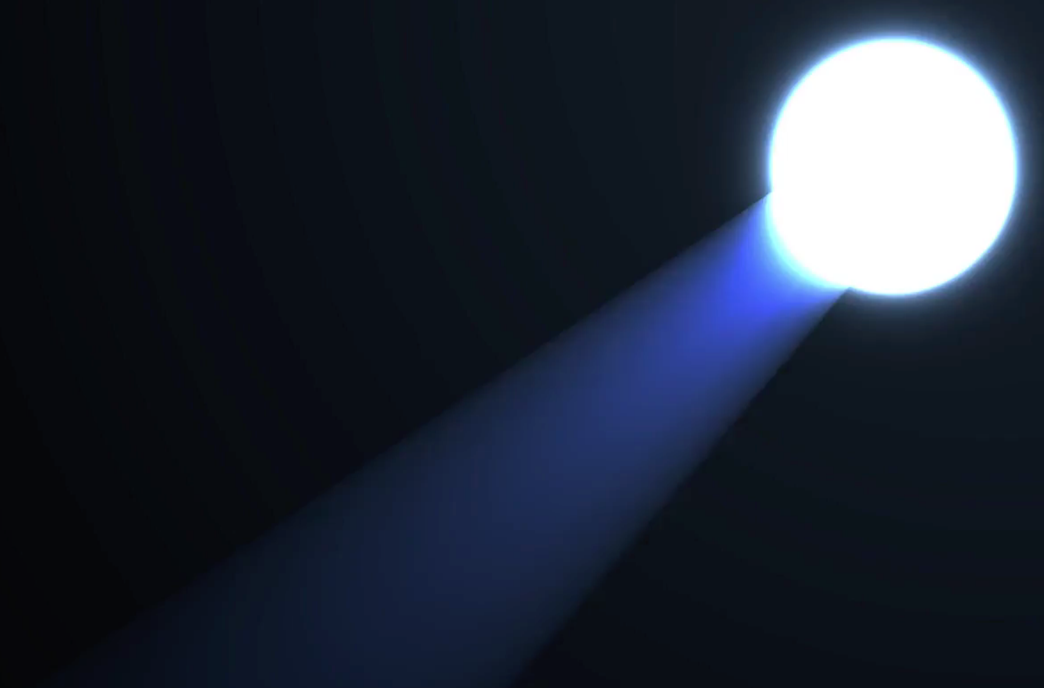
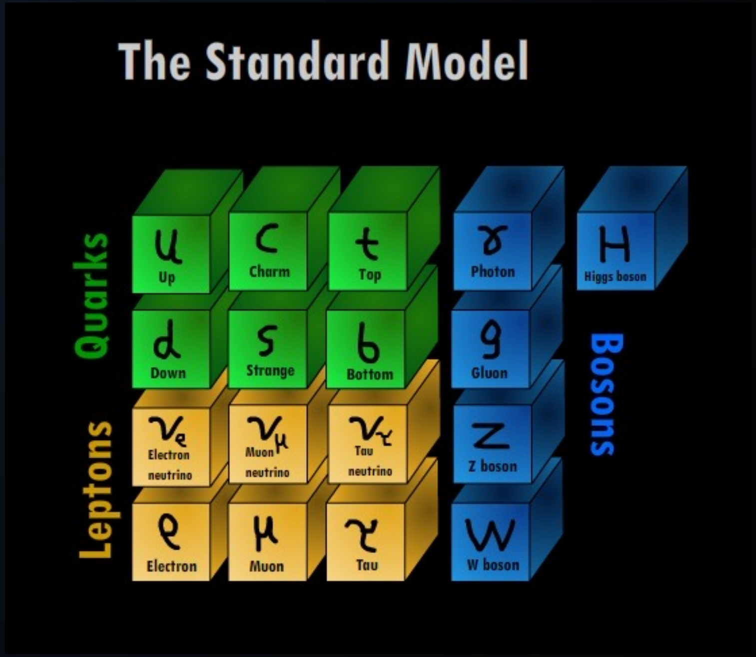
# PHY3004: Nuclear and Particle Physics

Marcel Merk, Jacco de Vries, …



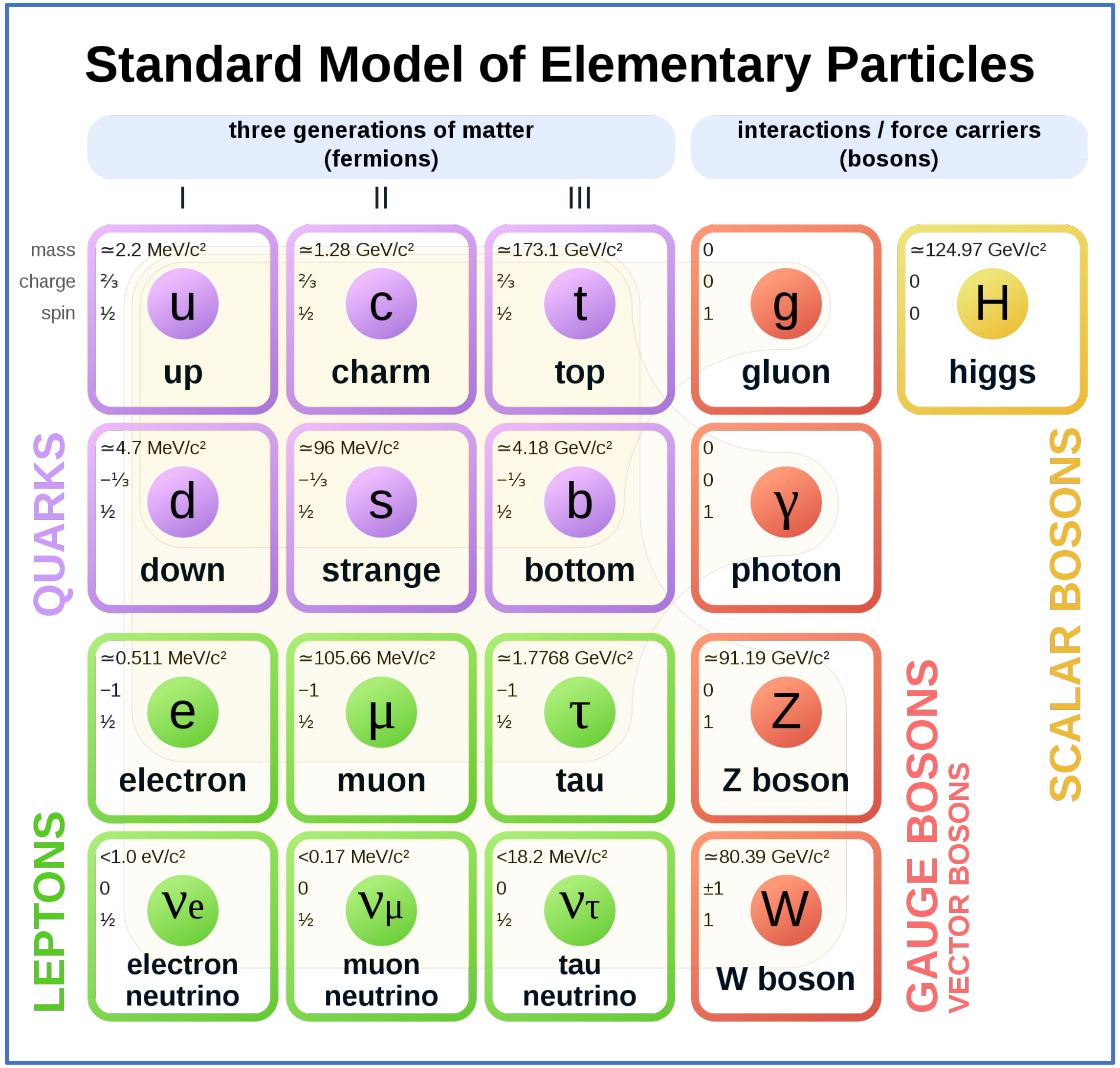


# Lecture 1: "Particles"

Griffiths chapter 1

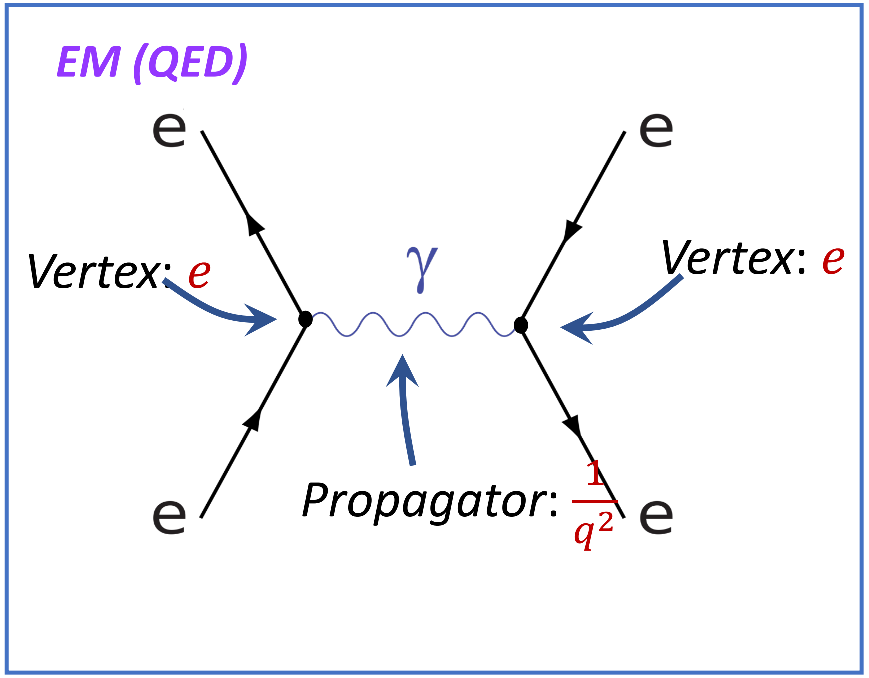
## *Classification of particles*

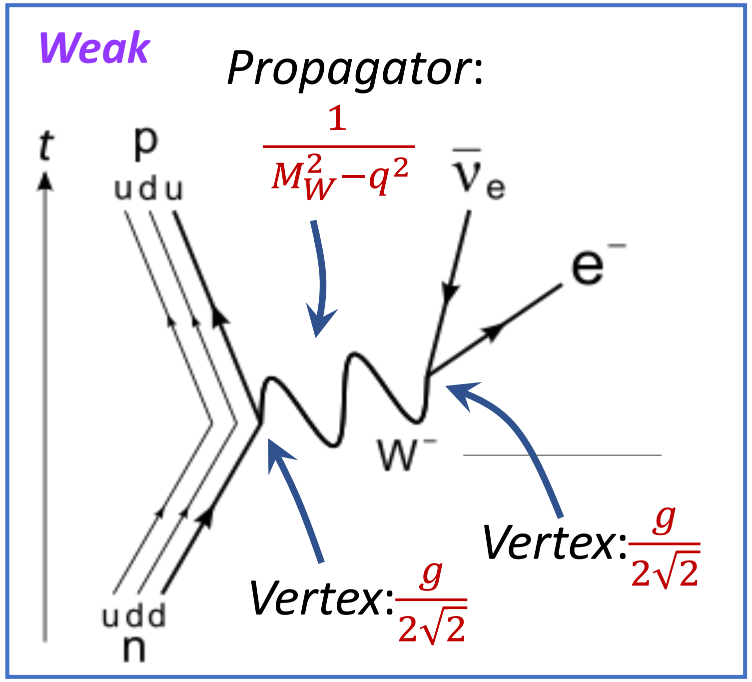
- Lepton: fundamental particle
- Hadron: consist of quarks
  - Meson: 1 quark + 1 antiquark ($\pi^+$, $B_S^0$, …)
  - Baryon: 3 quarks ($p$, $n$, $\Lambda$, …)
    - Anti-baryon: 3 anti-quarks
- ***Fermion***: particle with half-integer spin.
  - Antisymmetric wave function: obeys Pauli-exclusion principle and Pauli-Dirac statistics
  - All fundamental quarks and leptons are spin-½
  - Baryons (S=$^1/_2$, $^3/_2$)
- ***Boson***: particle with integer spin
  - Symmetric wave function: Bose-Einstein statistics
  - Mesons: (S=0, 1), Higgs (S=0)
  - Force carriers: $\gamma$, $W$, $Z$, $g$ (S=1); graviton(S=2)

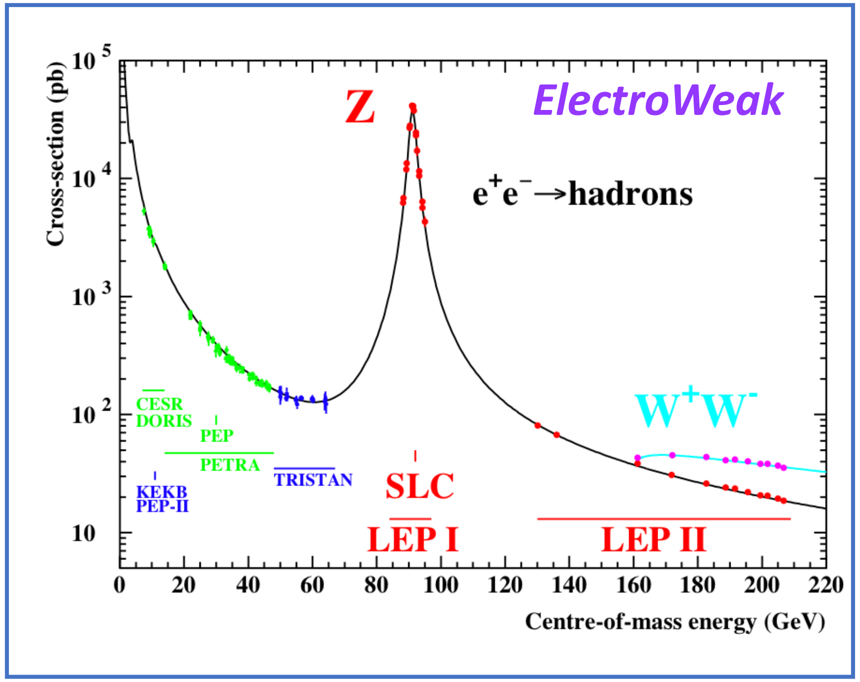


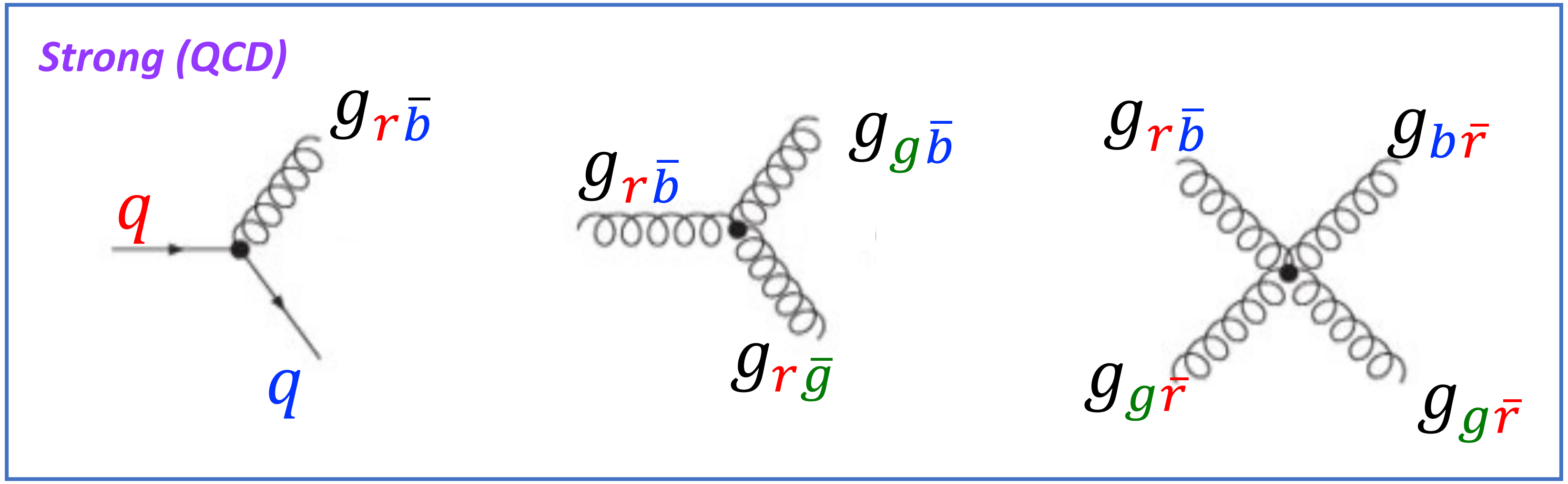
# Lecture 2: "Forces"

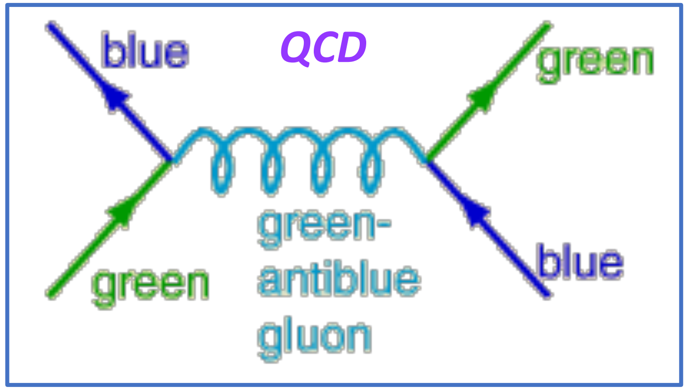
Griffiths chapter 2



# Lecture 3: "Waves" – wave equations

**Probability interpretation (Continuity equation)**

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

Quantum Mechanics: $E \rightarrow \hat{E} = i\hbar \frac{\partial}{\partial t}$ ; $p \rightarrow \hat{p} = -i\hbar\vec{\nabla}$

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Non-relativistic spin 0:

$$E = \frac{\vec{p}^2}{2m}$$

Schrödinger:

$$\boxed{i\hbar \frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi}$$

$$\psi = Ne^{i(\vec{p}\vec{x}-Et)}$$

$$\rho \equiv \psi^*\psi = |N|^2$$

$$\vec{j} \equiv \frac{i\hbar}{2m}\left(\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi\right) = \frac{|N|^2}{m}\vec{p}$$

