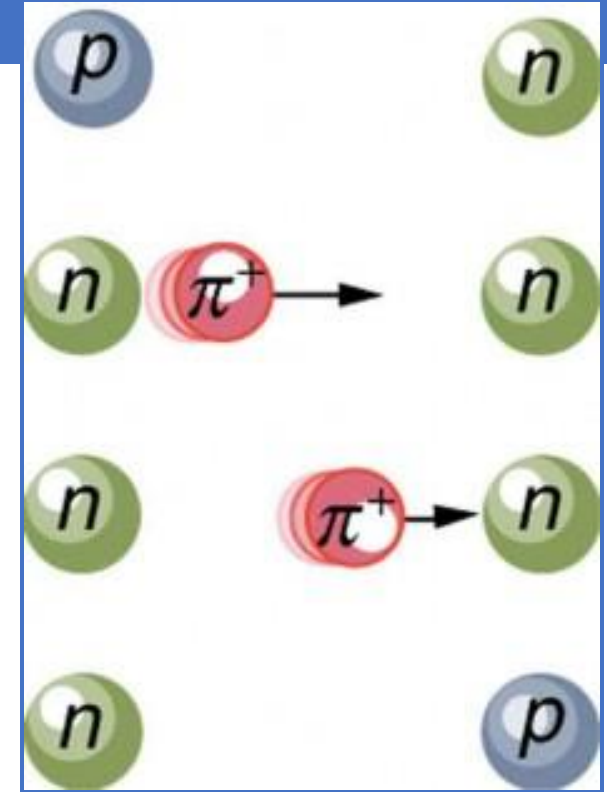
- Electric Force: inverse square law
 - Generated by exchange of massless photon quanta
- Nuclear Force
 - must be (much) stronger than electric force in the nucleus
 - must be (much) weaker than electric force outside the nucleus
- Pauli exclusion principle: Two fermions (protons, neutrons) cannot be at the same position
 - The potential must have a very strong repulsive core at small r
- Potential square well model
 - For protons there is a “Coulomb barrier” V_B
 - Beyond the range R it is negligible
 - With Rutherford scattering the range can be determined
 - $R \propto A^{1/3}$
- Nuclear force generated by exchange of massive pion



Yukawa's pion

Griffiths: problem 1.2, PP1 §i.2

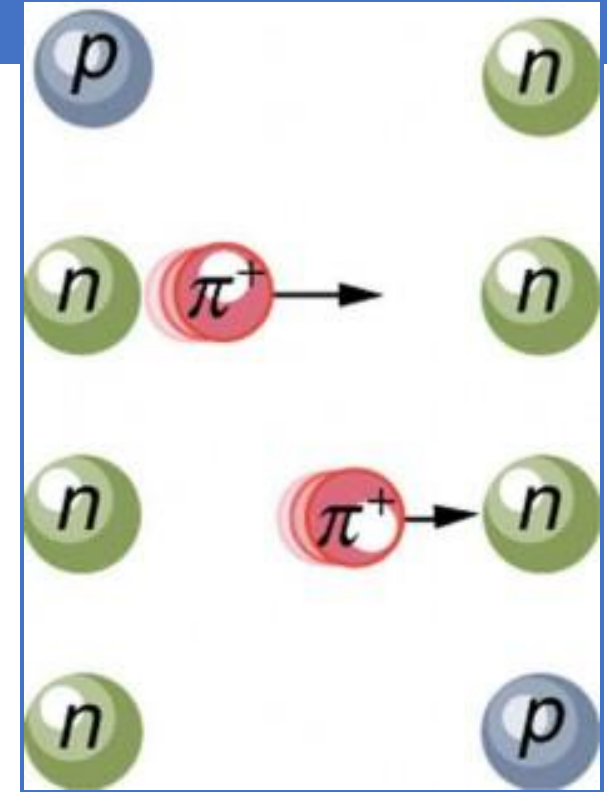
- Yukawa (1934) pi-meson gives attractive force between nucleons
- Heisenberg operator uncertainty in QM: $\Delta E \Delta t \geq \frac{\hbar}{2}$
- Translated to popular language:
 - You can violate energy conservation as long as:
 - time is shorter than: $\Delta t \leq \frac{\hbar}{2\Delta E}$ or energy is less than $\Delta E \leq \frac{\hbar}{2\Delta t}$



Yukawa's pion

Griffiths: problem 1.2, PP1 §i.2

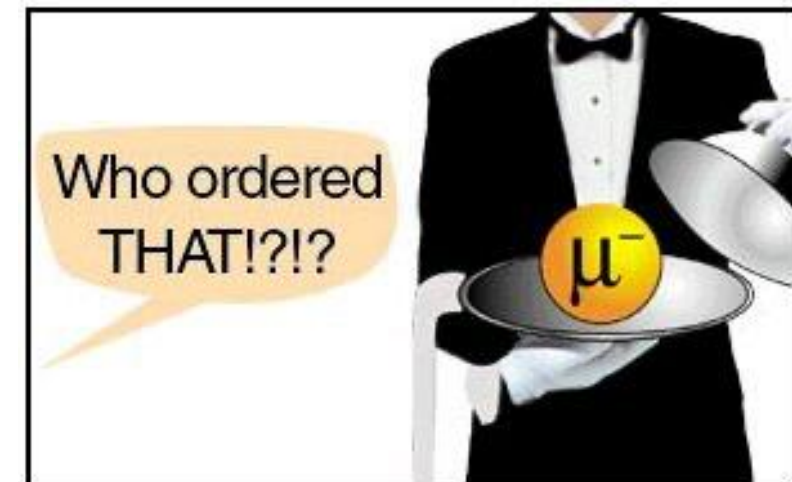
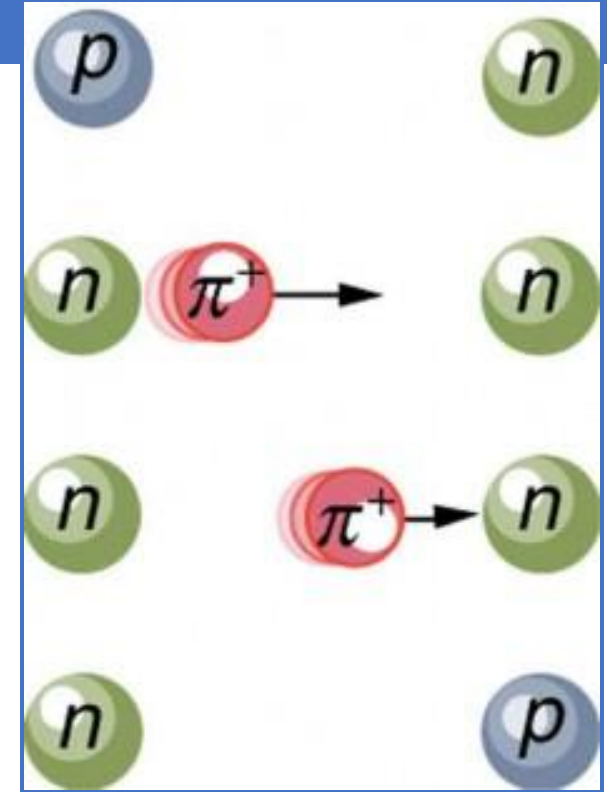
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 - You can violate energy conservation as long as:
 - time is shorter than: $\Delta t \leq \frac{\hbar}{2\Delta E}$ or energy is less than $\Delta E \leq \frac{\hbar}{2\Delta t}$
- Electromagnetic force is transmitted by photon quanta
 - $E = hf = \frac{hc}{\lambda}$ such that: $\Delta t \sim \frac{\lambda}{c}$ and the range $R \sim c\Delta t = \lambda$ is infinite
- Nuclear (“strong”) force is transmitted by massive pions:
 - $E = mc^2$ such that $\Delta t \sim \frac{h}{2mc^2}$ and $R \sim c\Delta t = \frac{h}{2mc}$
 - The heavier the particle, the more difficult to “violate energy conservation”, the shorter the range



Yukawa's pion

Griffiths: problem 1.2, PP1 §i.2

- Yukawa (1934) pi-meson gives attractive force between nucleons
- Heisenberg operator uncertainty in QM: $\Delta E \Delta t \geq \frac{\hbar}{2}$
- Translated to popular language:
 - You can violate energy conservation as long as:
 - time is shorter than: $\Delta t \leq \frac{\hbar}{2\Delta E}$ or energy is less than $\Delta E \leq \frac{\hbar}{2\Delta t}$
- Electromagnetic force is transmitted by photon quanta
 - $E = hf = \frac{hc}{\lambda}$ such that: $\Delta t \sim \frac{\lambda}{c}$ and the range $R \sim c\Delta t = \lambda$ is infinite
- Nuclear (“strong”) force is transmitted by massive pions:
 - $E = mc^2$ such that $\Delta t \sim \frac{h}{2mc^2}$ and $R \sim c\Delta t = \frac{h}{2mc}$
 - The heavier the particle, the more difficult to “violate energy conservation”, the shorter the range
- In 1937 a particle with the correct mass was found in cosmic rays, but it did not interact with nuclei!
 - “Who ordered that?” (Isidor Rabi). It was the muon.
- In 1947 the pion was discovered



The nucleon-nucleon potential

- Woods Saxon Potential, 1954

- $$V(r) = -\frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)}$$
- Nuclear Radius: $R = 1.25 \text{ fm} \times A^{1/3}$
 A is mass number, $V_0 \approx 50 \text{ MeV}$, $a \approx 0.5 \text{ fm}$

- Many models in 1970 – 1990

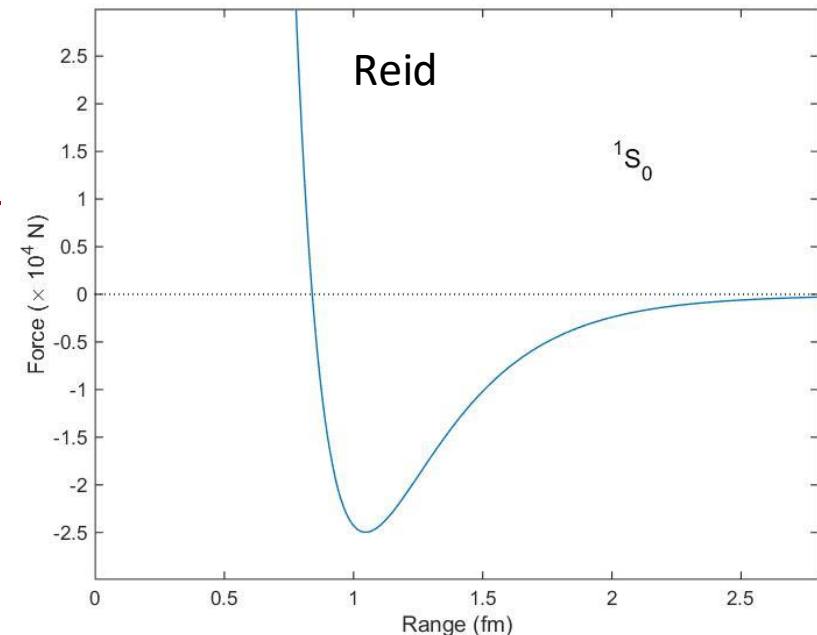
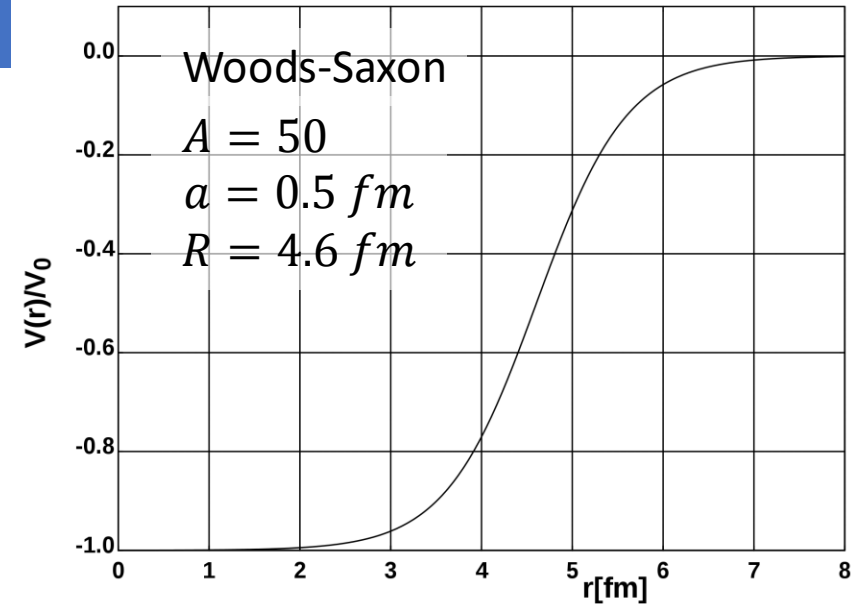
- Reid Potential, 1968, (semi-empirical) implements repulsive core:

- $$V_{Reid}(R) = -10.463 \frac{e^{-\mu r}}{\mu r} - 1650.6 \frac{e^{-4\mu r}}{\mu r} + 6482.2 \frac{e^{-7\mu r}}{\mu r}$$

in MeV with $\mu = 0.7 \text{ fm}^{-1}$

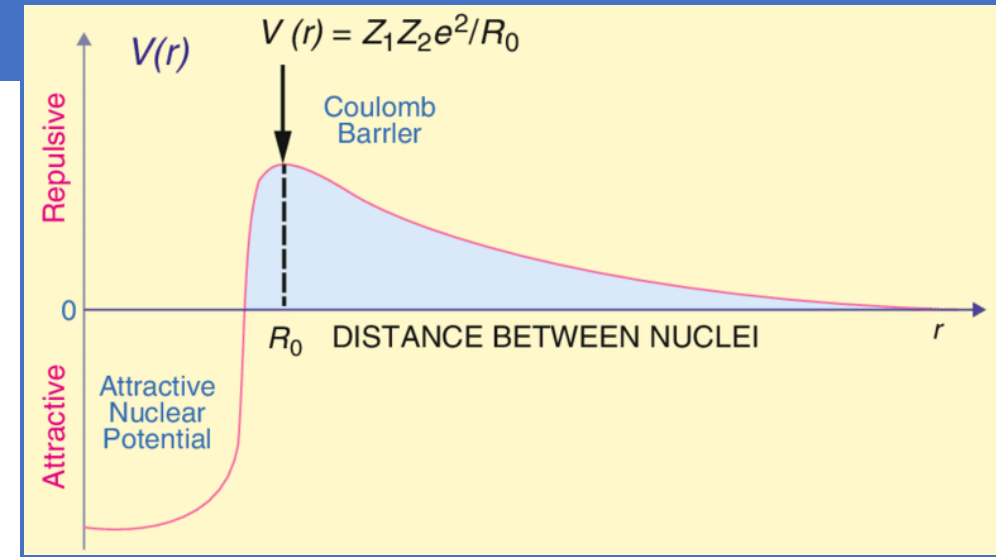
- Famous nucleon-nucleon potential is the Nijmegen potential:

- De Swart, Rijken et al., 1980 - 1990
- <https://arxiv.org/pdf/nucl-th/9509024.pdf>

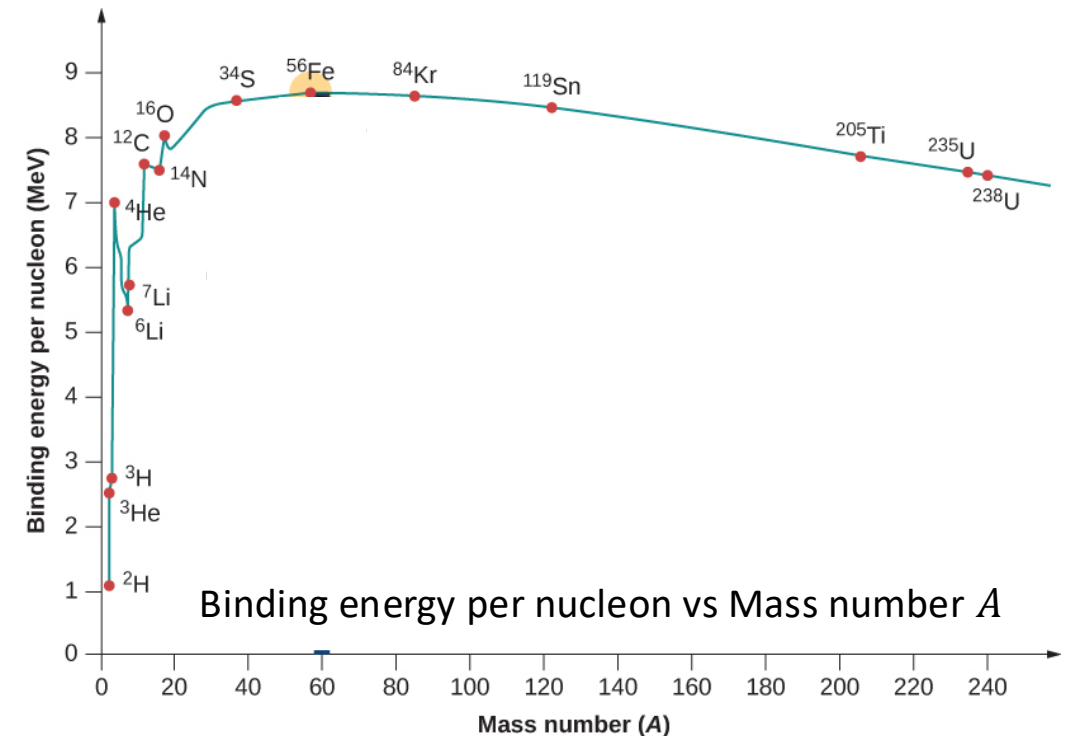


Binding Energy

- The energy of the state inside the potential well is lower than the energy of free particles
- If you “build” a nucleus from free particles energy becomes available: “binding energy”
- This appears by a reduction of mass: the nucleus is lighter than the sum of its components.



- Binding energy: $E = \Delta m c^2$
- $\Delta m = m_{nucleus} - (N_p M_p + N_n M_n)$
 $= m_{nucleus} - (Z M_p + (A - Z) M_n) < 0$
 - This holds for all bound systems
 - Hydrogen: $m_p + m_e = m_H + 13.6 \text{ eV}/c^2$
 - Deuterium: $m_p + m_n = M_D + 1.7 \text{ MeV}/c^2$
- ^{56}Fe has the highest binding energy

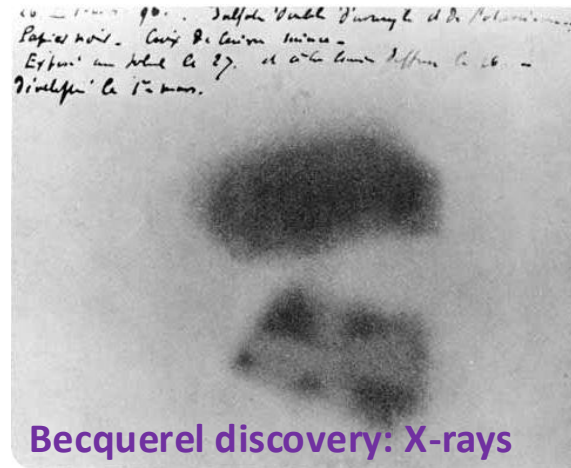


Nuclear decay (Radioactivity)

- Radioactivity (1896)

Henri Becquerel, Pierre and
Marie Curie:

- Unstable nucleus
- Quantum tunneling process



- Rutherford classification into: alpha, beta, gamma decays
 - alpha and beta decay changes chemical element (Z)

- In nature things decay exponentially:

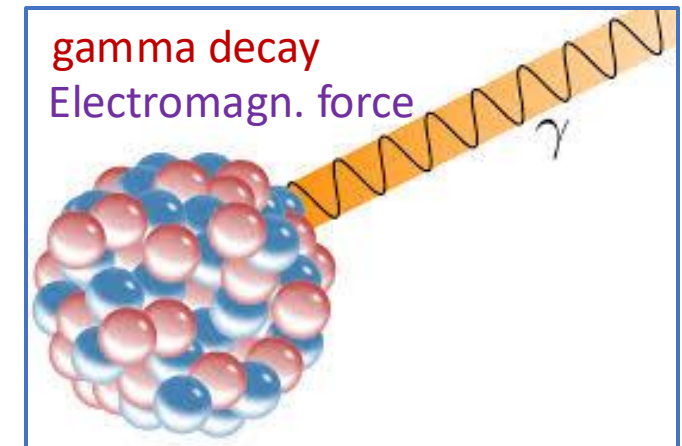
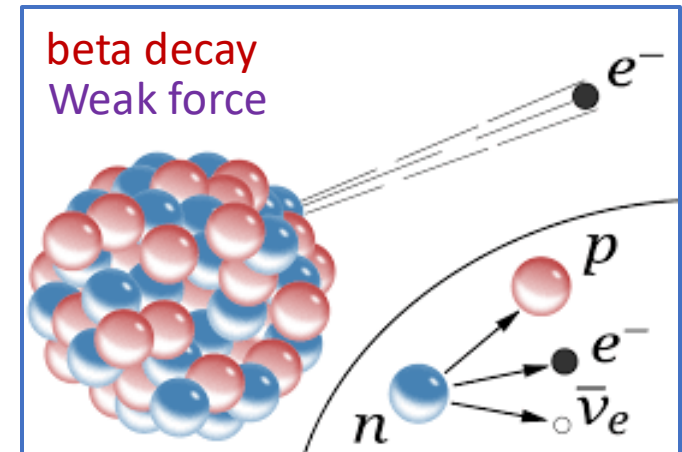
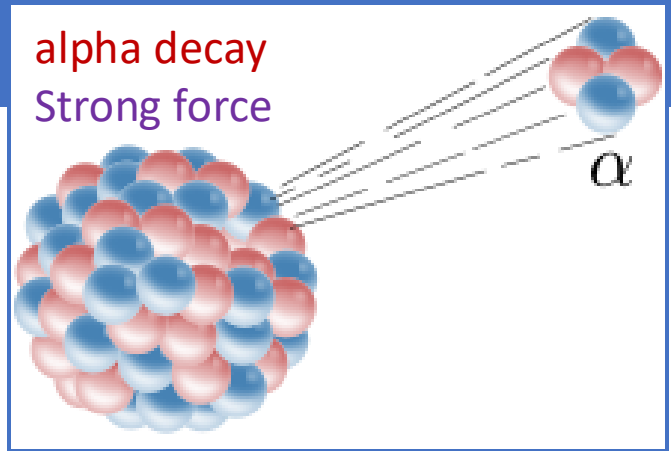
- Equal probability per unit time:

$$-dN = \lambda N dt$$

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}$$

- Halflife:

$$N_{1/2} = N_0/2 \rightarrow e^{-t_{1/2}/\tau} = \frac{1}{2} \rightarrow -t_{1/2}/\tau = \ln \frac{1}{2} \rightarrow t_{1/2} = \tau \ln 2$$



Nuclear decay (Radioactivity)

- Radioactivity (1896)

Henri Becquerel, Pierre and Marie Curie:

- Unstable nucleus
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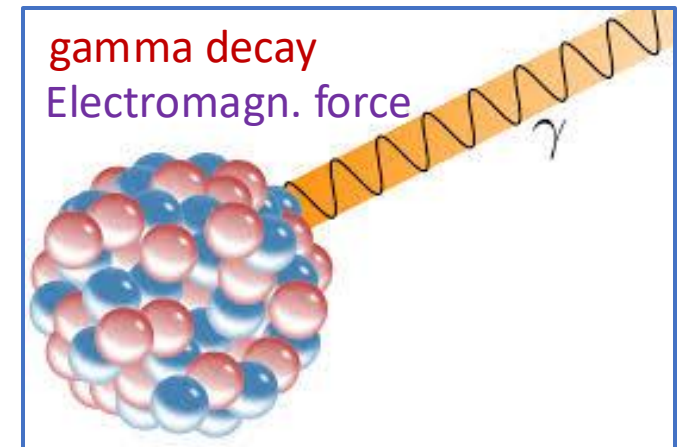
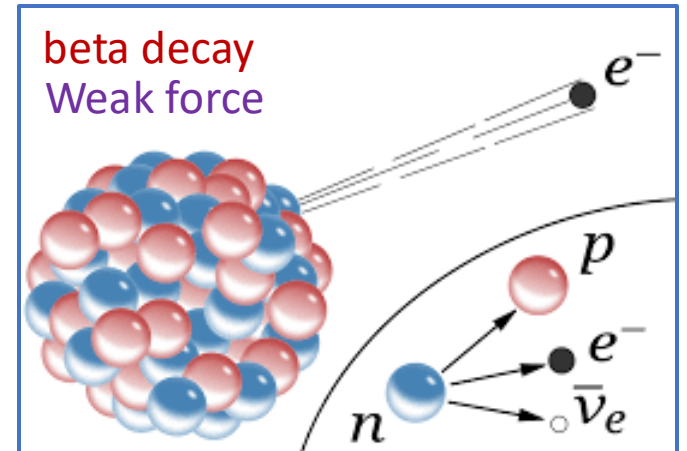
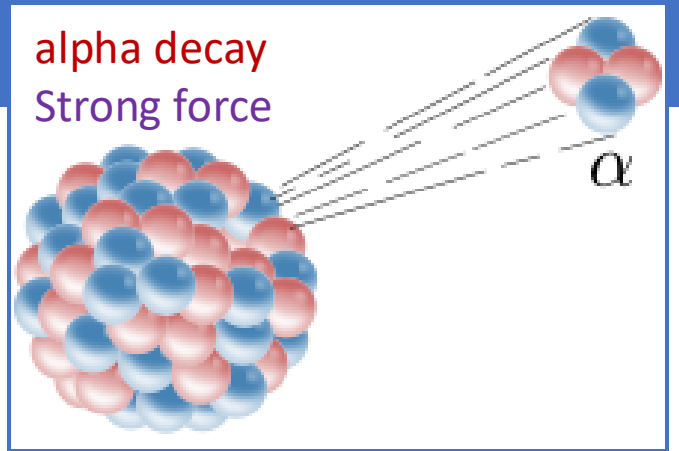
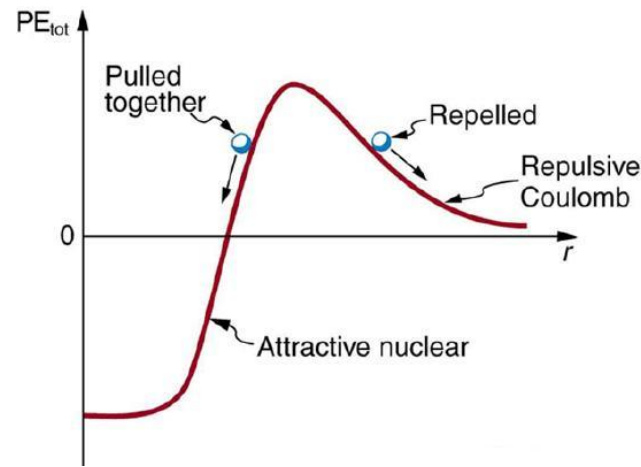
- Equal probability per unit time:

$$-dN = \lambda N dt$$

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}$$

- Halflife $t_{1/2}$ vs decay time τ :

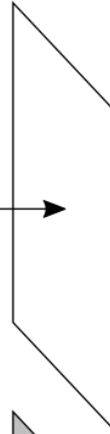
$$N_{1/2} = N_0/2 \rightarrow e^{-t_{1/2}/\tau} = \frac{1}{2} \rightarrow -t_{1/2}/\tau = \ln \frac{1}{2} \rightarrow t_{1/2} = \tau \ln 2$$



Penetrating power

- Alpha particles stopped by piece of paper
- Beta particles stopped by sheet of aluminium
- Gamma particles stopped by layer of lead

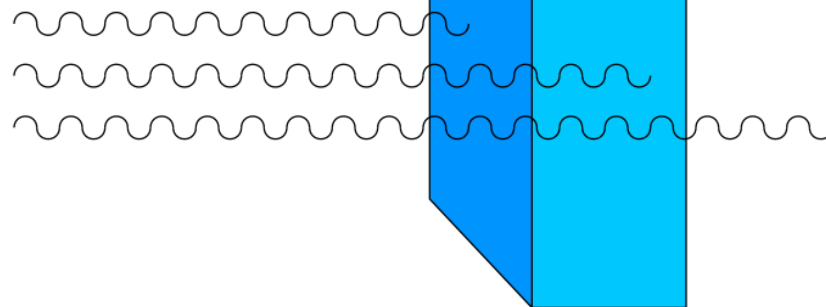
α



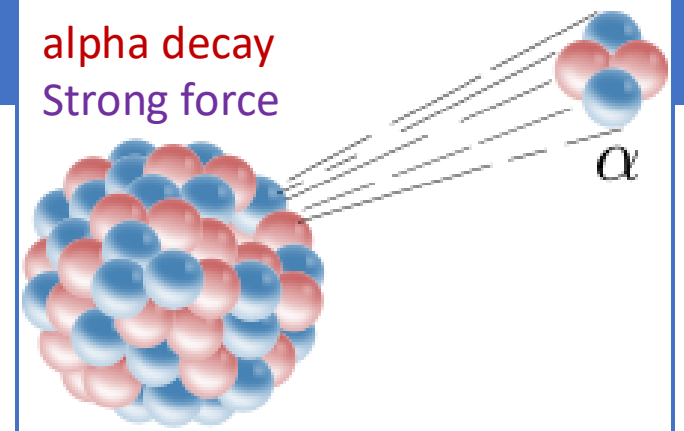
β



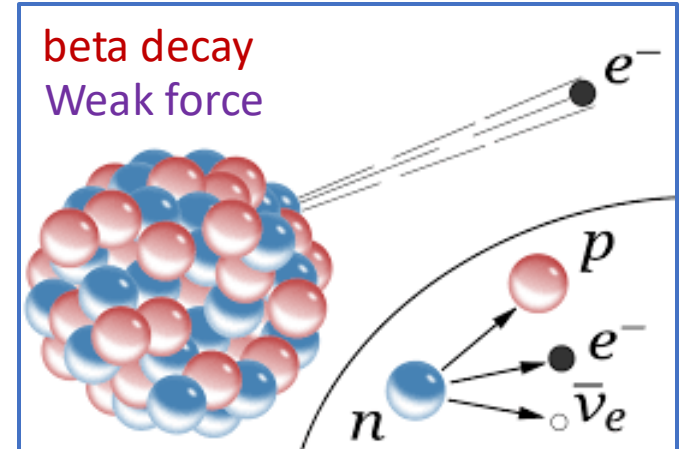
γ



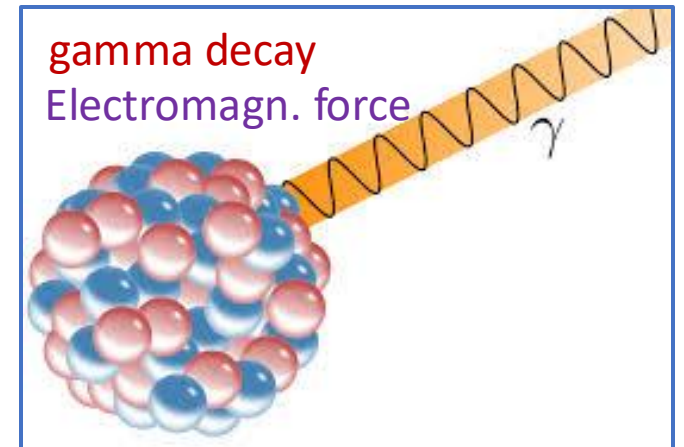
alpha decay
Strong force



beta decay
Weak force

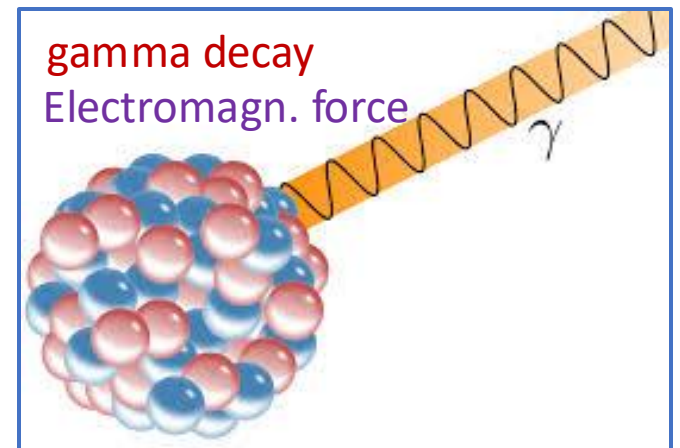
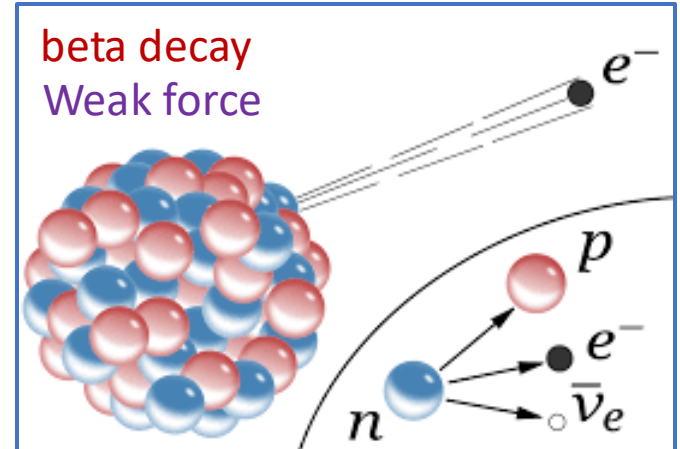
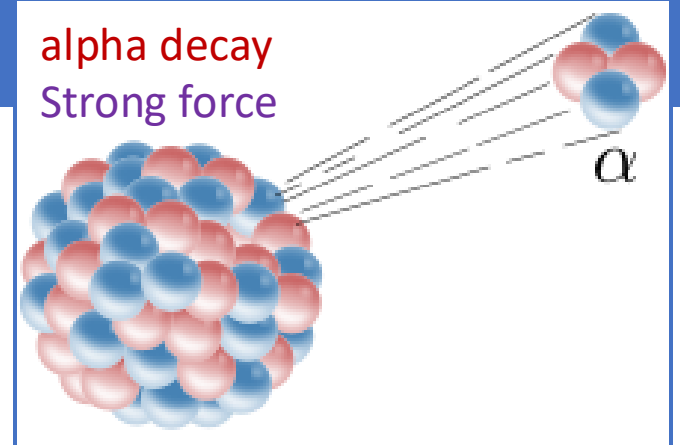
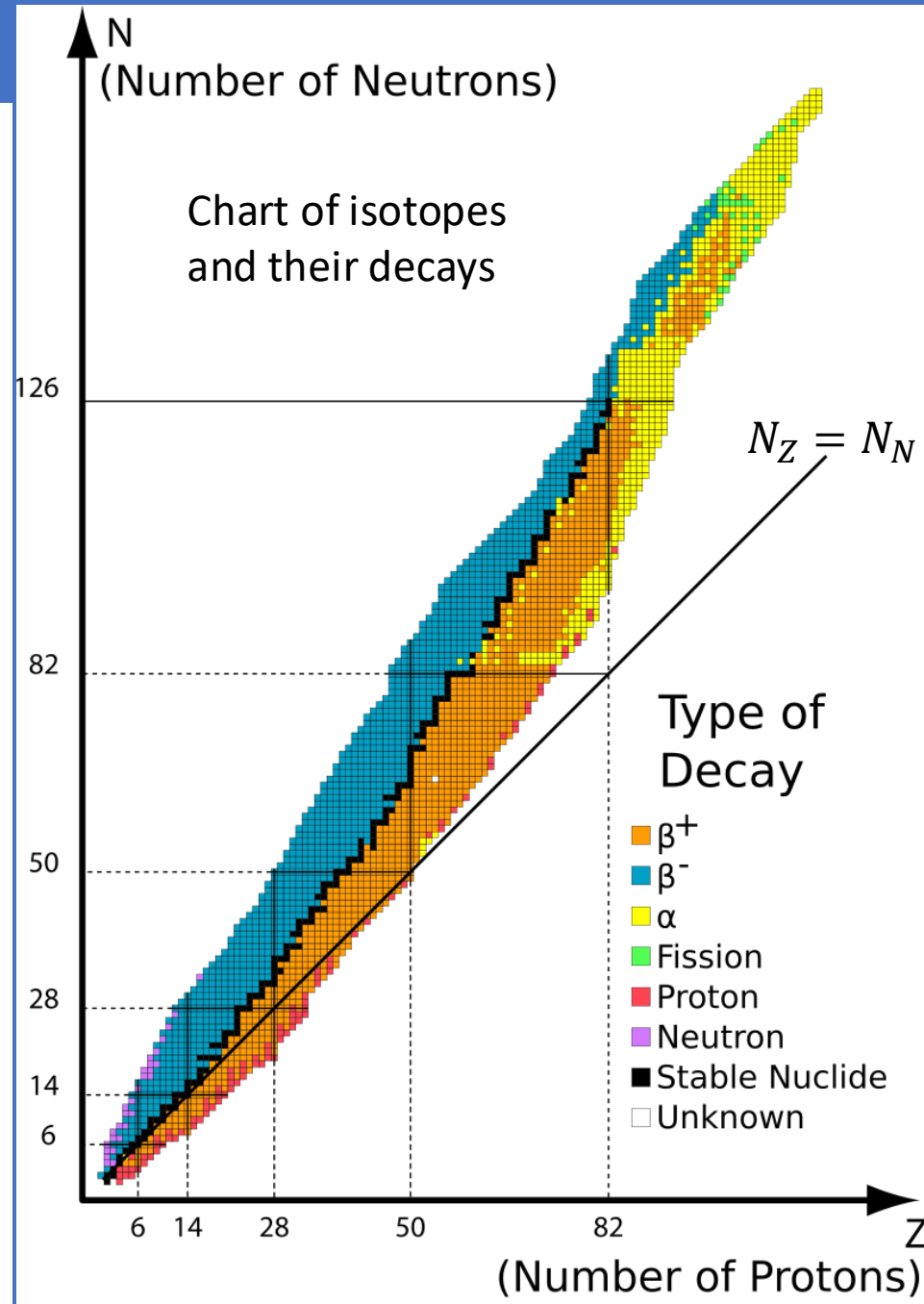


gamma decay
Electromagn. force



Nuclear Decays

- Alpha particles stopped by piece of paper
- Beta particles stopped by sheet of aluminium
- Gamma particles stopped by layer of lead



