PHY3004: Nuclear and Particle Physics

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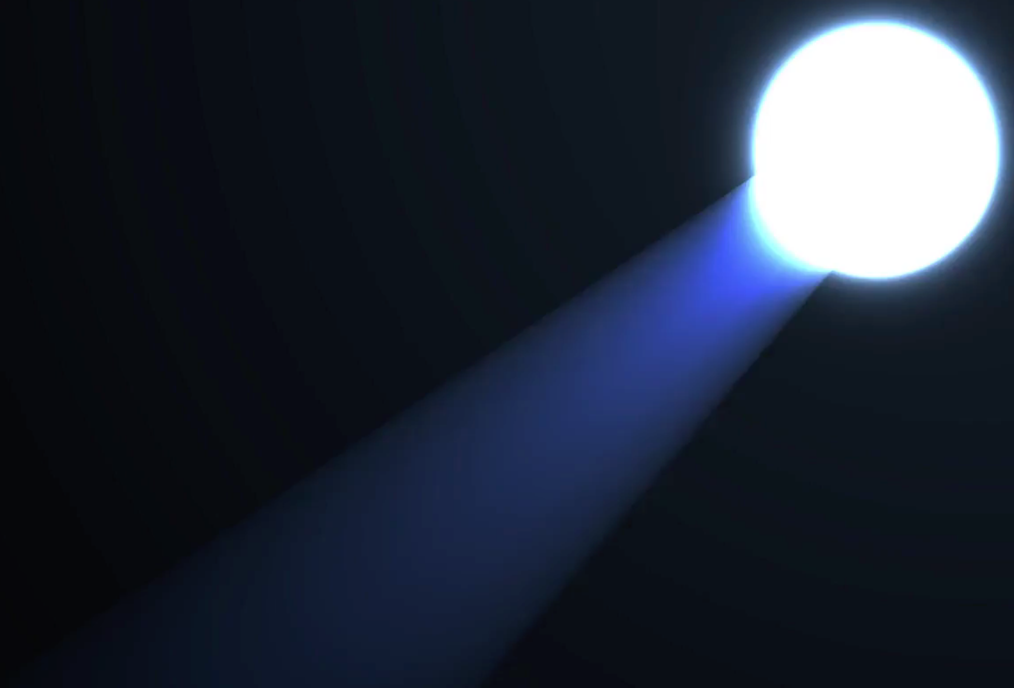
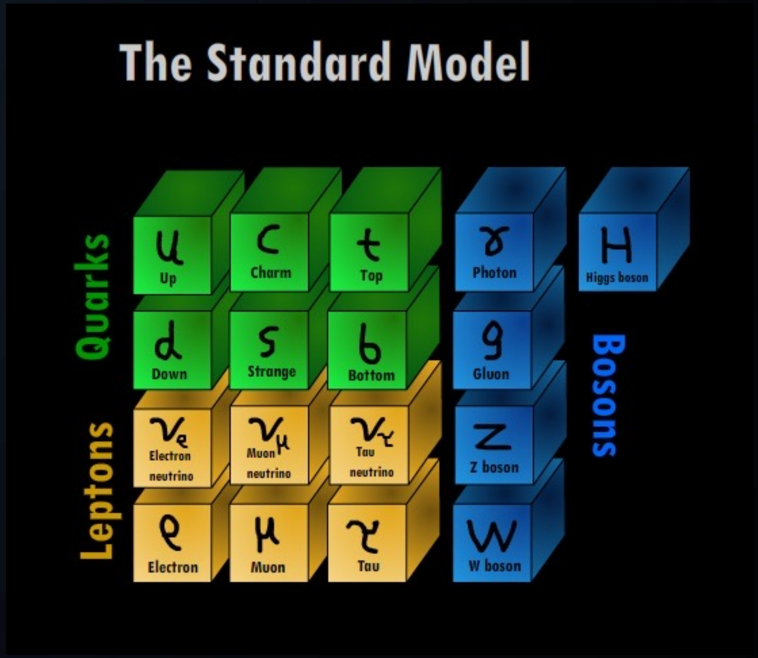
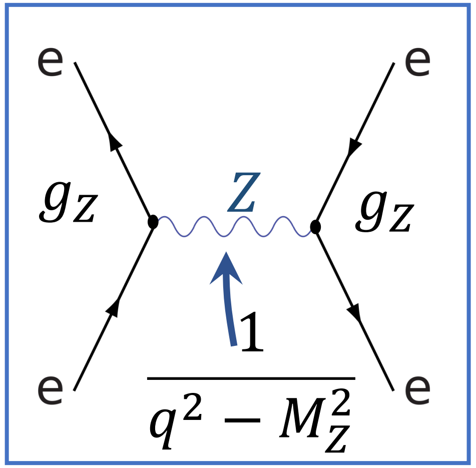
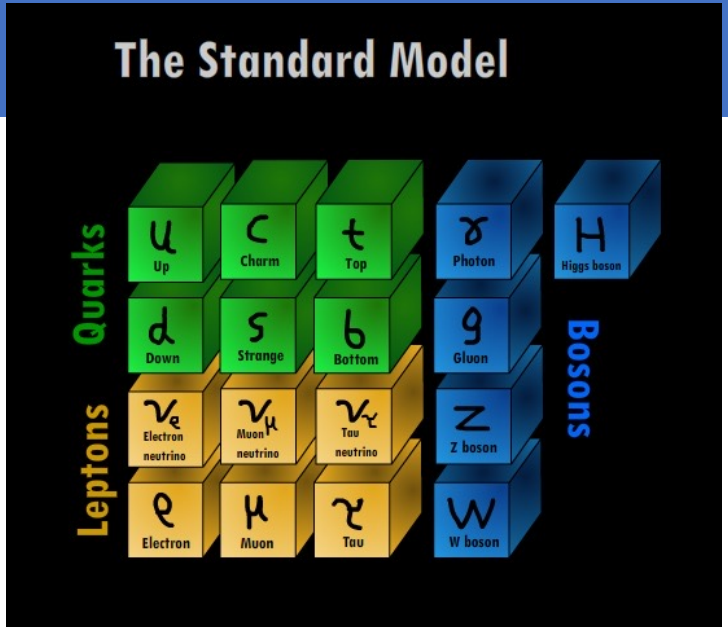


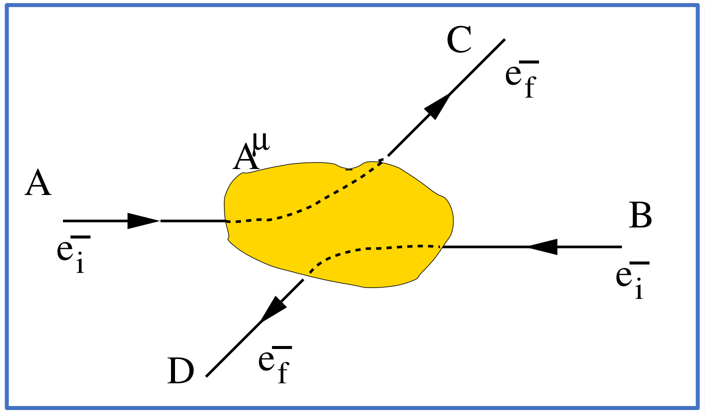


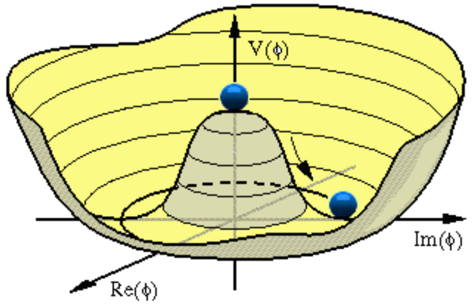
Recap: "Seeing the wood for the trees"

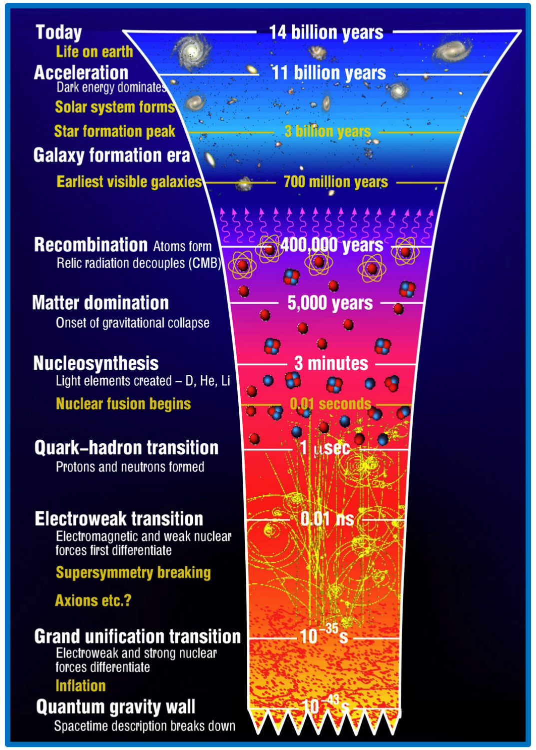
- Lecture 1: "Particles"
  - Zooming into constituents of matter
  - Skills: distinguish particle types, Spin
- Lecture 2: "Forces"
  - Exchange of quanta: EM, Weak, QCD
  - Skills: 4-vectors, Feynman diagrams
- Lecture 3: "Waves"
  - Quantum fields and gauge invariance
  - Dirac algebra, Lagrangian, co- & contra variant
- Lecture 4: "Symmetries"
  - Standard Model, Higgs, Discrete Symmetries
  - Skills: Lagrangians, Chirality & Helicity
- Lecture 5: "Scattering"
  - Cross section, decay, perturbation theory
  - Skills: Dirac-delta function, Feynman Calculus
- Lecture 6: "Detectors"
  - Energy loss mechanisms, detection technologies