---

Relativistic spin 0:

$$E^2 = p^2c^2 + m^2c^4$$

Example: pions

Klein-Gordon:

$$-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + \frac{m^2c^2}{\hbar^2}\phi$$

$$\boxed{\partial_\mu\partial^\mu\phi + m^2\phi = 0}$$

$$\phi = Ne^{i(\vec{p}\vec{x}-Et)}$$

$$j^\mu(\rho,\vec{j}) = i[\phi^*(\partial^\mu\phi) - \phi(\partial^\mu\phi^*)]$$

$$\rho = 2|N|^2E \qquad \vec{j} = 2|N|^2\vec{p} \qquad j^\mu = 2|N|^2p^\mu$$

---

Relativistic spin- ½:

$$H = (\vec{\alpha}\cdot\vec{p} + \beta m)$$

Fundamental quarks and leptons

Dirac:

$$i\frac{\partial}{\partial t}\psi = \left(-i\,\vec{\alpha}\cdot\vec{\nabla} + \beta m\right)\psi$$

$$\boxed{(i\gamma^\mu\partial_\mu - m)\psi = 0}$$

$$\psi = u(p)e^{i(\vec{p}\vec{x}-Et)}$$

$$u(p) = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

$$j^0 = \bar{\psi}\gamma^0\psi = \psi^\dagger\psi = \sum_{i=1}^{4}|\psi_i|^2$$

---

Relativistic spin-1:

Fundamental force carriers

Proca:

$$\boxed{\partial_\mu\partial^\mu A^\nu + m^2A^\nu = j^\nu}$$

EM: $A^\mu = \gamma \rightarrow m = 0$
QCD: $A^\mu = g \rightarrow m = 0$
Weak: $A^\mu = W, Z \rightarrow m \neq 0$

EM: Maxwell equations for $\vec{E}$ and $\vec{B}$ fields

# Lecture 3: "Waves" – gauge invariance

Lagrangians:

Spin 0 Scalar field ("pion"): $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$

Spin ½ Dirac fermion: $\mathcal{L} = i\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi$

Spin 1 gauge boson ("photon") : $\mathcal{L} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) - j^\mu A_\mu$

Euler Lagrange lead to the wave equations: $\frac{\partial \mathcal{L}}{\partial \phi(x)} = \partial_\mu \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi(x)\right)}$

***All forces result from requiring a symmetry principle***: Lagrangian should stay invariant

1) QED = U(1) symmetry

$\mathcal{L} = i\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi \longrightarrow \mathcal{L} = i\bar{\psi}\gamma_\mu D^\mu \psi - m\bar{\psi}\psi$

$\psi(x) \rightarrow \psi'(x) = e^{iq\alpha(x)}\psi(x)$

$A^\mu(x) \rightarrow A'^\mu(x) = A^\mu(x) - \partial^\mu \alpha(x)$

Covariant derivative: $\partial^\mu \rightarrow D^\mu \equiv \partial^\mu + iqA^\mu$

$$\mathcal{L} = \underbrace{i\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi}_{\text{“free”}} - \underbrace{q\bar{\psi}\gamma_\mu \psi A^\mu}_{\text{“interaction”}}$$

➔ 1 E.M. photon field: $A^\mu(x)$

2) Weak = SU(2) symmetry $\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$

$\psi(x) \rightarrow \psi'(x) = \exp\left(\frac{i}{2} g\vec{\tau}\cdot\vec{\alpha}(x)\right)\begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$

➔ 3 weak fields: $W^{\mu+}(x)$, $W^{\mu-}(x)$, $Z^\mu(x)$

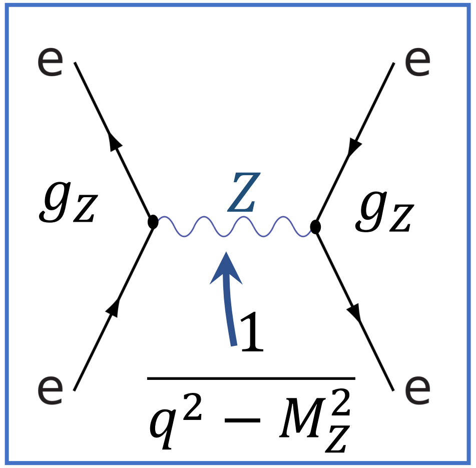
3) QCD = SU(3) symmetry $\psi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$

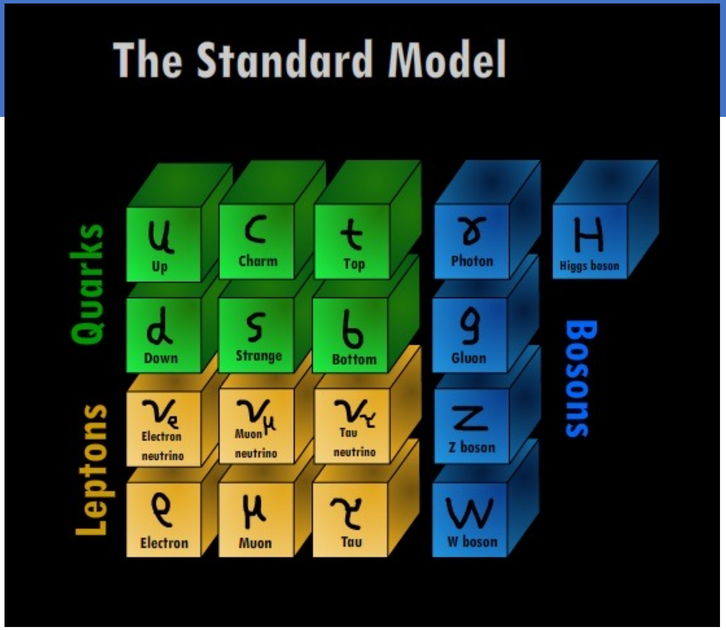
$\psi(x) \rightarrow \psi'(x) = \exp\left(\frac{i}{2} g_s\vec{\lambda}\cdot\vec{\alpha}(x)\right)\begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$

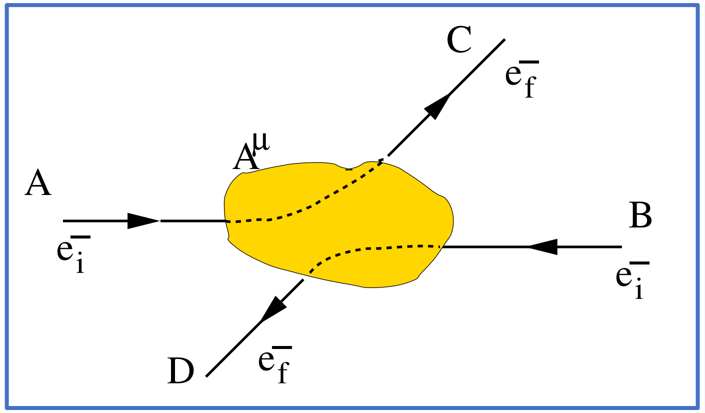
➔ 8 colored gluon fields: $g^\mu(x)$

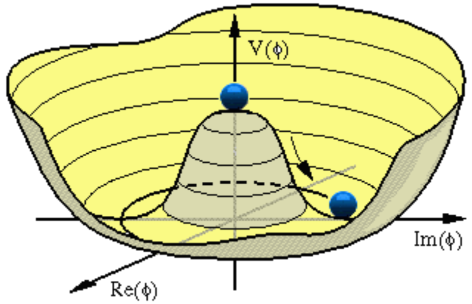
# Recap: "Seeing the wood for the trees"

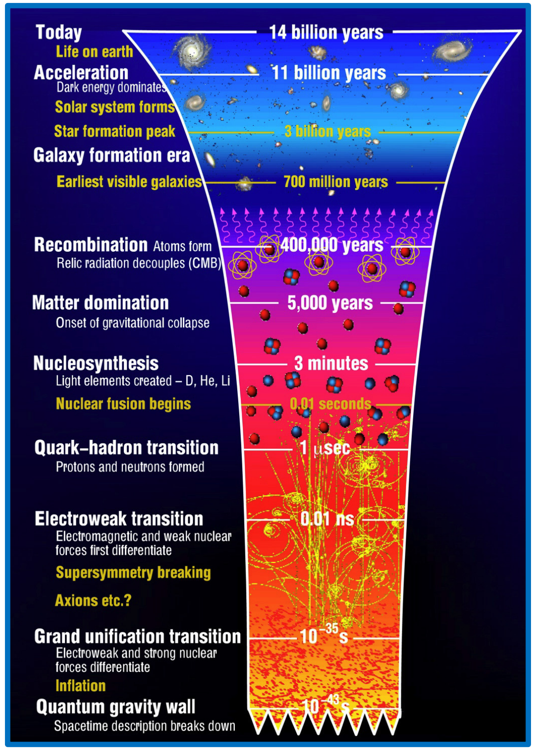
- Lecture 1: "Particles"
  - Zooming into constituents of matter
  - Skills: distinguish particle types, Spin
- Lecture 2: "Forces"
  - Exchange of quanta: EM, Weak, QCD
  - Skills: 4-vectors, Feynman diagrams
- Lecture 3: "Waves"
  - Quantum fields and gauge invariance
  - Dirac algebra, Lagrangian, co- & contra variant
- Lecture 4: "Symmetries"
  - Standard Model, Higgs, Discrete Symmetries
  - Skills: Lagrangians, Chirality & Helicity
- Lecture 5: "Scattering"
  - Cross section, decay, perturbation theory
  - Skills: Dirac-delta function, Feynman Calculus
- Lecture 6: "Detectors"
  - Energy loss mechanisms, detection technologies



# Lecture 4: "Symmetries"

- Gauge Symmetries: Standard Model
- Symmetry Breaking: Higgs Mechanism
- Discrete Symmetries

Griffiths 9.7, PP1 Lect 9

Griffiths 10.7-9, PP1 Lect 11

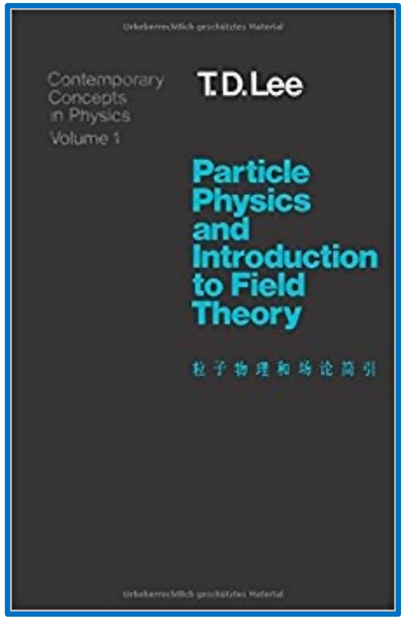
Griffiths chapter 4

# Symmetry and non-observables

T.D.Lee: "The root to all *symmetry* principles lies in the assumption that it is impossible to observe certain basic quantities; the *non-observables*"

There are four main types of symmetry:

- Permutation symmetry:
  - Bose-Einstein and Fermi-Dirac Statistics
- Continuous space-time symmetries:
  - translation, rotation, velocity, acceleration,…
- Discrete symmetries:
  - space inversion, time reversal, charge conjugation,…
- Unitary symmetries: gauge invariances:
  - $U_1$(charge), $SU_2$(isospin), $SU_3$(color),…




$\Rightarrow$ If a quantity is fundamentally non-observable it is related to an *exact* symmetry

$\Rightarrow$ If it could in principle be observed by an improved measurement; the symmetry is said to be *broken*

Noether Theorem: symmetry $\Leftrightarrow$ conservation law

# Symmetry and non-observables

| Non-observables | Symmetry Transformations | Conservation Laws or Selection Rule |
|---|---|---|
| Difference between identical particles | Permutation | B.-E. or F.-D. statistics |
| Absolute spatial position | Space translation: $\vec{r} \to \vec{r} + \vec{\Delta}$ | momentum |
| Absolute time | Time translation: $t \to t + \tau$ | energy |
| Absolute spatial direction | Rotation: $\vec{r} \to \vec{r}'$ | angular momentum |
| Absolute velocity | Lorentz transformation | generators of the Lorentz group |
| Absolute right (or left) | $\vec{r} \to -\vec{r}$ | parity |
| Absolute sign of electric charge | $e \to -e$ | charge conjugation |
| Relative phase between states of different charge Q | $\psi \to e^{i\theta Q}\psi$ | charge |
| Relative phase between states of different baryon number B | $\psi \to e^{i\theta N}\psi$ | baryon number |
| Relative phase between states of different lepton number L | $\psi \to e^{i\theta L}\psi$ | lepton number |
| Difference between different coherent mixture of p and n states | $\binom{p}{n} \to U\binom{p}{n}$ | isospin |

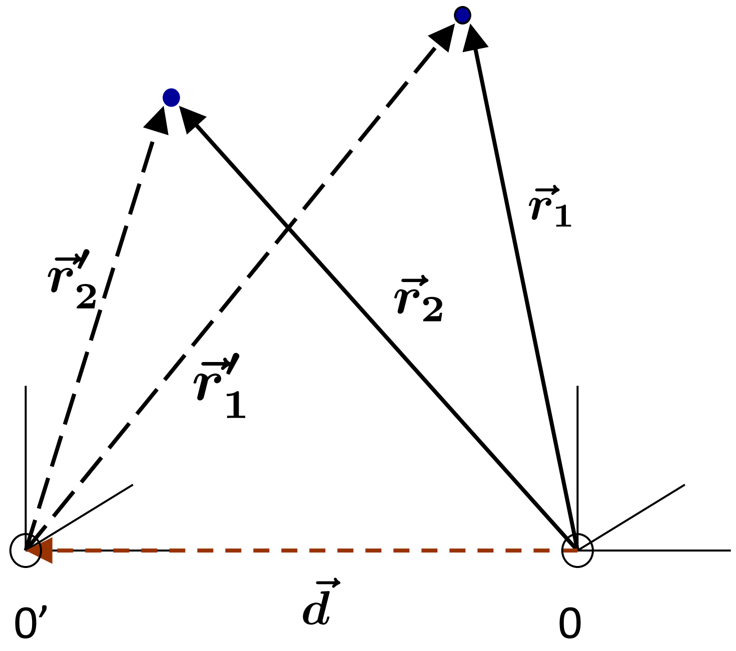
# Symmetry and non-observables: example

- Simple example: potential energy $V$ between two charged particles:

Absolute position is a non-observable:
The interaction is independent on the choice of the origin 0.

Symmetry:
$V$ is invariant under arbitrary space translations:

$$\vec{r_1} \rightarrow \vec{r}_1 + \vec{d} \qquad \vec{r_2} \rightarrow \vec{r}_2 + \vec{d}$$




Consequently:

$$V = V\left(\vec{r}_1 - \vec{r}_2\right)$$

Total momentum is conserved:

$$\frac{d}{dt}\underbrace{\left(\vec{p}_1 + \vec{p}_2\right)}_{\vec{p}_{\mathrm{tot}}} = \vec{F}_1 + \vec{F}_2 = -\left(\vec{\nabla}_1 + \vec{\nabla}_2\right) V = 0$$

# Lecture 4: “Symmetries”

## Part 1

## Gauge Symmetries in The Standard Model

Griffiths 9.7, PP1 Lect 9

# Standard Model

Griffiths §10.3, §10.4, §10.5

- The Lagrangian of the Standard Model includes electromagnetic, weak and strong interactions according to the gauge field principle

- Construction of the Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{free}} - \mathcal{L}_{\text{interaction}} = \mathcal{L}_{\text{Dirac}} - gJ^{\mu}A_{\mu}$
  - With $g$ a coupling constant, $J^{\mu}$ a *current* $(\bar{\psi}\mathrm{O}_i\psi)$ and $A_{\mu}$ a *force field*

A. Local $U(1)$ gauge invariance: symmetry under complex phase rotations
  - Conserved quantum number: (hyper-) charge
  - Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - q\,\underbrace{\bar{\psi}\gamma^{\mu}\psi}_{J^{\mu}_{EM}}\,A_{\mu}$

    $(\partial_{\mu} \rightarrow D_{\mu} \equiv \partial_{\mu} + iqA_{\mu})$

Note Spinor: $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$

B. Local $SU(2)$ gauge invariance: symmetry under transformations in isospin doublet space.
  - Conserved quantum number: weak isospin
  - Lagrangian: $\mathcal{L} = \bar{\Psi}(i\gamma^{\mu}D_{\mu} - m)\Psi = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi - \frac{g}{2}\underbrace{\bar{\Psi}\gamma^{\mu}\vec{\tau}\Psi}_{J^{\mu}_{Weak}}\,\vec{b}_{\mu}$

$(I\partial_{\mu} \rightarrow D_{\mu} = I\partial_{\mu} + igB_{\mu})$ ; $B_{\mu} = \frac{1}{2}\vec{\tau}\cdot\vec{b}_{\mu} = \frac{1}{2}\tau_1^a b_{\mu}^a = \frac{1}{2}\begin{pmatrix} b_3 & b_1 - ib_2 \\ b_1 + ib_2 & -b_3 \end{pmatrix}$

Note doublet spinors: $\Psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$

with $\psi_u, \psi_d$ spinors

C. Local $SU(3)$ gauge invariance: symmetry under transformations in colour triplet space
  - Conserved quantum number: color
  - Lagrangian: $\mathcal{L} = \bar{\Phi}(i\gamma^{\mu}D_{\mu} - m)\Phi = \bar{\Phi}(i\gamma^{\mu}\partial_{\mu} - m)\Phi - \frac{g_s}{2}\underbrace{\bar{\Phi}\gamma^{\mu}\vec{\lambda}\Phi}_{J^{\mu}_{QCD}}\,\vec{c}_{\mu}$

    $(I\partial_{\mu} \rightarrow D_{\mu} = I\partial_{\mu} + ig_sC_{\mu})$ $C_{\mu}$ are 3x3 matrices →gluon fields

Note triplet spinors: $\Phi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$

$\psi_r, \psi_g, \psi_b$ spinors

# Standard Model

Griffiths §10.3, §10.4, §10.5

- The Lagrangian of the Standard Model includes electromagnetic, weak and strong interactions according to the gauge field principle

- Construction of the Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{free}} - \mathcal{L}_{\text{interaction}} = \mathcal{L}_{\text{Dirac}} - gJ^{\mu}A_{\mu}$
  - With $g$ a coupling constant, $J^{\mu}$ a *current* ($\bar{\psi}\mathrm{O}_i\psi$) and $A_{\mu}$ a *force field*

Standard Model Lagrangian:

$$\boxed{\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - qJ^{\mu}_{EM}A_{\mu} - \frac{g}{2}J^{\mu}_{\text{Weak}}\vec{b}_{\mu} - \frac{g_s}{2}J^{\mu}_{QCD}\,\vec{c}_{\mu}}$$

Implements U(1), SU(2) and SU(3) symmetries simultaneous:

$$SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$$

Requiring the Lagrangian to be invariant (symmetry) implies that the EM, Weak and Strong force fields must exist and the interactions respectively conserve charge, weak isospin, and color.

