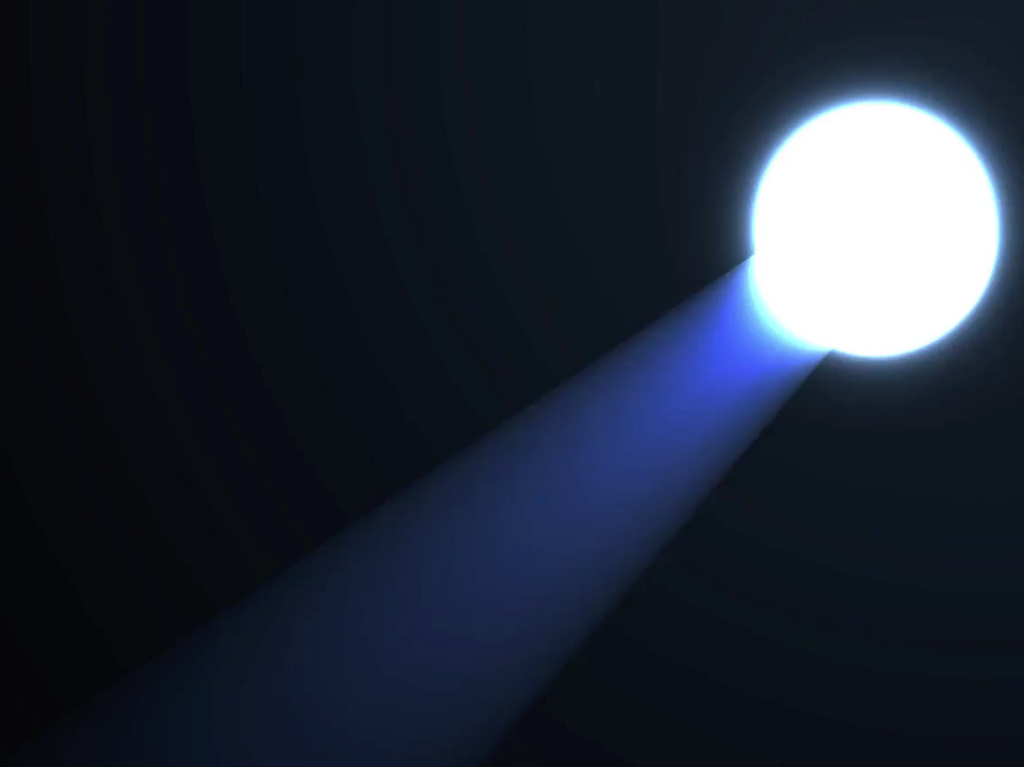
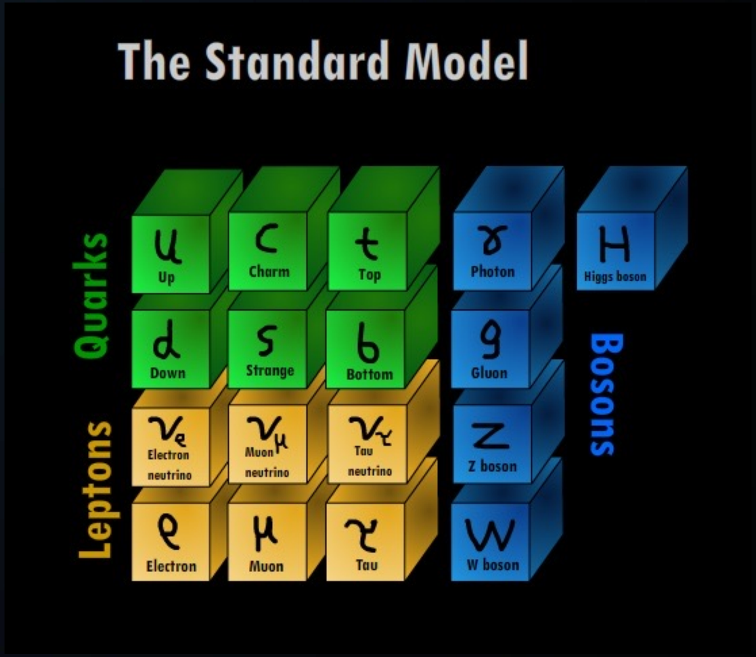
PHY3004: Nuclear and Particle Physics

Marcel Merk, Jacco de Vries





Introducing the lecturers

Lecturers:

Marcel Merk

Jacco de Vries

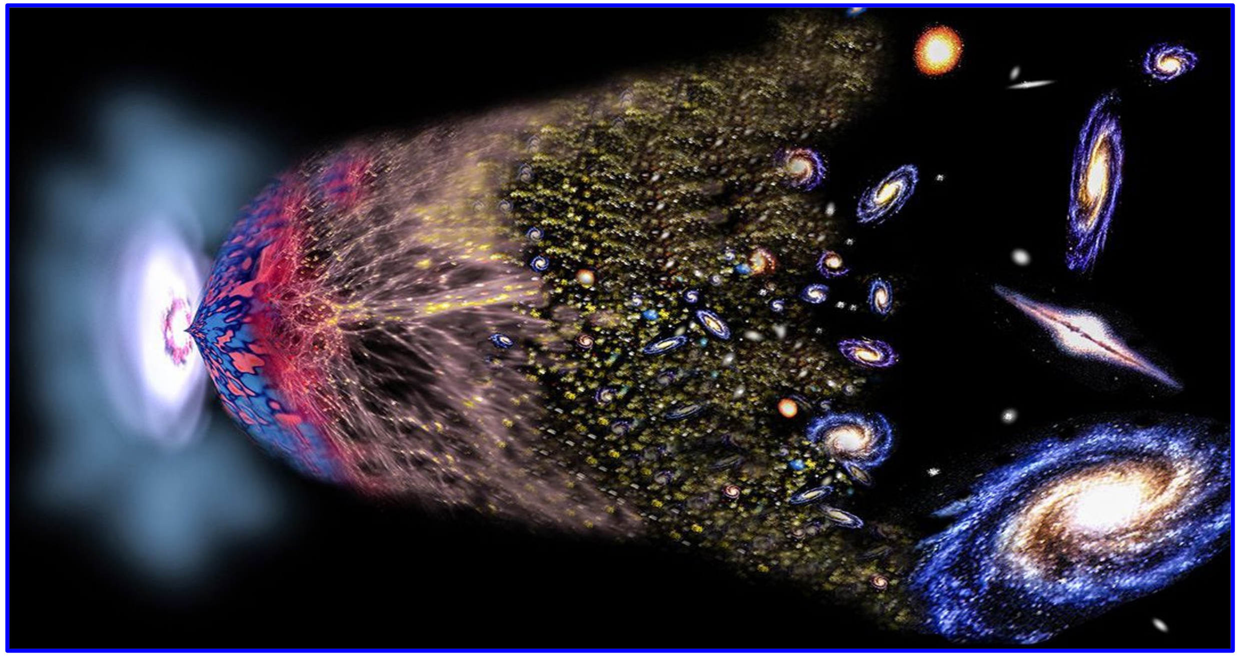
Tutors:

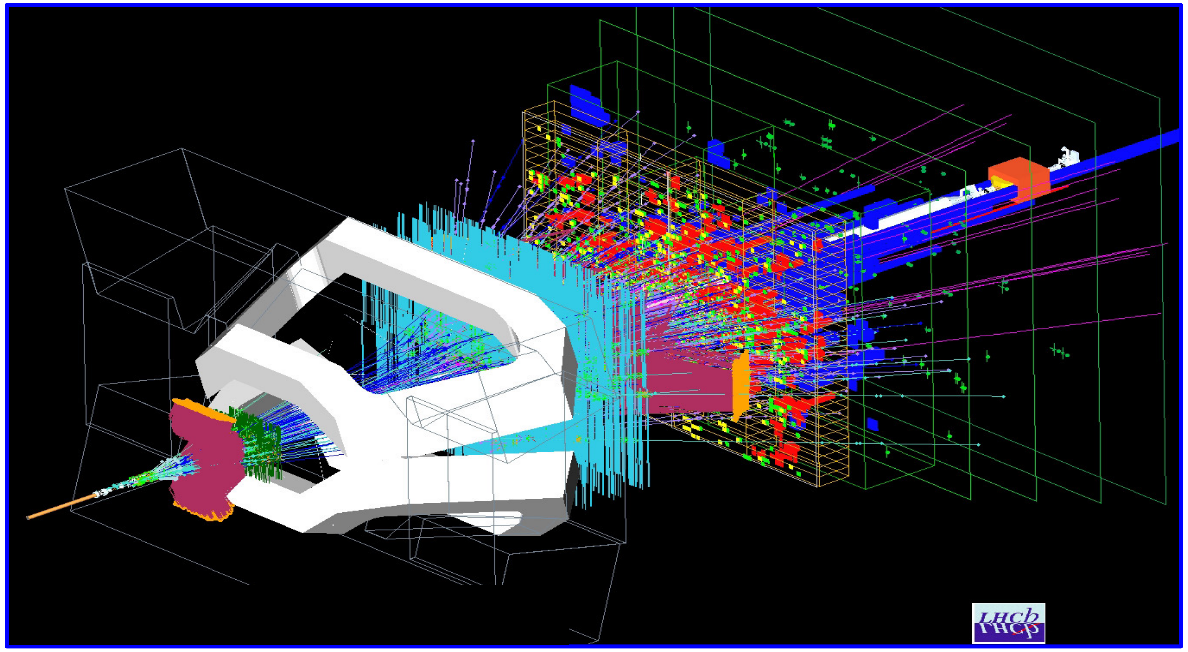
Miriam Lucio Martinez

Davide Nicotra

Research (theoretical):
- Why a *matter-vs-antimatter asymmetry* in nature?
- Why do we have *three generations* of particles?

Research (experimental):
- Detector technology at the *Large Hadron Collider.*
- *Measurements of CP violation rare decays*





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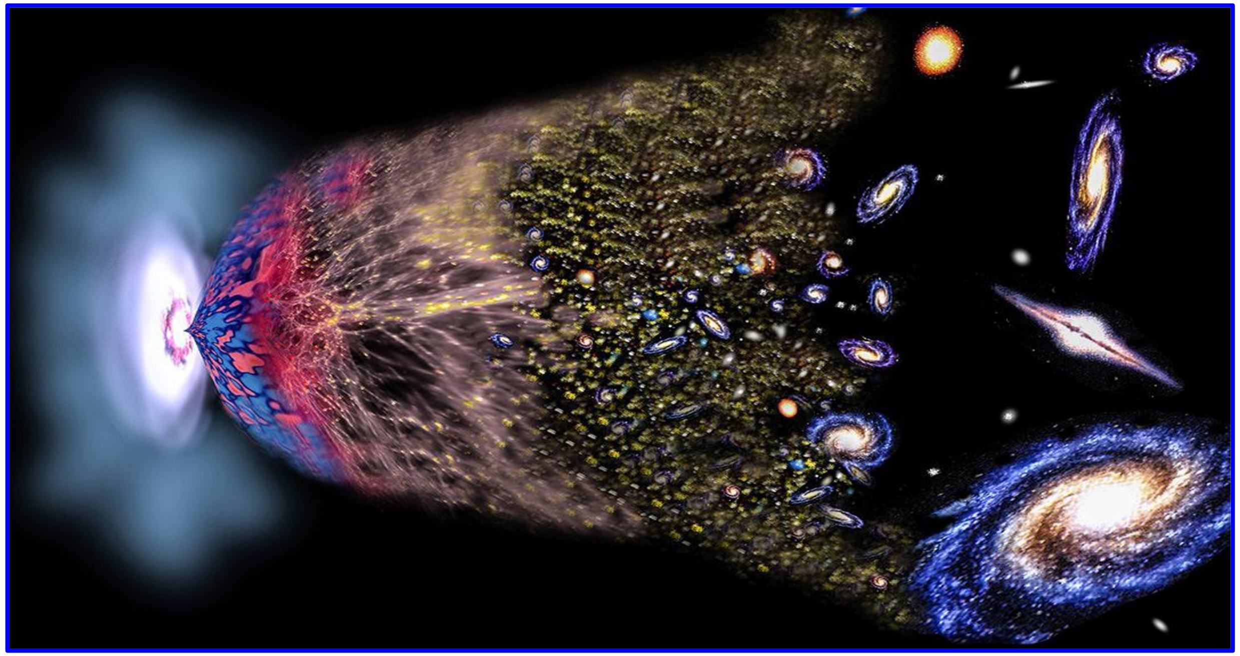
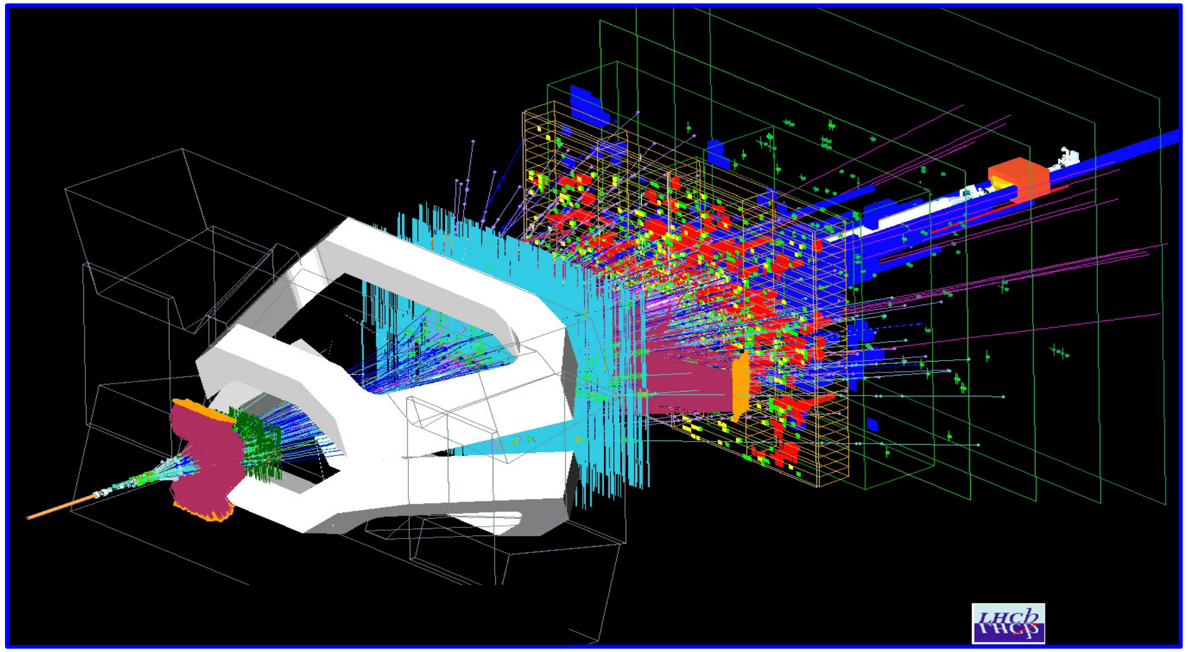
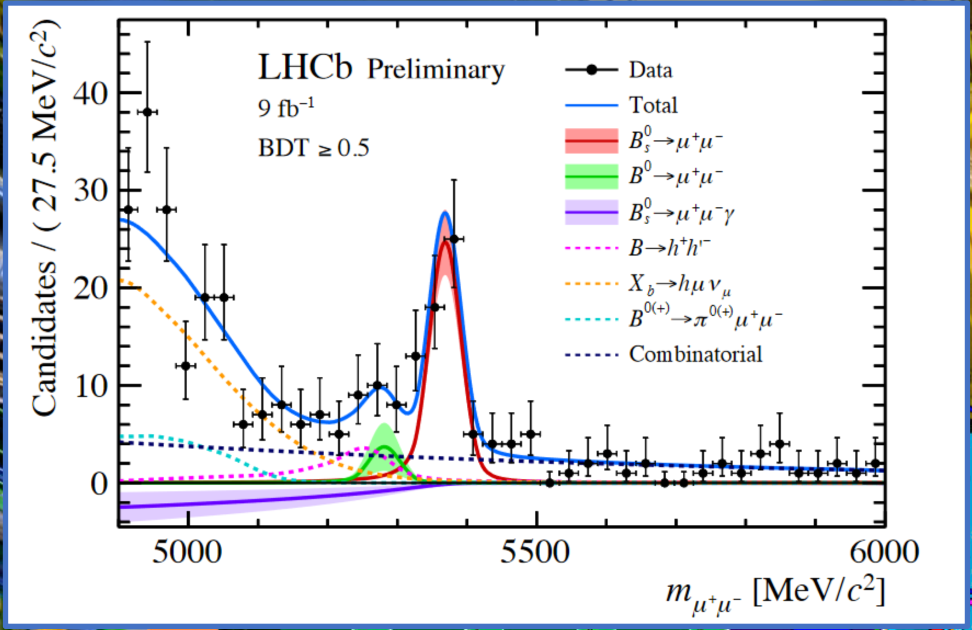
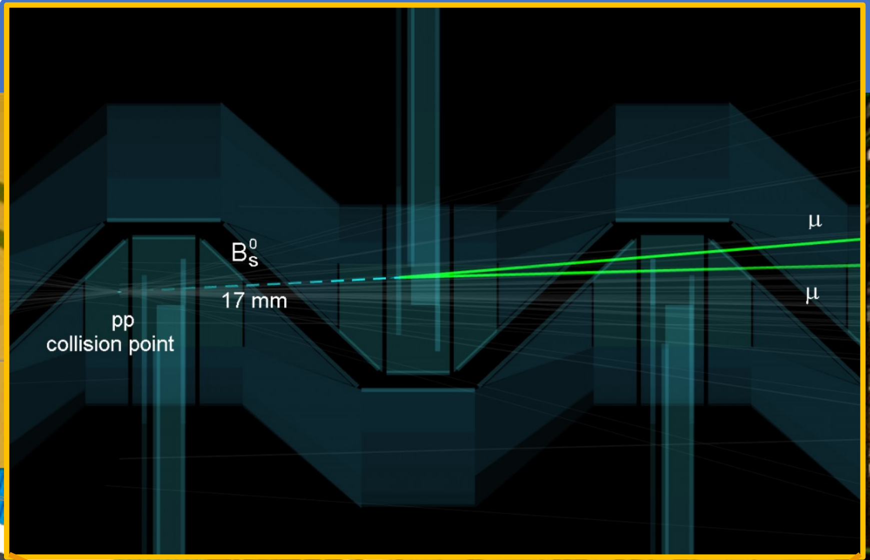
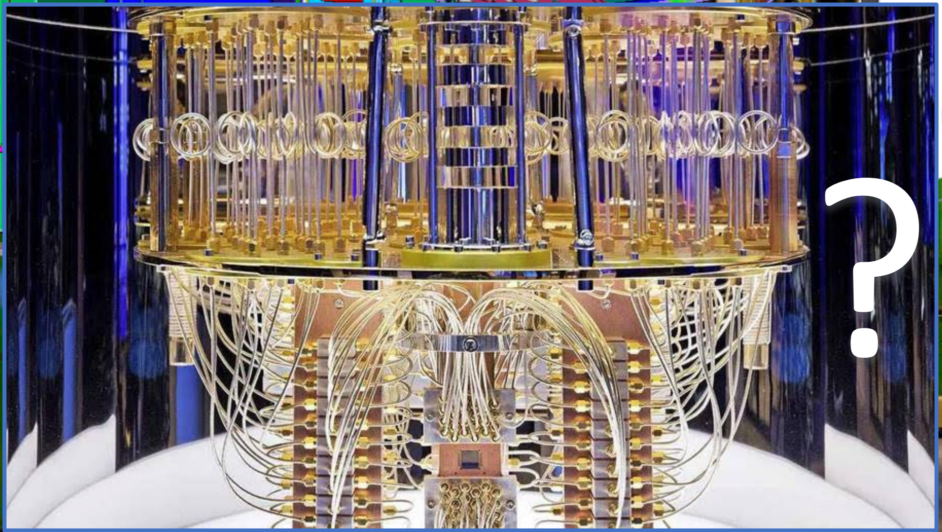


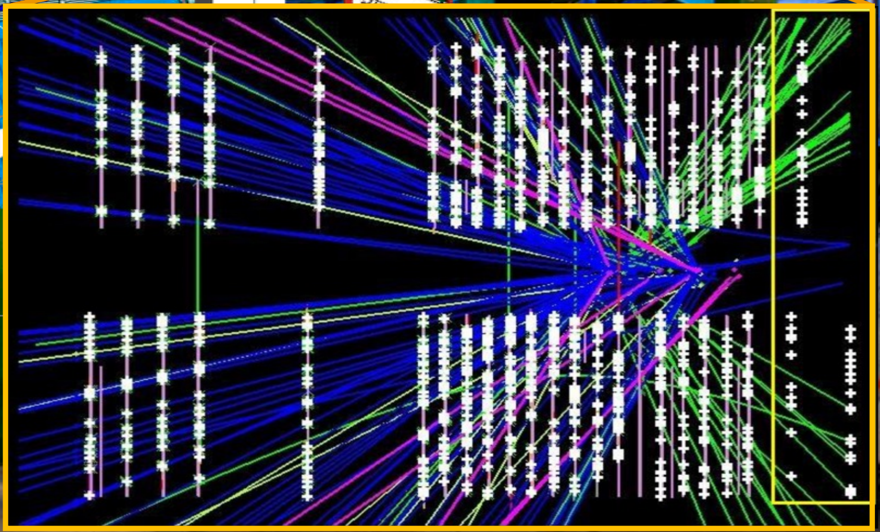


Our Research: “Forbidden decays & QC”









The Goal of these Lectures

- A first course in particle physics theory and experiment
  - Solid preparation for a master on particle physics
  - Pre-reqs are special relativity (“4-vectors”), Lagrange-Hamilton mechanics, Quantum Mechanics
    - Please give these topics attention in tutorials if you need to refresh them
- Various topics introduced
  - Cannot go to “too deep” in each topic
  - Aim is to give you a “feel” for the subject
    - Quite some mathematics later on, but focus on the concepts if you find the math difficult!
- Follow-up possibilities in master education:
  - Theoretical Particle Physics: Field theory courses
  - Theoretical and Experimental Particle Physics: Advanced particle physics courses
  - Experimental Particle Physics: Detection techniques for particles

Particle Physics – contents of the course

- Week 1: "Particles"
  - Nuclear Physics and Particle Physics
- Week 2: "Forces"
  - Electromagnetism, Weak force, Strong force
- Week 3: "Waves"
  - Wave Equations
    - Schrodinger, Klein – Gordon, Dirac, Photon-field
  - Lagrange Hamilton mechanics and Gauge Invariance
- Week 4: "Symmetries"
  - The Standard Model gauge symmetry
  - Discrete symmetries
  - Symmetry breaking: the Higgs Mechanism
- Week 5: "Scattering"
  - Scattering Theory
  - Feynman Calculus
- Week 6: "Experiments"
  - Experimental techniques, LHC detectors

- Week 1 – 5:
  - Homework exercises 20%
  - Tutor discussions 15%
- Week 6: Research Project
  - Presentation 15%
- Week 7: Open book Exam 50%

Videos: see https://www.nikhef.nl/~i93/Teaching

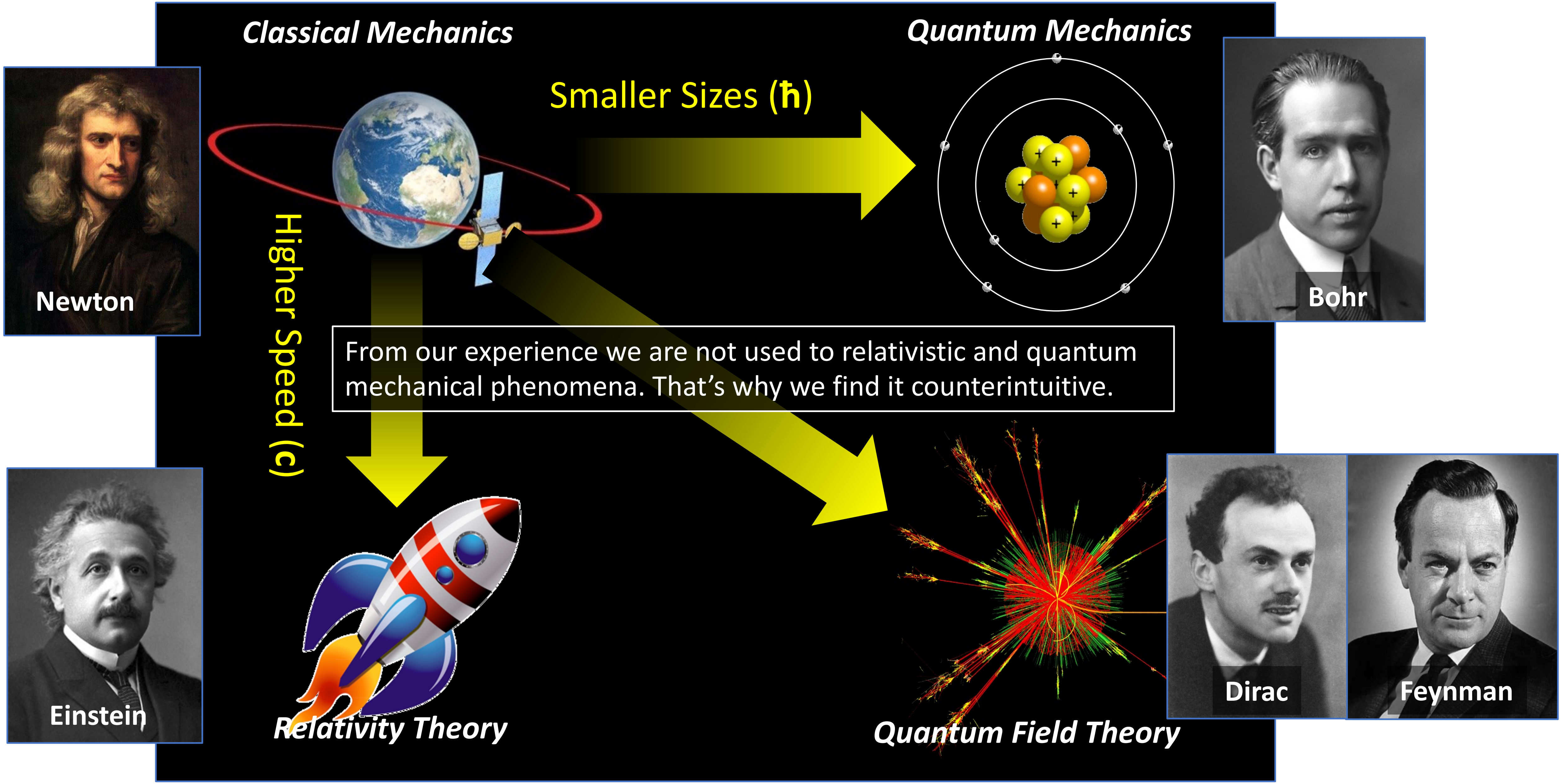
Literature belonging to the lectures:

- Griffiths: "Introduction to Elementary Particles"

Other good books (more advanced):

- Mark Thomson: "Modern Particle Physics"
- Halzen and Martin: "Quarks & Leptons: An introductory Course in Modern Particle Physics"
- Aitchison and Hey: "Gauge Theories in Particle Physics" – 2 volumes
- Lecture notes PP1 master course from Nikhef: https://www.nikhef.nl/~i93/Master/PP1/2017/Lectures/Lecture2017.pdf

Tools: Relativity and Quantum Mechanics



Classical Mechanics

Quantum Mechanics

Smaller Sizes (ħ)

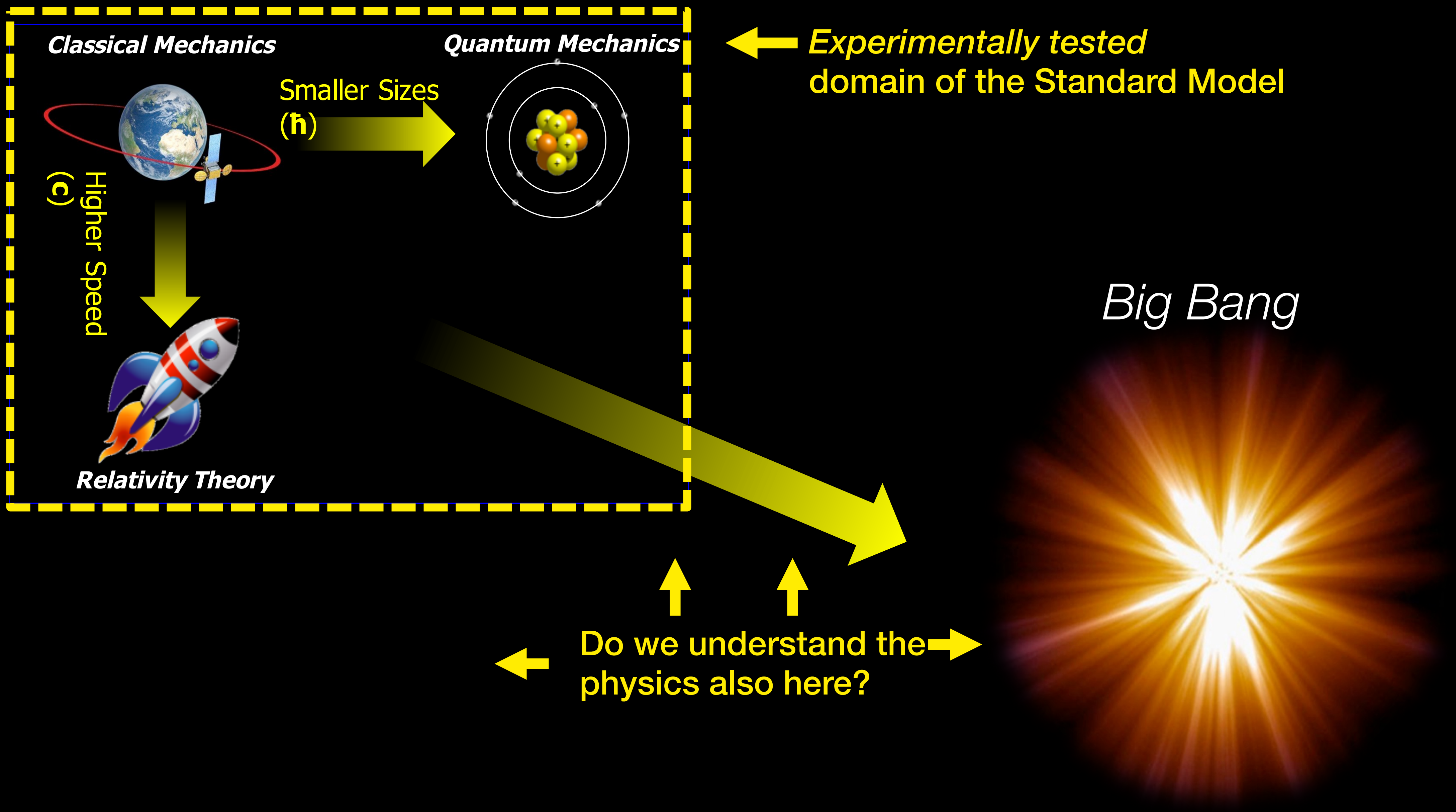
Higher Speed (c)

From our experience we are not used to relativistic and quantum mechanical phenomena. That’s why we find it counterintuitive.

Relativity Theory

Quantum Field Theory

How does nature behave in extreme conditions?



Classical Mechanics

Quantum Mechanics

Smaller Sizes
($\hbar$)

Higher Speed
(c)

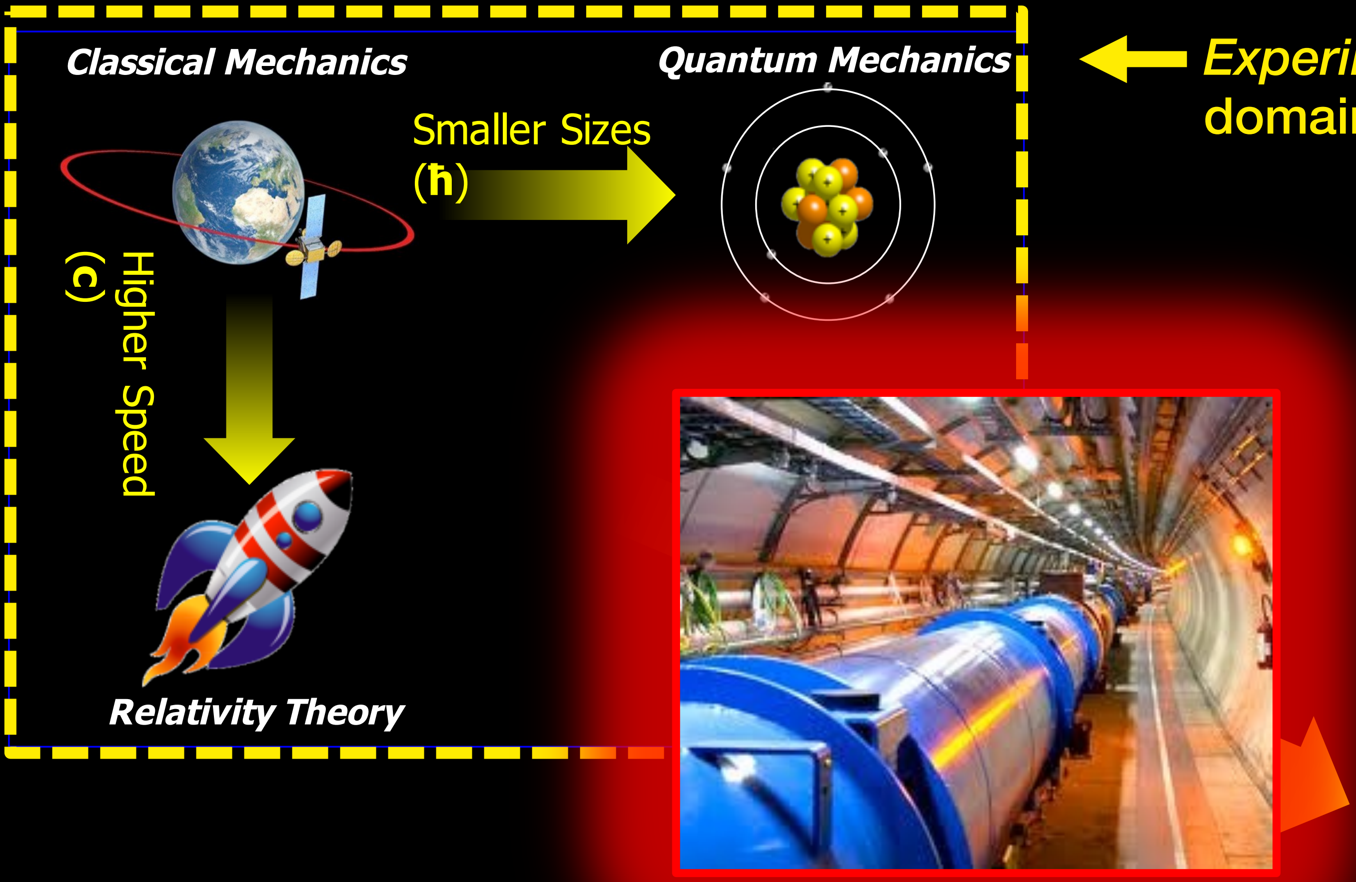
Relativity Theory

Experimentally tested
domain of the Standard Model

Big Bang

Do we understand the
physics also here?

How does nature behave in extreme conditions?



Classical Mechanics

Quantum Mechanics

Smaller Sizes
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Relativity Theory

Experimentally tested
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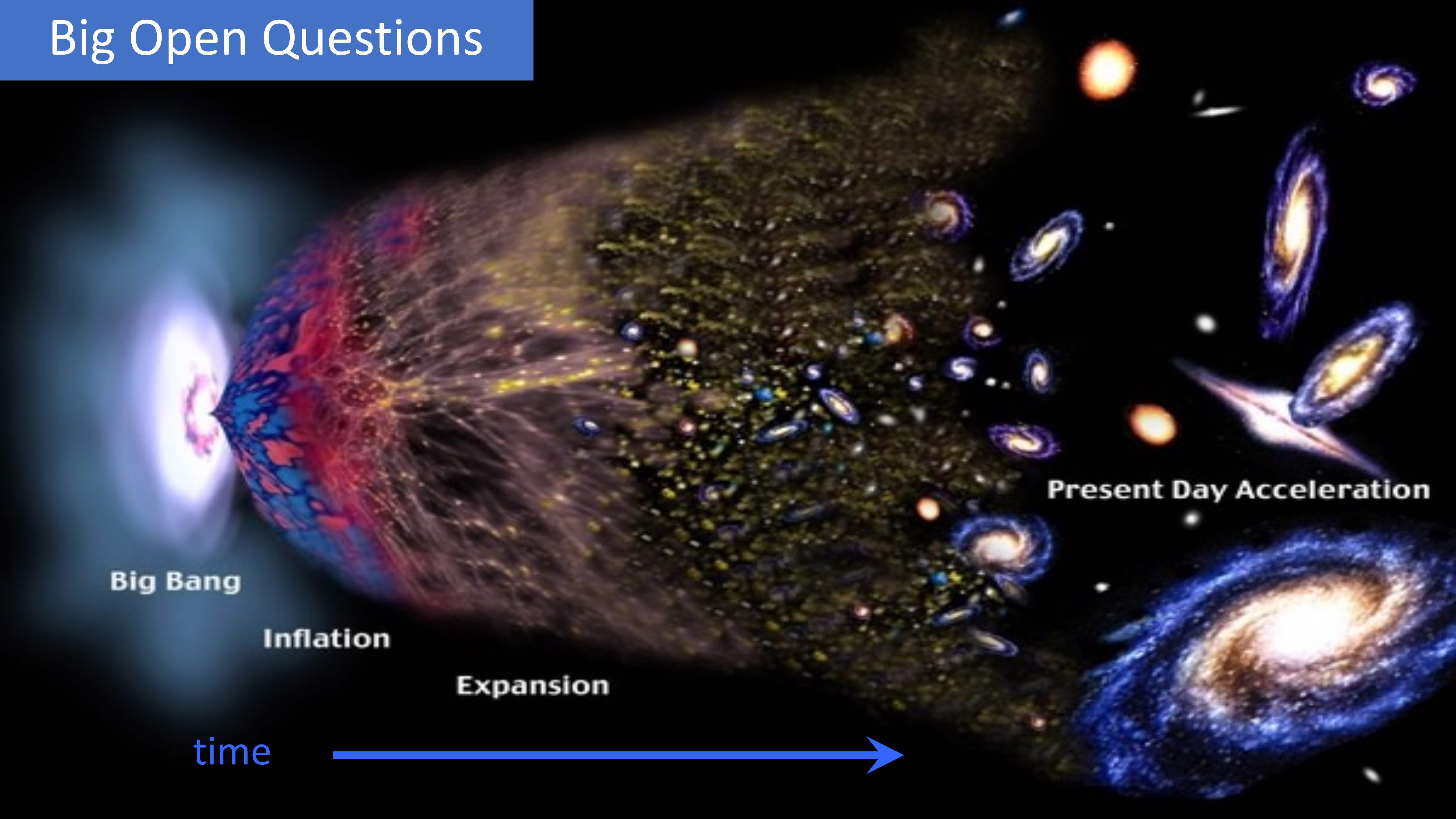
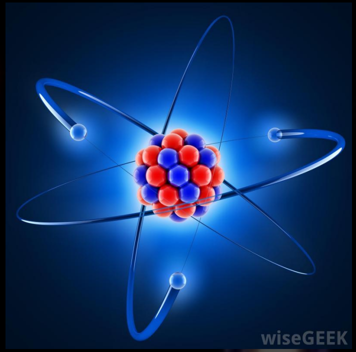
The Large Hadron Collider

Big Bang

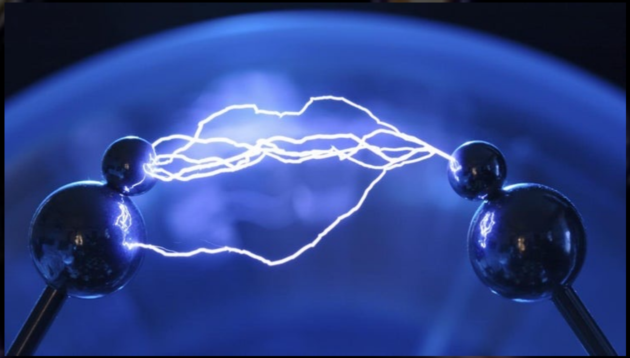
Big Open Questions



Big Open Questions

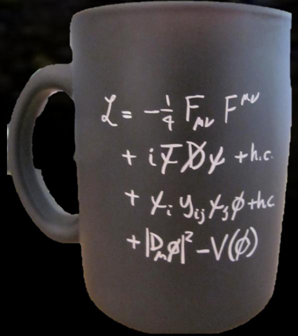


1. Which are the elementary building blocks of matter?
2. Which forces exist in nature and what are their essential differences?



3. Is empty space (‘the vacuum’) truly empty?
4. Can we explain the existance of our universe from the *big bang* using the known laws of nature?