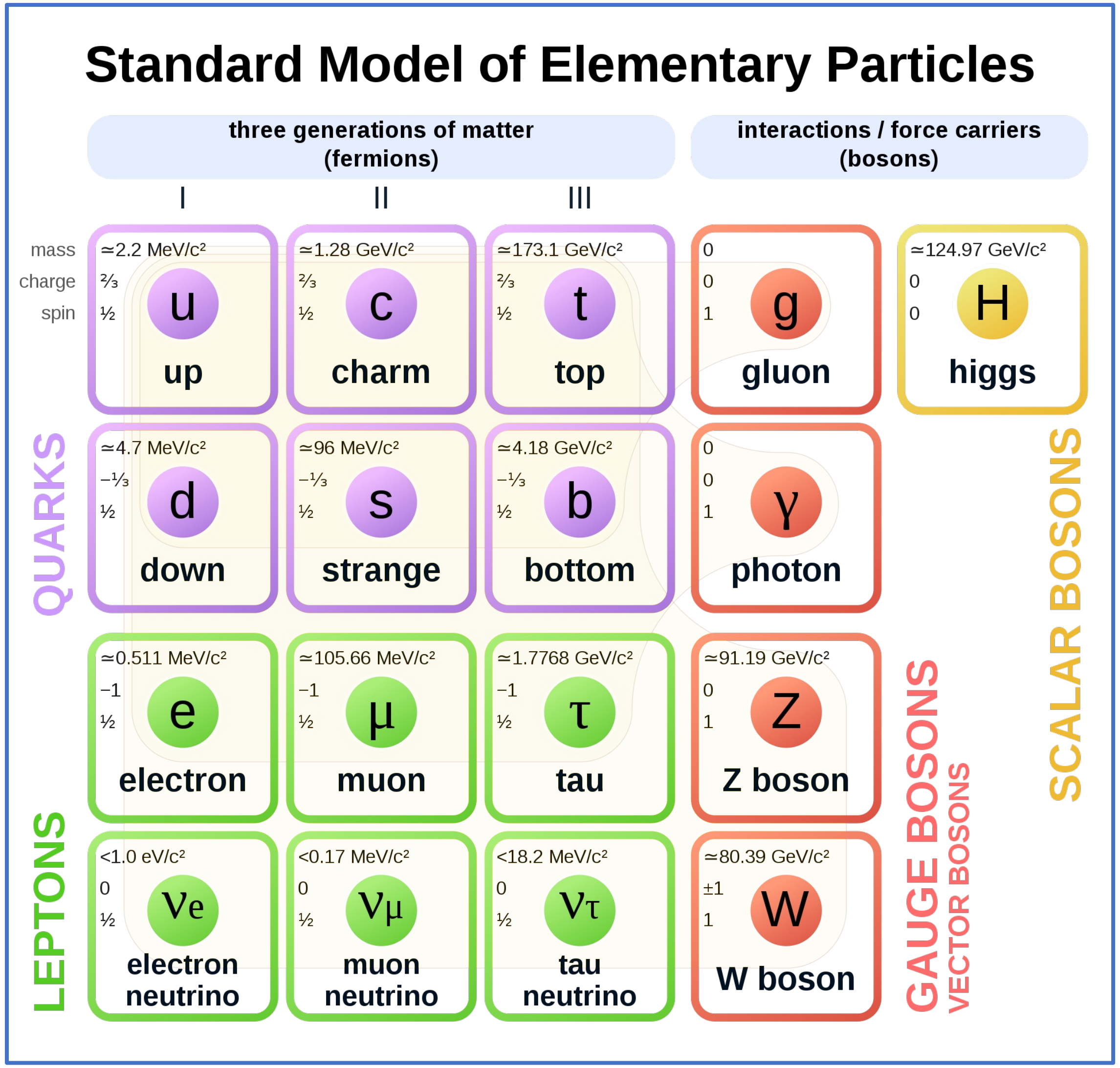


Lecture 1: “Particles”

Griffiths chapter 1

Classification of particles

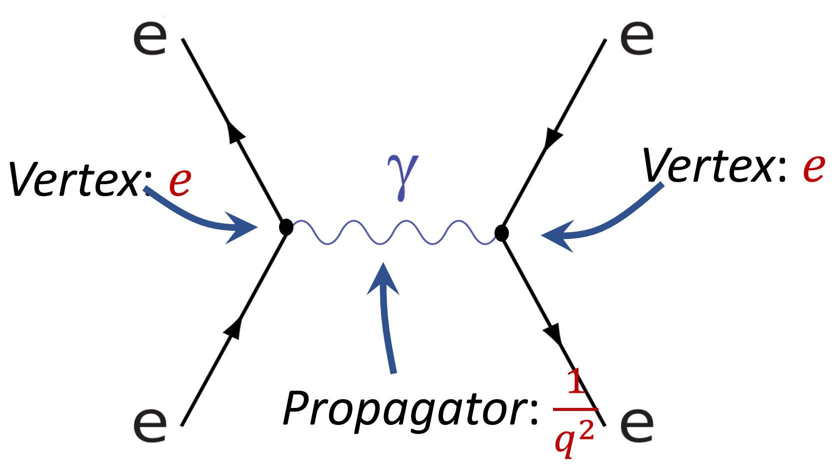
- Lepton: fundamental particle
- Hadron: consist of quarks
  - Meson: 1 quark + 1 antiquark (π^+, B_s^0, …)
  - Baryon: 3 quarks (p, n, Λ, …)
    - Anti-baryon: 3 anti-quarks
- ***Fermion***: particle with half-integer spin.
  - Antisymmetric wave function: obeys Pauli-exclusion principle and Pauli-Dirac statistics
  - All fundamental quarks and leptons are spin-½
  - Baryons (S=$^1/_2$, $^3/_2$)
- ***Boson***: particle with integer spin
  - Symmetric wave function: Bose-Einstein statistics
  - Mesons: (S=0, 1), Higgs (S=0)
  - Force carriers: γ, W, Z, g (S=1); graviton(S=2)



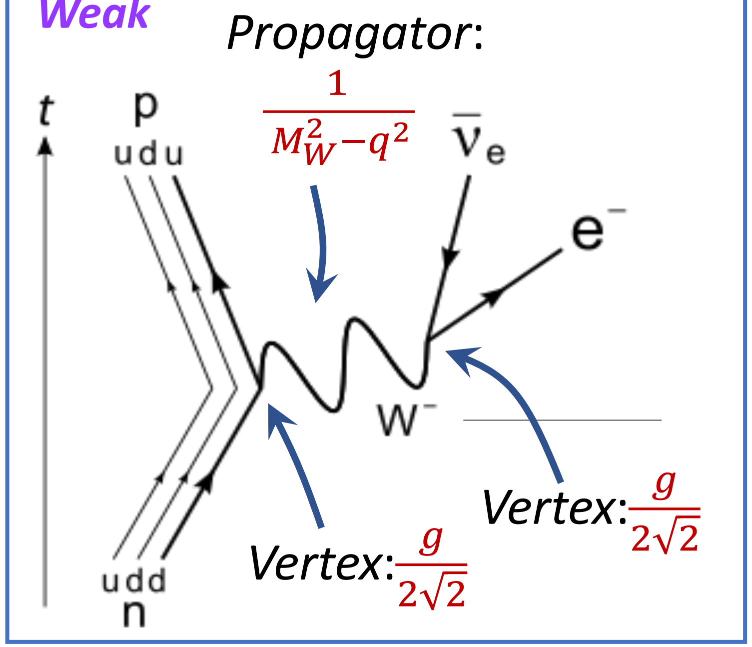
Lecture 2: “Forces”

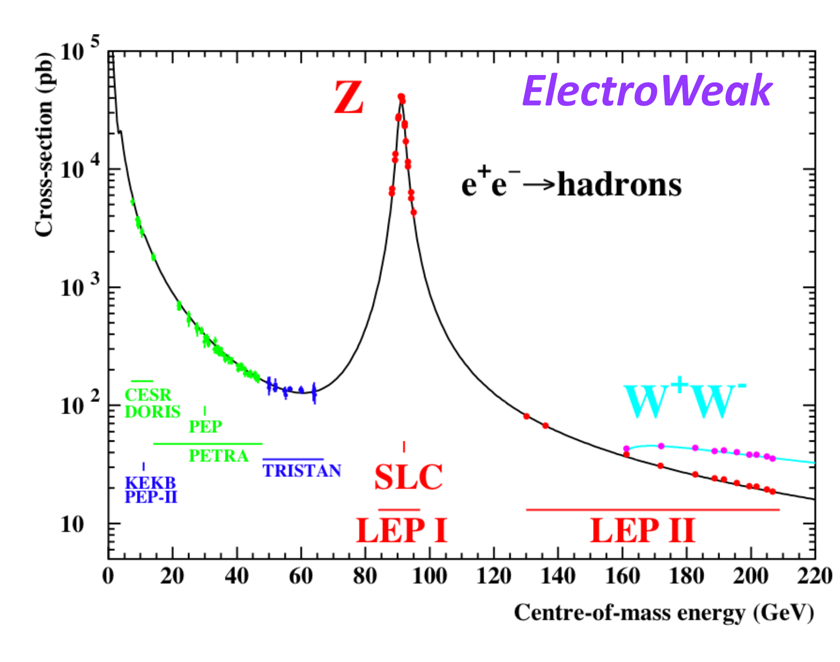
Griffiths chapter 2

EM (QED)