# Electromagnetism and Weak force

- U(1) gauge transformations require that the laws of physics (i.e. the Lagrangian) is invariant under:

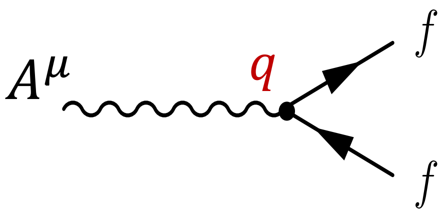
$$\psi(x) \rightarrow \psi'(x) = \mathrm{e}^{iq\alpha(x)}\psi(x)$$

$$A^{\mu}(x) \rightarrow A'^{\mu}(x) = A^{\mu}(x) - \partial^{\mu}\alpha(x)$$

$$(\partial_{\mu} \rightarrow D_{\mu} \equiv \partial_{\mu} + iqA_{\mu})$$

Electromagnetic field gauge transformation

- This leads to the interaction: $\mathcal{L}_{\text{int}} = -J_{\mu}A^{\mu}$ with $J_{\mu} = q\bar{\psi}\gamma_{\mu}\psi$




- SU(2) gauge transformations require that the laws of physics (i.e. the Lagrangian) is invariant under:

$$\Psi(x) \rightarrow \Psi'(x) = \mathrm{e}^{ig\frac{1}{2}\vec{\tau}\cdot\vec{\alpha}(x)}\Psi(x)$$

With doublets $\Psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$ and $\overline{\Psi} = (\psi_u, \psi_d)$

$$(I\partial_{\mu} \rightarrow D_{\mu} = I\partial_{\mu} + igB_{\mu}) \quad ; \qquad B_{\mu} = \frac{1}{2}\vec{\tau}\cdot\vec{b}_{\mu} = \frac{1}{2}\tau_1^a b_{\mu}^a = \frac{1}{2}\begin{pmatrix} b_3 & b_1 - ib_2 \\ b_1 + ib_2 & -b_3 \end{pmatrix}$$

- This leads to the interaction: $\mathcal{L}_{\text{int}} = -\vec{J}_{\mu}\vec{b}^{\mu}$ with $\vec{J}_{\mu} = \frac{g}{2}\,\overline{\Psi}\,\gamma_{\mu}\vec{\tau}\,\Psi$ Note: $\vec{J}_{\mu} = J_{\mu}^1, J_{\mu}^2, J_{\mu}^3$

Weak Isospin: $T_i = \frac{1}{2}\tau_i$

$\vec{\tau} = \tau_1, \tau_2, \tau_3$ are the Pauli matrices: $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

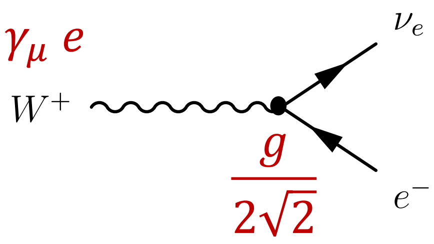
# The weak force

Griffiths §9.5

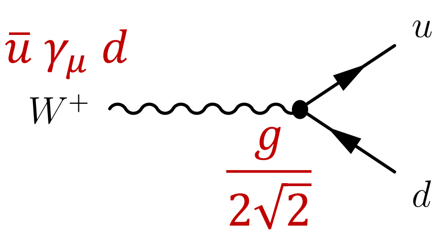
- The weak interaction includes charged ($J_\mu^1$ and $J_\mu^2$) and neutral ($J_\mu^3$) currents
- It turns out the following charge current fields are realized in Nature:
  - $W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(b_\mu^1 \mp i b_\mu^2\right)$ and $Z_\mu = b_\mu^3$ (see exercise)
- The charged current becomes
  - $J_\mu^\pm = \frac{1}{\sqrt{2}}\bar{\Psi}\gamma_\mu\tau^\pm\Psi$ with $\tau^\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$

$$\tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \tau^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

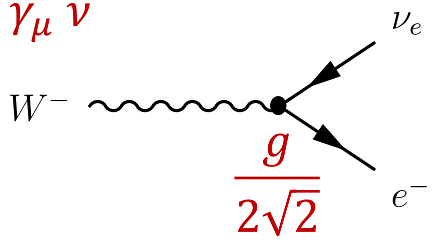
Charge raising interaction: $J_\mu^+ = \frac{g}{2\sqrt{2}}\,\bar{\nu}\,\gamma_\mu\,e$



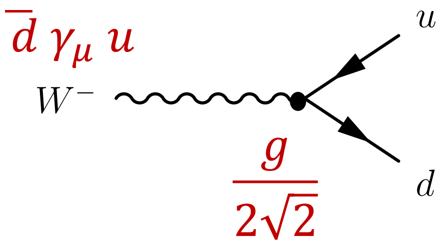

$J_\mu^+ = \frac{g}{2\sqrt{2}}\,\bar{u}\,\gamma_\mu\,d$




Charge lowering interaction: $J_\mu^- = \frac{g}{2\sqrt{2}}\,\bar{e}\,\gamma_\mu\,\nu$

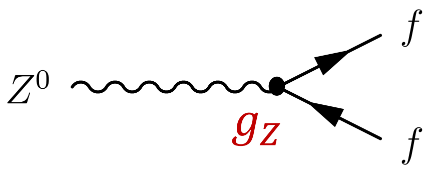



$J_\mu^- = \frac{g}{2\sqrt{2}}\,\bar{d}\,\gamma_\mu\,u$




- The neutral current is:
  - $J_\mu^3 = \frac{1}{2}\bar{\Psi}\gamma_\mu\tau^3\Psi$

$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



# Exercise – 18 : Charge Current

- Show that the definition $W_\mu^{\pm} = \frac{b_\mu^1 \mp i b_\mu^2}{\sqrt{2}}$ leads to the charged current:

$\mathcal{L} = -W_\mu^+ J^{\mu^+} - W_\mu^- J^{\mu^-}$ with $J^{\mu^+} = \frac{g}{\sqrt{2}} \bar{\Psi} \gamma_\mu \tau^+ \Psi$ and $J^{\mu^-} = \frac{g}{\sqrt{2}} \bar{\Psi} \gamma_\mu \tau^- \Psi$

# Electroweak unification

Griffiths §9.5 and §9.7

- A strange phenomenon for the neutral current
  - The $SU(2)$ gauge field $b_\mu^3$ and and the $U(1)$ gauge field $A_\mu$ are not physical
  - The physical fields are: $\gamma_\mu = A_\mu \cos\theta_W + b_\mu^3 \sin\theta_W$ ("*mixing*")

    $Z_\mu = -A_\mu \sin\theta_W + b_\mu^3 \cos\theta_W$
  - The electromagnetic and weak interaction are linear combinations of the $U(1)$ and $SU(2)$ symmetries – why??
    - We speak of a *unified electroweak force*

- The $U(1)$ symmetry is related to the quantity "hypercharge" $Y$
  - The charge of a particle is given by the relation: $Q = T_3 + \frac{1}{2}Y$

- The Standard Model of interactions implements the symmetry:

$$SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$$

  - Mystery 1: How do gauge bosons and fermions acquire a mass
  - Mystery 2: The weak interaction is only *left-handed*

# Electroweak Quantum Numbers

For weak isospin some people write $T_3$ while others write $I_3$

With : $Q = T_3 + \frac{1}{2}Y$

Or : $Q = I_3 + \frac{1}{2}Y$

| | Generation I | II | III | $I_3$ | $Y$ | $Q$ |
|---|---|---|---|---|---|---|
| Leptons | $\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L$ | $\begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L$ | $\begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L$ | $+1/2$<br>$-1/2$ | $-1$<br>$-1$ | $0$<br>$-1$ |
| | $e_R$ | $\mu_R$ | $\tau_R$ | $0$ | $-2$ | $-1$ |
| Quarks | $\begin{pmatrix}u \\ d\end{pmatrix}_L$ | $\begin{pmatrix}c \\ s\end{pmatrix}_L$ | $\begin{pmatrix}t \\ b\end{pmatrix}_L$ | $+1/2$<br>$-1/2$ | $+1/3$<br>$+1/3$ | $+2/3$<br>$-1/3$ |
| | $u_R$ | $c_R$ | $t_R$ | $0$ | $+4/3$ | $+2/3$ |
| | $d_R$ | $s_R$ | $b_R$ | $0$ | $-2/3$ | $-1/3$ |

# Lecture 4: ”Symmetries”

## Part 2

## Electroweak Symmetry Breaking

## The Higgs Mechanism

Griffiths 10.7-9, PP1 Lect 11

# Symmetry breaking

Griffiths §10.7

- Massive particles are forbidden in the SM Lagrangian
  - A hypothetical mass term in the Lagrangian for the photon is not gauge invariant under $A^\mu \rightarrow A^{\mu\prime}$:

$$m^2 A_\mu A^\mu \rightarrow m^2 \left(A_\mu + \frac{1}{e}\partial_\mu \alpha\right)\left(A^\mu + \frac{1}{e}\partial^\mu \alpha\right) \neq m^2 A_\mu A^\mu$$

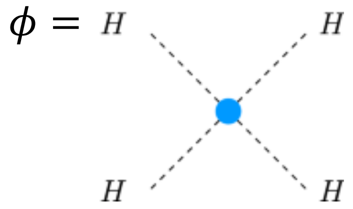
- The same holds (harder to show) for the weak mediators $W, Z$
  - However they **are** massive
  - ➔ SU(2)xU(1) symmetry is *broken*

- We will give an example how mass terms can be generated without destroying the symmetry of the Lagrangian

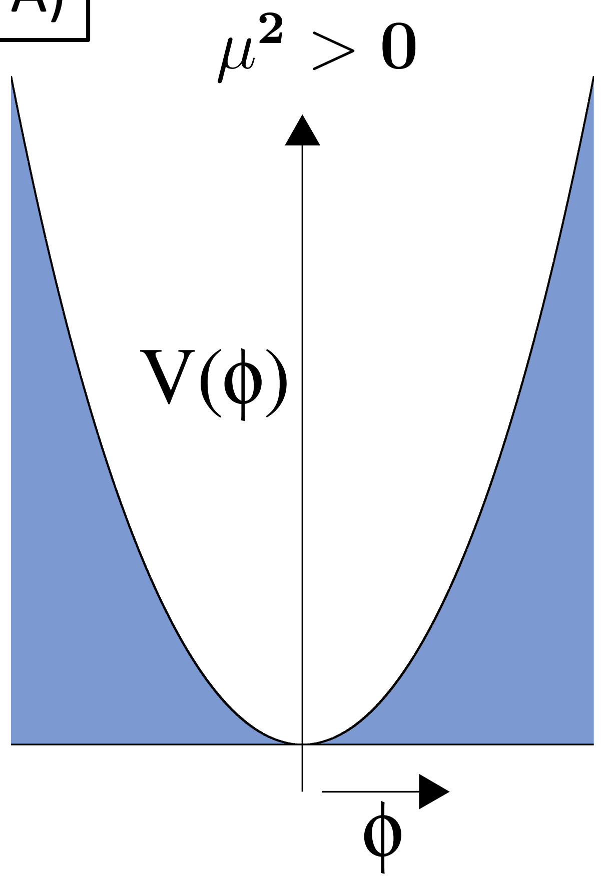
# The idea of symmetry breaking with a new field $\phi$

Griffiths §10.8

- Add a new field $\phi$ to the Lagrangian
  - Chose a scalar field ($S = 0$)
  - Include a potential $V(\phi)$: $\mathcal{L} = T - V$




Case A)



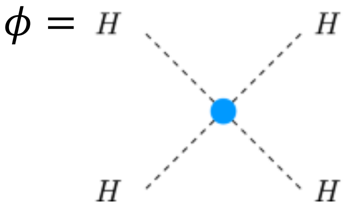

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2}_{\text{Massive Klein-Gordon term (Spin 0, mass }=\mu)} \underbrace{- \frac{1}{4}\lambda\phi^4}_{\text{Interaction term}}$$

- The vacuum (lowest energy state) has $\phi = 0$
  - This means no-field in the vacuum.
- The Lagrangian describes a new particle with $S = 0$ and $m = \mu$

# The idea of symmetry breaking with a new field $\phi$

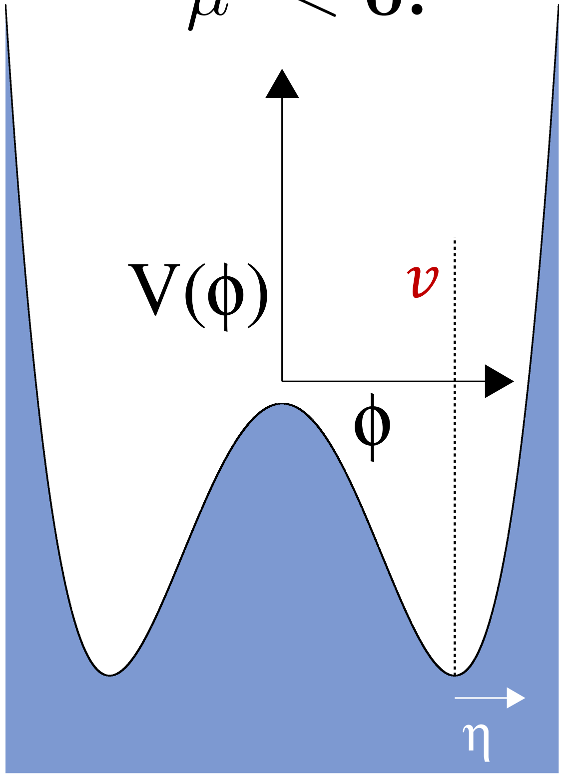
Griffiths §10.8

- Add a new field $\phi$ to the Lagrangian
  - Chose a scalar field ($S = 0$)
  - Include a potential $V(\phi)$: $\mathcal{L} = T - V$




Case B)

$\mu^2 < 0$:




$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2}_{\text{Massive Klein-Gordon term (Spin 0, mass } =\mu)} \underbrace{- \frac{1}{4}\lambda\phi^4}_{\text{Interaction term}}$$

- The particle has imaginary mass? $\mu^2 < 0$
- The Lagrangian has a minimum for $\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} \equiv v$ or $\mu^2 = -\lambda v^2$
  - The lowest energy (vacuum) includes a field with value $v$

Just do: $\frac{\partial V}{\partial \phi} = 0$

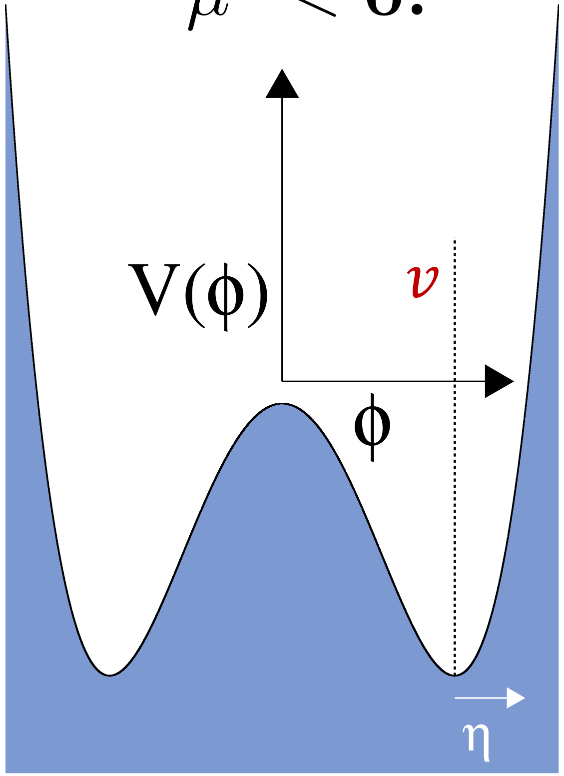
# Exercise – 19 : Symmetry breaking

Griffiths §10.8

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

Case B)

$\mu^2 < 0$:




- Redefine coordinates: $\eta \equiv \phi - v$ ( $\eta$ is the "shifted" field )
- Exercise: re-write the Lagrangian in $\eta$ and $v$ to show:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda v^4$$

  - Ignore the constant term $\frac{1}{4}\lambda v^4$ and neglect higher order $\eta^3$:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \underbrace{\lambda v^2}_{\text{mass}}\eta^2$$

  - This describes a new scalar field $\eta$ with a mass $\frac{1}{2}m_\eta^2 = \lambda v^2 \Rightarrow m_\eta = \sqrt{2\lambda v^2}$ $(= \sqrt{-2\mu^2})$
  - Price to pay: Lagrangian is no longer symmetric under $\eta \to -\eta$ in the new field.

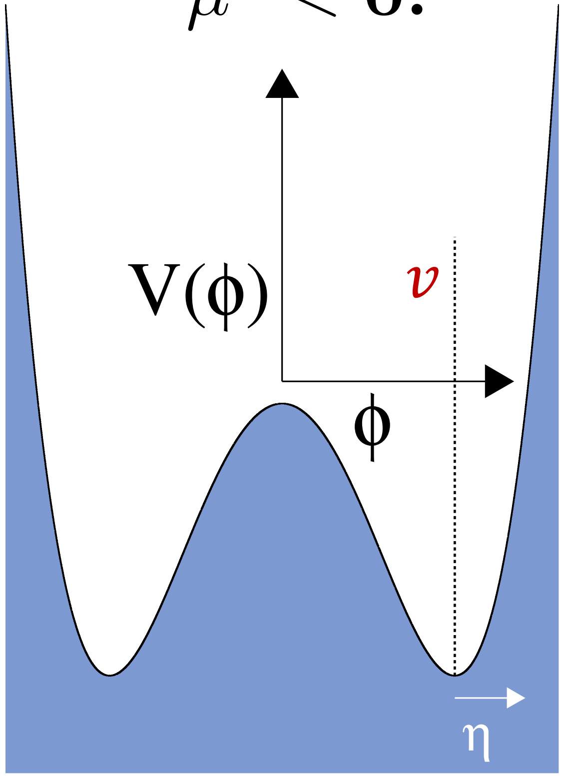
# The idea of symmetry breaking with a new field $\phi$

Griffiths §10.8

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

Case B)

$\mu^2 < 0$:




- Redefine coordinates: $\eta \equiv \phi - v$

Conclusion:

- The symmetry of the Lagrangian by adding a symmetric potential $\phi$ *has **not** been destroyed*
- The *vacuum is no longer* in a symmetric position

➔ The physical case includes a ***complex*** field $\phi$

- This describes a new scalar field $\eta$ with a mass $\frac{1}{2}m_\eta^2 = \lambda v^2 \Rightarrow m_\eta = \sqrt{2\lambda v^2}$ $(= \sqrt{-2\mu^2})$
- Price to pay: Lagrangian is no longer symmetric under $\eta \to -\eta$ in the new field.