- *Wanted: a consistent theory that can answer these questions*



Lecture 1: Particles

Part 1

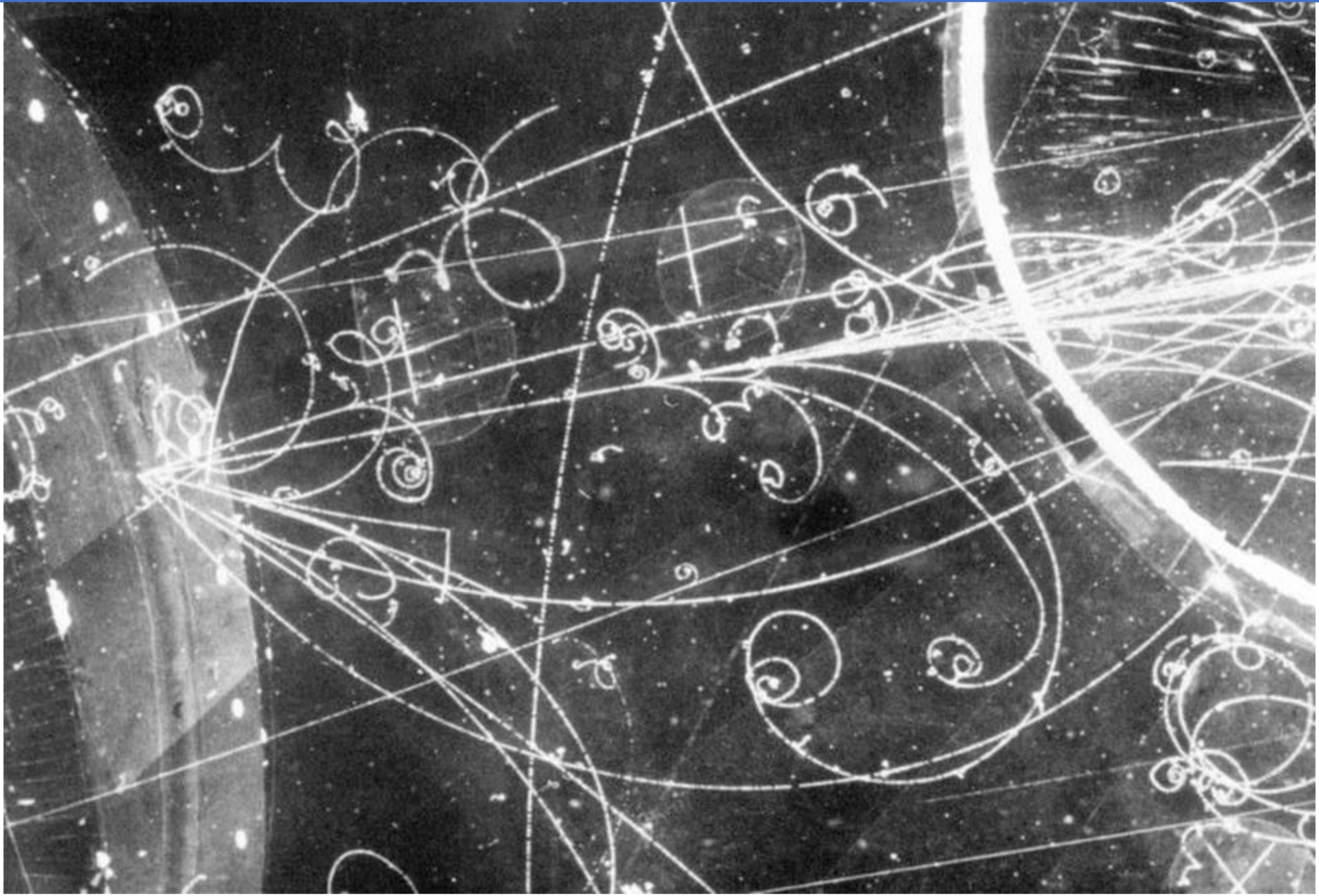
The Structure of Matter

(Nuclear Physics)

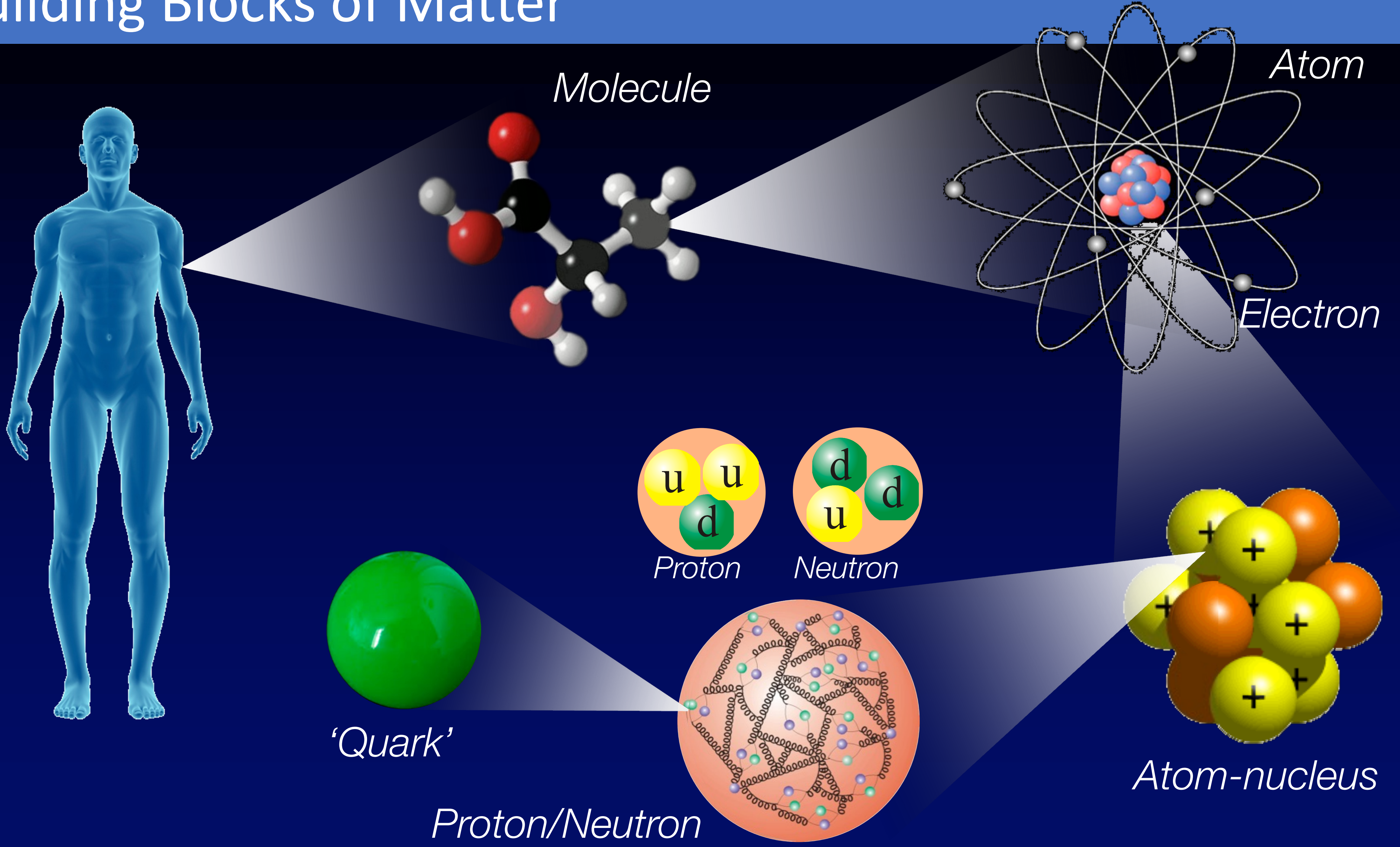
See the online slides

Not part of the exam material

Lecture 1: “Particles”



Building Blocks of Matter



Molecule

Atom

Electron

Proton

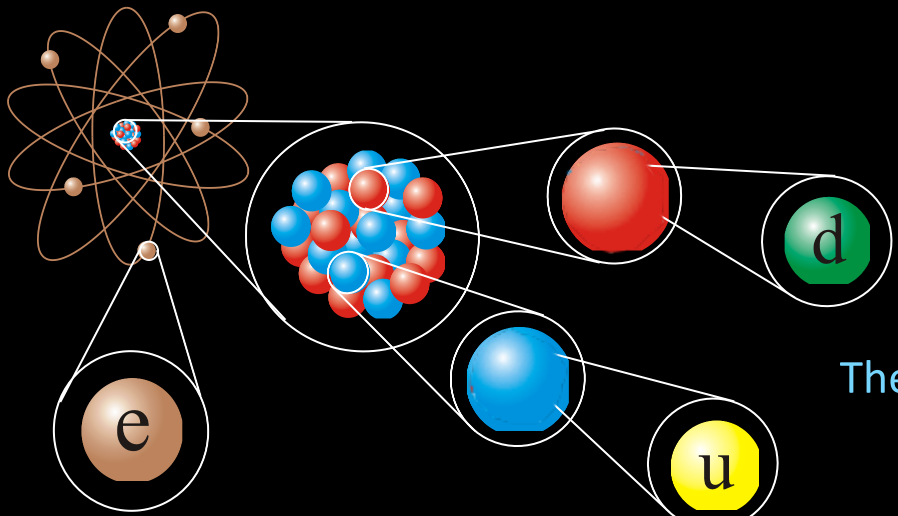
Neutron

‘Quark’

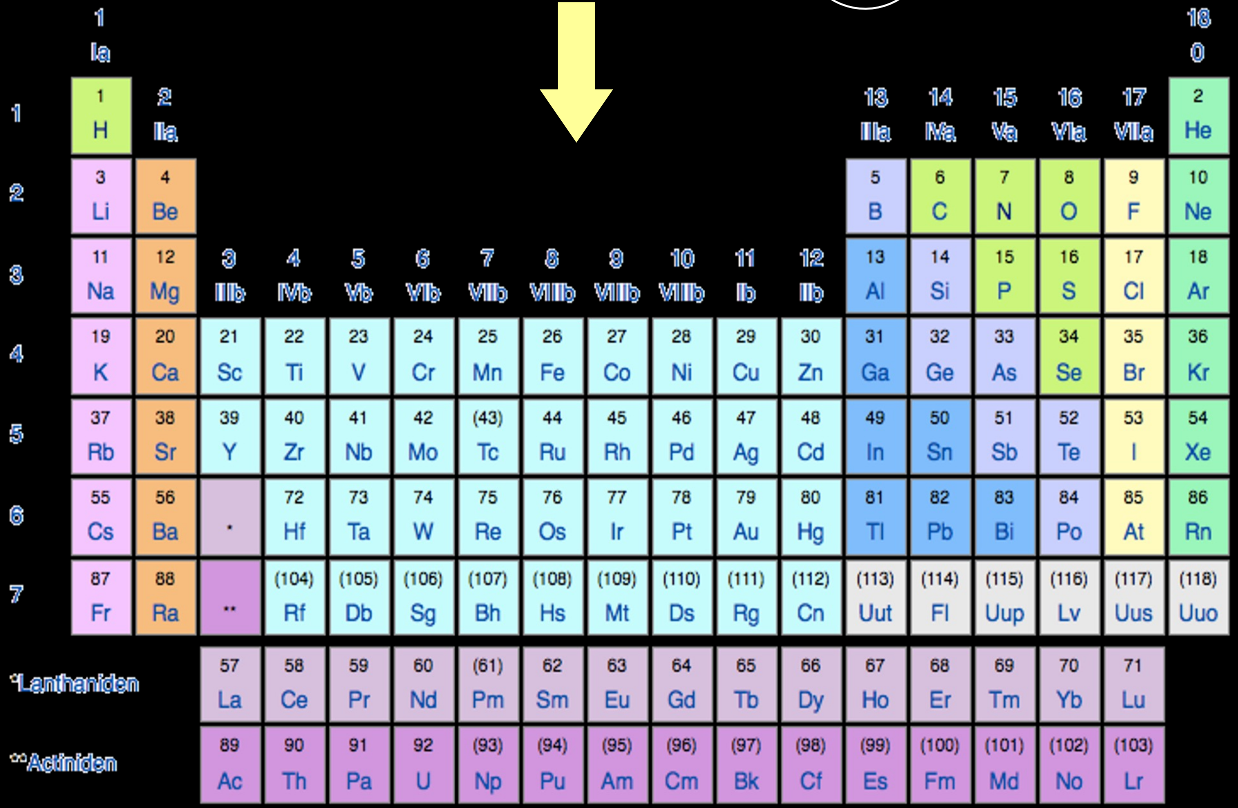
Proton/Neutron

Atom-nucleus

Stable Matter on Earth



The Lego blocks of nature

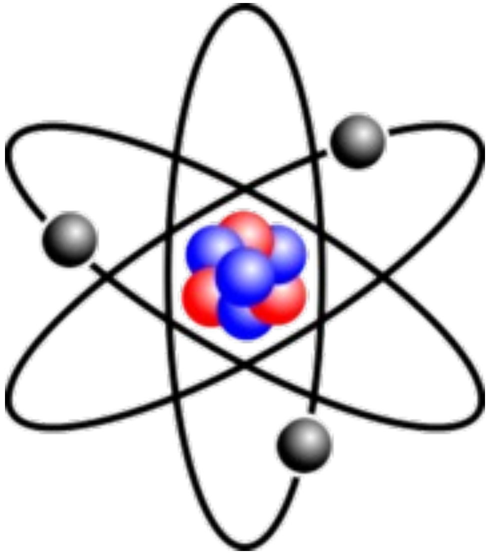


	1 Ia	2 IIa	3 IIIb	4 IVb	5 Vb	6 VIb	7 VIIb	8 VIIIb	9 VIIIb	10 VIIIb	11 Ib	12 IIb	13 IIIa	14 IVa	15 Va	16 VIa	17 VIIa	18 0
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	(43) Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	(104) Rf	(105) Db	(106) Sg	(107) Bh	(108) Hs	(109) Mt	(110) Ds	(111) Rg	(112) Cn	(113) Uut	(114) Fl	(115) Uup	(116) Lv	(117) Uus	(118) Uuo
*Lanthaniden			57 La	58 Ce	59 Pr	60 Nd	(61) Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
**Actiniden			89 Ac	90 Th	91 Pa	92 U	(93) Np	(94) Pu	(95) Am	(96) Cm	(97) Bk	(98) Cf	(99) Es	(100) Fm	(101) Md	(102) No	(103) Lr	

“Zoer-vleis”..?

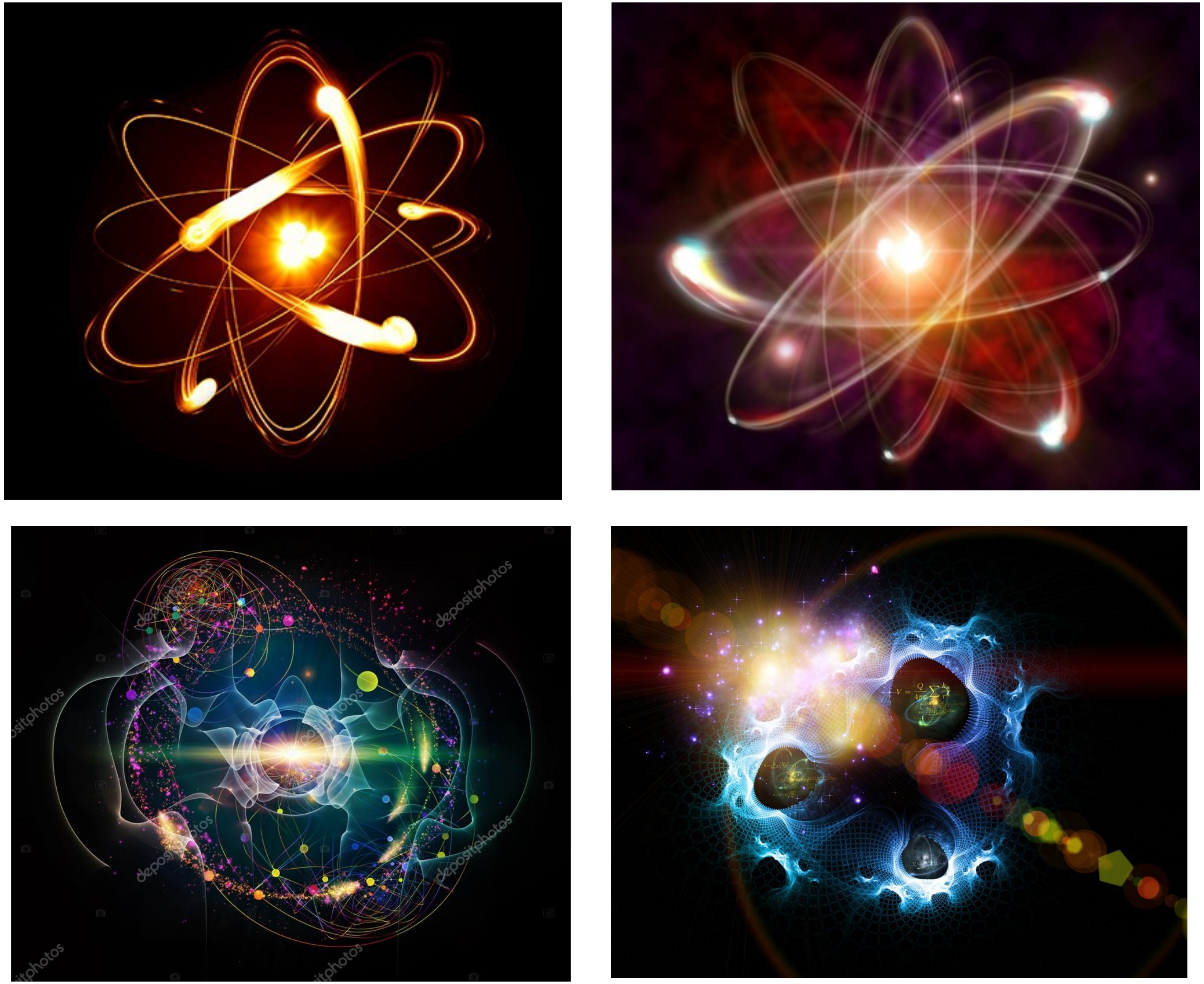
Which mental picture do you make of an atom?

Classic picture of the atom



- Does something “move”?
- How empty is it?
- How does an electron “exist”?

➔ Consider QM interpretations



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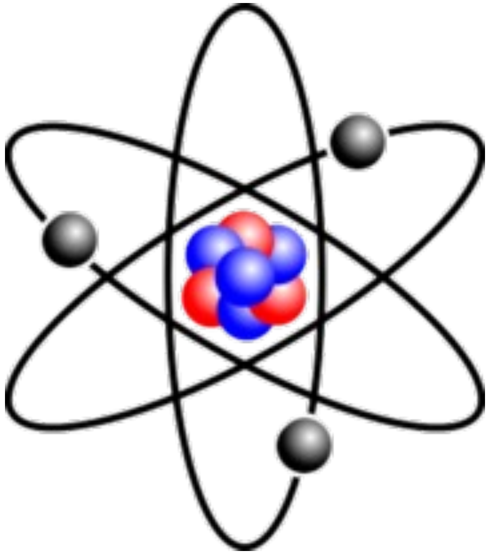

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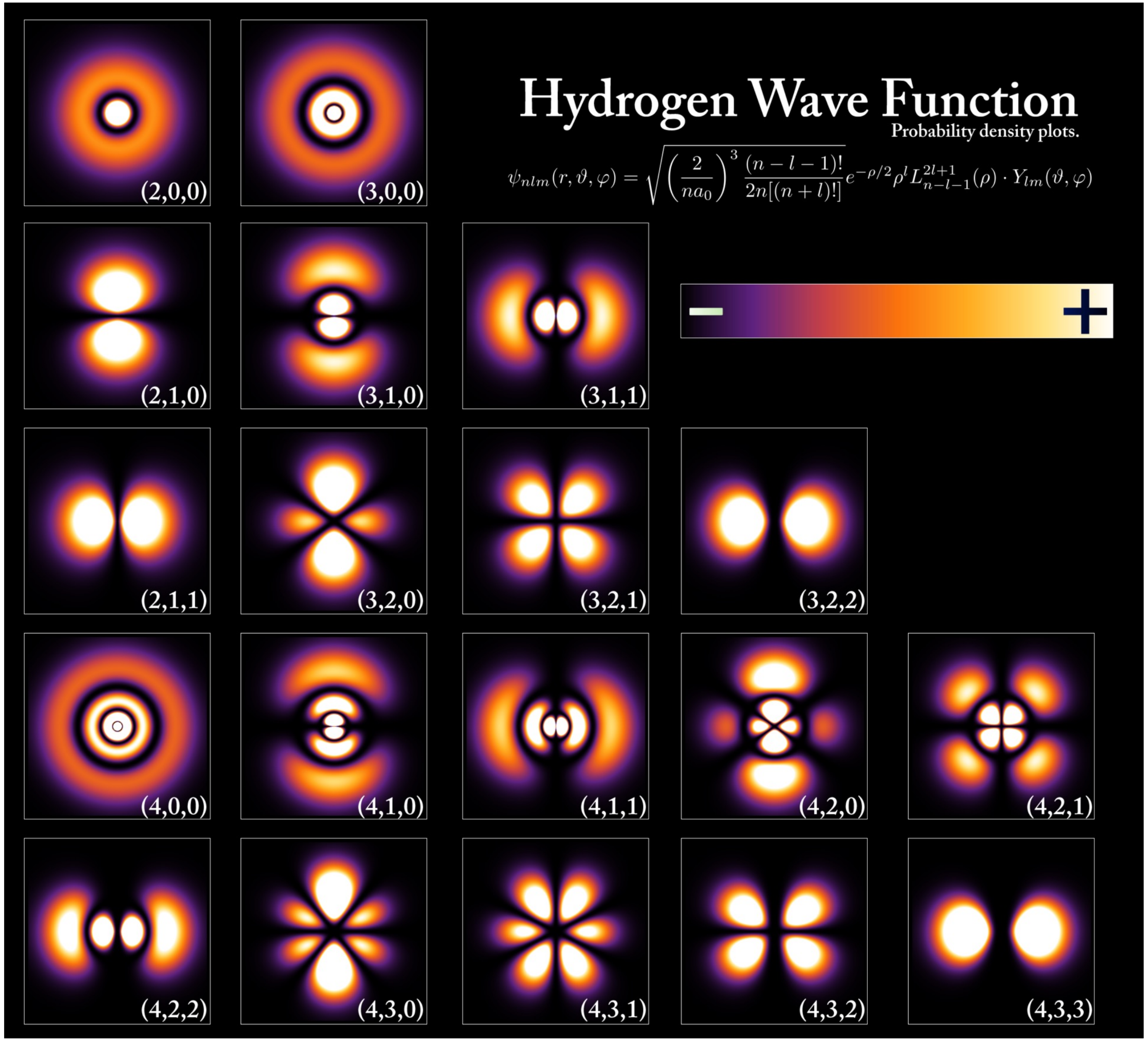
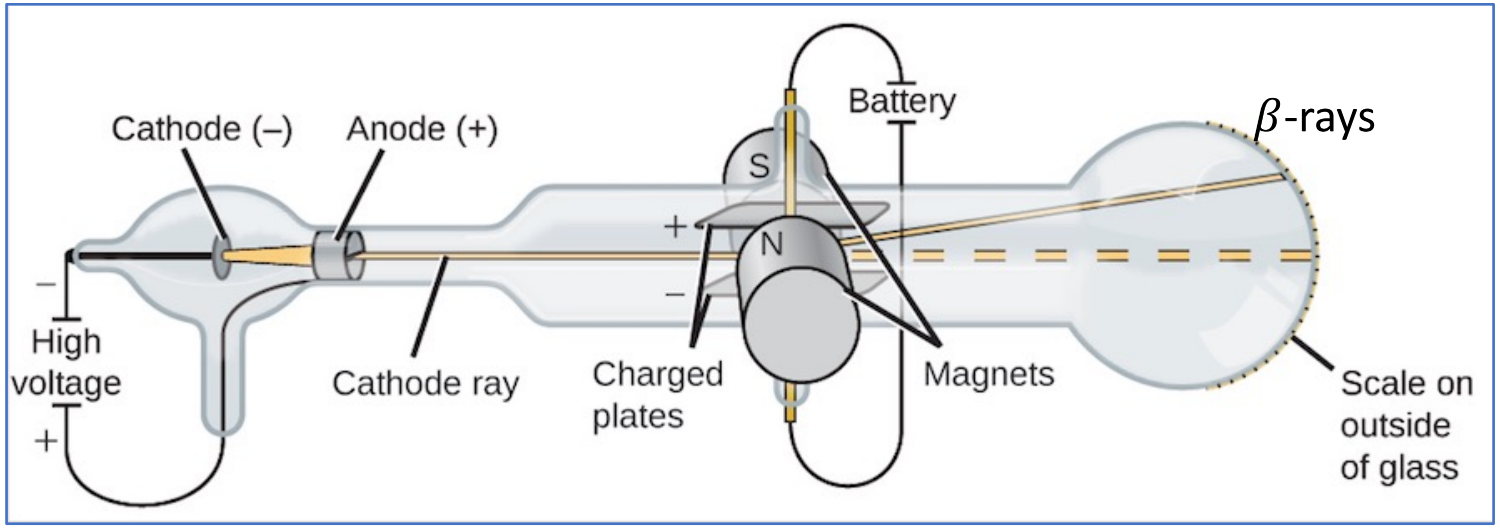


Discovery of the Electron

Griffiths §1.1

- JJ Thomson (1897): Cathode rays are bent by electric and magnetic field
  - Charged particles



1. Compensate Electric and Magnetic deflection:

$$F_E = F_B$$

$$qE = qvB \; ; \; v = {}^{E}/_{B}$$

2. Only B-field, cycle orbit:

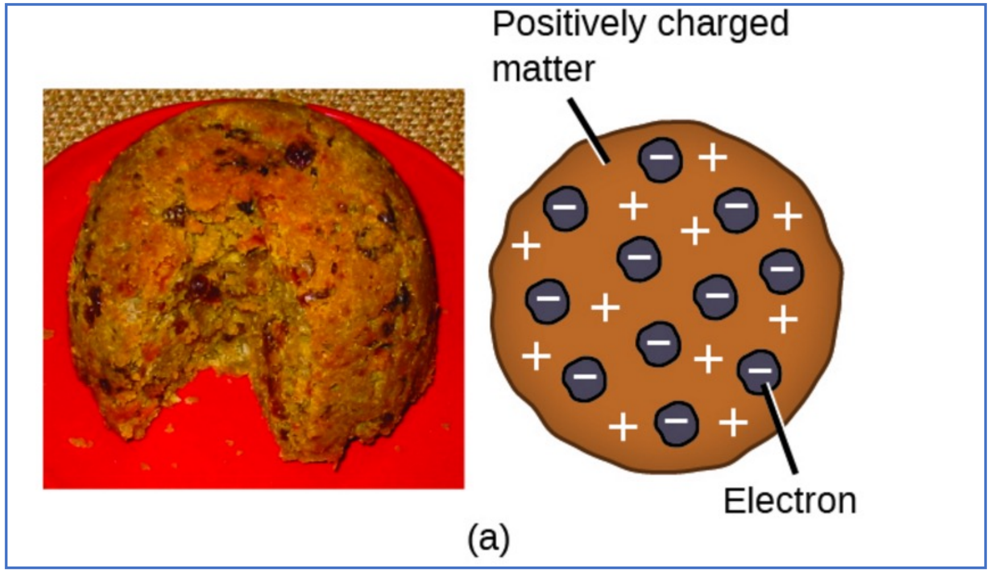
$$F_c = F_B$$

$$\frac{mv^2}{r} = qvB$$

$$\frac{q}{m} = \frac{E}{rB^2}$$

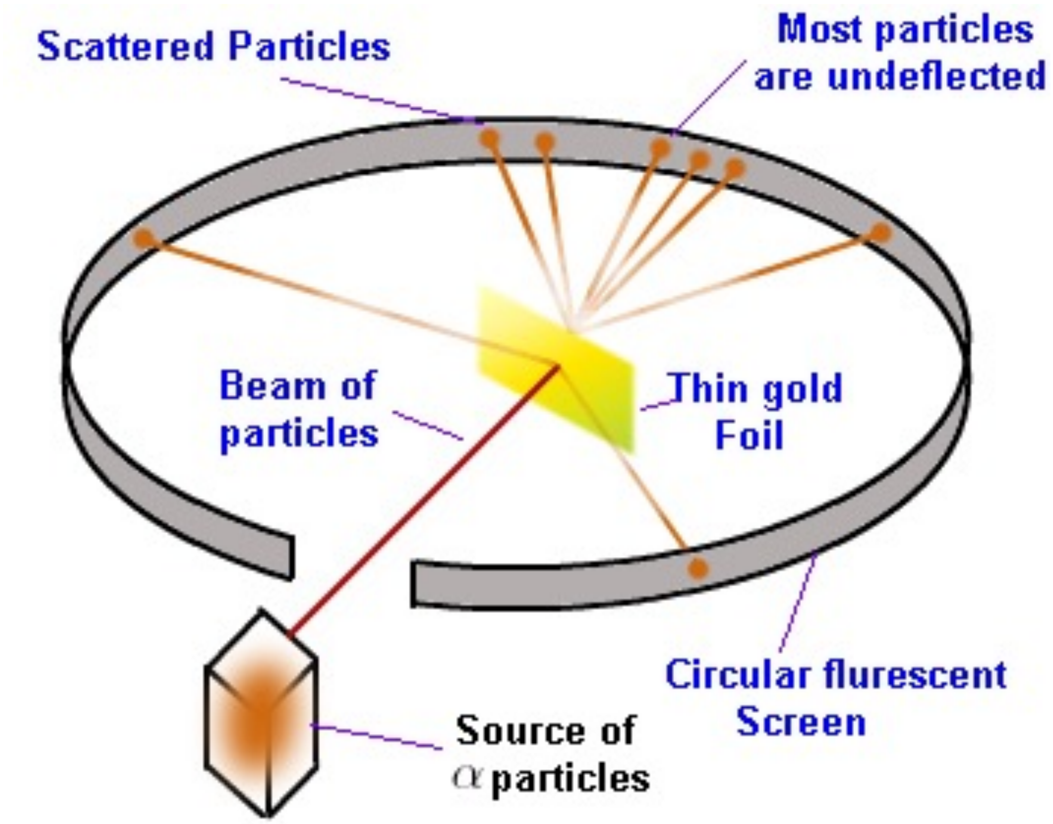
Electrons are much lighter than ions of same charge!

- Plum-pudding model of atoms:



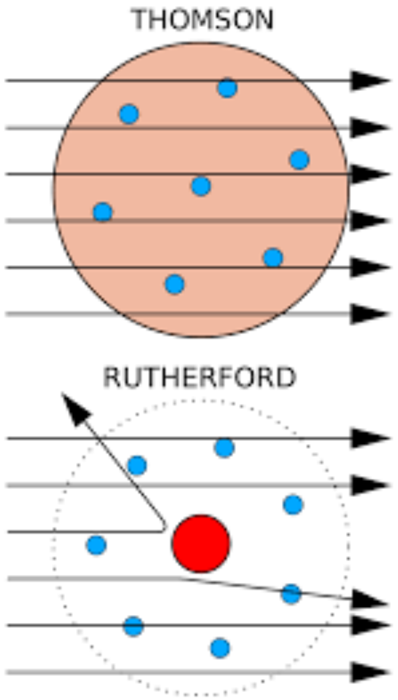
Rutherford Scattering

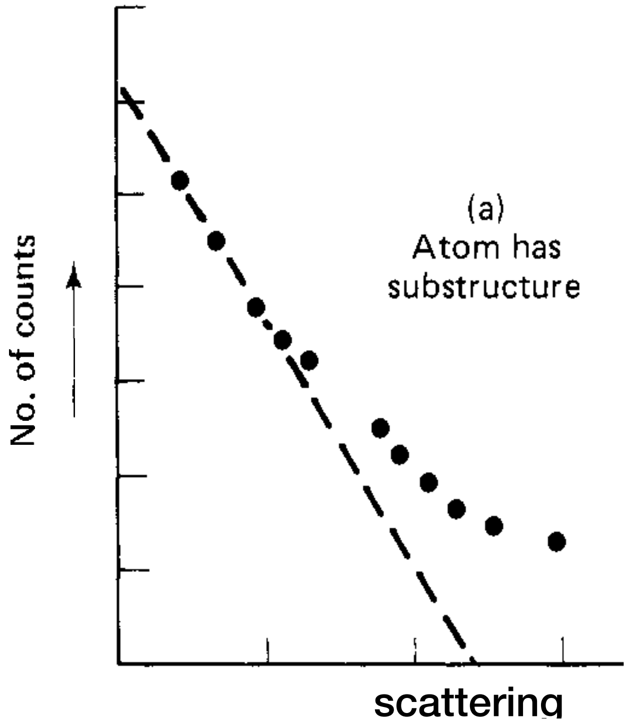
Griffiths §1.1



- Atom is basically just vacuum:
  - Atom size: ~ $10^{-10}\ m$
  - Nucleus size: ~ $10^{-15}\ m$

- Rutherford (1911)
  - α-particles at gold target
  - Most particle pass undisturbed, few have a “hard” collision
  - Atom has substructure: small heavy nucleus

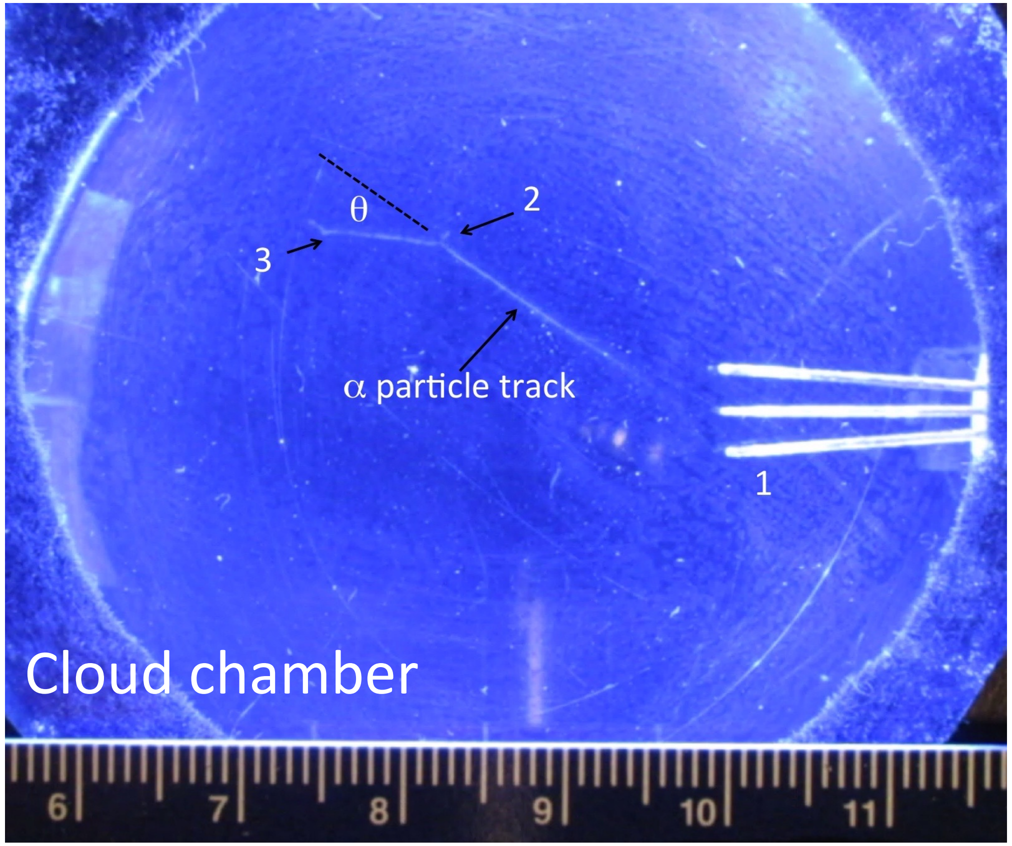




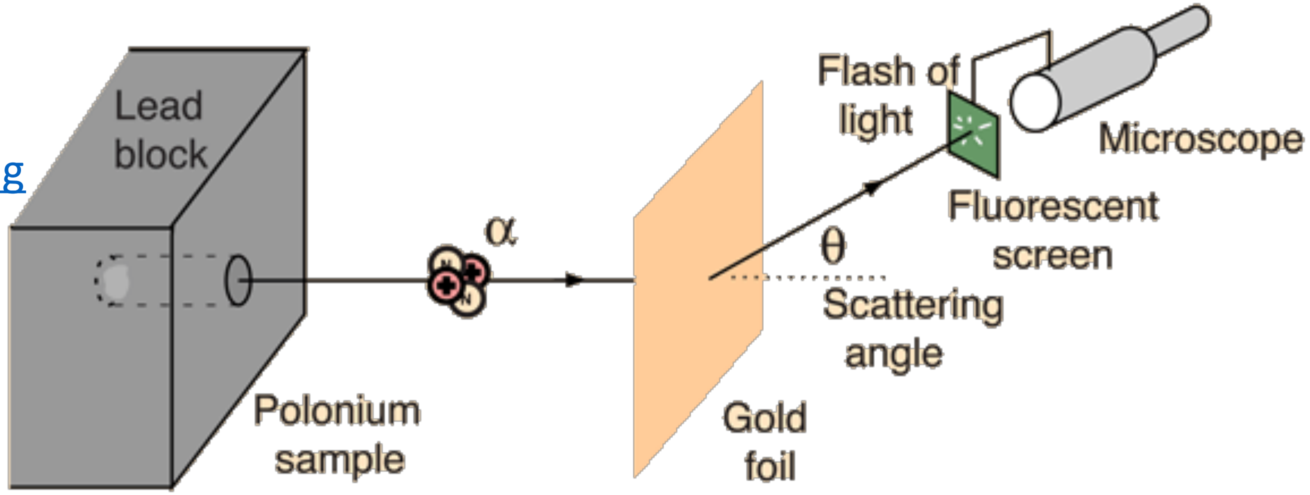
Example Rutherford scattering

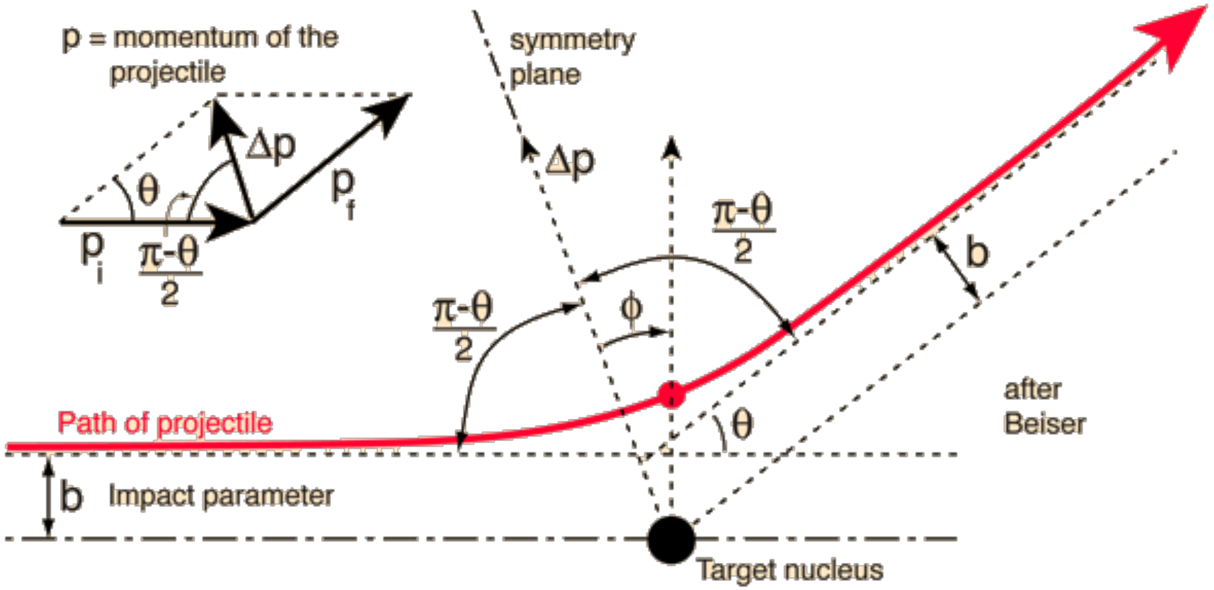
Griffiths §1.1

- Rutherford scattering on nucleus
  - Formula derivation: https://en.wikipedia.org/wiki/Rutherford_scattering
  - →not required



Cloud chamber





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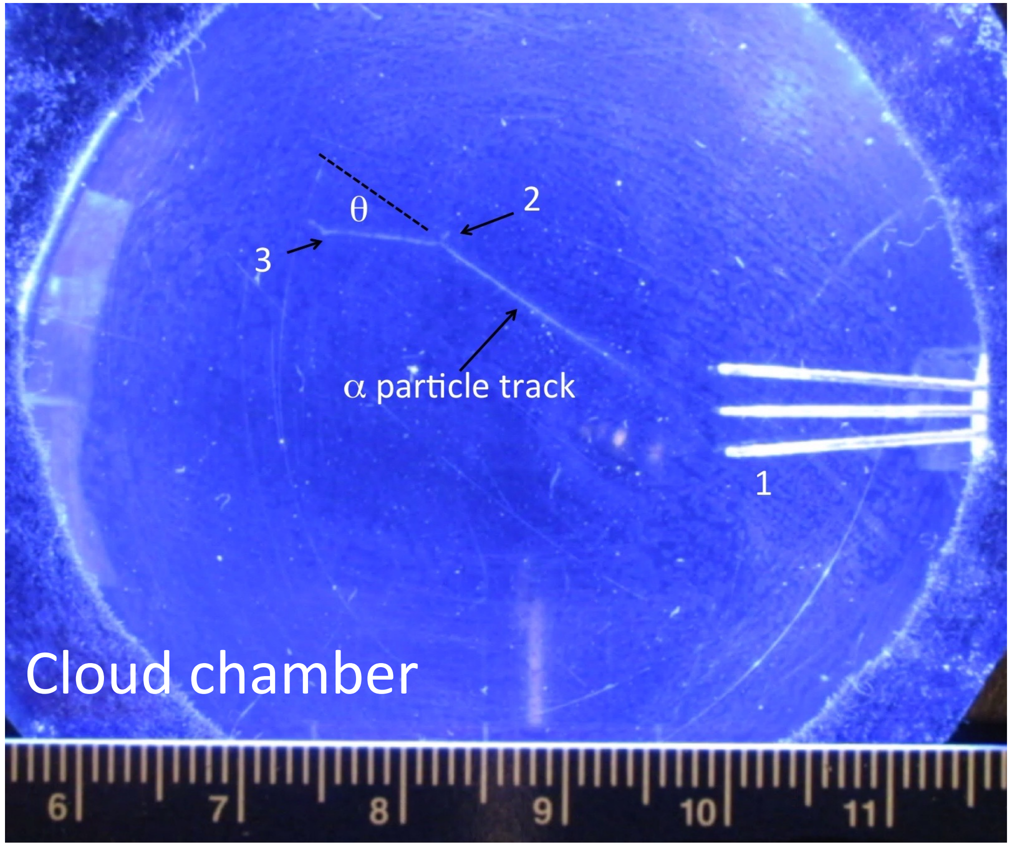
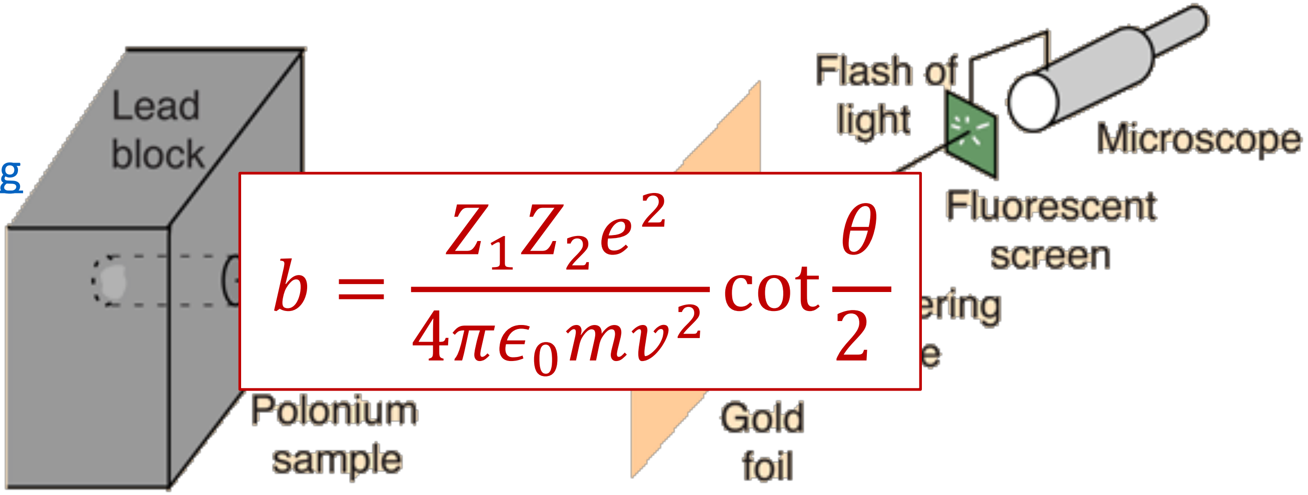
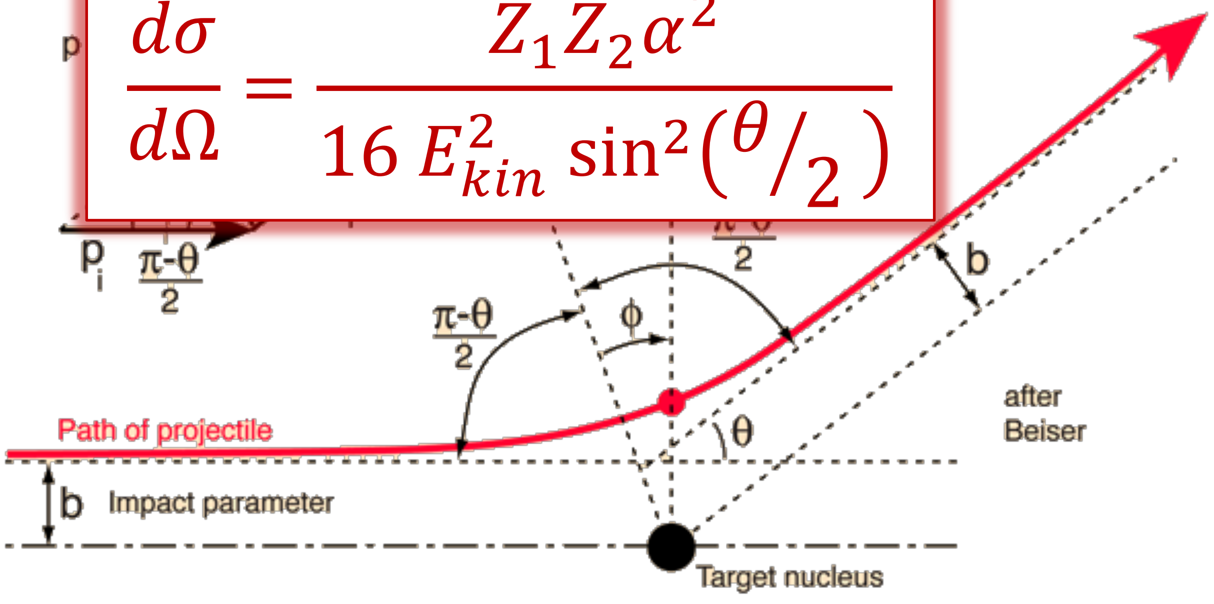




$$b = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 m v^2}\cot\frac{\theta}{2}$$

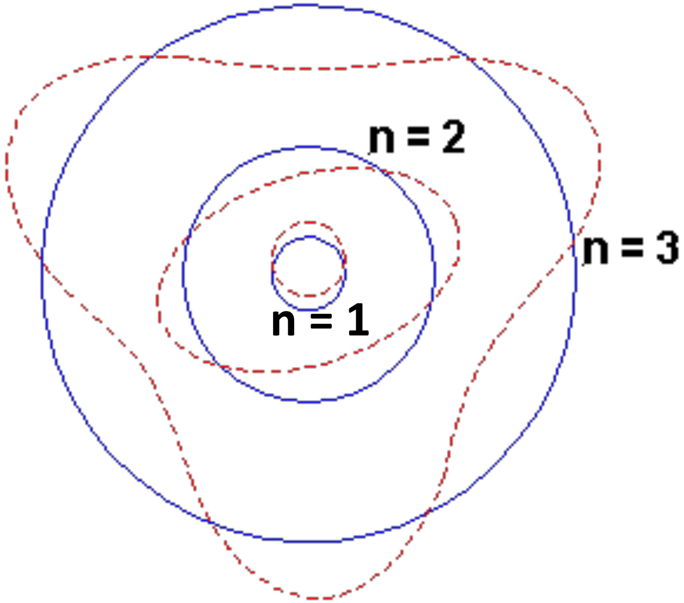


$$\frac{d\sigma}{d\Omega} = \frac{Z_1 Z_2 \alpha^2}{16\, E_{kin}^2 \sin^2\left({}^{\theta}/_{2}\right)}$$