Radioactive Elements

- Half-lives vary enormously

- At least one stable element exists
- Slightly radioactive $\tau_{1/2} \sim$ millions of years
- Radioactive: $\tau_{1/2} \sim$ thousands of years
- Highly radioactive: $\tau_{1/2} \sim$ minutes to weeks
- Extremely radioactive: $\tau_{1/2} \sim$ seconds

I

II

Extremely radioactive: $\tau_{1/2} \sim$ seconds

Period

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117

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Li

11

Na

19

K

37

Rb

55

Cs

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Fr

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Be

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Mg

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Ca

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Sr

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Ba

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Ra

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La

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Gd

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Er

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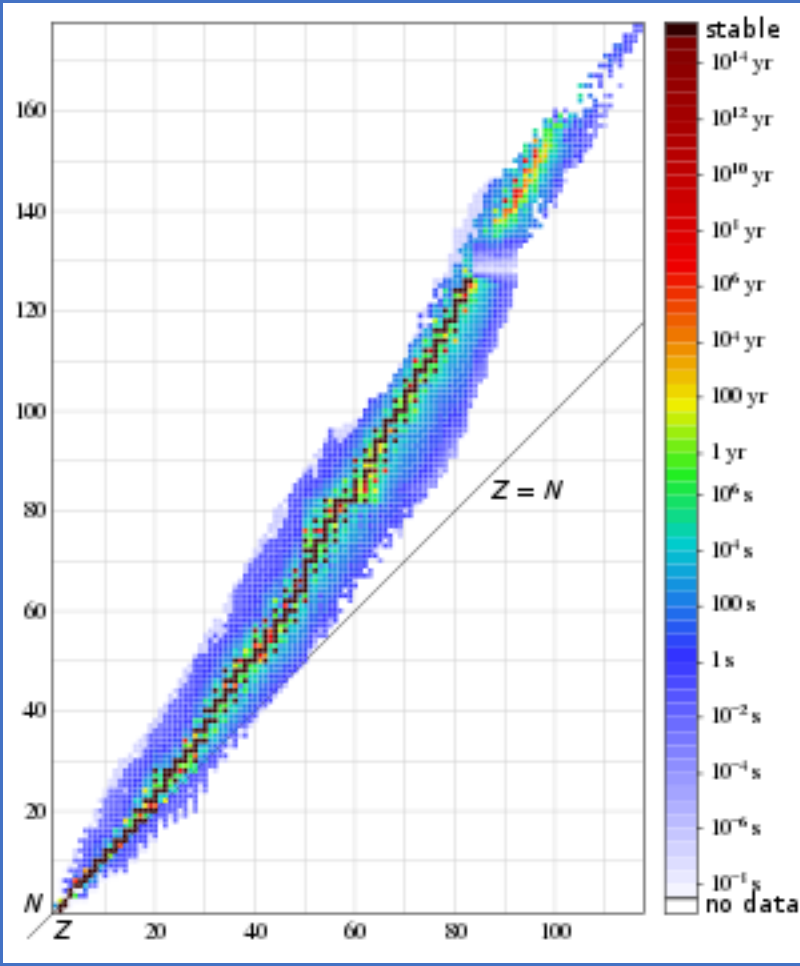
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Units in Radiation and Tissue

Inform yourself on doses related to Chernobyl and Fukushima.

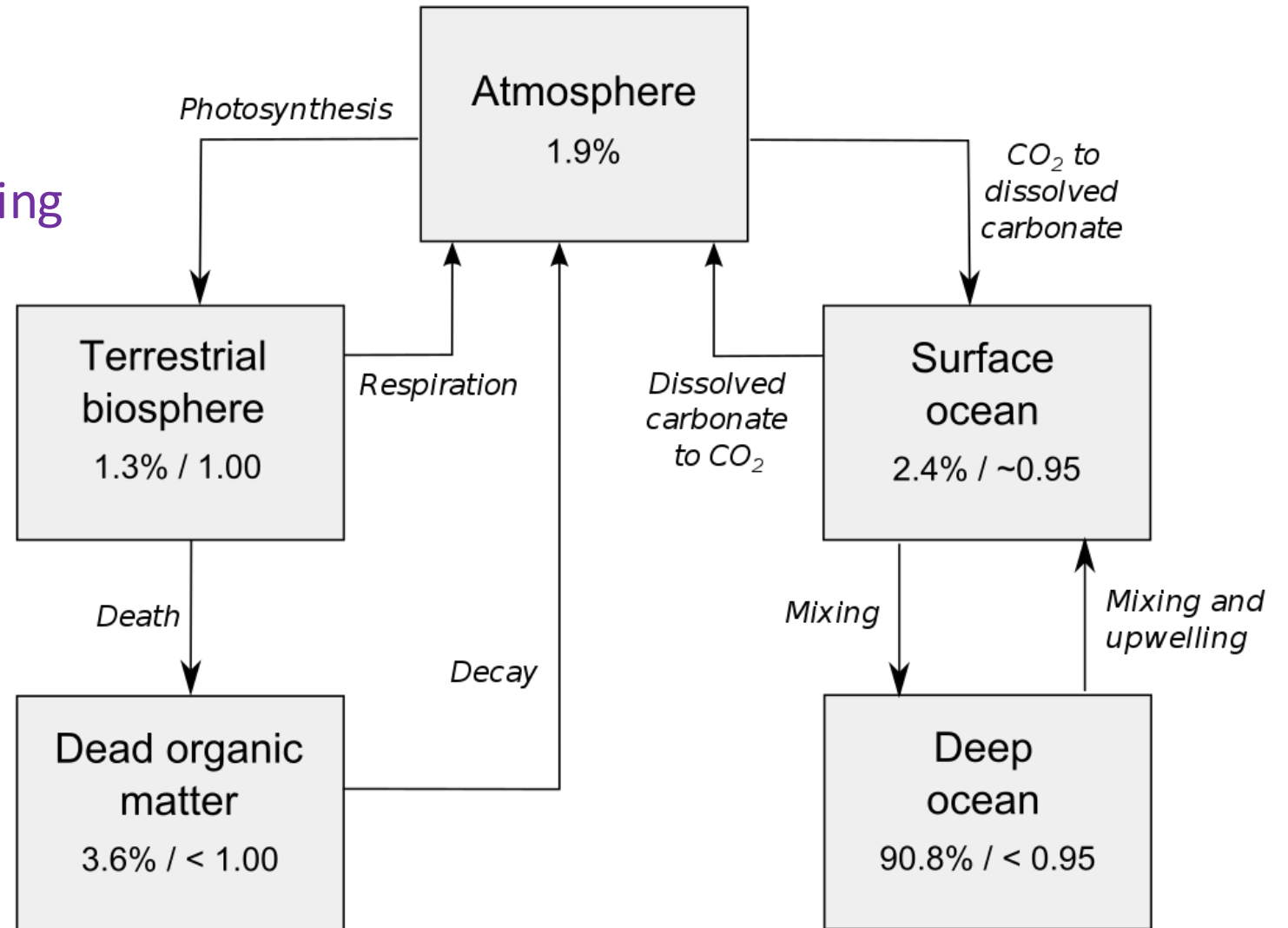
- Activity:
 - Number of disintegrations in a radioactive source is used to measure the intensity of the source
 - Main unit is Bequerel (Bq). 1 Bq is one event per second (typical kBq, MBq, GBq)
- Absorbed dose (exposure for X-rays):
 - The amount of energy coming from a source that is absorbed in a tissue. Used to assess the potential for biochemical changes in specific tissues.
 - Main unit is Gray (Gy). 1 Gy means that one kg of matter absorbed one joule of energy (typical mGy)
- Equivalent or biological dose:
 - Used to assess how much biological damage is expected from the absorbed dose. Different types of radiation have different damaging properties
 - Main unit is Sievert (Sv). 1 Sv equal 1 Gy multiplied by a weighting factor
- Effective dose:
 - Used to assess the potential for long-term effects that might occur in future. Takes into account all the organs in the body, the timespan of irradiation etc.

Units in Radiation and Tissue

Ionizing radiation related quantities view • talk • edit					
Quantity	Unit	Symbol	Derivation	Year	SI equivalence
Absorbed dose (<i>D</i>)	gray	Gy	$\text{J}\cdot\text{kg}^{-1}$	1974	SI unit
Absorbed dose (<i>D</i>)	erg per gram	erg/g	$\text{erg}\cdot\text{g}^{-1}$	1950	1.0×10^{-4} Gy
Absorbed dose (<i>D</i>)	rad	rad	$100 \text{ erg}\cdot\text{g}^{-1}$	1953	0.010 Gy
Activity (<i>A</i>)	becquerel	Bq	s^{-1}	1974	SI unit
Activity (<i>A</i>)	curie	Ci	$3.7 \times 10^{10} \text{ s}^{-1}$	1953	3.7×10^{10} Bq
Activity (<i>A</i>)	rutherford	Rd	10^6 s^{-1}	1946	1,000,000 Bq
Equivalent dose (<i>H</i>)	sievert	Sv	$\text{J}\cdot\text{kg}^{-1} \times \textcolor{blue}{W}_R$	1977	SI unit
Equivalent dose (<i>H</i>)	röntgen equivalent man	rem	$100 \text{ erg}\cdot\text{g}^{-1} \times \textcolor{blue}{W}_R$	1971	0.010 Sv
Exposure (<i>X</i>)	coulomb per kilogram	C/kg	$\text{C}\cdot\text{kg}^{-1}$ of air	1974	SI unit
Exposure (<i>X</i>)	röntgen	R	$\textcolor{blue}{\text{esu}}$ / 0.001293 g of air	1928	2.58×10^{-4} C/kg

How does Radiocarbon dating work?

- Exercise:
 - Explain how the Carbon dating method works



Percentages show the fraction of the total carbon reservoir of each type.
Numbers after slash show ratio of ^{14}C to ^{12}C as fraction of atmospheric ratio.

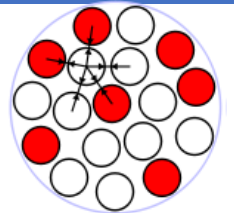
A model of the nucleus: Liquid drop model (~1935)

- Nucleus ~ uniform density, like an incompressible drop of water
- Strong nuclear force – pairs of nucleons: expect $\sim A^2$, but:
 - Very short ranges: sees only nearest neighbours
 - 'Volume' energy term $\propto A$
 - At surface less neighbours: a negative correction
 - 'Surface tension' term: $\propto A^{2/3}$, (since radius $\propto A^{1/3}$)
- Coulomb repulsion: Long range, all protons see all others. $R \propto A^{1/3}$
 - 'Coulomb term': $\propto Z^2 A^{-1/3}$ (think of sphere of charge density $\propto Q^2/R$)
- Pauli exclusion:
 - Asymmetry term, prefer to fill up low energy states first:
 - 'Pauli term': $\propto (N - Z)^2 / A$ or $(A - 2Z)^2 / A$
- Spin-spin coupling (pairing term), lowers energy if even #p or even #n:
 - $\propto \delta A^{-3/4}$ (empirical) where $\delta = \{-1, 0, 1\}$ if $= \{Z \text{ and } N = \text{even}, A = \text{odd}, Z \text{ and } N = \text{odd}\}$
- Binding energy (semi-empirical):

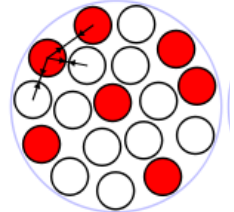
$$E_B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(N-Z)^2}{A} + \delta a_5 A^{-3/4}$$
- Bethe-Weizsacker empirical mass formula:

$$M(A, Z) = (A - Z)m_n + Z(m_p) - \frac{E_B}{c^2}$$

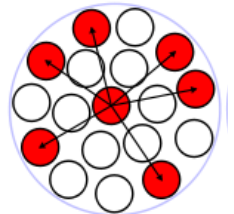
See Wikipedia:
Semi-empirical_mass_formula



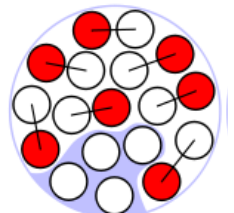
Volume



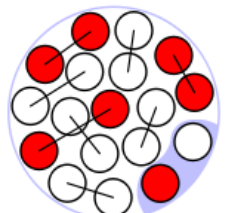
Surface



Coulomb



Asymmetry



Pairing

Values:

$$\begin{aligned} a_1 &\approx 15.6 \text{ MeV}, & a_2 &\approx 16.8 \text{ MeV}, \\ a_3 &\approx 0.72 \text{ MeV}, & a_4 &\approx 23.3 \text{ MeV} \\ a_5 &\approx 34 \text{ MeV} \end{aligned}$$

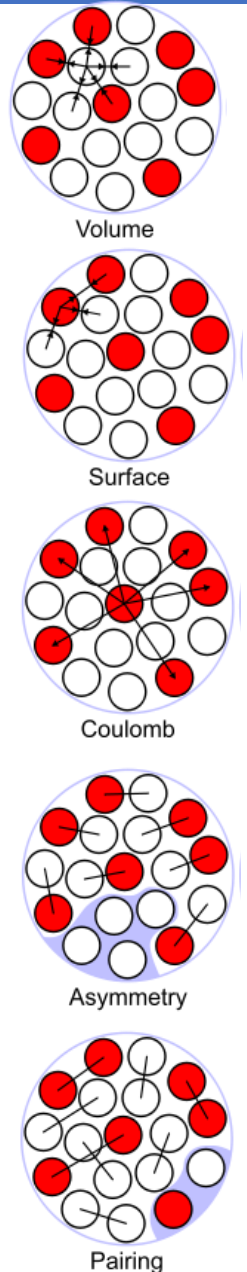
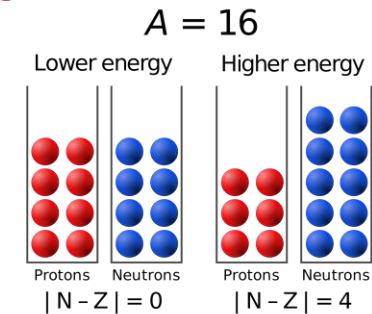
A model of the nucleus: Liquid drop model (~1935)

- Nucleus ~ uniform density, like an incompressible drop of water
- Strong nuclear force – pairs of nucleons: expect $\sim A^2$, but:
 - Very short ranges: sees only nearest neighbours
 - 'Volume' energy term $\propto A$
 - At surface less neighbours: a negative correction
 - 'Surface tension' term: $\propto A^{2/3}$, (since radius $\propto A^{1/3}$)
- Coulomb repulsion: Long range, all protons see all others. $R \propto A^{1/3}$
 - 'Coulomb term': $\propto Z^2 A^{-1/3}$ (think of sphere of charge density $\propto Q^2/R$)
- Pauli exclusion:
 - Asymmetry term, prefer to fill up low energy states first:
 - 'Pauli term': $\propto (N - Z)^2 / A$ or $(A - 2Z)^2 / A$
- Spin-spin coupling (pairing term), lowers energy if even #p or even #n:
 - $\propto \delta A^{-3/4}$ (empirical) where $\delta = \{-1, 0, 1\}$ if $= \{Z \text{ and } N = \text{even}, A = \text{odd}, Z \text{ and } N = \text{odd}\}$
- Binding energy (semi-empirical):

$$E_B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(N-Z)^2}{A} + \delta a_5 A^{-3/4}$$
- Bethe-Weizsacker empirical mass formula:

$$M(A, Z) = (A - Z)m_n + Z(m_p) - \frac{E_B}{c^2}$$

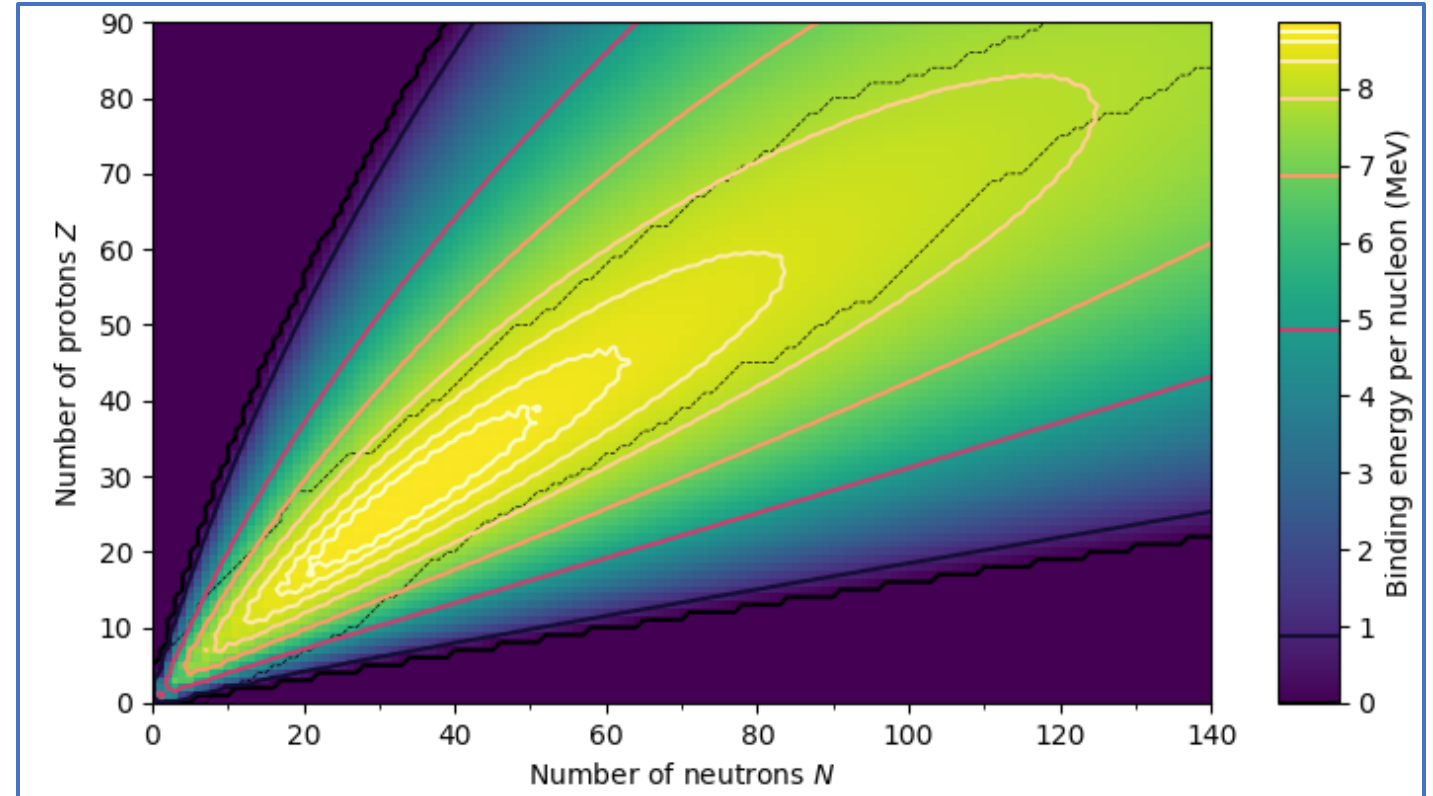
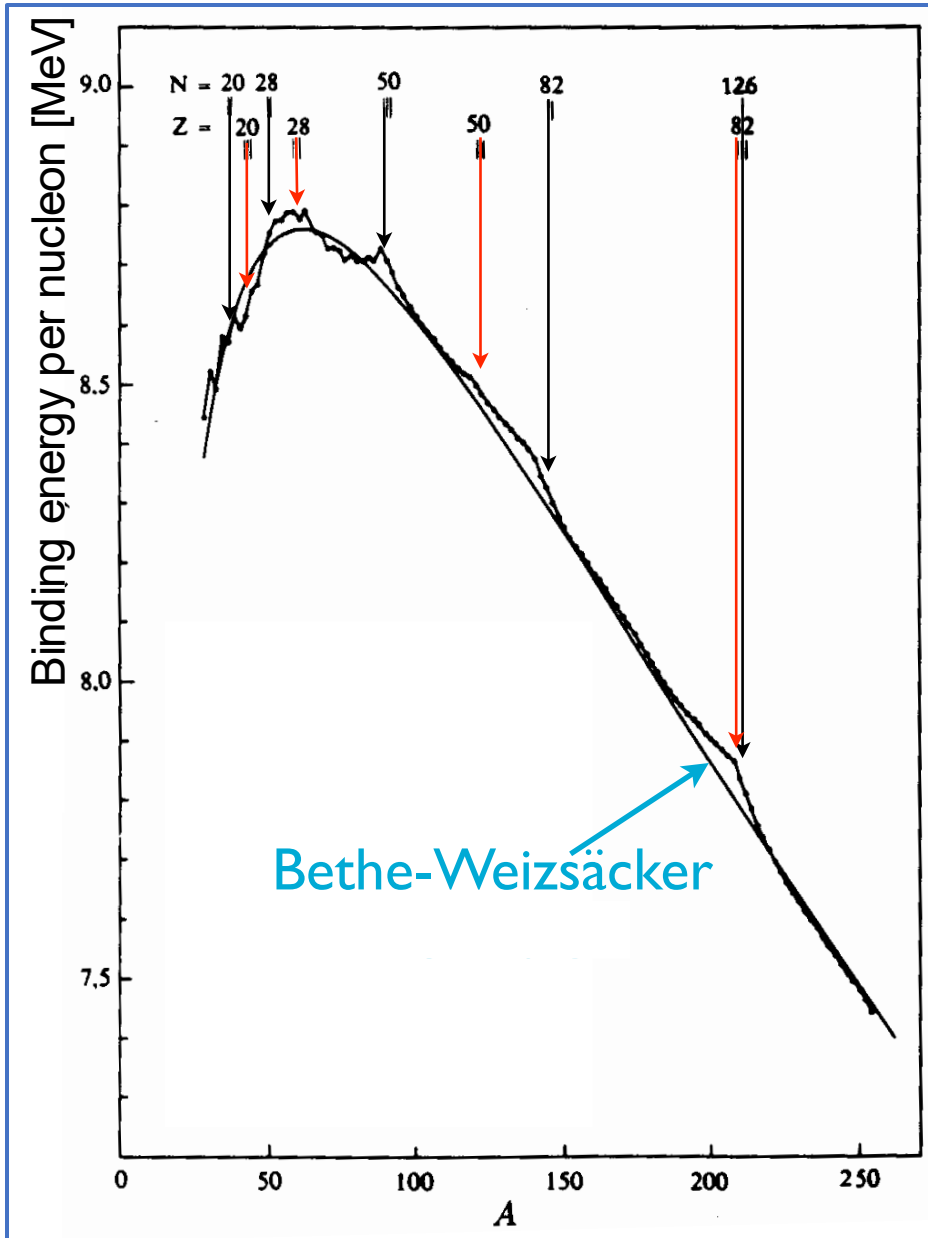
See Wikipedia:
Semi-empirical_mass_formula



Values:

$$\begin{aligned} a_1 &\approx 15.6 \text{ MeV}, a_2 \approx 16.8 \text{ MeV}, \\ a_3 &\approx 0.72 \text{ MeV}, a_4 \approx 23.3 \text{ MeV} \\ a_5 &\approx 34 \text{ MeV} \end{aligned}$$

Binding Energy: 'magic numbers'



- Fits quite well
- Cannot explain 'peaks' that occur at 'magic numbers' where the nucleus is strongly bound.
 - N or $Z = 2, 8, 20, 28, 50, 82, 126$
- Double magic nuclei (both N and Z are magic):
 - ${}^4\text{He}$, ${}^{16}\text{O}$, ${}^{40}\text{Ca}$, ${}^{48}\text{Ca}$, ${}^{48}\text{Ni}$, ${}^{208}\text{Pb}$

Nuclear Shell model (1949)

- Similar to the atomic shell model (remember hydrogen atom) for electrons):

- Atomic shells: Energy levels; n, l, s quantum numbers

- Solution of Schrodinger equation (QED)

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi \quad \text{with } V = -\frac{e^2}{4\pi\epsilon_0 r}$$

- Now strong force: which potential V ?

- Use the nuclear potential

- Try square well, try Harmonic oscillator, $V = \mu\omega^2 r^2 / 2$
Need something inbetween.

- Woods Saxon works well: $V(r) = -\frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)}$

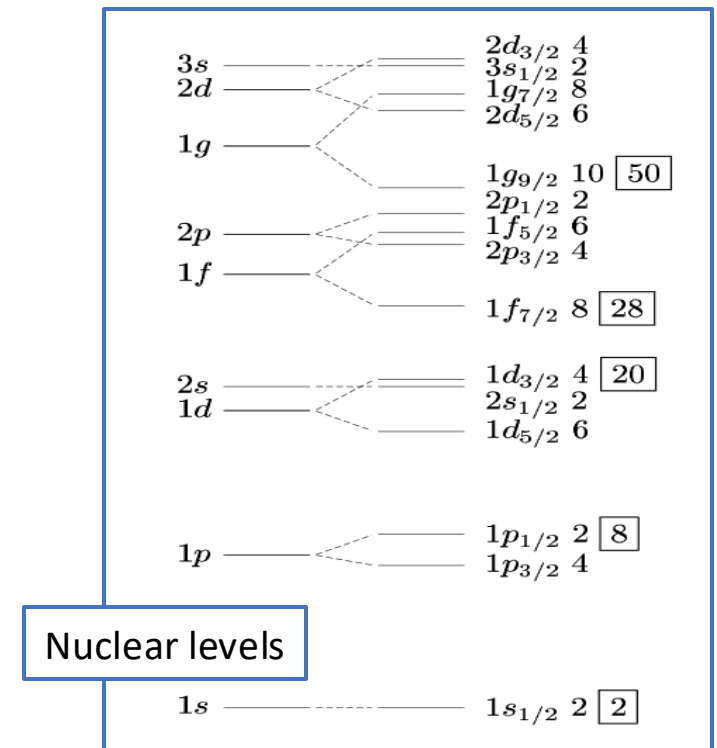
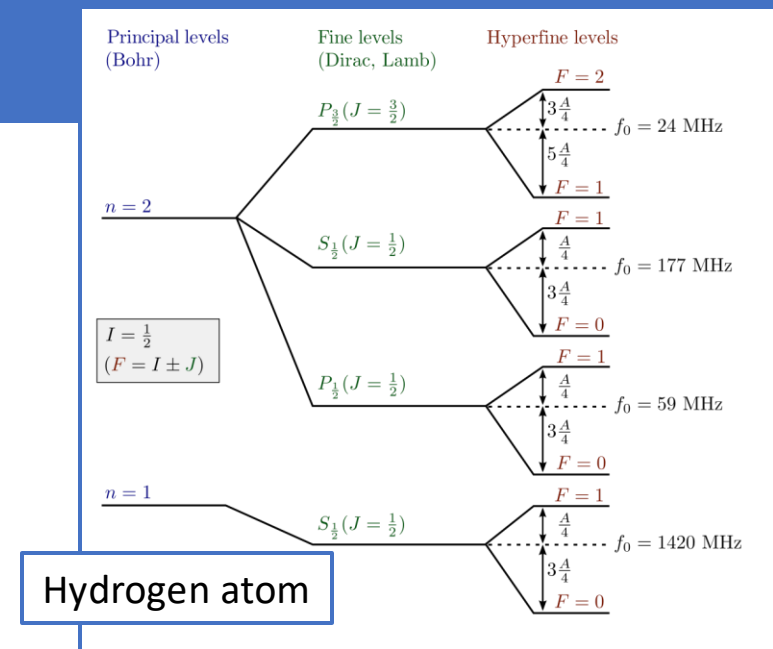
- Spin orbit interaction is stronger than in atom

- Nuclear shells “magic numbers”:

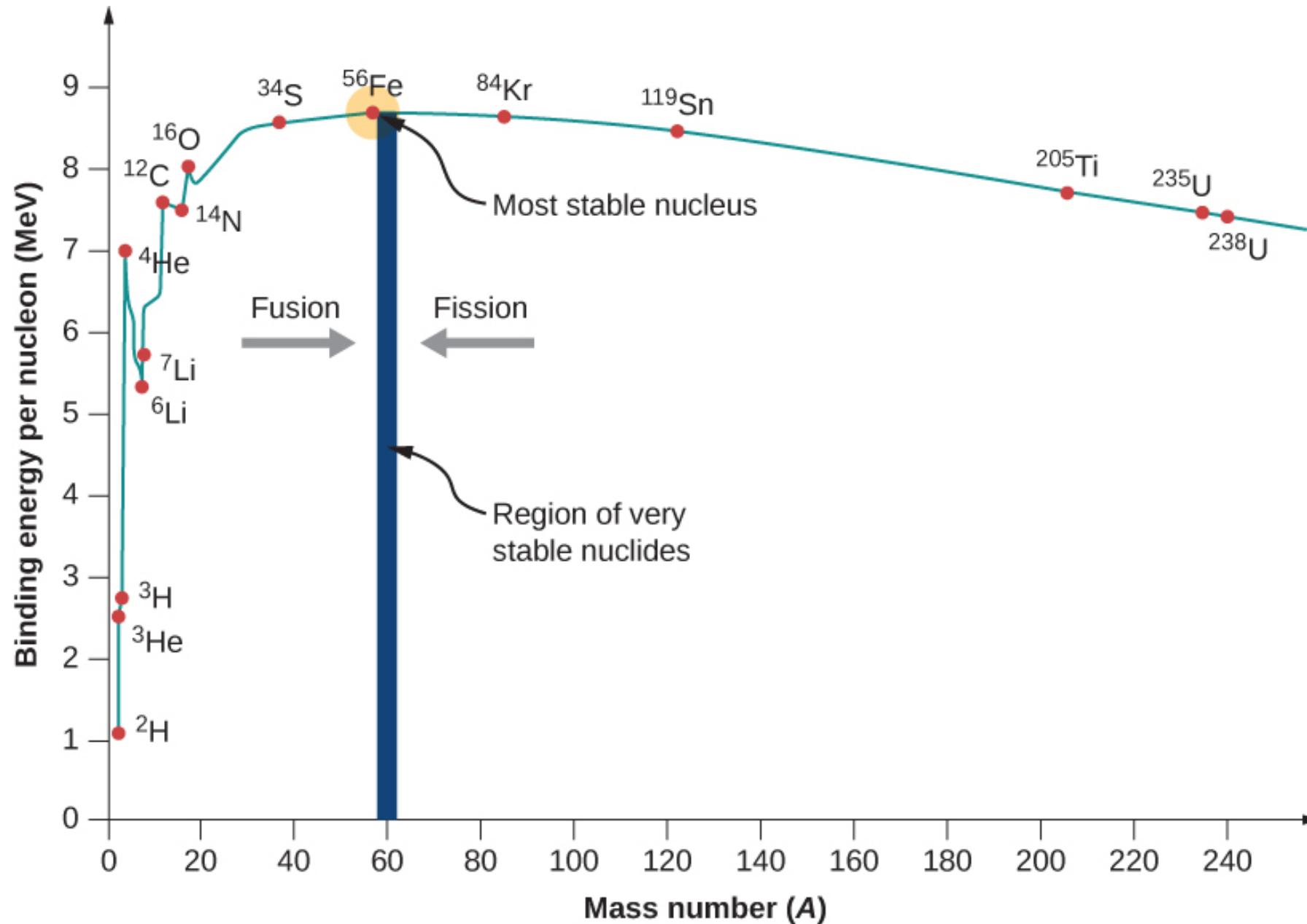
- Count energy levels of shells including spin:

- $2 \times 1 = 2, 2 \times (1+3) = 8, 2 \times (1+3+6) = 20, \dots$

- Filled shell $\rightarrow$ more tightly bound $\rightarrow$ larger E_B

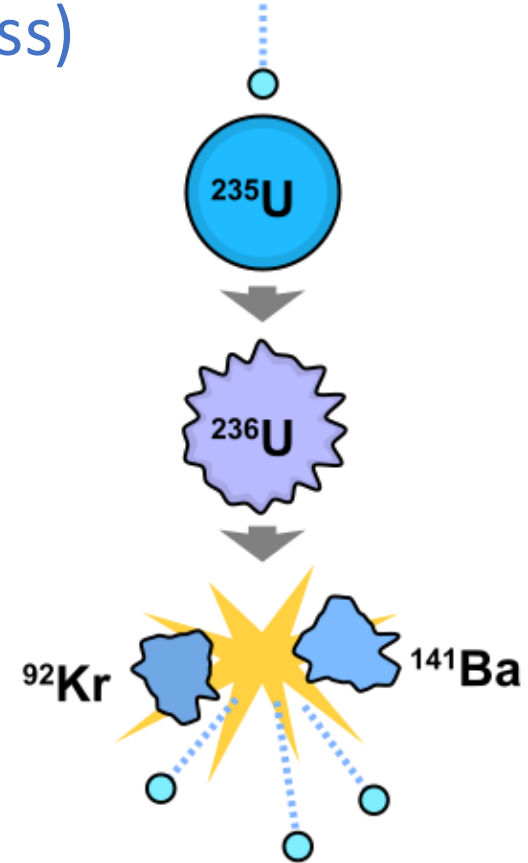


Nuclear Fission and Fusion



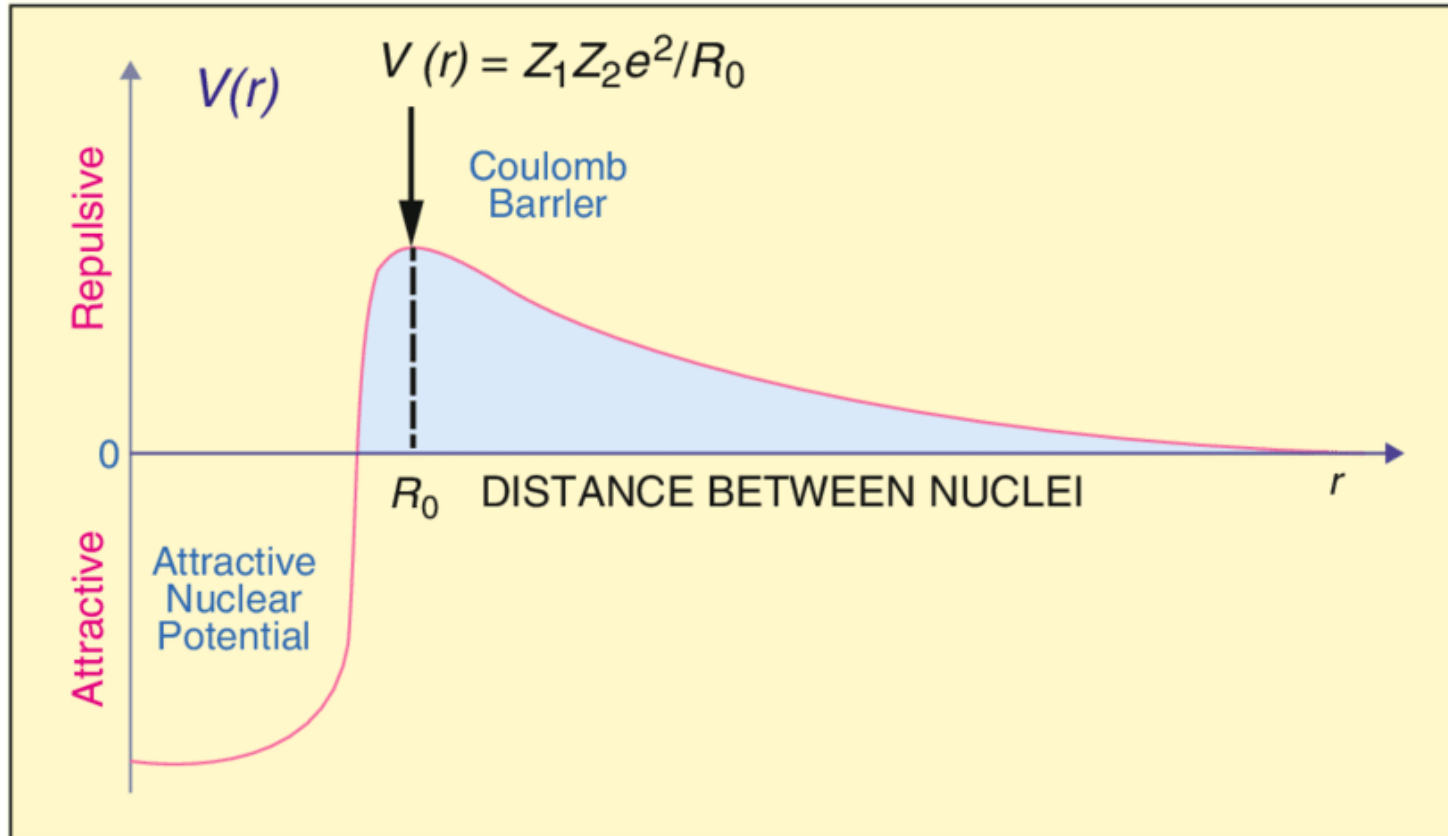
Nuclear Fission

- Heavy elements: less binding energy per nucleon (=more mass)
 - Transition heavy $\rightarrow$ light: free some mass $\rightarrow$ energy ($E = mc^2$)
 - Form high mass unstable isotope: eg. $^{235}_{92}\text{U} + 1 \text{ neutron}$. Reaction:
$$^{236}_{92}\text{U} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3^1_0\text{n}$$
$$236.045568u \rightarrow 140.914411u + 91.926156u + 3 \times 1.008644u$$
$$\Delta m = 0.18u \Rightarrow E = \Delta m c^2 = 2.68 \times 10^{-11} \text{J} = 168 \text{ MeV}$$
- Note: $^{235}_{92}\text{U}$ occurs as 0.72% natural Uranium
 - To use it as an energy source it must be enriched (centrifuge)
- E_B turned into kinetic energy $\rightarrow$ high temperature $\rightarrow$ steam
 - Nuclear reactors: controlled reaction, moderators for neutron absorption
 - Nuclear bombs: uncontrolled reaction



Nuclear Fusion

- Light elements fuse to free energy (Δm)
- Need to overcome Coulomb barrier in order to fuse together
 - Fuse H or D

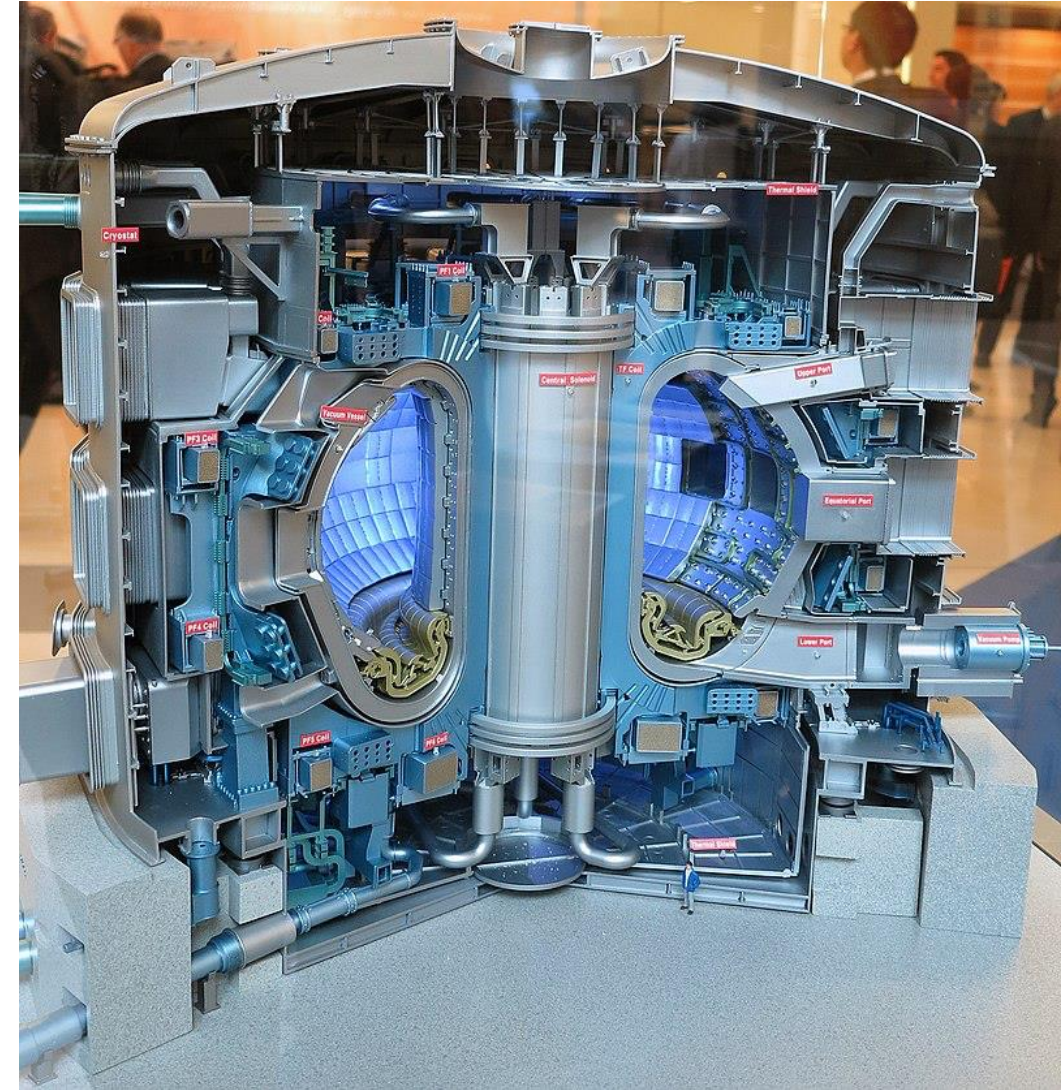
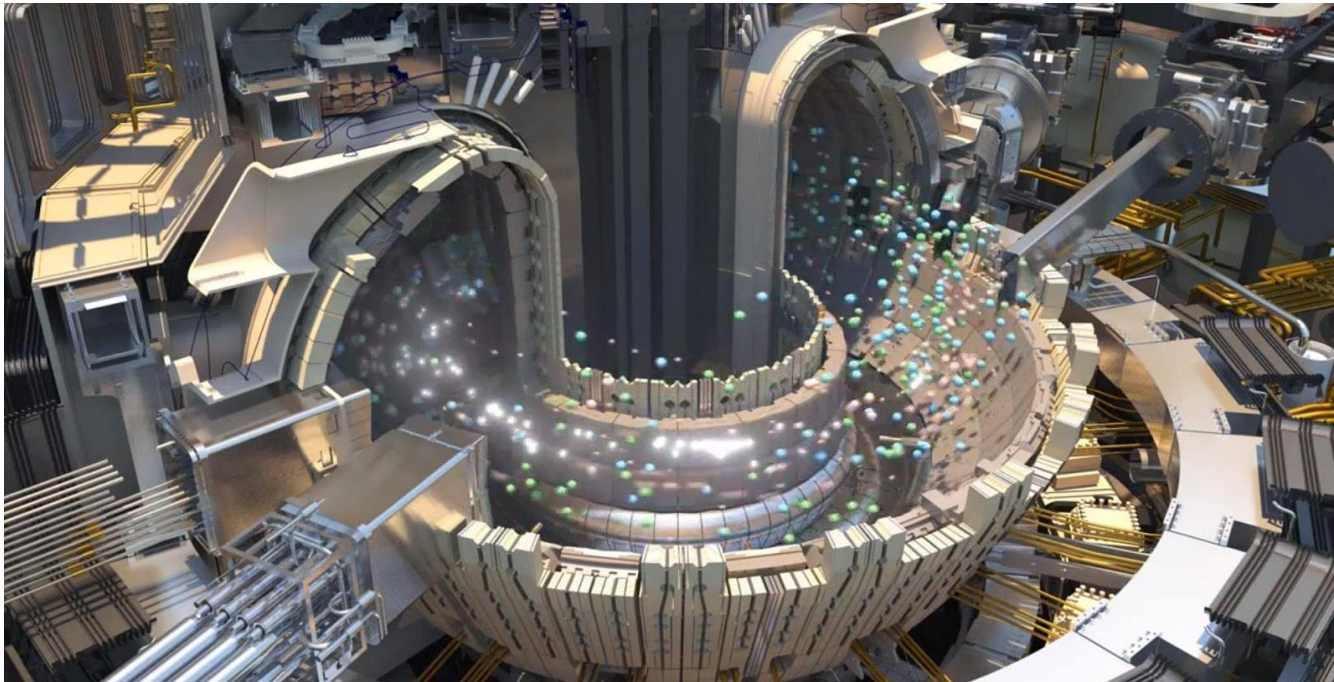


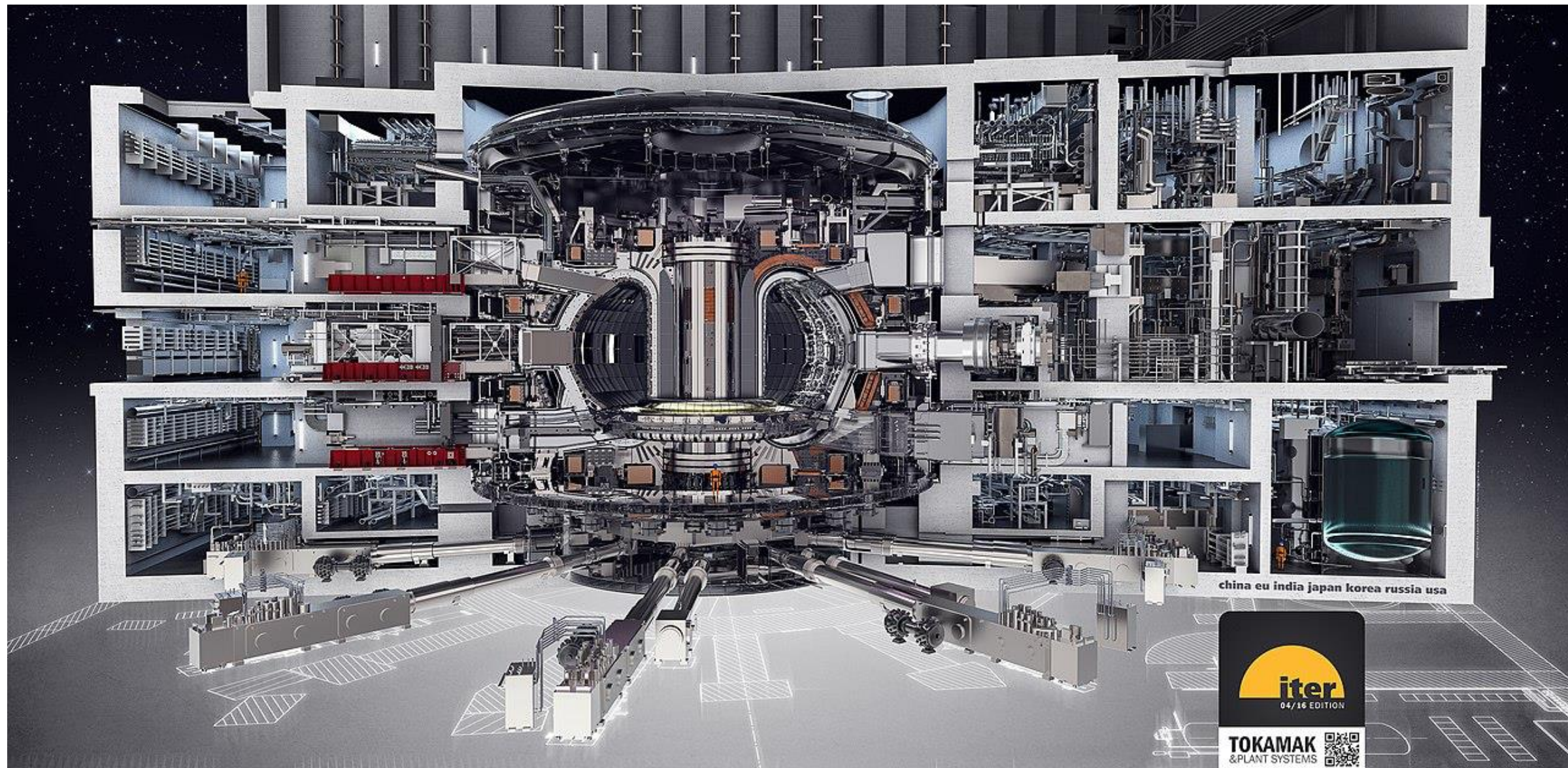
Coulomb barrier for charged-particle reactions

- Classically possible to fuse?
 - Would require extremely high kinetic energy $\rightarrow$ unrealistic temperature
 - Sun would not burn!
- Quantum mechanical tunneling helps:
 - Go through the barrier instead of over it
 - Gamow factor

Nuclear Fusion: lab

- Fusion for power: need high energies and densities
 - ITER in France
- $150 \times 10^6 \text{ } ^\circ\text{C}$
- Challenges:
 - keep confined
 - Keep sustained





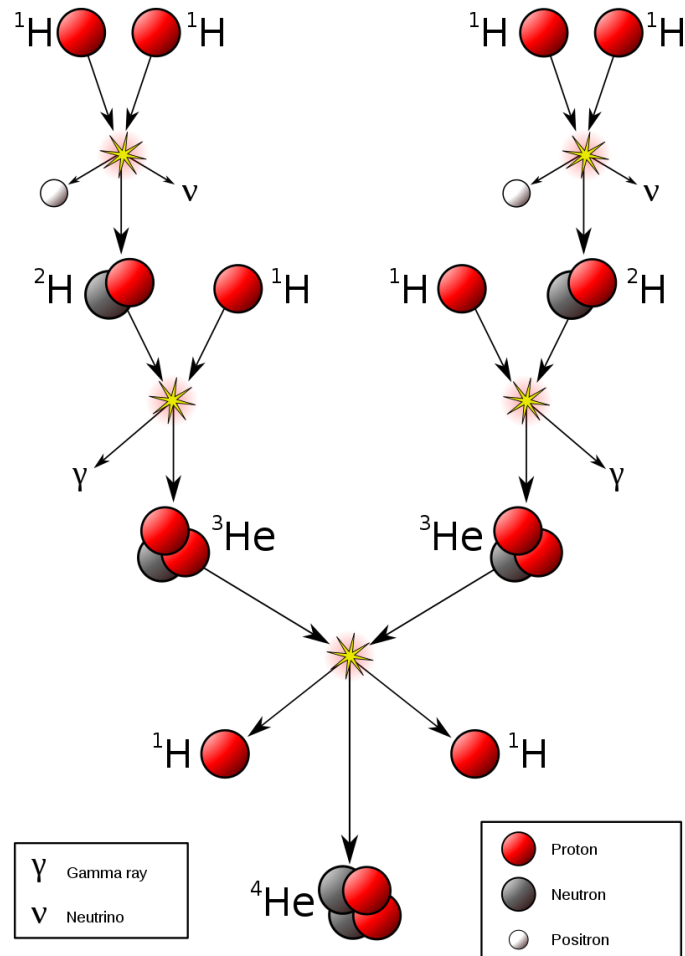
ITER in 2020



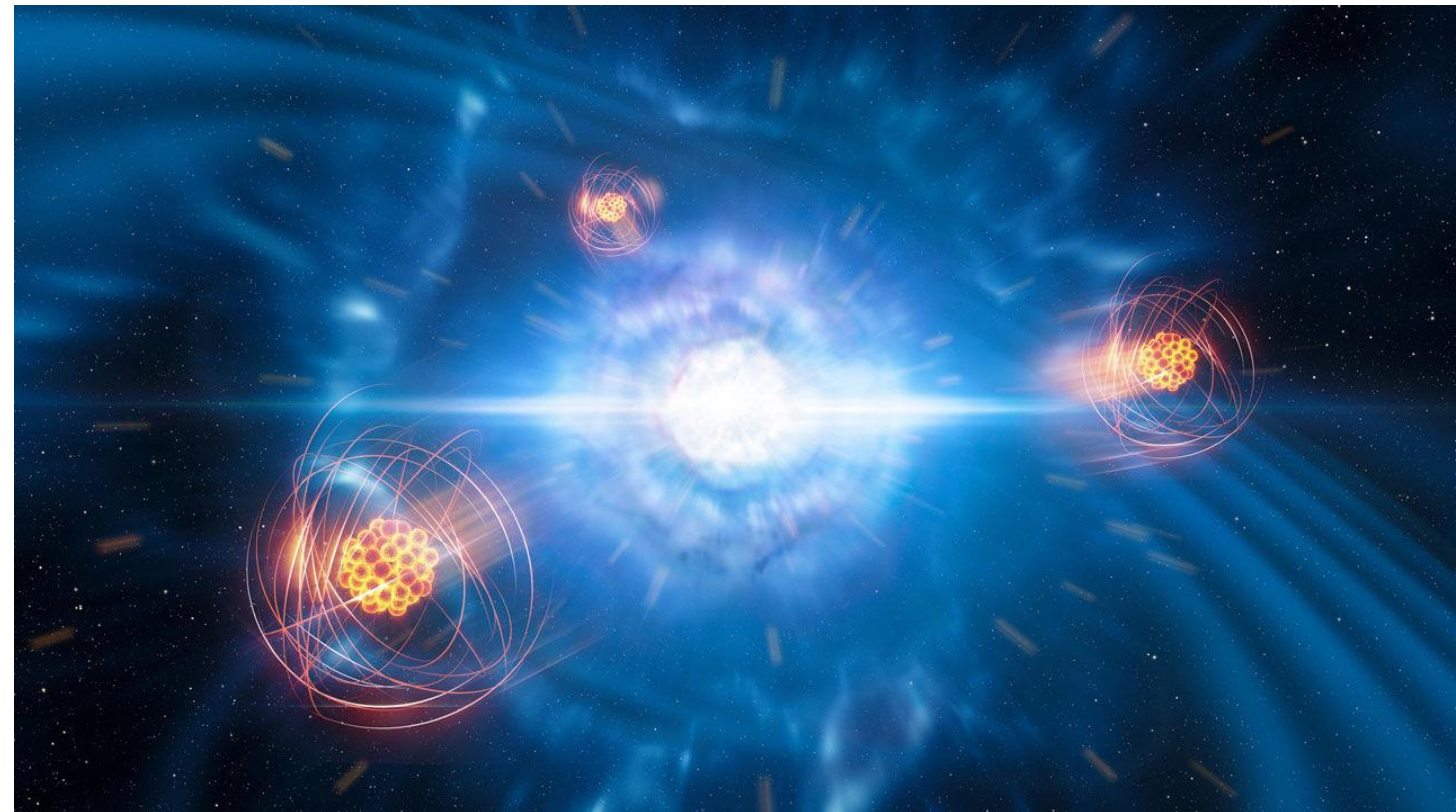
Nuclear Fusion: Universe

See Wikipedia: Nucleosynthesis

- Fusion in stars:
 - Proton-proton chain reaction (sun)
 - CNO catalytic cycle ($m_{star} > 1.3 m_{sun}$)



- How are heavy elements formed in the universe?
 - Colliding neutron stars



Exercise: Fission vs Fusion

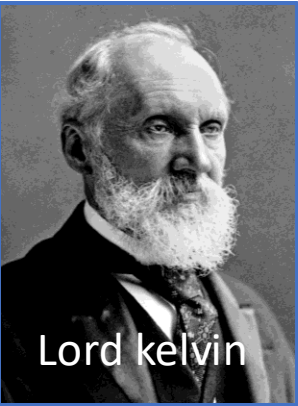
- a) Calculate the energy released if 1 gram of U-235 splits into La-148 + Br-87.
- b) Calculate the energy released in the fusion process of 0.5 grams of heavy water (D₂O) with 0.5 grams of superheavy water (T₂O), creating He-4 and a neutron. You may neglect the binding energies of the molecules.
- c) Compare the energies released per gram of fuel calculated above. Which would you prefer?

Lecture 1 : Particles

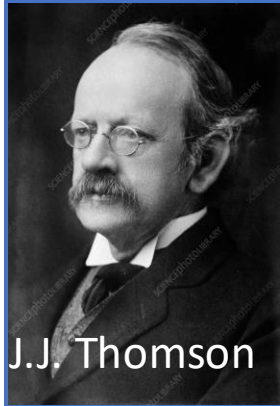
Part 2

Subatomic Particles

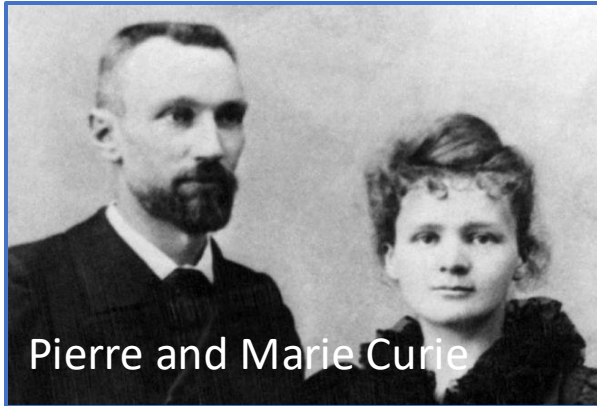
Particle Physics: Historical introduction – Griffiths chapter 1



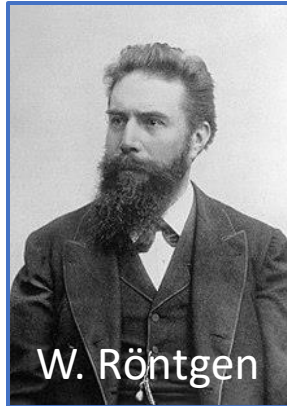
Lord kelvin



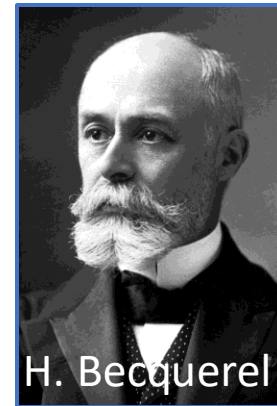
J.J. Thomson



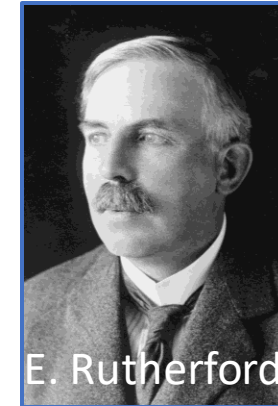
Pierre and Marie Curie



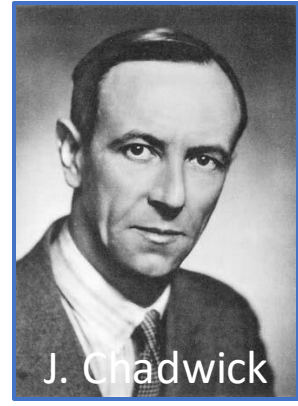
W. Röntgen



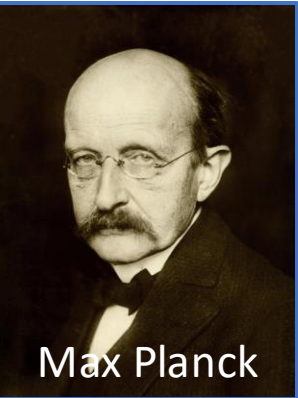
H. Becquerel



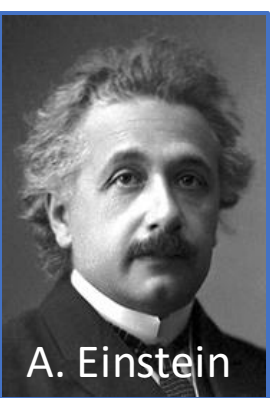
E. Rutherford



J. Chadwick



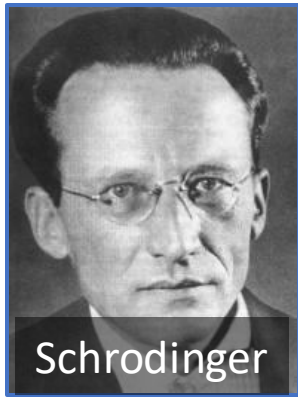
Max Planck



A. Einstein



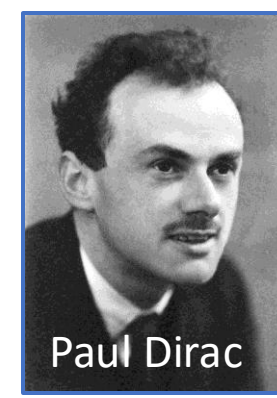
Niels Bohr



Schrodinger



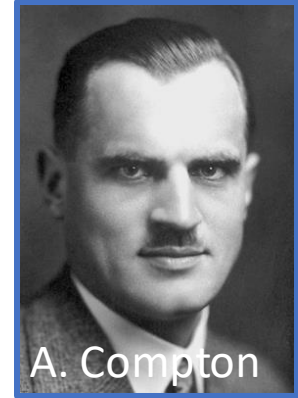
Heisenberg



Paul Dirac



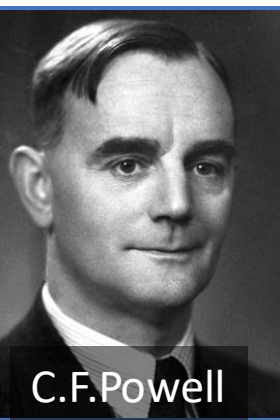
L. De Broglie



A. Compton



H. Yukawa



C.F. Powell



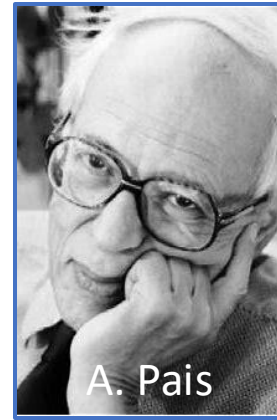
W. Pauli



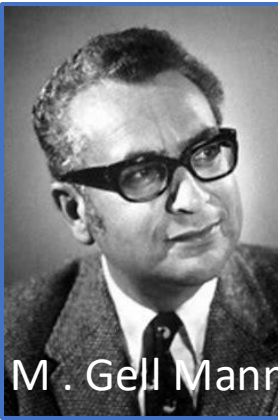
Anderson



J de Vries



A. Pais

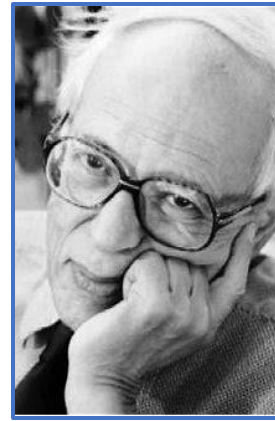
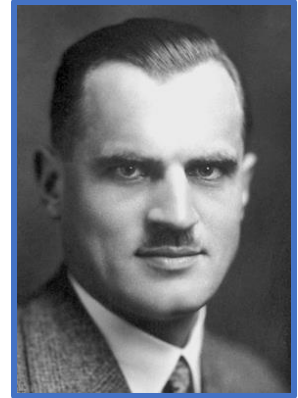
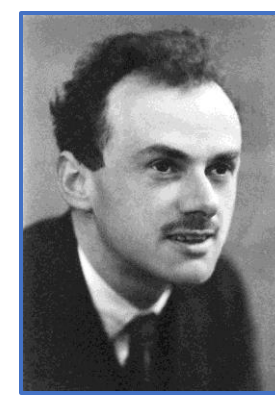
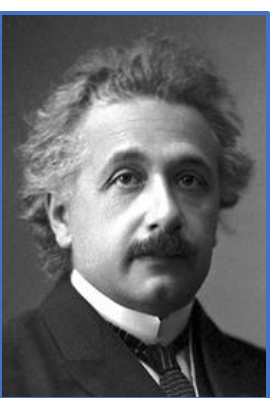
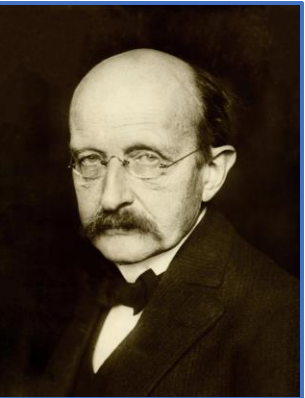
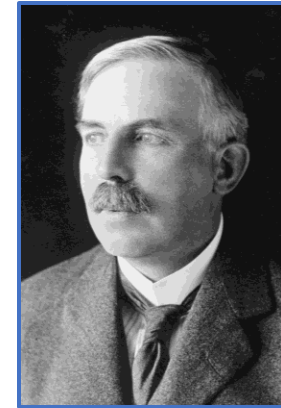
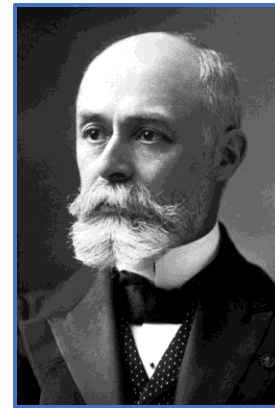
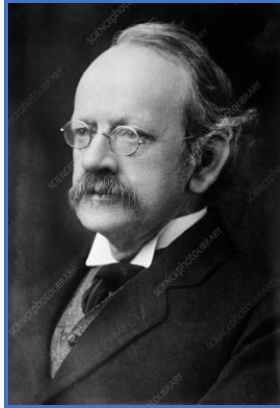
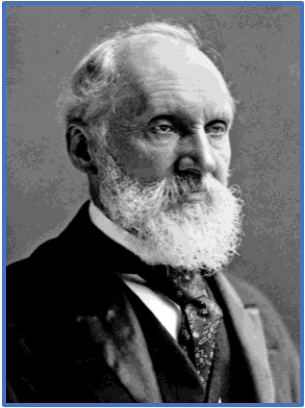