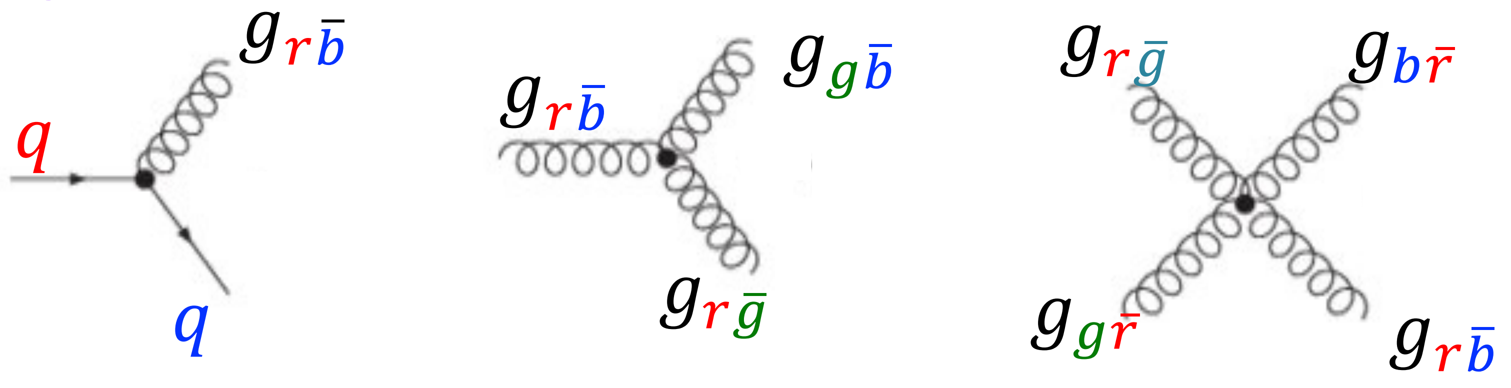


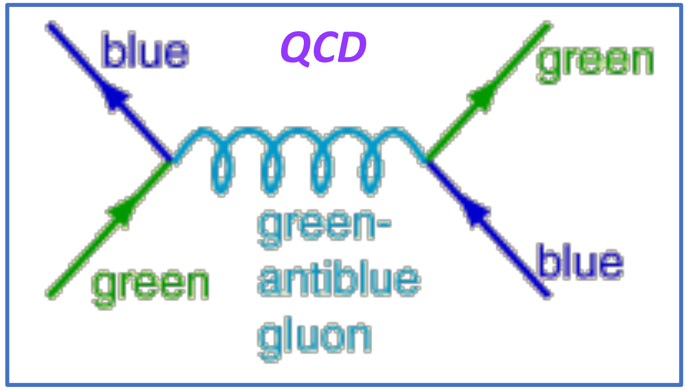
Weak





Strong (QCD)





Lecture 3: "Waves" – wave equations

Probability interpretation (Continuity equation)

Quantum Mechanics: $E \rightarrow \hat{E} = i\hbar\frac{\partial}{\partial t}$; $p \rightarrow \hat{p} = -i\hbar\vec{\nabla}$

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$$

Non-relativistic spin 0:

$$E = \frac{\vec{p}^2}{2m}$$

Schrödinger:

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi$$

$$\psi = Ne^{i(\vec{p}\vec{x}-Et)}$$

$$\rho \equiv \psi^*\psi = |N|^2$$

$$\vec{j} \equiv \frac{i\hbar}{2m}\left(\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi\right) = \frac{|N|^2}{m}\vec{p}$$

Relativistic spin 0:

$$E^2 = p^2c^2 + m^2c^4$$

Example: pions

Klein-Gordon:

$$-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + \frac{m^2c^2}{\hbar^2}\phi$$

$$\partial_\mu\partial^\mu\phi + m^2\phi = 0$$

$$\phi = Ne^{i(\vec{p}\vec{x}-Et)}$$

$$j^\mu(\rho,\vec{j}) = i[\phi^*(\partial^\mu\phi) - \phi(\partial^\mu\phi^*)]$$

$$\rho = 2|N|^2E \qquad \vec{j} = 2|N|^2\vec{p} \qquad j^\mu = 2|N|^2p^\mu$$

Relativistic spin- ½:

$$H = (\vec{\alpha}\cdot\vec{p} + \beta m)$$

Fundamental quarks and leptons

Dirac:

$$i\frac{\partial}{\partial t}\psi = \left(-i\,\vec{\alpha}\cdot\vec{\nabla} + \beta m\right)\psi$$

$$\left(i\gamma^\mu\partial_\mu - m\right)\psi = 0 \qquad \gamma^\mu = (\beta, \beta\vec{\alpha})$$

$$\psi = u(p)e^{i(\vec{p}\vec{x}-Et)}$$

$$u(p) = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

$$j^0 = \bar{\psi}\gamma^0\psi = \psi^\dagger\psi = \sum_{i=1}^{4}|\psi_i|^2$$

Relativistic spin-1:

Fundamental force carriers

Proca:

$$\partial_\mu\partial^\mu A^\nu + m^2A^\nu = j^\nu$$

EM: $A^\mu = \gamma \rightarrow m = 0$

QCD: $A^\mu = g \rightarrow m = 0$

Weak: $A^\mu = W, Z \rightarrow m \neq 0$

EM: Maxwell equations for $\vec{E}$ and $\vec{B}$ fields

Lecture 3: "Waves" – gauge invariance

$$S = \int d^4x\, \mathcal{L}(\phi(x), \partial\phi(x))$$
$$\delta S = 0$$

Lagrangians:

Spin 0 Scalar field: $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2$

Spin ½ Dirac fermion $\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$

Spin 1 gauge boson (photon) : $\mathcal{L} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) - j^\mu A_\mu$

Euler Lagrange lead to the wave equations: $$\frac{\partial\mathcal{L}}{\partial\phi(x)} = \partial_\mu \frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\phi(x)\right)}$$ (stationary action in field theory)

All forces result from requiring a symmetry principle: Lagrangian should stay invariant under transformations

1) QED = U(1) symmetry

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi \longrightarrow \mathcal{L} = i\bar{\psi}\gamma_\mu D^\mu\psi - m\bar{\psi}\psi$$

$\psi(x) \to \psi'(x) = e^{iq\alpha(x)}\psi(x)$

$A^\mu(x) \to A'^\mu(x) = A^\mu(x) - \partial^\mu\alpha(x)$

Covariant derivative: $\partial^\mu \to D^\mu \equiv \partial^\mu + iqA^\mu$

$$\mathcal{L} = \underbrace{i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi}_{\text{"free"}} - \underbrace{q\bar{\psi}\gamma_\mu\psi A^\mu}_{\text{"interaction"}}$$

➔ 1 E.M. photon field: $A^\mu(x)$

2) Weak = SU(2) symmetry $\psi = \begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix}$

$$\psi(x) \to \psi'(x) = \exp\left(\frac{i}{2}g\vec{\tau}\cdot\vec{\alpha}(x)\right)\begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix}$$

➔ 3 weak fields: $W^{\mu+}(x)$, $W^{\mu-}(x)$, $Z^\mu(x)$

3) QCD = SU(3) symmetry $\psi = \begin{pmatrix}\psi_r \\ \psi_g \\ \psi_b\end{pmatrix}$

$$\psi(x) \to \psi'(x) = \exp\left(\frac{i}{2}g_s\vec{\lambda}\cdot\vec{\alpha}(x)\right)\begin{pmatrix}\psi_r \\ \psi_g \\ \psi_b\end{pmatrix}$$

➔ 8 colored gluon fields: $g^\mu(x)$

Lecture 4: "Symmetries" – Standard Model

- The Lagrangian of the Standard Model includes electromagnetic, weak and strong interactions according to the gauge field principle

- Construction of the Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{free}} - \mathcal{L}_{\text{interaction}} = \mathcal{L}_{\text{Dirac}} - gJ^{\mu}A_{\mu}$
  - With g a coupling constant, J^{μ} a current ($\bar{\psi}\mathrm{O}_i\psi$) and A_{μ} a force field

A. Local $U(1)$ gauge invariance: symmetry under complex phase rotations
   - Conserved quantum number: (hyper-) charge
   - Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - q\,\underbrace{\bar{\psi}\gamma^{\mu}\psi}_{J^{\mu}_{EM}}\,A_{\mu}$ $\quad(\partial_{\mu} \to D_{\mu} \equiv \partial_{\mu} + iqA_{\mu})$

B. Local $SU(2)$ gauge invariance: symmetry under transformations in isospin doublet space.
   - Conserved quantum number: weak isospin
   - Lagrangian: $\mathcal{L} = \bar{\Psi}(i\gamma^{\mu}D_{\mu} - m)\Psi = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi - \frac{g}{2}\underbrace{\bar{\Psi}\gamma^{\mu}\vec{\tau}\Psi}_{J^{\mu}_{Weak}}\,\vec{b}_{\mu}$ $\quad(I\partial_{\mu} \to D_{\mu} = I\partial_{\mu} + igB_{\mu})$

$$B_{\mu} = \frac{1}{2}\vec{\tau}\cdot\vec{b}_{\mu} = \frac{1}{2}\tau_1^a b_{\mu}^a = \frac{1}{2}\begin{pmatrix} b_3 & b_1 - ib_2 \\ b_1 + ib_2 & -b_3 \end{pmatrix}$$