Bohr Atom

Griffiths §1.1

- Niels Bohr, 1914:
  - Calculate atomic energy levels using semi classical method



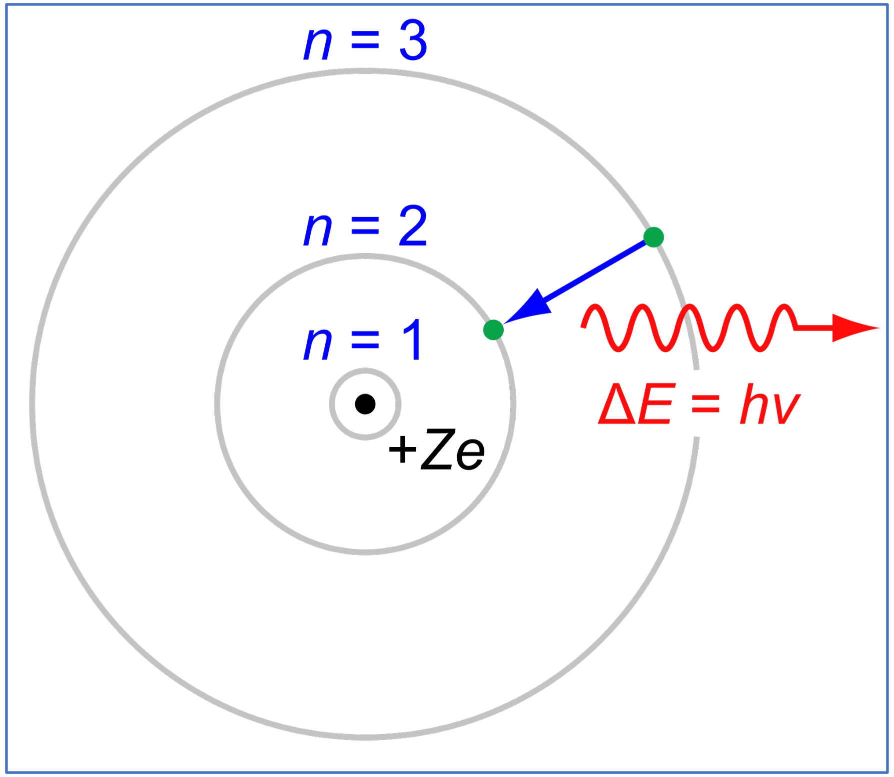
de Broglie: $\lambda = h / p$

$L = r\ p$

$L = r\ h/\lambda$

$L = r\ n\, h/(2\pi r)$

$L = n\, h/(2\pi) = n\hbar$



- Balmer spectrum of visible wavelengths in hydrogen:

Bohr Atom

Griffiths §1.1

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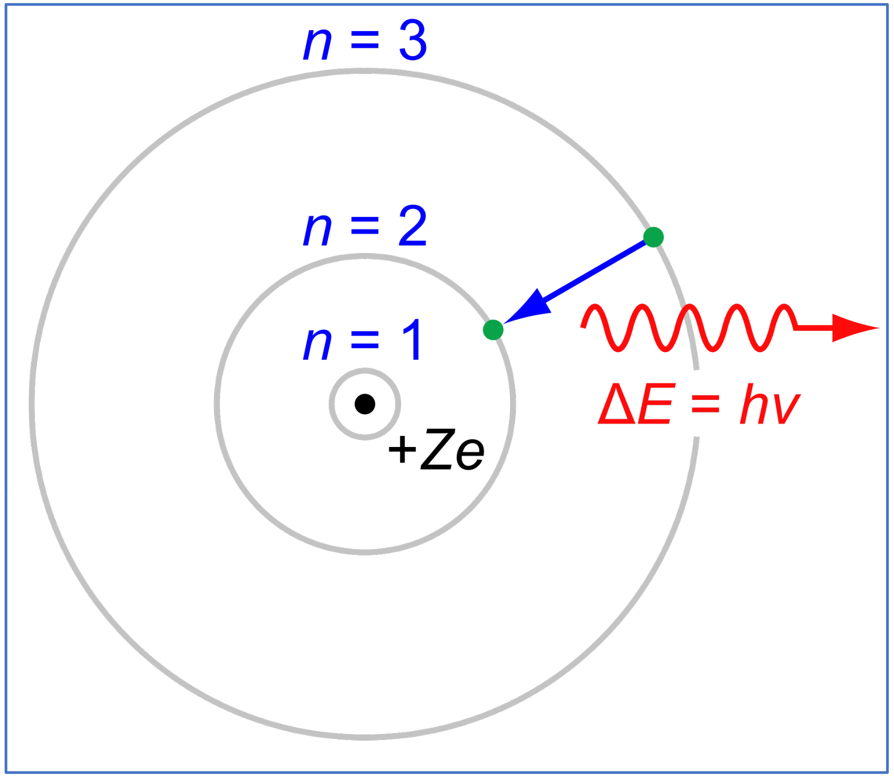
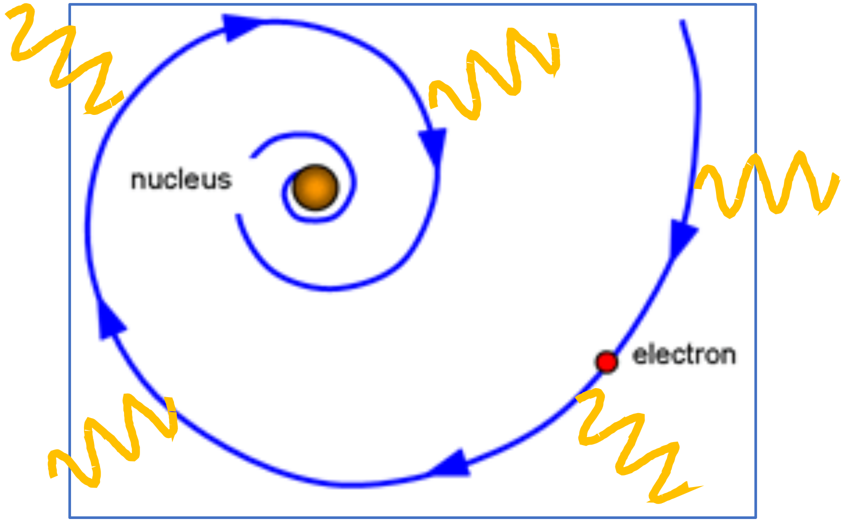


Exercise-2 “Pub Quizz”:

1: How come an atom is stable?

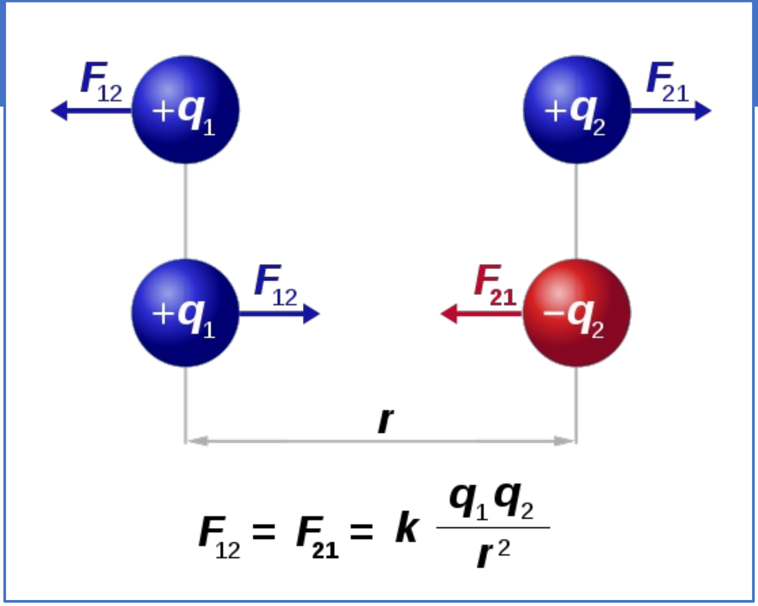
2: What hits each other when you clap your hands?

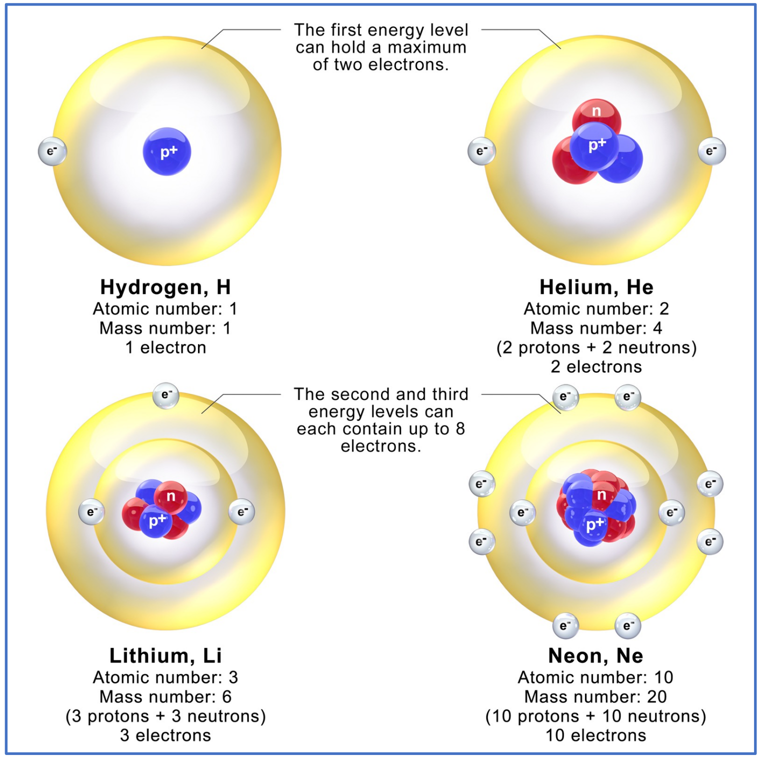
3: Why does a hydrogen electron not interact with the proton?



Nuclear Physics

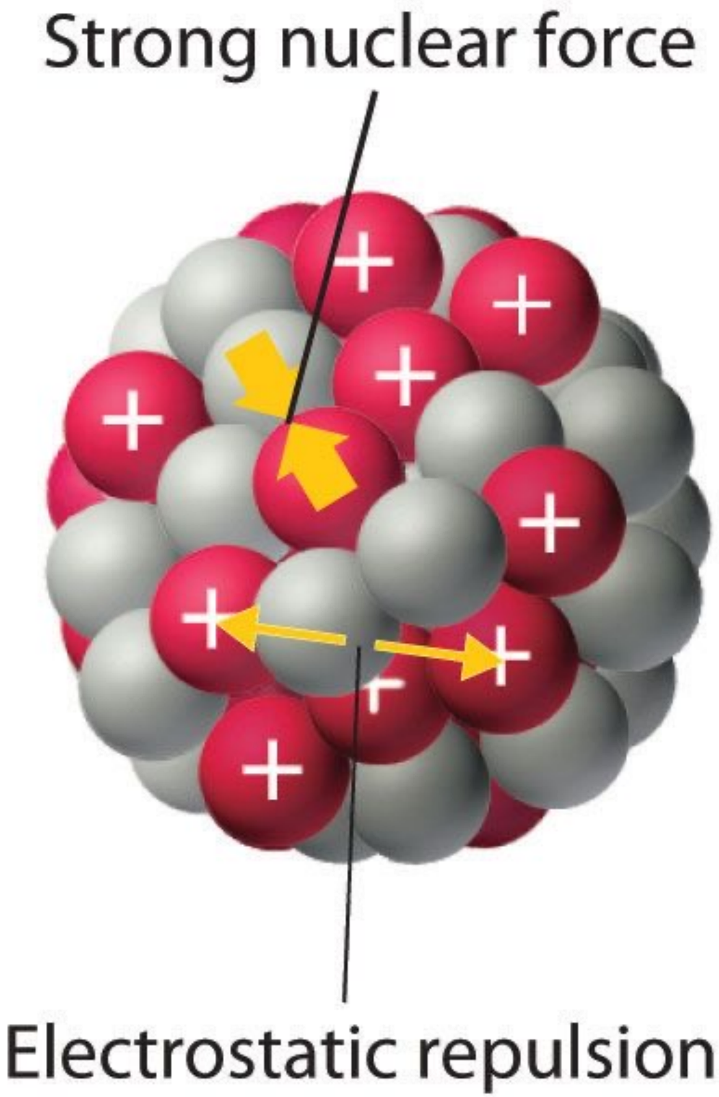
- Nuclei include protons, p
  - Masses of nuclei studied in chemistry (Avogadro)
    - Hydrogen = 1 proton: $M_H = 1\, GeV/c^2 = 200\, M_e$
- Study q/m of nuclei:
  - Masses of nuclei do not scale with charge, but with $\boldsymbol{A} \approx 2\times\boldsymbol{Z}$
    - $\boldsymbol{A}$= mass number, $\boldsymbol{Z}$=atomic number = nucleus charge
  - What keeps heavy nuclei together?
- Chadwick 1932: Discovery of neutron, n
  - Neutron has almost same mass as proton
  - Neutrons act as a glue to nucleons in the nucleus
    - Later we will see it is due to the gluon color force
- Notation: ${}^{A}_{Z}\text{MyAtom}$, with $Z = \#p$, $A = \#(p+n)$
  - Isotopes: same $\#p$, different $\#n$





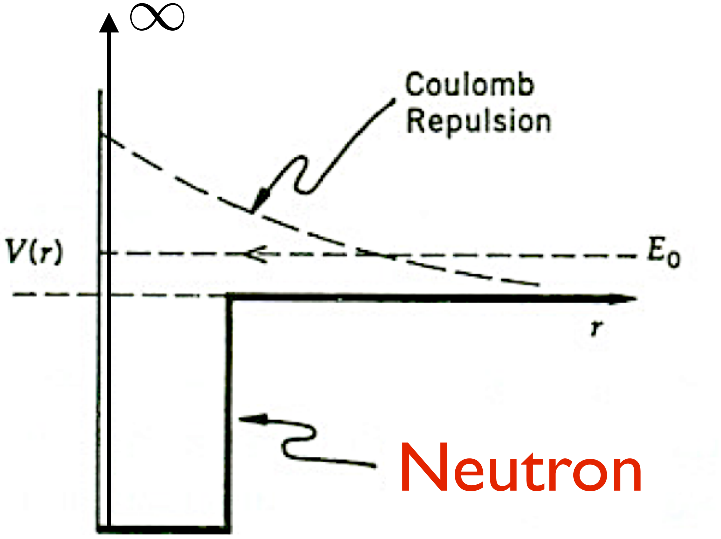
Electric Force vs Nuclear Force

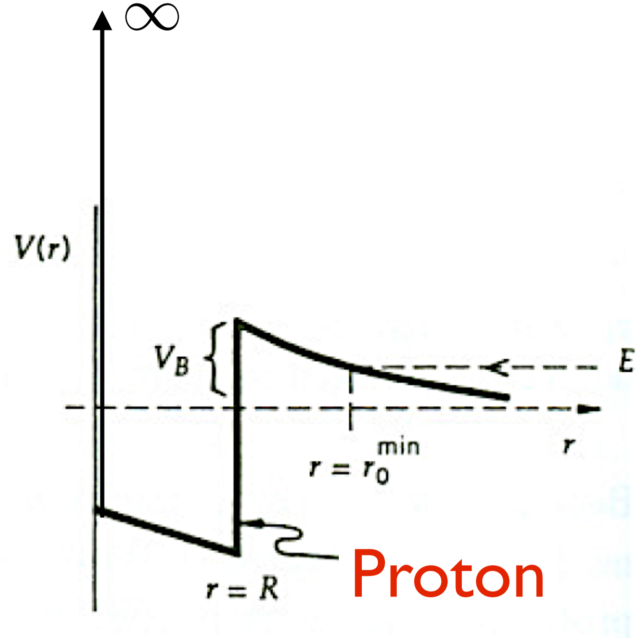
- Electric Force: inverse square law
  - Generated by exchange of massless photon quanta
- Nuclear Force
  - must be (much) stronger than electric force in the nucleus
  - must be (much) weaker than electric force outside the nucleus



Electric Force vs Nuclear Force

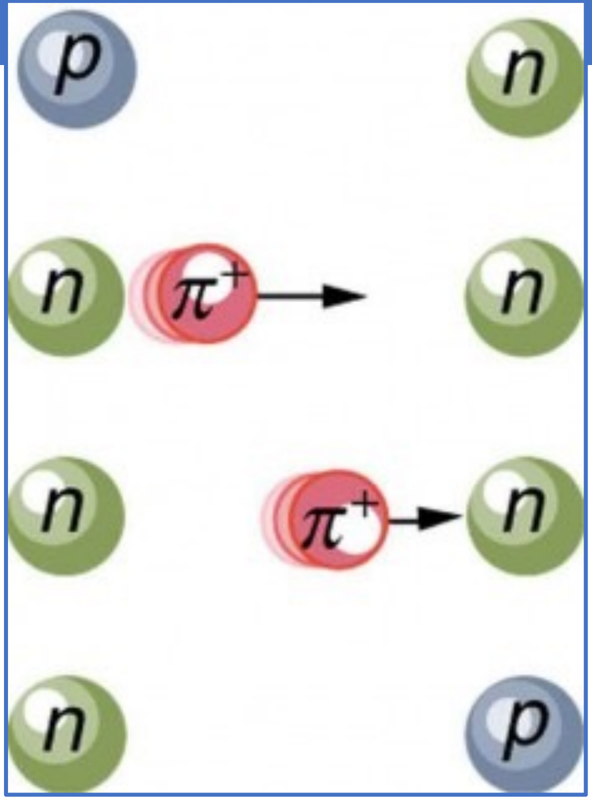
- Electric Force: inverse square law
  - Generated by exchange of massless photon quanta
- Nuclear Force
  - must be (much) stronger than electric force in the nucleus
  - must be (much) weaker than electric force outside the nucleus
- Pauli exclusion principle: Two fermions (protons, neutrons) cannot be at the same position
  - The potential must have a very strong repulsive core at small r
- Potential square well model
  - For protons there is a “Coulomb barrier” V_B
  - Beyond the range R it is negligible
  - With Rutherford scattering the range can be determined
    - $R \propto A^{1/3}$
- Nuclear force generated my exchange of massive pion





Yukawa's pion

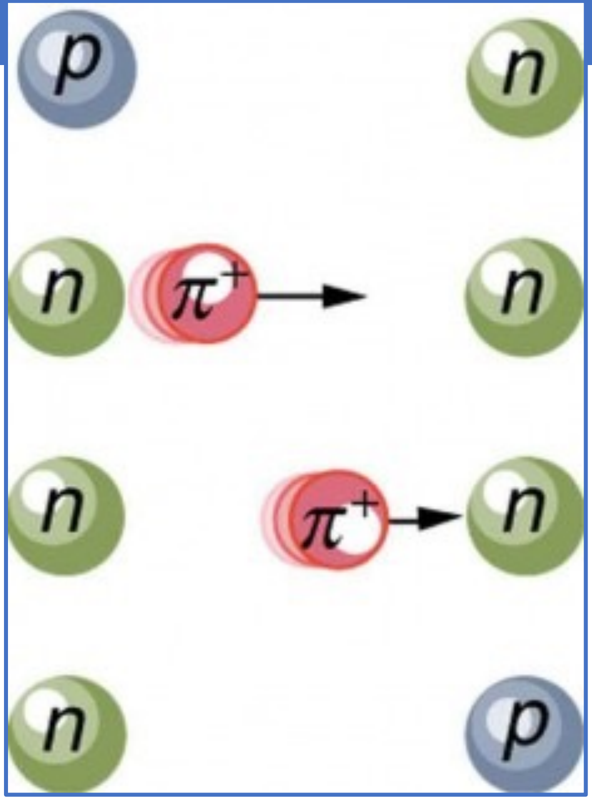
Griffiths: problem 1.2, PP1 §i.2



- Yukawa (1934) pi-meson gives attractive force between nucleons
- Heisenberg operator uncertainty in QM: $\Delta E \Delta t \geq \frac{\hbar}{2}$
- Translated to popular language:
  - You can violate energy conservation as long as:
    - time is shorter than: $\Delta t \leq \frac{\hbar}{2\Delta E}$ or energy is less than $\Delta E \leq \frac{\hbar}{2\Delta t}$

Yukawa’s pion

Griffiths: problem 1.2, PP1 §i.2



- Yukawa (1934) pi-meson gives attractive force between nucleons
- Heisenberg operator uncertainty in QM: $\Delta E \Delta t \geq \frac{\hbar}{2}$
- Translated to popular language:
  - You can violate energy conservation as long as:
    - time is shorter than: $\Delta t \leq \frac{\hbar}{2\Delta E}$ or energy is less than $\Delta E \leq \frac{\hbar}{2\Delta t}$
- Electromagnetic force is transmitted by photon quanta
  - $E = hf = \frac{hc}{\lambda}$ such that: $\Delta t \sim \frac{\lambda}{c}$ and the range $R \sim c\Delta t = \lambda$ is infinite
- Nuclear (“strong”) force is transmitted by massive pions:
  - $E = mc^2$ such that $\Delta t \sim \frac{h}{2mc^2}$ and $R \sim c\Delta t = \frac{h}{2mc}$
    - The heavier the particle, the more difficult to “violate energy conservation”, the shorter the range

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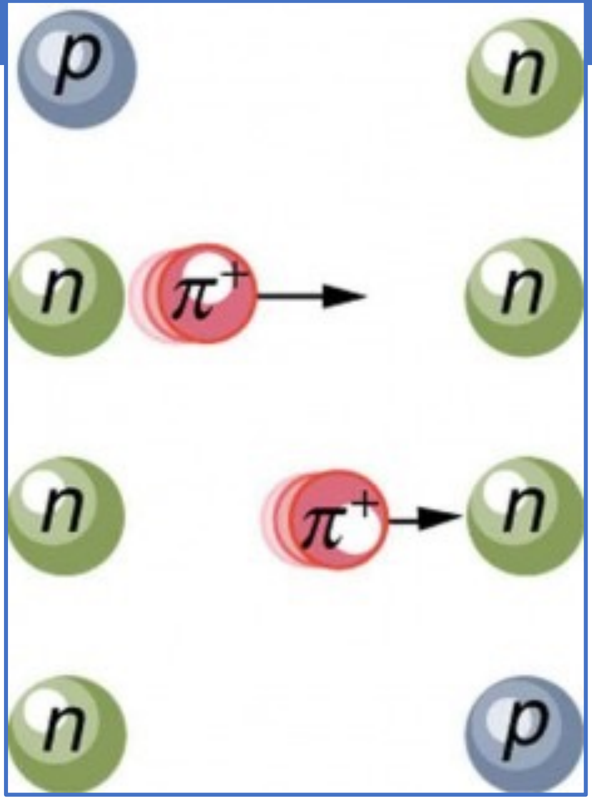
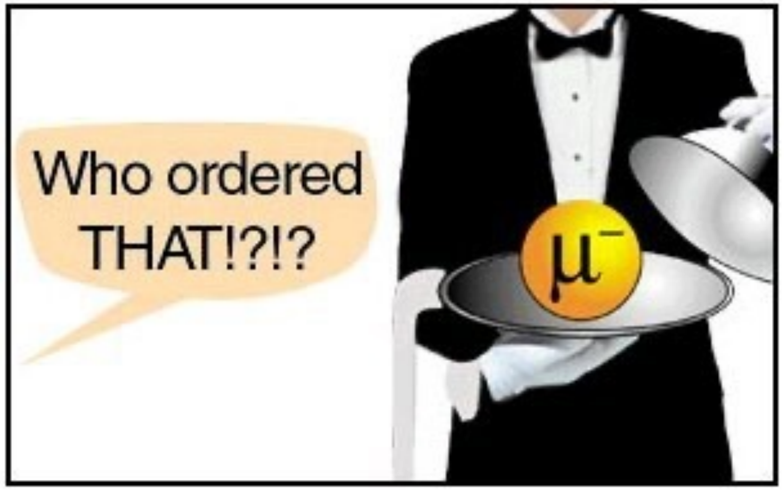
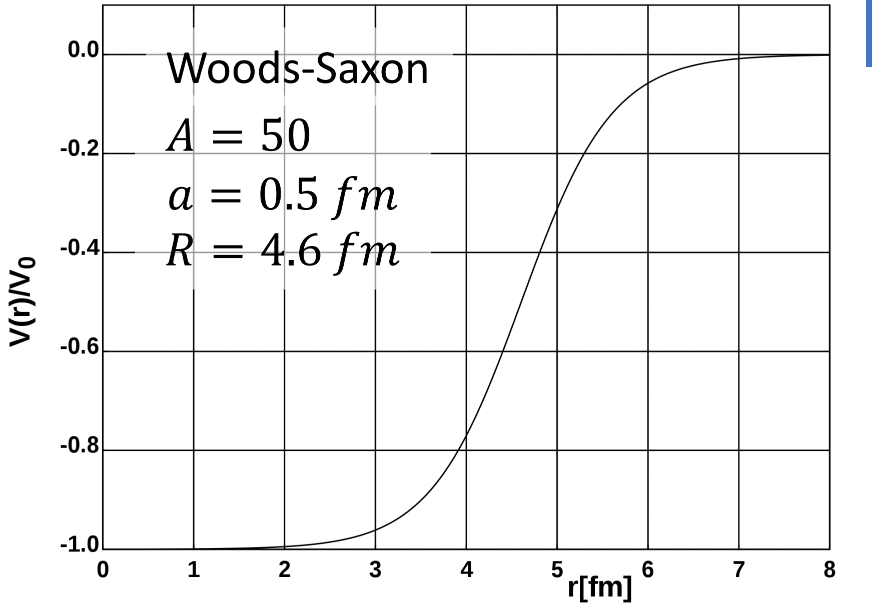
    - The heavier the particle, the more difficult to "violate energy conservation", the shorter the range
- In 1937 a particle with the correct mass was found in cosmic rays, but it did not interact with nuclei!
  - "Who ordered that?" (Isidor Rabi). It was the muon.
- In 1947 the pion was discovered



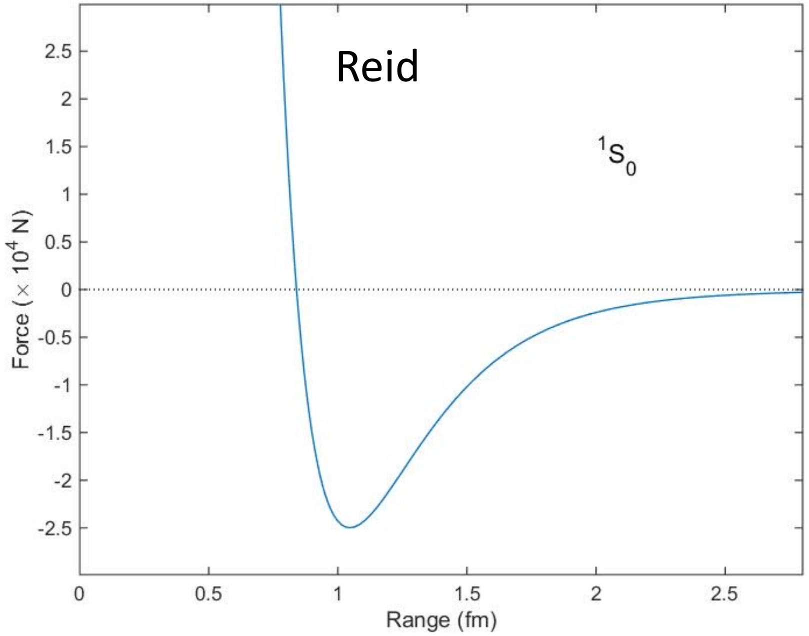


The nucleon-nucleon potential

- Woods Saxon Potential, 1954
  - $V(r) = -\frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)}$
  - Nuclear Radius: $R = 1.25 \text{ fm} \times A^{1/3}$
    A is mass number, $V_0 \approx 50 \text{ MeV}$, $a \approx 0.5\ fm$



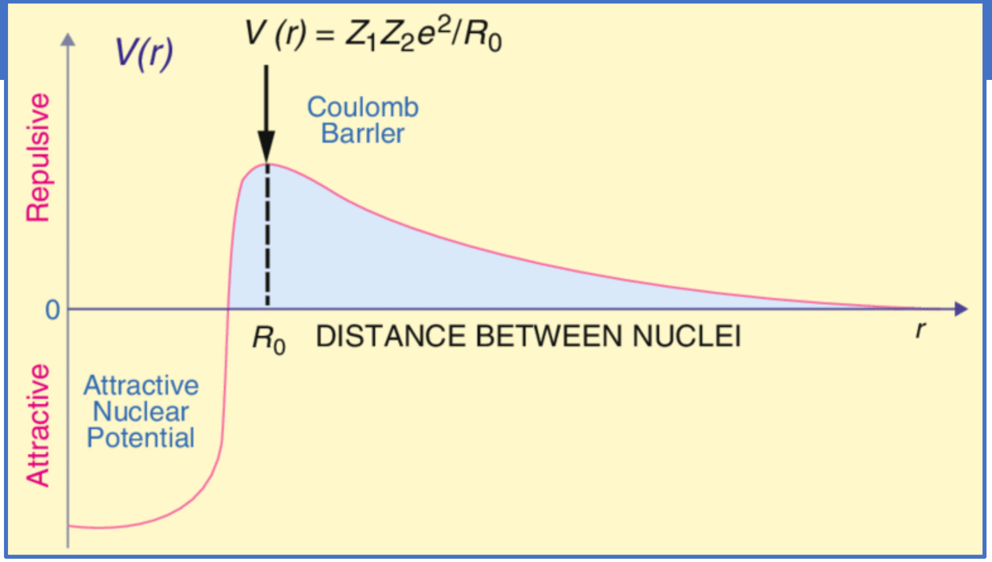
- Many models in 1970 – 1990
  - Reid Potential, 1968, (semi-empirical) implements repulsive core:
    - $V_{Reid}(R) = -10.463\frac{e^{-\mu r}}{\mu r} - 1650.6\frac{e^{-4\mu r}}{\mu r} + 6482.2\frac{e^{-7\mu r}}{\mu r}$
      in MeV with $\mu = 0.7\ fm^{-1}$
  - Famous nucleon-nucleon potential is the Nijmegen potential:
    - De Swart, Rijken et al., 1980 - 1990
    - https://arxiv.org/pdf/nucl-th/9509024.pdf

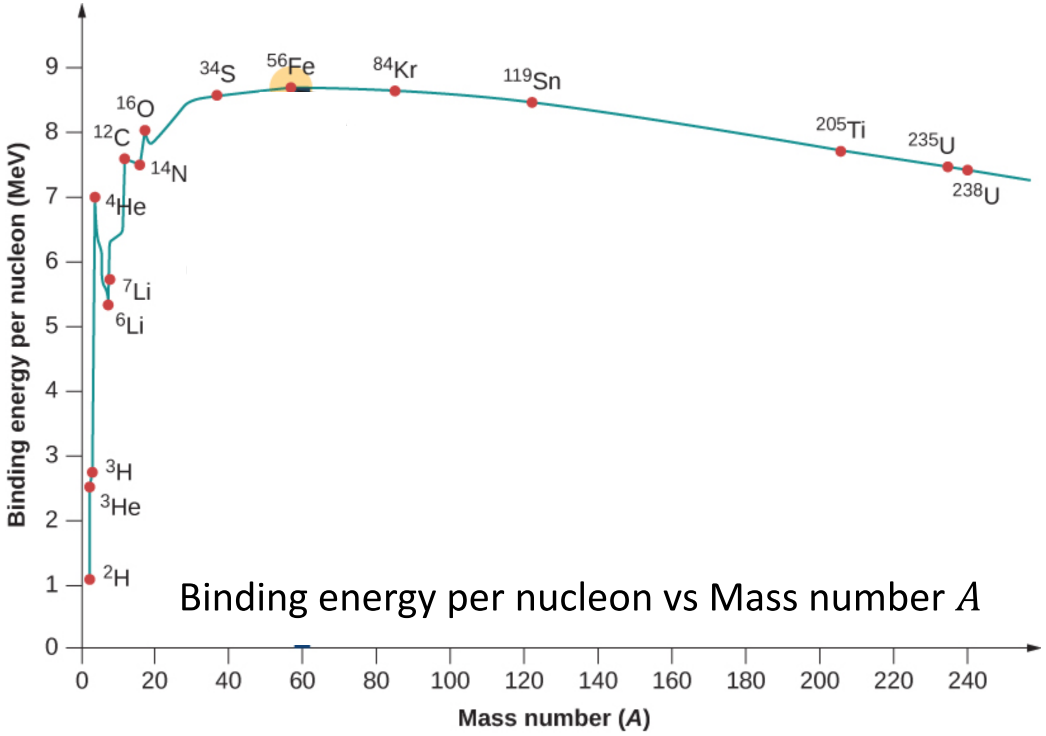


Binding Energy

- The energy of the state inside the potential well is lower than the energy of free particles
- If you “build” a nucleus from free particles energy becomes available: “binding energy”
- This appears by a reduction of mass: the nucleus is lighter than the sum of its components.
- Binding energy: $E = \Delta m \, c^2$
- $\Delta m = m_{nucleus} - (N_p M_p + N_n M_n)$

  $= m_{nucleus} - (Z M_p + (A - Z) M_n) < 0$
  - This holds for all bound systems
    - Hydrogen: $m_p + m_e = m_H + 13.6 \, eV/c^2$
    - Deuterium: $m_p + m_n = M_D + 1.7 \ MeV/c^2$
- ${}^{56}Fe$ has the highest binding energy



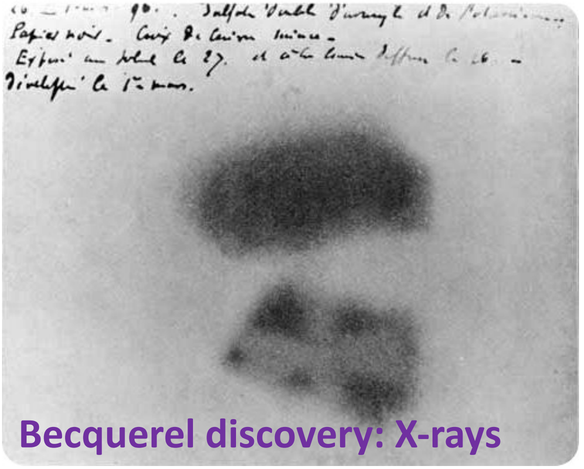


Binding energy per nucleon vs Mass number A

Nuclear decay (Radioactivity)

- Radioactivity (1896)

  Henri Becquerel, Pierre and Marie Curie:
  - Unstable nucleus
  - Quantum tunneling process



Becquerel discovery: X-rays

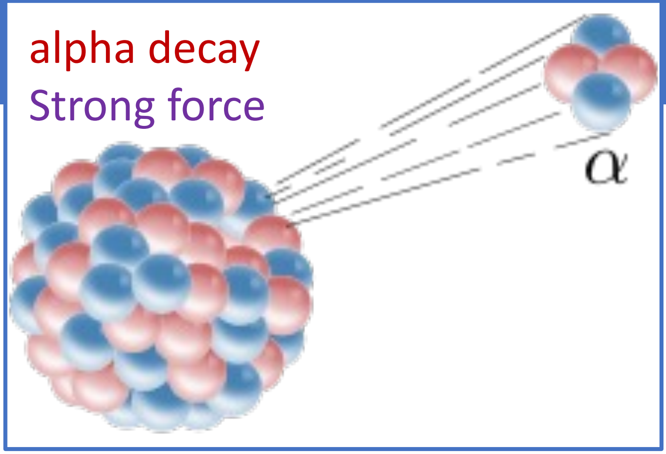
- Rutherford classification into: alpha, beta, gamma decays
  - alpha and beta decay changes chemical element (Z)
- In nature things decay exponentially:
  - Equal probability per unit time:

$$-dN = \lambda \mathrm{N} dt$$

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}$$

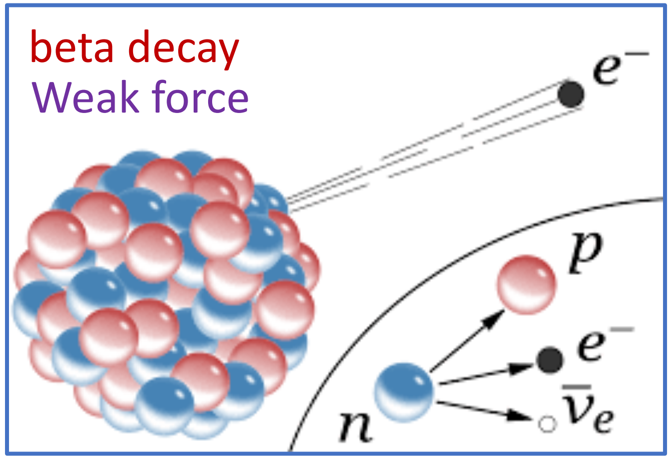
  - Halflife:

$$N_{1/2} = N_0/2 \;\rightarrow\; e^{-t_{1/2}/\tau} = \frac{1}{2} \;\rightarrow\; -t_{1/2}/\tau = \ln\frac{1}{2} \;\rightarrow\; t_{1/2} = \tau \ln 2$$



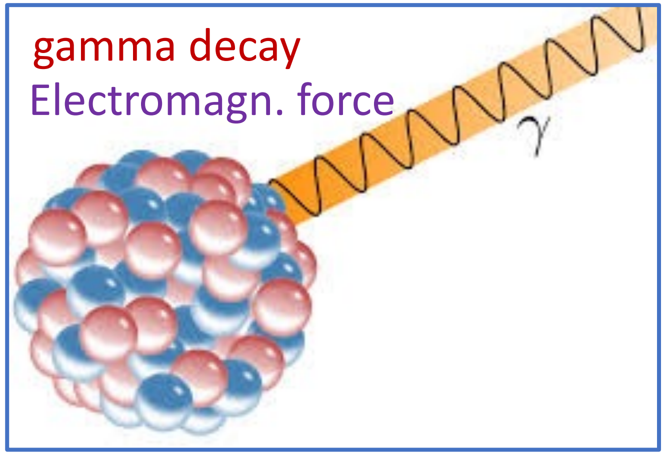
alpha decay

Strong force



beta decay

Weak force



gamma decay

Electromagn. force

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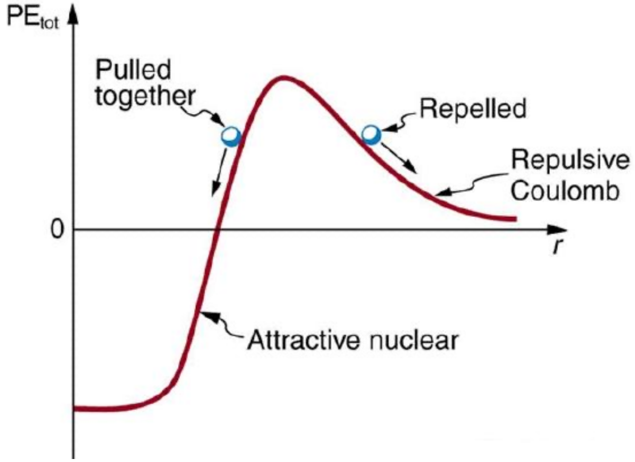

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$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}$$

  - Halflife $t_{1/2}$ vs decay time τ:

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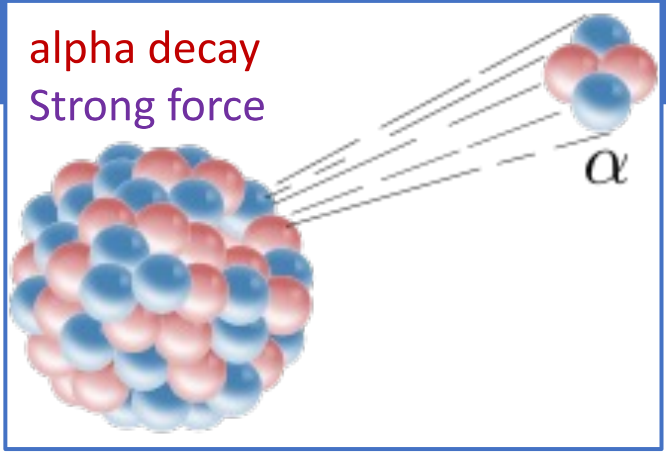
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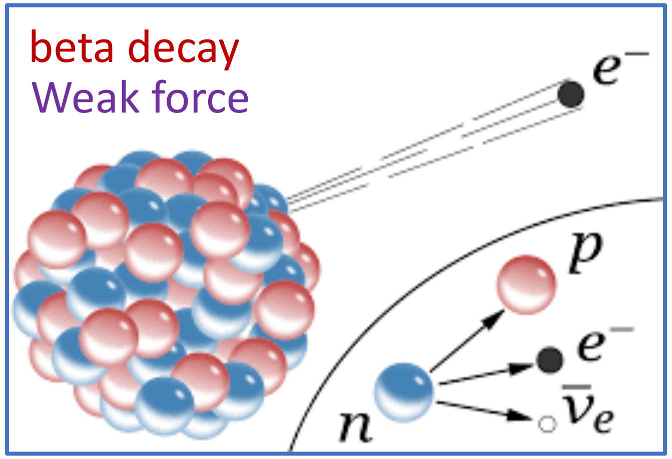
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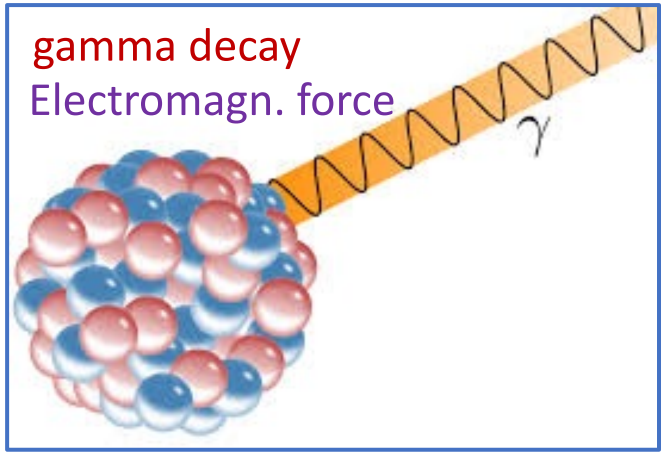
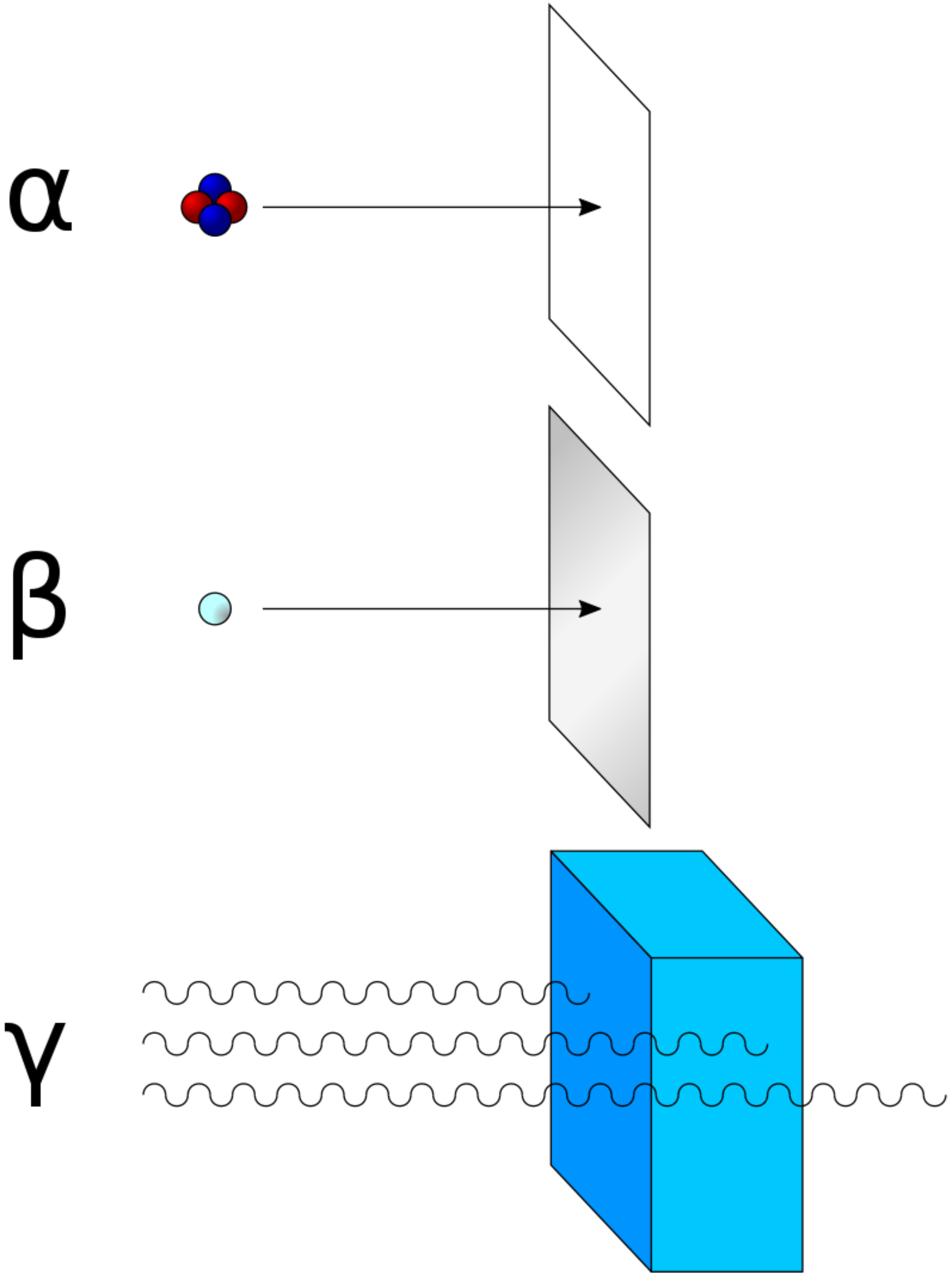
Weak force