M. Gell Mann



R. Feynman

Particle Physics: Historical introduction – Griffiths chapter 1



1927 Solvay Conference Brussels

“Possibly the most intelligent picture ever taken”

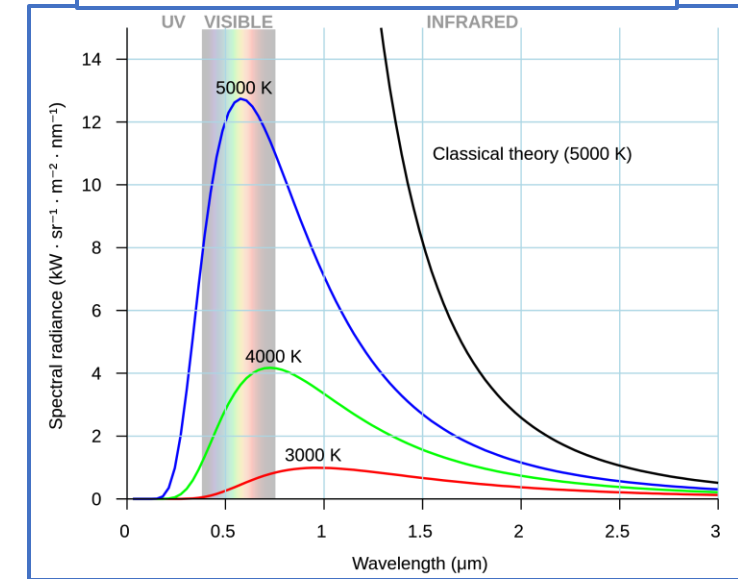


The classic era

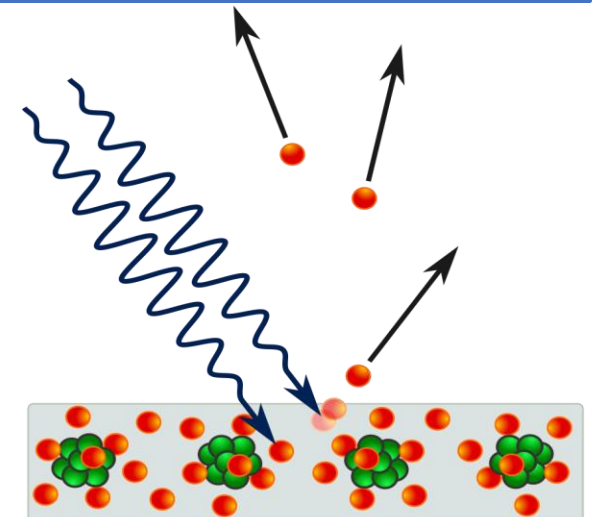
- ~1900: Lord Kelvin on physics:
 - “*There is nothing new to be discovered in physics now. All that remains is more and more precise measurements.*”
 - Enter Relativity Theory and Quantum Mechanics
- 1895-1898 Becquerel, Röntgen, Marie & Pierre Curie discover *radiation*
- 1897: J.J Thomson noted cathode rays are quantized particles, charge to mass ratio: discovery of *electrons*
- 1900: Planck: The blackbody spectrum can be explained by introducing emission of light in *quanta* $\hbar$ with $E = \hbar\nu$
- 1905: Einstein: photo electric effect: light is quantized: photon
 - Scattering behaviour verified in 1923 by Arthur Holly Compton
- 1911+1917: Rutherford: nucleus hydrogen is a proton
- 1932: Chadwick: discovery of the neutron
 - Neutral particle with the mass of a proton

Griffiths §1.1 and §1.2

Black body spectrum: Planck



Photoelectric Effect: Einstein



Mesons 1934 - 1947

Griffiths §1.3

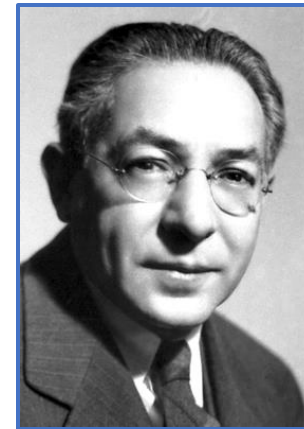
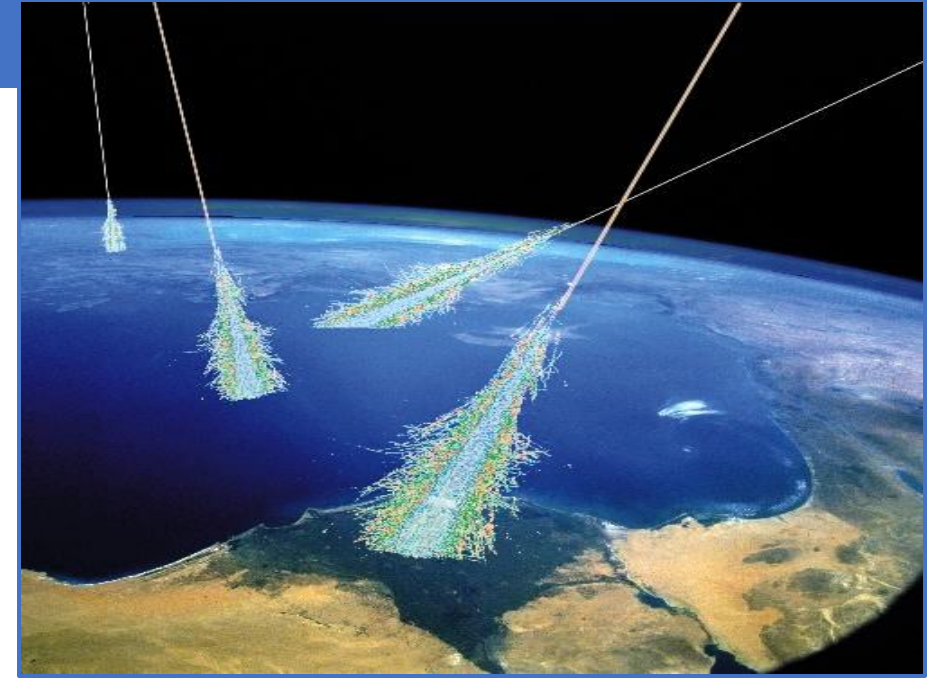
- The nucleus is held together by strong force, mediated by pions or pi-mesons.
- Can we see these pion quanta of mass $\sim 140 \text{ MeV}$?
- In 1937 particles were detected in cosmic rays:
 - Their mass was a bit too light: $m = 105 \text{ MeV}$
 - *They did not interact strongly with nuclei ?!*
- It turned out to be a muon, heavy version of the electron
 - “Who ordered that?”: Isaac Rabi
- Particles:
 - Baryons (p, n), Mesons (π), Leptons (e, μ)

Lepton: Greek “small” or “thin”: small mass, feels no nuclear force

Hadron: Greek “thick”: large mass; baryons and mesons

Baryon: Greek “heavy”: heavier than mesons

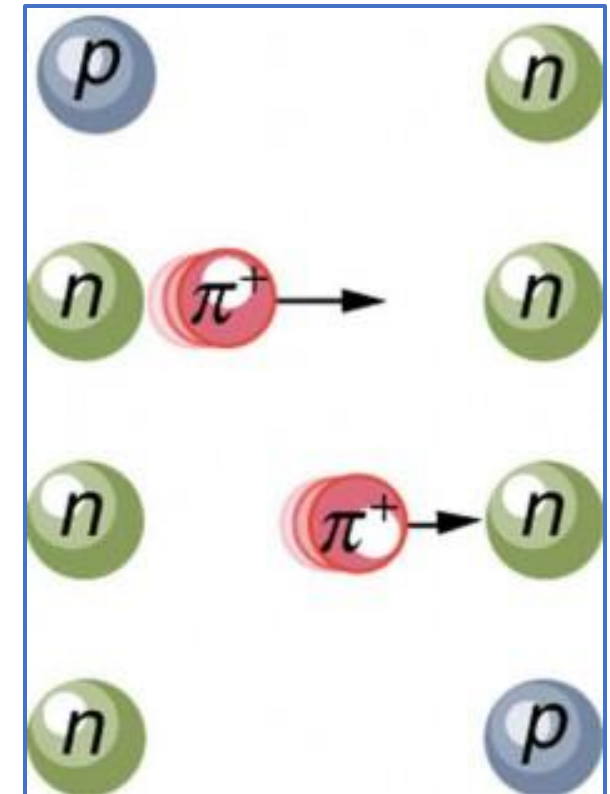
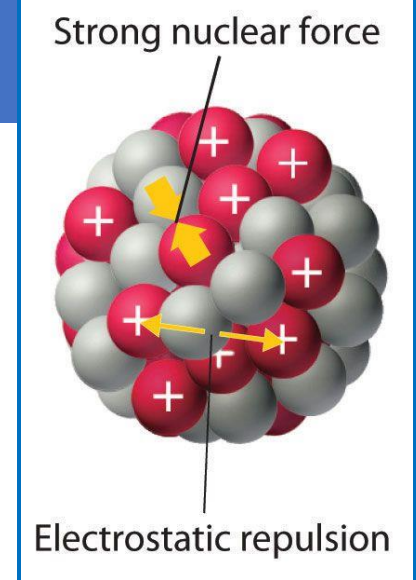
Meson: Greek “middle”: medium mass, feels nuclear interaction



Yukawa's pion

Griffiths: problem 1.2, PP1 §i.2

- Yukawa (1934) pi-meson gives attractive force between nucleons
- Heisenberg operator uncertainty in QM: $\Delta E \Delta t \geq \frac{\hbar}{2}$
- Translated to popular language:
 - You can violate energy conservation as long as:
 - time is shorter than: $\Delta t \leq \frac{\hbar}{2\Delta E}$ or energy is less than $\Delta E \leq \frac{\hbar}{2\Delta t}$
- Electromagnetic force is transmitted by *massless photon* quanta
 - $E = hf = \frac{hc}{\lambda}$ such that: $\Delta t \sim \frac{\lambda}{c}$ and the range $R \sim c\Delta t = \lambda$ is infinite
- Nuclear (“strong”) force is transmitted by *massive pions*:
 - $E = mc^2$ such that $\Delta t \sim \frac{h}{2mc^2}$ and $R \sim c\Delta t = \frac{h}{2mc}$ (Compton wavelength)
 - The heavier the particle, the more difficult to “violate energy conservation”, the shorter the range
- In 1947 the pion, with correct mass, was discovered



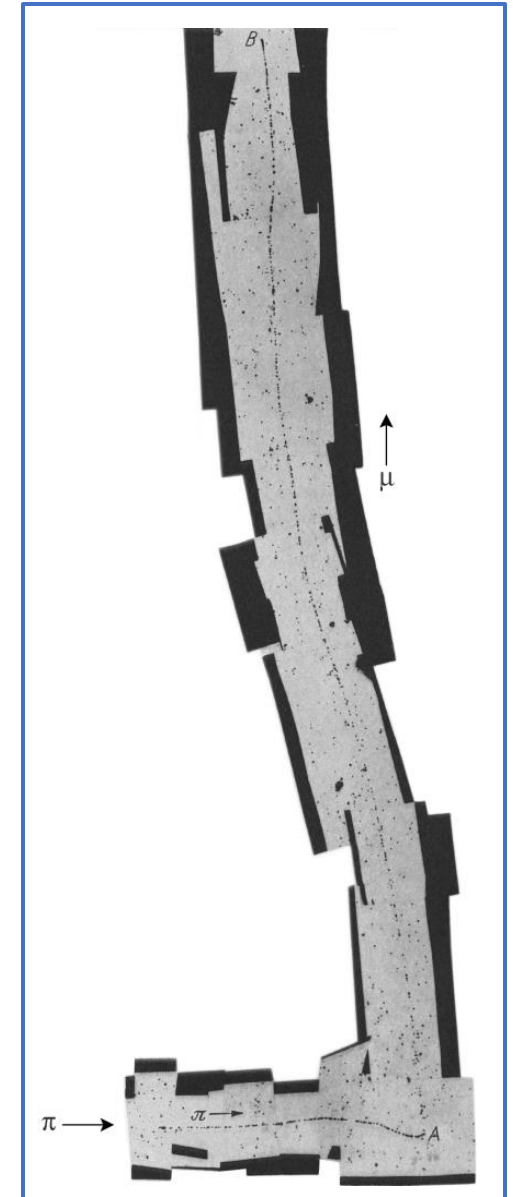
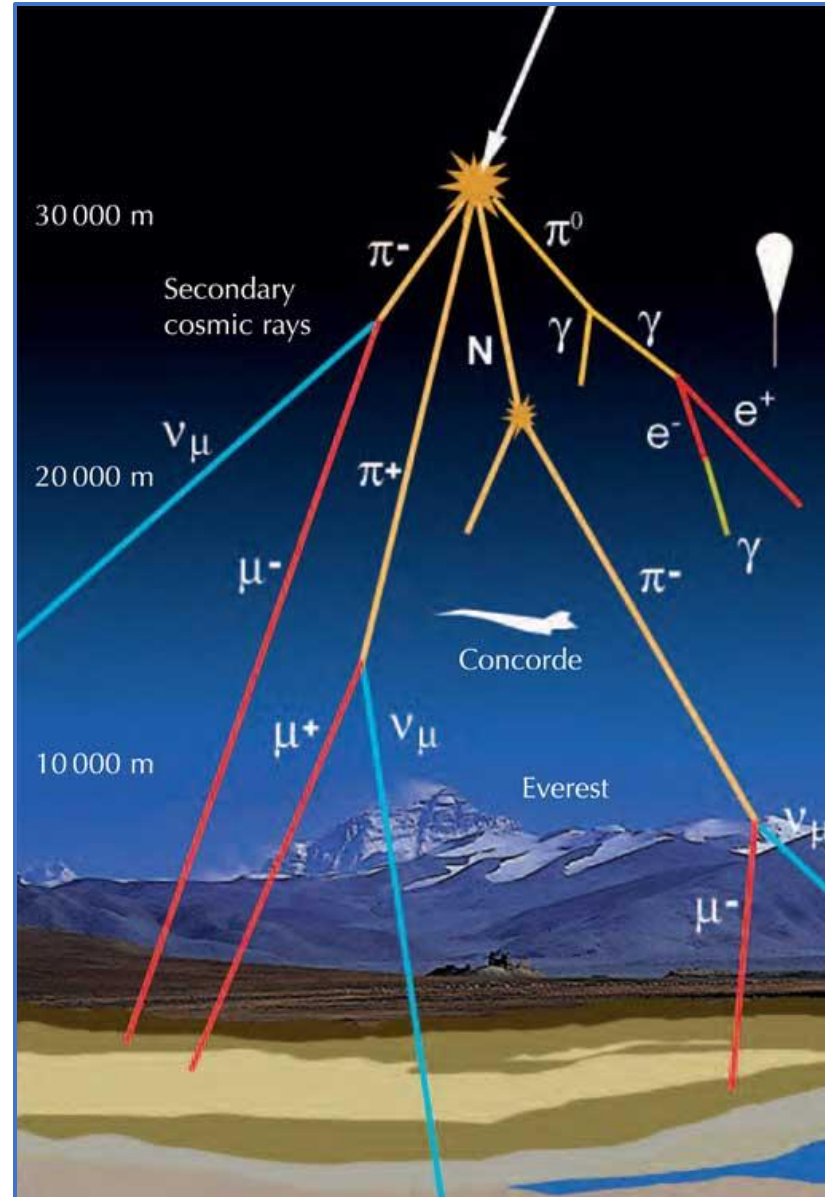
Exercise: The Yukawa Potential

- The electric force is transmitted by a photon with $m = 0$. The wave equation for a static electric field caused by pointlike charge e is: $\nabla^2 V(r) = 0$ (Laplace equation)
 - a) Show that the Coulomb potential $V(r) = -e^2 \frac{1}{r}$ fulfills this equation.
 - Note that the potential is **spherical symmetric**, ie. use spherical coordinates.
- The nuclear force is transmitted by a pi-meson with $m = m_\pi$. The wave equation for a static nuclear field caused by a pointlike color charge g is: $\nabla^2 U(r) = m^2 U(r)$ (Klein-Gordon equation)
 - b) Show that the Yukawa potential $U(r) = -g^2 \frac{e^{-r/R}}{r}$ fulfills this equation for a certain value of R , the **range** of the force. What is the relation between R and m_π ?
 - Again note that the potential is **spherical symmetric**.
 - This value is between the electron and proton mass, hence the particle was called a pi-meson or pion.
 - c) Calculate the range of the force from Heisenberg's uncertainty relation, using $R = c\Delta t$ and $\Delta E \Delta t \leq \frac{\hbar}{2}$ and $\Delta E = mc^2$.
 - d) The weak force is mediated by W (80 GeV) and Z (91 GeV) bosons. What is the estimated range of the weak force?

Discovery of the pion - 1947

Griffiths §1.3

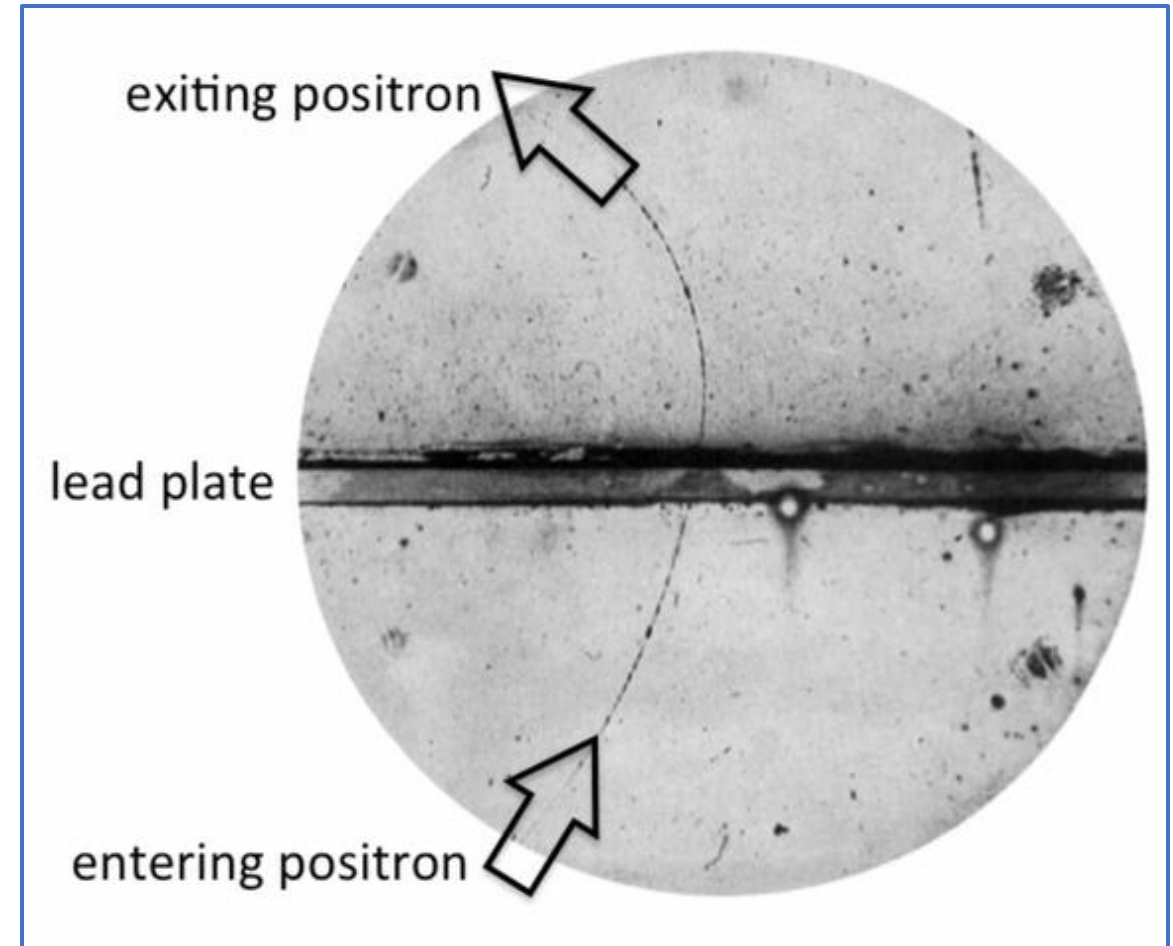
- Powell, 1947
 - Go to mountain top
 - Photographic emulsion
- Observes that cosmic rays include muons and pions
 - $m_\pi = 140 \text{ MeV}/c^2$
 - $m_\mu = 105 \text{ MeV}/c^2$
- A pion can decay into a muon, which again decays into an electron
- The pion was Yukawa's meson



Antimatter particles 1928 - 1956

Griffiths §1.4

- In 1928 Dirac predicted the existence of antimatter particles, combining relativity theory and quantum mechanics
 - We will learn about Dirac's equation later
- Exercise: convince yourself that Klein-Gordon equation: $E^2 = p^2c^2 + m^2c^4$ is in agreement with the relativistic formulae: $E = \gamma mc^2$ and $p = \gamma mv$
- Dirac: E can have positive and negative solutions:
 - Positive: matter particles (electron)
 - Negative: antimatter particles (positron)
- In 1932 the positron was discovered by Anderson



Question: How did he know which direction the particle went?

Antimatter particles 1928 - 1956

- Feynman Stückelberg interpretation of particles (more in lecture 3):
 - The *negative energy* solution of a *particle* corresponds with the *positive energy* solution of an *antiparticle* going backwards in time.

$$\psi_{\pm}(\vec{x}, t) = a_{\pm} e^{\frac{i}{\hbar}(\vec{x}\vec{p} - Et)}$$

$$\begin{array}{l} t \rightarrow -t ; E \rightarrow -E \\ \vec{x} \rightarrow -\vec{x} ; \vec{p} \rightarrow -\vec{p} \end{array}$$

- CPT theorem: an antiparticle *is* a particle going backwards in time
- For each particle p there exists a mirror anti-particle $\bar{p}$ in nature.
 - Berkeley Bevatron: discovery anti-proton (1954, Chamberlain, Segré) and discovery anti-neutron (1956, Cork).
 - The antiparticle has the *same mass, lifetime and spin, but opposite internal quantum numbers like charge*.
 - Matter and antimatter have identical interactions. Definition what is what?
- Wheeler: is there only one electron in the entire universe?
- Question: is there also an anti-photon? How about an anti-neutrino?

Antiparticle and Crossing Symmetry

- Crossing symmetry:

If: $A + B \rightarrow C + D$, then also possible:

$$A \rightarrow \bar{B} + C + D$$

$$A + \bar{C} \rightarrow \bar{B} + D$$

$$\bar{C} + \bar{D} \rightarrow \bar{A} + \bar{B}$$

Creation of an anti-particle
is destruction of a particle

Processes limited by energy conservation.

- Example:

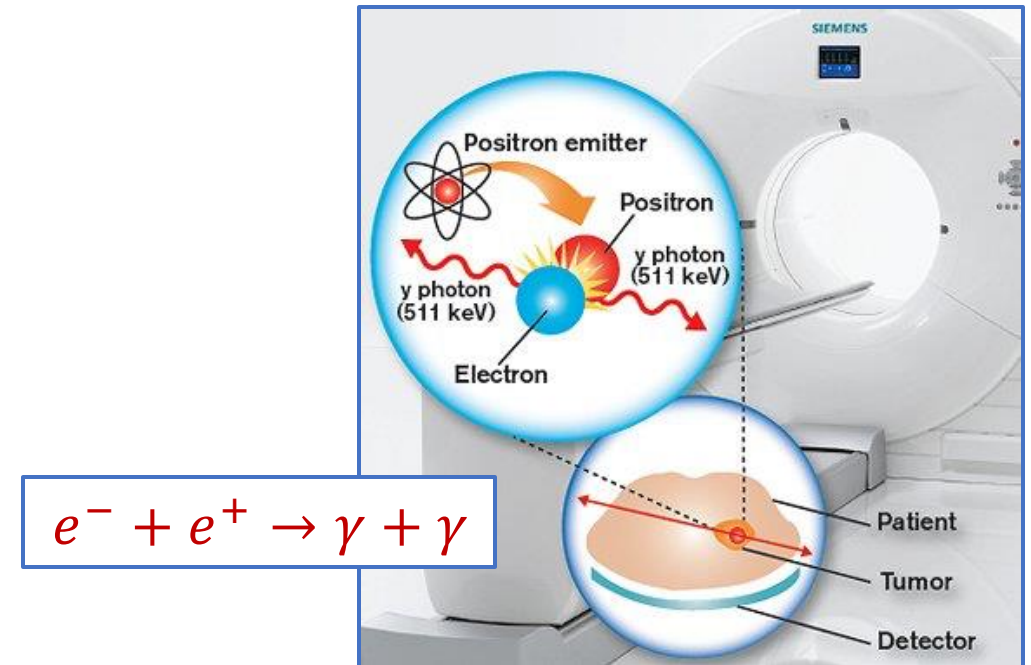
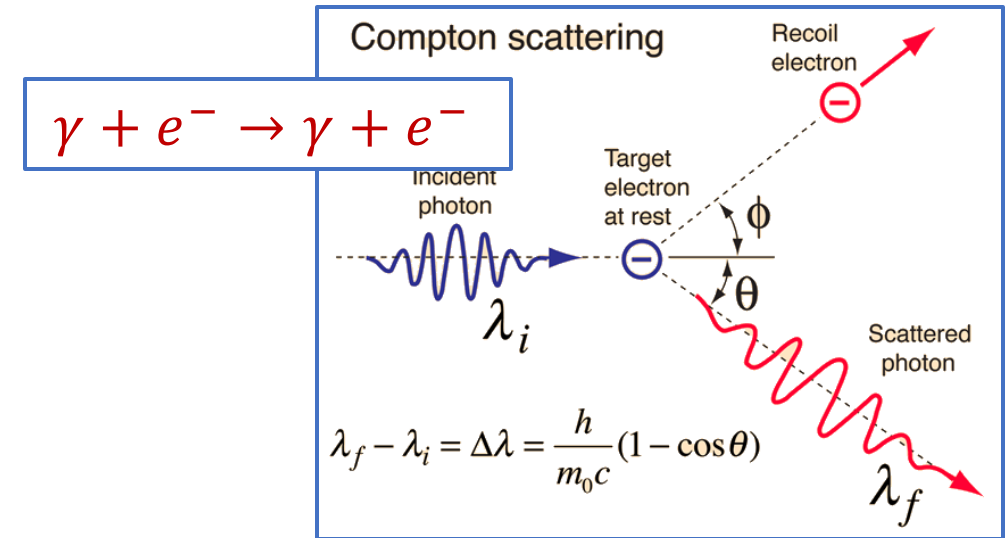
Compton scattering = pair annihilation

$$\gamma + e^- \rightarrow \gamma + e^- = e^- + e^+ \rightarrow \gamma + \gamma$$

- General symmetry between matter and antimatter (CPT theorem)

→ *Why is the universe matter dominated?*

→ *Subtle features in the weak interaction!*



Antiparticle and Crossing Symmetry

- Crossing symmetry:

If: $A + B \rightarrow C + D$, then also possible:

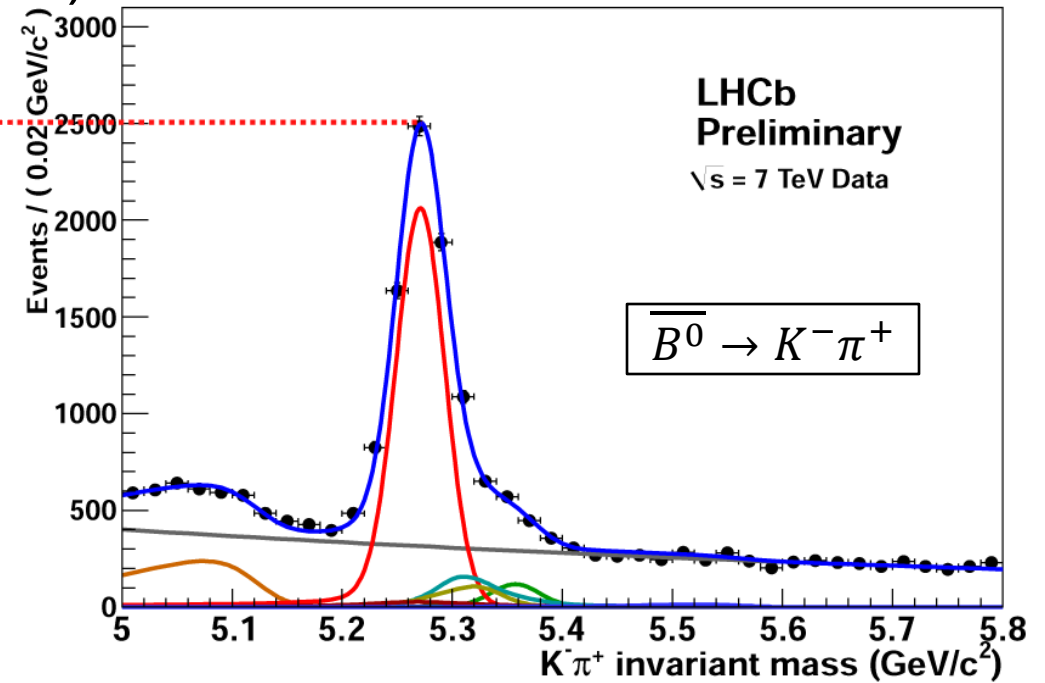
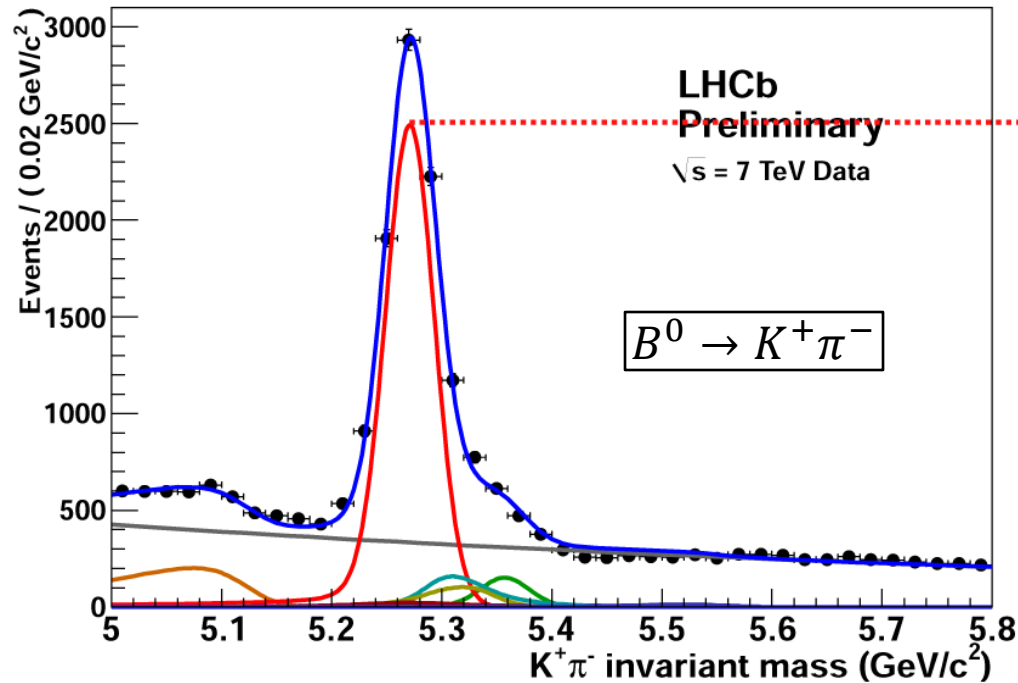
$$\gamma + e^- \rightarrow \gamma + e^-$$

Compton scattering

Recoil electron

Target

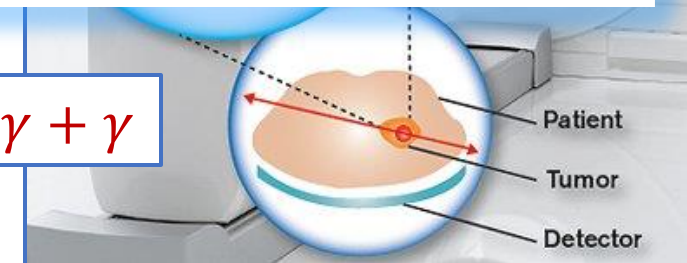
CP violation (2012)



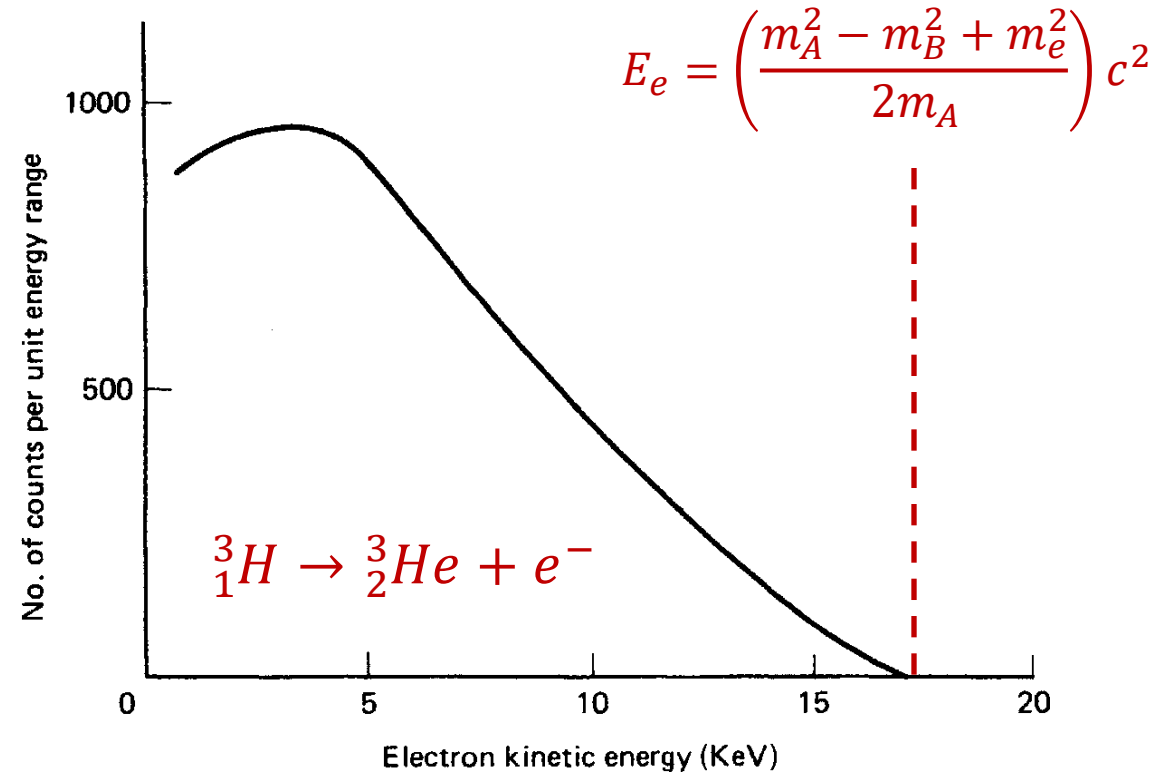
→ Why is the universe matter dominated?