# Symmetry breaking with a *complex* field $\phi$

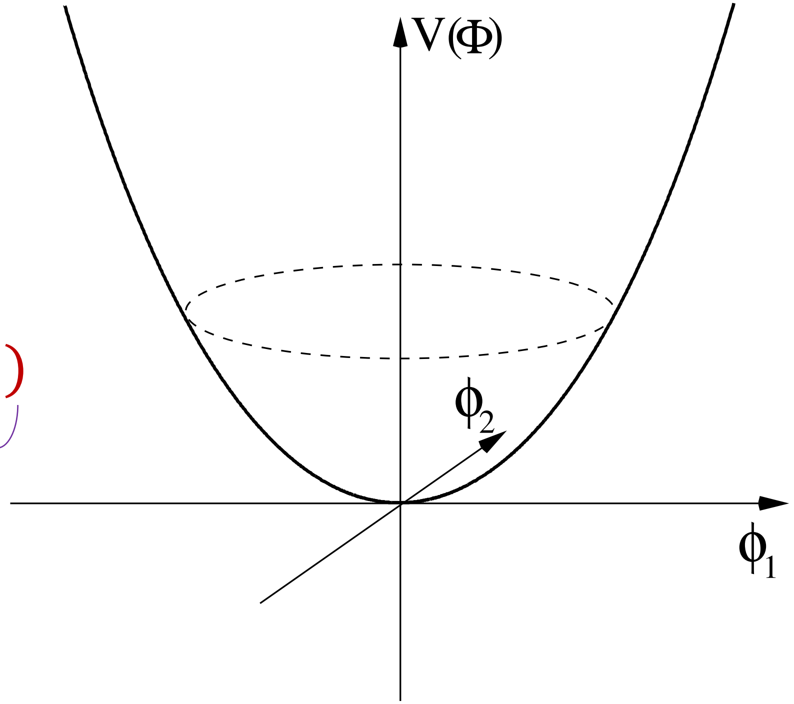
Griffiths §10.9

- Introduce a complex scalar field: $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$
- The Lagrangian term is: $\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi) - V(\phi)$ , with $V(\phi) = \mu^2(\phi^*\phi) + \lambda\,(\phi^*\phi)^2$
- Lagrangian:

$$\mathcal{L}(\phi_1, \phi_2) = \frac{1}{2}\,(\partial_\mu \phi_1)^2 + \frac{1}{2}\,(\partial_\mu \phi_2)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

Case A)

$\mu^2 > 0$




- Lagrangian:

$$\mathcal{L}(\phi_1, \phi_2) = \underbrace{\frac{1}{2}\,(\partial_\mu \phi_1)^2 - \frac{1}{2}\mu^2(\phi_1^2)}_{\text{Particle } \phi_1\text{, mass } \mu} + \underbrace{\frac{1}{2}\,(\partial_\mu \phi_2)^2 - \frac{1}{2}\mu^2(\phi_2^2)}_{\text{Particle } \phi_2\text{, mass } \mu}$$

+ interaction terms

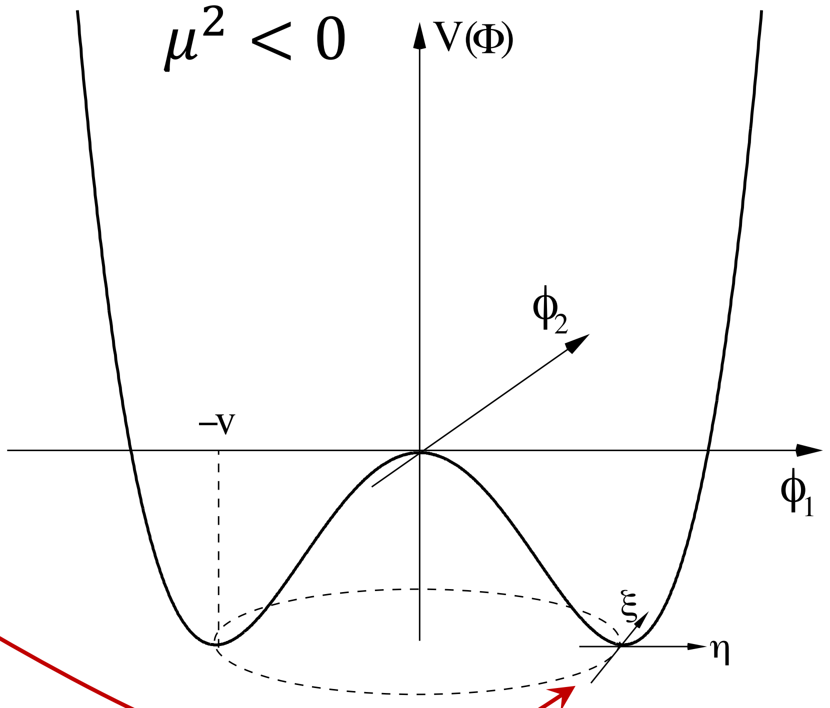
# Symmetry breaking with a *complex* field $\phi$

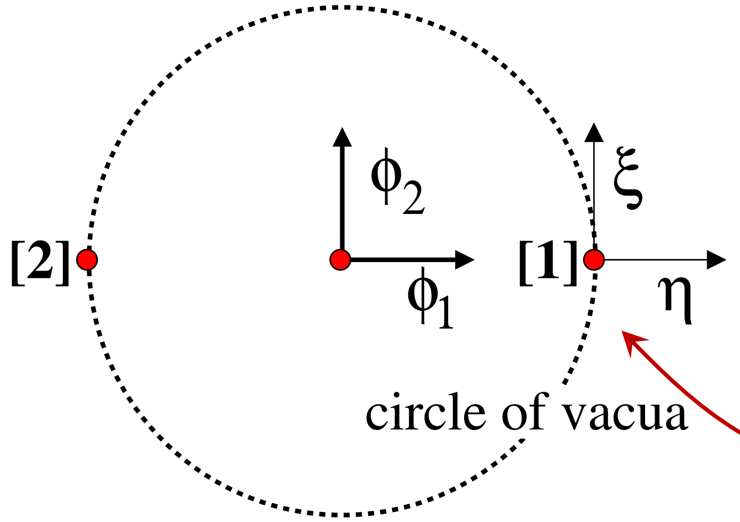
Griffiths §10.9

- Introduce a complex scalar field: $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$
- The Lagrangian term is: $\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - V(\phi)$ , with $V(\phi) = \mu^2(\phi^*\phi) + \lambda\,(\phi^*\phi)^2$
- Lagrangian:

$$\mathcal{L}(\phi_1, \phi_2) = \frac{1}{2}\,(\partial_\mu \phi_1)^2 + \frac{1}{2}\,(\partial_\mu \phi_2)^2$$
$$-\frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

Case B)




- We now have a whole circle of vacua to chose from:

$$\sqrt{\phi_1^2 + \phi_2^2} = \sqrt{\frac{-\mu^2}{\lambda}} = v$$

- Symmetry breaking: chose [1]: $\phi_0 = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$

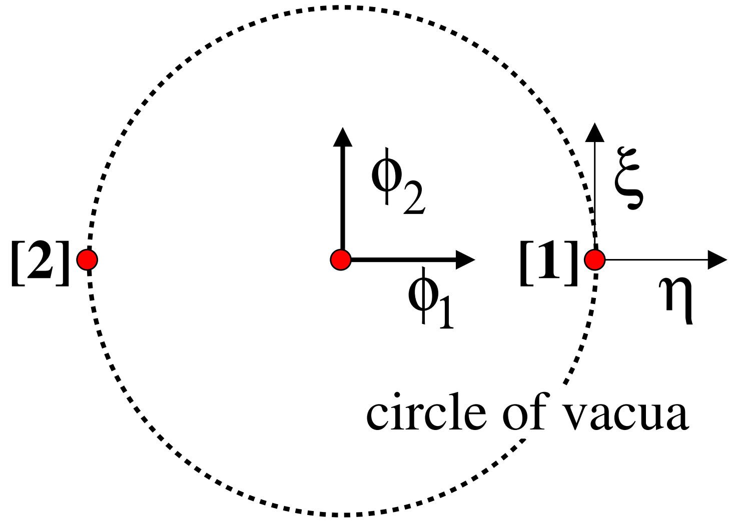
# Symmetry breaking with a *complex* field $\phi$

Griffiths §10.9

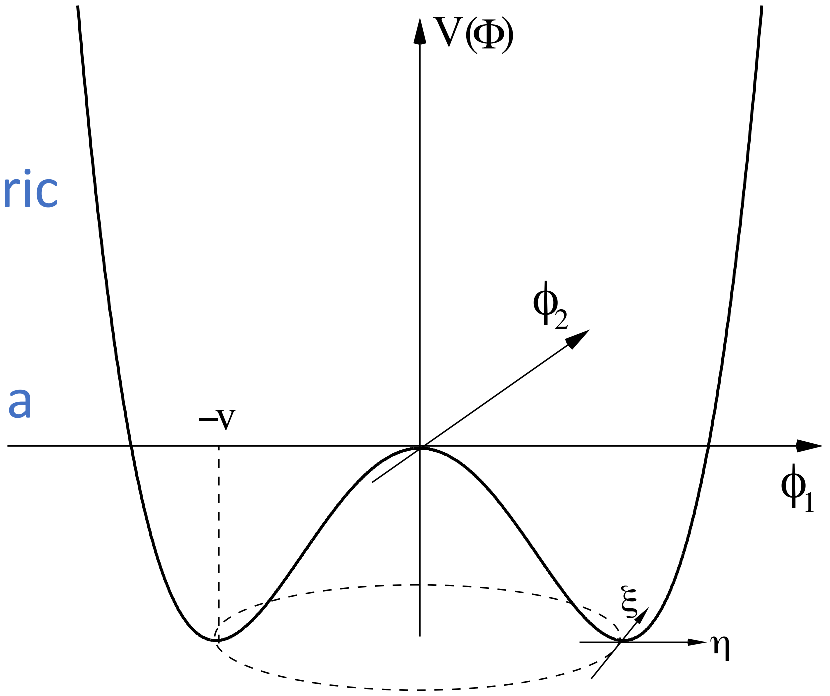
- Redefine coordinates: $\eta = \phi_1 - v \quad , \qquad \xi = \phi_2 \quad , \qquad \phi_0 = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$
- Exercise: rewrite the Lagrangian ignoring constant terms and higher order terms:

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu \eta)^2 - (\lambda v^2)\eta^2}_{\textit{massive}\text{ scalar particle } \mu} + \underbrace{\tfrac{1}{2}(\partial_\mu \xi)^2 - 0 \cdot \xi^2}_{\textit{massless}\text{ scalar particle } \xi} + \text{higher order terms}$$

Case B)




- The Lagrangian is still symmetric
- The vacuum is no longer symmetric
- We have a *massive* scalar and a *massless* scalar
  - The latter is called a Goldstone boson.




- Symmetry breaking: chose [1]: $\phi_0 = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$

# Higgs Mechanism

Griffiths §10.9

- The Higgs mechanism breaks the symmetry of the (electro-)weak interaction
  - Works along the lines as described in previous slides; introduce a complex SU(2) doublet
  - Details beyond the scope these lectures, idea as follows:
- Electroweak Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi + (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi)$
  - Where the covariant derivatives:

U(1): $\psi(x) \to \psi'(x) = e^{i\alpha(x)}\psi(x)$

$$A^\mu(x) \to A'^\mu(x) = A^\mu(x) - \frac{1}{q}\partial^\mu\alpha(x)$$

$$\Rightarrow \quad D^\mu = \partial^\mu + iqA^\mu$$

and SU(2): $\psi(x) \to \psi'(x) = G(x)\psi(x)$

with $G(x) = \exp\left(\frac{i}{2}\vec{\tau}\cdot\vec{\alpha}(x)\right)$

$$B'_\mu = GB_\mu G^{-1} + \frac{i}{g}(\partial_\mu G)G^{-1}$$

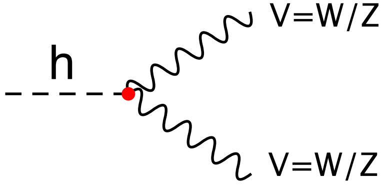
$$\Rightarrow \quad D_\mu = I\partial_\mu + igB_\mu$$

- Higgs field is weak isospin doublet: $\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2\\ \phi_3 + i\phi_4\end{pmatrix}$ ; $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$
  - With the potential: $V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$ where: $\mu^2 < 0$

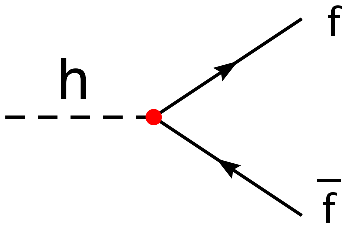
# Higgs Mechanism : Resulting phenomenology

Griffiths §10.9

- The Higgs mechanism breaks the symmetry of the (electro-)weak interaction
  - The Higgs choses a preferred direction in weak isospin space
  - One massive Higgs scalar field remains – due to field excitations around $v$; the earlier $\eta$ term
  - Three massless Goldstone bosons appear, but they are re-written as massterms for the gauge fields of the broken symmetry.
    - The $W^+, W^-, Z^0$ bosons acquire mass.
  - The photon remains massless




- Higgs and fermions:
  - The SM allows to couple the Higgs field to fermion isospin doublets:
    - The vacuum expectation value of the Higgs gives rise the fermion masses
    - Mass term: $m_f = Y_f \cdot \frac{1}{\sqrt{2}} v$ where $Y_f$ is a particle constant.
      - For the top quark: $Y_f = 1$ ?!




- Mass eigenstates and interaction eigenstates:
  - The Higgs and the $W$ boson do not agree on the "generation" eigenstates, see lecture 2.
  - The Higgs couplings give rise to the $CKM$ elements

# Exercise – 20 : Mass of the proton

Besides giving mass to the weak vector bosons, it was briefly flashed that the same Higgs mechanism is responsible for giving mass to the fermion masses in the Standard Model, through ad-hoc Yukawa couplings. The mass of a 'naked' quark can be estimated through models of soft QCD, where it enters as a parameter for e.g. the binding energy of a meson. For up and down, they are found to be roughly 2 resp. 5 MeV/c .

a) What fraction of the proton mass is due to the Higgs mass of the constituent quarks?

b) Can you find out where the other part of the proton mass comes from?

- See also:
- https://en.wikipedia.org/wiki/Mathematical_formulation_of_the_Standard_Model

# Lecture 4: ”Symmetries”

## Part 3

## Discrete Symmetries

Griffiths chapter 4

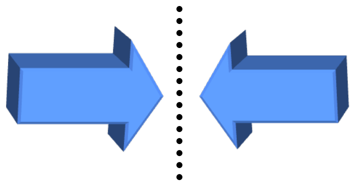
# Discrete Symmetries

- Is nature invariant if we look at it through a mirror?

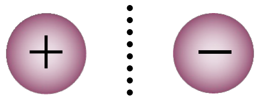
# Discrete $C, P, T$ Symmetries



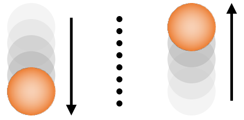
- Parity, $P$: *unobservable: (absolute handedness)*
  - Reflects a system through the origin. Converts right-handed to left-handed.
    - $\vec{\boldsymbol{x}} \rightarrow -\vec{\boldsymbol{x}}$ , $\vec{\boldsymbol{p}} \rightarrow -\vec{\boldsymbol{p}}$ (vectors) but $\vec{\boldsymbol{L}} = \vec{\boldsymbol{x}} \times \vec{\boldsymbol{p}}$ (axial vectors)



- Charge Conjugation, $C$: *unobservable: (absolute charge)*
  - Turns internal charges to opposite sign.
    - $\boldsymbol{e}^{+} \rightarrow \boldsymbol{e}^{-}$ , $\boldsymbol{K}^{-} \rightarrow \boldsymbol{K}^{+}$



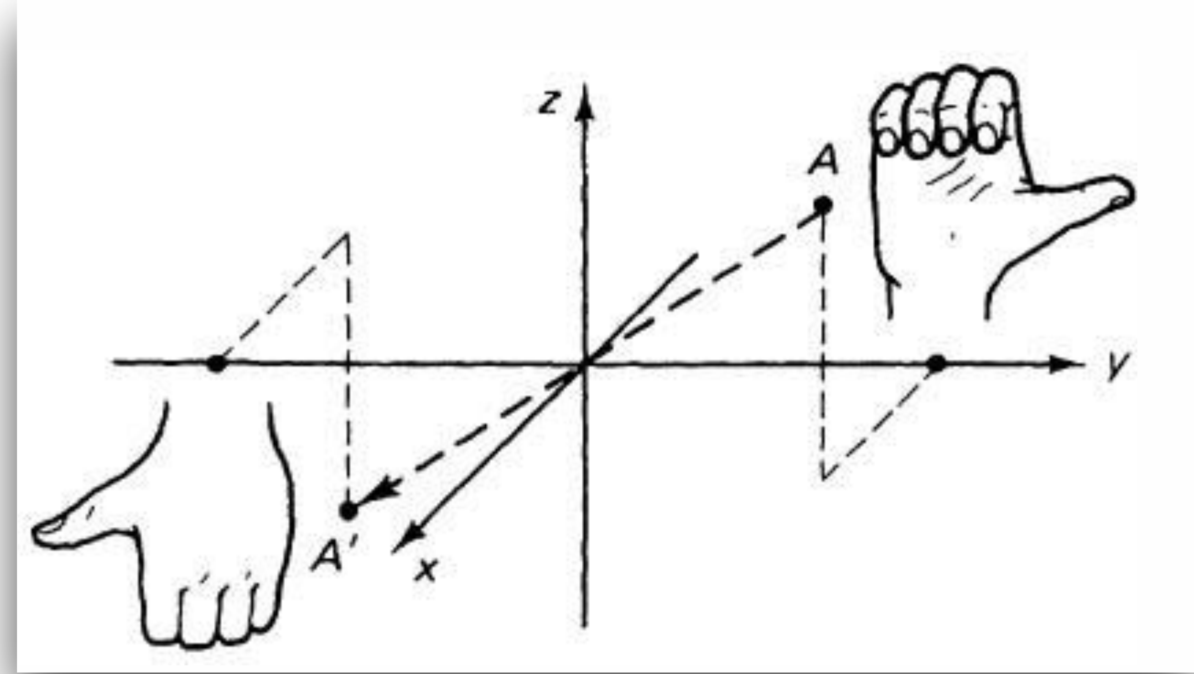
- Time Reversal, $T$: *unobservable: (direction of time)*
  - Changes direction of motion of particles
    - $\boldsymbol{t} \rightarrow -\boldsymbol{t}$



- $CPT$ Theorem:
  - All interactions are invariant under combined $C$, $P$ and $T$ operation
  - A particle ***is*** an antiparticle travelling backward in time
  - Implies e.g. particle and anti-particle have equal masses and lifetimes

# Parity: Helicity and Chirality

- Parity image
  - $\vec{L} = \vec{r} \times \vec{p} \quad \rightarrow \quad -\vec{r} \times -\vec{p} = \vec{L}$
  - Same for spin $\vec{S}$
- Helicity: spin projection on momentum
  - $\lambda = \vec{\sigma} \cdot \vec{p} \quad \rightarrow \quad \vec{\sigma} \cdot -\vec{p} = -\lambda$
  - The mirror of left-handed = right-handed
- Chirality:
  - If you, as observer overtake the electron, it changes from left handed to right-handed
  - How is it for a neutrino – zero mass?
    - You cannot overtake it.
    - Chirality is the helicity in the relativistic limit: $m \rightarrow 0 \; ; v \rightarrow c$




(a) Right-handed (b) Left-handed

# Exercise – 21 : Helicity vs Chirality

a) Write out the chirality operator $\gamma^5$ in the Dirac-Pauli representation.

b) The helicity operator is defined as $\lambda = \frac{1}{2}\vec{\Sigma}\cdot\hat{p}$ . Show that helicity operator and the chirality operator have the same effect on a spinor solution, i.e.

$$\gamma^5\psi = \gamma^5\begin{pmatrix}\chi^{(s)}\\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)}\end{pmatrix} \approx \lambda\begin{pmatrix}\chi^{(s)}\\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)}\end{pmatrix} = \lambda\psi \qquad \text{with: } \chi^{(1)} = \begin{pmatrix}1\\0\end{pmatrix};\ \chi^{(2)} = \begin{pmatrix}0\\1\end{pmatrix}$$

in the relativistic limit where $E \gg m$

c) Show explicitly that for a Dirac spinor:

$\bar{\psi}\gamma^\mu\psi = \overline{\psi_L}\gamma^\mu\psi_L + \overline{\psi_R}\gamma^\mu\psi_R$ making use of $\psi = \psi_L + \psi_R$ and the projection operators: $\psi_L = \frac{1}{2}(1-\gamma^5)$ and $\psi_R = \frac{1}{2}(1+\gamma^5)$

d) Explain why the weak interaction is called left-handed.