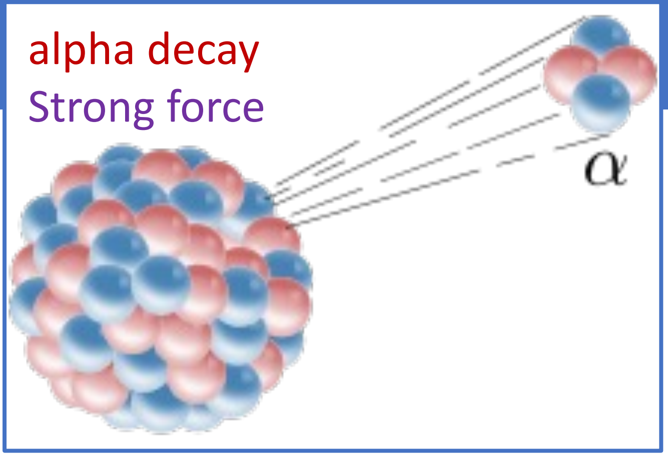

gamma decay

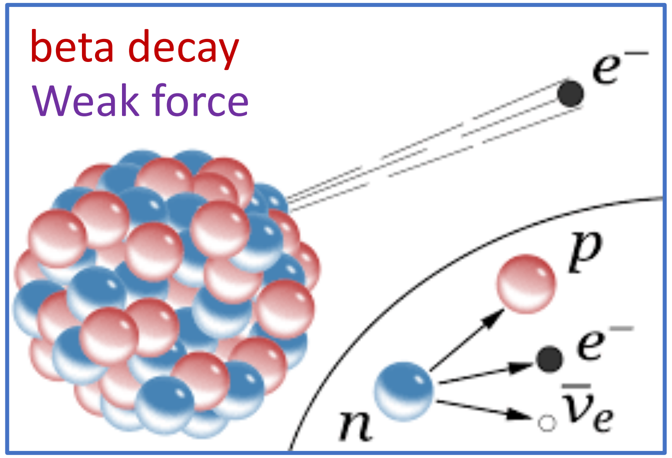
Electromagn. force

Penetrating power

- Alpha particles stopped by piece of paper
- Beta particles stopped by sheet of aluminium
- Gamma particles stopped by layer of lead




alpha decay
Strong force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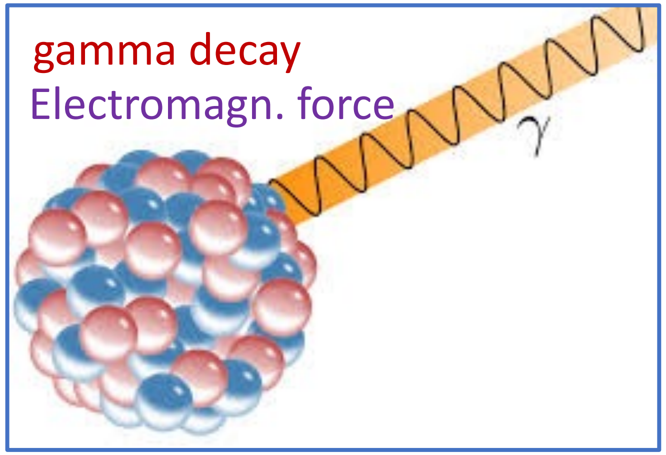
gamma decay
Electromagn. force

Nuclear Decays

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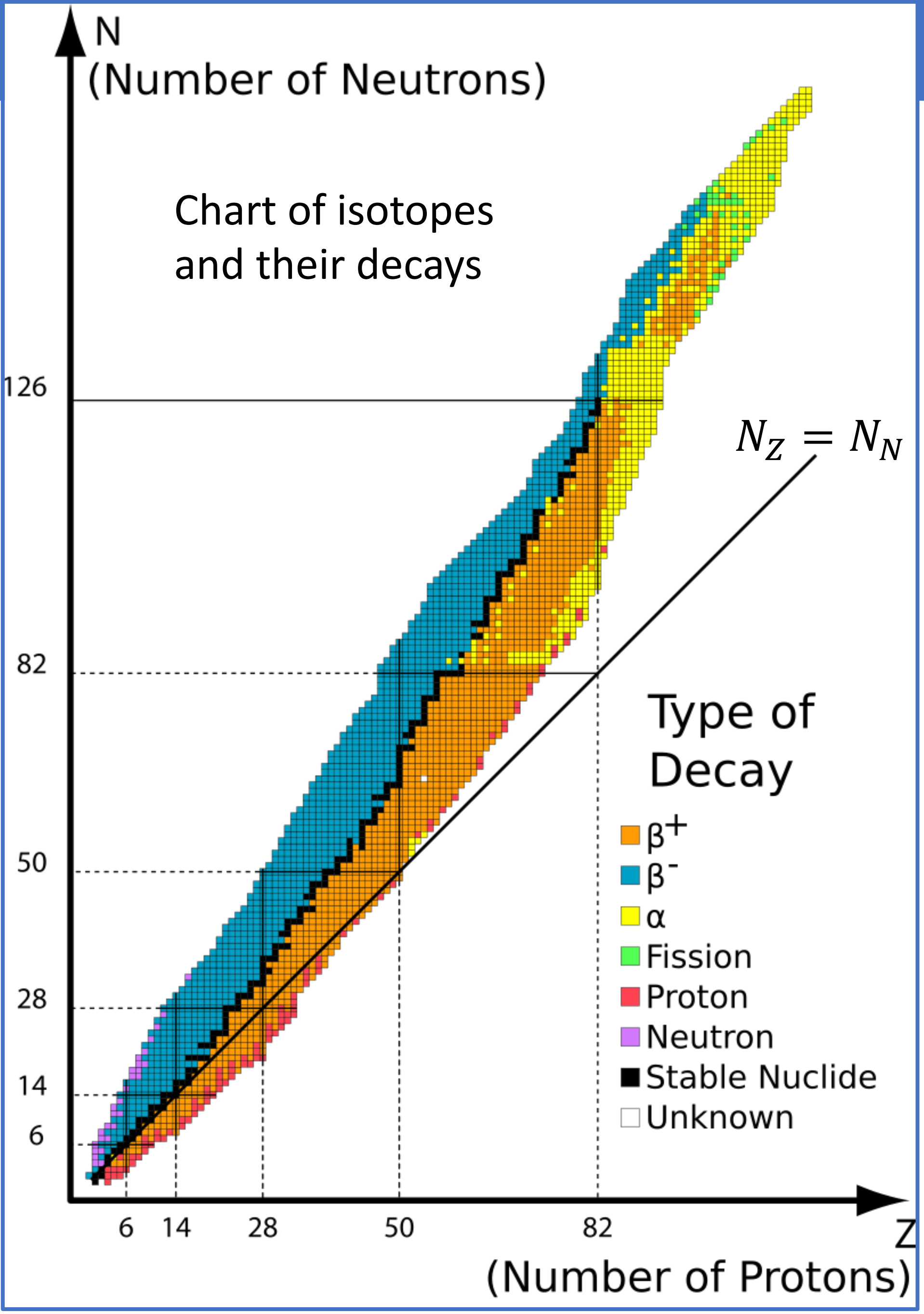

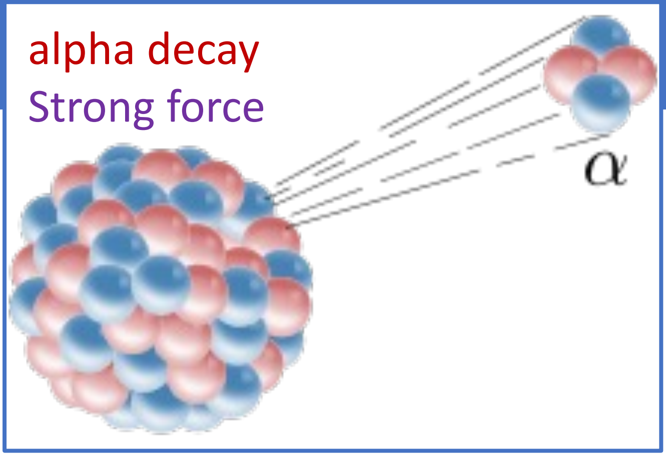
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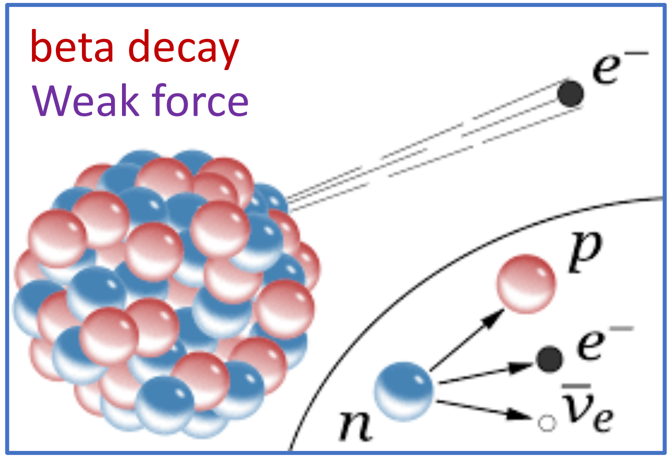
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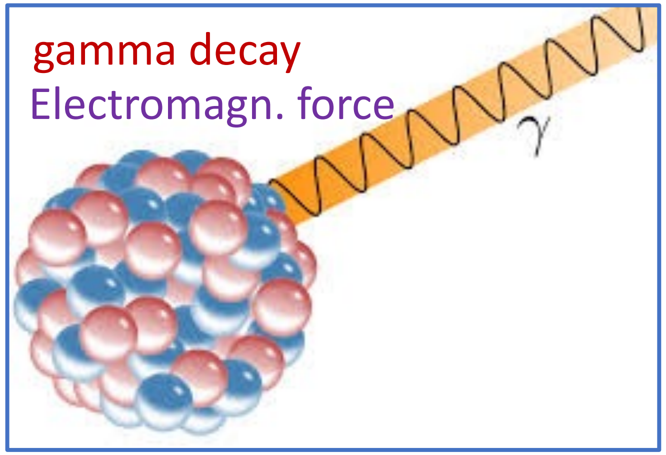

gamma decay
Electromagn. force

Radioactive Elements

- Half-lives vary enormously

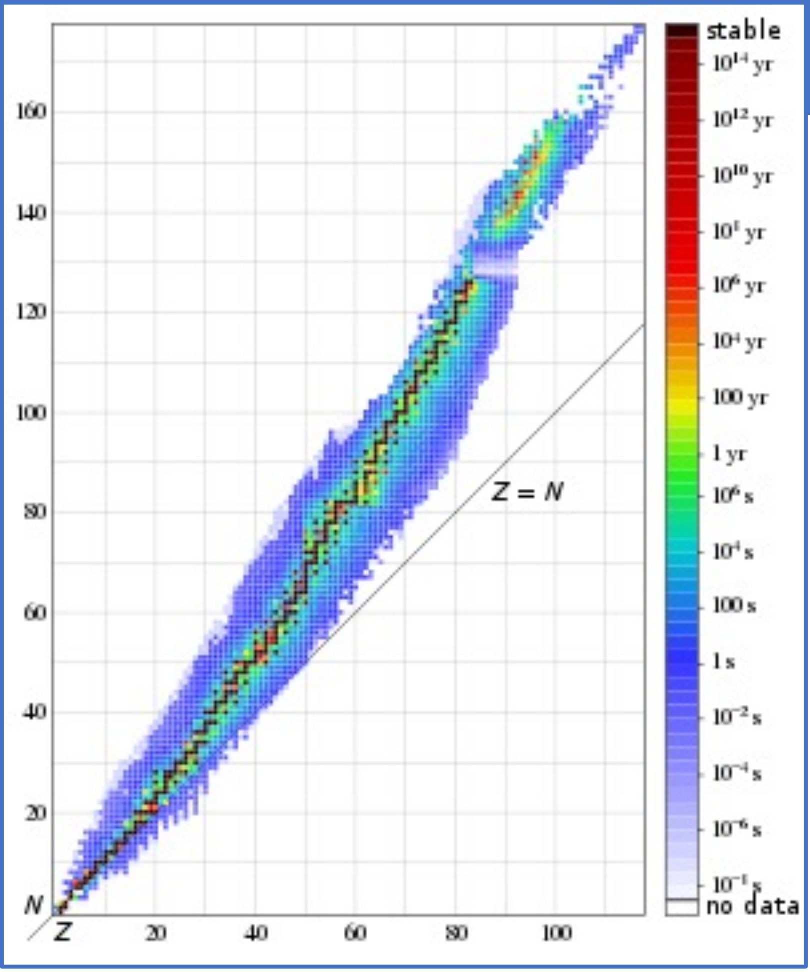
🟦 At least one stable element exists

🟩 Slightly radioactive $\tau_{1/2}$ ~ millions of years

🟨 Radioactive: $\tau_{1/2}$ ~ thousands of years

🟥 Highly radioactive: $\tau_{1/2}$ ~ minutes to weeks

🟪 Extremely radioactive: $\tau_{1/2}$ ~seconds



Period	I	II																									III	IV	V	VI	VII	VIII
1	🟦 1 H																															🟦 2 He
2	🟦 3 Li	🟦 4 Be																									🟦 5 B	🟦 6 C	🟦 7 N	🟦 8 O	🟦 9 F	🟦 10 Ne
3	🟦 11 Na	🟦 12 Mg																									🟦 13 Al	🟦 14 Si	🟦 15 P	🟦 16 S	🟦 17 Cl	🟦 18 Ar
4	🟦 19 K	🟦 20 Ca															🟦 21 Sc	🟦 22 Ti	🟦 23 V	🟦 24 Cr	🟦 25 Mn	🟦 26 Fe	🟦 27 Co	🟦 28 Ni	🟦 29 Cu	🟦 30 Zn	🟦 31 Ga	🟦 32 Ge	🟦 33 As	🟦 34 Se	🟦 35 Br	🟦 36 Kr
5	🟦 37 Rb	🟦 38 Sr															🟦 39 Y	🟦 40 Zr	🟦 41 Nb	🟦 42 Mo	🟩 43 Tc	🟦 44 Ru	🟦 45 Rh	🟦 46 Pd	🟦 47 Ag	🟦 48 Cd	🟦 49 In	🟦 50 Sn	🟦 51 Sb	🟦 52 Te	🟦 53 I	🟦 54 Xe
6	🟦 55 Cs	🟦 56 Ba	🟦 57 La	🟦 58 Ce	🟦 59 Pr	🟦 60 Nd	🟧 61 Pm	🟦 62 Sm	🟦 63 Eu	🟦 64 Gd	🟦 65 Tb	🟦 66 Dy	🟦 67 Ho	🟦 68 Er	🟦 69 Tm	🟦 70 Yb	🟦 71 Lu	🟦 72 Hf	🟦 73 Ta	🟦 74 W	🟦 75 Re	🟦 76 Os	🟦 77 Ir	🟦 78 Pt	🟦 79 Au	🟦 80 Hg	🟦 81 Tl	🟦 82 Pb	🟩 83 Bi	🟧 84 Po	🟥 85 At	🟧 86 Rn
7	🟥 87 Fr	🟨 88 Ra	🟧 89 Ac	🟩 90 Th	🟨 91 Pa	🟩 92 U	🟩 93 Np	🟩 94 Pu	🟨 95 Am	🟩 96 Cm	🟨 97 Bk	🟨 93 Cf	🟧 99 Es	🟧 100 Fm	🟧 101 Md	🟥 102 No	🟥 103 Lr	🟥 104 Rf	🟧 105 Db	🟥 106 Sg	🟥 107 Bh	🟥 108 Hs	🟪 109 Mt	🟪 110 Ds	🟪 111 Rg	🟪 112 Cn	🟪 113 Uut	🟪 114 Fl	🟪 115 Uup	🟪 116 Lv	🟪 117 Uus	🟪 118 Uuo

Units in Radiation and Tissue

Inform yourself on doses related to Chernobyl and Fukushima.

- Activity:
  - Number of disintegrations in a radioactive source is used to measure the intensity of the source
    - Main unit is Bequerel (Bq). 1 Bq is one event per second (typical kBq, MBq, GBq)
- Absorbed dose (exposure for X-rays):
  - The amount of energy coming from a source that is absorbed in a tissue. Used to assess the potential for biochemical changes in specific tissues.
    - Main unit is Gray (Gy). 1 Gy means that one kg of matter absorbed one joule of energy (typical mGy)
- Equivalent or biological dose:
  - Used to assess how much biological damage is expected from the absorbed dose. Different types of radiation have different damaging properties
    - Main unit is Sievert (Sv). 1 Sv equal 1 Gy multiplied by a weighting factor
- Effective dose:
  - Used to assess the potential for long-term effects that might occur in future. Takes into account all the organs in the body, the timespan of irradiation etc.

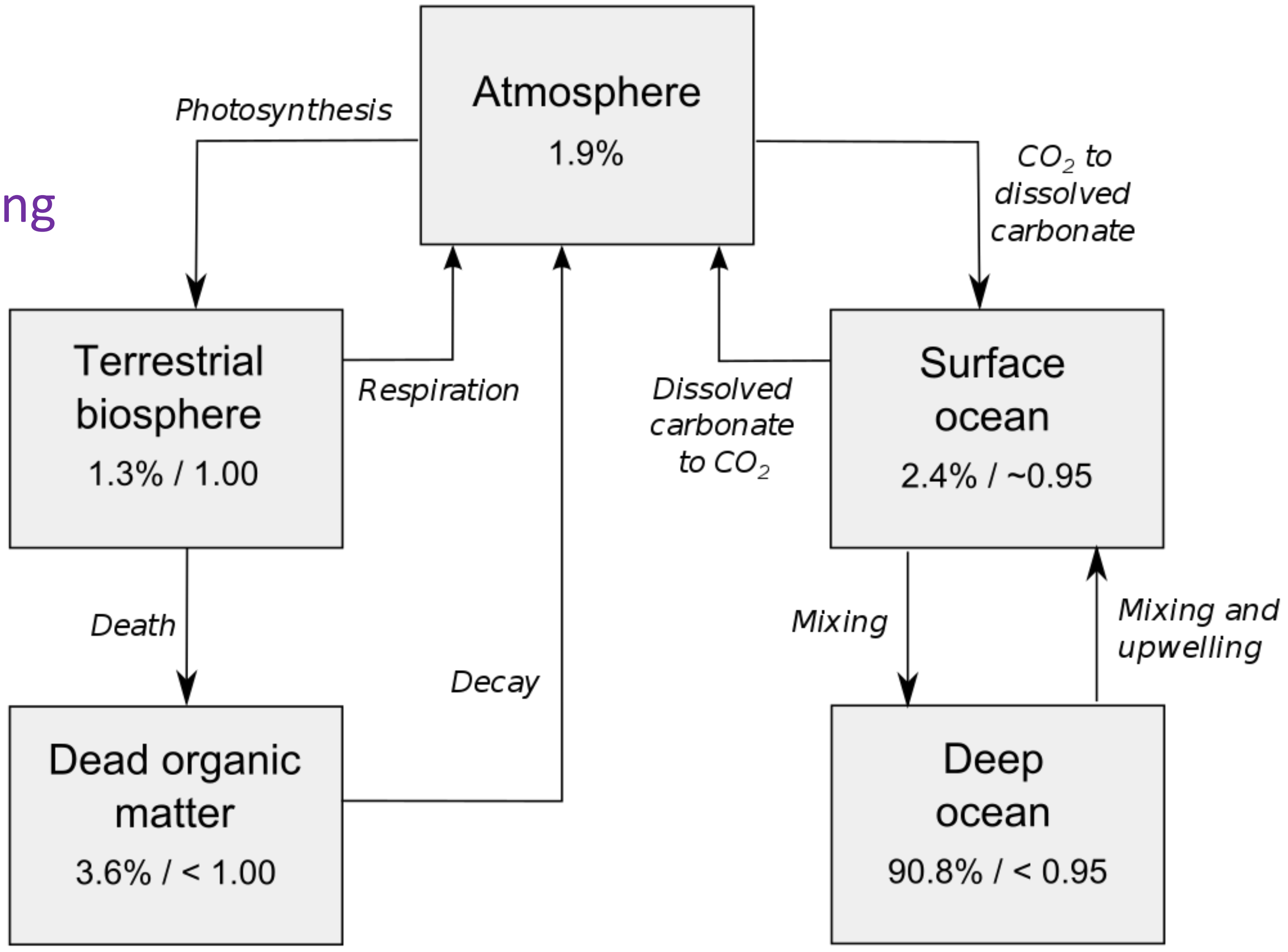
Units in Radiation and Tissue

Ionizing radiation related quantities view · talk · edit

Quantity	Unit	Symbol	Derivation	Year	SI equivalence
Absorbed dose (*D*)	gray	Gy	J·kg^{-1}	1974	SI unit
Absorbed dose (*D*)	erg per gram	erg/g	erg·g^{-1}	1950	1.0×10^{-4} Gy
Absorbed dose (*D*)	rad	rad	100 erg·g^{-1}	1953	0.010 Gy
Activity (*A*)	becquerel	Bq	s^{-1}	1974	SI unit
Activity (*A*)	curie	Ci	3.7×10^{10} s^{-1}	1953	3.7×10^{10} Bq
Activity (*A*)	rutherford	Rd	10^{6} s^{-1}	1946	1,000,000 Bq
Equivalent dose (*H*)	sievert	Sv	J·kg^{-1} × W_R	1977	SI unit
Equivalent dose (*H*)	röntgen equivalent man	rem	100 erg·g^{-1} x W_R	1971	0.010 Sv
Exposure (*X*)	coulomb per kilogram	C/kg	C·kg^{-1} of air	1974	SI unit
Exposure (*X*)	röntgen	R	esu / 0.001293 g of air	1928	2.58×10^{-4} C/kg

How does Radiocarbon dating work?

- Exercise:
  - Explain how the Carbon dating method works



Percentages show the fraction of the total carbon reservoir of each type.
Numbers after slash show ratio of ^{14}C to ^{12}C as fraction of atmospheric ratio.

A model of the nucleus: Liquid drop model (~1935)

- Nucleus ~ uniform density, like an incompressable drop of water
- Strong nuclear force – pairs of nucleons: expect $\sim A^2$, but:
  - Very short ranges: sees only nearest neighbours
    - ’Volume’ energy term $\propto A$
  - At surface less neighbours: a negative correction
    - ‘Surface tension’ term: $\propto A^{2/3}$, (since radius $\propto A^{1/3}$)
- Coulomb repulsion: Long rage, all protons see all others. $R \propto A^{1/3}$
  - ’Coulomb term’: $\propto Z^2 A^{-1/3}$ (think of sphere of charge density $\propto Q^2/R$)
- Pauli exclusion:
  - Asymmetry term, prefer to fill up low energy states first:
    - ‘Pauli term’:$\propto (N-Z)^2/A$ or $(A-2Z)^2/A$
- Spin-spin coupling (pairing term), lowers energy if even $\#p$ or even $\#n$:
  - $\propto \delta A^{-3/4}$ (empirical) where $\delta = \{-1, 0, 1\}$ if $= \{Z \text{ and } N = \text{even}\,, A = \text{odd},\ Z \text{ and } N = \text{odd}\}$
- Binding energy (semi-empirical):

$$E_B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(N-Z)^2}{A} + \delta a_5 A^{-3/4}$$

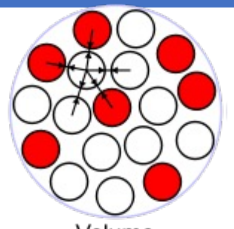
- Bethe-Weizsacker empirical mass formula:

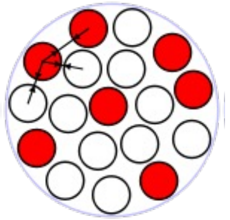
$$M(A,Z) = (A-Z)m_n + Z(m_p) - \frac{E_B}{c^2}$$

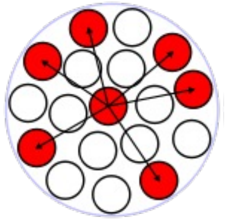
See Wikipedia:
Semi-empirical_mass_formula

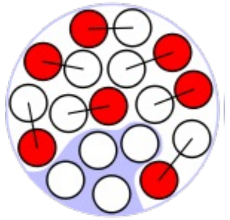
Values:

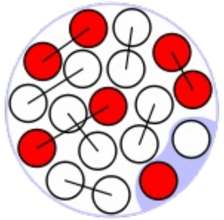
$a_1 \approx 15.6\text{ MeV}$, $a_2 \approx 16.8\text{ MeV}$,
$a_3 \approx 0.72\text{ MeV}$, $a_4 \approx 23.3\text{ MeV}$
$a_5 \approx 34\text{ MeV}$


Volume


Surface


Coulomb


Asymmetry


Pairing

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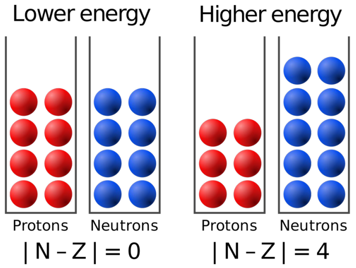
See Wikipedia:
Semi-empirical_mass_formula

$A = 16$



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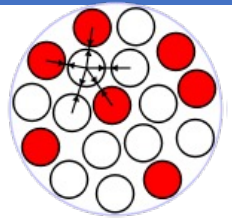
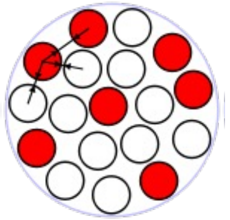
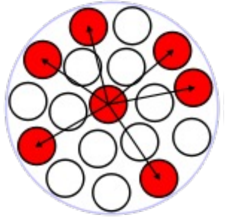
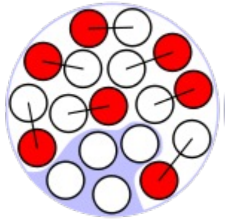
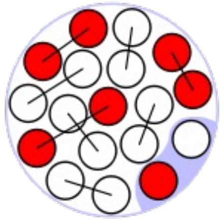

Volume

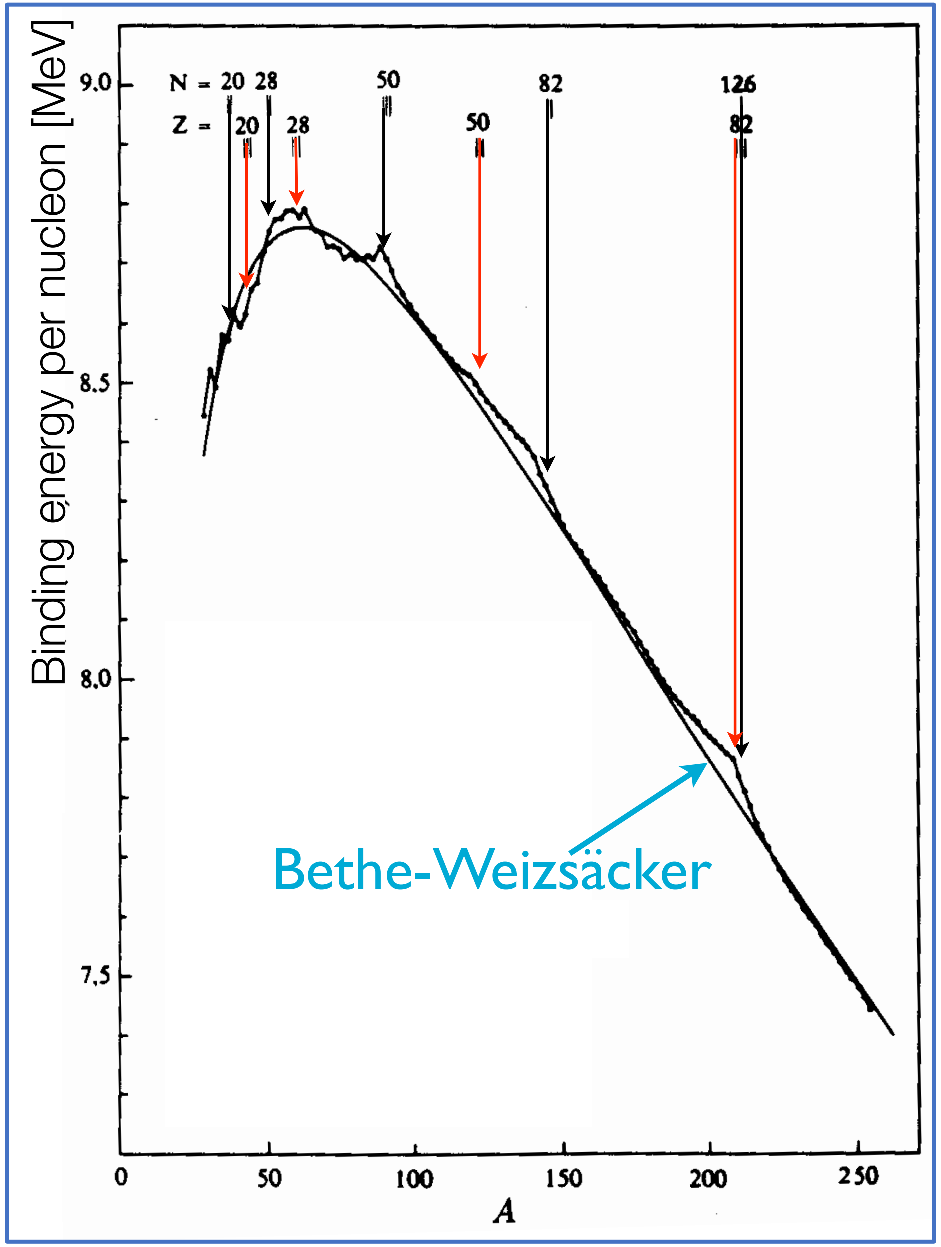

Surface

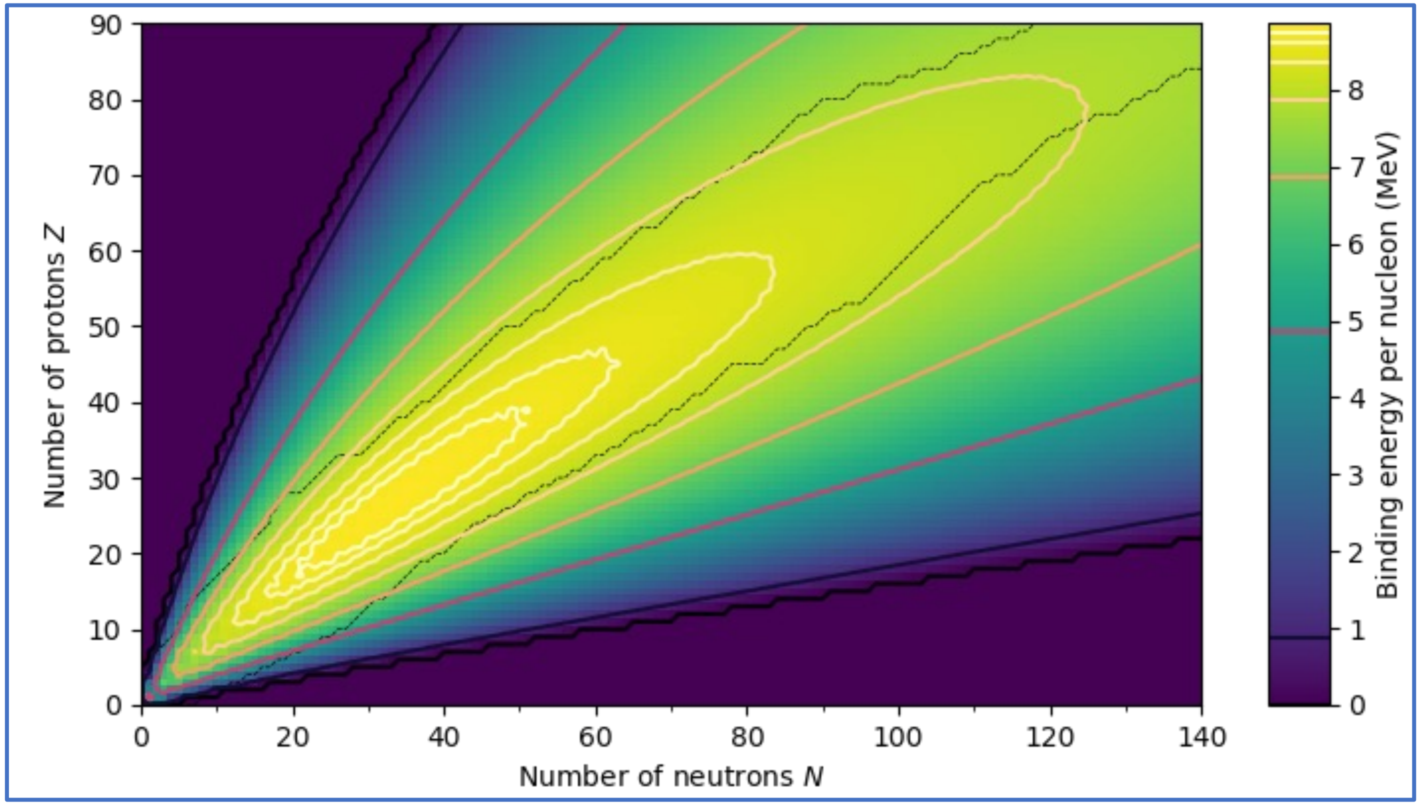

Coulomb


Asymmetry


Pairing

Binding Energy: ‘magic numbers’

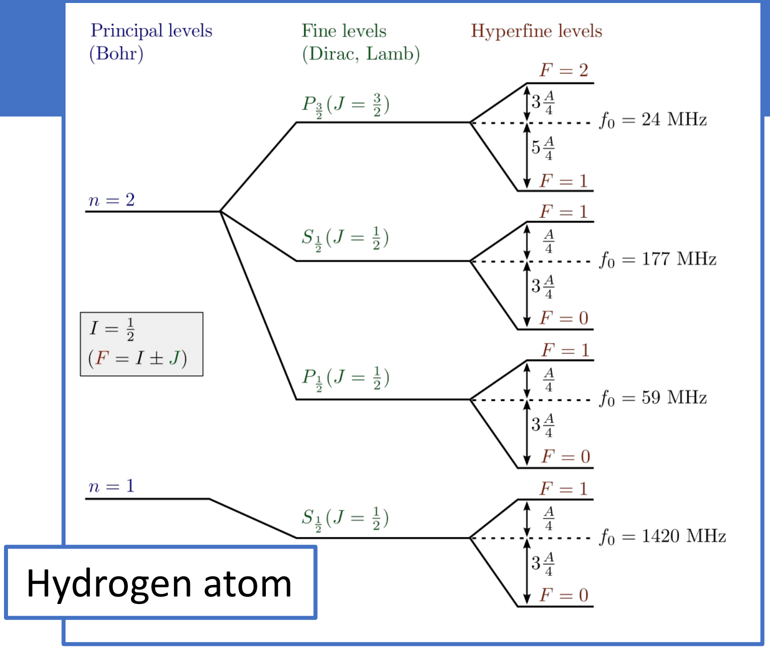




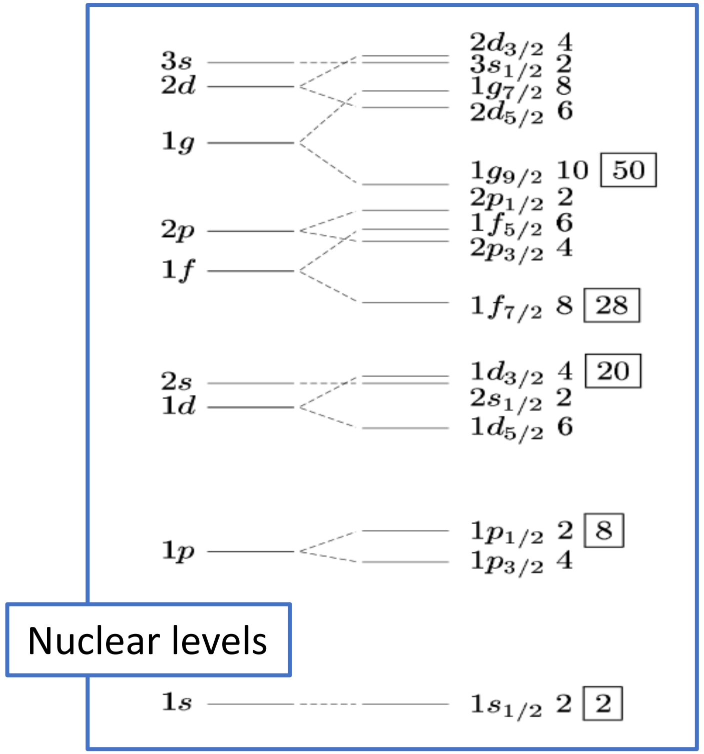
- Fits quite well
- Cannot explain ‘peaks’ that occur at ‘magic numbers’ where the nucleus is strongly bound.
  - $N \text{ or } Z = 2, 8, 20, 28,\ 50,\ 82,\ 126$
- Double magic nuclei (both N and Z are magic):
  - $^{4}\mathrm{He},\ ^{16}\mathrm{O},\ ^{40}\mathrm{Ca}\,,\ ^{48}\mathrm{Ca}\,,\ ^{48}\mathrm{Ni}\,,\ ^{208}\mathrm{Pb}$

Nuclear Shell model (1949)

- Similar to the atomic shell model (remember hydrogen atom) for electrons):
  - Atomic shells: Energy levels; n, l, s quantum numbers
  - Solution of Schrodinger equation (QED)
    - $i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$ with $V = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$
  - Now strong force: which potential V?
    - Use the nuclear potential
    - Try square well, try Harmonic oscillator, $V = \mu\omega^2 r^2/2$ Need something inbetween.
    - Woods Saxon works well: $V(r) = -\frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)}$
    - Spin orbit interaction is stronger than in atom
- Nuclear shells "magic numbers":
  - Count energy levels of shells including spin:
    - 2x1=2, 2x(1+3)=8, 2x(1+3+6)=20,…
  - Filled shell → more tightly bound →larger E_B

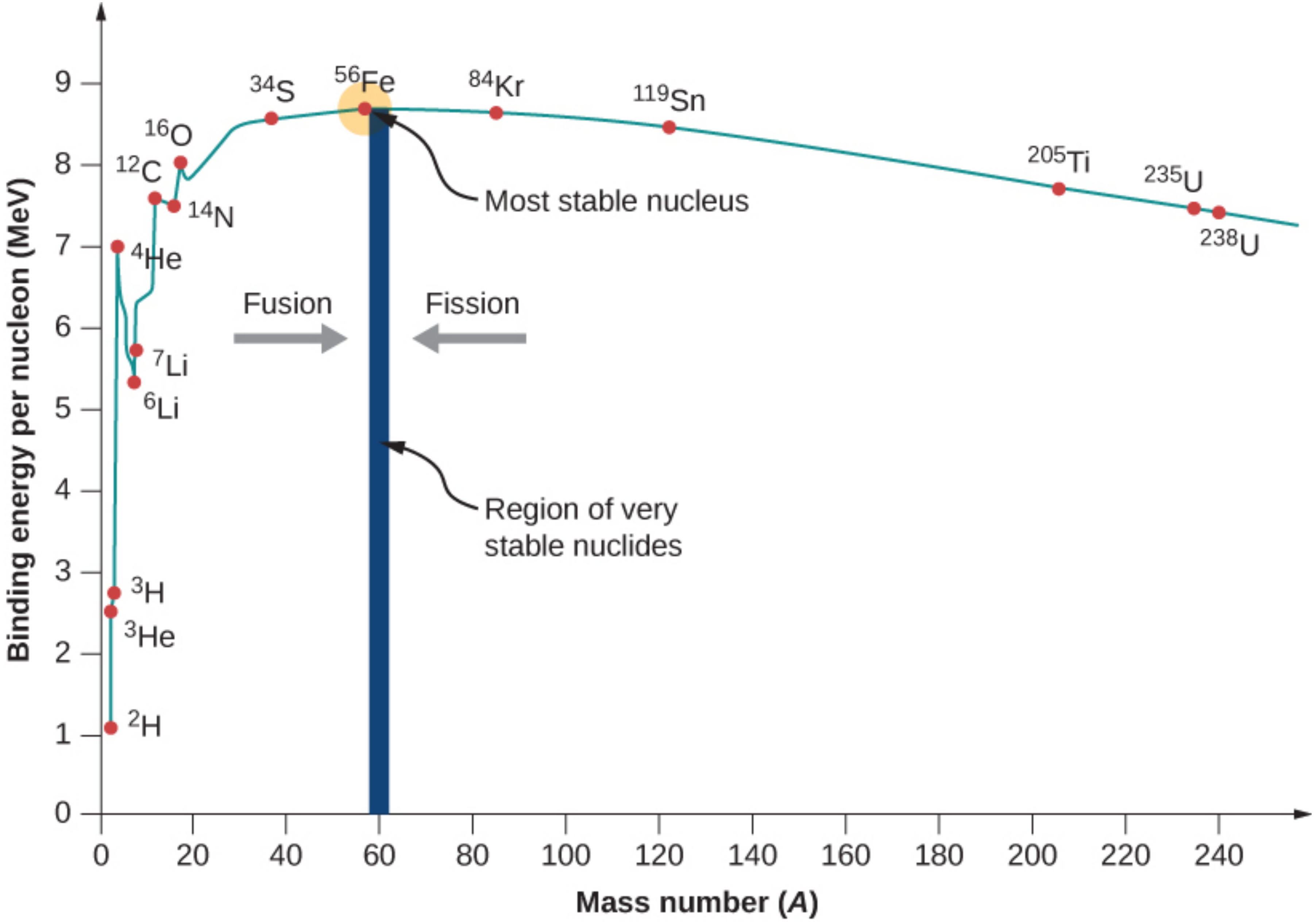


Hydrogen atom

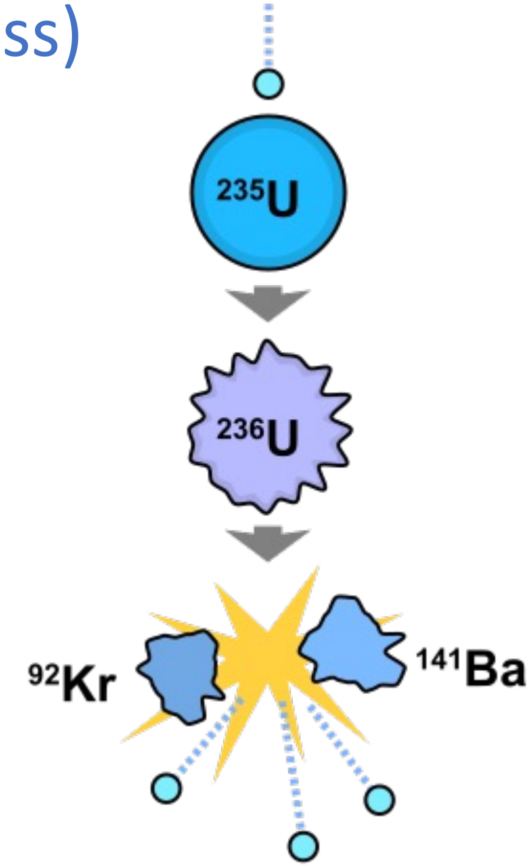


Nuclear levels

Nuclear Fission and Fusion



Nuclear Fission



- Heavy elements: less binding energy per nucleon (=more mass)
  - Transition heavy → light: free some mass → energy $(E = mc^2)$
  - Form high mass unstable isotope: eg. $^{235}_{92}U$ + 1 neutron. Reaction:

$$^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{o}n$$

$$236.045568u \rightarrow 140.914411u + 91.926156u + 3\times 1.008644u$$

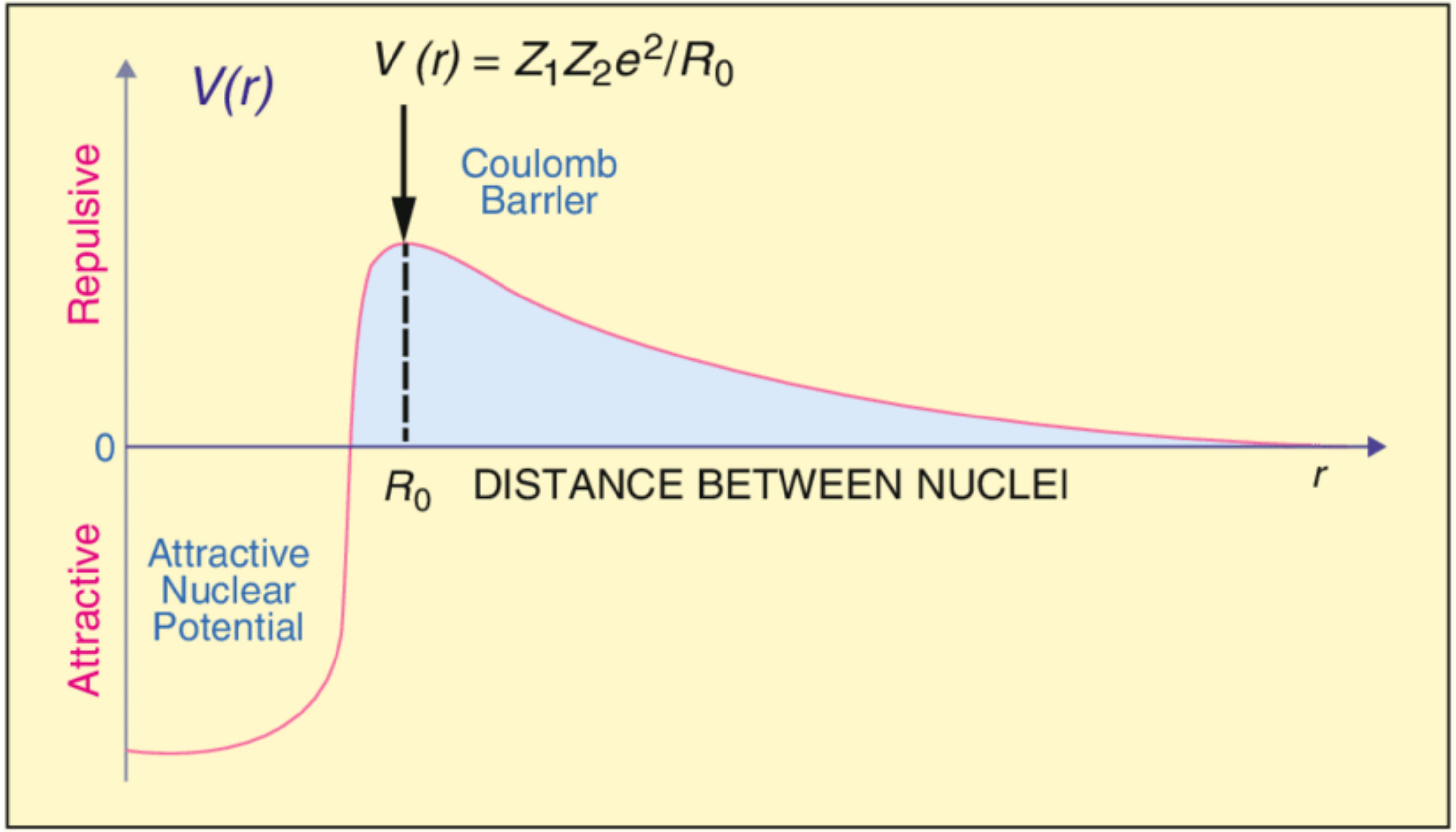
$$\Delta m = 0.18u \Rightarrow E = \Delta m\, c^2 = 2.68\times 10^{-11} J = 168\ MeV$$