→ Subtle features in the weak interaction!

$$e^- + e^+ \rightarrow \gamma + \gamma$$



- Nuclear beta decay: $A \rightarrow B + e^-$
 - Exercise: calculate the kinetic energy of the electron.
 - Observed kinetic energy in tritium decay is a *spectrum*
- Niels Bohr was thinking to abandon the law of conservation of energy(!)

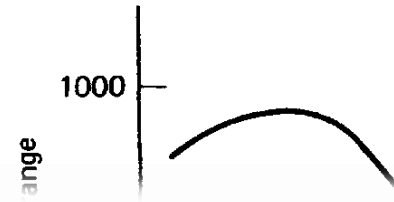


- Pauli proposed that an undetected neutral particle was produced
 - Proposed the name neutron
 - Fermi presented beta decay theory, neutrino: $n \rightarrow p + e^- + \bar{\nu}$

- Nuclear beta decay: $A \rightarrow B + e^-$

- Exercise: calculate the kinetic energy of the electron

- Observed spectrum is a



$$E_e = \left(\frac{m_A^2 - m_B^2 + m_e^2}{2m_A} \right) c^2$$

- Niels Bohr proposed the law

The power of 4-vector notation (look it up after completing four-vector notation):

$$p_A^\mu = \left(\frac{E_A}{c}, \vec{0} \right), \quad p_B^\mu = \left(\frac{E_B}{c}, \vec{p}_B \right), \quad p_e^\mu = (E_e, \vec{p}_e)$$

Conservation of energy and momentum: $p_A^\mu = p_B^\mu + p_e^\mu$

Invariant mass: $p_i^\mu p_{i\mu} = m_i^2 c^2$ for $i = A, B, e$

Then use: $(p_A - p_e)^2 = p_A^2 + p_e^2 - 2p_A \cdot p_e = m_A^2 c^2 + m_e^2 c^2 - 2p_A \cdot p_e = p_B^2 = m_B^2 c^2$

Then: $2p_A \cdot p_e = m_A^2 c^2 - m_e^2 c^2 - m_B^2 c^2$

Now go to the rest frame of decaying particle A:

$$p_A^\mu = (m_A c^2, \vec{0}) \text{ so that } p_A \cdot p_e = \frac{m_A c^2 E_e}{c^2} - \vec{0} \cdot \vec{p}_e = m_A E_e$$

- Pauli

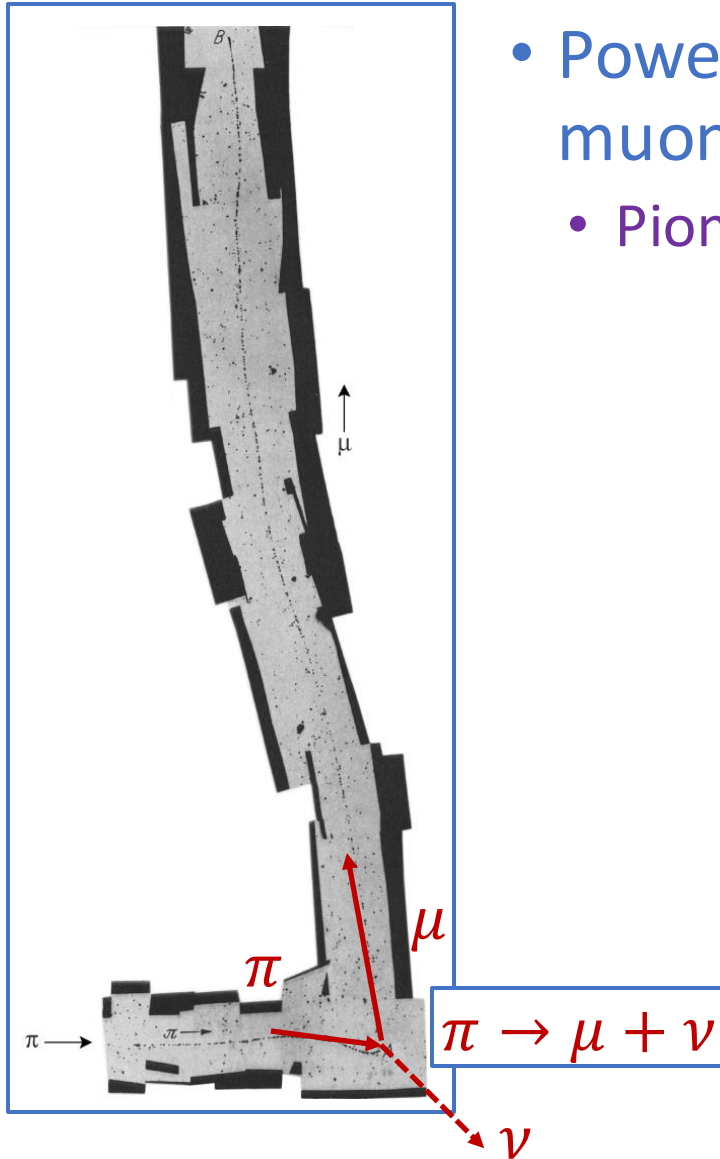
Plugging it in: $2p_A \cdot p_e = 2m_A E_e = m_A^2 c^2 + m_e^2 c^2 - m_B^2 c^2$

- Proposed

- Fermi

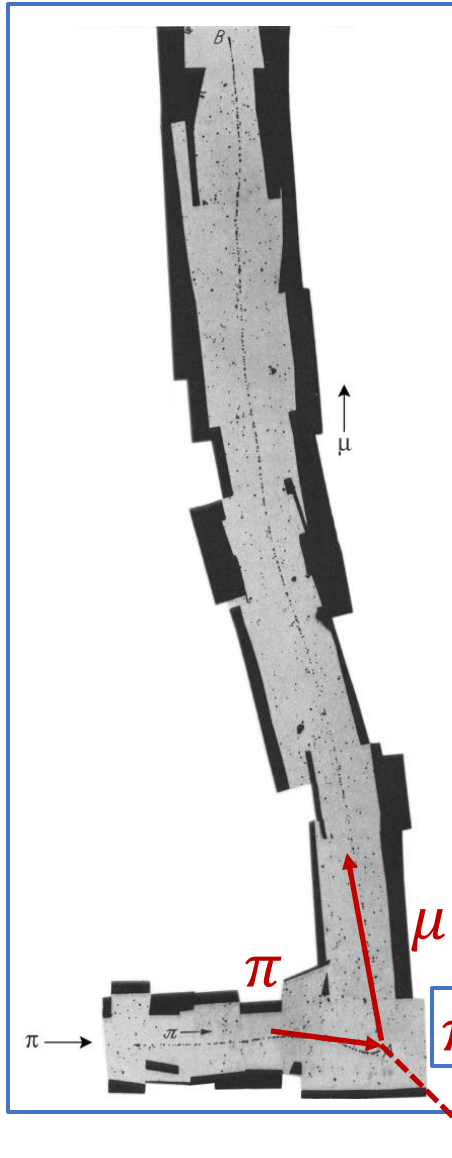
So that finally the expected energy is: $E_e = \frac{m_A^2 + m_e^2 - m_B^2}{2m_A} c^2$

Neutrinos 1930 - 1962

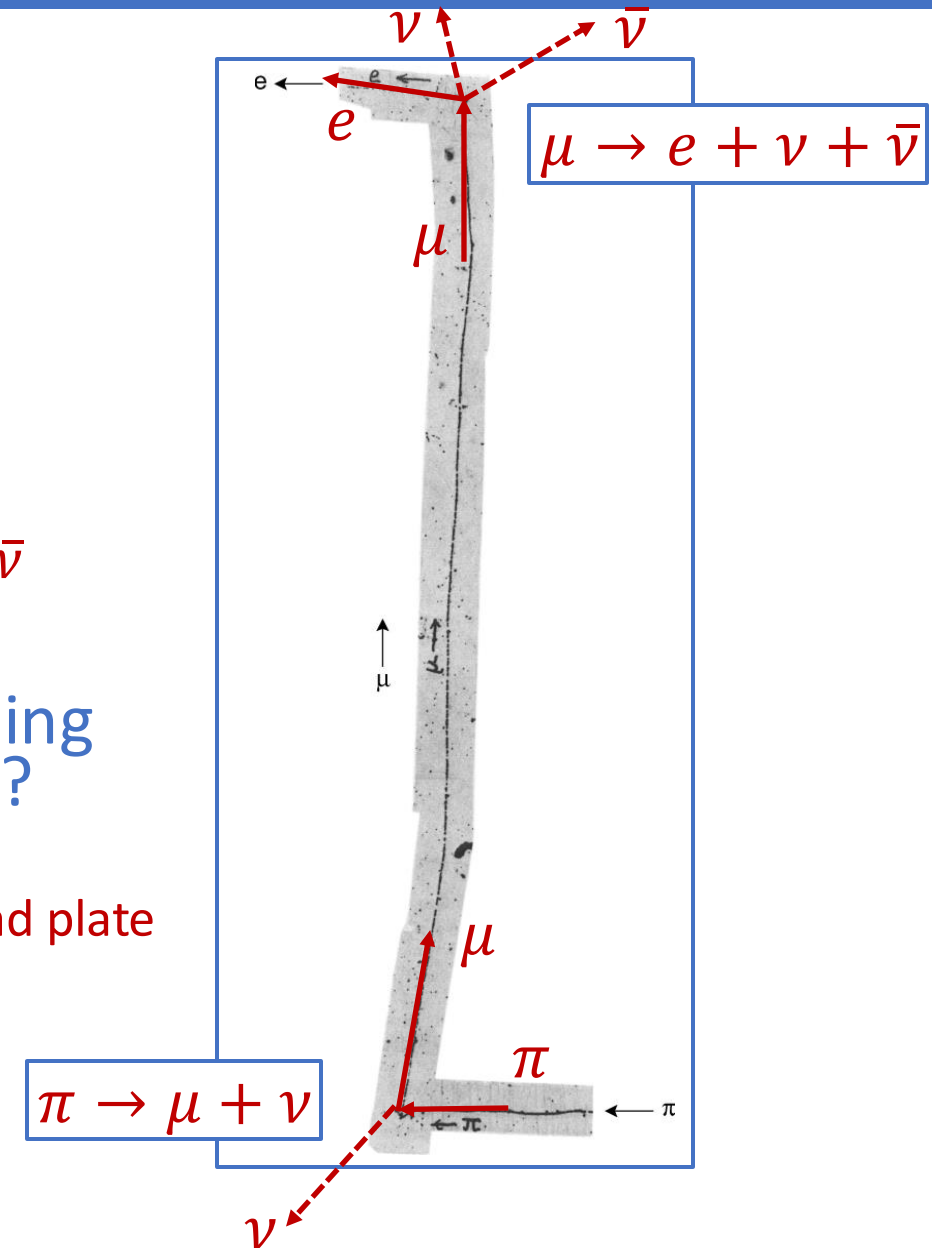


- Powell's discovery of muon and pion
 - Pion decay: $\pi \rightarrow \mu + \nu$

Neutrinos 1930 - 1962



- Powell's discovery of muon and pion
 - Pion decay: $\pi \rightarrow \mu + \nu$
- Later discovery of Powell
 - Pion *and* muon decays: $\pi \rightarrow \mu + \nu$
followed by: $\mu \rightarrow e + \nu + \bar{\nu}$
- Neutrinos: are they a bookkeeping device or are they real particles?
 - *Very weak* interaction
 - Can penetrate 1000 lightyears thick lead plate
 - What is their mass? $m_\nu \neq 0$?



Observation of neutrinos

- 1955 Cowan and Reines observe “inverse beta decay”
 - Water tank next to Nuclear Reactor
 - Antineutrinos colliding with protons: $\bar{\nu} + p^+ \rightarrow n + e^+$
 - Inverse beta decay: $n \rightarrow p^+ + e^- + \bar{\nu}$
- Are neutrinos their own antiparticles, like photons?
 - From Cowan and Reines and crossing we know that: $\nu + n \rightarrow p^+ + e^-$ must exist
 - Davis and Hamer looked for: $\bar{\nu} + n \rightarrow p^+ + e^-$ It does **not** exist
 - Neutrino is not its own antiparticle. Or is there more going on?
- What distinguishes a neutrino from an anti-neutrino?
 - A nice research topic: “Dirac” neutrinos vs “Majorana” neutrinos

Lepton Number and Lepton Flavor

- 1953, Introduction Lepton *number*

- Leptons (e^- , μ^- , ν) have quantum number $L = 1$
- Antileptons ($\bar{e} = e^+$, $\bar{\mu} = \mu^+$, $\bar{\nu}$) have quantum number $L = -1$
- Lepton number is conserved in reactions, eg:

$$\begin{array}{ccccccc} \pi^- & \rightarrow & \mu^- & + & \bar{\nu} & & \text{or} & \pi^+ & \rightarrow & \mu^+ & + & \nu \\ L: & 0 & 1 & & -1 & & & 0 & & -1 & & 1 \end{array}$$

- Lepton *Flavor*: how about: $\mu^- \rightarrow e^- + \gamma$? It is not observed!

- Each lepton ‘family’ has its own lepton number conservation, eg:

$$\begin{array}{cccc} \mu^- & \rightarrow & e^- & + & \bar{\nu}_e & + & \nu_\mu \\ L_\mu: & 1 & 0 & & 0 & & 1 \\ L_e: & 0 & 1 & & -1 & & 0 \end{array}$$

Lepton number is conserved for each type (‘generation’) separately

- Proven in 1962: $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$ was observed,

while: $\bar{\nu}_\mu + p \rightarrow e^+ + n$ was not!

- How did they know? Because these neutrino’s were produced by pion decays into a muon at Brookhaven lab.

Lepton summary 1962 - 1976

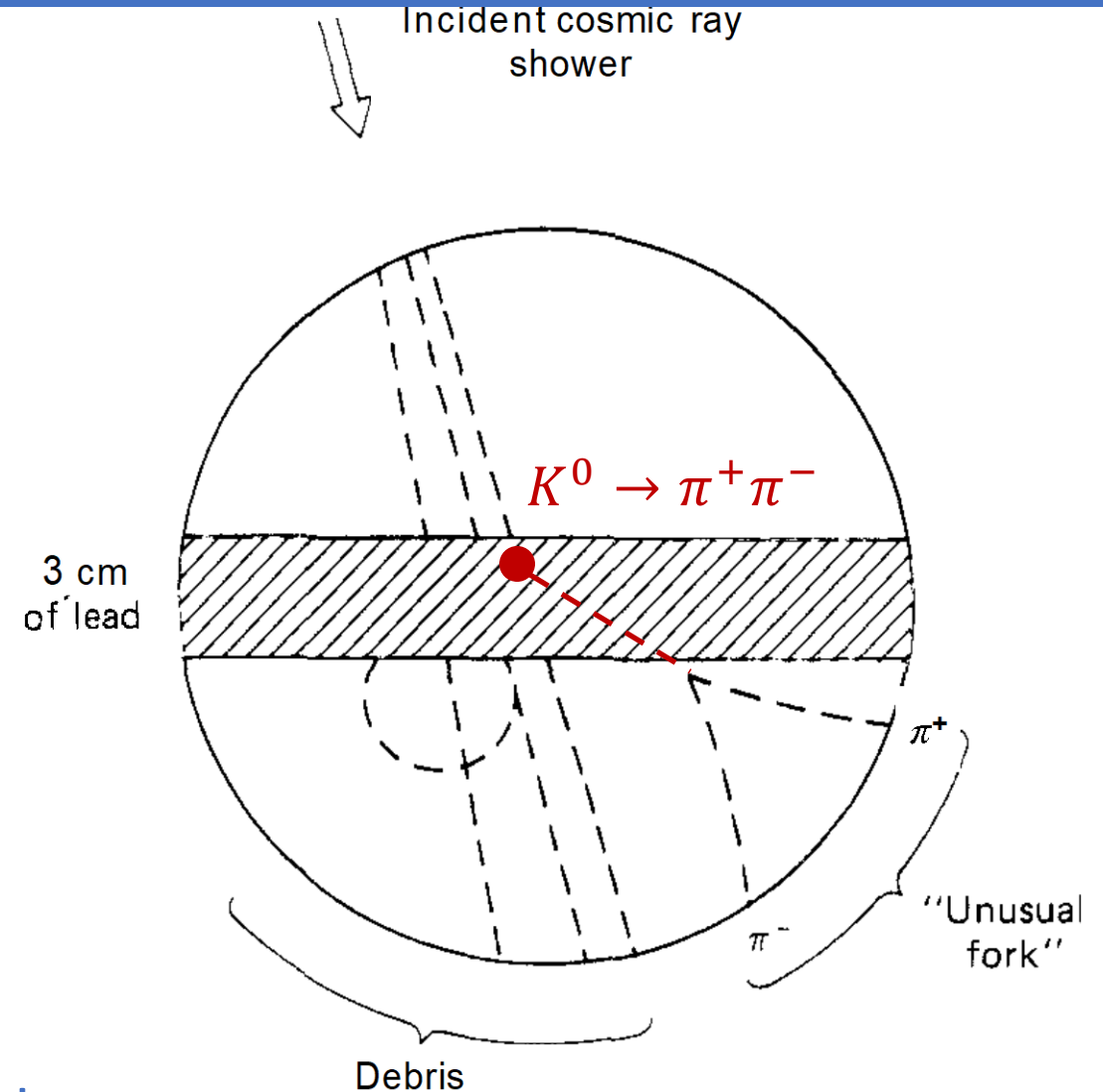
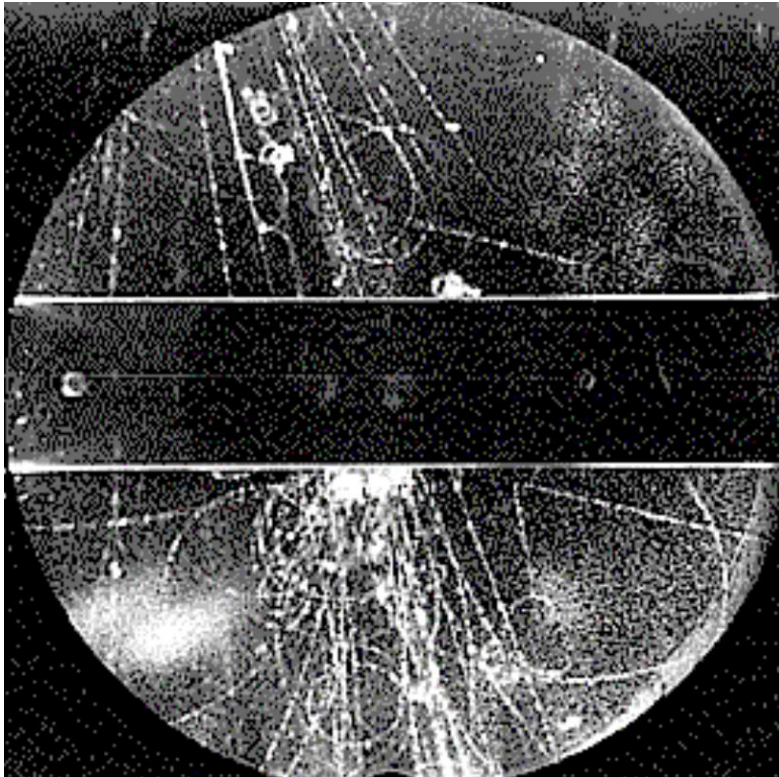
	Lepton number	Electron number	Muon number
Leptons			
e^-	1	1	0
ν_e	1	1	0
μ^-	1	0	1
ν_μ	1	0	1
Antileptons			
e^+	-1	-1	0
$\bar{\nu}_e$	-1	-1	0
μ^+	-1	0	-1
$\bar{\nu}_\mu$	-1	0	-1

- Are neutrinos massless? Not exactly → Neutrino oscillations
 - Neutrino's can oscillate to different neutrino type and back! (good research topic)

Strangeness (1947 – 1960)

Griffiths §1.6

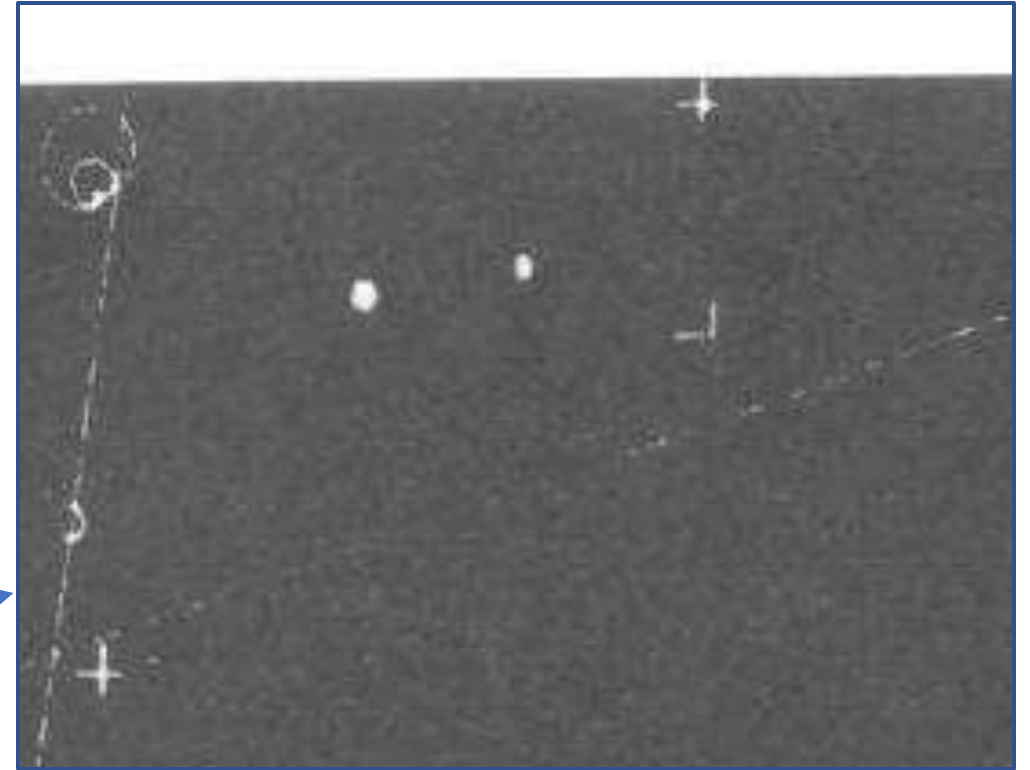
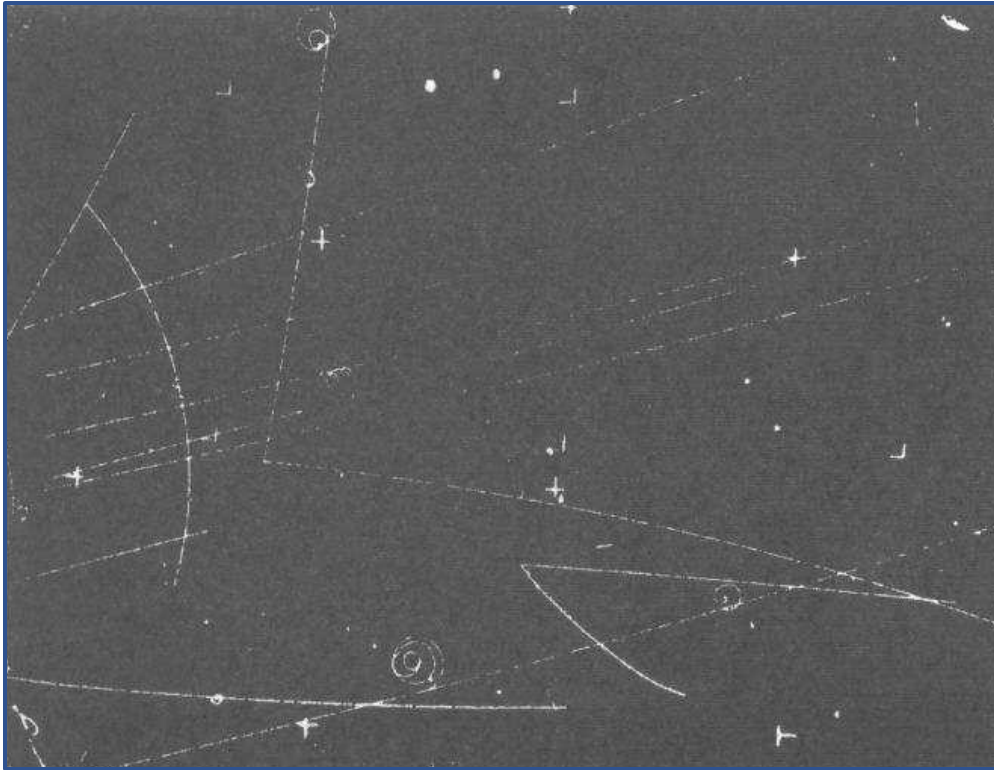
- Rochester and Butler observed strange “V-prongs” in cosmic rays:



- Strange: produced copiously, but decay slowly!
 - Abraham Pais: They are produced with the strong force, but they decay with the weak force.

Similarly observed: $\Lambda \rightarrow p^+ \pi^-$

Example of associated strangeness production



Bubble chamber picture of the associated reaction $\pi^- + p \rightarrow K^0 + \Lambda$. Incoming pion is indicated by the arrow. The unseen neutral particles K^0 and Λ are detected by their “ V^0 ” decays: $K^0 \rightarrow \pi^+ + \pi^-$ and $\Lambda \rightarrow \pi^- + p$.

It turns out these strange particles are always produced in pairs.

Strangeness

- Also observed: $K^+ \rightarrow \pi^+ + \pi^+ + \pi^- \rightarrow M_K \sim 500 \text{ MeV}$; it's a meson
- 1952, Brookhaven cosmotron: new strange *baryons*: Σ ($\sim 1.2 \text{ GeV}$), Ξ ($\sim 1.3 \text{ GeV}$)
- In production (strong interaction) conserve *baryon number* as well as *strangeness*

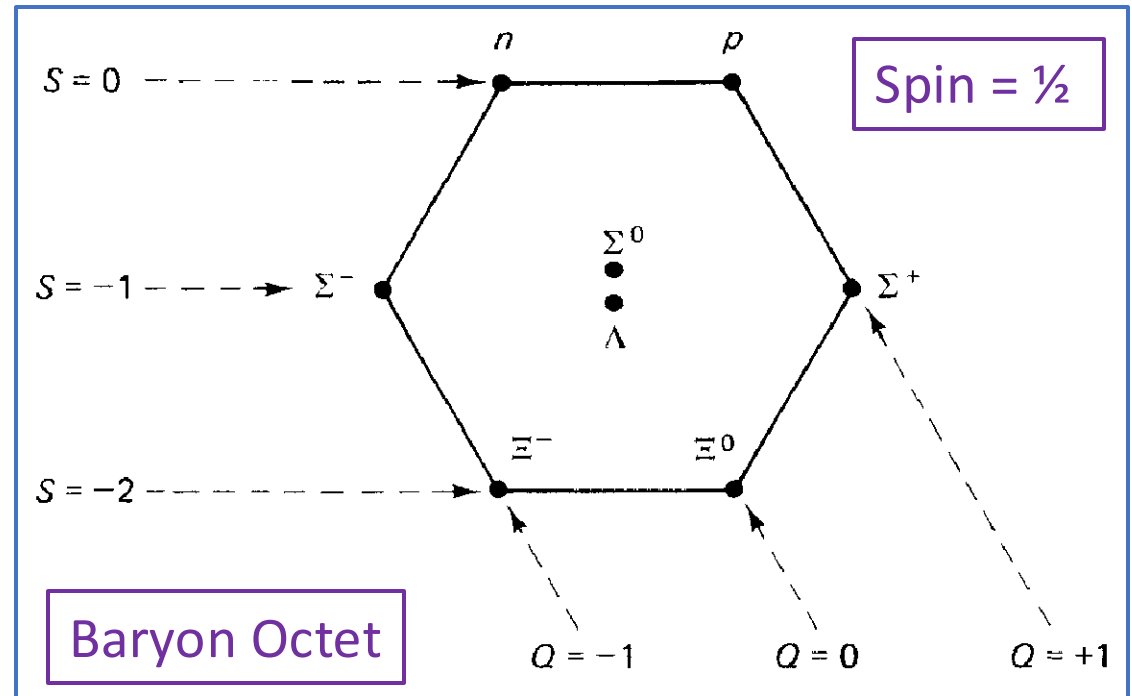
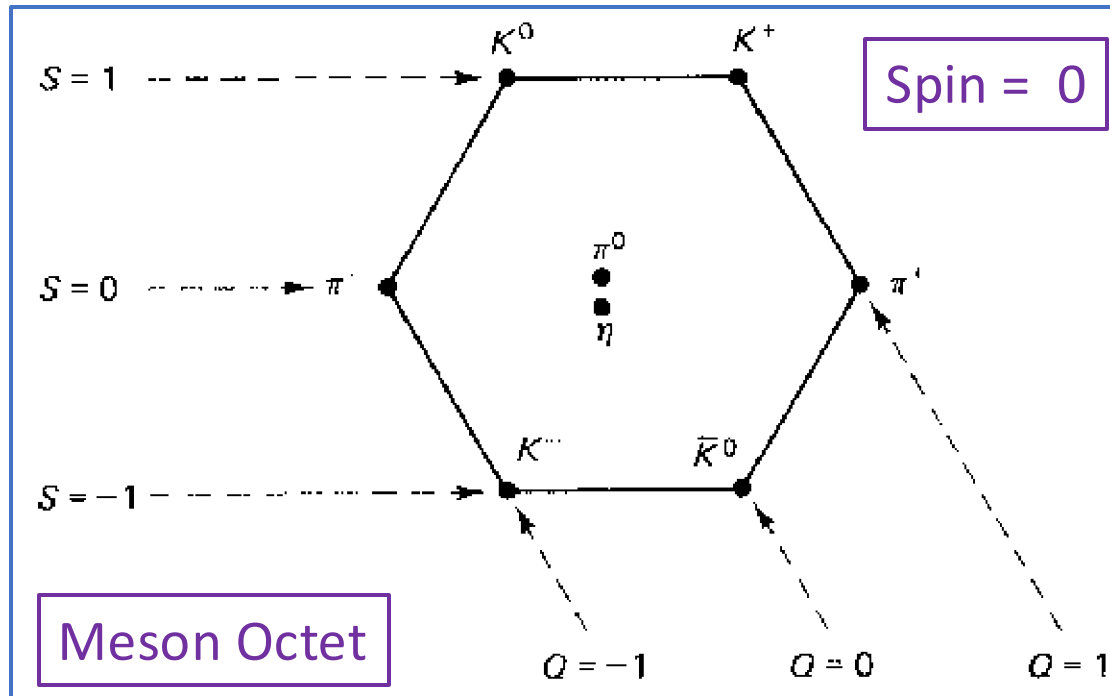
	B:	S:	OK?
$\pi^- + p \rightarrow K^+ + \Sigma^-$	$0 + 1 \rightarrow 0 + 1$	$0 + 0 \rightarrow 1 + -1$	Yes
$\rightarrow K^0 + \Lambda$	$0 + 1 \rightarrow 0 + 1$	$0 + 0 \rightarrow 1 + -1$	Yes
$\rightarrow \pi^+ + \Sigma^-$	$0 + 1 \rightarrow 0 + 1$	$0 + 0 \rightarrow 0 + -1$	No
$\rightarrow K^+ + K^-$	$0 + 1 \rightarrow 0 + 0$	$0 + 0 \rightarrow 1 + -1$	No

- In decay (weak force) *strangeness is not conserved*: long lifetimes
eg. $\Delta s = 1$ decays: $\Lambda \rightarrow p^+ + \pi^-$; $\Sigma^+ \rightarrow n + \pi^+$
- Electric charge is always conserved!