Wolfgang Pauli

*"I cannot believe God is a weak left-hander."*

# The weak interaction of particles is “left-handed”

- Look at the weak pion decay: $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ in the pion:

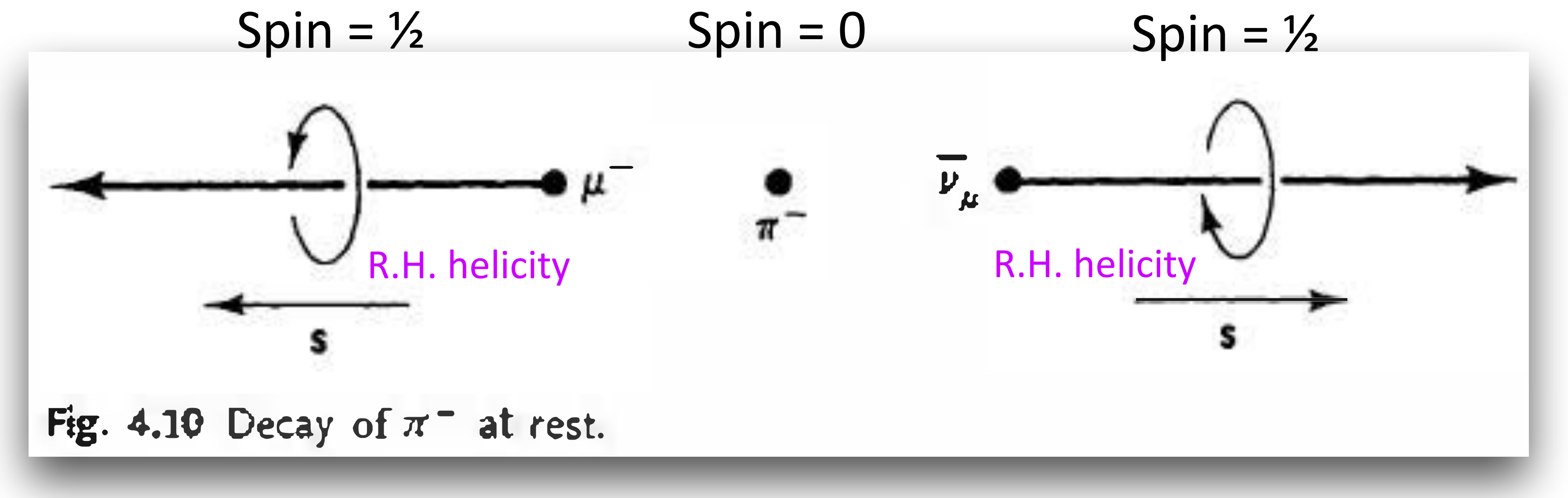


Fig. 4.10 Decay of $\pi^-$ at rest.

  - $\pi^- \rightarrow \mu^- \bar{\nu}$ : muon spin was found right handed: anti-neutrino is also right handed (R.H.)
- Compare to the decay: $\pi^+ \rightarrow \mu^+ \nu$ and measure the spin of the muon:
  - $\pi^+ \rightarrow \mu^+ \nu$ : anti-muon spin was found left-handed: neutrino is also left handed (L.H.)
- Since neutrino’s are ultra-relativistic ($m \approx 0$): *neutrino’s are always left-handed*

  *anti-neutrino’s are always right handed*

➔ The weak interaction maximally violates parity symmetry!

# Classical Mirror Worlds → Invariant!

- Parity $P$: $\vec{x} \rightarrow -\vec{x}$ , $\vec{p} \rightarrow -\vec{p}$
  - Mass $m$ — $P\,m = m$ : scalar
  - Force $\vec{F}$ $(\vec{F} = d\vec{p}/dt)$ — $P\,\vec{F} = P\,d\vec{p}/dt = -\,d\vec{p}/dt = -\vec{F}$ : vector
  - Acceleration $\vec{a}$ $(\vec{a} = d^2\vec{x}/dt^2)$ — $P\,\vec{a} = -\,d^2x/dt^2 = -\vec{a}$ : vector
  - Angular momentum $\vec{L}, \vec{S}, \vec{J}$ $(\vec{L} = \vec{x} \times \vec{p})$ — $P\ \vec{L} = -\vec{x} \times -\vec{p} = \ \vec{L}$ : axial vector

- Parity: Newton's law is *invariant* under $P$-operation (i.e. the same in the mirror world):

$$\vec{F} = m\,\vec{a} \xrightarrow{P} -\vec{F} = -m\vec{a} \quad \Leftrightarrow \quad \vec{F} = m\,\vec{a}$$

- Charge: Lorentz Force in the $C$-mirror world is *invariant*:

$$\vec{F} = q\,[\vec{E} + \vec{v} \times \vec{B}] \xrightarrow{C} \vec{F} = -q[-\vec{E} + \vec{v} \times -\vec{B}]$$

- Time: laws of physics are also *invariant* unchanged under $T$-reversal, since:

$$\vec{F} = m\,\vec{a} = m\,\frac{d^2\vec{x}}{dt^2} \xrightarrow{T} \vec{F} = m\,\frac{d^2\vec{x}}{d(-t)^2} \quad \Leftrightarrow \quad \vec{F} = m\,\vec{a}$$

  - QM: Consider Schrodinger's equation $(t \rightarrow -t)$ : $i\hbar\dfrac{\partial\psi}{\partial t} = -\dfrac{\hbar^2}{2m}\vec{\nabla}^2\psi$

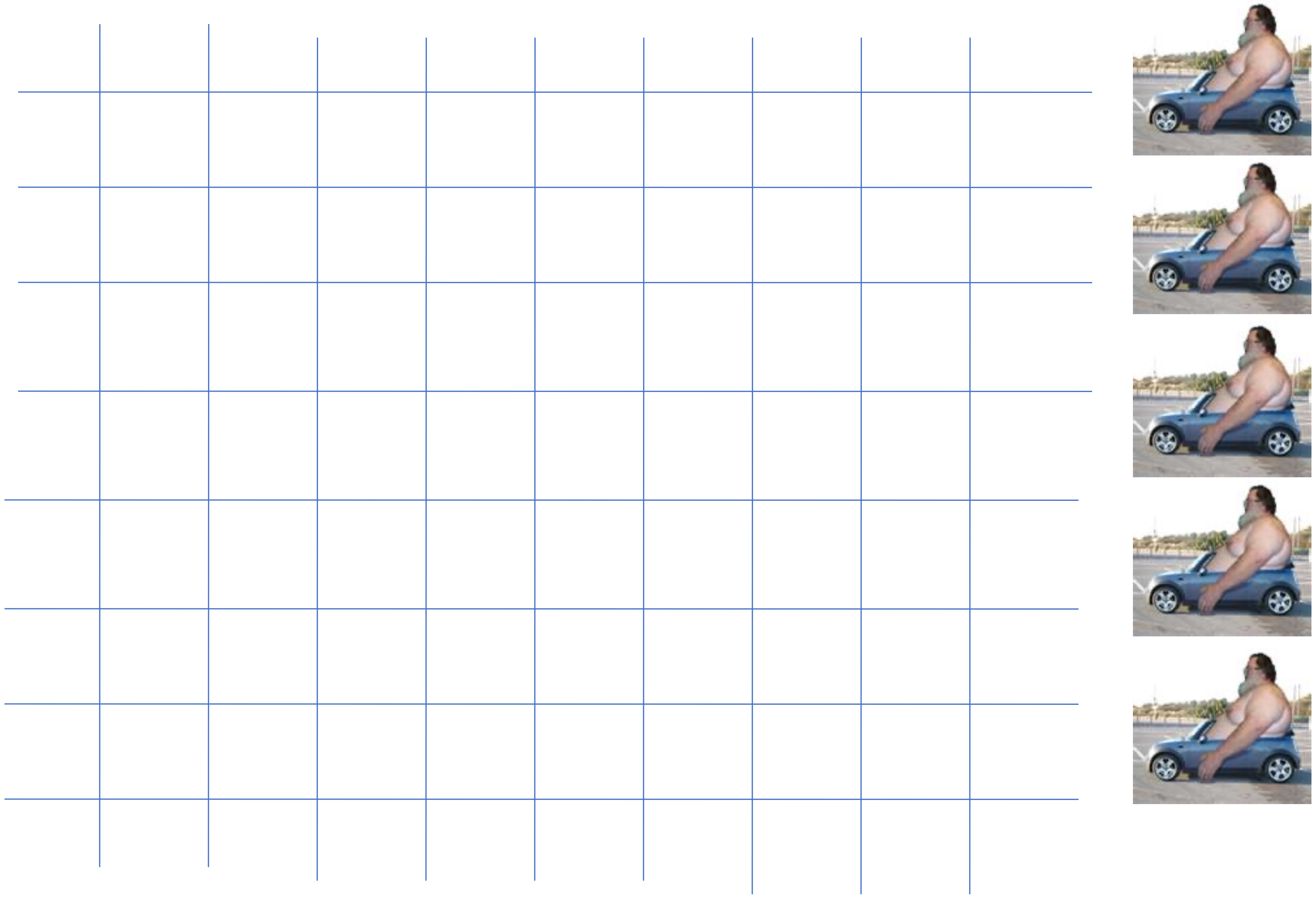
    Complex conjugation is required to stay invariant: $\psi \xrightarrow{T} \psi^*$

# $C$-, $P$-, $T$- Symmetry

- Classical Theory is invariant under $C$, $P$, $T$ operations; i.e. they conserve $C$, $P$, $T$ symmetry
  - Newton mechanics, Maxwell electrodynamics.

- Suppose we watch some physical event. Can we determine unambiguously whether:
  - We are watching the event where all ***charges are reversed*** or not?
  - We are watching the event ***in a mirror*** or not?
    - Macroscopic biological asymmetries are considered *accidents of evolution* rather than fundamental asymmetry in the laws of physics.
  - We are watching the event in a ***film running backwards*** or not?
    - The arrow of time is due to thermodynamics: i.e. the realization of a macroscopic final state is *statistically more probable* than the initial state

# Macroscopic time reversal (T.D. Lee)

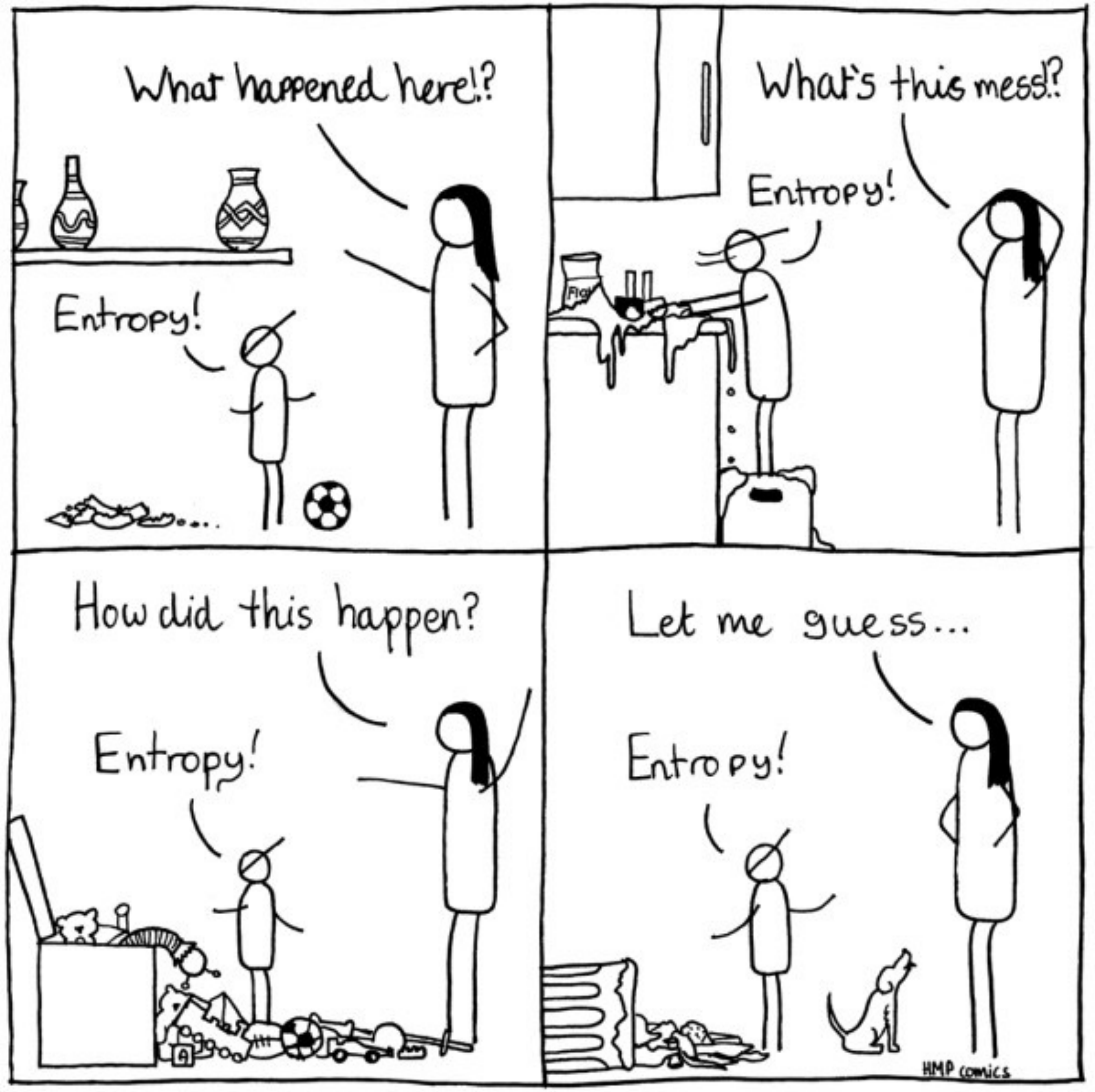


- At each crossing: 50% - 50% choice to go left or right
- After many decisions: reverse the velocity of the final state and return
- Do we end up with the initial state?

# Macroscopic time reversal (T.D. Lee)


- At each crossing: 50% - 50% choice to go left or right
- After many decisions: reverse the velocity of the final state and return
- Do we end up with the initial state?

# Macroscopic time reversal

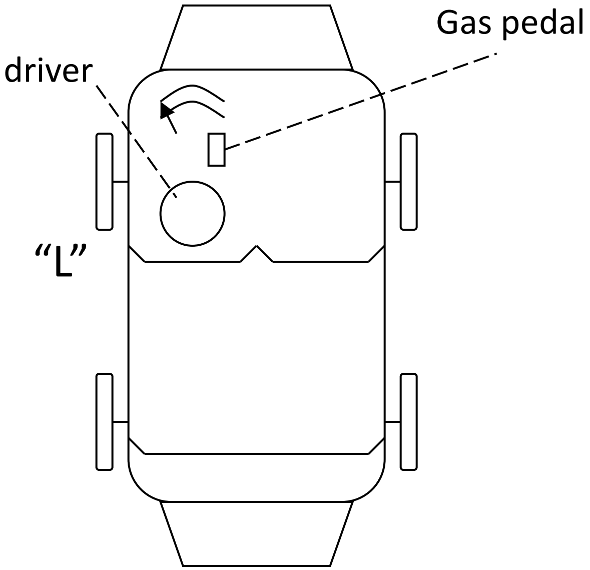



This is why we don't teach our children about entropy until much later...

# Parity Violation

Before 1956 physicists were **<u>convinced</u>** that the laws of nature were left-right symmetric. Strange?

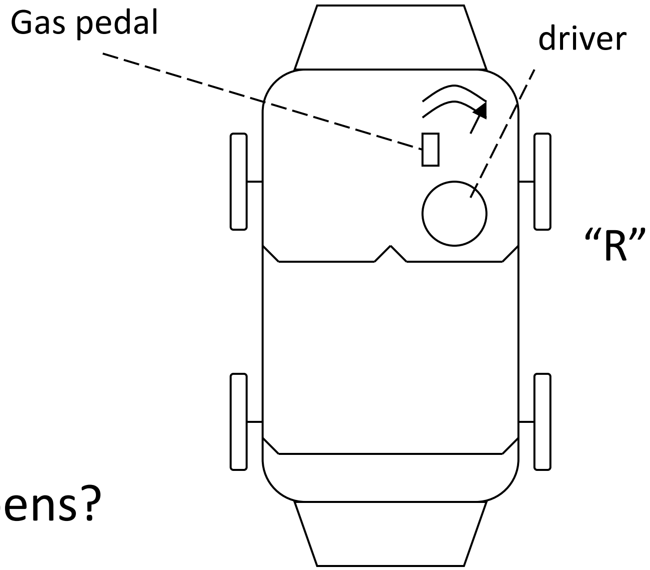
A "gedanken" experiment: consider two perfectly mirror symmetric cars:




"L" and "R" are fully symmetric,
Each nut, bolt, molecule etc.
However the engine is a black box

Person "L" gets in, starts, ….. 60 km/h

Person "R" gets in, starts, ….. What happens?




What happens in case the ignition mechanism uses, say, $Co^{60}$ $\beta$ decay?