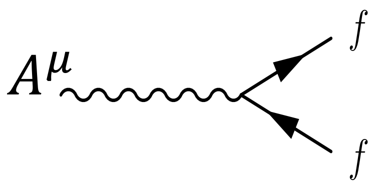
C. Local $SU(3)$ gauge invariance: symmetry under transformations in colour triplet space
   - Conserved quantum number: color
   - Lagrangian: $\mathcal{L} = \bar{\Phi}(i\gamma^{\mu}D_{\mu} - m)\Phi = \bar{\Phi}(i\gamma^{\mu}\partial_{\mu} - m)\Phi - \frac{g_s}{2}\underbrace{\bar{\Phi}\gamma^{\mu}\vec{\lambda}\Phi}_{\vec{J}^{\mu}_{QCD}}\,\vec{c}_{\mu}$ $\quad(I\partial_{\mu} \to D_{\mu} = I\partial_{\mu} + ig_sC_{\mu})$

   C_{μ} are 3x3 matrices →gluon fields

Lecture 4: "Symmetries" – Standard Model

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - qJ^\mu_{EM}A_\mu - \frac{g}{2}J^\mu_{\text{Weak}}\vec{b}_\mu - \frac{g_s}{2}J^\mu_{QCD}\,\vec{c}_\mu$$

QED U(1) $\mathcal{L}_{\text{int}} = -J_\mu A^\mu$ with $J_\mu = q\bar{\psi}\gamma_\mu\psi$

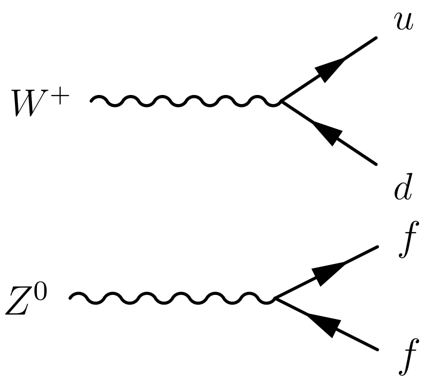


Electromagnetic interaction

Weak SU(2) : $\mathcal{L}_{\text{int}} = -\vec{J}_\mu \vec{b}^\mu$ with $\vec{J}_\mu = \frac{g}{2}\,\bar{\Psi}\,\gamma_\mu\vec{\tau}\,\Psi$

$W^\pm_\mu \equiv \frac{1}{\sqrt{2}}(b^1_\mu \mp i b^2_\mu)$ $\quad J^\pm_\mu = \frac{1}{\sqrt{2}}\bar{\Psi}\gamma_\mu\tau^\pm\Psi$ $\quad$ with $\tau^\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$

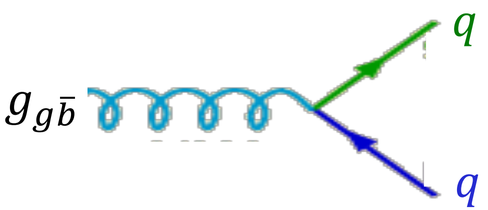
$Z_\mu \sim b^3_\mu$ $\quad J^3_\mu = \frac{1}{2}\bar{\Psi}\gamma_\mu\tau^3\Psi$ $\quad$ with $\tau^\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$



Weak interaction

Electroweak mixing SU(2)xU(1):

$$\gamma_\mu = A_\mu \cos\theta_W + b^3_\mu \sin\theta_W$$
$$Z_\mu = -A_\mu \sin\theta_W + b^3_\mu \cos\theta_W$$



Strong interaction

Standard Model: $SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$

Lecture 4: Electroweak Quantum Numbers

For weak isospin some people write T_3 while others write I_3

With : $Q = T_3 + \frac{1}{2}Y$

Or : $Q = I_3 + \frac{1}{2}Y$

	Generation I	Generation II	Generation III	I_3	Y	Q
Leptons	$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L$	$\begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L$	$\begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L$	$+1/2$ $-1/2$	-1 -1	0 -1
	e_R	μ_R	τ_R	0	-2	-1
Quarks	$\begin{pmatrix}u \\ d\end{pmatrix}_L$	$\begin{pmatrix}c \\ s\end{pmatrix}_L$	$\begin{pmatrix}t \\ b\end{pmatrix}_L$	$+1/2$ $-1/2$	$+1/3$ $+1/3$	$+2/3$ $-1/3$
	u_R	c_R	t_R	0	$+4/3$	$+2/3$
	d_R	s_R	b_R	0	$-2/3$	$-1/3$

Symmetry breaking with a *real* field ϕ

Griffiths §10.9

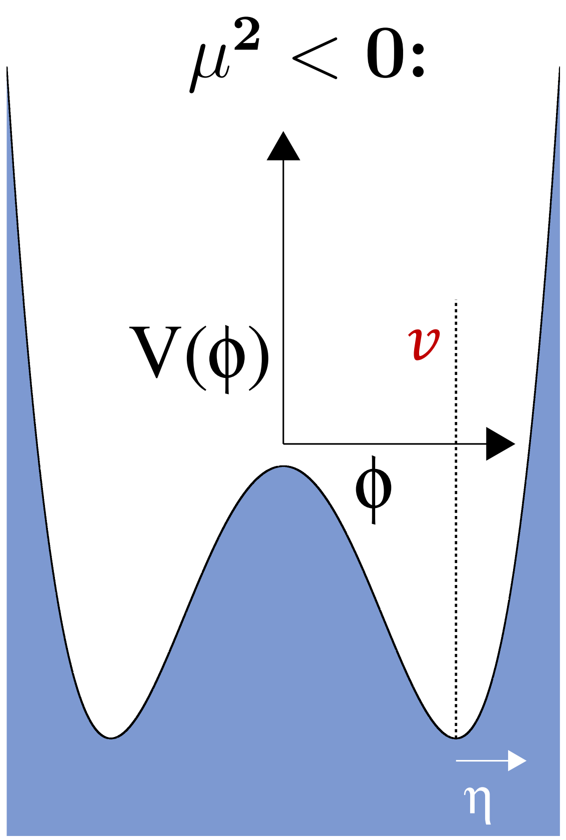
- Explicit mass terms violate the symmetry: $m^2 A_\mu A^\mu \rightarrow m^2\left(A_\mu + \frac{1}{e}\partial_\mu \alpha\right)\left(A^\mu + \frac{1}{e}\partial^\mu \alpha\right) \neq m^2 A_\mu A^\mu$
- Add a new field to the Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2}_{\text{Massive Klein-Gordon term (Spin 0, mass }=\mu)} \underbrace{- \frac{1}{4}\lambda\phi^4}_{\text{Interaction term}}$$

Massive Klein-Gordon term (Spin 0, mass $=\mu$)

Interaction term

The Lagrangian has a minimum for $\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} = v$ or $\mu^2 = -\lambda v^2$



Conclusion:

- The symmetry of the Lagrangian by adding a symmetric potential ϕ *has not been destroyed*
- The *vacuum is no longer* in a symmetric position

Symmetry breaking with a *complex* field ϕ

Griffiths §10.9

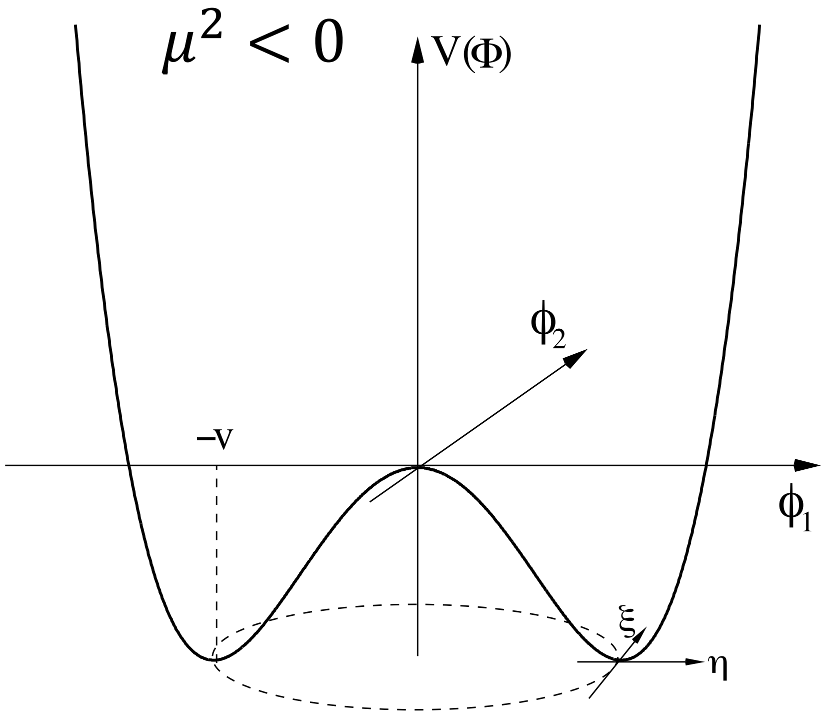
- Introduce a complex scalar field: $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$
- The Lagrangian term is: $\mathcal{L} = \left(\partial_\mu \phi\right)^* (\partial^\mu \phi) - V(\phi)$, with $V(\phi) = \mu^2(\phi^*\phi) + \lambda\,(\phi^*\phi)^2$
- Lagrangian:

$$\mathcal{L}(\phi_1, \phi_2) = \frac{1}{2}\left(\partial_\mu \phi_1\right)^2 + \frac{1}{2}\left(\partial_\mu \phi_2\right)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

Conclusion:

- The symmetry of the Lagrangian by adding a symmetric potential ϕ *has not been destroyed*
- The *vacuum is no longer* in a symmetric position

The real case includes a complex (isospin doublet) field ϕ
- ϕ degrees of freedom lead to mass terms for the W^+, W^-, Z^0
- ϕ can also couple to fermions → particle masses



- Symmetry breaking:

$$\phi_0 = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$$

Lecture 5 : “Scattering” – non-Rel.