The Particle Zoo and the Eightfold Way (1961 - 1964)

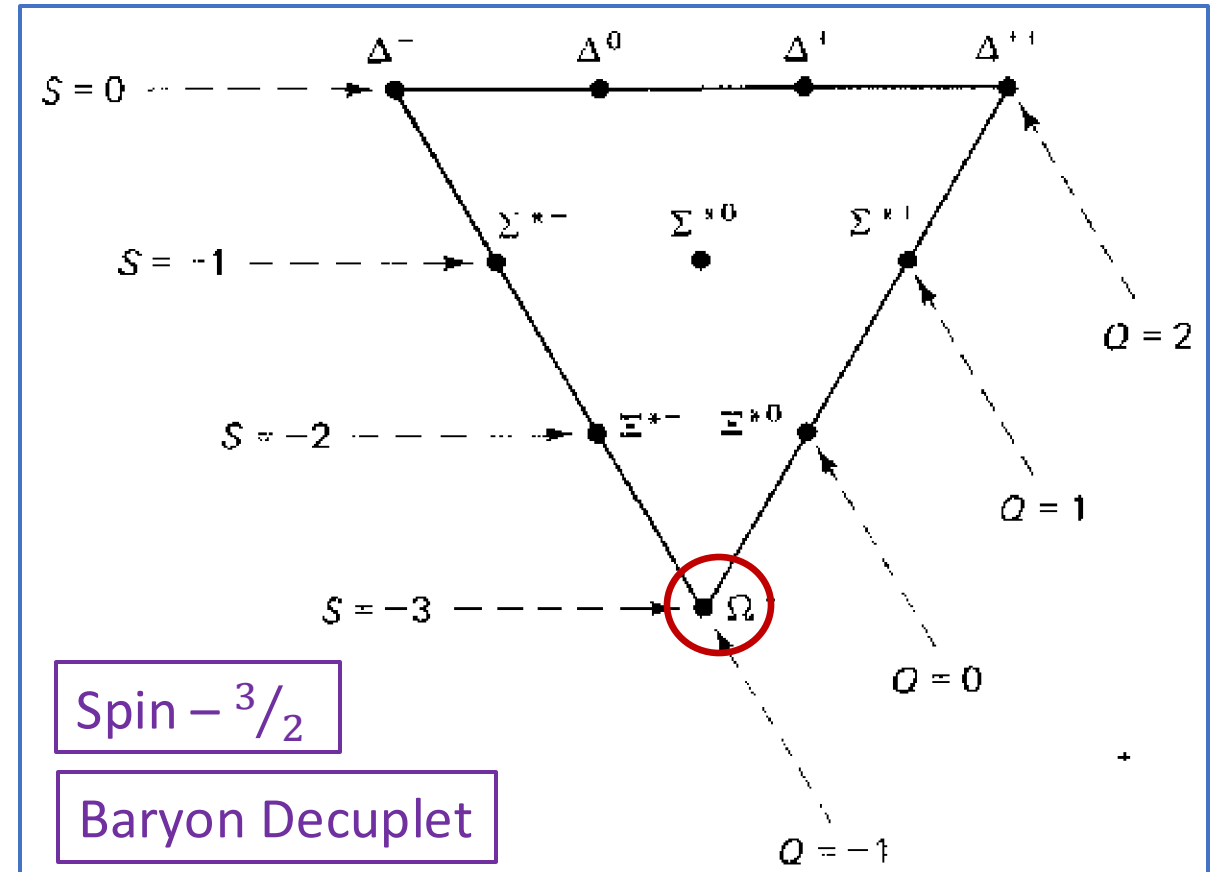
Griffiths §1.7

- So many particles! *"The finder of a new elementary particle should be punished by a 10,000 \$ fine!"* - Willis Lamb, 1955.
- Gell-Mann (1961): Make a classification a la Periodic Table

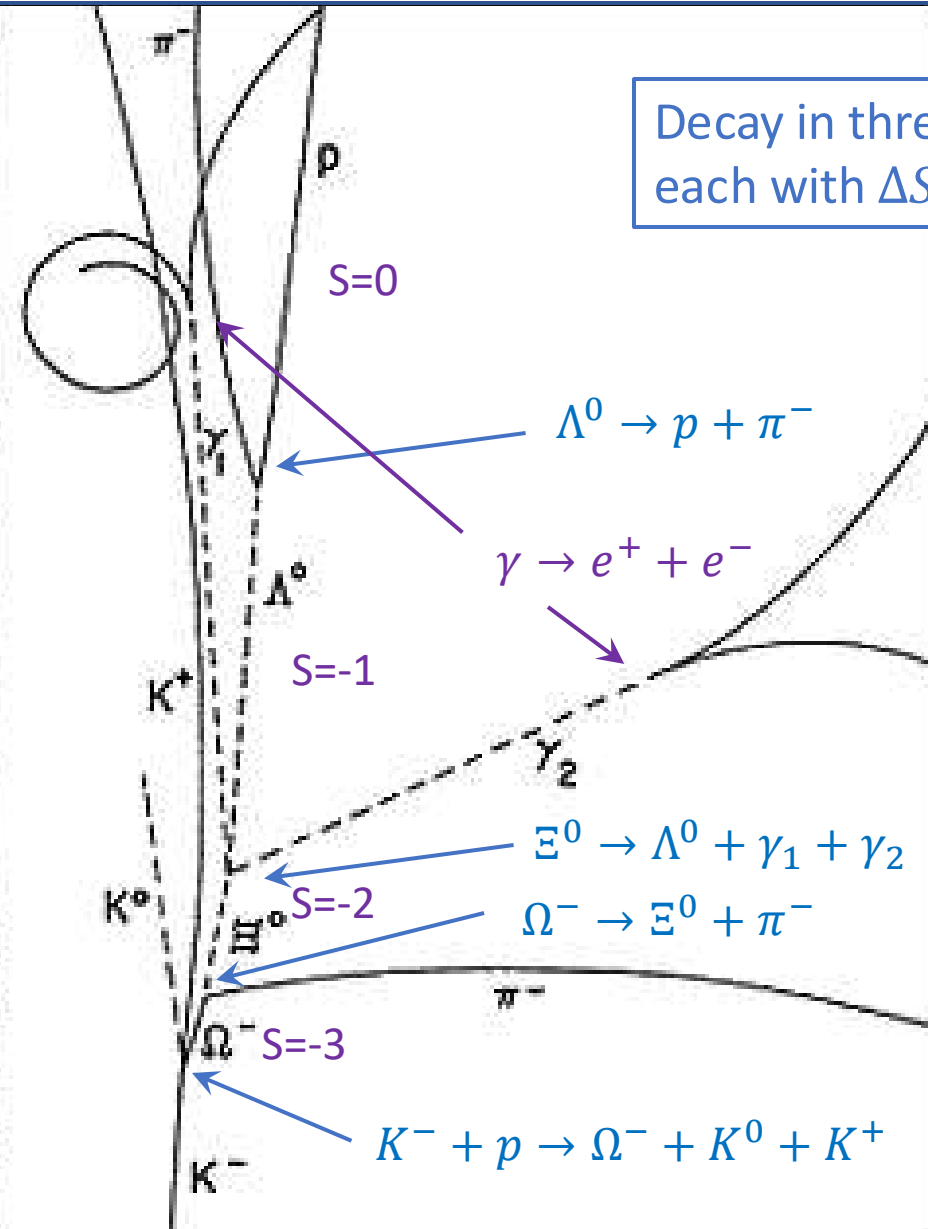


The Particle Zoo and the Eightfold Way (1961 - 1964)

- So many particles! *“The finder of a new elementary particle should be punished by a 10,000 \$ fine!”* - Willis Lamb, 1955.
- Gell-Mann (1961): Make a classification a la Periodic Table
- For heavier baryon particles (spin = $3/2$) there is a decuplet
 - The reason that spin- $1/2$ baryons form an octet and spin- $3/2$ a decuplet is far from trivial.
- The Ω particle was not known yet:
 - Mass and lifetime was predicted
 - Observed in 1964
- Octets and decuplets suggest an underlying structure! – quarks!



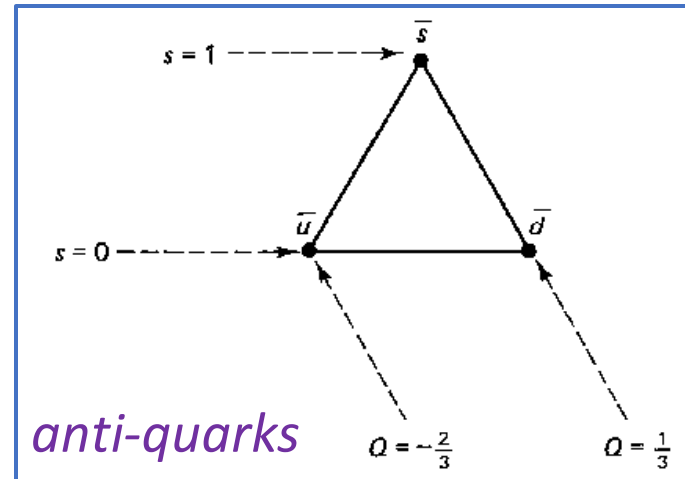
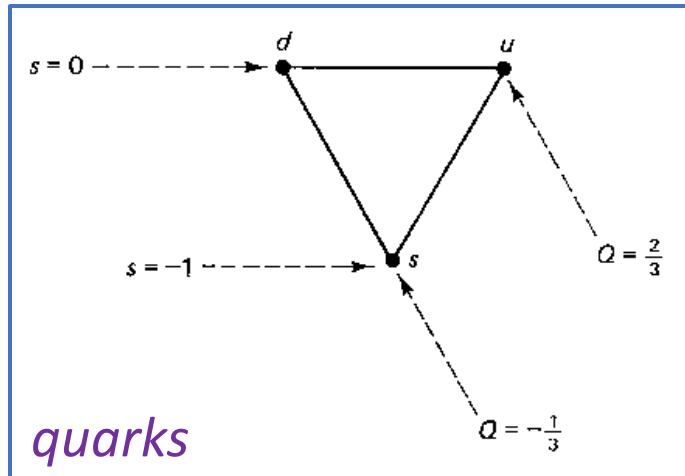
Discovery of the Omega particle (1964)



The Quark Model (1964)

Griffiths §1.8

- Gell-Mann and Zweig independently proposed that elementary particles are composed of spin- $\frac{1}{2}$ *quarks: up, down, strange*:



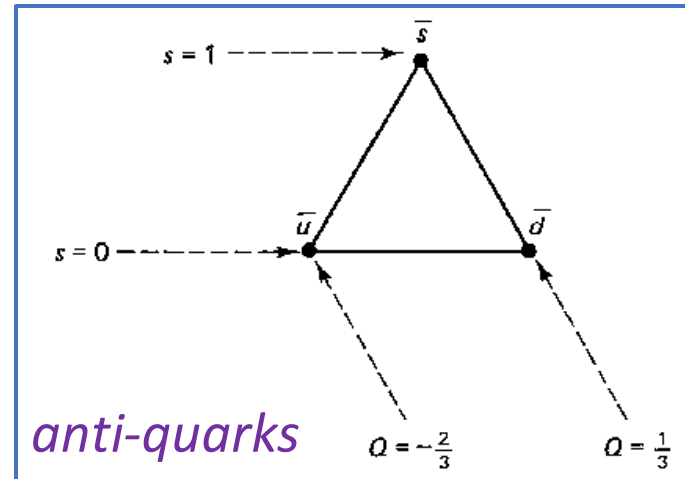
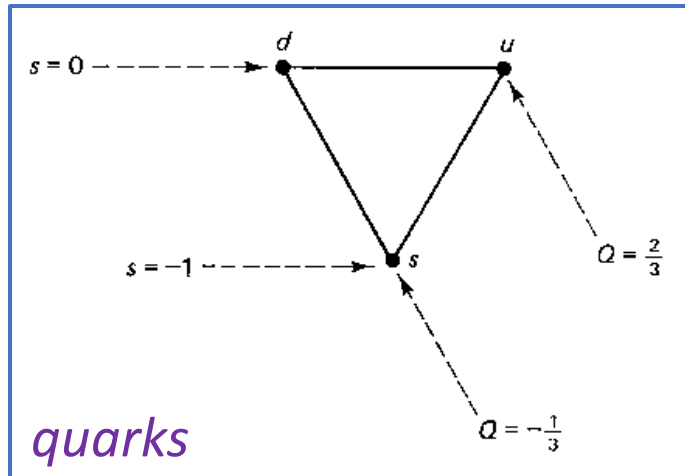
- Baryons: particle consisting of 3 quarks or antiquarks ($q q q$) or ($\bar{q} \bar{q} \bar{q}$)
 - Proton= uud , neutron= udd , etc
- Mesons: particles consisting of 1 quark and 1 anti-quarks: ($q \bar{q}$)
 - Pions: $\pi^+ = u\bar{d}$, $\pi^0 = u\bar{u} + d\bar{d}$, $\pi^- = d\bar{u}$
 - Kaons: $K^+ = u\bar{s}$, etc

Explains “elementary particles” as quark combinations, spin, strangeness etc.

The Quark Model (1964)

Details in Griffiths §5.5 and §5.6

- Gell-Mann and Zweig independently proposed that elementary particles are composed of spin- $\frac{1}{2}$ *quarks: up, down, strange*:



The meson nonet			
$q\bar{q}$	Q	S	Meson
$u\bar{u}$	0	0	π^0
$u\bar{d}$	1	0	π^+
$d\bar{u}$	-1	0	π^-
$d\bar{d}$	0	0	η
$u\bar{s}$	1	1	K^+
$d\bar{s}$	0	1	K^0
$s\bar{u}$	-1	-1	K^-
$s\bar{d}$	0	-1	$\bar{K}^0$
$s\bar{s}$	0	0	??

The baryon decuplet			
qqq	Q	S	Baryon
uuu	2	0	Δ^{++}
uud	1	0	Δ^+
udd	0	0	Δ^0
ddd	-1	0	Δ^-
uus	1	-1	Σ^{*+}
uds	0	-1	Σ^{*0}
dds	-1	-1	Σ^{*-}
uss	0	-2	Ξ^{*0}
dss	-1	-2	Ξ^{*-}
sss	-1	-3	Ω^-

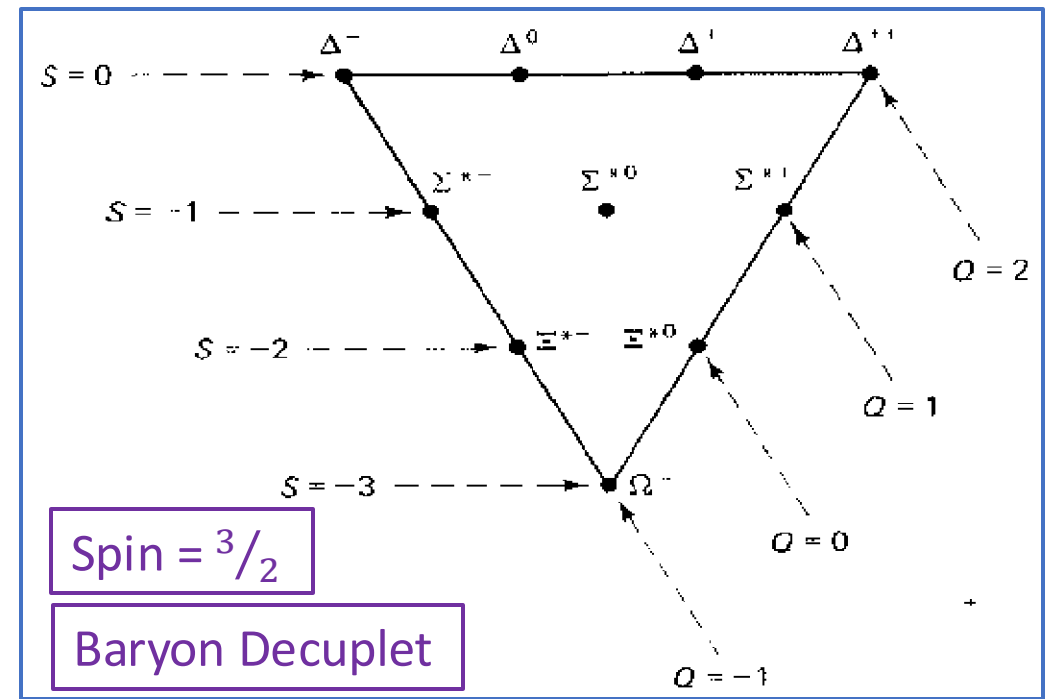
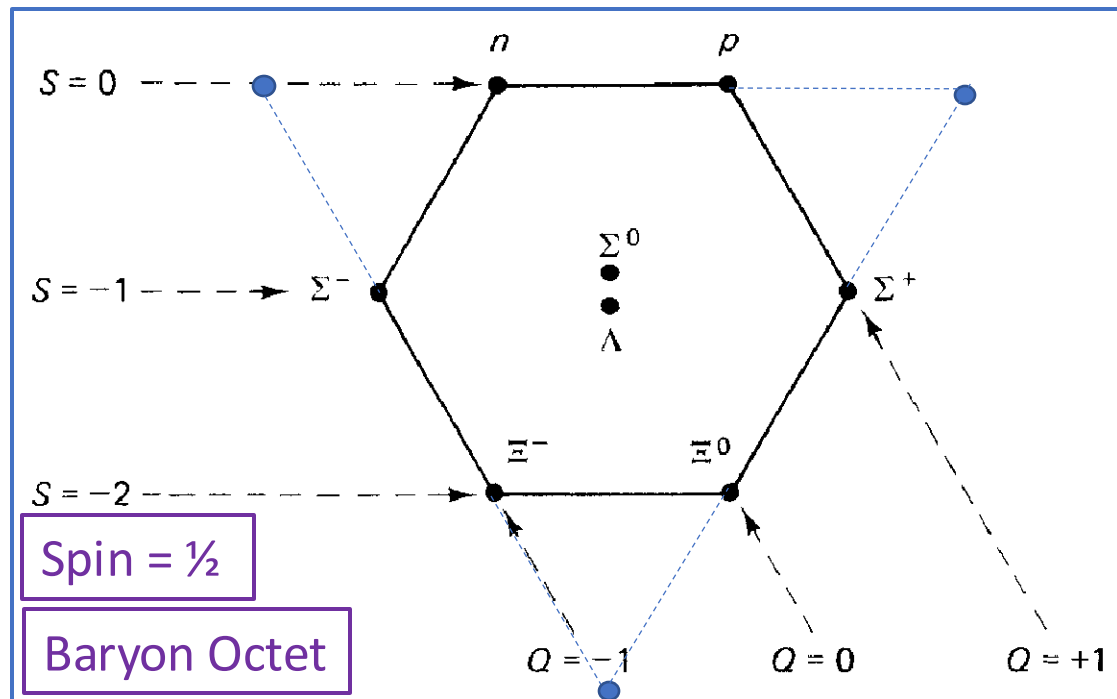
- Complications: η' singlet, baryon octet:

Griffiths §5.5

- Spin and (anti-)symmetrization of wave functions!*

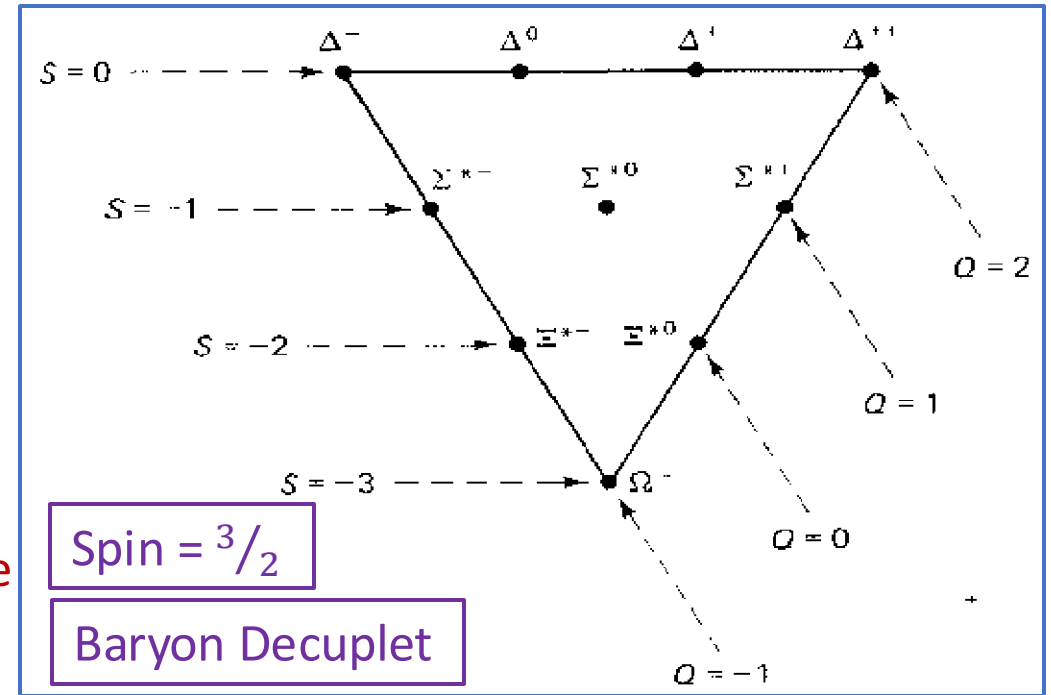
Complications

- The spin- $\frac{1}{2}$ octet is like the spin- $\frac{3}{2}$ decuplet with the corners missing.
- Wave function has:
 - 3 identical quarks ($u u u$), ($d d d$), or ($s s s$)
 - 3 identical spin directions of quarks: $\uparrow\uparrow\uparrow$ or $\downarrow\downarrow\downarrow$



Desperate Measures

- A quantum mechanical problem: check Δ^{++} :
 - Wave function has:
 - 3 identical quarks ($u u u$), ($d d d$), or ($s s s$)
 - 3 identical spin directions of quarks: $\uparrow\uparrow\uparrow$ or $\downarrow\downarrow\downarrow$
 - Not allowed by Pauli-exclusion principle:
 - This wave function is symmetric under particle exchange
 - *Identical* fermion particles must have an asymmetric wave function under exchange of the particles



- Quarks differ by having an additional quantum number: **color**
 - A quark can have quantum number: “red”, “green”, “blue”
 - An anti-quark can carry quantum number: “anti-red”, “anti-green”, “anti-blue”
- All physical particles must be colorless (“confinement”):
 - baryons $\rightarrow r g b$, mesons $\rightarrow r \bar{r}$, $g \bar{g}$, or $b \bar{b}$

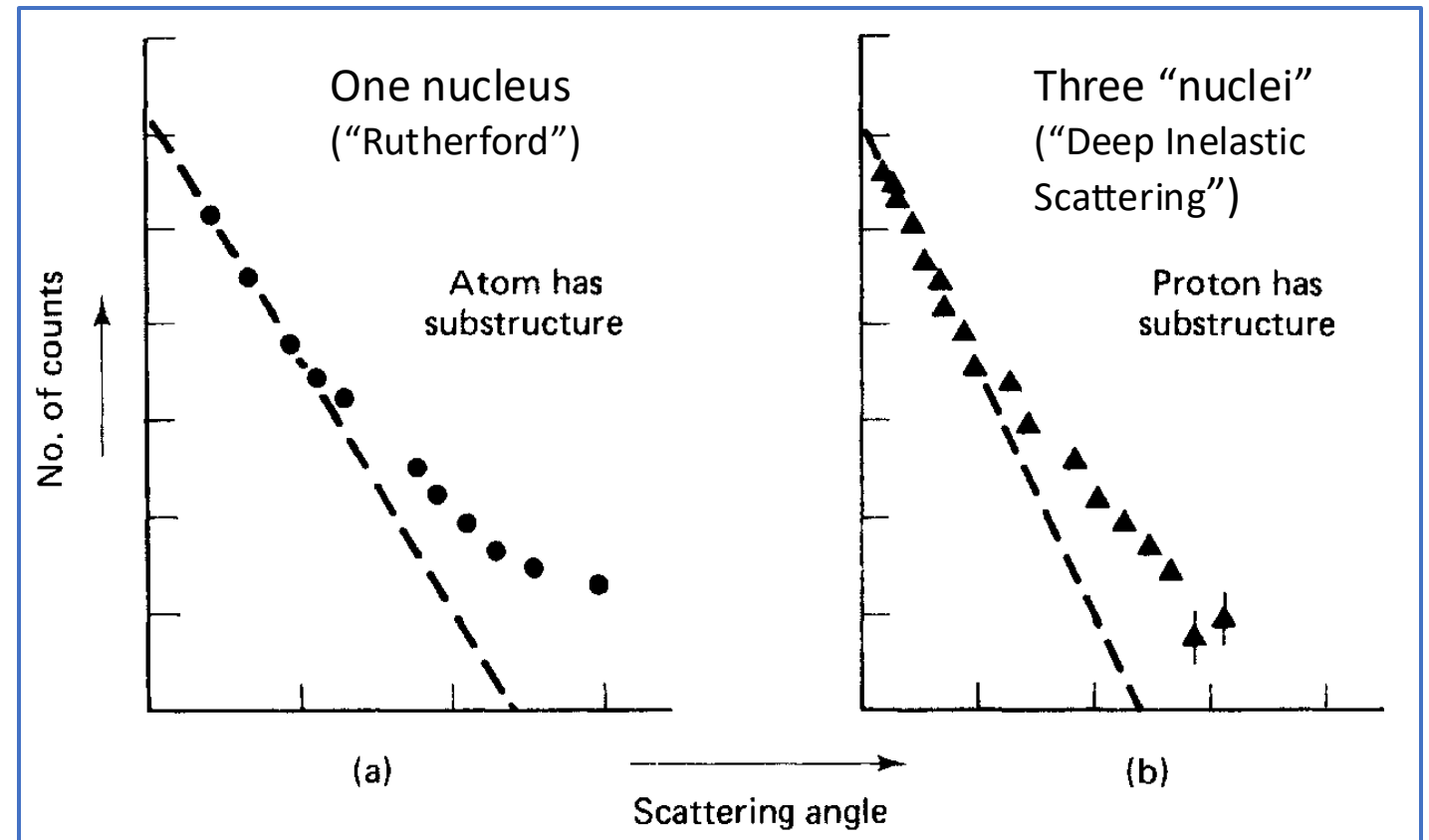
No free quarks in nature!

Deep Inelastic Scattering (“DIS”)

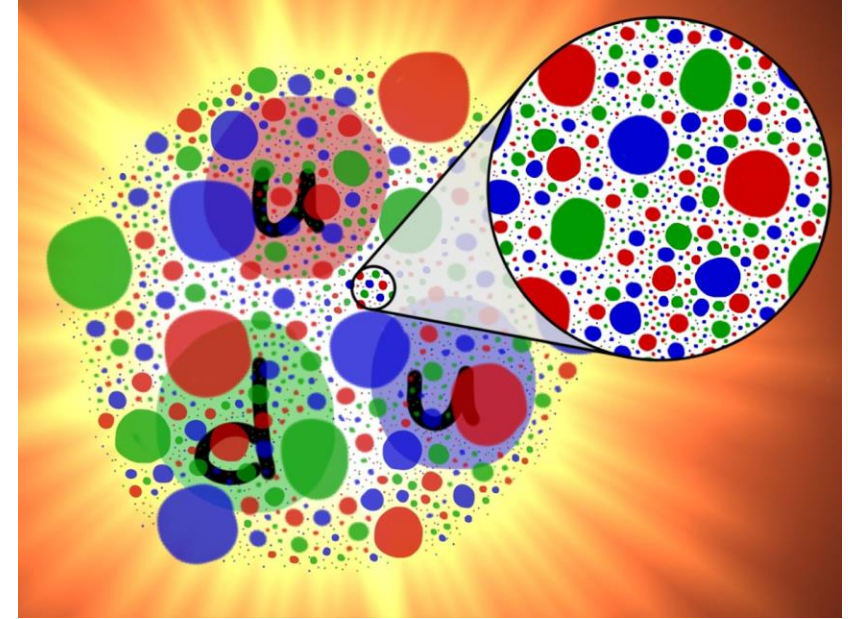
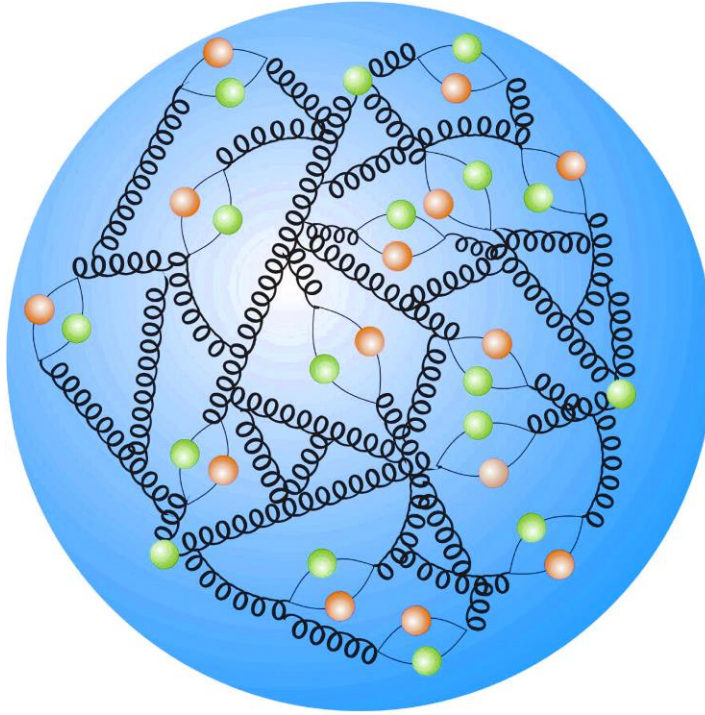
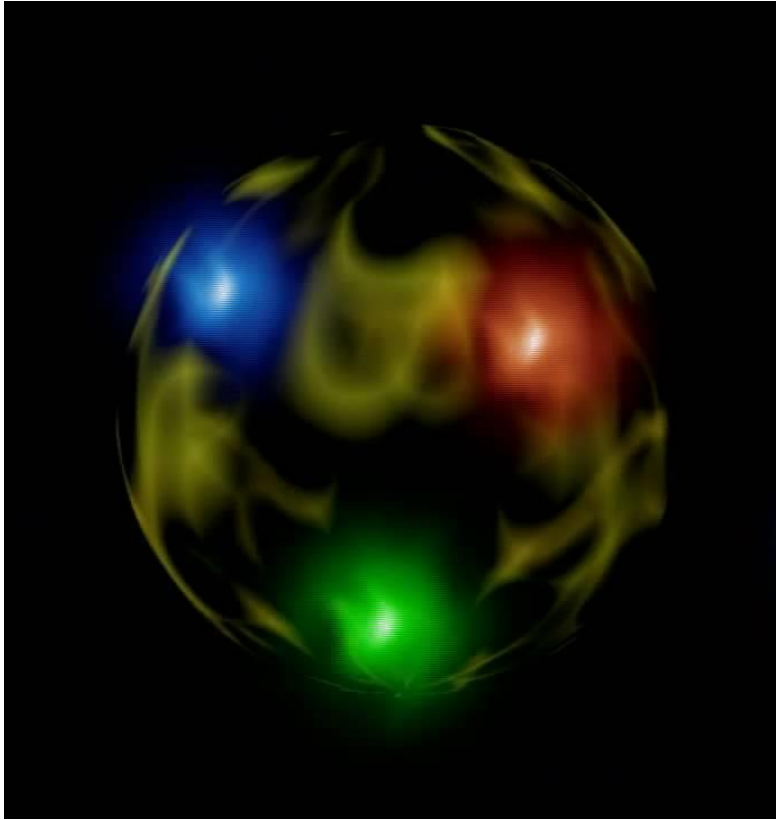
- Indeed individual quarks have never been observed.
 - Do they actually exist or are they just a mathematical bookkeeping tool?
- ~1970 SLAC: “DIS”
 - Shoot electrons at protons
Proton substructure is seen
similar to Rutherford scattering
- Quarks are real and they carry an additional quantum number which can have three values
 - “Color” is just a name

Research:

what is the principle behind confinement, ie. being colorless?



How to make a mental picture of a proton?