  - Note: $^{235}_{92}U$ occurs as 0.72% natural Uranium
    - To use it as an energy source it must be enriched (centrifuge)

- E_B turned into kinetic energy →high temperature → steam
  - Nuclear reactors: controlled reaction, moderators for neutron absorption
  - Nuclear bombs: uncontrolled reaction

Nuclear Fusion

- Light elements fuse to free energy (Δm)
- Need to overcome Coulomb barrier in order to fuse together
  - Fuse H or D

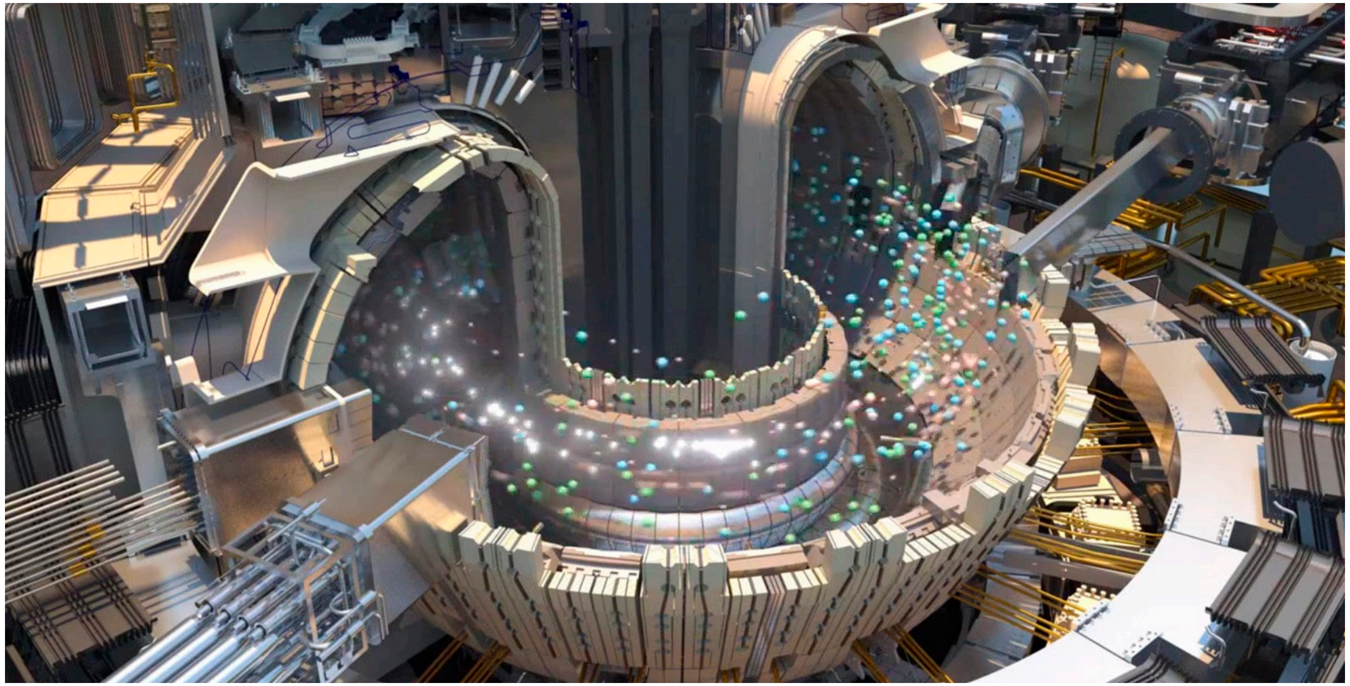


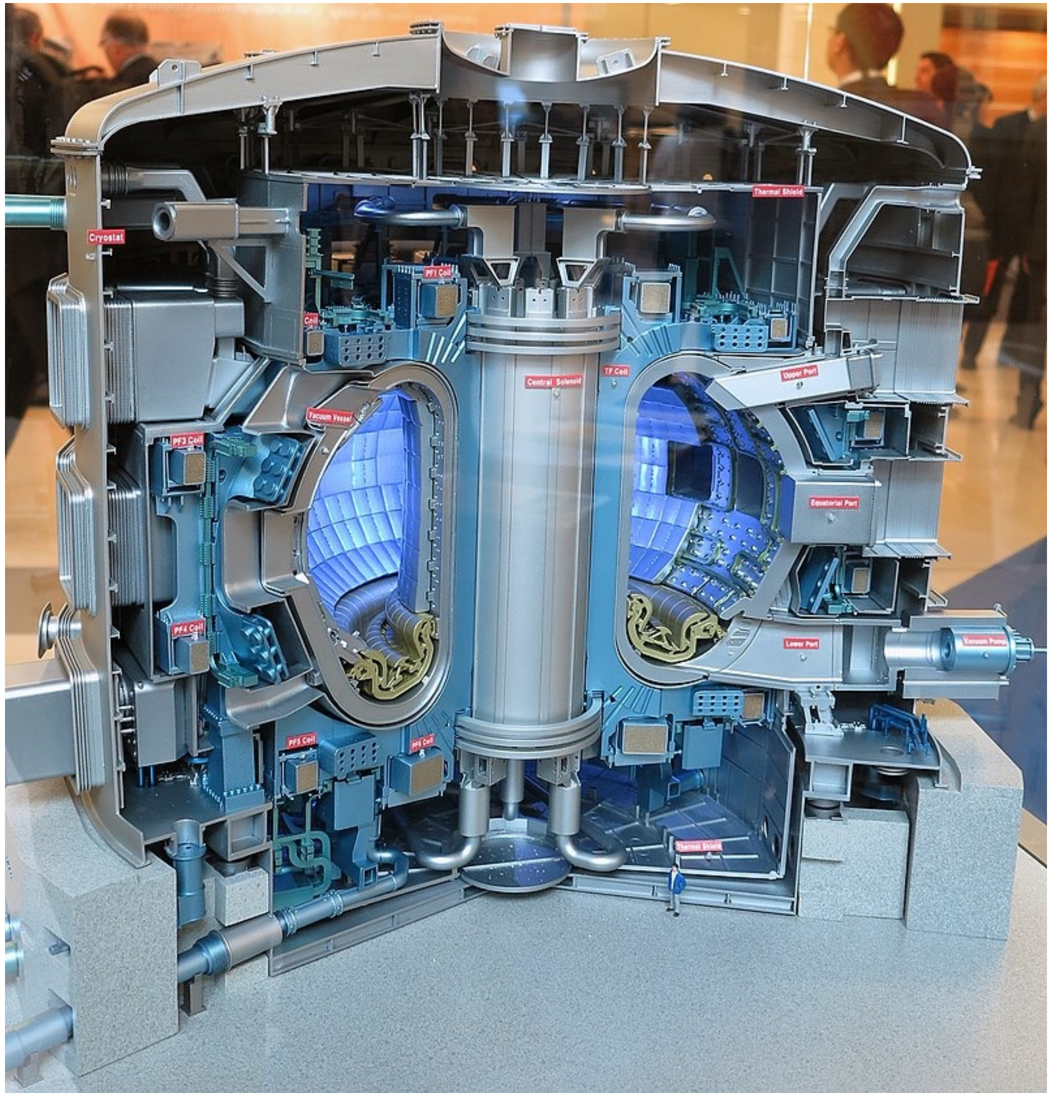
Coulomb barrier for charged-particle reactions

- Classically possible to fuse?
  - Would require extremely high kinetic energy → unrealistic temperature
  - Sun would not burn!
- Quantum mechanical tunneling helps:
  - Go through the barrier instead of over it
  - Gamow factor

Nuclear Fusion: lab

- Fusion for power: need high energies and densities
  - ITER in France
- $150 \times 10^{6}\ {}^{0}C$
- Challenges:
  - keep confined
  - Keep sustained





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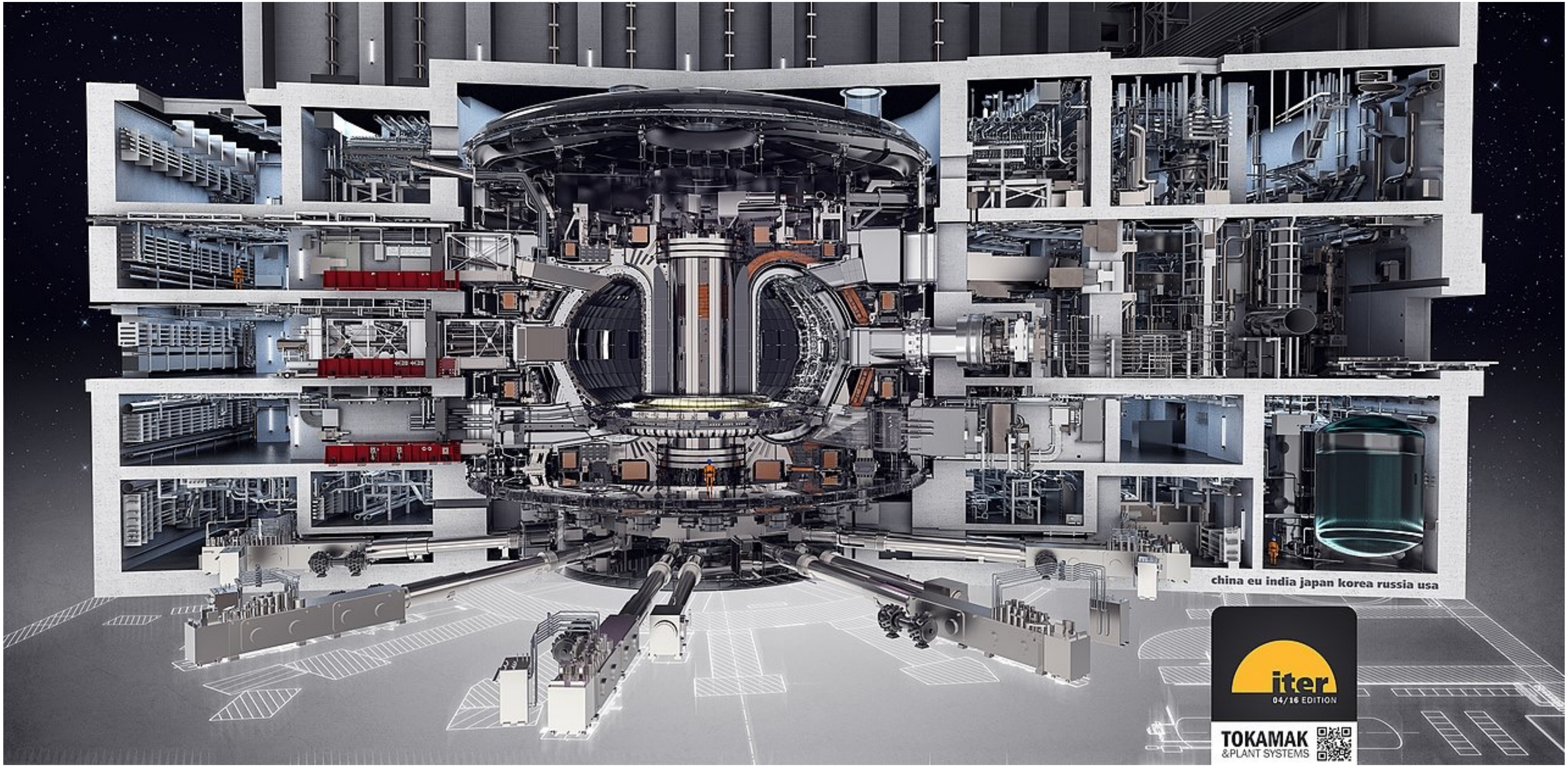
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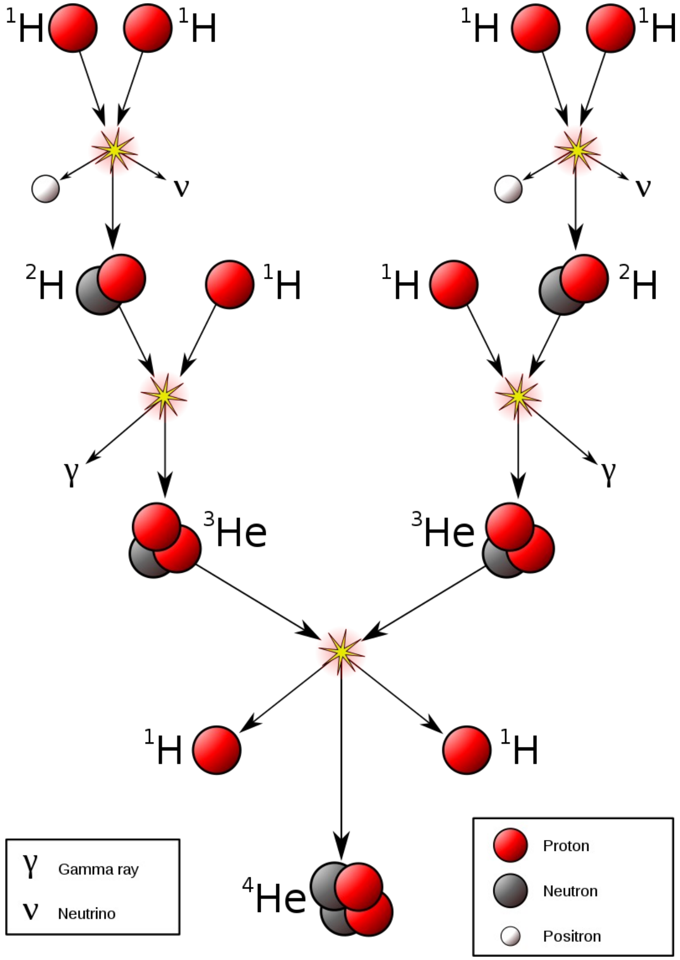
04/16 EDITION
TOKAMAK
&PLANT SYSTEMS

ITER in 2020

Nuclear Fusion: Universe

See Wikipedia: Nucleosynthesis

- Fusion in stars:
  - Proton-proton chain reaction (sun)
  - CNO catalytic cycle ($m_{star} > 1.3\ m_{sun}$)



- How are heavy elements formed in the universe?
  - Colliding neutron stars

Exercise: Fission vs Fusion

a) Calculate the energy released if 1 gram of U-235 splits into La-148 + Br-87.

b) Calculate the energy released in the fusion process of 0.5 grams of heavy water (D2O) with 0.5 grams of superheavy water (T2O), creating He-4 and a neutron. You may neglect the binding energies of the molecules.

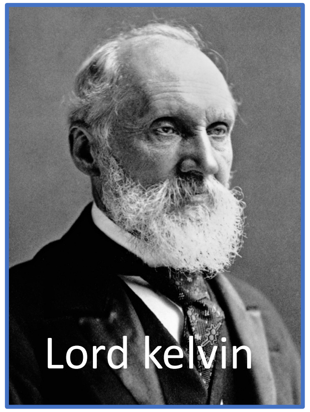
c) Compare the energies released per gram of fuel calculated above. Which would you prefer?

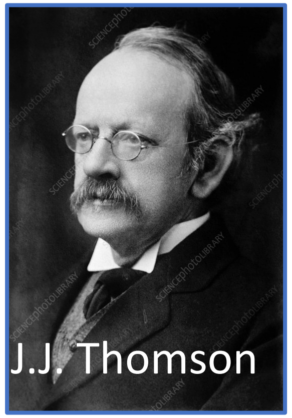
Lecture 1 : Particles

Part 2

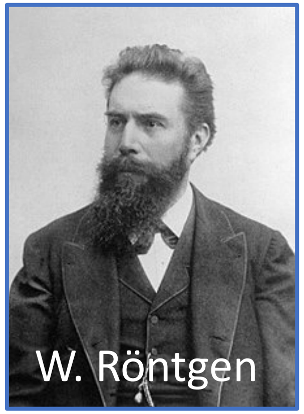
Subatomic Particles

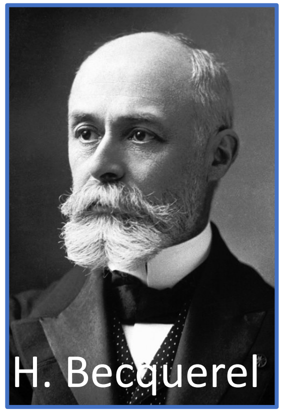
Particle Physics: Historical introduction – Griffiths chapter 1

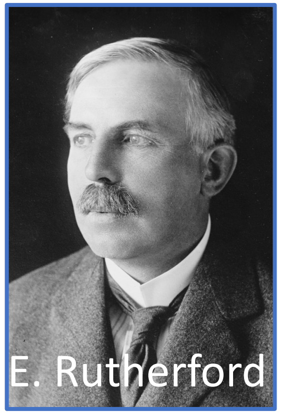

Lord kelvin


J.J. Thomson

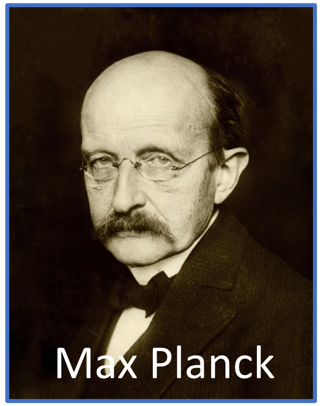

Pierre and Marie Curie

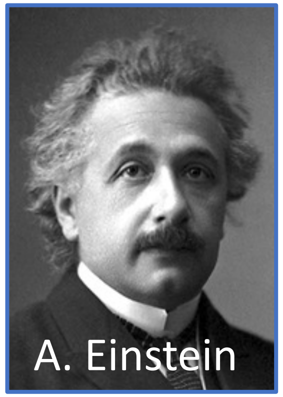

W. Röntgen

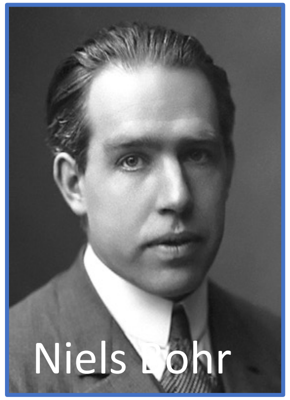

H. Becquerel

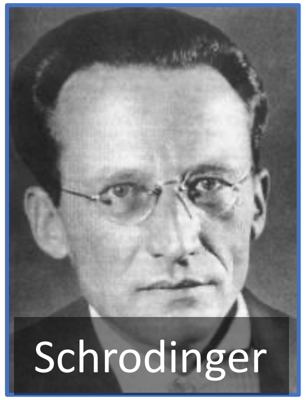

E. Rutherford

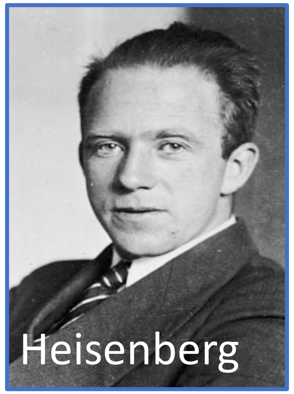

J. Chadwick

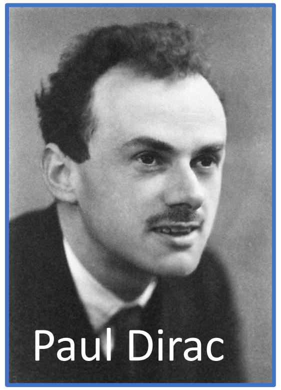

Max Planck

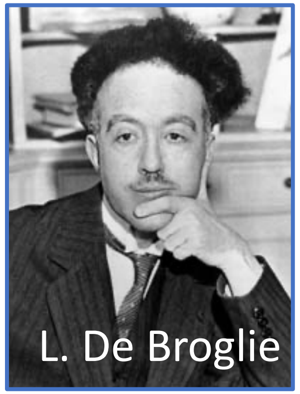

A. Einstein


Niels Bohr


Schrodinger

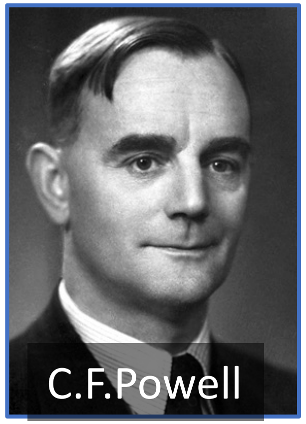

Heisenberg


Paul Dirac


L. De Broglie


A. Compton

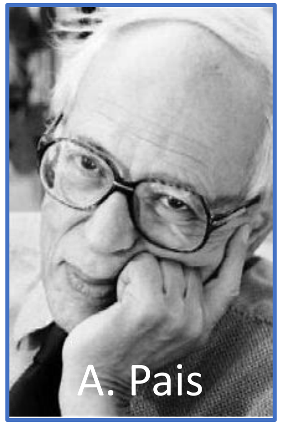

H. Yukawa

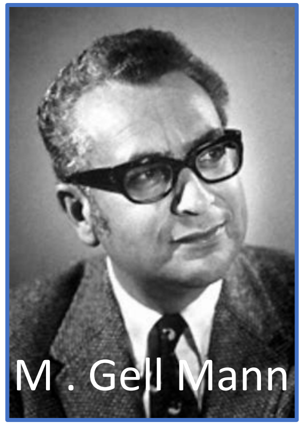

C.F.Powell

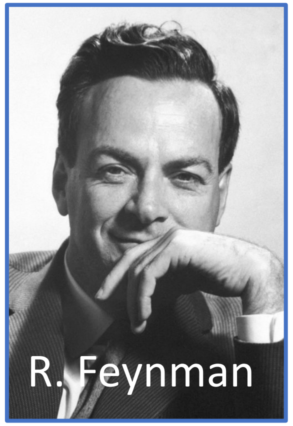

W. Pauli


Anderson


J de Vries


A. Pais


M . Gell Mann


R. Feynman

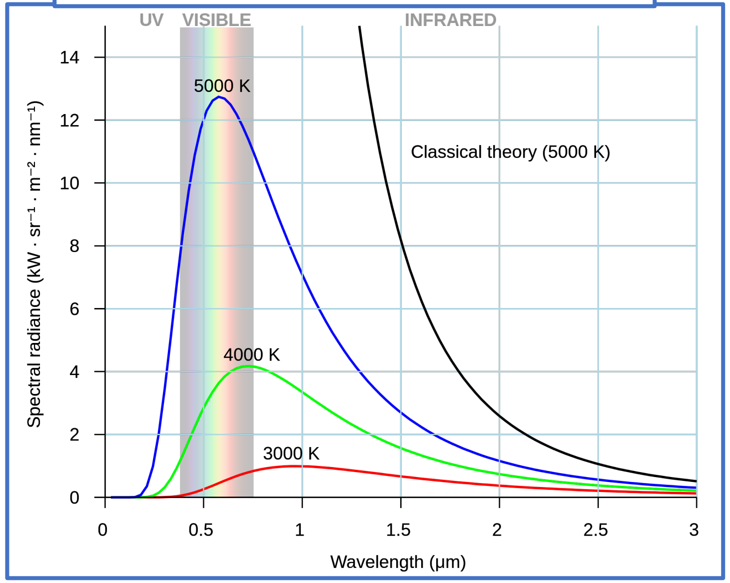
Particle Physics: Historical introduction – Griffiths chapter 1

The classic era

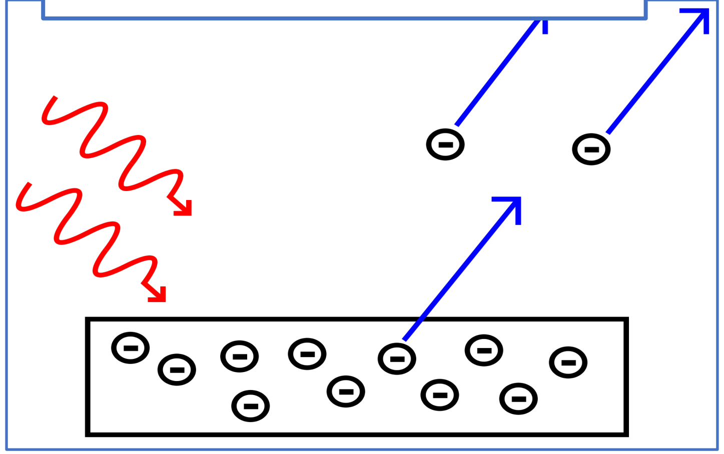
Griffiths §1.1 and §1.2

- ~1900: Lord Kelvin on physics:
  - *“There is nothing new to be discovered in physics now. All that remain is more and more precise measurements.”*
    - Enter Relativity Theory and Quantum Mechanics
- 1896-1897 Becquerel, Röntgen, Marie & Pierre Curie discover *radiation*
- 1897: J.J Thomson noted cathode rays are quantized particles, charge to mass ratio: discovery of *electrons*
- 1900: Planck: The blackbody spectrum can be explained by introducing emission of light in *quanta* $\hbar$ with $E = \hbar\nu$
- 1905: Einstein: photo electric effect: light is quantized: photon
  - Scattering behaviour verified in 1923 by Arthur Holly Compton
- 1911: Rutherford: nucleus hydrogen is a proton
- 1932: Chadwick: discovery of the neutron
  - Neutral particle with the mass of a proton

Black body spectrum: Planck

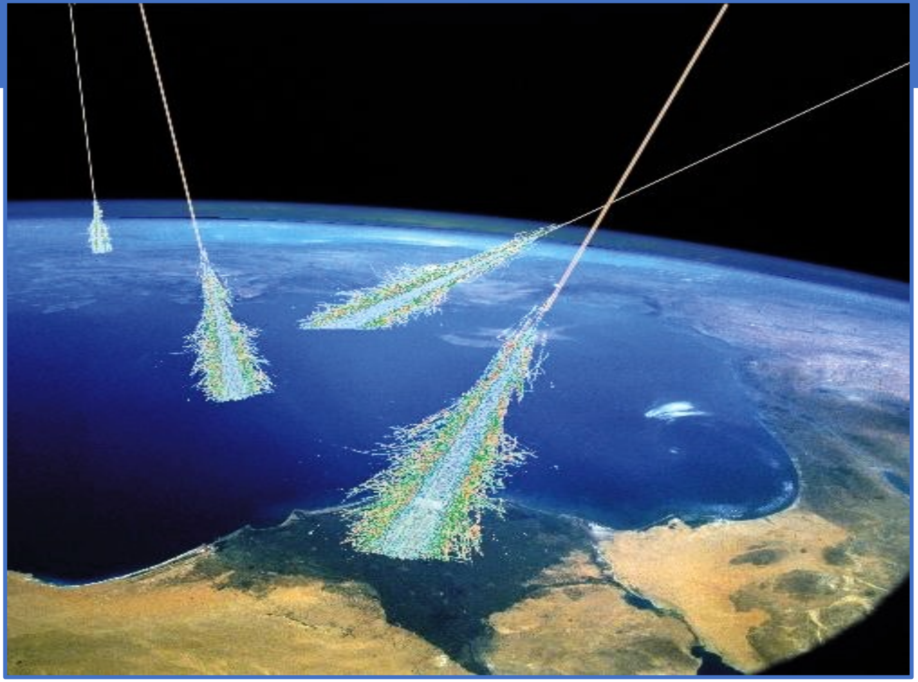


Photoelectric Effect: Einstein



Mesons 1934 - 1947

Griffiths §1.3



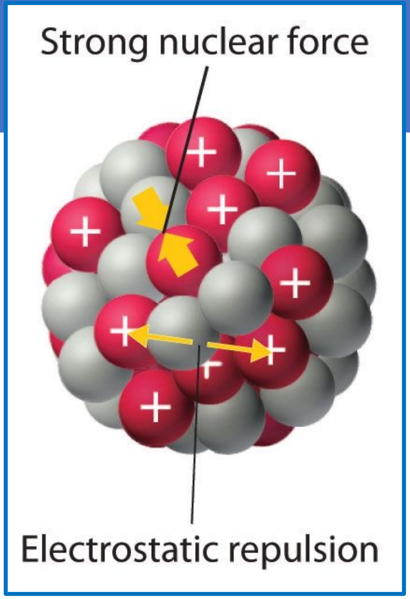
- The nucleus is held together by strong force, mediated by pions or pi-mesons.
- Can we see these pion quanta of mass ~ 150 MeV?
- In 1937 particles where detected in cosmic rays:
  - There mass was a bit too light: $m = 105\ MeV$
  - *They did not interact strongly with nuclei ?!*
- It turned out to be a muon, heavy version of the electron
  - “Who ordered that?”: Isaac Rabi
- Particles:
  - Baryons (p,n), Mesons (π), Leptons (e, μ)

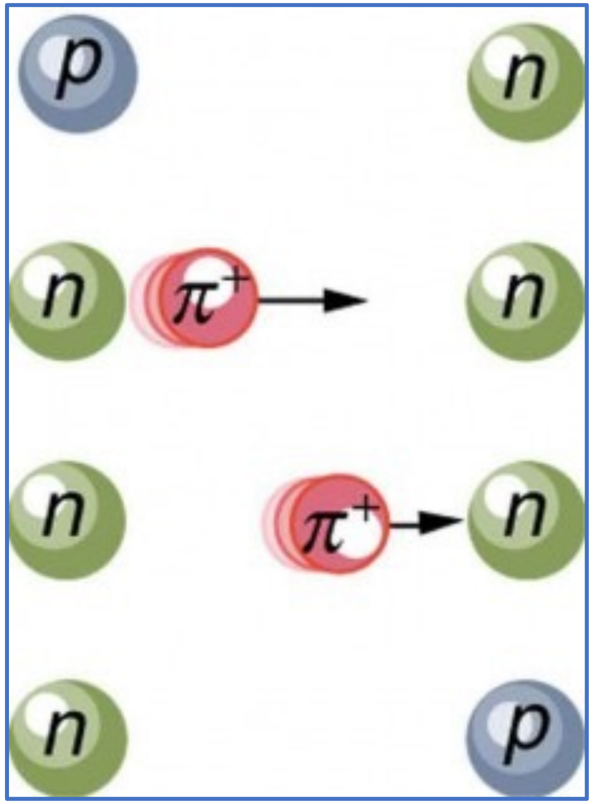
Lepton: Greek “small” or “thin”: small mass, feels no nuclear force
Hadron: Greek “thick” : large mass, feels nuclear interaction
Meson: Greek “middle”: medium mass, feels nuclear interaction

Yukawa's pion

Griffiths: problem 1.2, PP1 §i.2

- Yukawa (1934) pi-meson gives attractive force between nucleons
- Heisenberg operator uncertainty in QM: $\Delta E \Delta t \geq \frac{\hbar}{2}$
- Translated to popular language:
  - You can violate energy conservation as long as:
    - time is shorter than: $\Delta t \leq \frac{\hbar}{2\Delta E}$ or energy is less than $\Delta E \leq \frac{\hbar}{2\Delta t}$
- Electromagnetic force is transmitted by photon quanta
  - $E = hf = \frac{hc}{\lambda}$ such that: $\Delta t \sim \frac{\lambda}{c}$ and the range $R \sim c\Delta t = \lambda$ is infinite
- Nuclear ("strong") force is transmitted by massive pions:
  - $E = mc^2$ such that $\Delta t \sim \frac{h}{2mc^2}$ and $R \sim c\Delta t = \frac{h}{2mc}$
    - The heavier the particle, the more difficult to "violate energy conservation", the shorter the range
- In 1947 the pion, with correct mass, was discovered





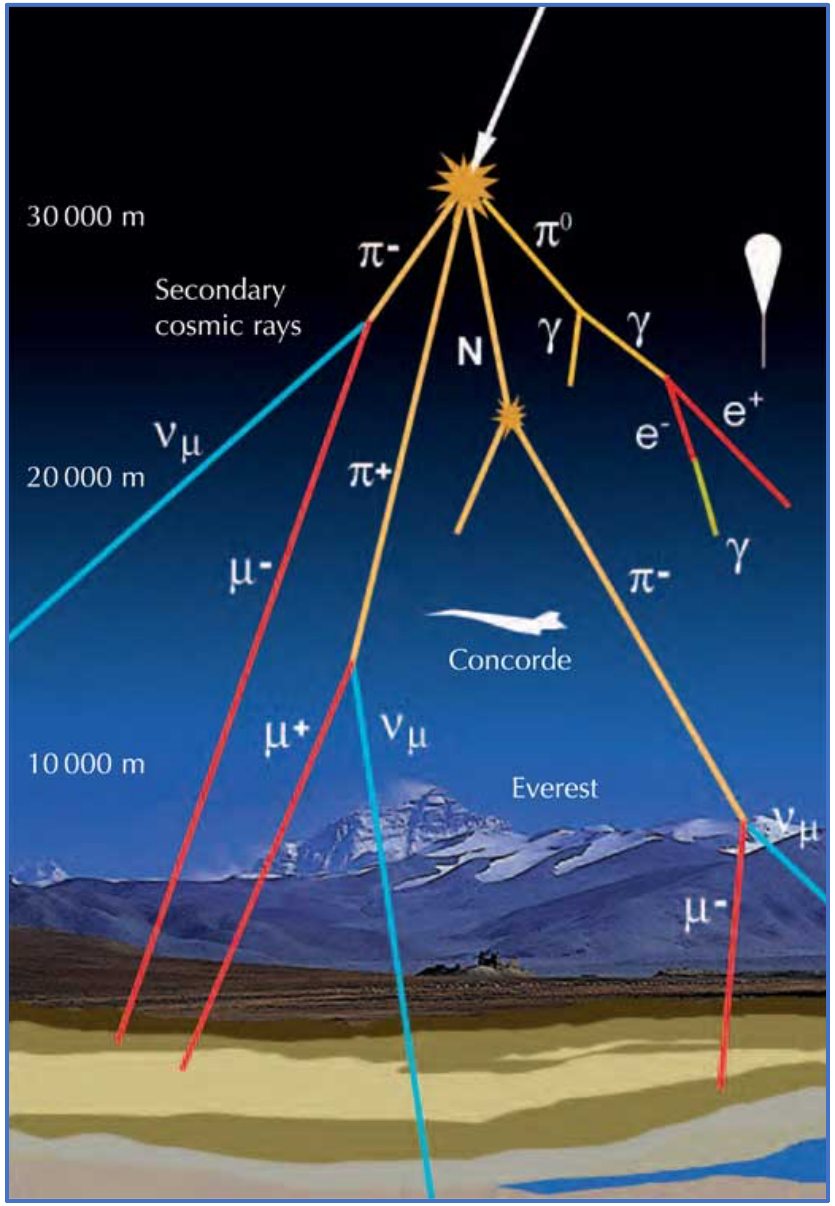
Exercise: The Yukawa Potential

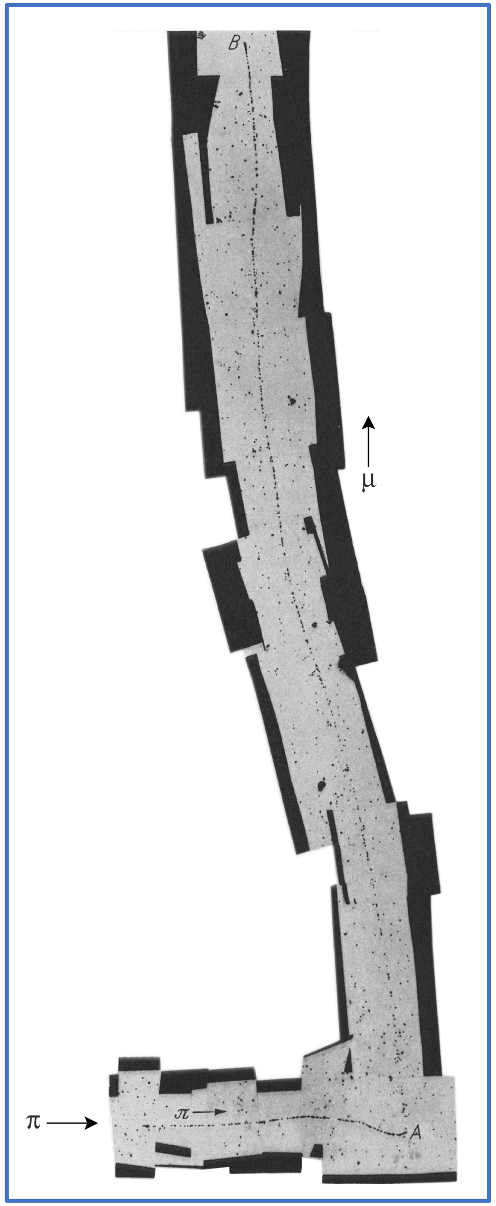
- The electric force is transmitted by a photon with $m = 0$. The wave equation for a static electric field caused by pointlike charge e is: $\nabla^2 V(r) = 0$ (Laplace equation)
  - a) Show that the Coulomb potential $V(r) = -e^2 \frac{1}{r}$ fulfills this equation.
    - Note that the potential is ***spherical symmetric,*** ie. use spherical coordinates.
- The nuclear force is transmitted by a pi-meson with $m = m_\pi$. The wave equation for a static nuclear field caused by a pointlike color charge g is: $\nabla^2 U(r) = m^2 U(r)$ (Klein-Gordon equation)
  - b) Show that the Yukawa potential $U(r) = -g^2 \frac{e^{-r/R}}{r}$ fulfills this equation for a certain value of R, the ***range*** of the force. What is the relation between R and m_π?
    - Again note that the potential is ***spherical symmetric.***
    - This value is between the electron and proton mass, hence the particle was called a pi-meson or pion.
  - c) Calculate the range of the force from Heisenberg's uncertainty relation, using $R = c\Delta t$ and $\Delta E \Delta t \leq \frac{\hbar}{2}$ and $\Delta E = mc^2$.
  - d) The weak force is mediated by W (80 GeV) and Z (91 GeV) bosons. What is the estimated range of the weak force?

Discovery of the pion - 1947

Griffiths §1.3

- Powell, 1947
  - Go to mountain top
  - Photographic emulsion
- Observes that cosmic rays include muons and pions
  - $m_\pi = 140\ MeV/c^2$
  - $m_\mu = 105\ MeV/c^2$
- A pion can decay into a muon, which can again decay into an electron
- The pion was Yukawa's meson

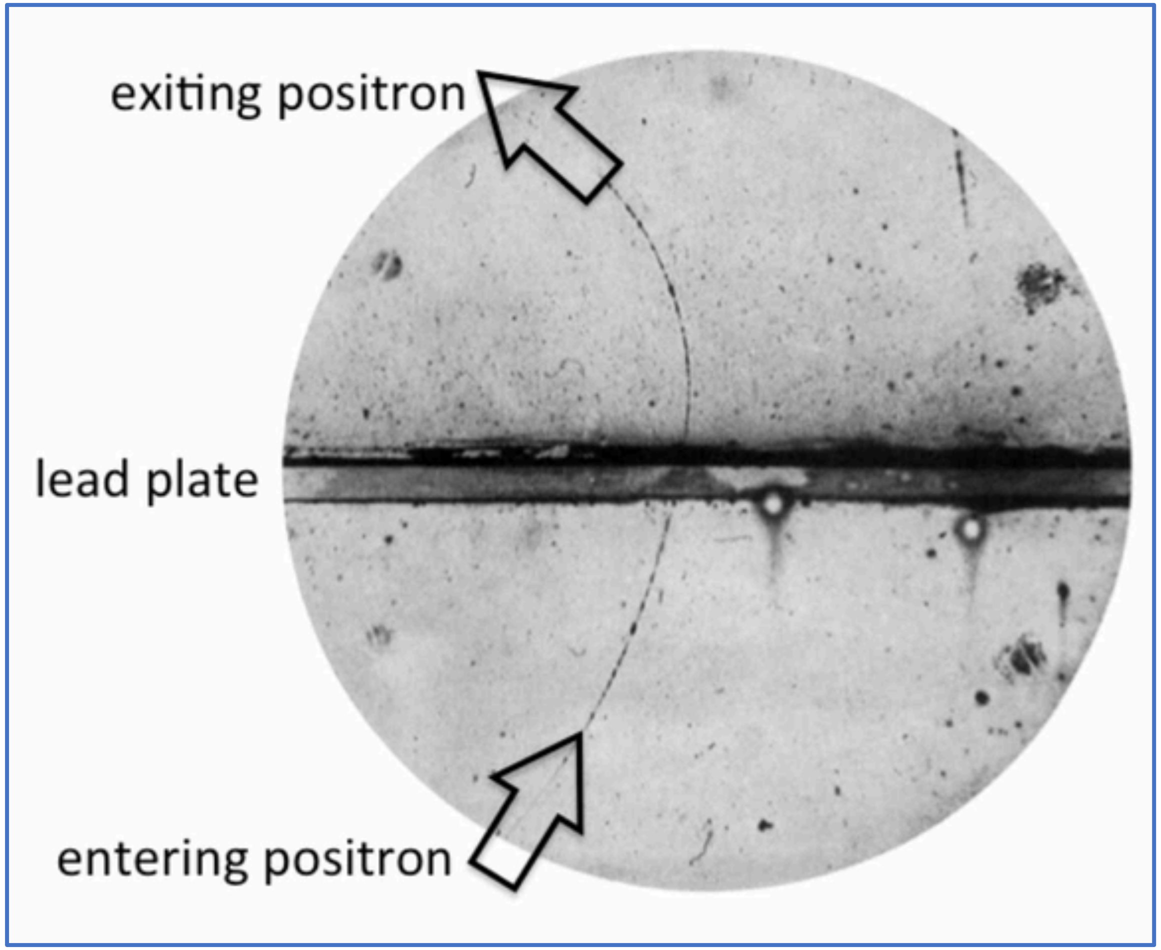




Antimatter particles 1928 - 1956

Griffiths §1.4

- In 1928 Dirac predicted the existence of antimatter particles, combining relativity theory and quantum mechanics
  - We will learn about Dirac's equation later
- Exercise: convince yourself that Klein-Gordon equation: $E^2 = p^2c^2 + m^2c^4$ is in agreement with the relativistic formulae: $E = m\gamma c^2$ and $p = m\gamma v$
- Dirac: E can have positive and negative solutions:
  - Positive: matter particles (electron)
  - Negative antimatter particles (positron)
- In 1932 the positron was discovered by Anderson



Question: How did he know which direction the particle went?