<u>Perturbation theory</u>

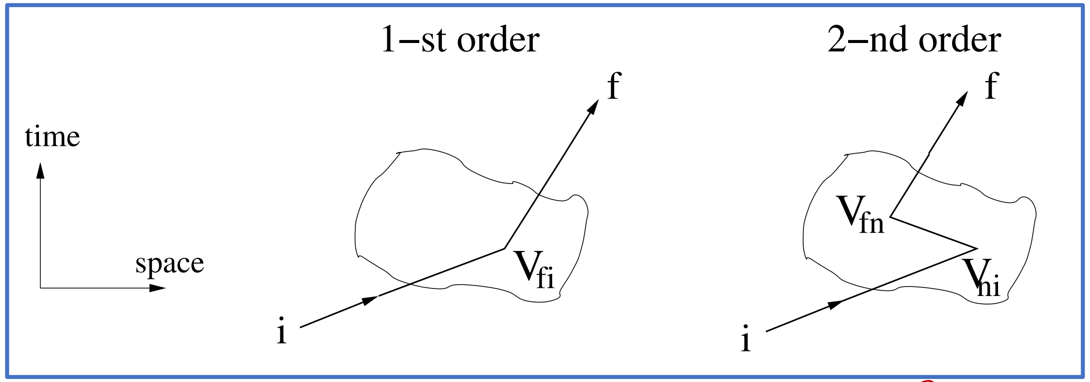
1) $V(x,t)$ is fixed

Solve wave equation Iteratively…

$$i\frac{\partial\psi}{\partial t} = \left(H_0 + V(\vec{x},t)\right)\psi$$

…use plane waves

$$\psi = \sum_{n=0}^{\infty} a_n(t)\phi_n(\vec{x})e^{-iE_nt}$$



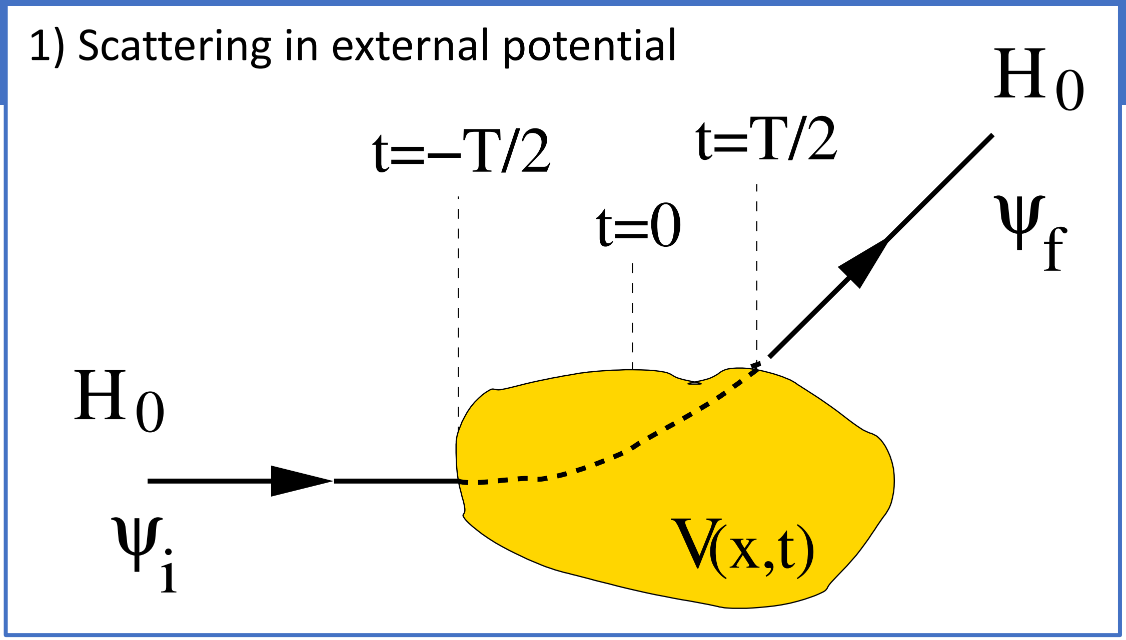
$$\sigma = N/\mathcal{L} \qquad \mathrm{d}\sigma = \frac{W_{fi}}{\mathrm{flux}}\mathrm{d}\Phi \qquad W_{fi} \equiv \lim_{T\to\infty}\frac{|T_{fi}|^2}{T}$$

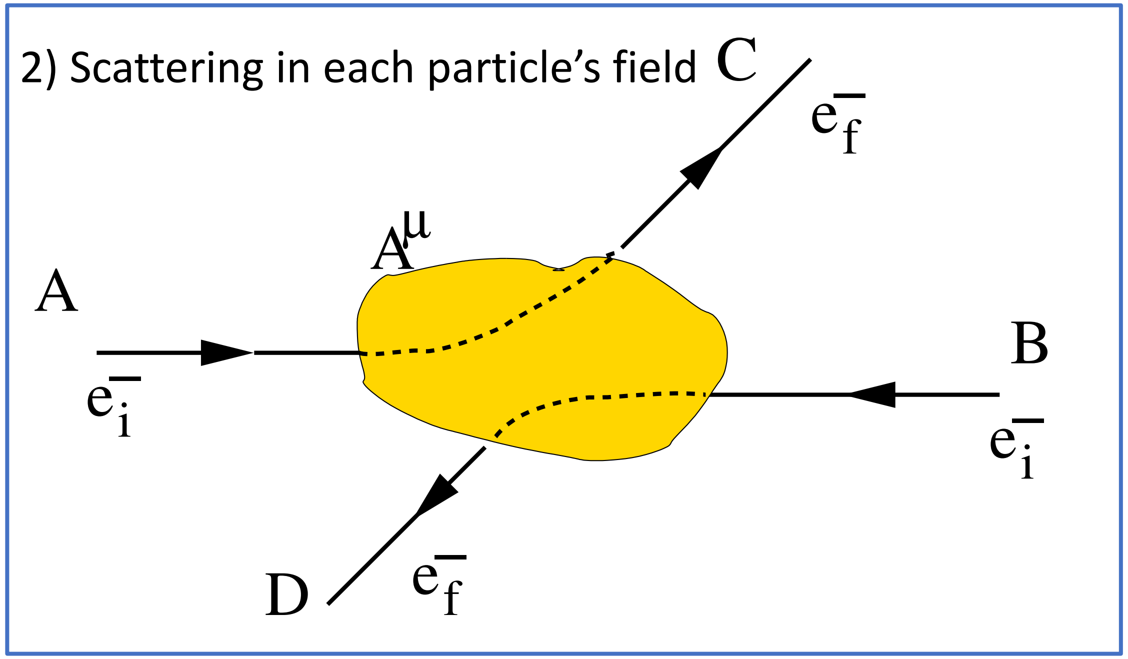
$$T_{fi} = -i\int \mathrm{d}^4x\, \psi_f^*(x)V(x)\psi_i(x) \;= -2\pi V_{fi}\delta\left(E_f - E_i\right)$$

Energy conservation

2) Determine V from A field scattering particles
(Solve Maxwell equation)

Relativistic: $V_{fi} \rightarrow \mathcal{M}$ “matrix element”