# The weak interaction is left handed

- Look again at pion decay

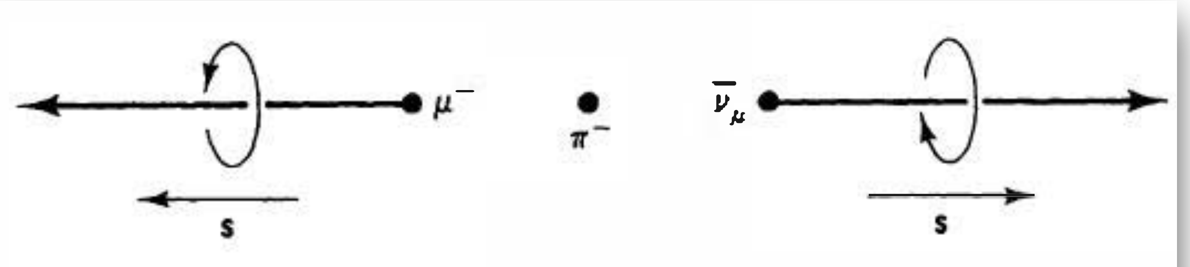
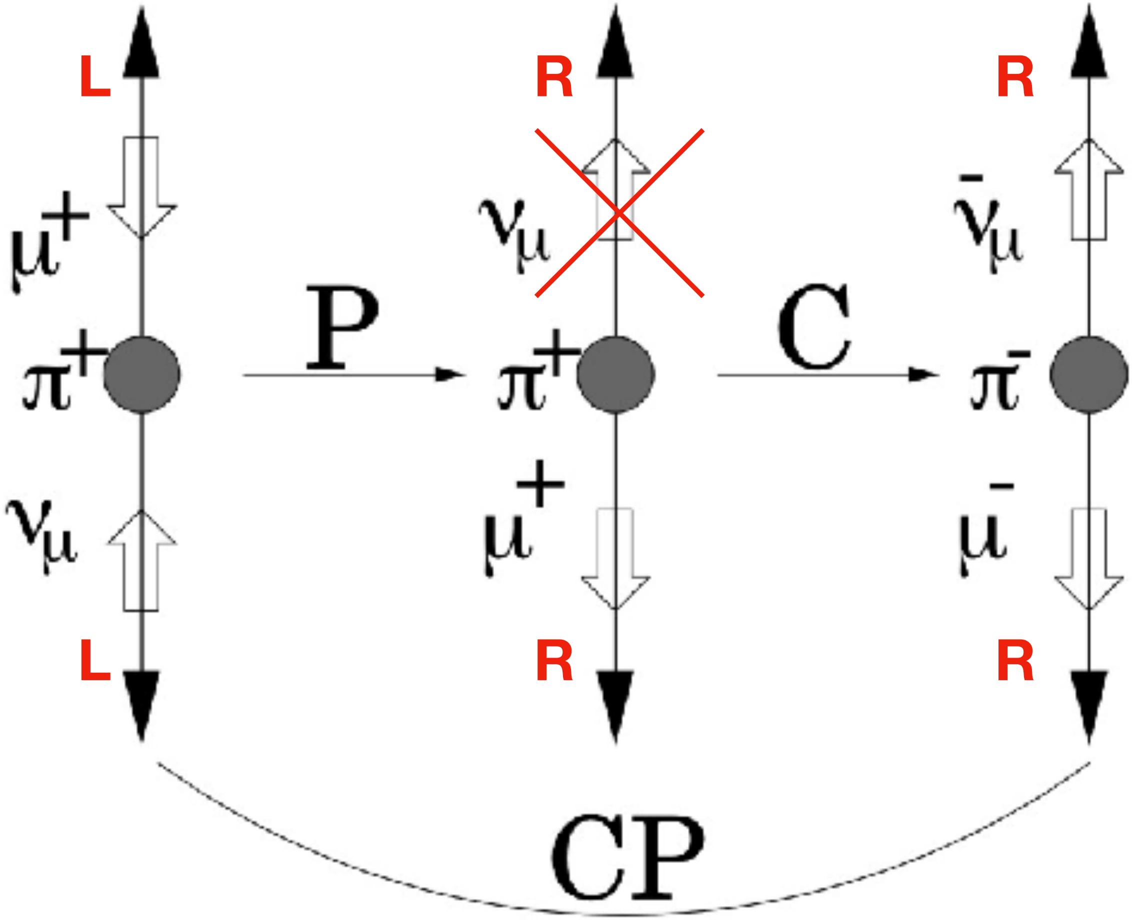
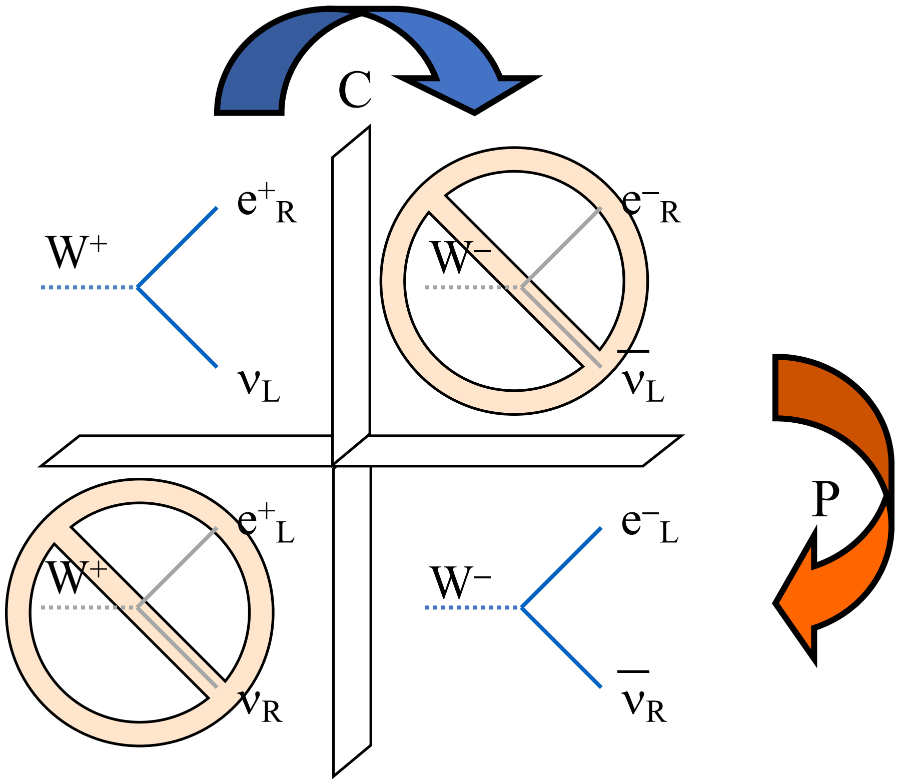



Fig. 4.10 Decay of $\pi^-$ at rest.




- Both Parity $P$ as well as charge conjugation $C$ symmetry are violated
  - But happens if we do both: $CP$?

# Weak Force breaks $C$ and $P$, is $CP$ really OK?

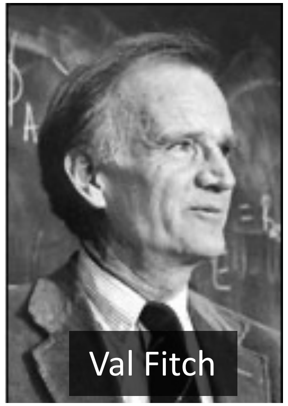


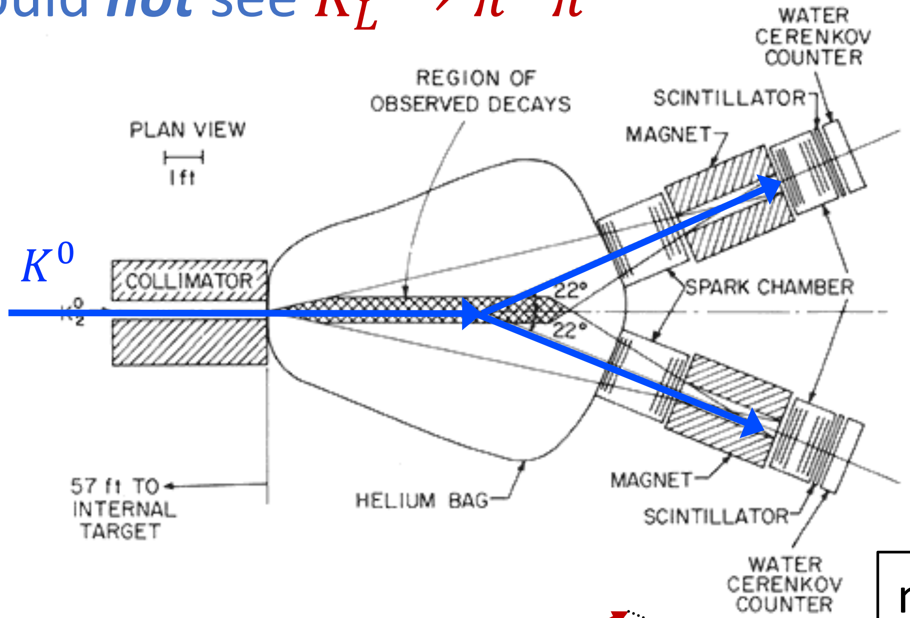

- Weak interaction breaks ***C*** and ***P*** symmetry maximally!
  - Nature is left-handed for matter and right-handed for antimatter.
- Despite *maximal* violation of ***C*** and ***P***, combined ***CP*** seemed *conserved*…
- But in 1964, Christenson, Cronin, Fitch and Turlay observed ***CP*** *violation* in decays of neutral kaons!

# Discovery of $CP$-Violation with Kaons

- Create a pure $K_L$ beam ("wait" for $K_S$ to decay)
- If $CP$ is conserved, should ***not*** see $K_L \rightarrow \pi^+\pi^-$

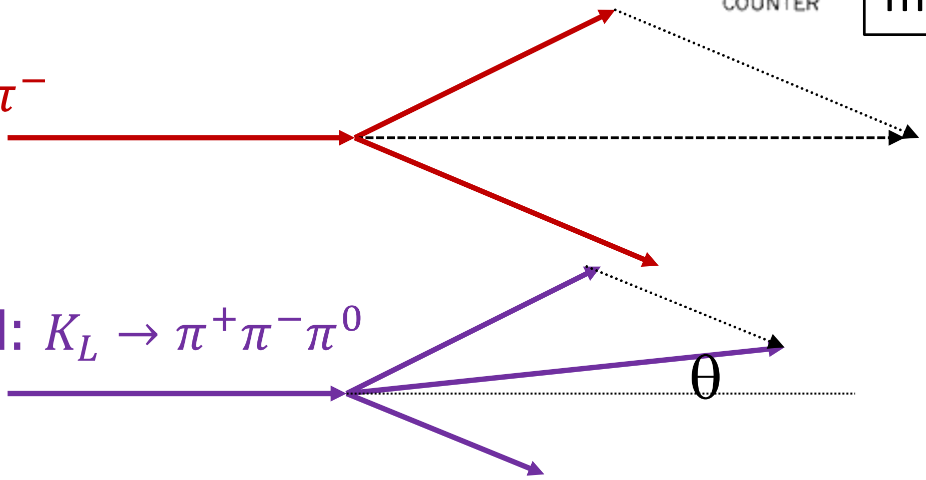
$K_S$: Short-lived is $CP$ even:
$K_1^0 \rightarrow \pi^+\pi^-$ (fast)
$K_L$: Long-lived is $CP$ odd:
$K_2^0 \rightarrow \pi^+\pi^-\pi^0$ (slow)

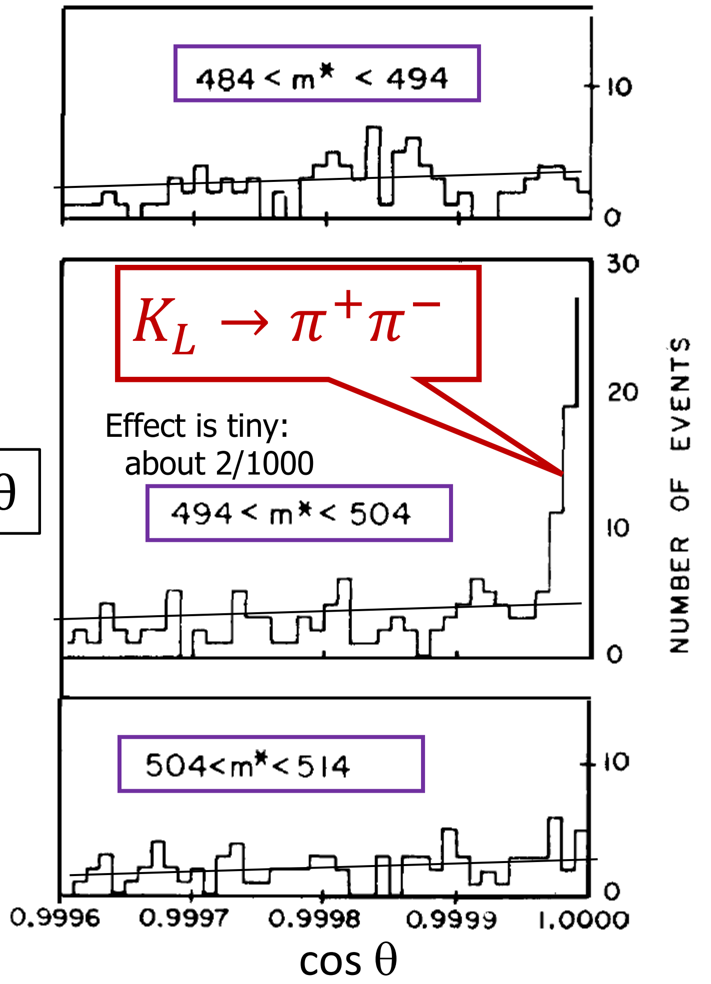




mass, θ

CPV Signal: $K_L \rightarrow \pi^+\pi^-$



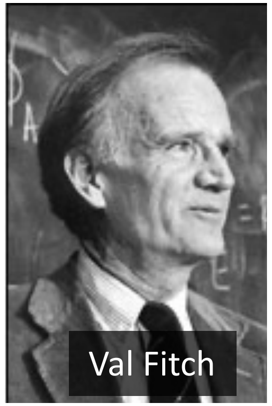

Expected Background: $K_L \rightarrow \pi^+\pi^-\pi^0$

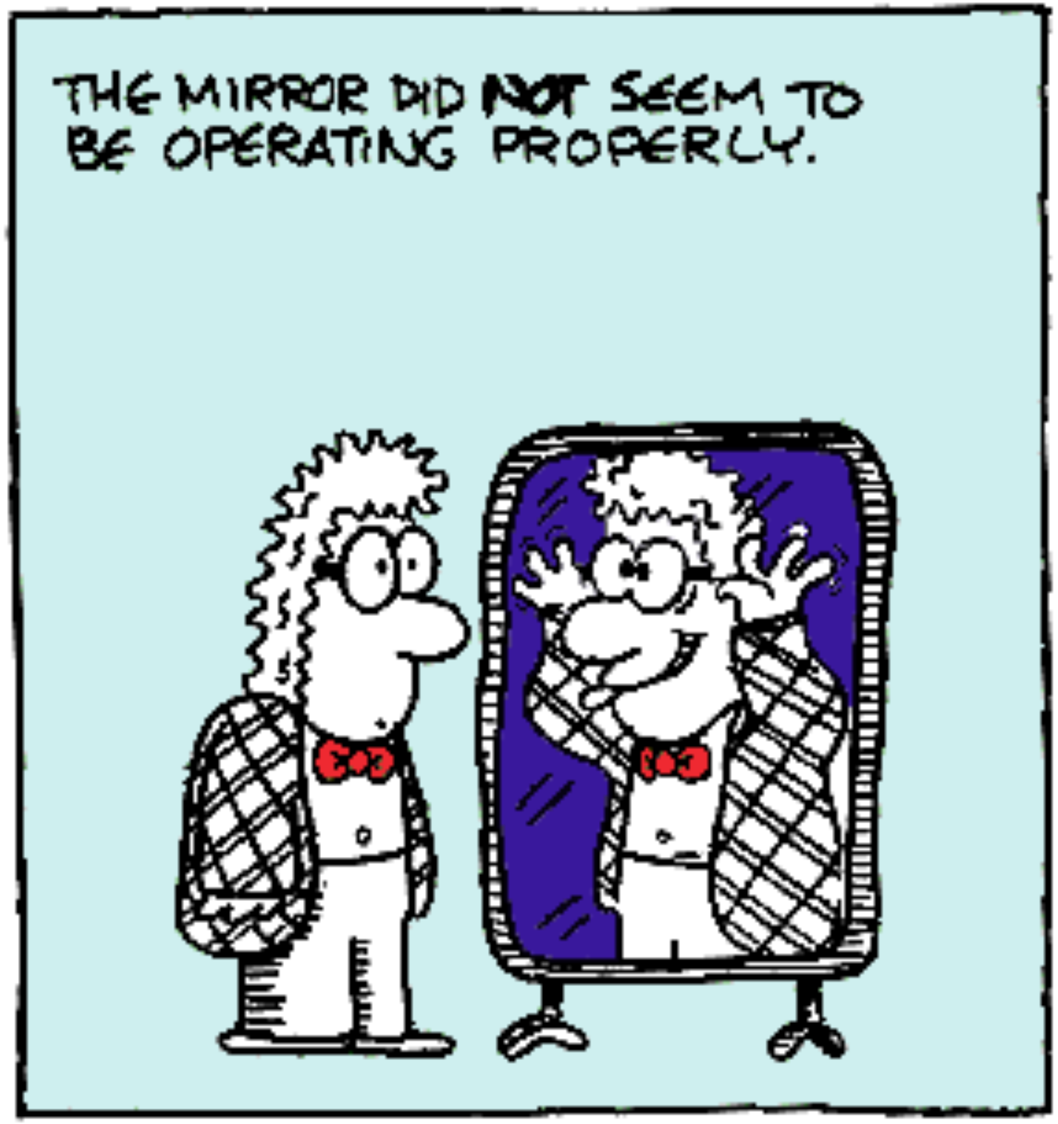


# Discovery of $CP$-Violation with Kaons

$K_S$: Short-lived is $CP$ even:
$K_1^0 \to \pi^+\pi^-$ (fast)
$K_L$: Long-lived is $CP$ odd:
$K_2^0 \to \pi^+\pi^-\pi^0$ (slow)

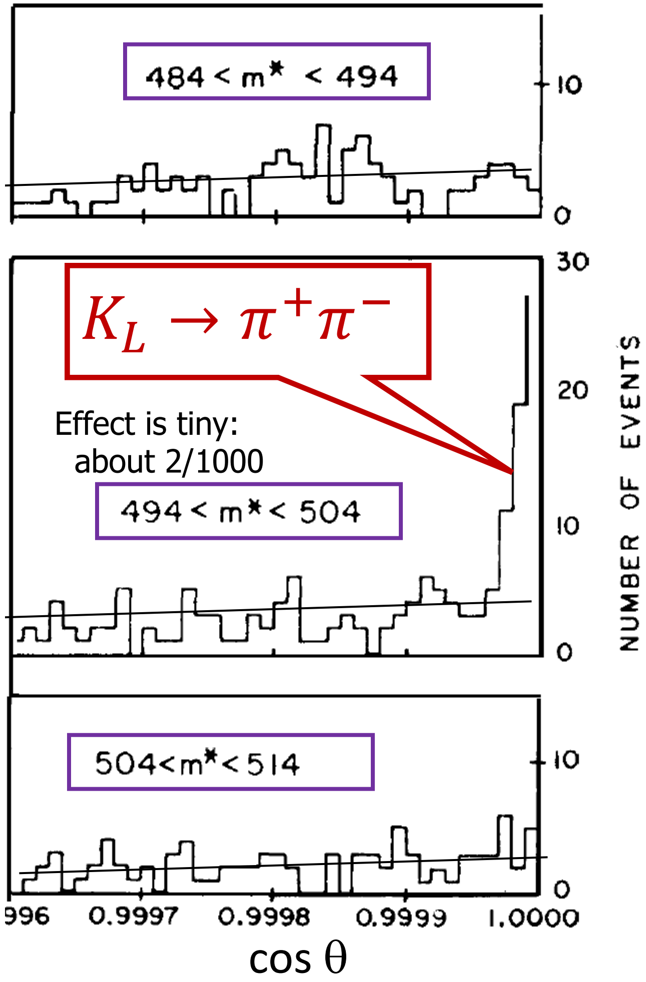
- Create a pure $K_L$ be
- If $CP$ is conserved,