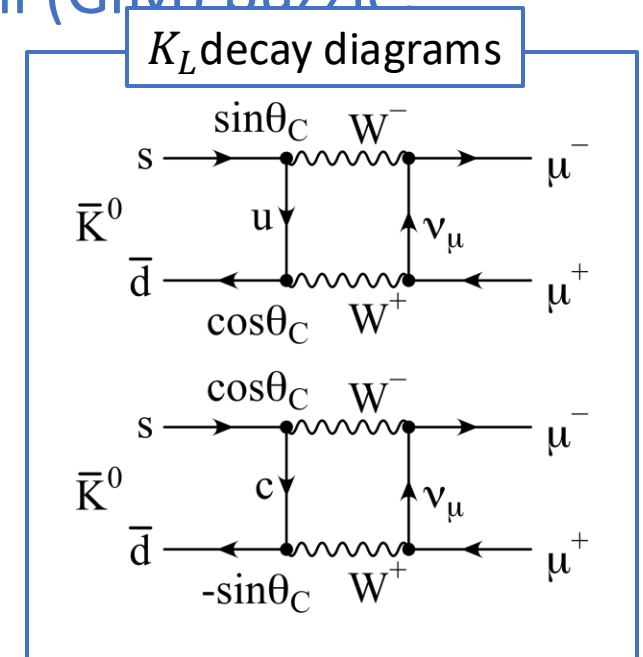


- Looking from "afar" there are three colored quarks: red, green, blue
- Zooming in it becomes a quantum foam of particles + antiparticles + force particles (gluons)

The November Revolution: 1974 - 1983

Griffiths §1.9

- Fundamental particles:
 - Why 4 Leptons: (e, ν_e) (μ, ν_μ) and 3 Quarks : (u, d, s) ?
 - 1974 Two groups discover a new particle: Ting at BNL (“ J ”) and Richter at SLAC (“ ψ ”)
 - J/ψ particle is special: very heavy $M \sim 3000 \text{ MeV}$, long lifetime $\tau \sim 1000 \times$ longer than heavy hadrons
 - ➔ new meson consisting of heavy charm quarks: $c\bar{c}$ ➔ Enter: **charm** quantum number
- Existence of charm also solved the Glashow, Iliopoulos, Maiani (GIM) puzzle
 - It explained why the K_L particle had a very long lifetime
- Other new particles were discovered soon afterwards
 - Mesons: $D^0 (c\bar{u})$, $D^+ (c\bar{d})$, $D_s^+ (c\bar{s})$, ...
 - Baryons: $\Sigma_c (uuc)$, $\Lambda_c (udc)$, $\Xi_c (usc)$, $\Omega_c (ssc)$, ...
- Similar to “strangeness”, “charm” is conserved in production (strong force), and violated in (weak force) decay



GIM: theory research project

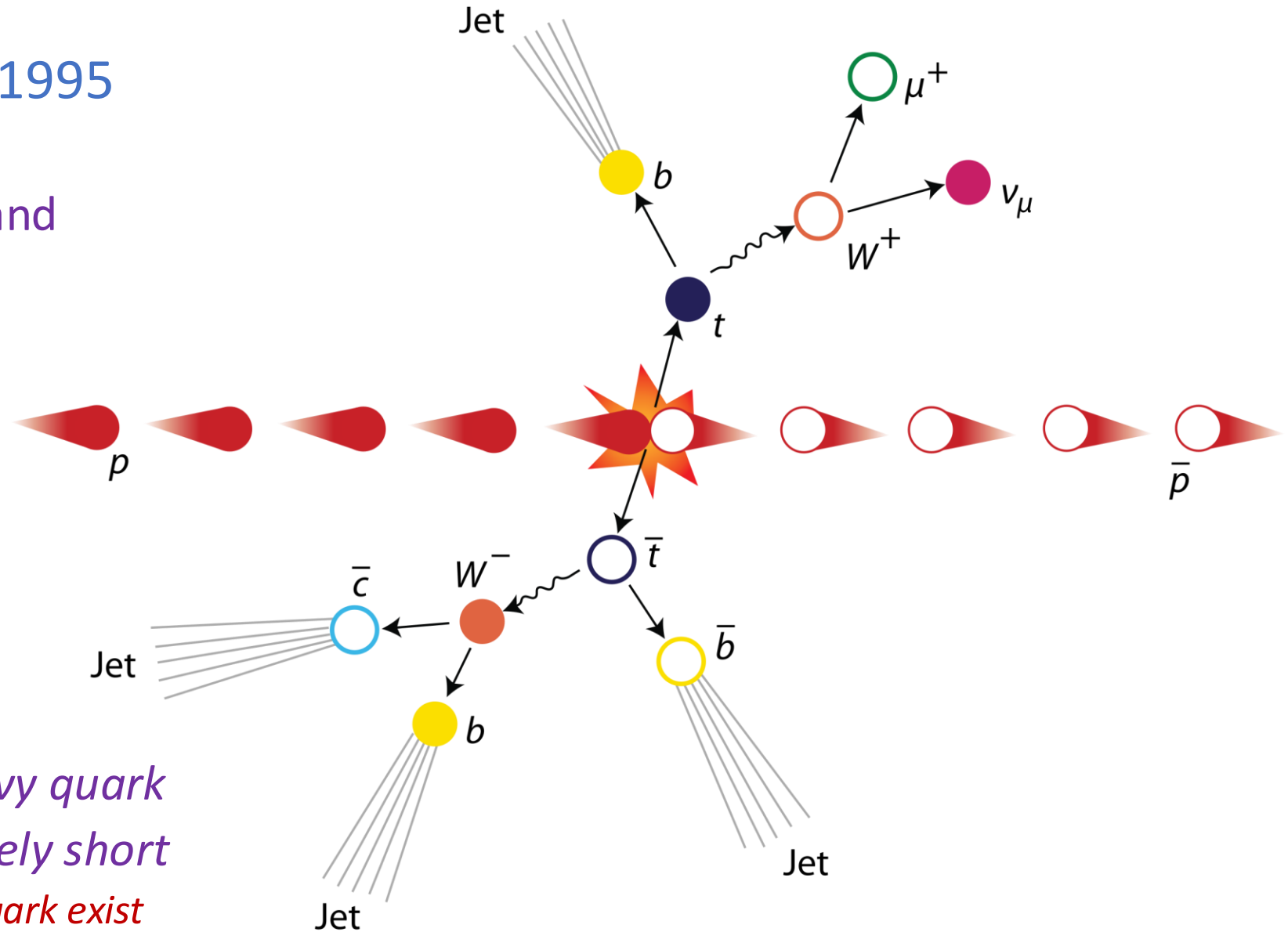
The November Revolution: 1974 - 1983

- 1974 Discovery of charm brings Lepton-Quark symmetry:
 - (ν_e, e) (ν_μ, μ) and (u, d) (c, s)
- 1975 Discovery of the τ - lepton breaks symmetry again
 - Also a corresponding ν_τ neutrino
- 1977 Discovery Υ ($b\bar{b}$) particle, consisting of *bottom* or *beauty* quarks
 - *Baryons*: 1980: $\Lambda_b^0(udb)$, 2006: $\Sigma_b^+(uub)$, 2007: $\Xi_b^-(dsb)$,
 - *Mesons*: 1983: $B^0(\bar{b}d)$, $B^-(b\bar{u})$, later also: $B_s^0(\bar{b}s)$, $B_c^+(\bar{b}c)$
 - First $B_c^+ \rightarrow B_s^0$ weak decay observed by J.A. de Vries *et al.*
- Terminology: upness, downness, strangeness, charm, beauty = “*Flavour*”
 - Quarks have *charge*, *flavor* and *color* quantum numbers
- Now 6 leptons and 5 quarks?
 - (ν_e, e) (ν_μ, μ) (ν_τ, τ) and (u, d) (c, s) $(b, \dots)$

Top quark

- Discovered at Fermilab 1995

- $p + \bar{p} \rightarrow t + \bar{t}$
- Top decays immediately and form so-called *jets*



- Top quark:

- $m_t = 176 \text{ GeV}$, very heavy quark
- $\tau_t = 5 \times 10^{-25} \text{ s}$, extremely short
 - No bound states with top quark exist

Top quark : $t\bar{t}$ resonance? Toponium?

Available on the CERN CDS information server

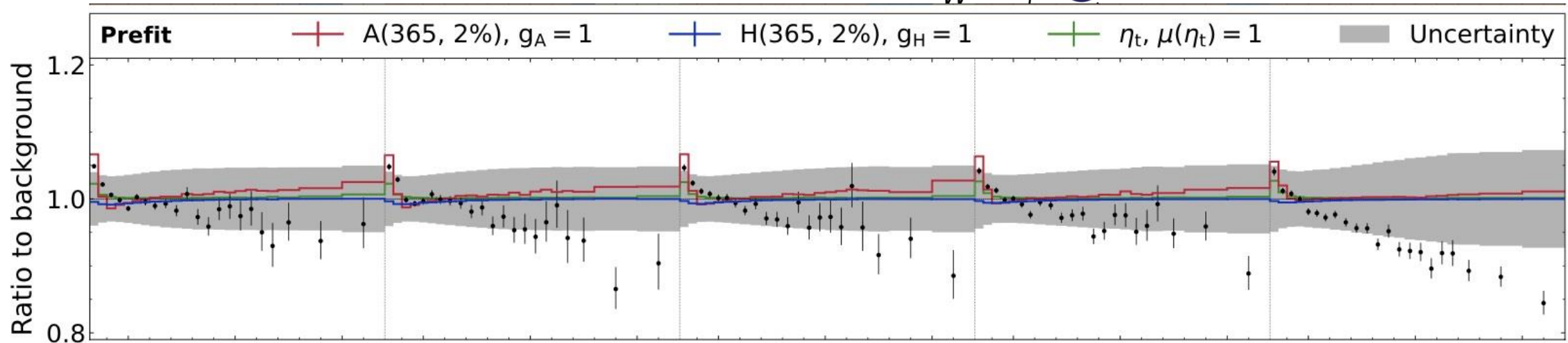
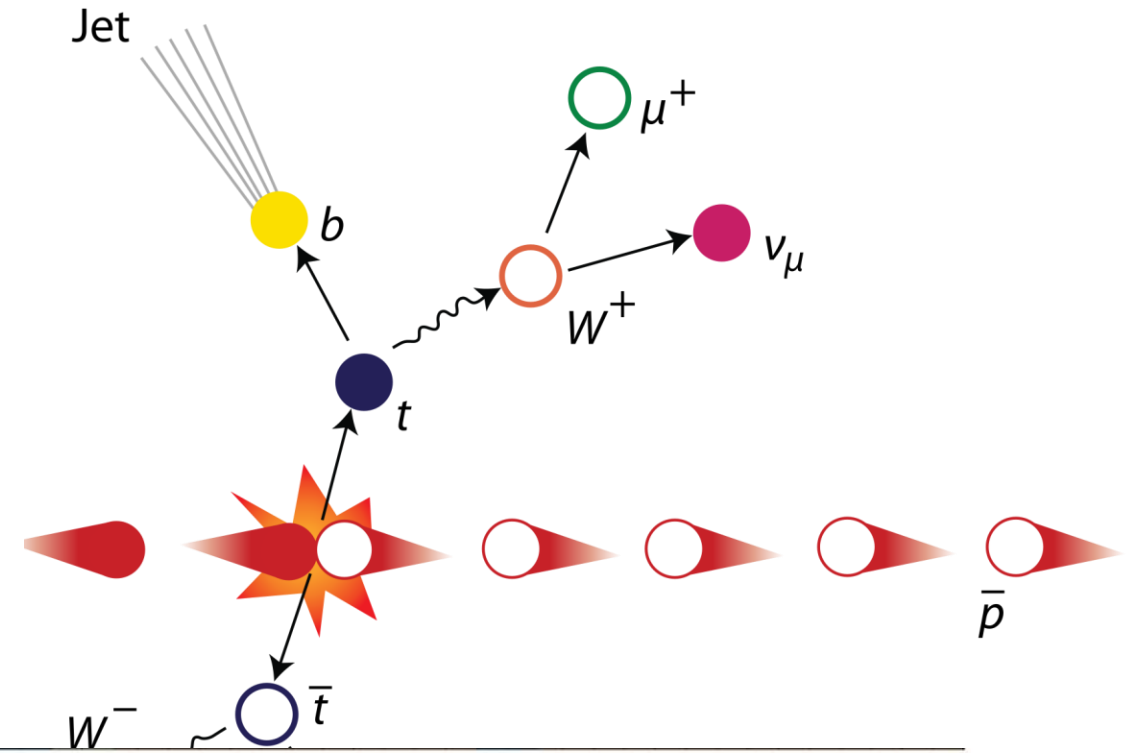
CMS PAS HIG-22-013

CMS Physics Analysis Summary

Contact: cms-pag-conveners-higgs@cern.ch

2024/09/28

Search for heavy pseudoscalar and scalar bosons decaying
to top quark pairs in proton-proton collisions at
 $\sqrt{s} = 13\text{ TeV}$



Standard Model of particles

Griffiths §1.10 and §1.11

Standard Model of Elementary Particles

three generations of matter (elementary fermions)			three generations of antimatter (elementary antifermions)			interactions / force carriers (elementary bosons)		
	I	II	III	I	II	III		
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 124.97 \text{ GeV}/c^2$
	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0
charge	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
spin	u	c	t	$\bar{u}$	$\bar{c}$	$\bar{t}$	g	H
	up	charm	top	antiup	anticharm	antitop	gluon	higgs
mass	$\approx 4.7 \text{ MeV}/c^2$	$\approx 96 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	$\approx 4.7 \text{ MeV}/c^2$	$\approx 96 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	
charge	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
spin	d	s	b	$\bar{d}$	$\bar{s}$	$\bar{b}$	γ	
	down	strange	bottom	antidown	antistrange	antibottom	photon	
mass	$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 91.19 \text{ GeV}/c^2$	
	-1	-1	-1	1	1	1	0	
charge	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
spin	e	μ	τ	e^+	$\bar{\mu}$	$\bar{\tau}$	Z	
	electron	muon	tau	positron	antimuon	antitau	Z ⁰ boson	
mass	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$
	0	0	0	0	0	0	1	-1
charge	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1
spin	ν_e	ν_μ	ν_τ	$\bar{\nu}_e$	$\bar{\nu}_\mu$	$\bar{\nu}_\tau$	W^+	W^-
	electron neutrino	muon neutrino	tau neutrino	electron antineutrino	muon antineutrino	tau antineutrino	W ⁺ boson	W ⁻ boson

QUARKS

LEPTONS

GAUGE BOSONS
VECTOR BOSONS

SCALAR BOSONS

Current Fundamental Questions in Particle physics

- Why are there three generations of particles?
- Why are the masses of the particles what they are?
- Why is there no antimatter in our universe?
- Why is the charge of the electron exactly opposite to that of the proton?
Or: why is the sum of the charge of leptons + quarks exactly equal to zero?
- Is a neutrino its own anti-particle?
- Can all forces be described in a single theory? Unification
- Are quarks and leptons truly fundamental
- What is the source of dark matter?
- What is the source of dark energy?

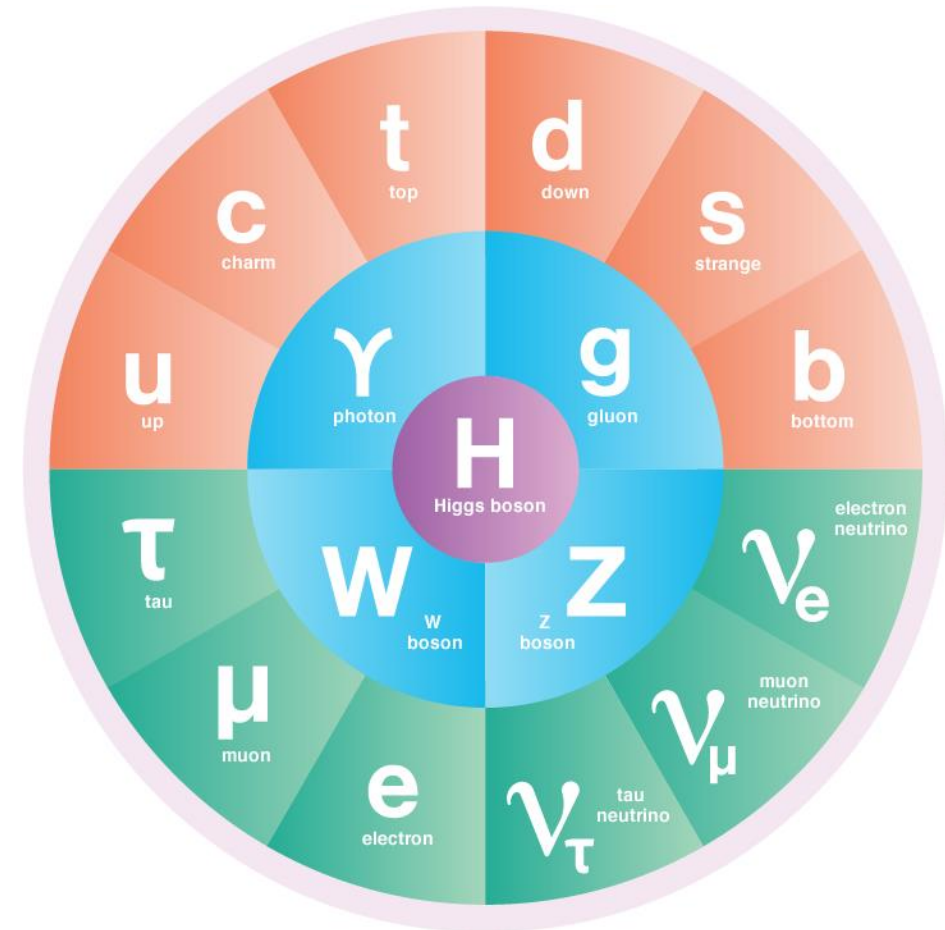
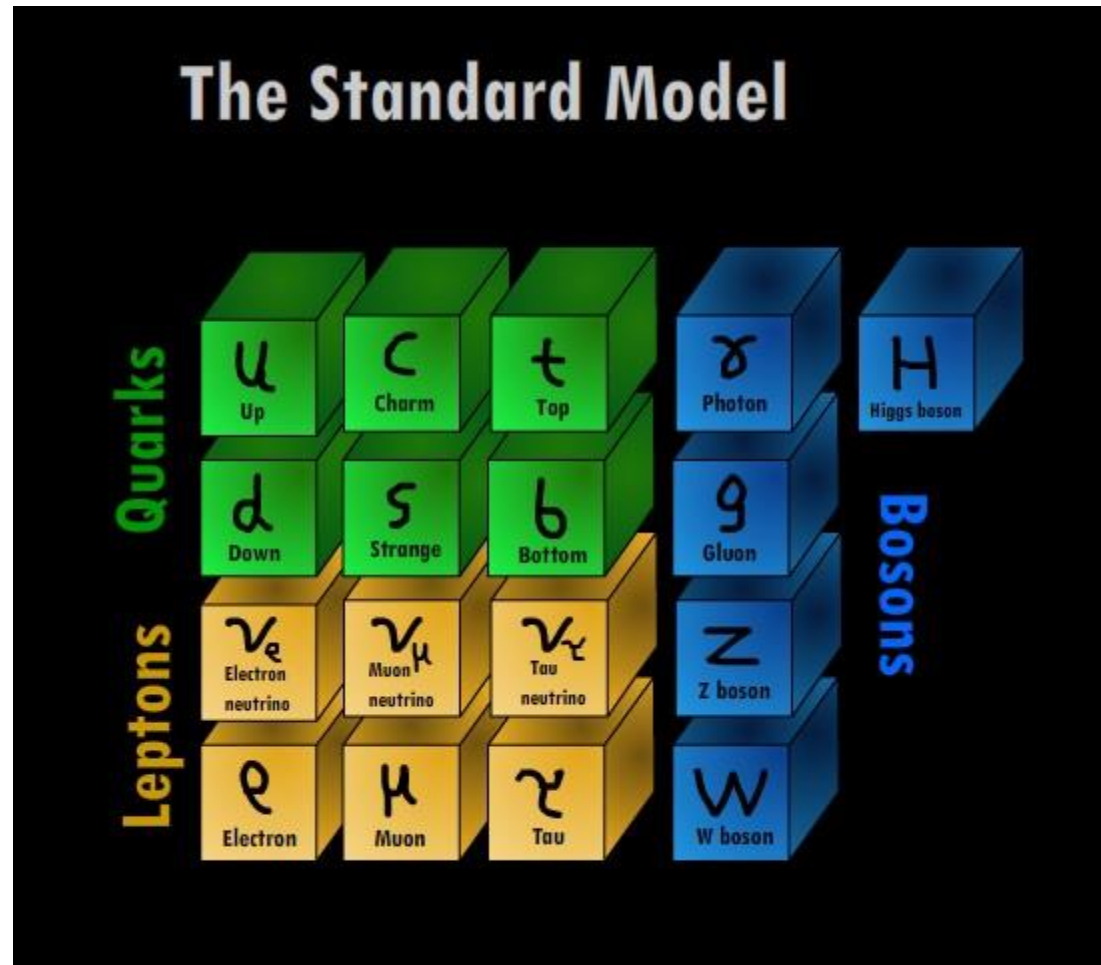
Lecture 1: Extra Material

Just some random slides

Graphical views of the Standard Model of particles

$2.2\text{MeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$ u	$1.3\text{GeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$ c	$173.1\text{GeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$ t	$2.2\text{MeV}/c^2$ $-\frac{2}{3}$ $\frac{1}{2}$ $\bar{u}$	$1.3\text{GeV}/c^2$ $-\frac{2}{3}$ $\frac{1}{2}$ $\bar{c}$	$173.1\text{GeV}/c^2$ $-\frac{2}{3}$ $\frac{1}{2}$ $\bar{t}$	0 0 1 g	0 0 2 G
$4.7\text{MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ d	$96\text{MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ s	$4.2\text{GeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ b	$4.7\text{MeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$ $\bar{d}$	$96\text{MeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$ $\bar{s}$	$4.2\text{GeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$ $\bar{b}$	0 0 1 γ	
$511\text{keV}/c^2$ -1 $\frac{1}{2}$ e	$105.7\text{MeV}/c^2$ -1 $\frac{1}{2}$ μ	$177.7\text{GeV}/c^2$ -1 $\frac{1}{2}$ τ	$511\text{keV}/c^2$ 1 $\frac{1}{2}$ $\bar{e}$	$105.7\text{MeV}/c^2$ 1 $\frac{1}{2}$ $\bar{\mu}$	$177.7\text{GeV}/c^2$ 1 $\frac{1}{2}$ $\bar{\tau}$	$91.2\text{GeV}/c^2$ 0 1 Z^0	
$<1\text{eV}/c^2$ 0 $\frac{1}{2}$ ν_e	$<0.2\text{MeV}/c^2$ 0 $\frac{1}{2}$ ν_μ	$<18.2\text{MeV}/c^2$ 0 $\frac{1}{2}$ ν_τ	$<1\text{eV}/c^2$ 0 $\frac{1}{2}$ $\bar{\nu}_e$	$<0.2\text{MeV}/c^2$ 0 $\frac{1}{2}$ $\bar{\nu}_\mu$	$<18.2\text{MeV}/c^2$ 0 $\frac{1}{2}$ $\bar{\nu}_\tau$	$80.4\text{GeV}/c^2$ ± 1 1 $W^\pm$	$125\text{GeV}/c^2$ 0 0 H^0

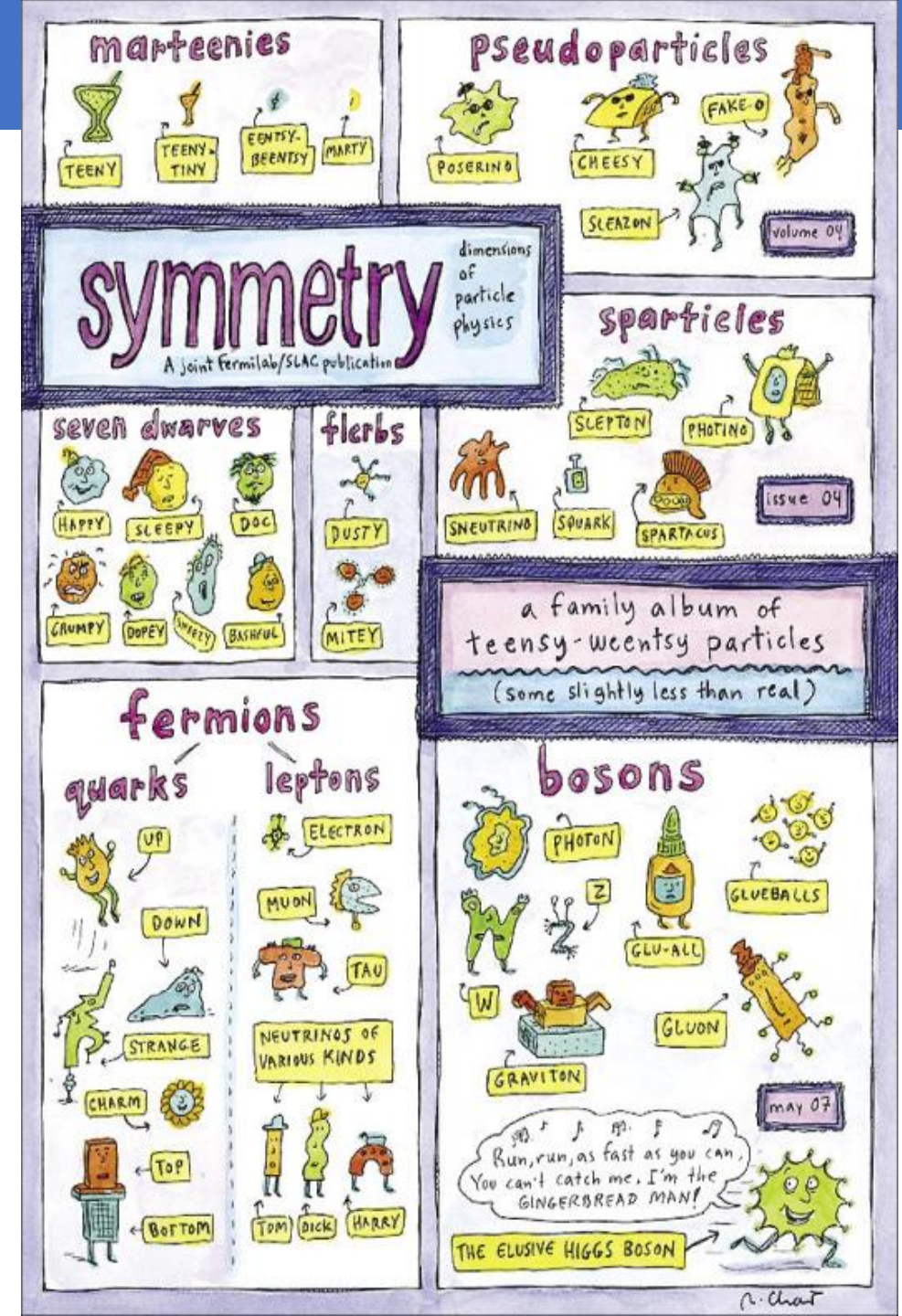
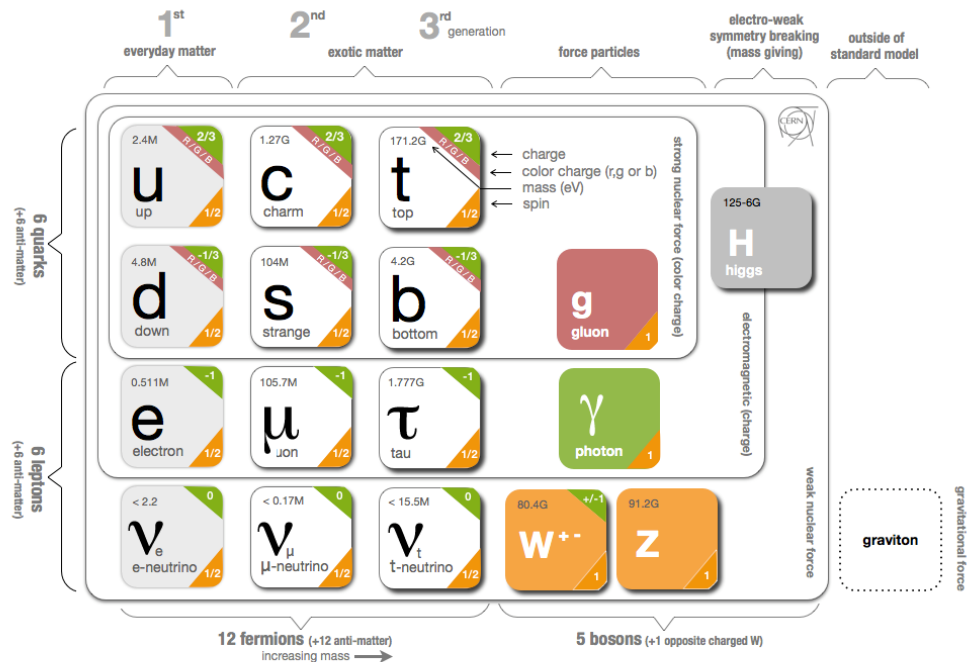
Graphical views of the Standard Model of particles



Graphical views of the SM

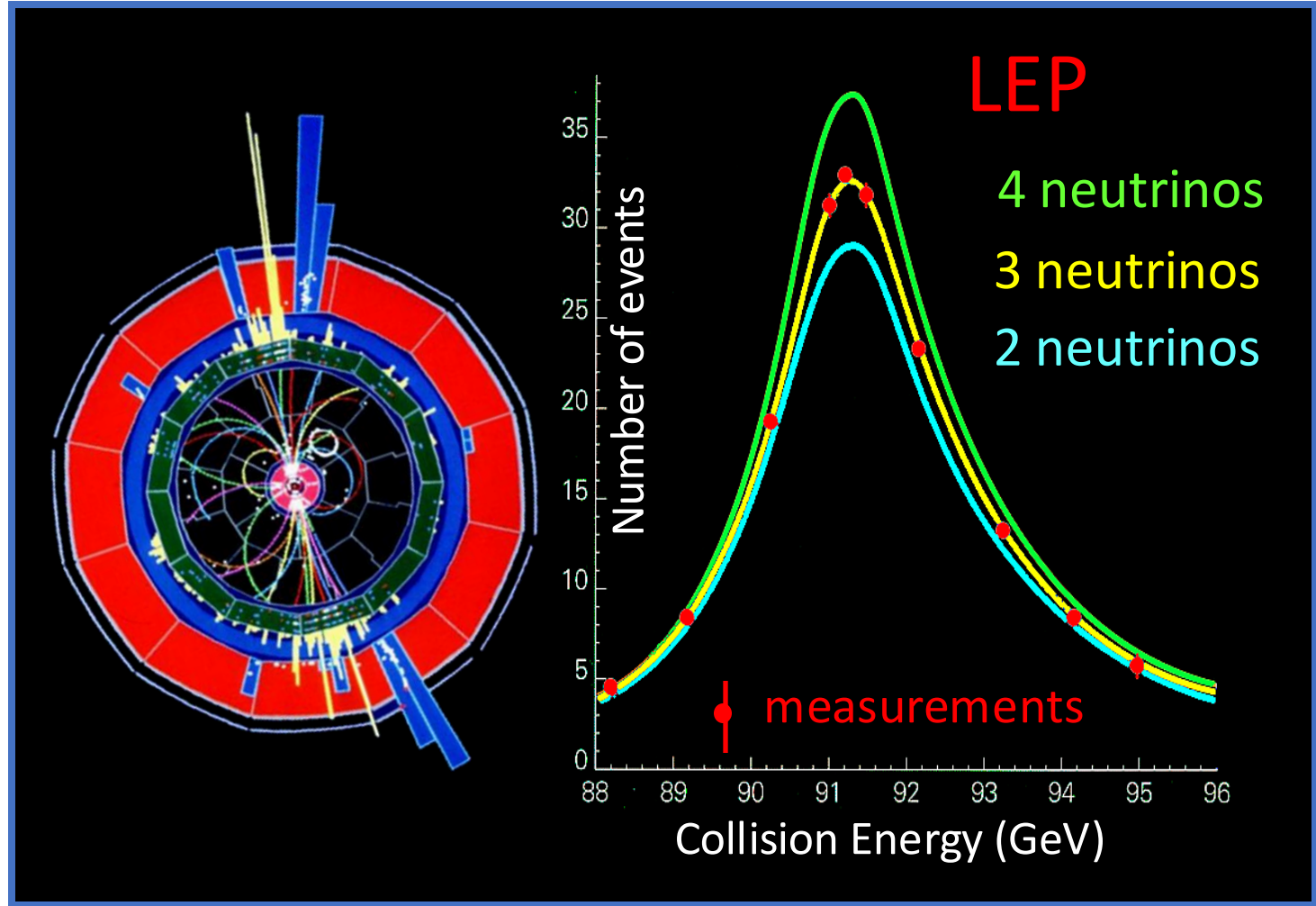
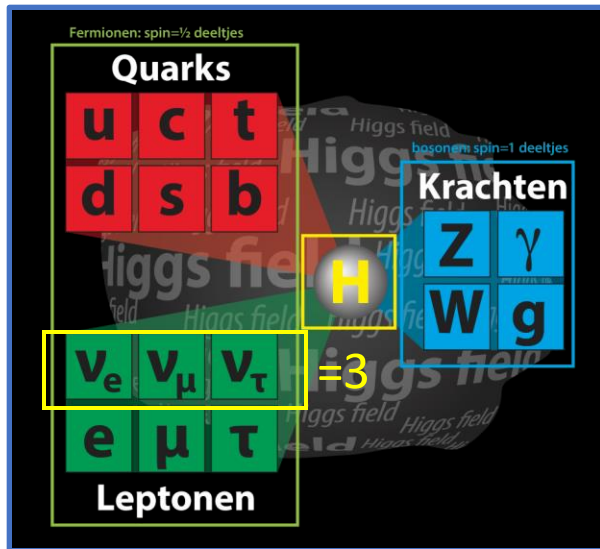
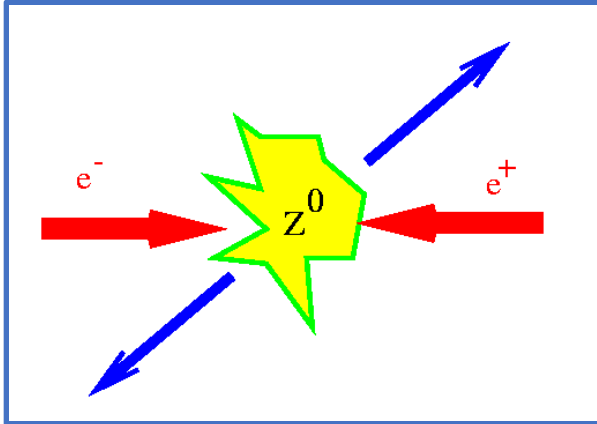
STANDARD MODEL OF ELEMENTARY PARTICLES

QUARKS	UP			GLUON			HIGGS BOSON
	mass $2.3 \text{ MeV}/c^2$	CHARM $1,275 \text{ GeV}/c^2$	TOP $173.07 \text{ GeV}/c^2$	0		126 GeV/c^2	
	charge $\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0		0	
	spin $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	g	0	
	u	c	t			H	
LEPTONS	DOWN			PHOTON			GAUGE BOSONS
	mass $4.8 \text{ MeV}/c^2$	STRANGE $95 \text{ MeV}/c^2$	BOTTOM $4.18 \text{ GeV}/c^2$	0			
	charge $-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0			
	spin $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	γ		
	d	s	b				
	ELECTRON			Z BOSON			
	mass $0.511 \text{ MeV}/c^2$	MUON $105.7 \text{ MeV}/c^2$	TAU $1,777 \text{ GeV}/c^2$	91.2 GeV/c^2			
	-1	-1	-1	0			
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	Z		
	e	μ	τ				
	ELECTRON NEUTRINO			W BOSON			
	mass $< 2.2 \text{ eV}/c^2$	MUON NEUTRINO $< 0.17 \text{ MeV}/c^2$	TAU NEUTRINO $< 15.5 \text{ MeV}/c^2$	80.4 GeV/c^2			
	0	0	0	± 1			
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	W		
	ν_e	ν_μ	ν_τ				



3 Generations of particles – How do we know?

LEP: The heavy Z boson decays into 3 light neutrino types.