Lecture 5 : “Scattering” – non-Relativistic

Perturbation theory

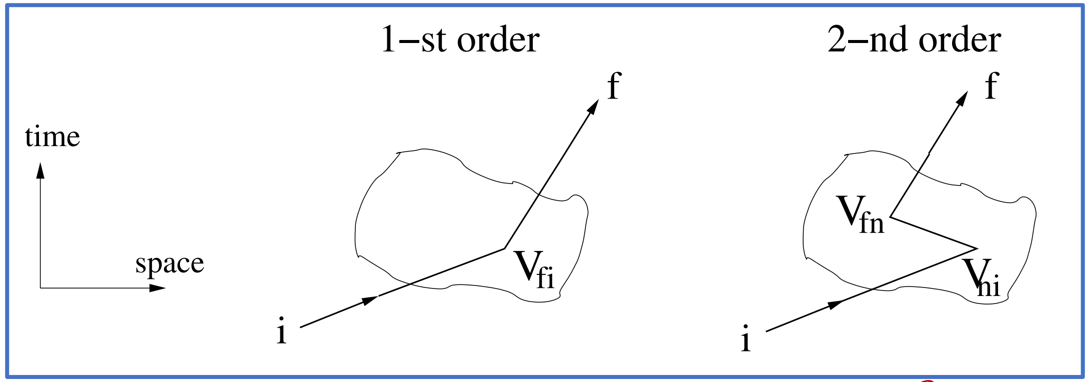
1) $V(x,t)$ is fixed

Solve wave equation Iteratively…

$$i\frac{\partial\psi}{\partial t} = \left(H_0 + V(\vec{x},t)\right)\psi$$

…use plane waves

$$\psi = \sum_{n=0}^{\infty} a_n(t)\phi_n(\vec{x})e^{-iE_n t}$$



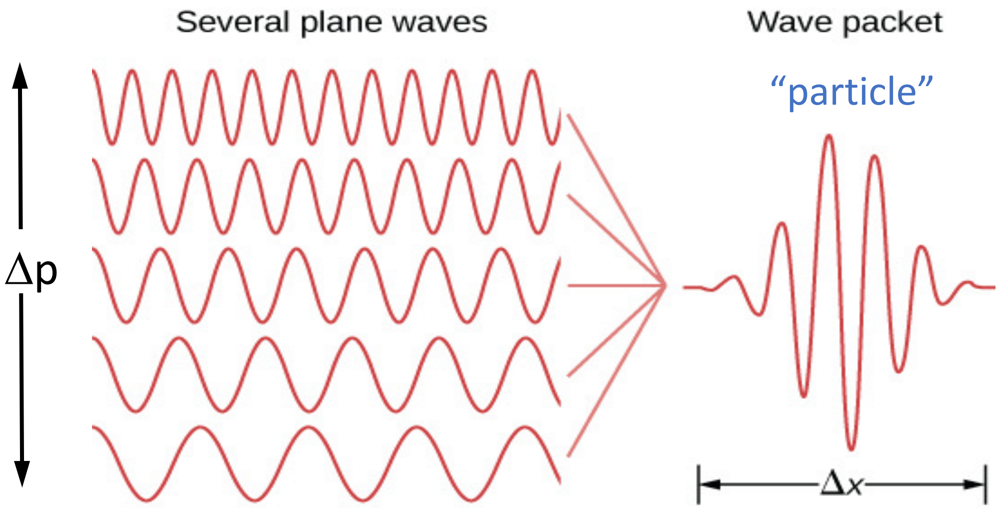
$$\sigma = N/\mathcal{L} \qquad \mathrm{d}\sigma = \frac{W_{fi}}{\mathrm{flux}}\mathrm{d}\Phi \qquad W_{fi} \equiv \lim_{T\to\infty}\frac{\left|T_{fi}\right|^2}{T}$$

$$T_{fi} = -i\int \mathrm{d}^4x\,\psi_f^*(x)V(x)\psi_i(x) = -2\pi V_{fi}\delta\left(E_f - E_i\right)$$

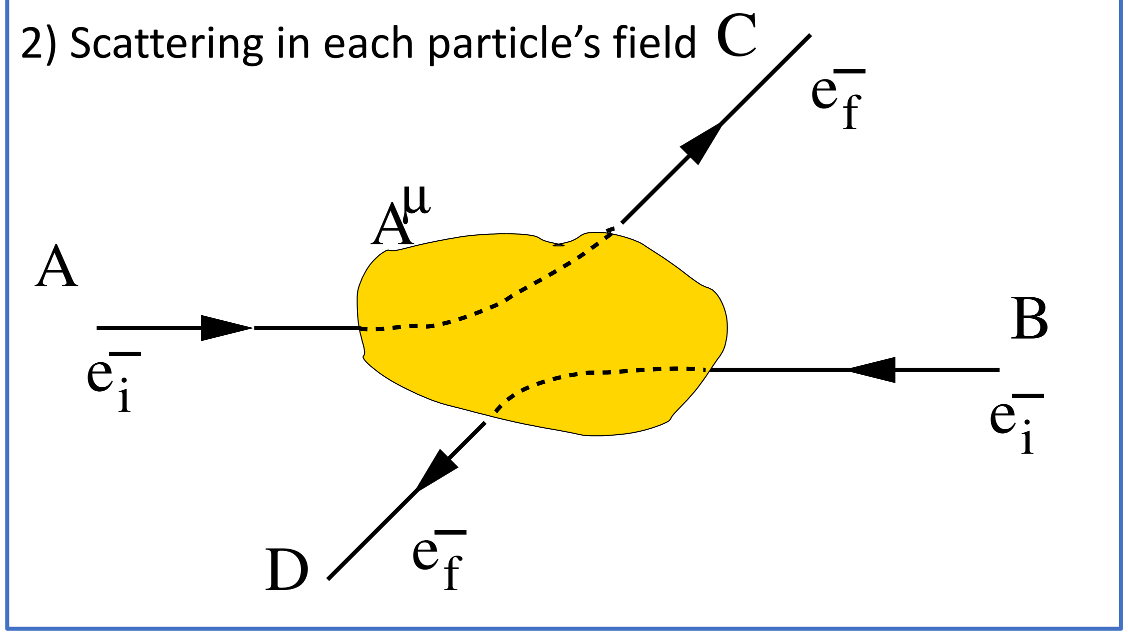
Energy conservation

2) Determine V from A field scattering particles
(Solve Maxwell equation)

Relativistic: V_{fi} ➔ $\mathcal{M}$ “matrix element”



2) Scattering in each particle’s field

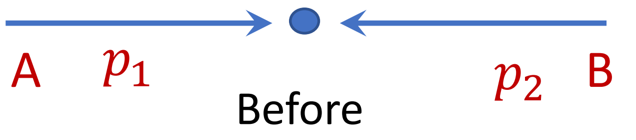


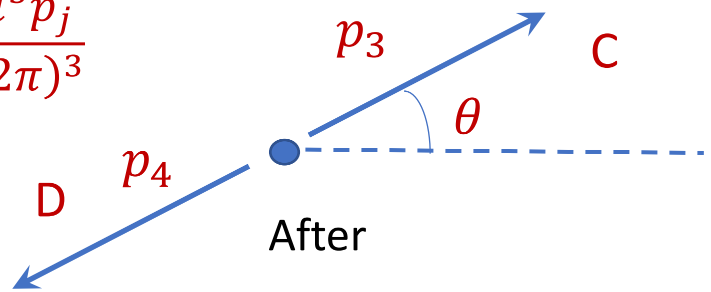
Lecture 5 : “Scattering” - Relativistic

Cross section:

$$\sigma = \frac{S}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 \ldots - p_n) \times \prod_{j=3}^{n} \frac{1}{2E_j} \frac{d^3\vec{p}_j}{(2\pi)^3}$$

Eg: “2-to-2” scattering:





$$\sigma = \frac{S}{64\pi^2 (E_1 + E_2)|\vec{p}_1|} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \frac{d^3\vec{p}_3}{E_3} \frac{d^3\vec{p}_4}{E_4}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

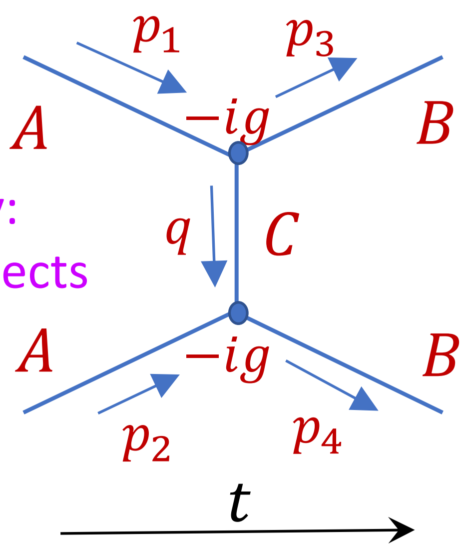
How to determine $\mathcal{M}$? → Feynman rules (depend on actual theory/interaction):

Feynman rules (ABC theory):

1. Diagram: see sketch
2. Labels: see sketch
3. Two vertices: $(-ig)^2 = -g^2$
4. Propagators: one internal line: $\frac{i}{q^2 - m_C^2}$
5. Conservation of $E, \vec{p}$ twice: $(2\pi)^4 \delta^4(p_1 - p_3 - q)$ and $(2\pi)^4 \delta^4(p_2 + q - p_4)$
6. Integrate: one integral: $1/(2\pi)^4 \, d^4q$
7. Erase delta-function and multiply by i to find: $\mathcal{M} = \frac{g^2}{(p_4 - p_2)^2 - m_C^2}$

Example diagram:
$A + A \rightarrow B + B$

From ABC → Standard Model theory:
More complicated rules and spin objects
→ Master level education



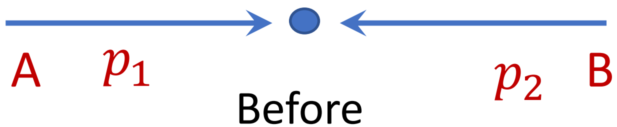
Lecture 5 : “Scattering” - Relativistic

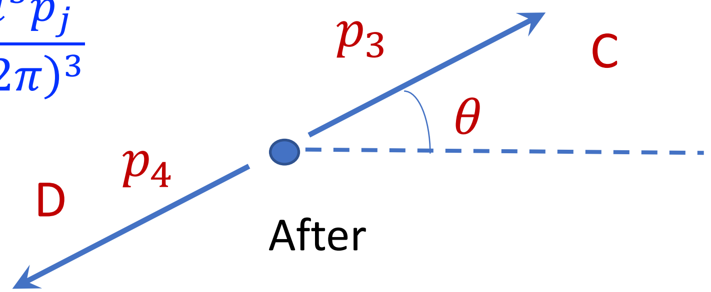
$$d\sigma = \frac{W_{fi}}{\text{flux}} d\Phi \quad \Rightarrow \quad \sigma = \frac{1}{\text{flux}} \int W_{fi}\, d\Phi$$

Cross section: Flux, Matrix element, Energy momentum conservation, Phase space

$$\sigma = \frac{S}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 \, (2\pi)^4 \delta^4(p_1 + p_2 - p_3 \dots - p_n) \times \prod_{j=3}^{n} \frac{1}{2E_j} \frac{d^3\vec{p}_j}{(2\pi)^3}$$

Eg: “2-to-2” scattering:





$$\sigma = \frac{S}{64\pi^2 (E_1 + E_2)|\vec{p}_1|} \int |\mathcal{M}|^2 \, (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \frac{d^3\vec{p}_3}{E_3} \frac{d^3\vec{p}_4}{E_4}$$