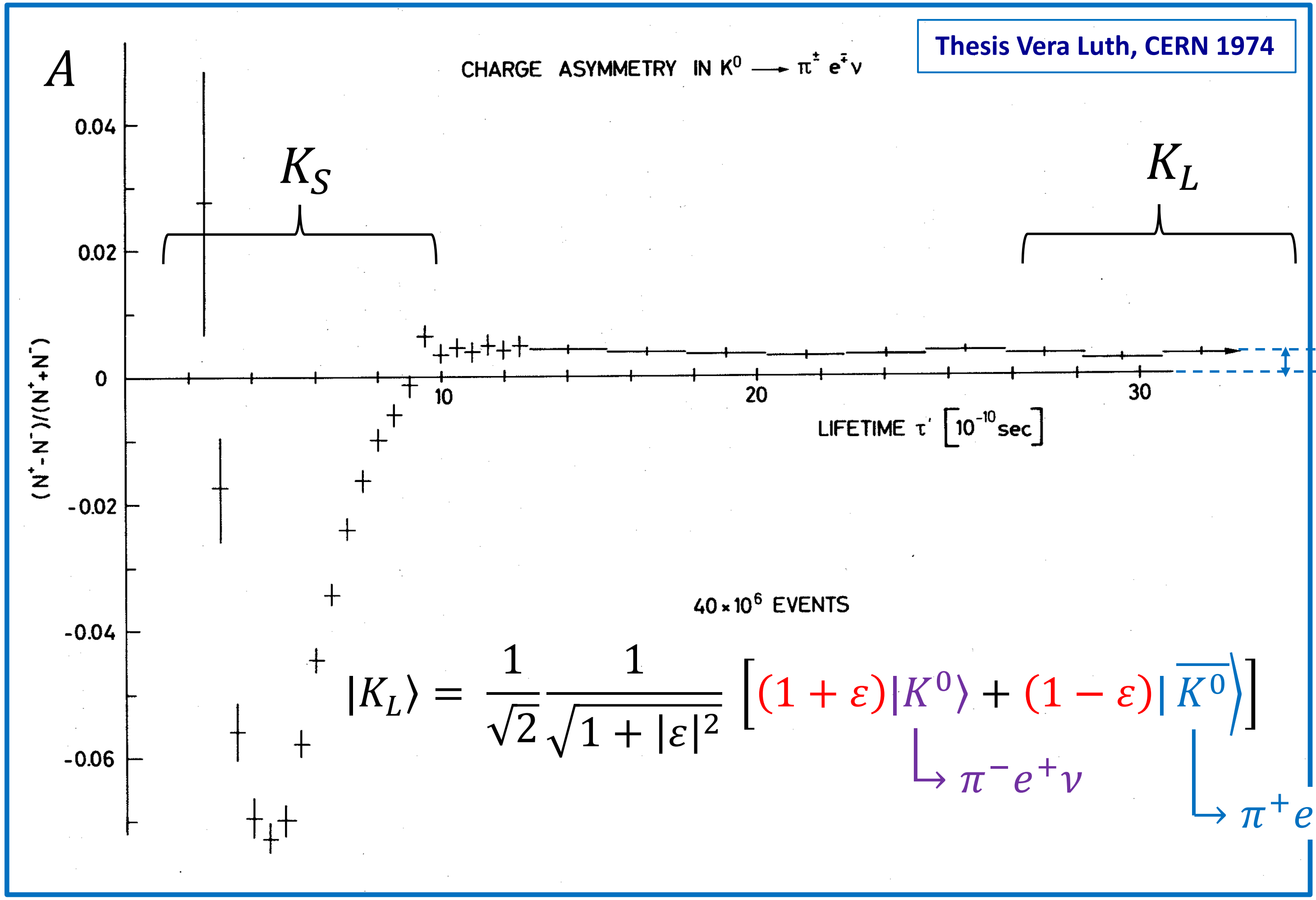

CPV Signal: $K_L \to$

Expected Backgr



# Alternative: Charge Asymmetry in $K^0$ decays

Measure $A = \dfrac{N^+ - N^-}{N^+ + N^-}$ with $N^+ = K^0 \to \pi^- e^+ \nu$, $N^- = \overline{K^0} \to \pi^+ e^- \bar{\nu}$ vs the $K^0$ decay time




$$|K_L\rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[ (1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\overline{K^0}\rangle \right]$$

$\hookrightarrow \pi^- e^+ \nu$

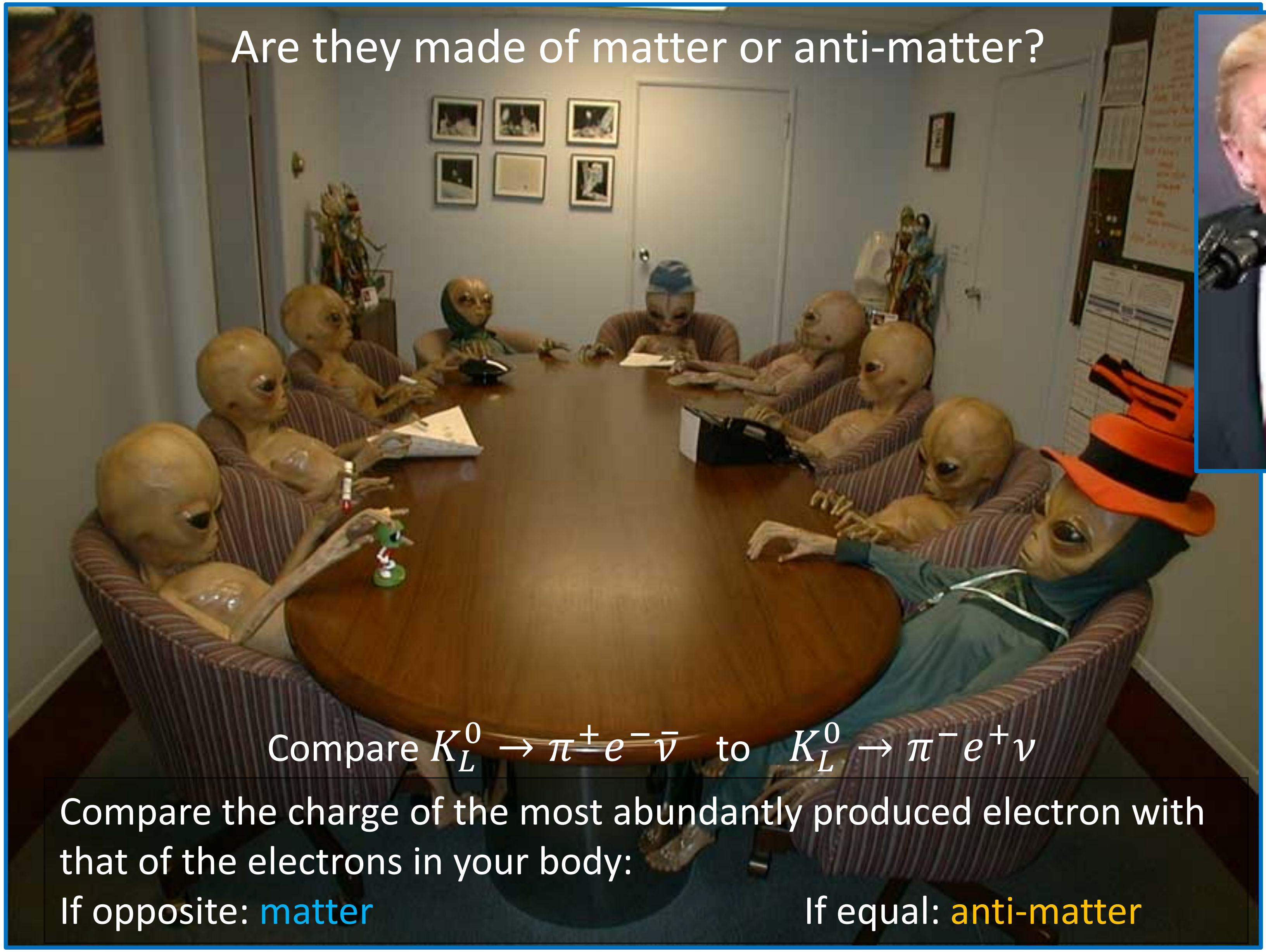
$\hookrightarrow \pi^+ e^- \bar{\nu}$

$$\boxed{4\Re(\varepsilon)} = \left| \frac{(1+\varepsilon)}{(1-\varepsilon)} \right|^2$$

$K^0 \to \pi^- e^+ \nu$ happens a bit ($\varepsilon$) more than $\overline{K^0} \to \pi^+ e^- \bar{\nu}$

$CP$ violation in meson mixing.

# Contact with Aliens !



Are they made of matter or anti-matter?

Compare $K_L^0 \rightarrow \pi^+ e^- \bar{\nu}$ to $K_L^0 \rightarrow \pi^- e^+ \nu$

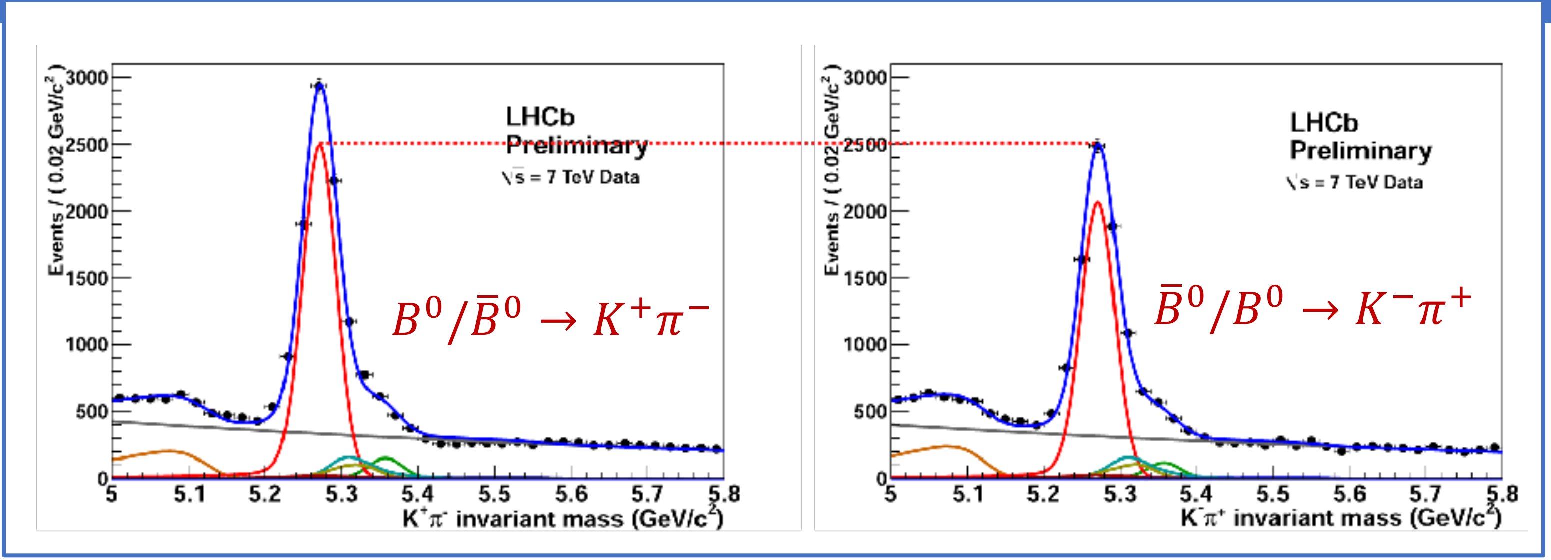
Compare the charge of the most abundantly produced electron with that of the electrons in your body:

If opposite: matter

If equal: anti-matter

# CP Violation & B-mesons

- Example: $B^0 \to K^+\pi^-$
  - Two quantum amplitudes (Feynman diagrams)
  - Interference gives rise to CP violation
    - Requires "strong" and "weak" phases



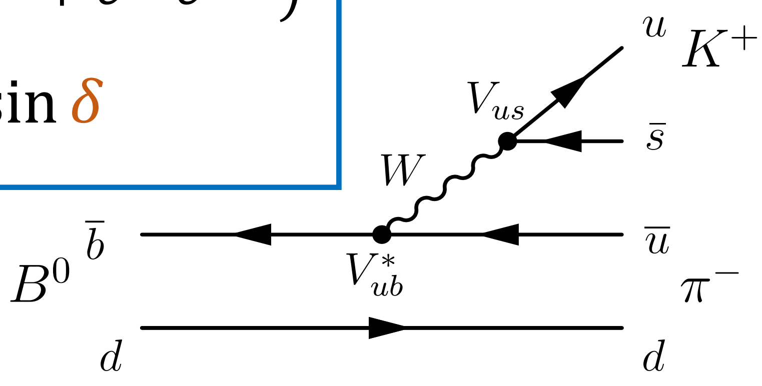

$$A = A_1 + A_2 e^{i\phi} e^{i\delta} \qquad \bar{A} = A_1 + A_2 e^{-i\phi} e^{i\delta}$$

$$|A|^2 = |A_1|^2 + |A_2|^2 + A_1 A_2\left(e^{i\phi}e^{i\delta} + e^{-i\phi}e^{-i\delta}\right)$$

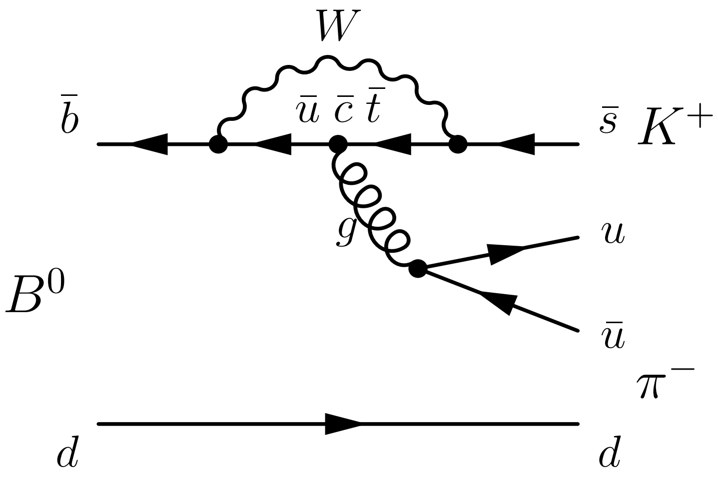
$$|\bar{A}|^2 = |A_1|^2 + |A_2|^2 + A_1 A_2\left(e^{-i\phi}e^{i\delta} + e^{i\phi}e^{\ i\delta}\right)$$

$$|A - \bar{A}|^2 = 4\, A_1 A_2 \sin\phi \sin\delta$$

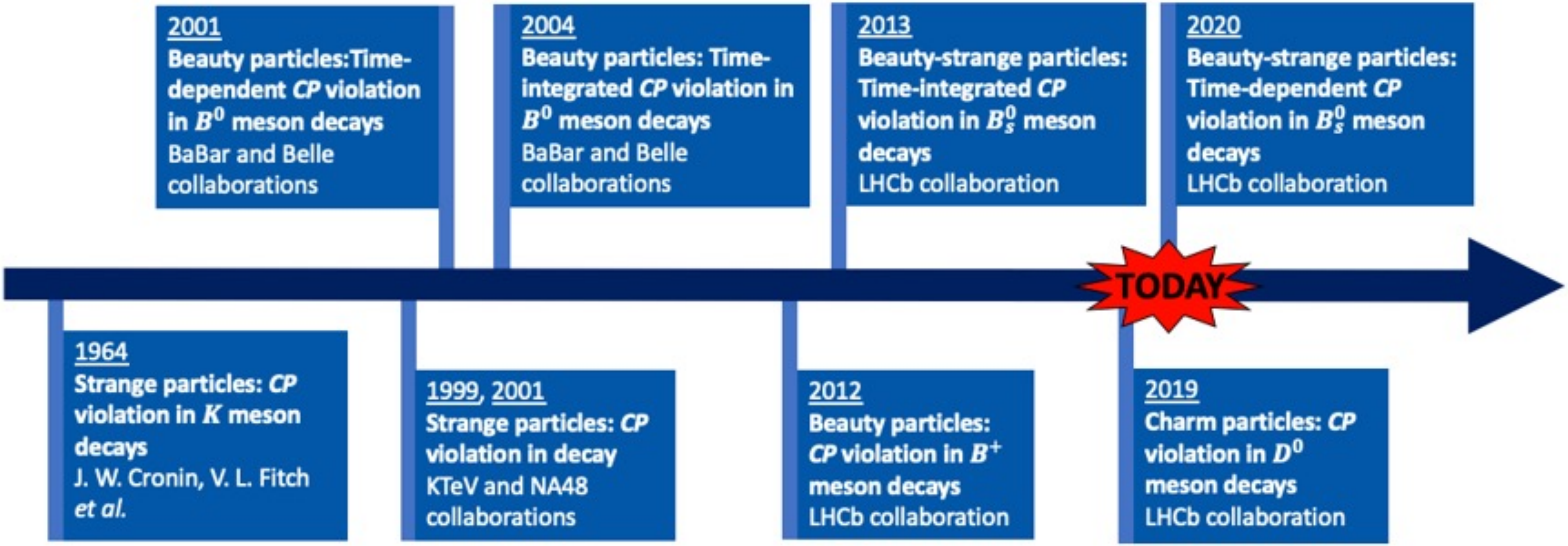
*"Tree"= $A_1$*




*"Penguin"= $A_2$*



# CP Violation is a hot topic at the LHCb experiment



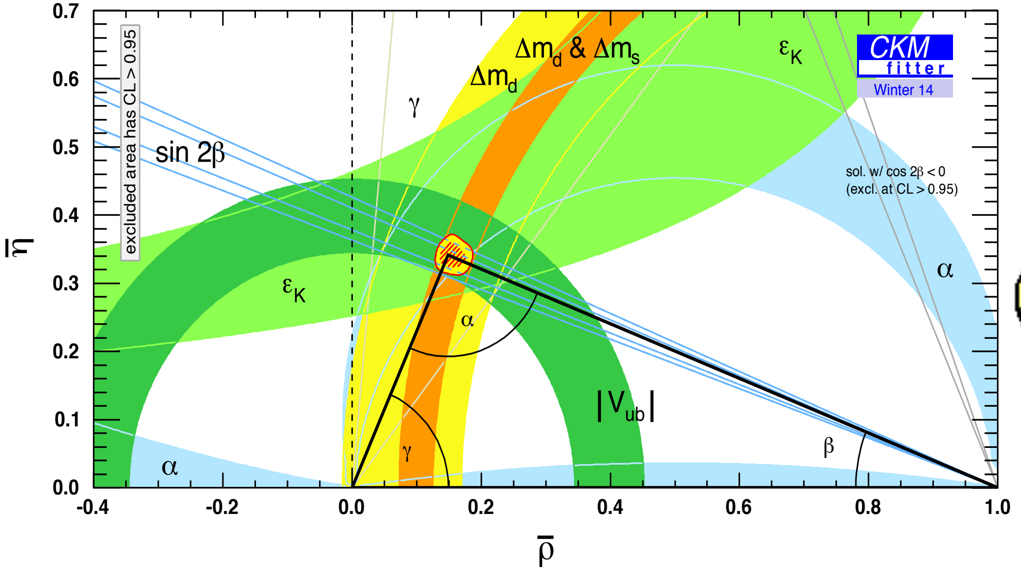
# $CPT$ Violation...