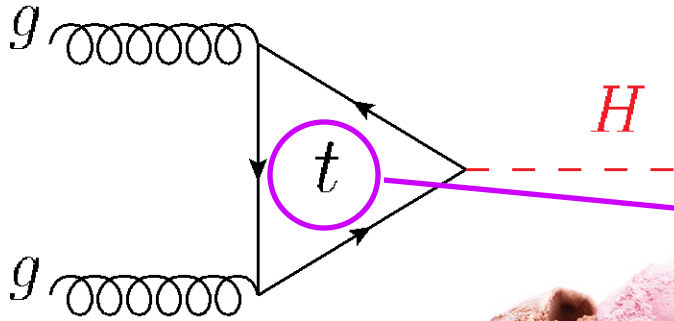


- No additional weakly interacting *light fermion* generations.

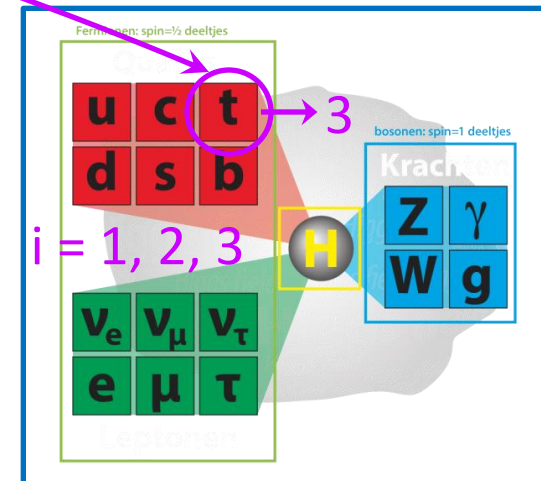
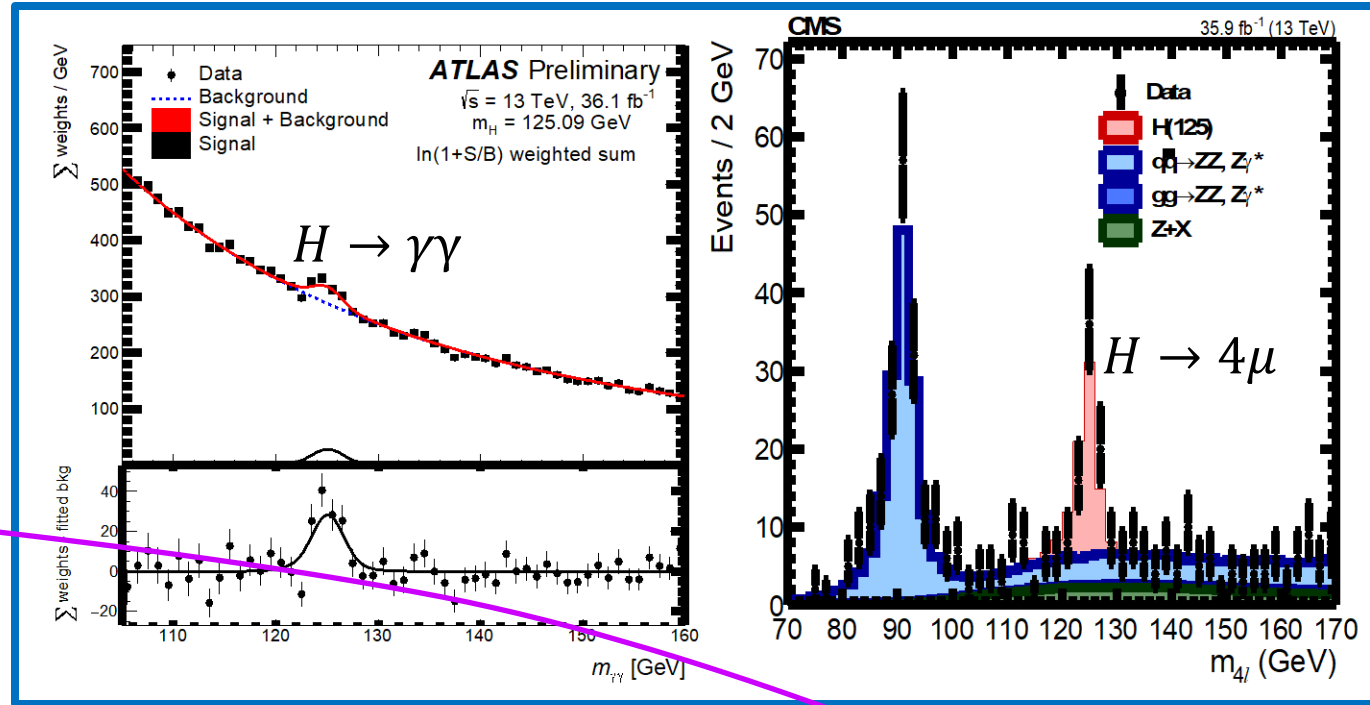
3 Generations of particles – How do we know?

LHC: Higgs production:

Loop diagram is proportional to the mass of the heaviest fermion.



- Top is the **heaviest fermion flavour**.
- 3 Flavour generations



Lecture 1: Discussion Topics

Discussions Topics belonging to
Lecture 1

- Redefine the unit $\hbar = \frac{h}{2\pi} \approx 1.055 \times 10^{-34} \text{ Js}$ to be: $\hbar \equiv 1$
- Redefine the unit $c = 2.998 \times 10^8 \text{ m/s}$ to be: $c \equiv 1$
- Explain how Energy, mass, distance, time can then be expressed in the unit **GeV**
- How can you get answers that can be compared with measurements?
- What are the advantages of doing this?

quantity	symbol in natural units	equivalent symbol in ordinary units
space	x	$x/\hbar c$
time	t	$t/\hbar$
mass	m	mc^2
momentum	p	pc
energy	E	E
positron charge	e	$e\sqrt{\hbar c/\epsilon_0}$

Conversion of basic quantities between natural and ordinary units.

quantity	conversion factor	natural unit	normal unit
mass	$1 \text{ kg} = 5.61 \times 10^{26} \text{ GeV}$	GeV	GeV/c^2
length	$1 \text{ m} = 5.07 \times 10^{15} \text{ GeV}^{-1}$	GeV^{-1}	$\hbar c / \text{GeV}$
time	$1 \text{ s} = 1.52 \times 10^{24} \text{ GeV}^{-1}$	GeV^{-1}	$\hbar / \text{GeV}$

Conversion factors from natural units to ordinary units.

Topic-2: The Lorentz Transformation

- Why are space and time coordinates not universal (ie not the same for each observer)?
- Explain the Lorentz transformation
- When does this effect become noticeable?

Topic-2: The Lorentz Transformation

A reference system or coordinate system is used to determine the time and position of an event.

Reference system S is linked to observer Alice at position $(x,y,z) = (0,0,0)$

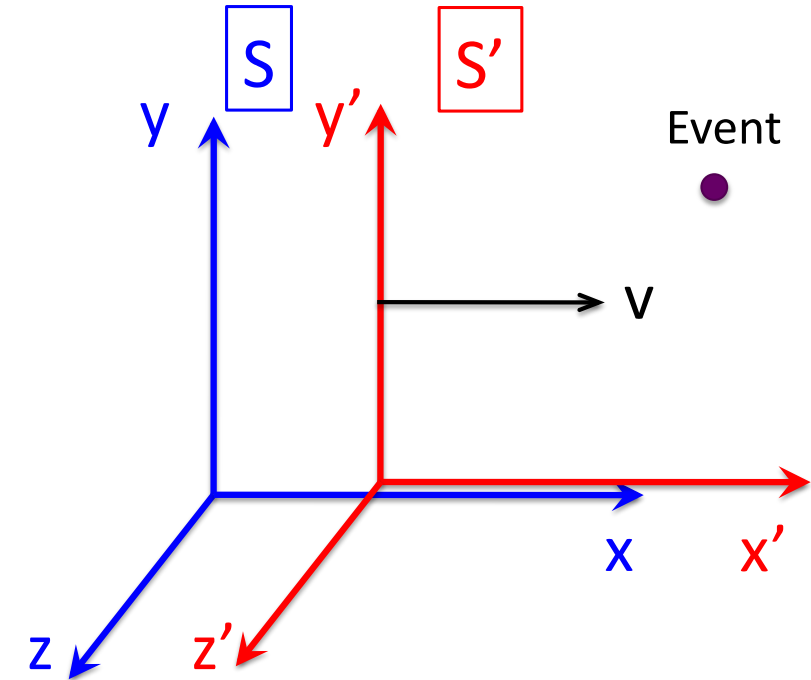
An event is fully specified by giving its coordinates and time: (t, x, y, z)

Reference system S' is linked to observer Bob who is moving with velocity v with respect to Alice.

The event has: (t', x', y', z')

How are the coordinates of an event, say a lightning strike in a tree, expressed in coordinates for Alice and for Bob?

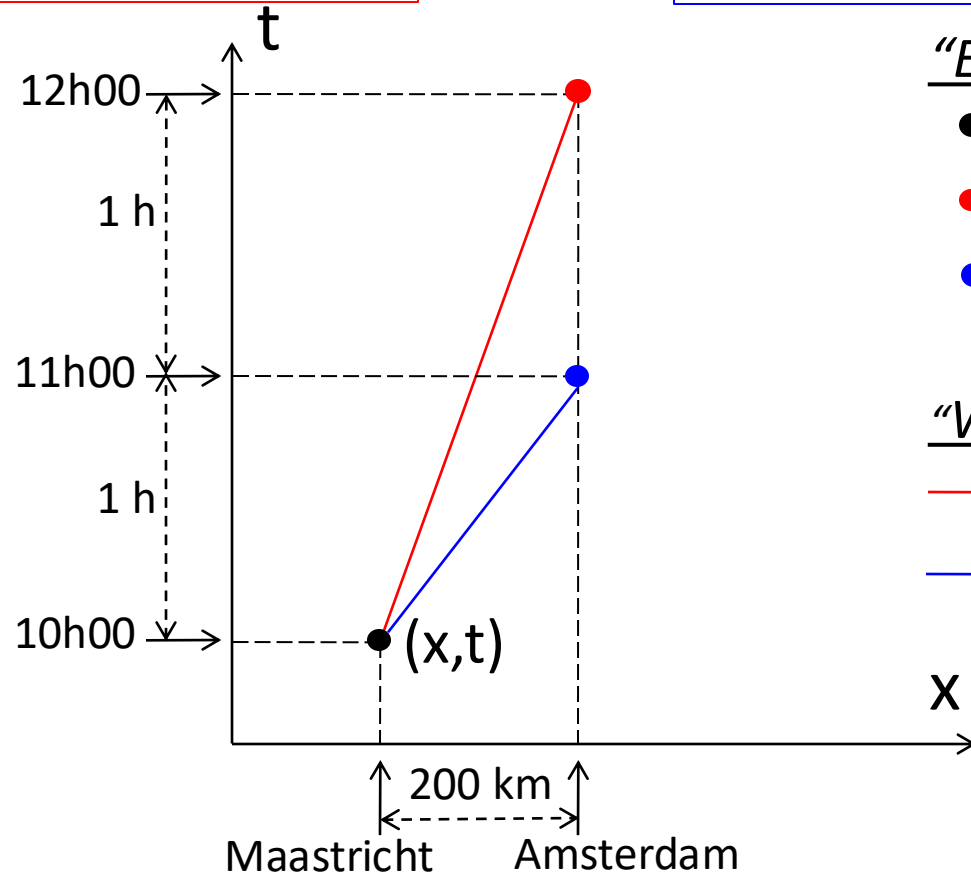
$$(t, x, y, z) \rightarrow (t', x', y', z')$$



Topic-2: Space-time diagram

Bob drives from Maastricht to Amsterdam with 100 km/h.

Alice drives from Maastricht to Amsterdam with 200 km/h.



"Events":

- Departure Alice & Bob
- Arrival Bob
- Arrival Alice

"World lines":

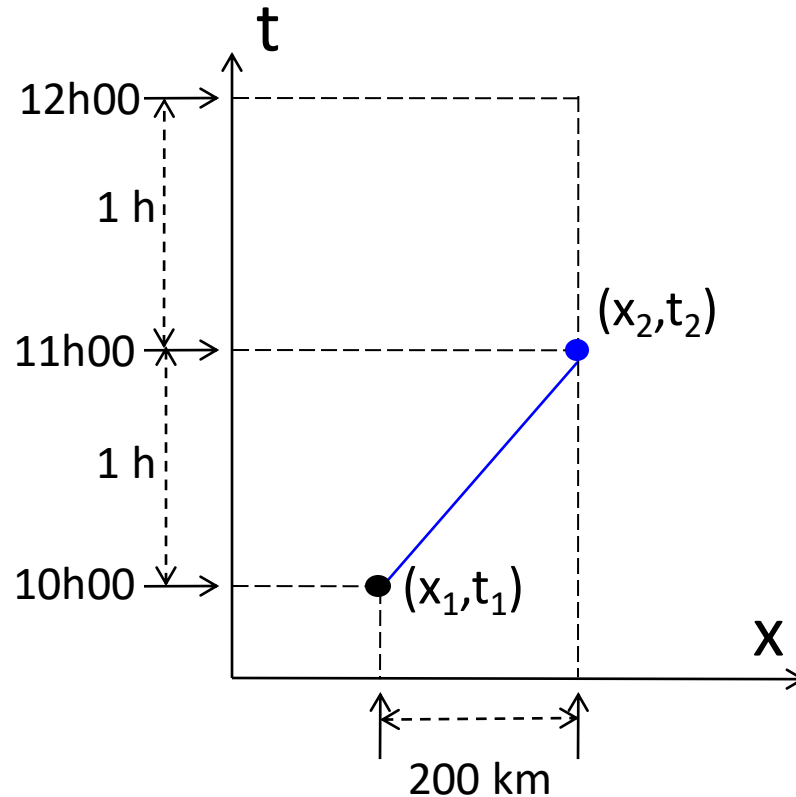
- Bob's world-line
- Alice's world-line

Events with space-time coordinates: (x, t)

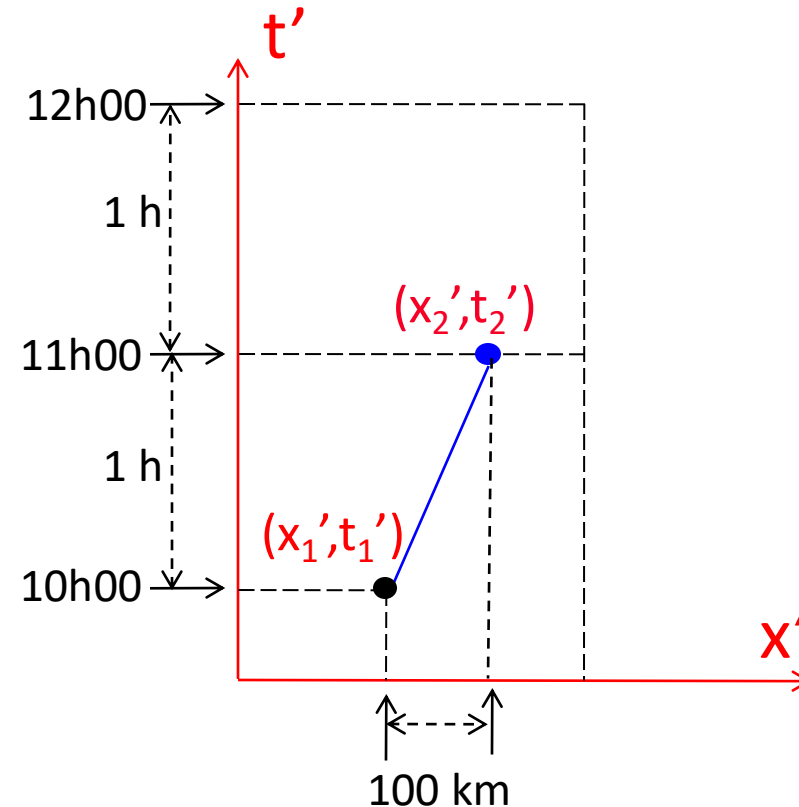
More general: it is a 4-dimensional space: (x, y, z, t)

Topic-2: Coordinate transformation

How does **Alice's** trip look like in the coordinates of the reference system of **Bob**?



Alice as seen from Maastricht
 S = fixed reference system in Maastricht

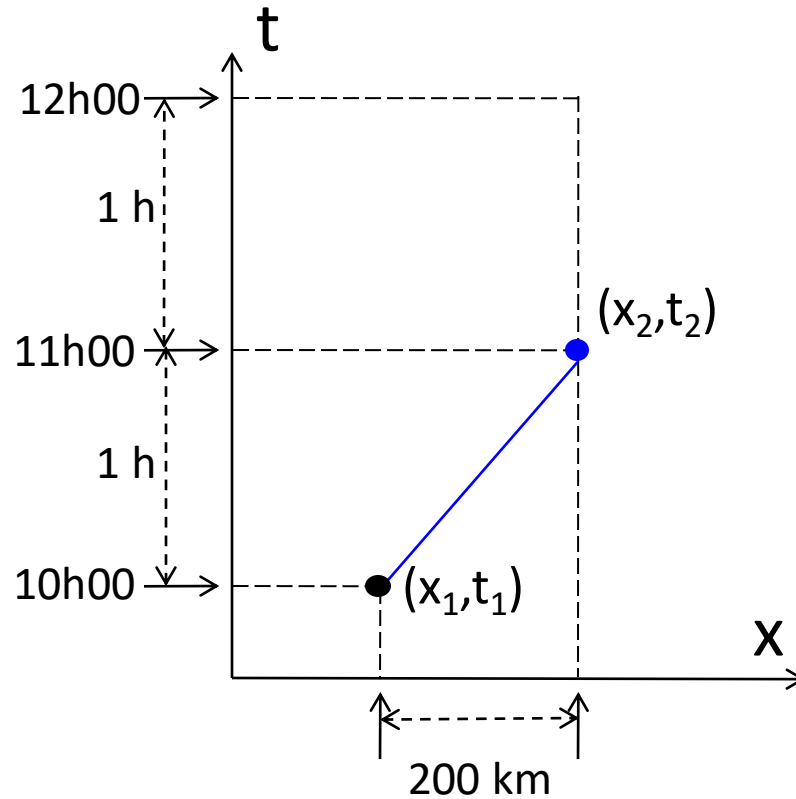


Alice as seen from **Bob**
 S' = fixed reference to **Bob**

Bob's frame moves with velocity v (100km/h) with respect to Maastricht

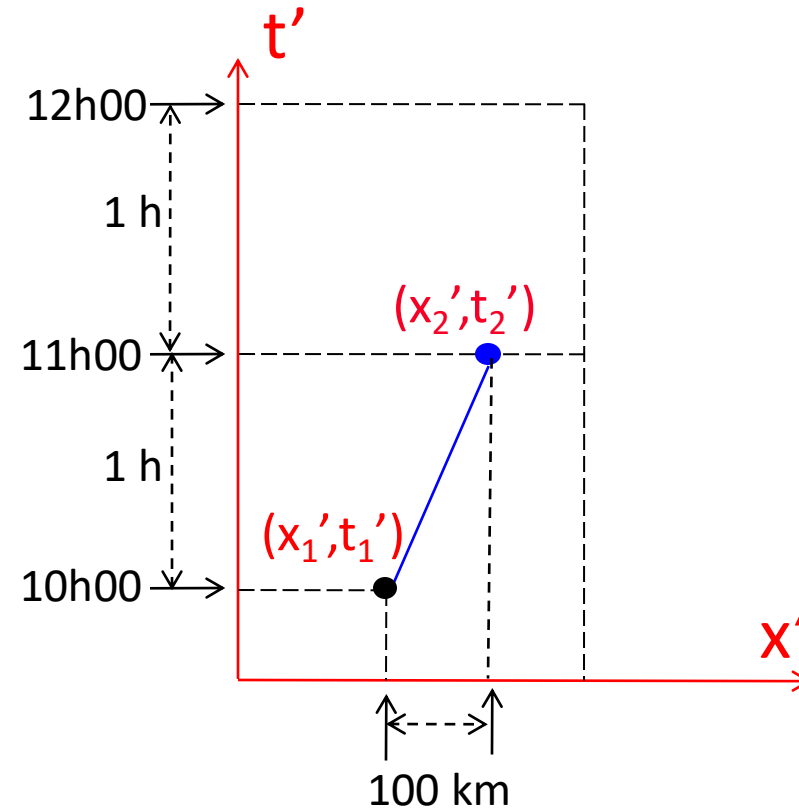
Topic-2: Coordinate transformation

How does **Alice's** trip look like in the coordinates of the reference system of **Bob**?



Classical (Galilei Transformation):

$$\begin{aligned} t' &= t \\ x' &= x - v t \end{aligned}$$



Relativistic (Lorentz Transformation):

$$\begin{aligned} t' &= \gamma \left(t - \frac{v}{c^2} x \right) \\ x' &= \gamma (x - v t) \end{aligned} \quad \text{with: } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Topic-2: Lorentz Transformations

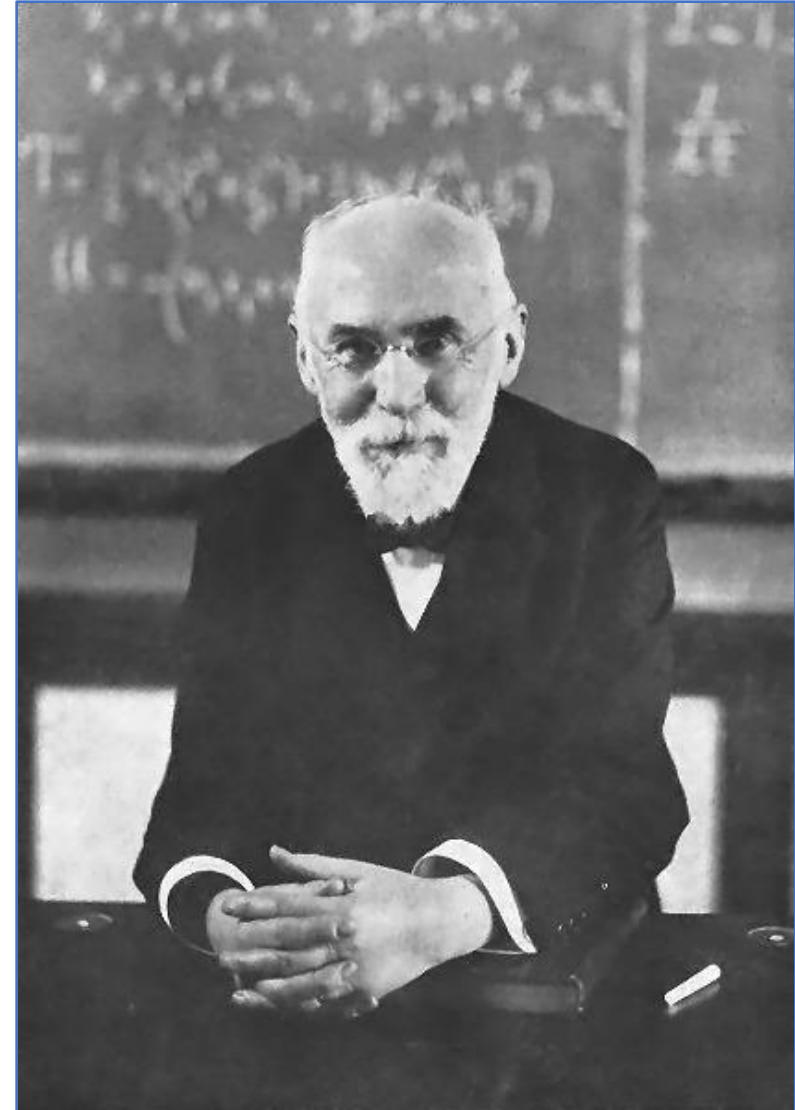
Hendrik Anton Lorentz (1853 – 1928)

Dutch Physicist in Leiden
(Nobelprize 1902 with Pieter Zeeman)

To explain the Michelson-Morley experiment he assumed that bodies contracted due to intermolecular forces as they were moving through the aether.

(He believed in the existence of the aether)

Einstein derived it from the relativity principle and also saw that time has to be modified.



Topic-2: Let's go crazy and derive them...

15

Start with classical Galilei Transformation:

$$x' = x - vt$$

$$x = x' + vt'$$

Let's try a modification by including a factor:

$$x' = f(x - vt)$$

$$x = f(x' + vt')$$

For light: $x = ct$ and $x' = ct'$, so:

$$ct' = f(ct - vt)$$

$$ct = f(ct' + vt')$$

Then: $t' = f\left(\frac{c - v}{c}\right) t$

$$t = f\left(\frac{c + v}{c}\right) t'$$

Substitute first into second:

$$t = f\left(\frac{c + v}{c}\right) f\left(\frac{c - v}{c}\right) t$$

Divide by t : $1 = \left(\frac{c+v}{c}\right) \left(\frac{c-v}{c}\right) f^2 = \left(\frac{c^2 - v^2}{c^2}\right) f^2$

It follows then that: $f^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - v^2/c^2}$

So that we find: $f = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$

Therefor we have derived the

Lorentz transformation:

$$x' = \gamma(x - vt)$$

Similarly we find the Lorentz transformation for time:
(see lecture notes)

$$t' = \gamma\left(t - \frac{v}{c^2} x\right)$$

whereas the Galilei translation was:

$$t' = t$$

Topic-3: Four vectors & co- and contra-variance

- Explain the so-called covariant (ie 4-vector) notation.
- What is the difference between contra-variant and co-variant?
- Explain Einstein's summation convention for indices

Topic-3: Four vectors & co- and contra-variance

See Griffiths: chapter 3

- Four vector: $x^\mu = (x^0, x^1, x^2, x^3)$ with $x^0 = ct \Rightarrow x^0 = t$ (use $c \equiv 1$ convention)
- We call this a **contravariant** vector and: $x^\mu = (x^0, \vec{x})$
- Lorentz transformation along x^1 axis using $\beta = v/c$ and $\gamma = 1/\sqrt{1 - \beta^2}$ is:

$$x^{0'} = \gamma(x^0 - \beta x^1)$$

$$x^{1'} = \gamma(x^1 - \beta x^0)$$

$$x^{2'} = x^2$$

$$x^{3'} = x^3$$

Write it in Matrix notation:

$$x^{\mu'} = \Lambda^\mu_{\nu'} x^\nu ; \Lambda^\mu_{\nu'} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Lorentz transformations leave the “length” s invariant $s = x \cdot x = |x|^2 = x^{0^2} - |\vec{x}|^2$
 - Explicitly: $(ct)^2 - |\vec{x}|^2 = (ct')^2 - |\vec{x}'|^2 = s$ is invariant.

- This can be written as:

$$s = (x^0, x^1, x^2, x^3) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} (x^0, x^1, x^2, x^3)$$

→ “Metric tensor”

Topic-3: Scalar product and co- and contra-variant

- Define **co-variant** vectors: $x_\mu = (x^0, -\vec{x})$

- Define the metric tensor:
$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Definition:

A contravariant vector transforms like x^μ and a covariant vector transforms like x_μ . Note the Einstein summation convention!

- To obtain a co-variant from contra-variant vector:

$$x_\mu = \sum_\nu g_{\mu\nu} x^\nu = g_{\mu\nu} x^\nu$$

Note the Einstein summation convention!

- Then the invariance distance $s = x^\mu x_\mu = \sum_{\mu\nu} x^\mu g_{\mu\nu} x^\nu = x^{0^2} - |\vec{x}|^2$
- We speak of a scalar product: $A \cdot B = A^\mu B_\mu = g_{\mu\nu} A^\mu B^\nu$, where a sum is always implicit over *contravariant* and *covariant* indices.
- The **scalar product** or **inproduct** of Lorentz 4-vectors is always Lorentz invariant:
 - $I = a_\mu b^\mu = a \cdot b$ for any Lorentz 4-vectors a^μ and b^μ
 - Example are the space-time vectors a^μ , but also the 4-momentum vector $p^\mu = (E, \vec{p})$
 - $E^2 = \vec{p}^2 c^2 + m^2 c^4 \Rightarrow p^\mu = (E, \vec{p}) \Rightarrow p_\mu p^\mu = E^2 - \vec{p}^2 = m^2$ **the invariant mass**

- Contravariant vector:

$$x^\mu = (ct, \vec{x})$$

But contravariant derivative:

$$\partial^\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, -\vec{\nabla} \right)$$

- Covariant vector:

$$x_\mu = (ct, -\vec{x})$$

But covariant derivative:

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, \vec{\nabla} \right)$$

Note that the minus sign is “opposite” to the case of the coordinate four-vectors.

Lecture 1: Exercises

Exercises belonging to Lecture 1

Exercise-1: Natural Units

Griffiths: p. 9-10, PP1: p 12-13

- In particle physics we make often use of natural units
 - Very confusing at first but very convenient when you are used to it (“sloppy”)
Set $c = 2.998 \times 10^8 \text{ m/s} \equiv 1$ and $\hbar = 1.055 \times 10^{-34} \text{ Js} \equiv 1$
(Just leave them out and put them back at very end of any calculation)
 - Consequence: there is only one basic unit for length, time, mass and energy: GeV
- Exercise: derive the numbers on the conversion table on the next page

PS:

Remember: $1 \text{ Joule} = 1 \text{ Newton} \times 1 \text{ m}$

Whereas: $1 \text{ Newton} = 1 \text{ kg m s}^{-2}$

So that: $1 \text{ Joule} = 1 \text{ kg m}^2 \text{ s}^{-2}$

Out of energy (*Joule* or *GeV*), length (*m*), time (*s*)

Natural units make two constraints:

$c = 1$ relates *m* to *s* ; $\hbar = 1$ relates *s* to *GeV*

→ All (length, time, energy) can be expressed in *GeV*

quantity	symbol in natural units	equivalent symbol in ordinary units
space	x	$x/\hbar c$
time	t	$t/\hbar$
mass	m	mc^2
momentum	p	pc
energy	E	E
positron charge	e	$e\sqrt{\hbar c/\epsilon_0}$

Conversion of basic quantities between natural and ordinary units.

quantity	conversion factor	natural unit	normal unit
mass	$1 \text{ kg} = 5.61 \times 10^{26} \text{ GeV}$	GeV	GeV/c^2
length	$1 \text{ m} = 5.07 \times 10^{15} \text{ GeV}^{-1}$	GeV^{-1}	$\hbar c / \text{GeV}$
time	$1 \text{ s} = 1.52 \times 10^{24} \text{ GeV}^{-1}$	GeV^{-1}	$\hbar / \text{GeV}$

Conversion factors from natural units to ordinary units.

Exercise-2 : The Yukawa Potential

- The electric force is transmitted by a photon with $m = 0$. The wave equation for a static electric field caused by pointlike charge e is: $\nabla^2 V(r) = 0$ (Laplace equation)
 - a) Show that the Coulomb potential $V(r) = -e^2 \frac{1}{r}$ fulfills this equation.
 - Note that the potential is **spherical symmetric**, ie. use spherical coordinates.
- The nuclear force is transmitted by a pi-meson with $m = m_\pi$. The wave equation for a static nuclear field caused by a pointlike color charge g is: $\nabla^2 U(r) = m^2 U(r)$ (Klein-Gordon equation)
 - b) Show that the Yukawa potential $U(r) = -g^2 \frac{e^{-r/R}}{r}$ fulfills this equation for a certain value of R , the **range** of the force. What is the relation between R and m_π ?
 - Again note that the potential is **spherical symmetric**.
 - This value is between the electron and proton mass, hence the particle was called a pi-meson or pion.
 - c) Calculate the range of the force from Heisenberg's uncertainty relation, using $R = c\Delta t$ and $\Delta E \Delta t \leq \frac{\hbar}{2}$ and $\Delta E = mc^2$.
 - d) The weak force is mediated by W (80 GeV) and Z (91 GeV) bosons. What is the estimated range of the weak force?

Exercise-3: Kinematics: Z -boson production

- The Z -boson particle is a carrier of the weak force. It has a mass of 91.1 GeV. It can be produced experimentally by annihilation of an electron and a positron. The mass of an electron, as well as that of a positron, is 0.511 MeV.
 - a) Assume that an electron and a positron are accelerated in opposite directions and collide head-on to produce a Z -boson in the lab frame. Calculate the minimal beam energy required for the electron and the positron in order to produce a Z -boson.
 - b) Assume that a beam of positron particles is shot on a target containing electrons. Calculate the beam energy required for the positron beam in order to produce Z -bosons.
 - c) This experiment was carried out in the 1990's. Which method do you think was used? Why?