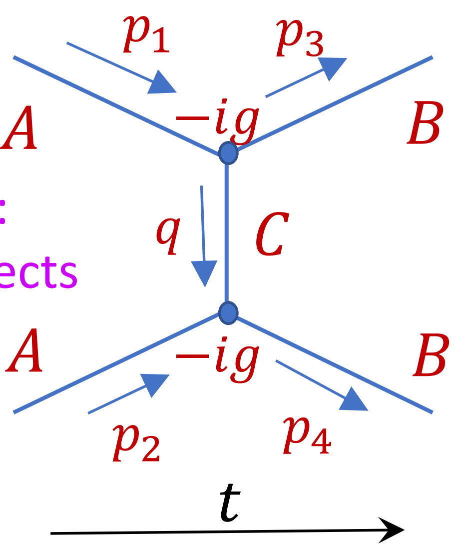
$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

How to determine $\mathcal{M}$? → Feynman rules (depend on actual theory/interaction):

Feynman rules (ABC theory):

1. Diagram: see sketch
2. Labels: see sketch
3. Two vertices: $(-ig)^2 = -g^2$
4. Propagators: one internal line: $\frac{i}{q^2 - m_C^2}$
5. Conservation of $E, \vec{p}$ twice: $(2\pi)^4 \, \delta^4(p_1 - p_3 - q)$ and $(2\pi)^4 \, \delta^4(p_2 + q - p_4)$
6. Integrate: one integral: $1/(2\pi)^4 \, d^4 q$
7. Erase delta-function and multiply by i to find: $\mathcal{M} = \frac{g^2}{(p_4 - p_2)^2 - m_C^2}$

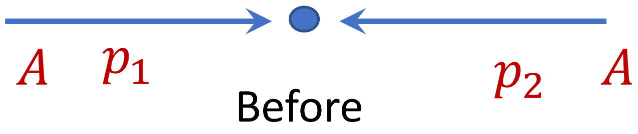
Example diagram:
$A + A \rightarrow B + B$

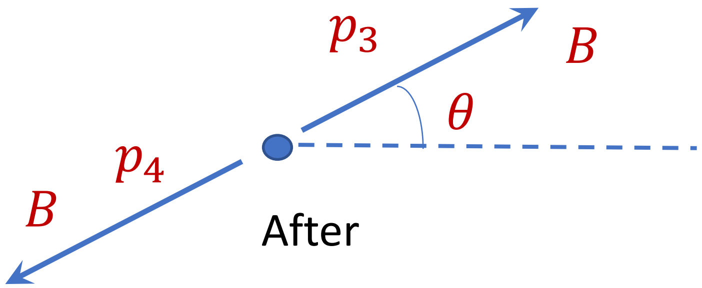


+ diagram $p_3 \leftrightarrow p_4$

From ABC → Standard Model theory:
More complicated rules and spin objects
→ Master level education

$A + A \to B + B$ Scattering: $d\sigma/d\Omega$





- Look at the matrix element and assume that $m_A = m_B = m$ and $m_C = 0$ (eg. a photon):

$$\mathcal{M} = \frac{g^2}{(p_3 - p_2)^2} + \frac{g^2}{(p_4 - p_2)^2}$$

$$\begin{aligned}(p_4 - p_2)^2 - m_C^2 &= p_4^2 + p_2^2 - 2p_2 \cdot p_4 \\ &= m_4^2 + m_2^2 - 2p_2 \cdot p_4 \\ &= 2m^2 - 2E_2E_4 + 2(\vec{p}_2 \cdot \vec{p}_4) \\ &= 2m^2 - 2\left(\sqrt{m^2 + \vec{p}^2}\right)\left(\sqrt{m^2 + \vec{p}^2}\right) + 2\vec{p}^2 \cos\theta \\ &= -2\vec{p}^2(1 - \cos\theta)\end{aligned}$$

$$(p_3 - p_2)^2 - m_C^2 = -2\vec{p}^2(1 + \cos\theta)$$

Note that for 4-vectors:
$p_i \cdot p_j = p_{i_\mu} p_j^\mu = E_iE_j - \vec{p}_i \cdot \vec{p}_j$
and that $p^2 = p_\mu p^\mu = E^2 - \vec{p}^2 = m^2$

(Invariant mass)

$$\mathcal{M} = \frac{g^2}{-2\vec{p}^2(1 - \cos\theta)} + \frac{g^2}{-2\vec{p}^2(1 + \cos\theta)} = -\frac{g^2}{2\vec{p}^2 \sin^2\theta}$$

- Plug in: ($S = 1/2$)

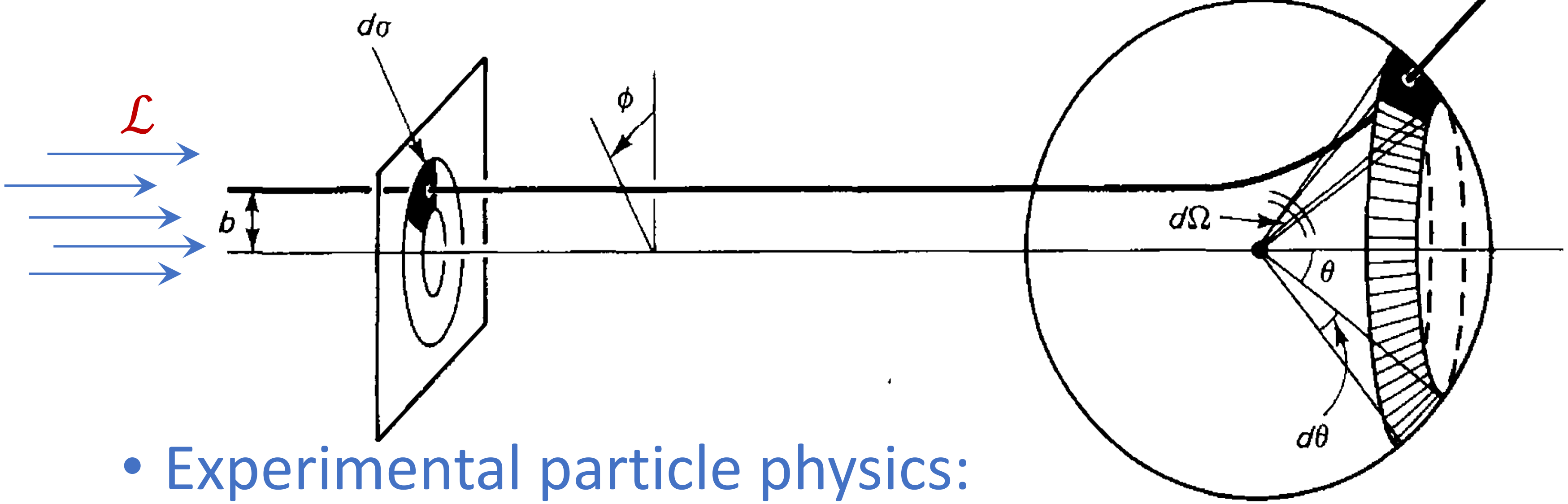
$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{g^2}{16\pi E\vec{p}^2 \sin^2\theta}\right)^2$$

Lecture 5 : Towards Experimental

Griffiths §6.1

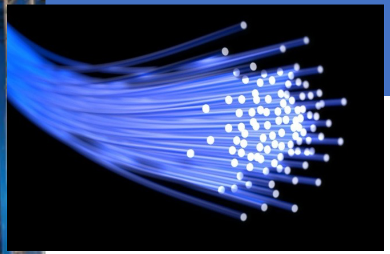
- Consider beam of particles on a target
  - Luminosity $\mathcal{L}$ is number of particles per unit time, per unit area.
  - Number of particles passing through area dσ: $dN = \mathcal{L}\, \mathrm{d}\sigma$
  - Number of particles scattering into solid angle dΩ : $dN = \mathcal{L}\, \mathrm{d}\sigma = \mathcal{L}\, D(\theta)\, \mathrm{d}\Omega$
  - By counting one can measure the *differential cross section*: $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = D(\theta) = \frac{\mathrm{d}N}{\mathcal{L}\, \mathrm{d}\Omega}$
  - Alternatively the total cross section: $N = \mathcal{L}\, \sigma$

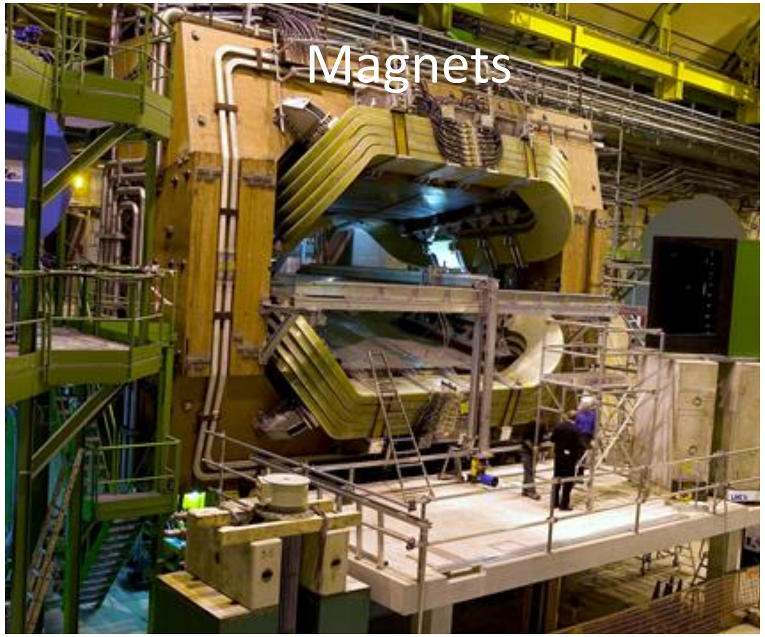


These aspects are needed when you Compare theory with experiments.