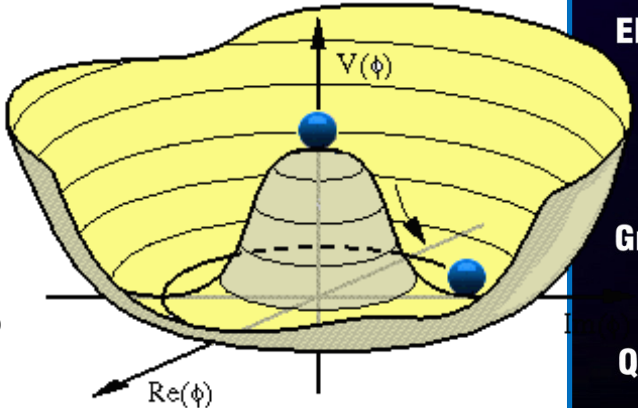

$CPT$ symmetry implies that an antiparticle is ***identical*** to a particle travelling backwards in time.

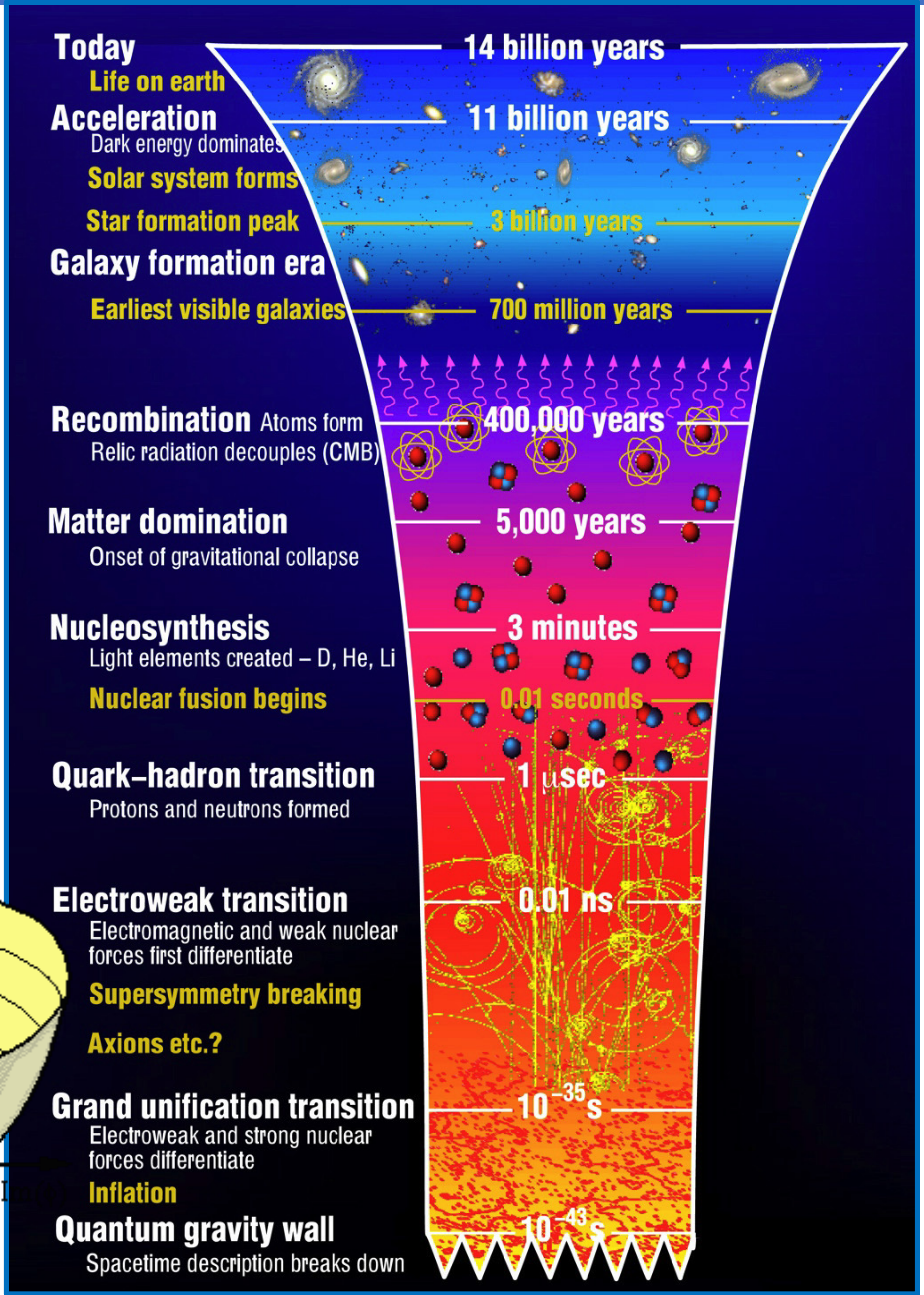
# Symmetry breaking in the early universe

- Higgs mechanism generates mass
  - For the weak bosons
  - For the fermions
- Higgs couplings lead to CKM couplings
- 3 generations allow for CP violation

- Can it explain the matter anti-matter asymmetry?
  - So far: no!



# Exercise – 22 : Symmetries

a) What do you think is the difference between an exact and a broken symmetry?

b) Can you explain the name *spontaneous* symmetry breaking means?

c) Which symmetry is involved in the gauge theories below? Which of these gauge symmetries are exact? Why/Why not?

   i. U1(Q) symmetry
   ii. SU2(u-d-flavour) symmetry
   iii. SU3(u-d-s-flavour) symmetry
   iv. SU6(u-d-s-c-b-t) symmetry
   v. SU3(colour) symmetry
   vi. SU5(Grand unified) symmetry
   vii. SuperSymmetry

# Lecture 4: Discussion Topics

Discussions Topics belonging to
Lecture 4

# Topic-10: Symmetry and non-observables

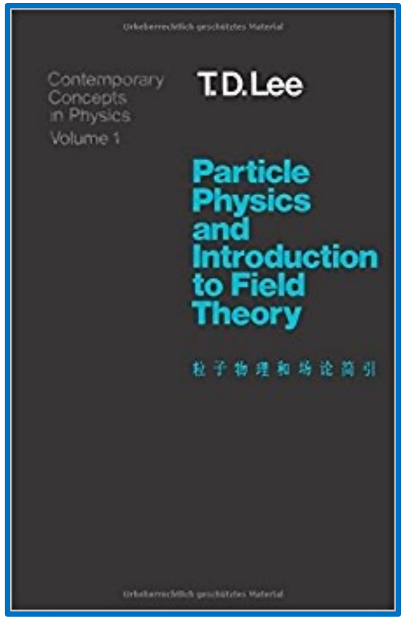
- Explain the idea behind non observables
- What are the symmetries and non-observables related to:
  - Electromagnetism
  - Weak interaction
  - Strong interaction
  - C-violation
  - P-Violation
  - T-Violation

# Topic-10: Symmetry and non-observables

T.D.Lee: “The root to all *symmetry* principles lies in the assumption that it is impossible to observe certain basic quantities; the *non-observables*”

There are four main types of symmetry:

- Permutation symmetry:
  Bose-Einstein and Fermi-Dirac Statistics
- Continuous space-time symmetries:
  translation, rotation, velocity, acceleration,…
- Discrete symmetries:
  space inversion, time reversal, charge conjugation,…
- Unitary symmetries: gauge invariances:
  $U_1$(charge), $SU_2$(isospin), $SU_3$(color),…




$\Rightarrow$ If a quantity is fundamentally non-observable it is related to an *exact* symmetry

$\Rightarrow$ If it could in principle be observed by an improved measurement; the symmetry is said to be *broken*

Noether Theorem: symmetry $\Leftrightarrow$ conservation law

# Topic-10: Symmetry and non-observables

| Non-observables | Symmetry Transformations | Conservation Laws or Selection Rule |
|---|---|---|
| Difference between identical particles | Permutation | B.-E. or F.-D. statistics |
| Absolute spatial position | Space translation: $\vec{r} \to \vec{r} + \vec{\Delta}$ | momentum |
| Absolute time | Time translation: $t \to t + \tau$ | energy |
| Absolute spatial direction | Rotation: $\vec{r} \to \vec{r}'$ | angular momentum |
| Absolute velocity | Lorentz transformation | generators of the Lorentz group |
| Absolute right (or left) | $\vec{r} \to -\vec{r}$ | parity |
| Absolute sign of electric charge | $e \to -e$ | charge conjugation |
| Relative phase between states of different charge Q | $\psi \to e^{i\theta Q}\psi$ | charge |
| Relative phase between states of different baryon number B | $\psi \to e^{i\theta N}\psi$ | baryon number |
| Relative phase between states of different lepton number L | $\psi \to e^{i\theta L}\psi$ | lepton number |
| Difference between different coherent mixture of p and n states | $\binom{p}{n} \to U\binom{p}{n}$ | isospin |

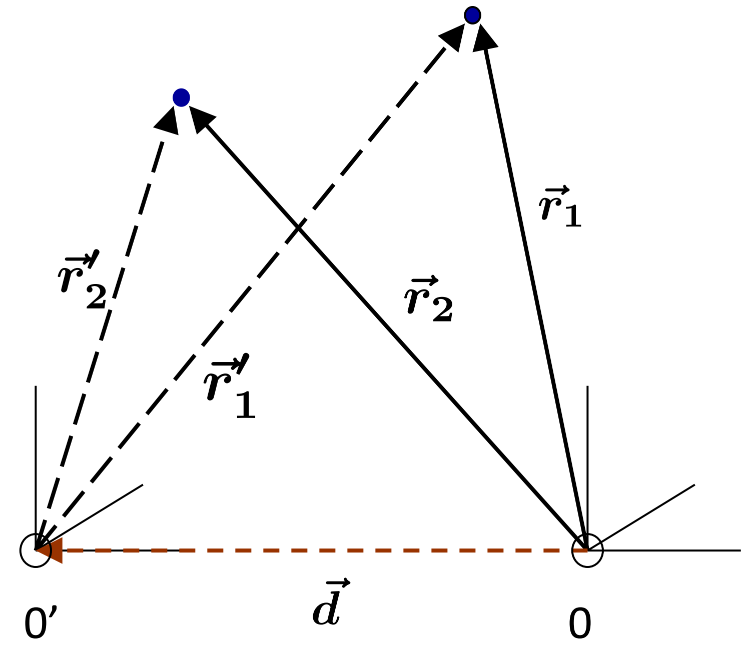
# Topic-10: Symmetry and non-observables: example

- Simple example: potential energy $V$ between two charged particles:

Absolute position is a non-observable:
The interaction is independent on the choice of the origin 0.

Symmetry:
$V$ is invariant under arbitrary space translations:

$$\vec{r_1} \to \vec{r}_1 + \vec{d} \qquad \vec{r_2} \to \vec{r}_2 + \vec{d}$$




Consequently:

$$V = V\left(\vec{r}_1 - \vec{r}_2\right)$$

Total momentum is conserved:

$$\frac{d}{dt}\underbrace{\left(\vec{p}_1 + \vec{p}_2\right)}_{\vec{p}_{\text{tot}}} = \vec{F}_1 + \vec{F}_2 = -\left(\vec{\nabla}_1 + \vec{\nabla}_2\right) V = 0$$

# Topic-11: Broken symmetries

a) What do you think is the difference between an exact and a broken symmetry?

b) Can you explain the name *spontaneous* symmetry breaking means?

c) Which symmetry is involved in the gauge theories below? Which of these gauge symmetries are exact? Why/Why not?

   i. U1(Q) symmetry
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   vi. SU5(Grand unified) symmetry
   vii. SuperSymmetry

# Lecture 4: Exercises

Exercises belonging to Lecture 4

# Exercise – 13 : Charge Current

- Show that the definition $W_\mu^\pm = \frac{b_\mu^1 \mp i b_\mu^2}{\sqrt{2}}$ leads to the charged current:

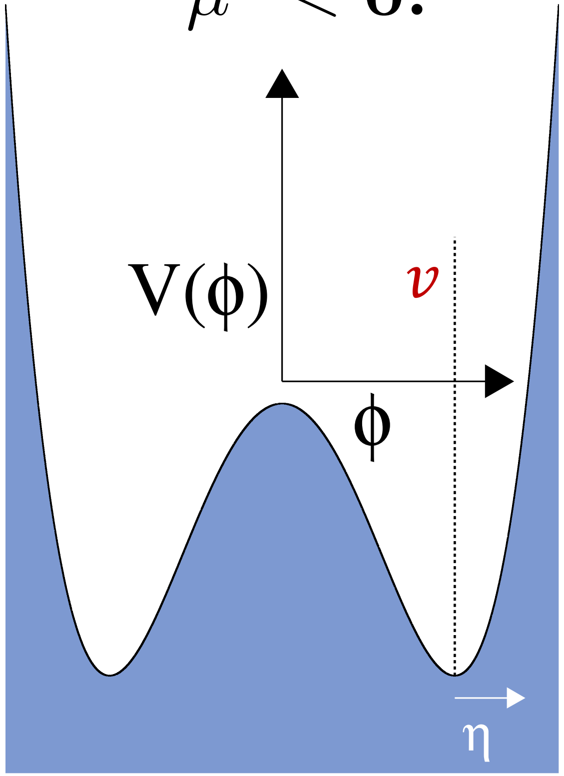
$\mathcal{L} = -W_\mu^+ J^{\mu^+} - W_\mu^- J^{\mu^-}$ with $J^{\mu^+} = \frac{g}{\sqrt{2}} \bar{\Psi} \gamma_\mu \tau^+ \Psi$ and $J^{\mu^-} = \frac{g}{\sqrt{2}} \bar{\Psi} \gamma_\mu \tau^- \Psi$

# Exercise – 14 : Symmetry breaking

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

Case B)

$\mu^2 < 0$:




- Redefine coordinates: $\eta \equiv \phi - v$
- Exercise: re-write the Lagrangian in $\eta$ and $v$ to show:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda v^4$$

  - Ignore the constant term $\frac{1}{4}\lambda v^4$ and neglect higher order $\eta^3$:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2\eta^2$$

  - This describes a new scalar field $\eta$ with a mass $\frac{1}{2}m_\eta^2 = \lambda v^2 \Rightarrow m_\eta = \sqrt{2\lambda v^2}$ $(= \sqrt{-2\mu^2})$
  - Price to pay: Lagrangian is no longer symmetric under $\eta \to -\eta$ in the new field.

# Exercise – 15 : Mass of the proton

Besides giving mass to the weak vector bosons, it was briefly flashed that the same Higgs mechanism is responsible for giving mass to the fermion masses in the Standard Model, through ad-hoc Yukawa couplings. The mass of a 'naked' quark can be estimated through models of soft QCD, where it enters as a parameter for e.g. the binding energy of a meson. For up and down, they are found to be roughly 2 resp. 5 MeV/c .

a) What fraction of the proton mass is due to the Higgs mass of the constituent quarks?

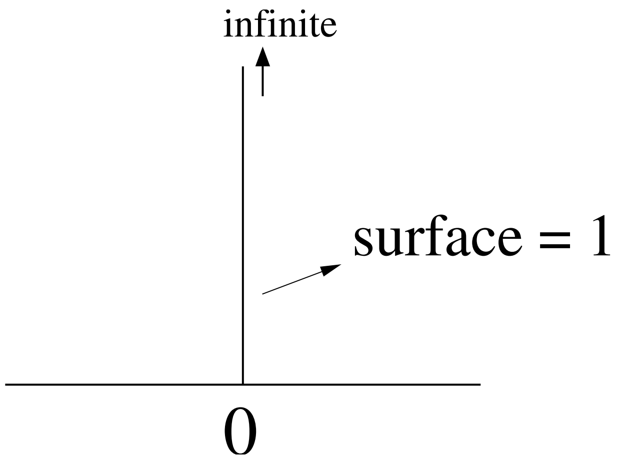
b) Can you find out where the other part of the proton mass comes from?

# Exercise – 16: Dirac delta function (1)

See Griffiths Appendix A

- Consider a function defined by the following prescription:

$$\delta(x) = \lim_{\Delta\to 0} \begin{cases} 1/\Delta & \text{for } |x| < \Delta/2 \\ 0 & \text{otherwise} \end{cases}$$




- The integral of this function is normalized: $\int_{-\infty}^{\infty} \delta(x)\,\mathrm{d}x = 1$
- For a function $f(x)$ we have: $f(x)\delta(x) = f(0)\delta(x)$

…and therefore: $\int_{-\infty}^{\infty} f(x)\delta(x)\,\mathrm{d}x = f(0)\int_{-\infty}^{\infty} \delta(x)\,\mathrm{d}x = f(0)$

- Exercise:
  a) Prove that: $\delta(kx) = \frac{1}{|k|}\delta(x)$
  b) Prove that: $\delta\big(g(x)\big) = \sum_{i=1}^{n} \frac{1}{|g'(x_i)|}\delta(x - x_i)$ , where $g(x_i) = 0$ are the zero-points
     - Hint: make a Taylor expansion of $g$ around the 0-points.

# Exercise – 16: Dirac delta function (2)

- The delta function has many forms. One of them is: $\delta(x) = \lim_{\alpha\to\infty} \frac{1}{\pi} \frac{\sin^2 \alpha x}{\alpha x^2}$

c) Make this plausible by sketching the function $\sin^2(\alpha x)\,/(\pi\alpha x^2)$ for two relevant values of $\alpha$

- Remember the Fourier transform:

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(k)\, e^{ikx}\, \mathrm{d}k$$

$$g(k) = \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, \mathrm{d}x$$

d) Use this to show that another (important!) representation of the Dirac delta-function is given by:

$$\boxed{\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}\, \mathrm{d}k}$$

← We will use this later in the lecture!