Antimatter particles 1928 - 1956

- Feynman Stückelberg interpretation of particles (more in lecture 3):
  - The *negative energy* solution of a *particle* corresponds with the *positive energy* solution of an *antiparticle* going backwards in time.

$$\psi_{\pm}(\vec{x},t) = a_{\pm} e^{\frac{i}{\hbar}(\vec{x}\vec{p}-Et)}$$

$t \to -t\,;E \to -E$
$\vec{x} \to -\vec{x}\,;\vec{p} \to -\vec{p}$

    - CPT theorem: an antiparticle ***is*** a particle going backwards in time

- For each particle p there exists a mirror anti-particle $\overline{p}$ in nature.
  - Berkeley Bevatron: discovery anti-proton (1954, Chamberlain, Segré) and discovery anti-neutron (1956, Cork).
  - The antiparticle has the *same mass, lifetime and spin, but opposite internal quantum numbers like charge*.
  - Matter and antimatter have identical interactions. Definition what is what?

- Wheeler: is there only one electron in the entire universe?

- Question: is there also an anti-photon? How about an anti-neutrino?

Antiparticle and Crossing Symmetry

- Crossing symmetry:

  If: $A + B \rightarrow C + D$, then also possible:

  $$A \rightarrow \bar{B} + C + D$$
  $$A + \bar{C} \rightarrow \bar{B} + D$$
  $$\bar{C} + \bar{D} \rightarrow \bar{A} + \bar{B}$$

  Limited by energy conservation.

- Example:

  Compton scattering = pair annihilation

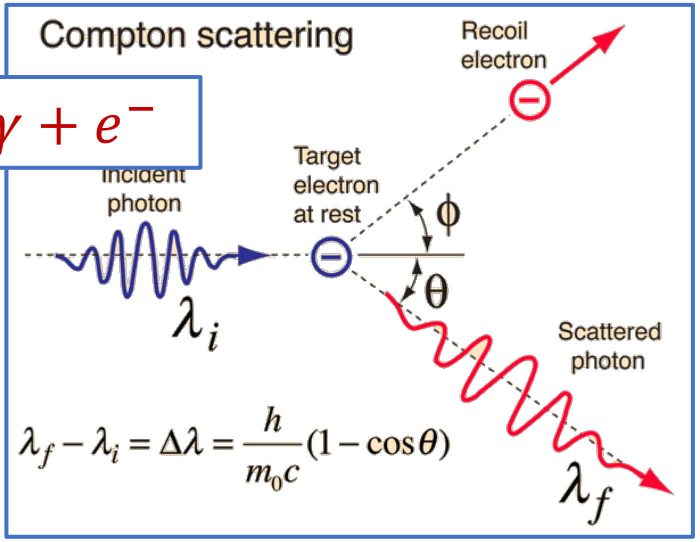
  $$\gamma + e^- \rightarrow \gamma + e^- \quad = \quad e^- + e^+ \rightarrow \gamma + \gamma$$

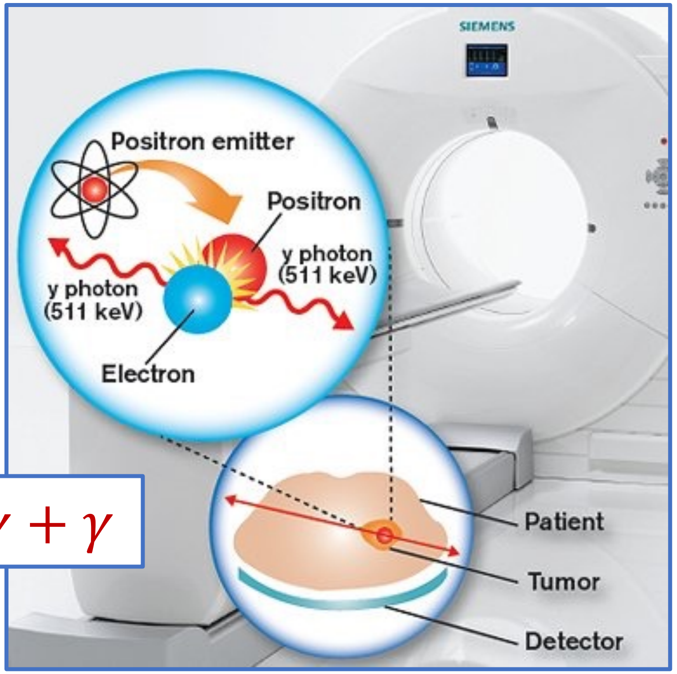
- General symmetry between matter and antimatter (CPT theorem)

  ➜***Why is the universe matter dominated?***

  ➜***Subtle features in the weak interaction!***

$$\gamma + e^- \rightarrow \gamma + e^-$$





$$e^- + e^+ \rightarrow \gamma + \gamma$$

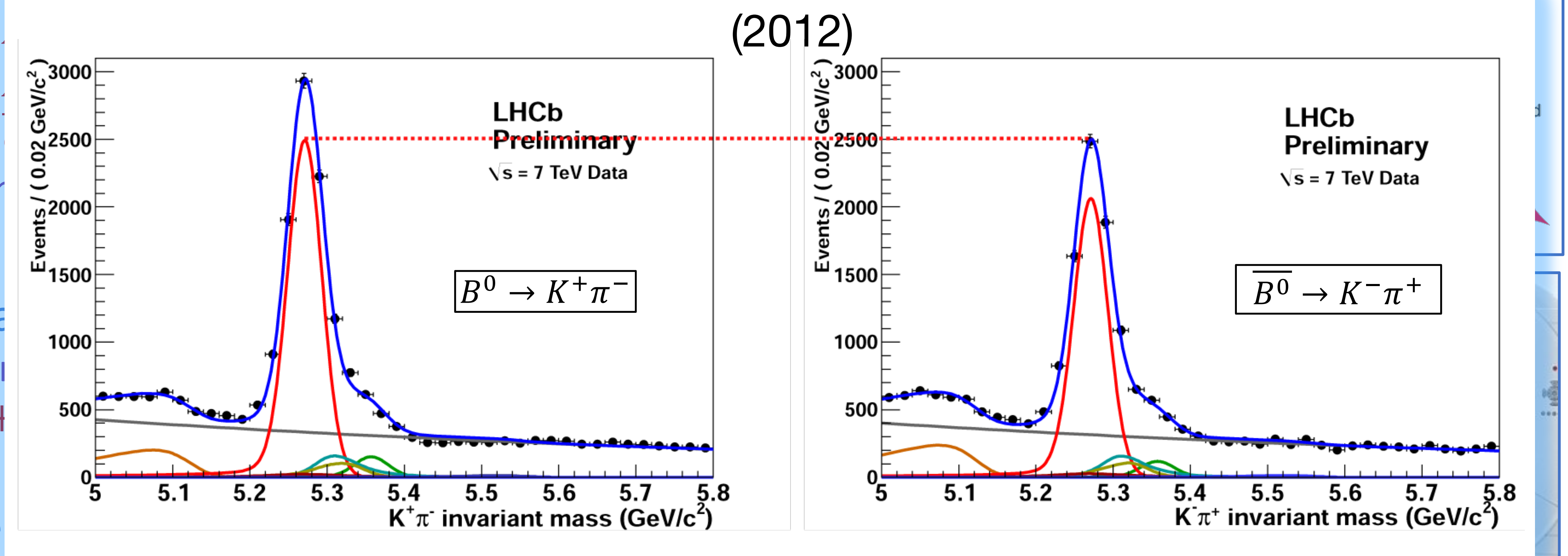
Antiparticle and Crossing Symmetry

- Crossing symmetry:

  If: $A + B \to C + D$, then also possible:

$\gamma + e^- \to \gamma + e^-$

(2012)



Lim

- Exa

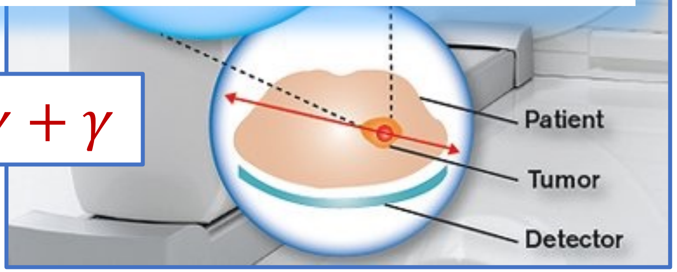
  Co

  $\gamma +$

- Ge
  antimatter (CPT theorem)

  ➔***Why is the universe matter dominated?***

  ➔***Subtle features in the weak interaction!***

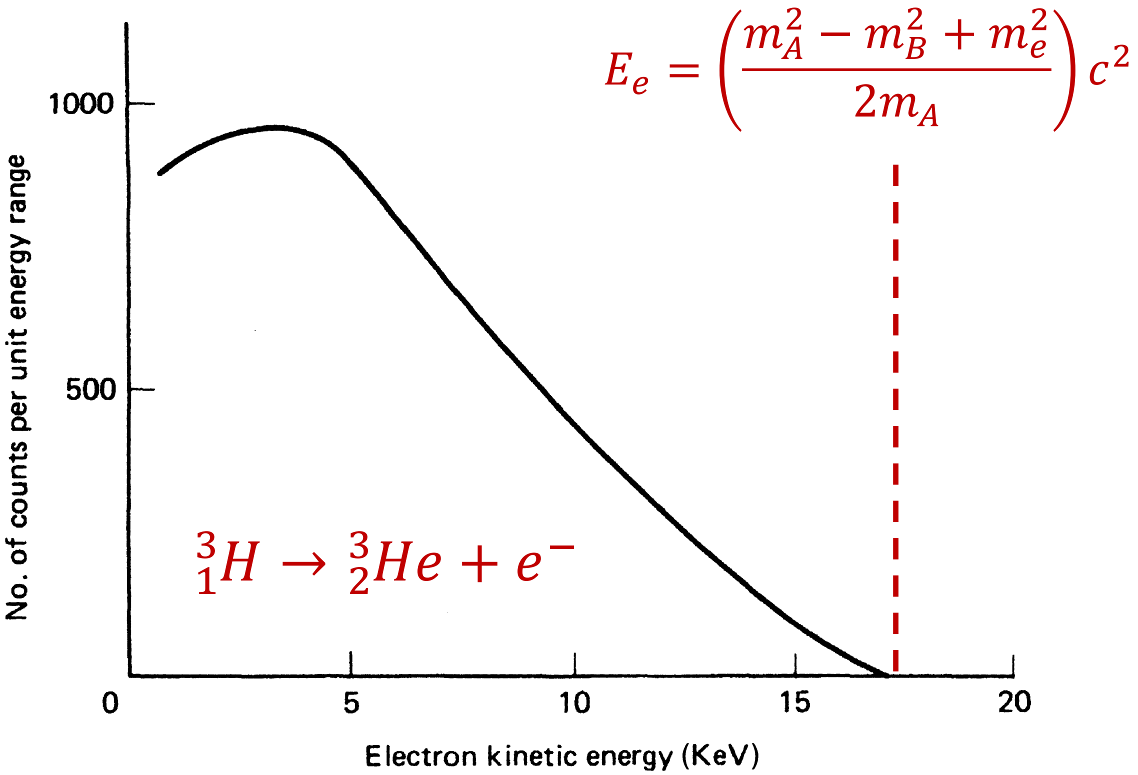


$e^- + e^+ \to \gamma + \gamma$

Neutrinos 1930 - 1962

Griffiths §1.5

- Nuclear beta decay: $A \rightarrow B + e^-$
  - Exercise: calculate the kinetic energy of the electron.
  - Observed kinetic energy in tritium decay is a *spectrum*
- Niels Bohr was thinking to abandon the law of conservation of energy(!)



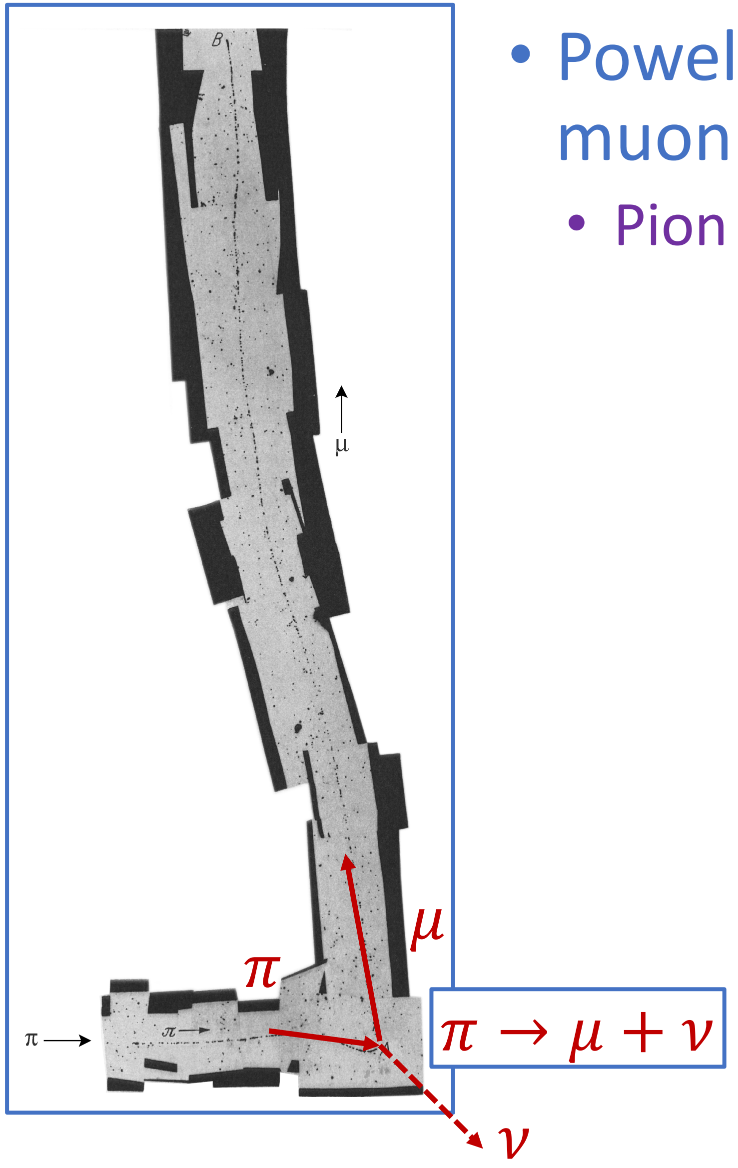
$$E_e = \left(\frac{m_A^2 - m_B^2 + m_e^2}{2m_A}\right)c^2$$

$${}^3_1H \rightarrow {}^3_2He + e^-$$

- Pauli proposed that an undetected neutral particle was produced
  - Proposed the name neutron
  - Fermi presented beta decay theory, neutrino: $n \rightarrow p + e^- + \bar{\nu}$

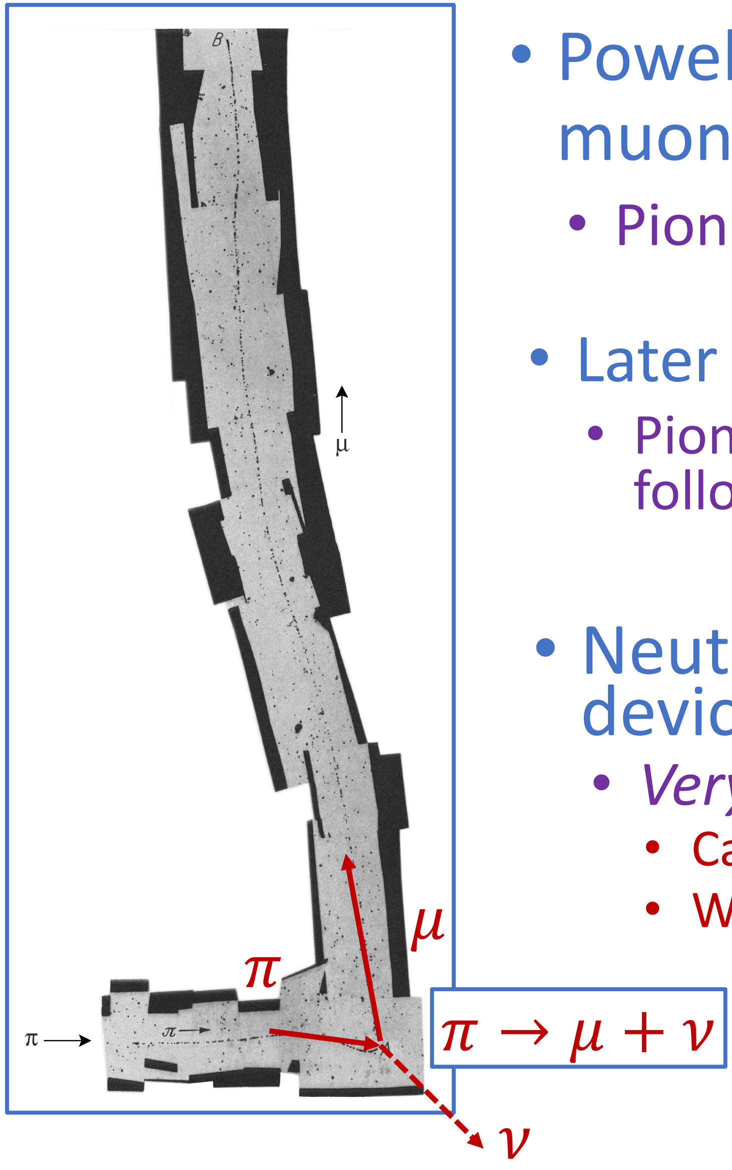
Neutrinos 1930 - 1962

- Powell’s discovery of muon and pion
  - Pion decay: $\pi \rightarrow \mu + \nu$



$\pi \rightarrow \mu + \nu$

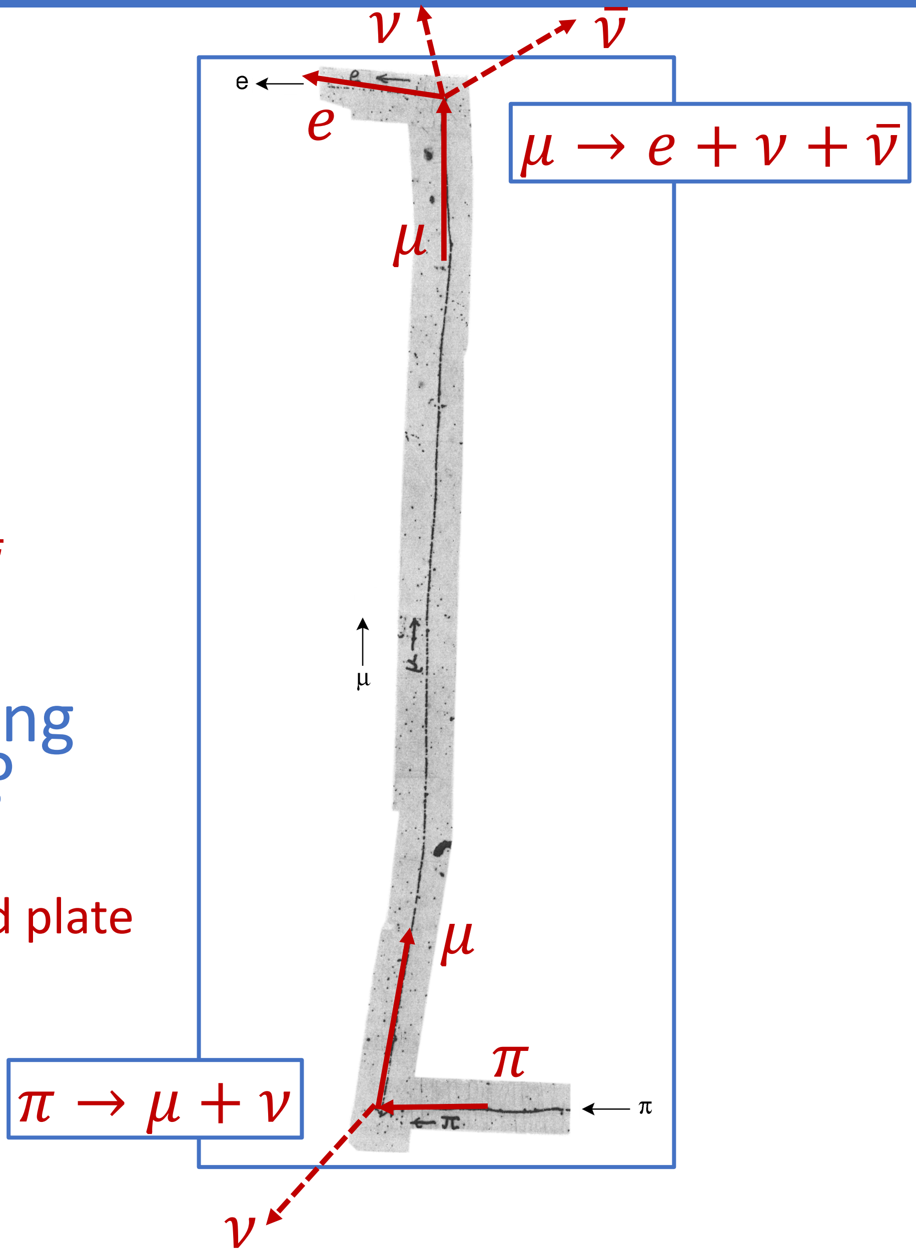
Neutrinos 1930 - 1962



- Powell’s discovery of muon and pion
  - Pion decay: $\pi \rightarrow \mu + \nu$

- Later discovery of Powell
  - Pion *and* muon decays: $\pi \rightarrow \mu + \nu$
    followed by: $\mu \rightarrow e + \nu + \bar{\nu}$

- Neutrinos: are they a bookkeeping device or are they real particles?
  - *Very weak* interaction
    - Can penetrate 1000 lightyears thick lead plate
    - What is their mass? $m_\nu \neq 0$?



Observation of neutrinos

- 1955 Cowan and Reines observe “inverse beta decay”
  - Water tank next to Nuclear Reactor
  - Antineutrinos colliding with protons: $\overline{\nu} + p^+ \rightarrow n + e^+$
    - Inverse beta decay

- Are neutrinos their own antiparticles, like photons?
  - From Cowan and Reines and crossing we know that: $\nu + n \rightarrow p^+ + e^-$ must exist
  - Davis and Hamer looked for: $\overline{\nu} + n \rightarrow p^+ + e^-$ It does ***not*** exist
    - Neutrino is not its own antiparticle. Or is there more going on?

- What distinguishes a neutrino from an anti-neutrino?
  - A nice research topic: “Dirac” neutrinos vs “Majorana” neutrinos

Lepton Number and Lepton Flavor

- 1953, Introduction Lepton number
  - Leptons (e^-, μ^-, ν) have quantum number $L = 1$
  - Antileptons $(\overline{e} = e^+,\ \overline{\mu} = \mu^+,\ \overline{\nu})$ have quantum number $L = -1$
  - Lepton number is conserved in reactions, eg:

$$\begin{array}{lccccccccccc}
 & \pi^- & \rightarrow & \mu^- & + & \overline{\nu} & \quad\text{or}\quad & \pi^+ & \rightarrow & \mu^+ & + & \nu \\
L: & 0 & & 1 & & -1 & & 0 & & -1 & & 1
\end{array}$$

- Lepton Flavor: how about: $\mu^- \rightarrow e^- + \gamma$? It is not observed!
  - Each lepton 'family' has its own lepton number conservation, eg:

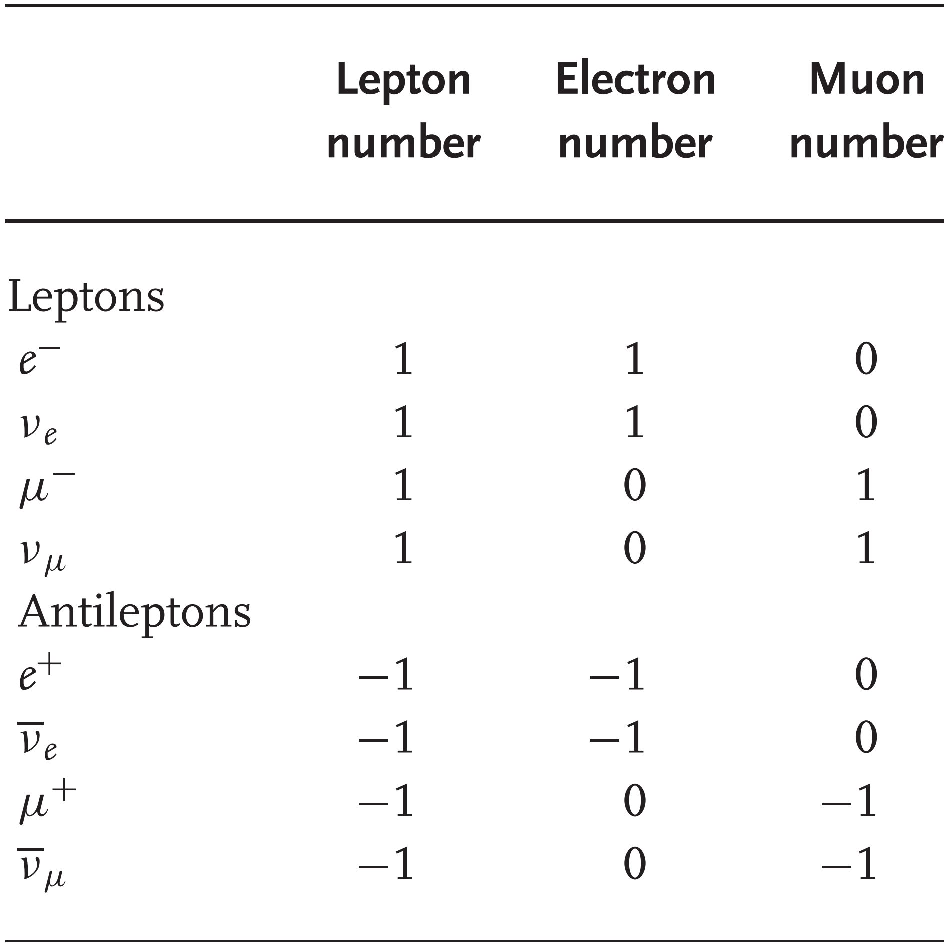
$$\begin{array}{lccccccc}
 & \mu^- & \rightarrow & e^- & + & \overline{\nu_e} & + & \nu_\mu \\
L_\mu: & 1 & & 0 & & 0 & & 1 \\
L_e: & 0 & & 1 & & -1 & & 0
\end{array}$$

Lepton number is conserved for each type ('generation') separately

- Proven in 1962: $\overline{\nu_\mu} + p \rightarrow \mu^+ + n$ was observed,

  while: $\overline{\nu_\mu} + p \rightarrow e^+ + n$ was not!
  - How did they know? Because these neutrino's were produced by pion decays into a muon at Brookhaven lab.

Lepton summary 1962 - 1976



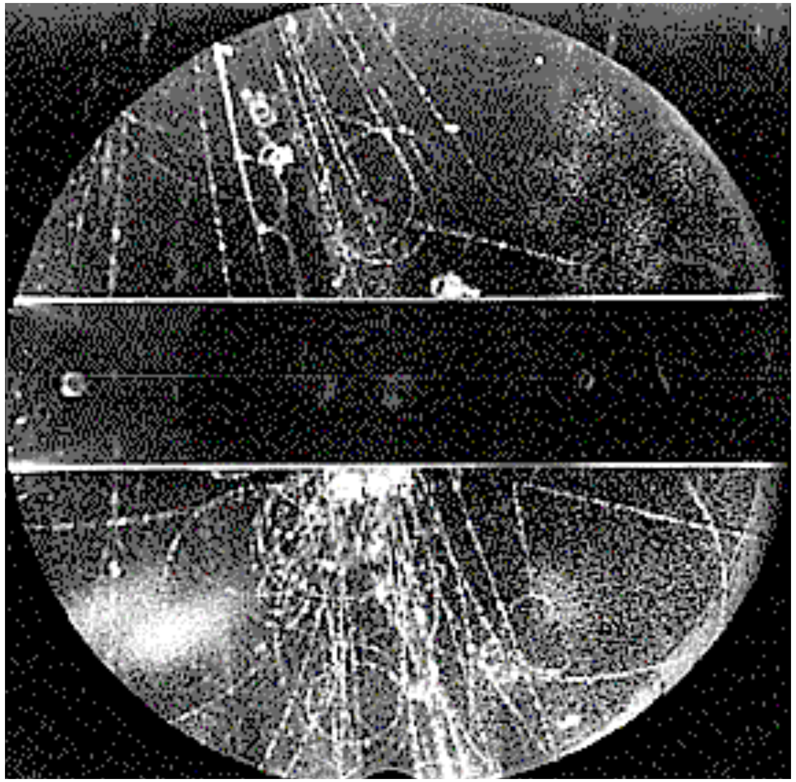
	Lepton number	**Electron number**	**Muon number**
Leptons			
e^-	1	1	0
ν_e	1	1	0
μ^-	1	0	1
ν_μ	1	0	1
Antileptons			
e^+	−1	−1	0
$\overline{\nu}_e$	−1	−1	0
μ^+	−1	0	−1
$\overline{\nu}_\mu$	−1	0	−1

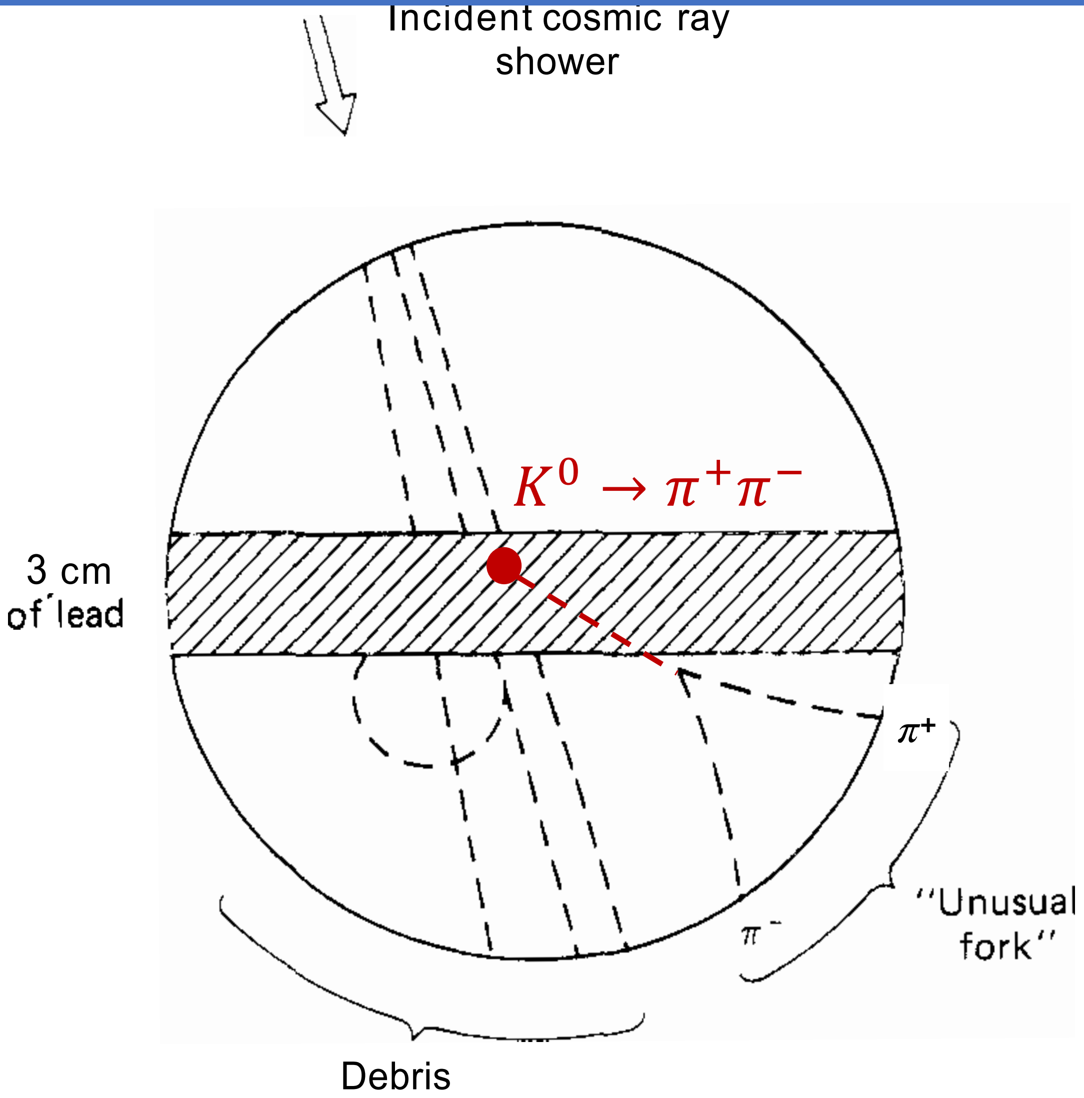
- Are neutrinos massless? Not exactly → Neutrino oscillations
  - Neutrino's can oscillate to different neutrino type and back! (good research topic)

Strangeness (1947 – 1960)

Griffiths §1.6

- Rochester and Butler observed strange "V-prongs" in cosmic rays:

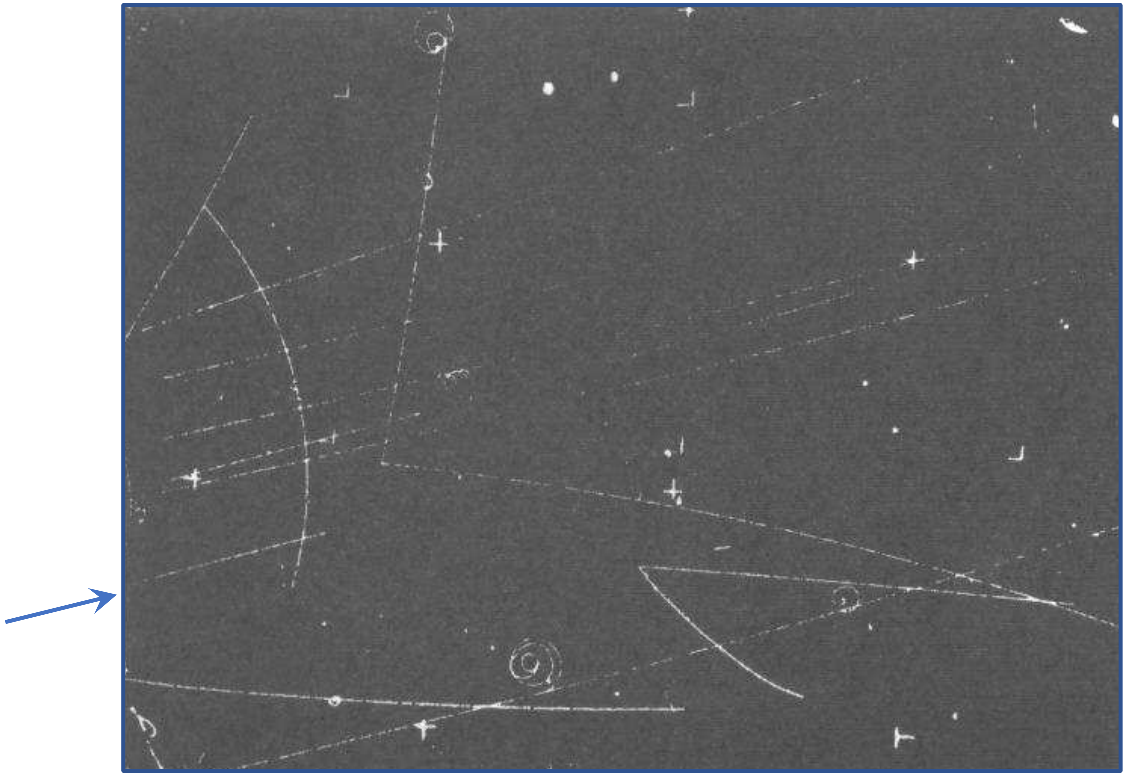


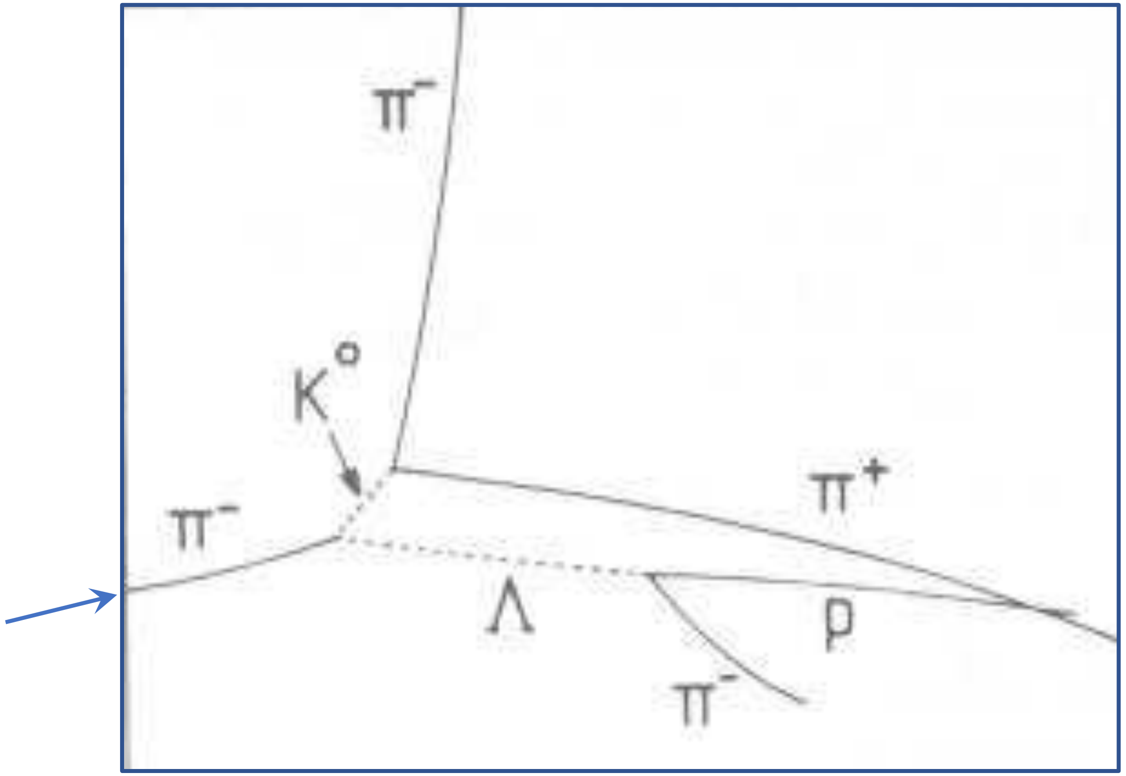


Similarly observed: $\Lambda \rightarrow p^+\pi^-$

- Strange: produced copiously, but decay slowly!
  - Abraham Pais: They are produced with the strong force, but they decay with the weak force.

Example of associated strangeness production





Bubble chamber picture of the associated reaction $\pi^- + p \rightarrow K^0 + \Lambda$. Incoming pion is indicated by the arrow. The unseen neutral particles K^0 and Λ are detected by their “V^0” decays: $K^0 \rightarrow \pi^0 + \pi^-$ and $\Lambda \rightarrow \pi^- + p$.

It turns out these strange particles are always produced in pairs.

Strangeness

- Also observed: $K^+ \to \pi^+ + \pi^+ + \pi^-$ ➔ $M_K \sim 500\ MeV$; it's a meson
- 1952, Brookhaven cosmotron: new strange *baryons*: $\Sigma\ (\sim 1.2\ GeV)$, $\Xi\ (\sim 1.3\ GeV)$
- In production (strong interaction) conserve *baryon number* as well as *strangeness*

	B:	**S:**	OK?
$\pi^- + p \to K^+ + \Sigma^-$	$0 + 1 \to 0 + 1$	$0 + 0 \to 1 + -1$	Yes
$\to K^0 + \Lambda$	$0 + 1 \to 0 + 1$	$0 + 0 \to 1 + -1$	Yes
$\to \pi^+ + \Sigma^-$	$0 + 1 \to 0 + 1$	$0 + 0 \to 0 + -1$	No
$\to K^+ + K^-$	$0 + 1 \to 0 + 0$	$0 + 0 \to 1 + -1$	No

- In decay (weak force) *strangeness is not conserved*: long lifetimes

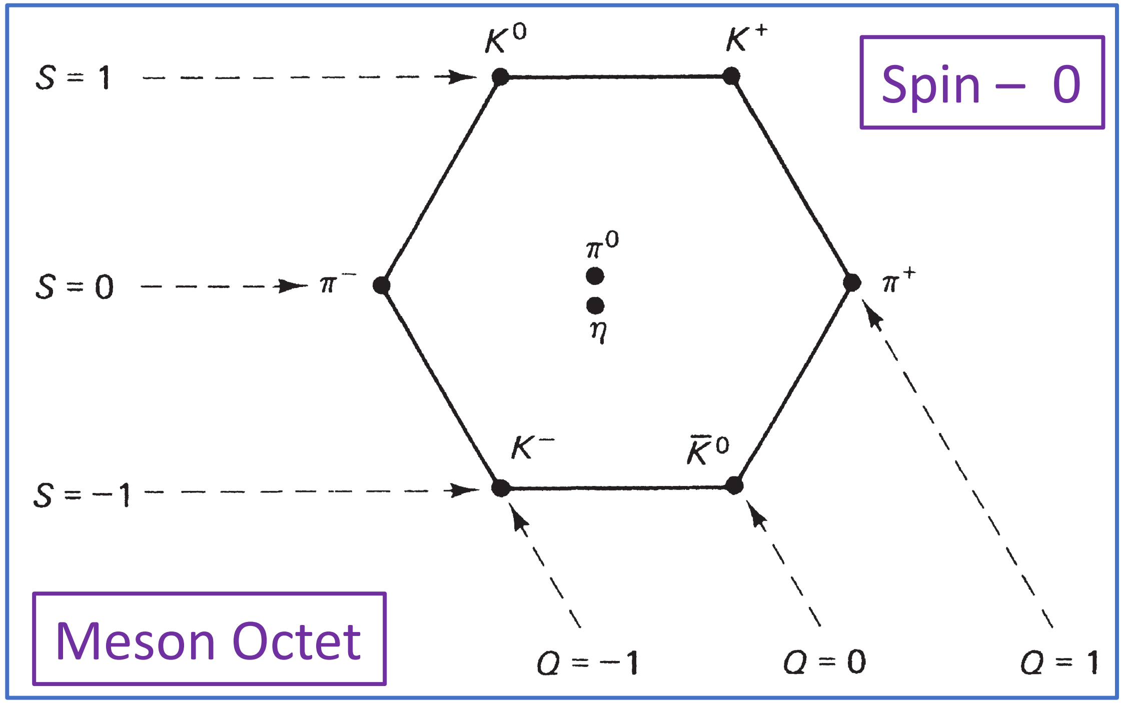
  eg. decays: $\Lambda \to p^+ + \pi^-$; $\Sigma^+ \to n + \pi^+$

- Electric charge is always conserved!