- Experimental particle physics:
  - Measure number of events N and the luminosity $\mathcal{L}$ to find cross section $\sigma = N/\mathcal{L}$
  - Compare with theoretical calculation of σ (or $\frac{d\sigma}{d\Omega}$) using e.g. Standard Model

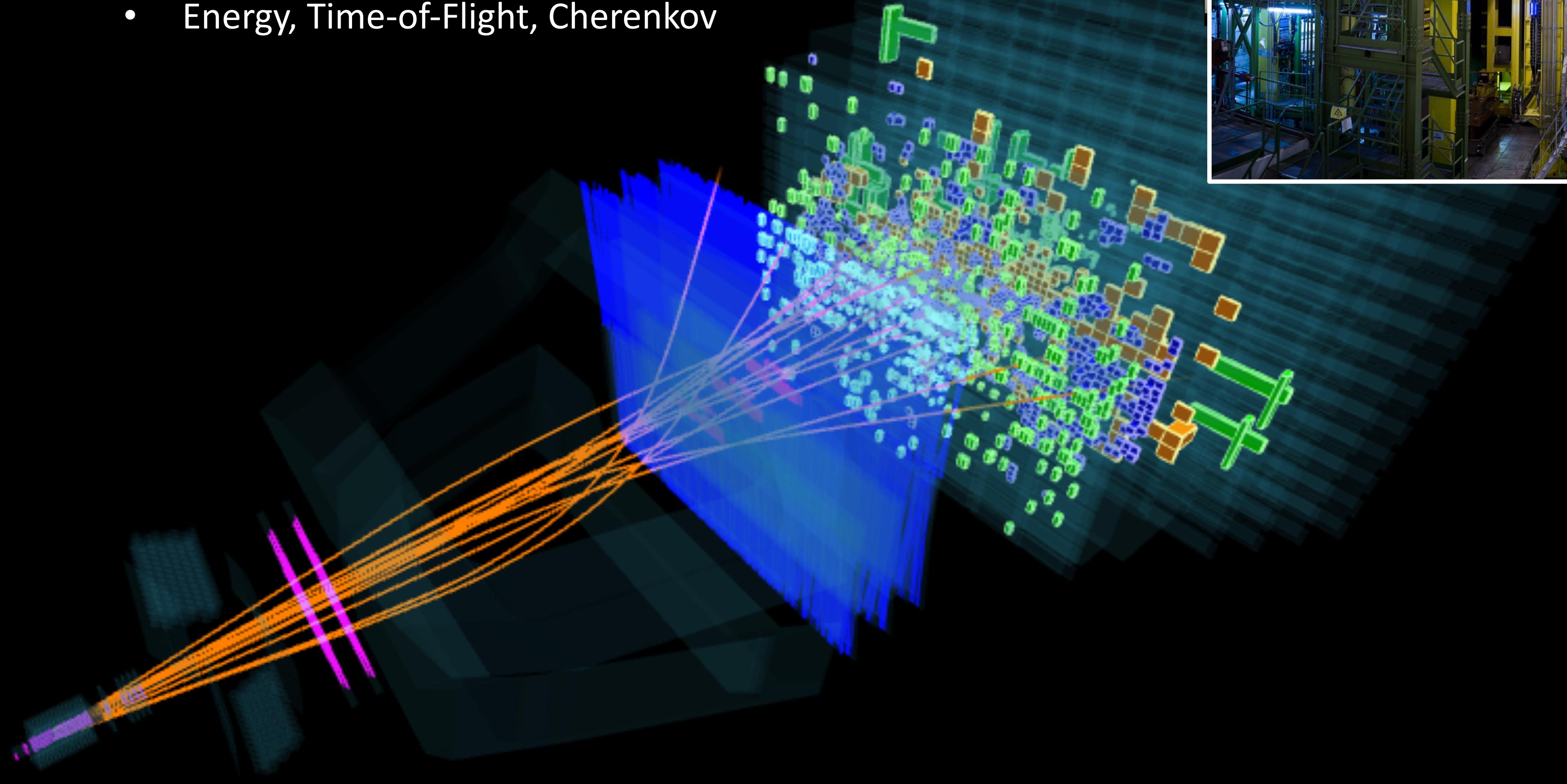
Lecture 6 : “Detectors” - Technologies





Lecture 6 : “Detectors” - Combination

- Charged particle tracking:
  - Position & Momentum measurement
- Particle Identification:
  - Energy, Time-of-Flight, Cherenkov



Skill: Four vectors & co- and contra-variance

Griffiths: chapter 3

- Four vector: $x^\mu = (x^0, x^1, x^2, x^3)$ with $x^0 = ct \Rightarrow x^0 = t$ (since $c \equiv 1$)
- We call this a ***contravariant*** vector and: $x^\mu = (x^0, \vec{x})$
- Lorentz transformation:

$$x^{\mu\prime} = \Lambda^\mu_\nu x^\nu \ ; \ \Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\beta & 0 & 0 \\ -\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x^{0\prime} = \gamma(x^0 - \beta x^1)$$
$$x^{1\prime} = \gamma(x^1 - \beta x^0)$$
$$x^{2\prime} = x^2$$
$$x^{3\prime} = x^3$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Lorentz transformation leaves the following invariant: $|x|^2 = {x^0}^2 - |\vec{x}|^2$
  - $|x|^2 = {x^0}^2 - |\vec{x}|^2 = (ct)^2 - |\vec{x}|^2 = (ct')^2 - |\vec{x}'|^2$
- Introduce ***covariant*** vectors $x_\mu = \sum_\mu g_{\mu\nu} x^\nu = g_{\mu\nu} x^\nu$

Note the Einstein summation convention

- Inproduct invariants: $I = a_\mu b^\mu = a \cdot b = a'_\mu b'^\mu$ for any Lorentz 4-vectors a^μ and b^μ
  - Example invariant mass: $E^2 = \vec{p}^2 c^2 + m^2 c^4 \Rightarrow p^\mu = (E, \vec{p}) \Rightarrow p_\mu p^\mu = E^2 - \vec{p}^2 = m^2$

Skill: Four vectors & co- and contra-variance

- Contravariant vector:

$$x^\mu = (ct, \vec{x})$$
$$p^\mu = (E, \vec{p})$$

But covariant derivative:

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}\right)$$

- Covariant vector:

$$x_\mu = (ct, -\vec{x})$$
$$p_\mu = (E, -\vec{p})$$

But covariant derivative:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

Note that the minus sign is "opposite" to the case of the coordinate four-vectors.

- Use cases:

$$\partial_\mu A^\mu = \partial^\mu A_\mu = \frac{\partial A^0}{\partial t} + \frac{\partial A^1}{\partial x} + \frac{\partial A^2}{\partial y} + \frac{\partial A^3}{\partial z}$$

$$\partial_\mu \partial^\mu \phi = \frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial y} - \frac{\partial \phi}{\partial z}$$

Skill: Dirac Gamma Matrices

- Dirac γ matrices: $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ in 4x4 matrices.
  - We will use the Dirac-Pauli representation

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Note the indices: (confusing!)

$\mu, \nu = 0,1,2,3$ are the ***Lorentz indices in space-time***:

Dirac matrix indices: 1,2,3,4 Have to do with the row and column indices of the matrix (and spinors)

Or: $\gamma^0 = \beta = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix}$ and $\gamma^k = \beta\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$ with Pauli matrices σ_k

Define also the chirality matrix: $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

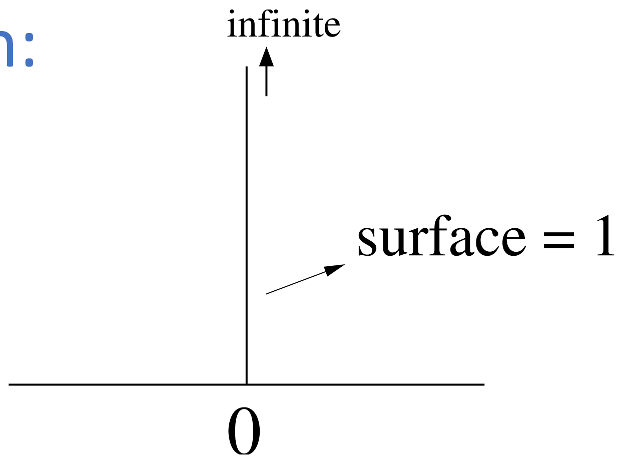
- Note: although the gamma indices are Lorentz-indices ("space-time", the gamma-matrices are not 4-vectors! (They are simply constants.)

Skills: Dirac delta function

See Griffiths Appendix A

- Consider a function defined by the following prescription:

$$\delta(x) = \lim_{\Delta \to 0} \begin{cases} 1/\Delta & \text{for } |x| < \Delta/2 \\ 0 & \text{otherwise} \end{cases}$$



- The integral of this function is normalized: $\int_{-\infty}^{\infty} \delta(x)\, \mathrm{d}x = 1$

- For a function $f(x)$ we have: $f(x)\delta(x) = f(0)\delta(x)$

…and therefore: $\int_{-\infty}^{\infty} f(x)\delta(x)\, \mathrm{d}x = f(0) \int_{-\infty}^{\infty} \delta(x)\, \mathrm{d}x = f(0)$

- An important representation of the Dirac delta function is:

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx}\, \mathrm{d}k$$