The Particle Zoo and the Eightfold Way

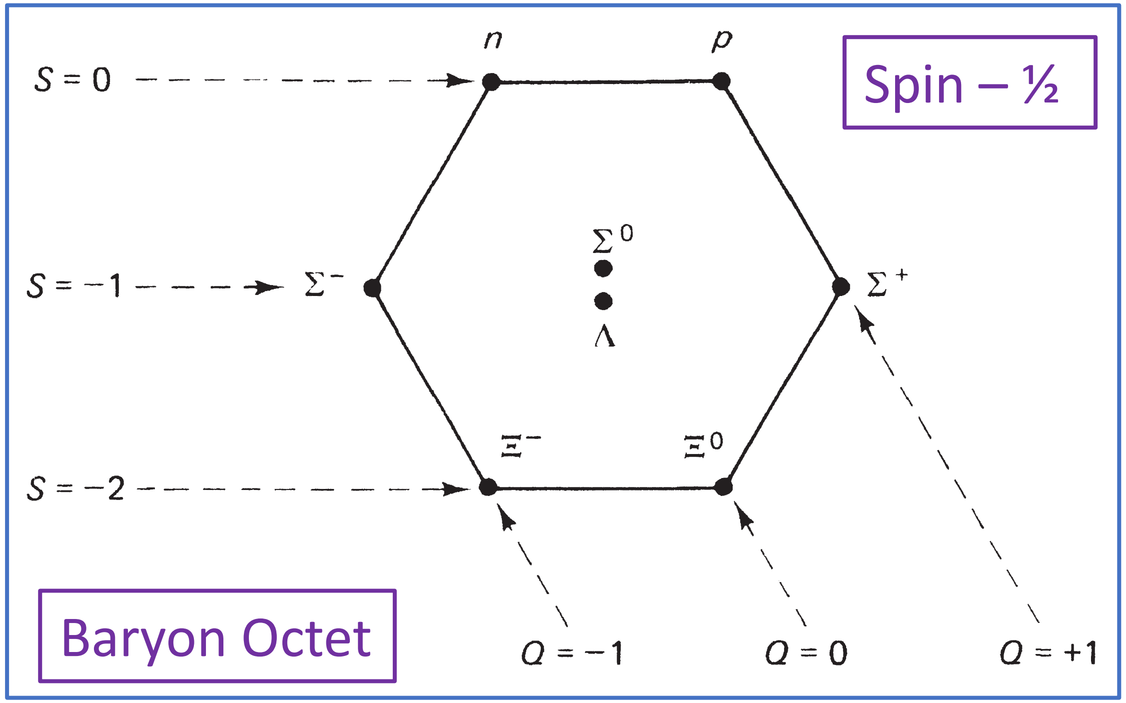
Griffiths §1.7

- So many particles! *"The finder of a new elementary particle should be punished by a 10,000 $ fine!"* - Willis Lamb, 1955.
- Gell-Mann (1961): Make a classification a la Periodic Table



Spin – 0

Meson Octet

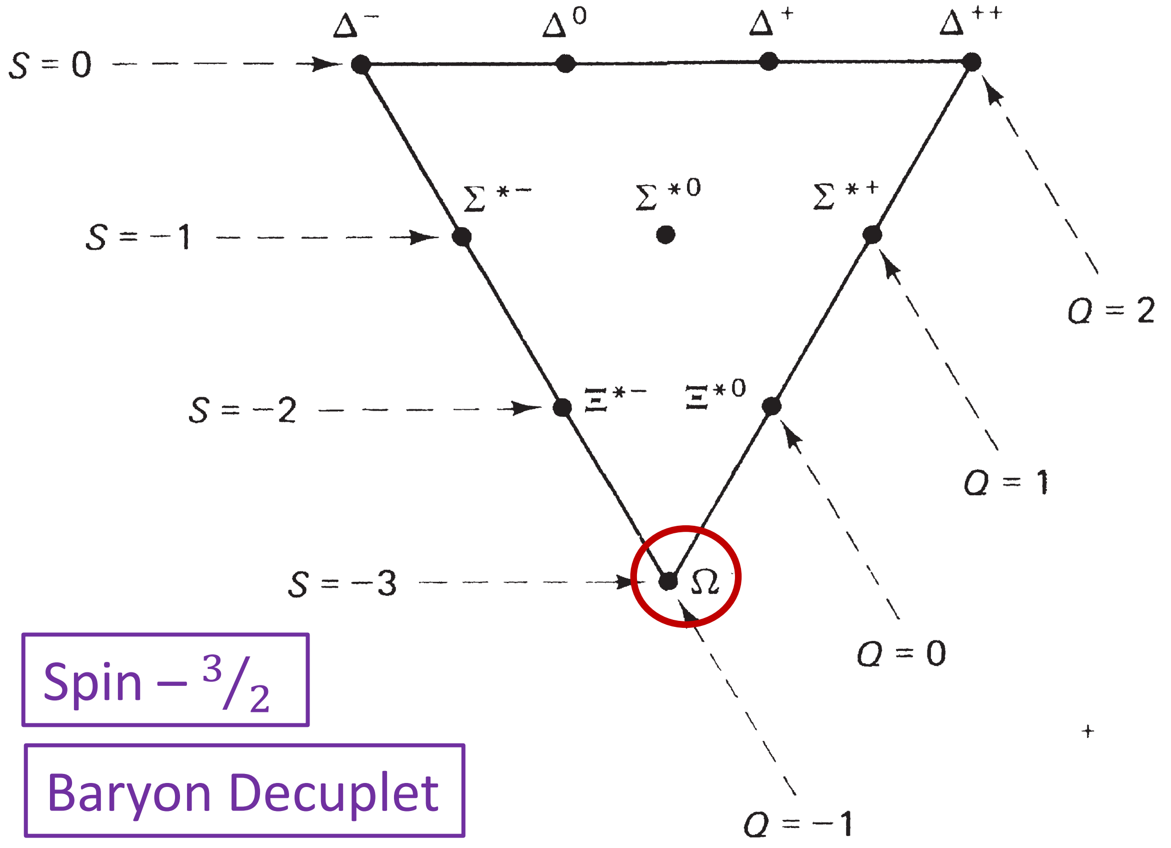


Spin – ½

Baryon Octet

The Particle Zoo and the Eightfold Way

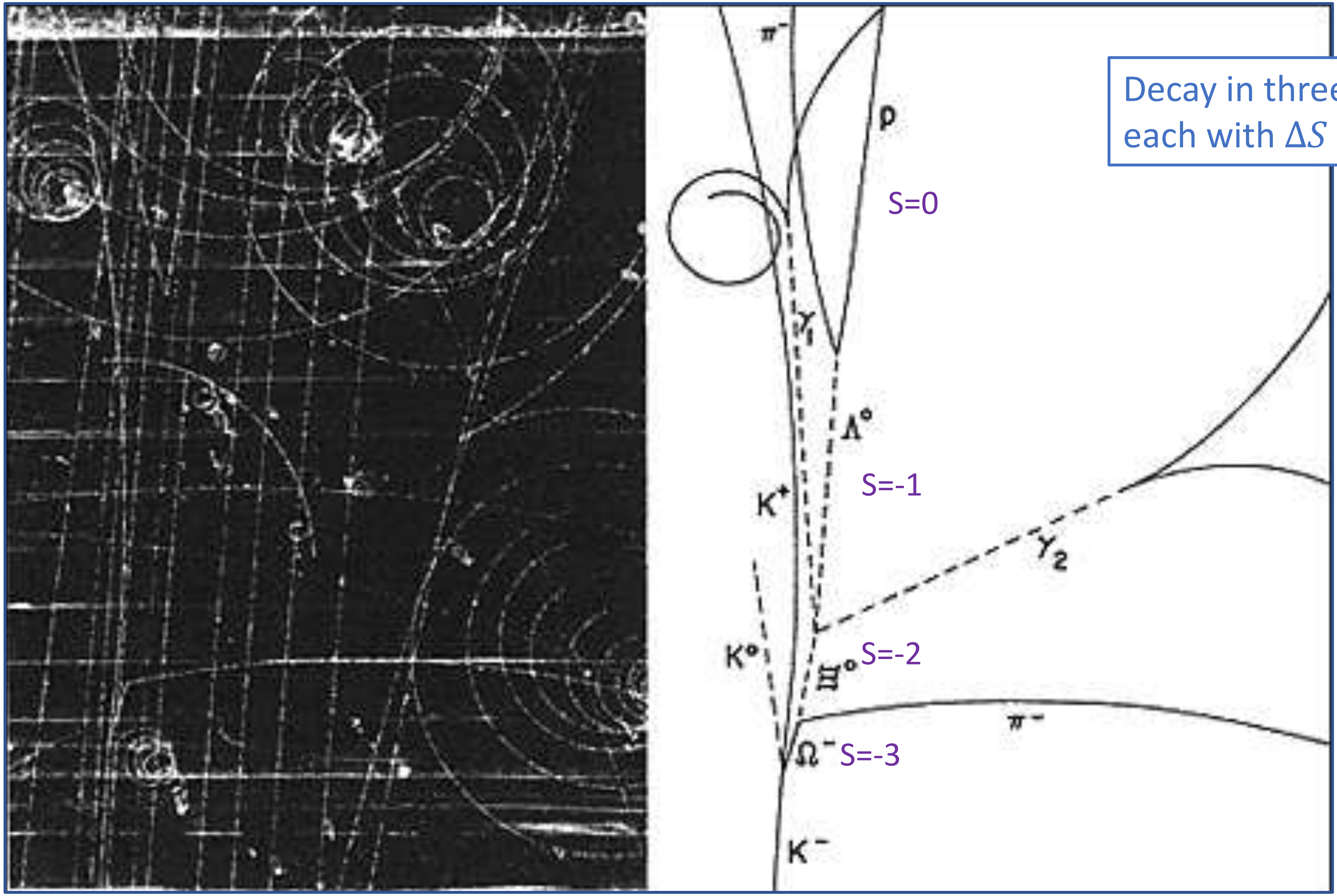
- So many particles! *"The finder of a new elementary particle should be punished by a 10,000 $ fine!"* - Willis Lamb, 1955.
- Gell-Mann (1961): Make a classification a la Periodic Table
- For heavier baryon particles (spin- $^{3}/_{2}$) there is a decuplet
  - The reason that spin- ½ baryons form an octet and spin-$^{3}/_{2}$ a decuplet is far from trivial.
- The Ω particle was not known yet:
  - Mass and lifetime was predicted
  - Observed in 1964
- Octets and decuplets suggest an underlying structure! – quarks!



Spin – $^{3}/_{2}$

Baryon Decuplet

Discovery of the Omega particle

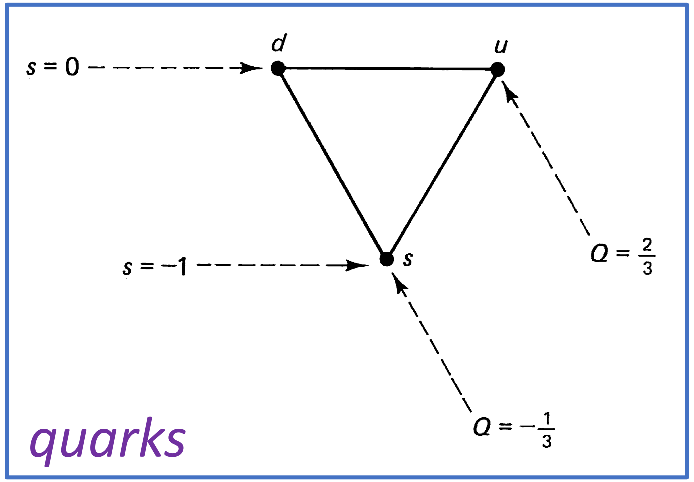


Decay in three steps, each with $\Delta S = -1$

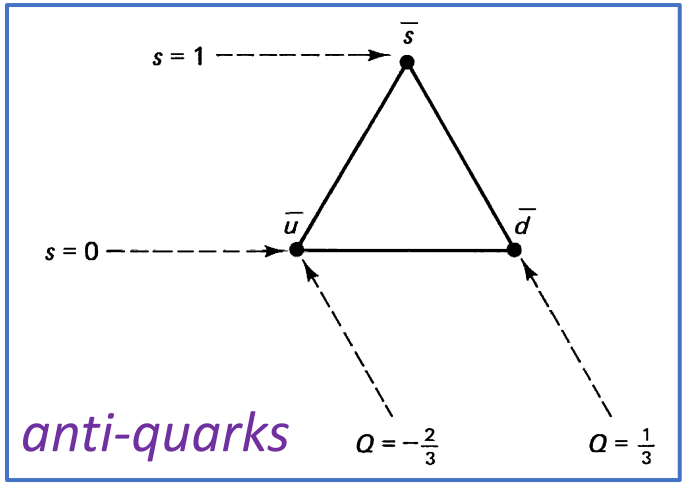
The Quark Model (1964)

Griffiths §1.8

- Gell-Mann and Zweig independently proposed that elementary particles are composed of spin-½ *quarks: up, down, strange:*



quarks



anti-quarks

- *Baryons*: particle consisting of 3 quarks or antiquarks $(q\ q\ q)$ or $(\bar{q}\ \bar{q}\ \bar{q})$
  - Proton$=uud$, neutron$=udd$, etc

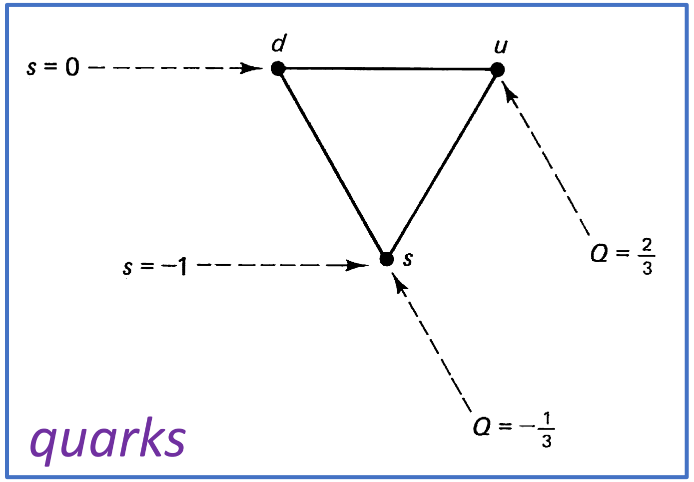
- *Mesons*: particles consisting of 1 quark and 1 anti-quarks: $(q\ \bar{q})$
  - *Pions:* $\pi^+ = u\bar{d}$, $\pi^0 = u\bar{u}+d\bar{d}$, $\pi^- = d\bar{u}$
    Kaons: $K^+ = u\bar{s}$ *, etc*

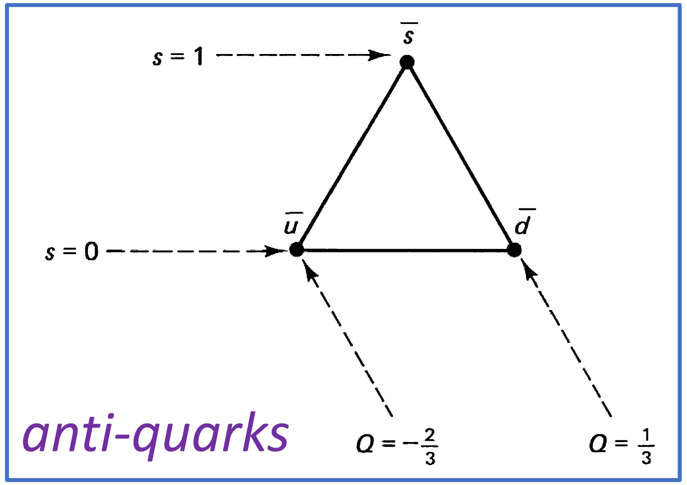
Explains "elementary particles" as quark combinations, spin, strangeness etc.

The Quark Model (1964)

Details in Griffiths §5.5 and §5.6

- Gell-Mann and Zweig independently proposed that elementary particles are composed of spin-½ *quarks: up, down, strange:*





The meson nonet

$q\bar{q}$	Q	S	Meson
$u\bar{u}$	0	0	π^0
$u\bar{d}$	1	0	π^+
$d\bar{u}$	−1	0	π^-
$d\bar{d}$	0	0	η
$u\bar{s}$	1	1	K^+
$d\bar{s}$	0	1	K^0
$s\bar{u}$	−1	−1	K^-
$s\bar{d}$	0	−1	$\bar{K}^0$
$s\bar{s}$	0	0	??

The baryon decuplet

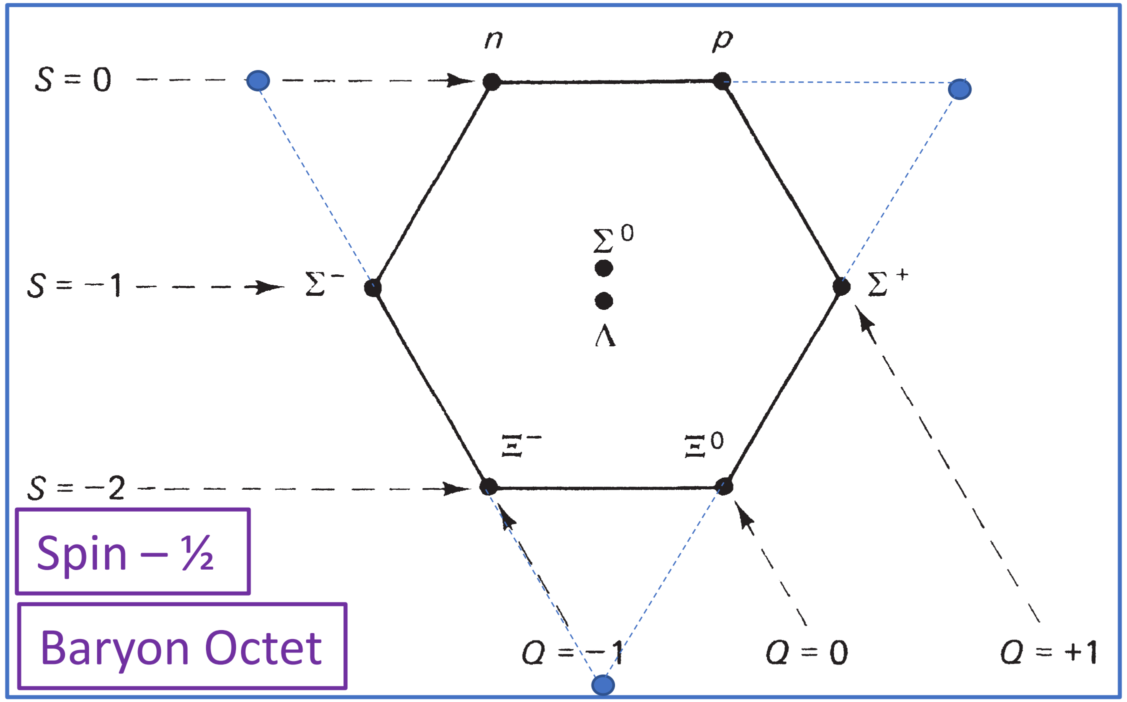
qqq	Q	S	Baryon
uuu	2	0	Δ^{++}
uud	1	0	Δ^+
udd	0	0	Δ^0
ddd	−1	0	Δ^-
uus	1	−1	Σ^{*+}
uds	0	−1	Σ^{*0}
dds	−1	−1	Σ^{*-}
uss	0	−2	Ξ^{*0}
dss	−1	−2	Ξ^{*-}
sss	−1	−3	Ω^-

- Complications: η' singlet, baryon octet:
  - *Spin and (anti-)symmetrization of wave functions!*

Griffiths §5.5

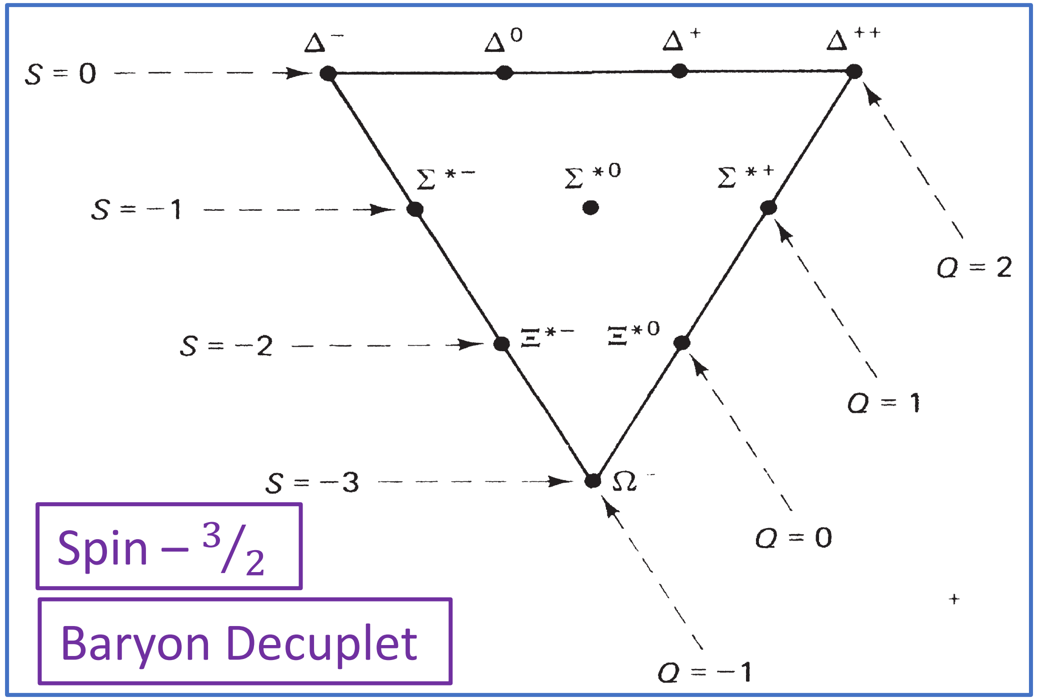
Complications

- The spin-½ octet is like the spin-$^3/_2$ decuplet with the corners missing.
  - Wave function has:
    - 3 identical quarks $(u\ u\ u)$, $(d\ d\ d)$, or $(s\ s\ s)$
    - 3 identical spin directions of quarks: ↑↑↑ or ↓↓↓



Spin – ½

Baryon Octet

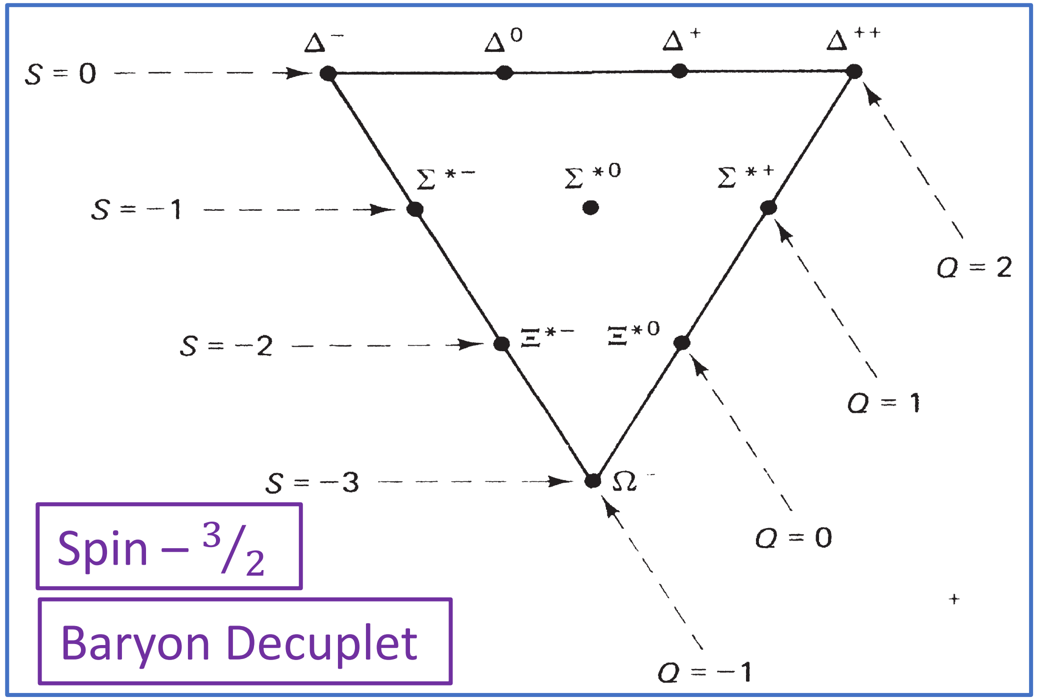


Spin – $^3/_2$

Baryon Decuplet

Desperate Measures

- A quantum mechanical problem: check Δ^{++}:
  - Wave function has:
    - 3 identical quarks ($u\ u\ u$), ($d\ d\ d$), or ($s\ s\ s$)
    - 3 identical spin directions of quarks: ↑↑↑ or ↓↓↓
  - Not allowed by Pauli-exclusion principle:
    - This wave function is symmetric under particle exchange
    - *Identical* spin-½ particles must have an asymmetric wave function under exchange of the particles



Spin – $^3/_2$

Baryon Decuplet

- Quarks differ by having an additional quantum number: color
  - A quark can have quantum number: "red", "green", "blue"
  - An anti-quark can carry quantum number: "anti-red", "anti-green", "anti-blue"
- All physical particles must be colorless ("*confinement*"):
  - baryons → $r\ g\ b$, mesons → $r\ r$, $g\ g$, or $b\ b$

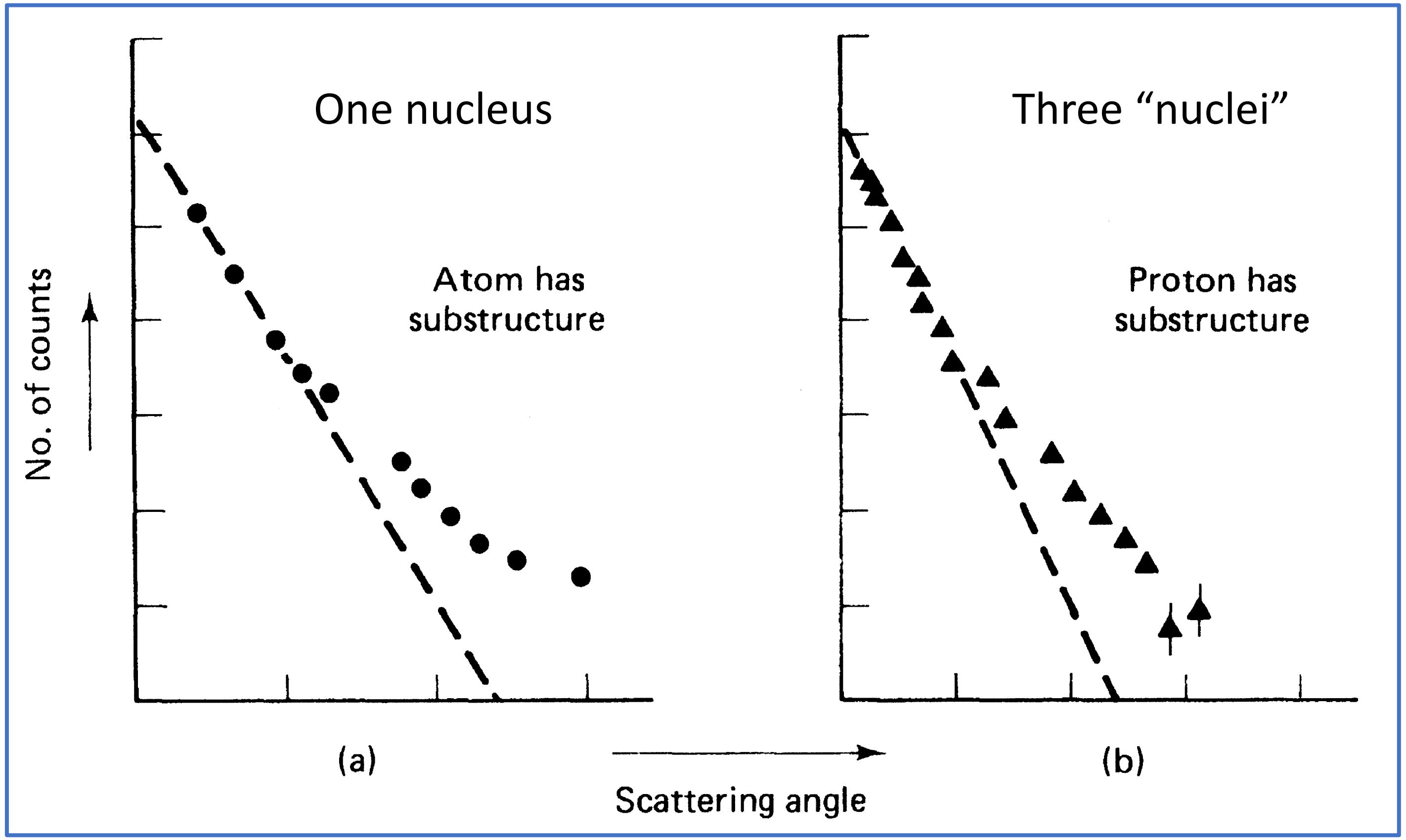
No free quarks in nature!

Deep Inelastic Scattering ("DIS")

- Indeed individual quarks have never been observed.
  - Do they actually exist or are they just a mathematical bookkeeping tool?

- ~1970 SLAC: "DIS"
  - Proton substructure is seen similar to Rutherford scattering

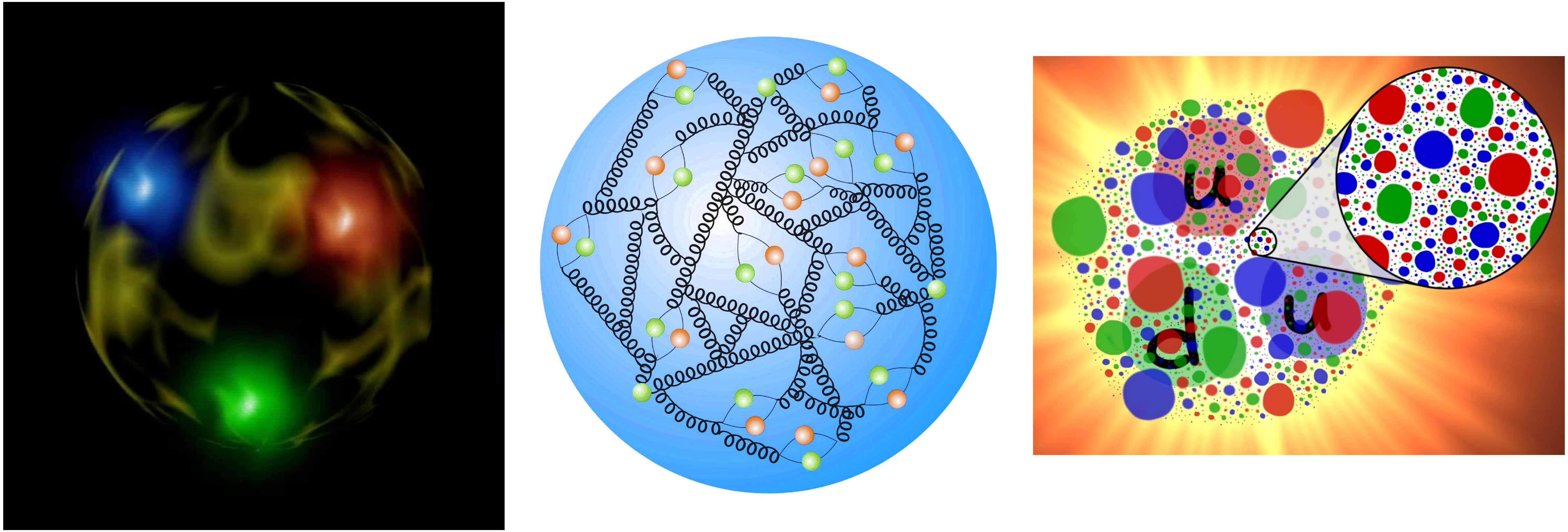
- Quarks are real and they carry an additional quantum number which can have three values
  - *"Color" is just a name*



<u>Research:</u>

what is the principle behind confinement, ie. being colorless?

How to make a mental picture of a proton?



- Looking from ”afar” there are three colored quarks: red, green, blue
- Zooming in it becomes a quantum foam of particles + antiparticles + force particles (gluons)

The November Revolution: 1974 - 1983

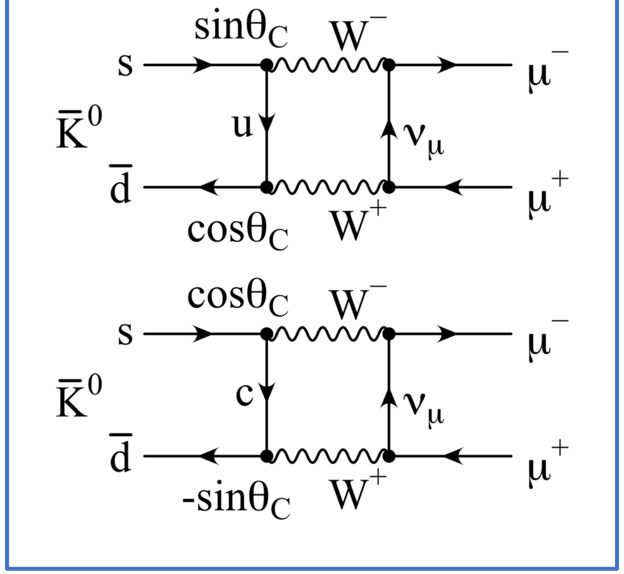
Griffiths §1.9

- Fundamental particles:
  - Why 4 Leptons: (e, ν_e) (μ, ν_μ) and 3 Quarks : (u, d, s) ?
  - 1974 Two groups discover a new particle: Ting at BNL ("J") and Richter at SLAC ("ψ")
    - J/ψ particle is special: very heavy $M \sim 3000\ MeV$, long lifetime $\tau \sim 1000 \times$ longer than heavy hadrons

    ➔new meson consisting of heavy charm quarks: $c\bar{c}$ ➔ Enter: ***charm*** quantum number

- Existence of charm also solved the Glashow, Iliopoulos, Maiani (GIM) puzzle.
  - It explained why the K_L particle had a very long lifetime

K_L decay diagrams



- Other new particles were discovered soon afterwards
  - Mesons: D^0 $(c\bar{u})$, D^+ $(c\bar{d})$, D_s^+ $(c\bar{s})$, …
  - Baryons: Σ_c (uuc) , $\Lambda_c (udc)$, Ξ_c (usc) , Ω_c (ssc) , …

- Similar to "strangeness", "charm" is conserved in production (strong force), and violated in (weak force) decay

The November Revolution: 1974 - 1983

- 1974 Discovery of charm brings Lepton-Quark symmetry:
  - (ν_e, e) (ν_μ, μ) and (u, d) (c, s)
- 1975 Discovery of the τ- lepton breaks symmetry again
  - Also a corresponding ν_τ neutrino
- 1977 Discovery Υ $(b\bar{b})$ particle, consisting of *bottom* or *beauty* quarks

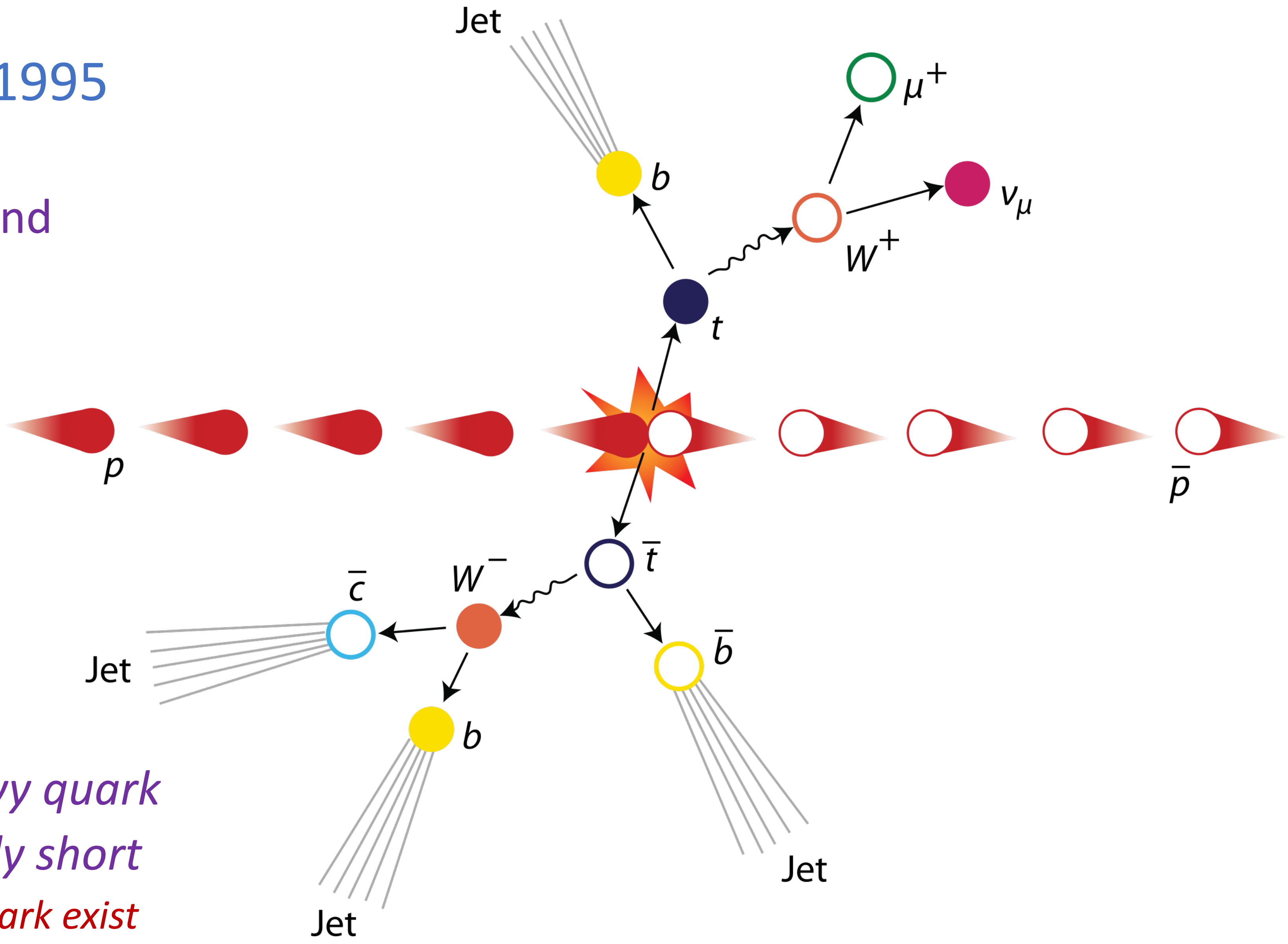
  ➔ *Baryons: 1980: $\Lambda_b^0(udb)$, 2006: $\Sigma_b^+(uub)$*, 2007: Ξ_b^-(ds$b$),

  ➔ *Mesons*: 1983: $B^0(\bar{b}d)$, B$^-$ $(b\bar{u})$, later also: $B_s^0(\bar{b}s)$, B_c^+ $(\bar{b}c)$

  First $B_c^+ \rightarrow B_s^0$ weak decay observed by J.A. de Vries *et al.*
- Terminology: upness, downness, strangeness, charm, beauty = ***"Flavour"***
  - Quarks have *charge*, *flavor* and *color* quantum numbers
- Now 6 leptons and 5 quarks?
  - (ν_e, e) (ν_μ, μ) (ν_τ, τ) and (u, d) (c, s) (b, …)

Top quark

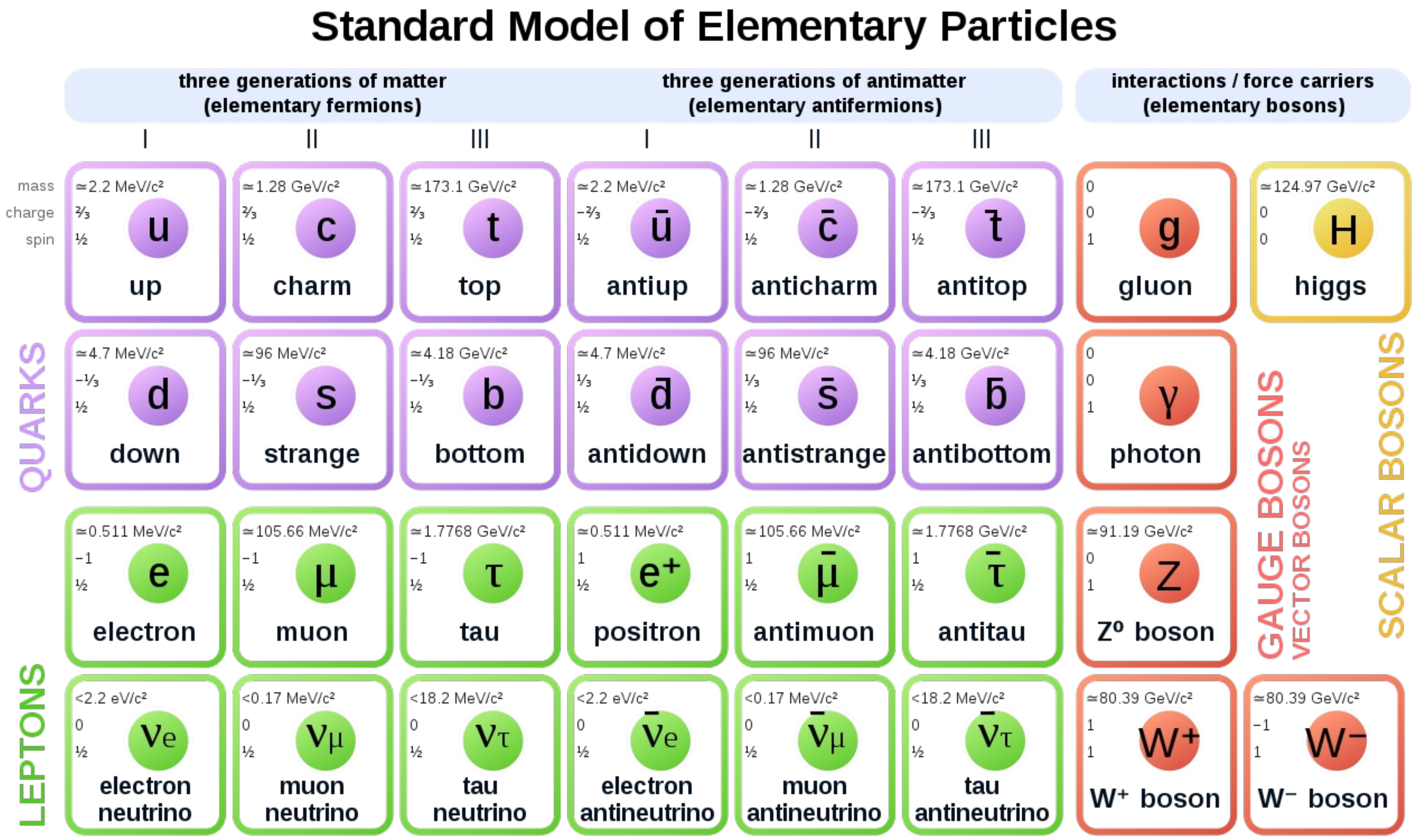
- Discovered at Fermilab 1995
  - $p + \bar{p} \rightarrow t + \bar{t}$
  - Top decays immediately and form so-called *jets*



- Top quark:
  - $m_t = 176\ GeV$, *very heavy quark*
  - $\tau_t = 5\times10^{-25} s$, *extremely short*
    - *No bound states with top quark exist*

Standard Model of particles

Griffiths §1.10 and §1.11



Standard Model of Elementary Particles

	three generations of matter (elementary fermions)			three generations of antimatter (elementary antifermions)			interactions / force carriers (elementary bosons)	
	I	II	III	I	II	III		
mass charge spin	≃2.2 MeV/c^2 ⅔ ½ u **up**	≃1.28 GeV/c^2 ⅔ ½ c **charm**	≃173.1 GeV/c^2 ⅔ ½ t **top**	≃2.2 MeV/c^2 −⅔ ½ $\bar{u}$ **antiup**	≃1.28 GeV/c^2 −⅔ ½ $\bar{c}$ **anticharm**	≃173.1 GeV/c^2 −⅔ ½ $\bar{t}$ **antitop**	0 0 1 g **gluon**	≃124.97 GeV/c^2 0 0 H **higgs**
QUARKS	≃4.7 MeV/c^2 −⅓ ½ d **down**	≃96 MeV/c^2 −⅓ ½ s **strange**	≃4.18 GeV/c^2 −⅓ ½ b **bottom**	≃4.7 MeV/c^2 ⅓ ½ $\bar{d}$ **antidown**	≃96 MeV/c^2 ⅓ ½ $\bar{s}$ **antistrange**	≃4.18 GeV/c^2 ⅓ ½ $\bar{b}$ **antibottom**	0 0 1 γ **photon**	GAUGE BOSONS VECTOR BOSONS SCALAR BOSONS
	≃0.511 MeV/c^2 −1 ½ e **electron**	≃105.66 MeV/c^2 −1 ½ μ **muon**	≃1.7768 GeV/c^2 −1 ½ τ **tau**	≃0.511 MeV/c^2 1 ½ e^+ **positron**	≃105.66 MeV/c^2 1 ½ $\bar{\mu}$ **antimuon**	≃1.7768 GeV/c^2 1 ½ $\bar{\tau}$ **antitau**	≃91.19 GeV/c^2 0 1 Z **Z^0 boson**	
LEPTONS	<2.2 eV/c^2 0 ½ ν_e **electron neutrino**	<0.17 MeV/c^2 0 ½ ν_μ **muon neutrino**	<18.2 MeV/c^2 0 ½ ν_τ **tau neutrino**	<2.2 eV/c^2 0 ½ $\bar{\nu}_e$ **electron antineutrino**	<0.17 MeV/c^2 0 ½ $\bar{\nu}_\mu$ **muon antineutrino**	<18.2 MeV/c^2 0 ½ $\bar{\nu}_\tau$ **tau antineutrino**	≃80.39 GeV/c^2 1 1 W^+ **W^+ boson**	≃80.39 GeV/c^2 −1 1 W^- **W^- boson**

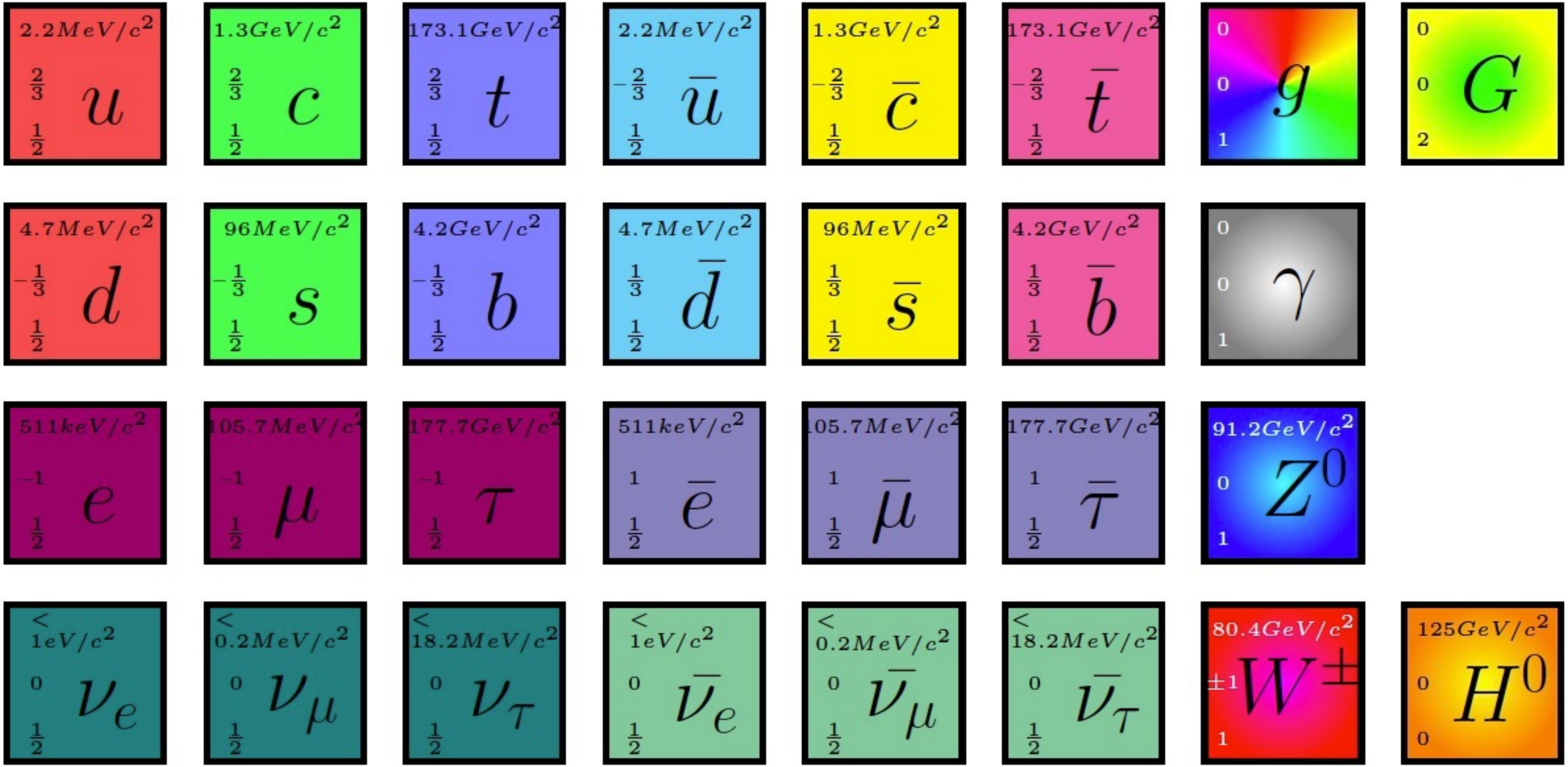
Current Fundamental Questions in Particle physics

- Why are there three generations of particles?
- Why are the masses of the particles what they are?
- Why is there no antimatter in our universe?
- Why is the charge of the electron exactly opposite to that of the proton? Or: why is the sum of the charge of leptons + quarks exactly equal to zero?
- Is a neutrino its own anti-particle?
- Can all forces be described in a single theory? Unification
- Are quarks and leptons truly fundamental
- What is the source of dark matter?
- What is the source of dark energy?

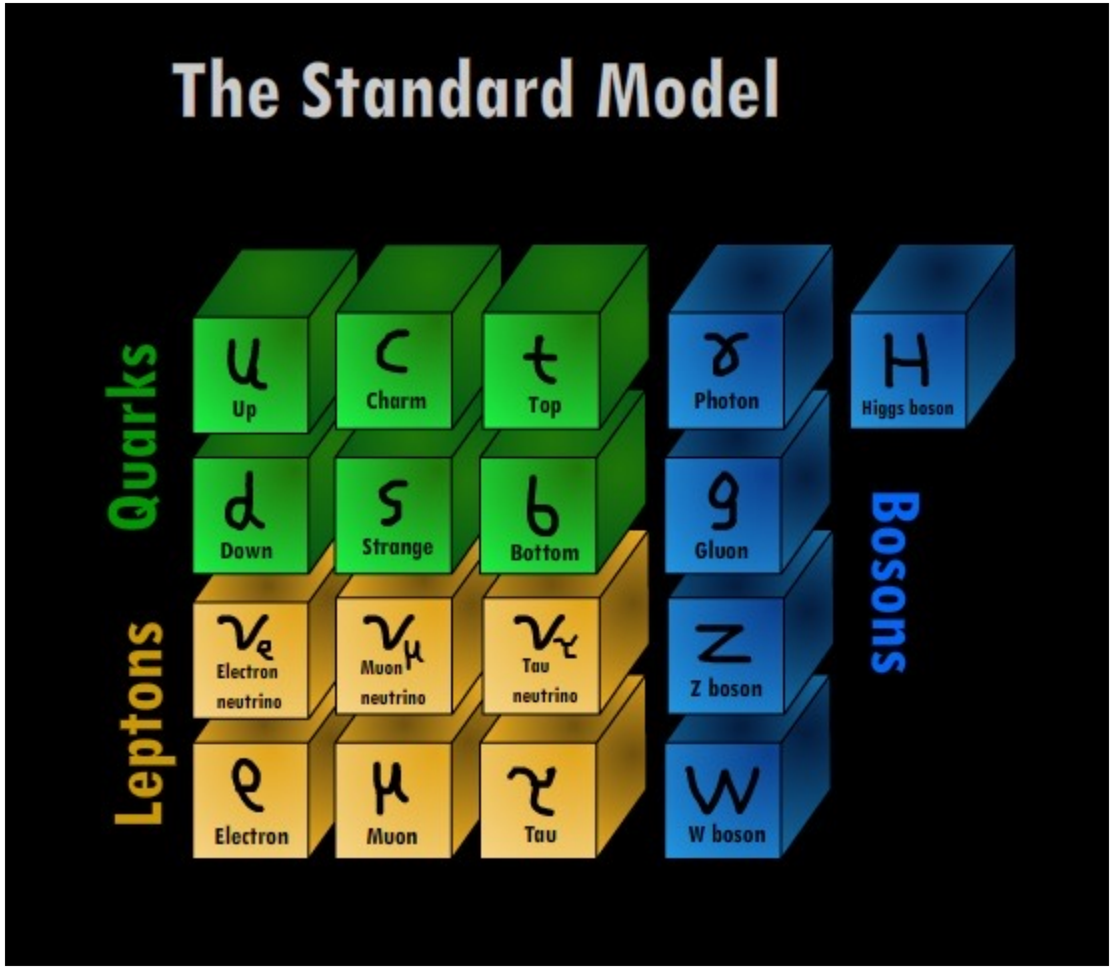
Lecture 1: Extra Material

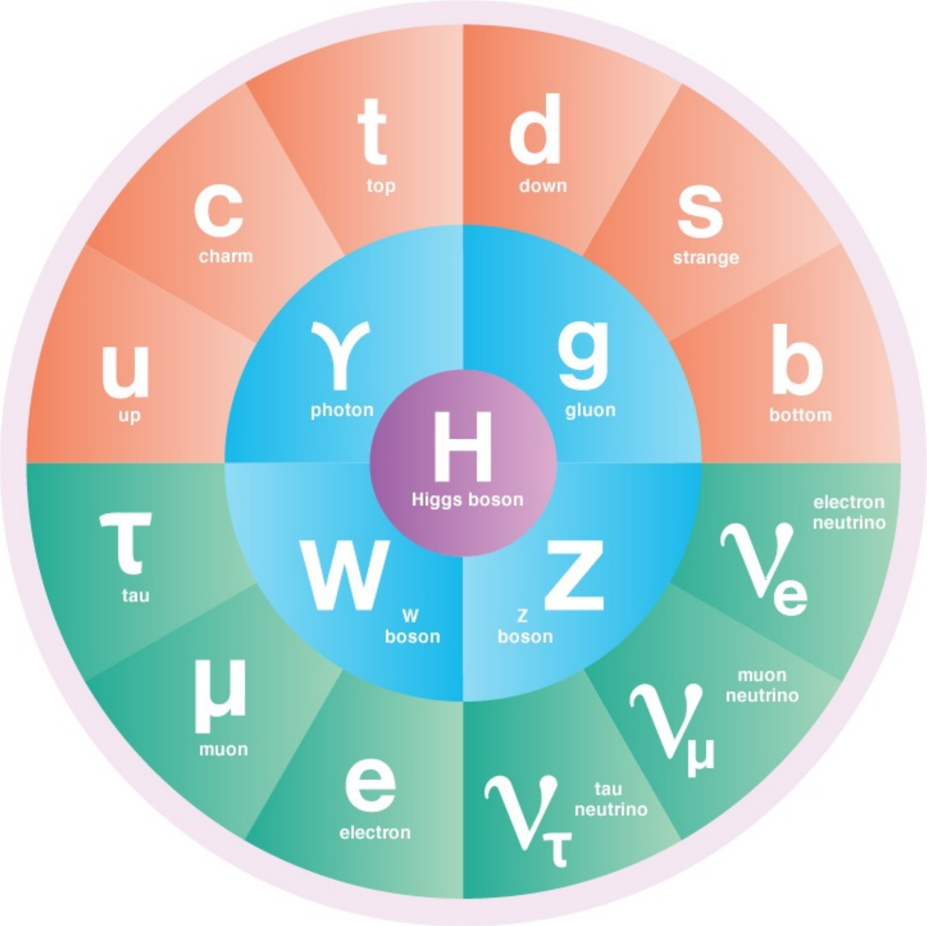
Just some random slides

Graphical views of the Standard Model of particles

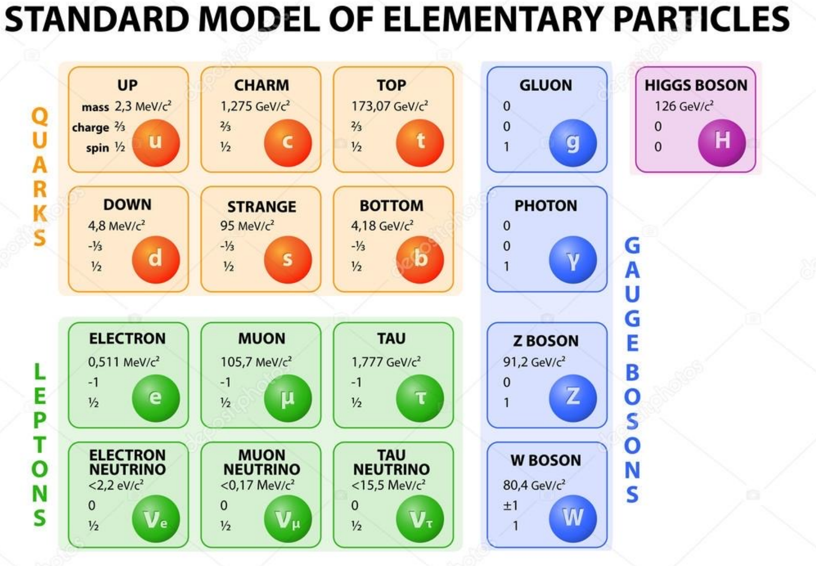


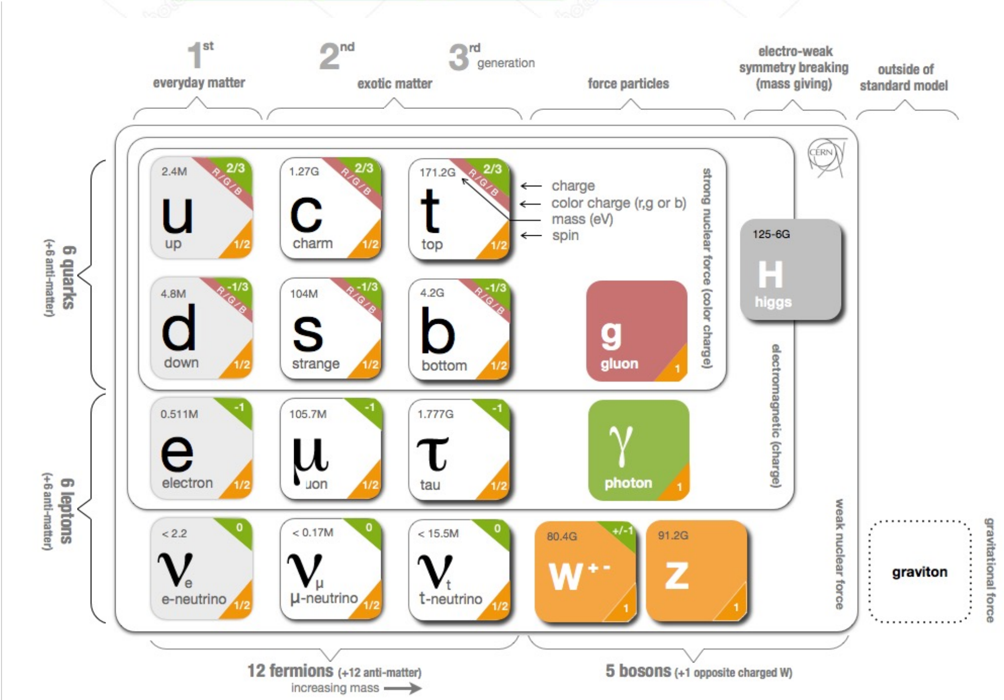
Graphical views of the Standard Model of particles

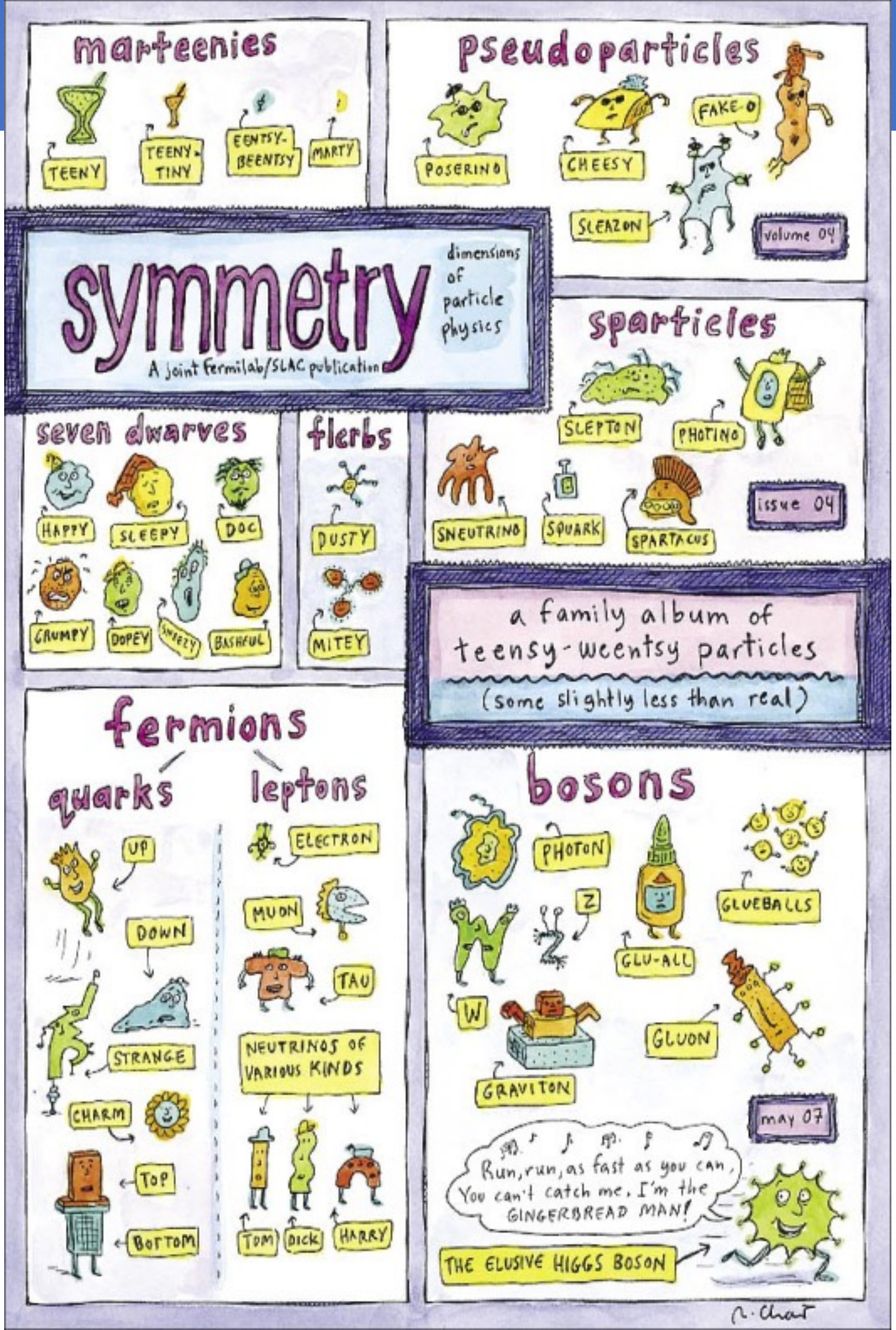




Graphical views of the SM

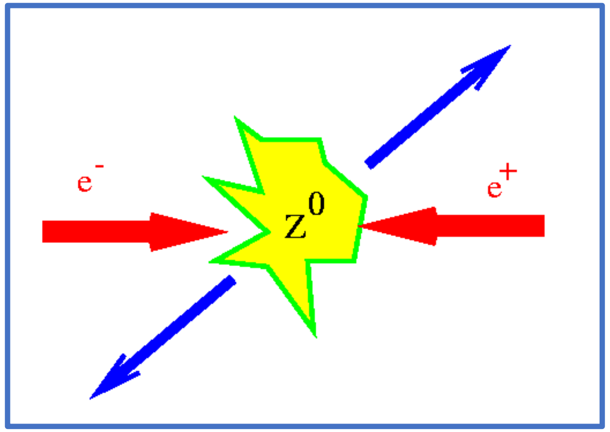


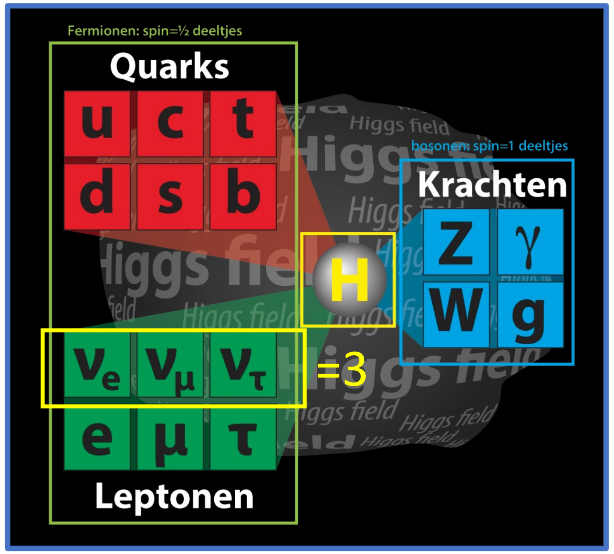


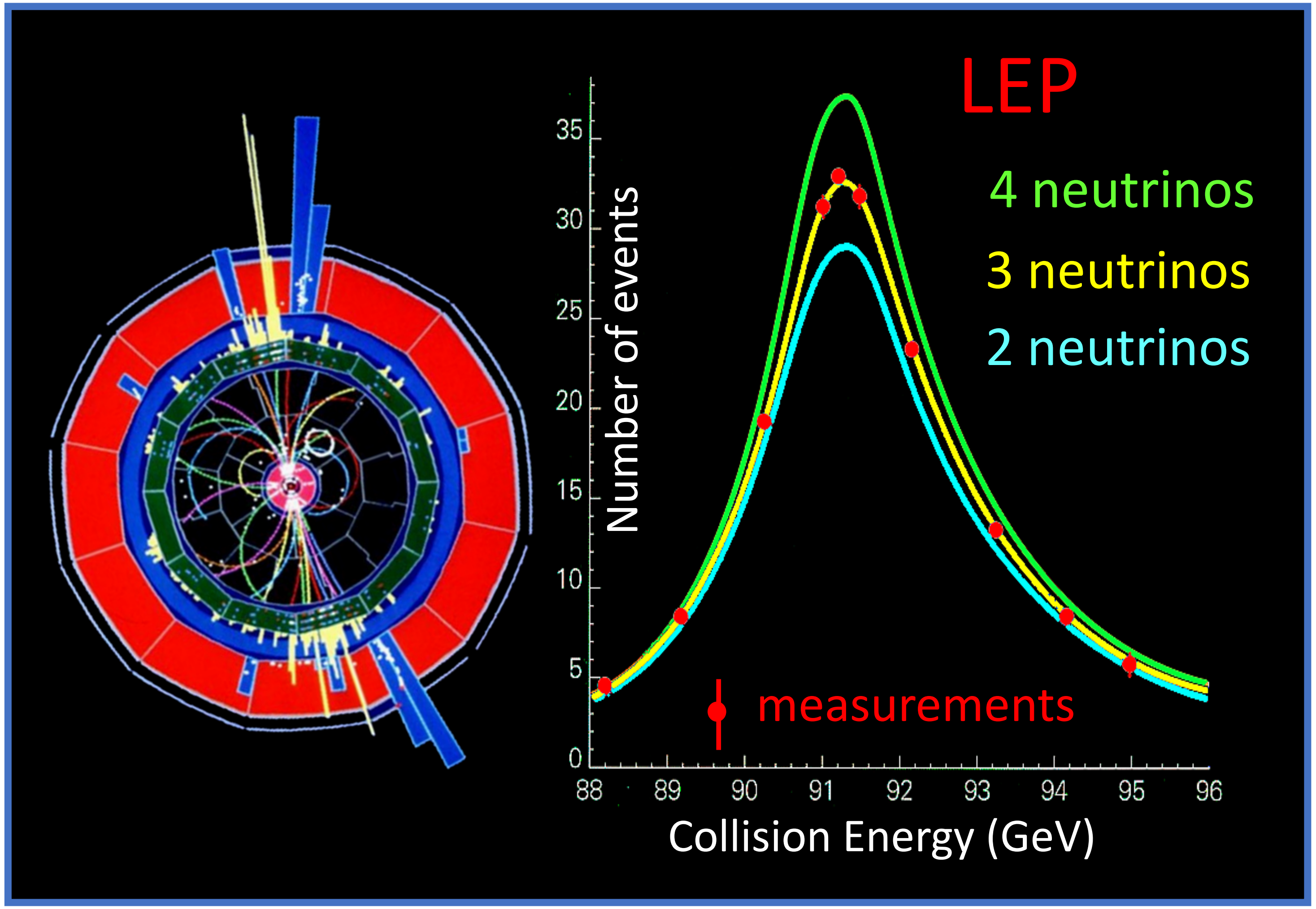


3 Generations of particles – How do we know?

LEP: The heavy Z boson decays into 3 light neutrino types.





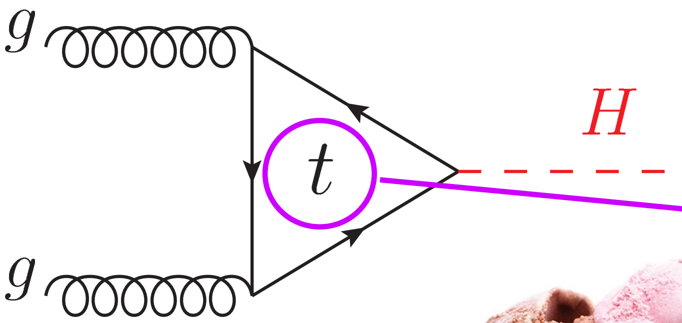


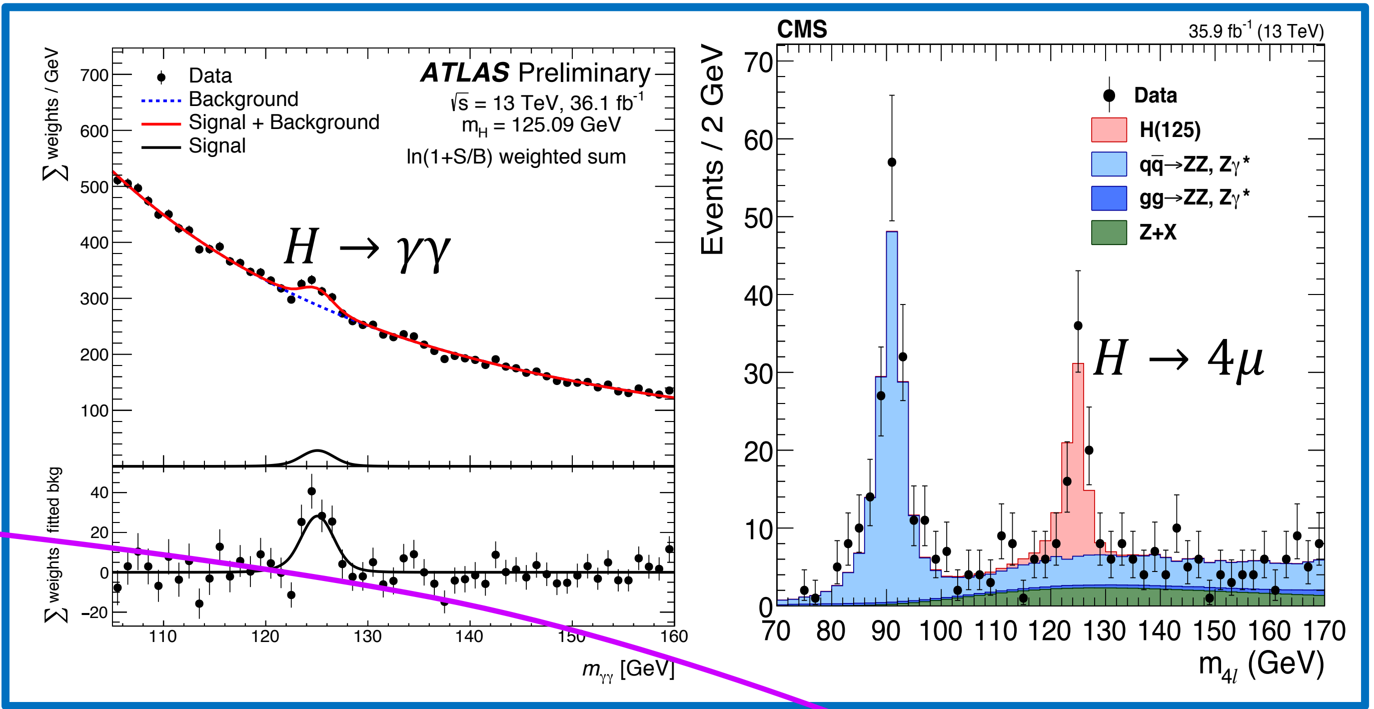
- *No additional weakly interacting **light fermion** generations.*

3 Generations of particles – How do we know?

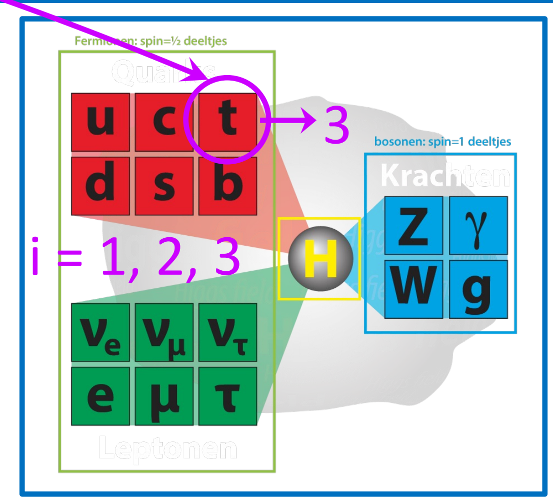
LHC: Higgs production:

Loop diagram is proportional to the mass of the heaviest fermion.





- *Top is the **heaviest fermion** flavour.*
  - *3 Flavour generations*



Lecture 1: Discussion Topics

Discussions Topics belonging to
Lecture 1

Topic-1: Natural Units:

Griffiths: p. 9-10

- Redefine the unit $\hbar = \frac{h}{2\pi} \approx 1.055 \times 10^{-34}$ Js to be: $\hbar \equiv 1$
- Redefine the unit $c = 2.998 \times 10^{8}$ m/s to be: $c \equiv 1$
- Explain how Energy, mass, distance, time can then be expressed in the unit GeV
- How can you get answers that can be compared with measurements?
- What are the advantages of doing this?

Topic-1: Natural Units:

Griffiths: p. 9-10

quantity	symbol in natural units	equivalent symbol in ordinary units
space	x	$x/\hbar c$
time	t	$t/\hbar$
mass	m	mc^2
momentum	p	pc
energy	E	E
positron charge	e	$e\sqrt{\hbar c/\epsilon_0}$

Conversion of basic quantities between natural and ordinary units.

quantity	conversion factor	natural unit	normal unit
mass	$1\ \mathrm{kg} = 5.61 \times 10^{26}\ \mathrm{GeV}$	GeV	GeV/c^2
length	$1\ \mathrm{m} = 5.07 \times 10^{15}\ \mathrm{GeV}^{-1}$	GeV^{-1}	$\hbar c/\ \mathrm{GeV}$
time	$1\ \mathrm{s} = 1.52 \times 10^{24}\ \mathrm{GeV}^{-1}$	GeV^{-1}	$\hbar/\ \mathrm{GeV}$

Conversion factors from natural units to ordinary units.

Topic-2: The Lorentz Transformation

- Why are space and time coordinates not universal (ie not the same for each observer)?
- Explain the Lorentz transformation
- When does this effect become noticeable?

Topic-2: The Lorentz Transformation

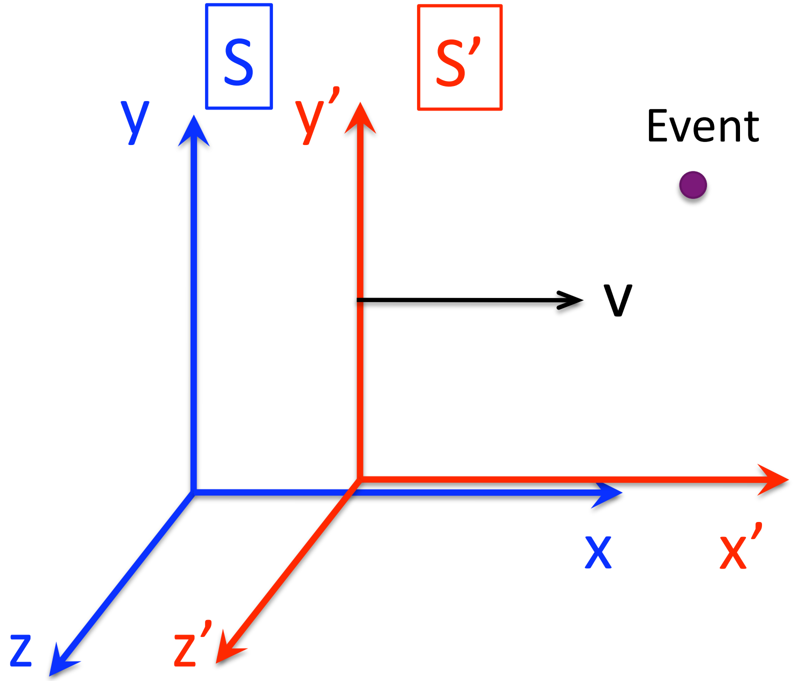
A reference system or coordinate system is used to determine the time and position of an event.

Reference system S is linked to observer Alice at position (x,y,z) = (0,0,0)
An event is fully specified by giving its coordinates and time: (t, x, y, z)

Reference system S' is linked to observer Bob who is moving with velocity v with respect to Alice.
The event has: (t', x', y', z')

How are the coordinates of an event, say a lightning strike in a tree, expressed in coordinates for Alice and for Bob?

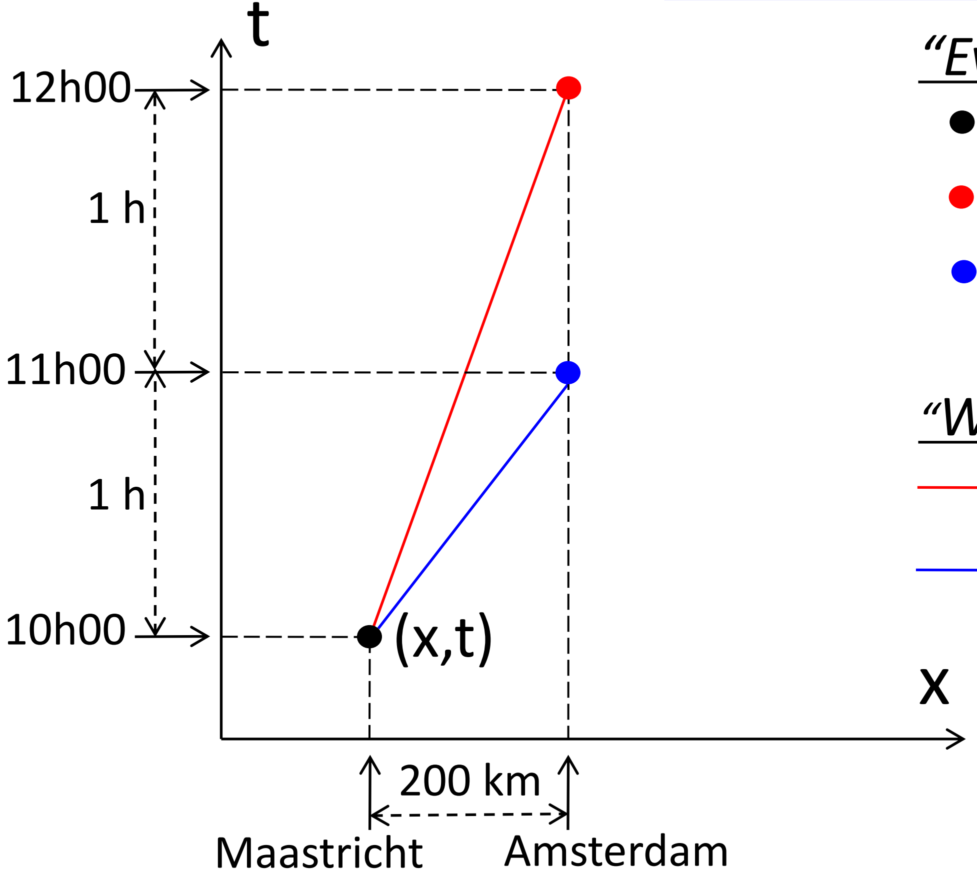
(t, x, y, z) → (t', x', y', z')



Topic-2: Space-time diagram

Bob drives from Maastricht to Amsterdam with 100 km/h.

Alice drives from Maastricht to Amsterdam with 200 km/h.



“Events”:

- ● Departure Alice & Bob
- ● Arrival Bob
- ● Arrival Alice

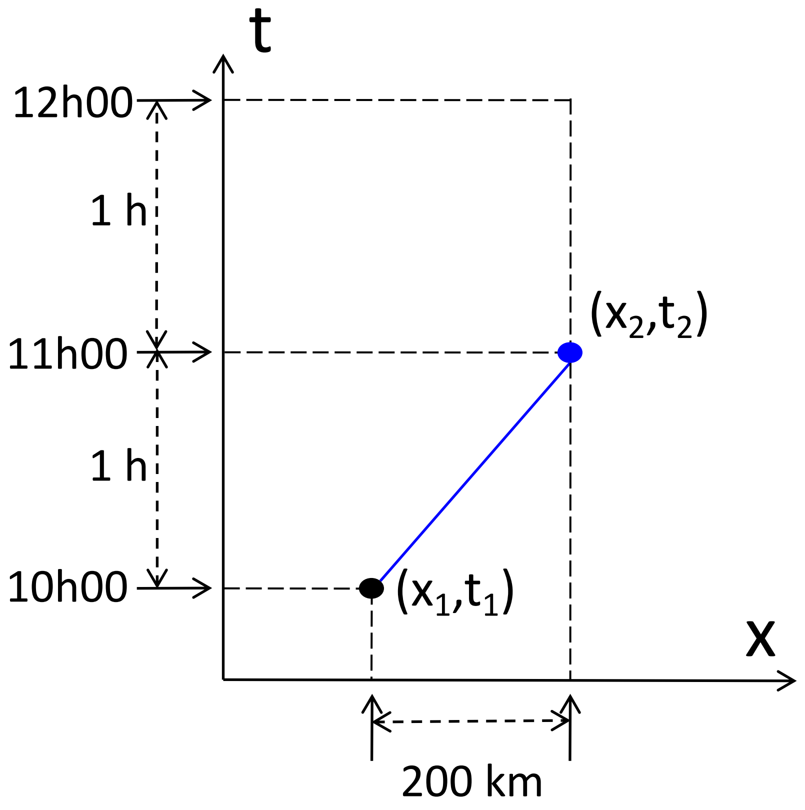
“World lines”:

- — Bob’s world-line
- — Alice’s world-line

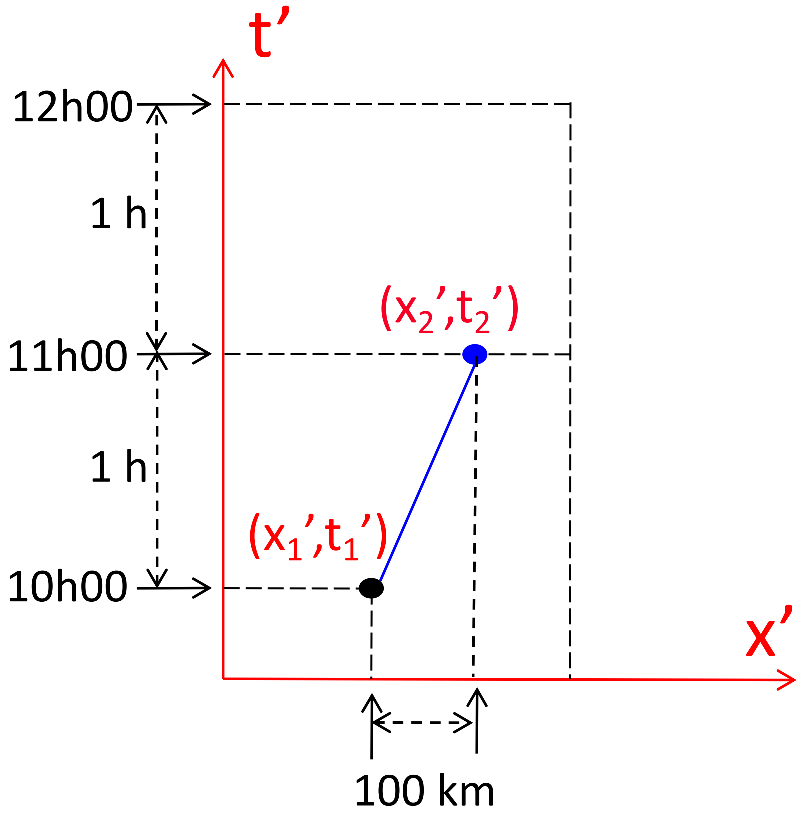
Events with space-time coordinates: (x,t)
More general: it is a 4-dimensional space: (x,y,z,t)

Topic-2: Coordinate transformation

How does Alice's trip look like in the coordinates of the reference system of Bob?



Alice as seen from Maastricht
S = fixed reference system in Maastricht

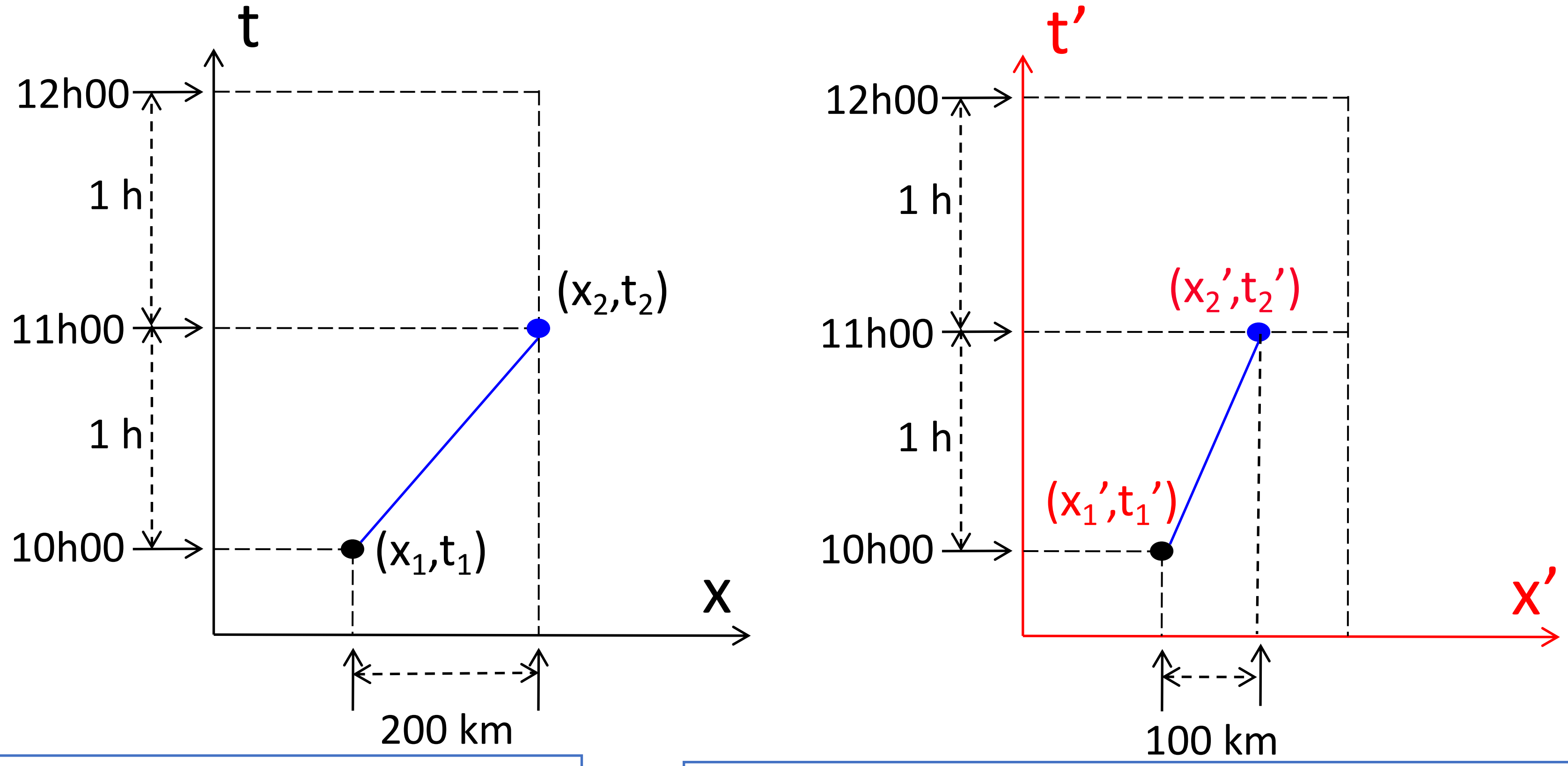


Alice as seen from Bob
S' = fixed reference to Bob

Bob's frame moves with velocity v (100km/h) with respect to Maastricht

Topic-2: Coordinate transformation

How does Alice's trip look like in the coordinates of the reference system of Bob?



Classical (Gallilei Transformation):

$$\begin{aligned} t' &= t \\ x' &= x - v\,t \end{aligned}$$

Relativistic (Lorentz Transformation):

$$\begin{aligned} t' &= \gamma\left(t - \frac{v}{c^2}\,x\right) \\ x' &= \gamma\,(x - v\,t) \end{aligned}$$

with: $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

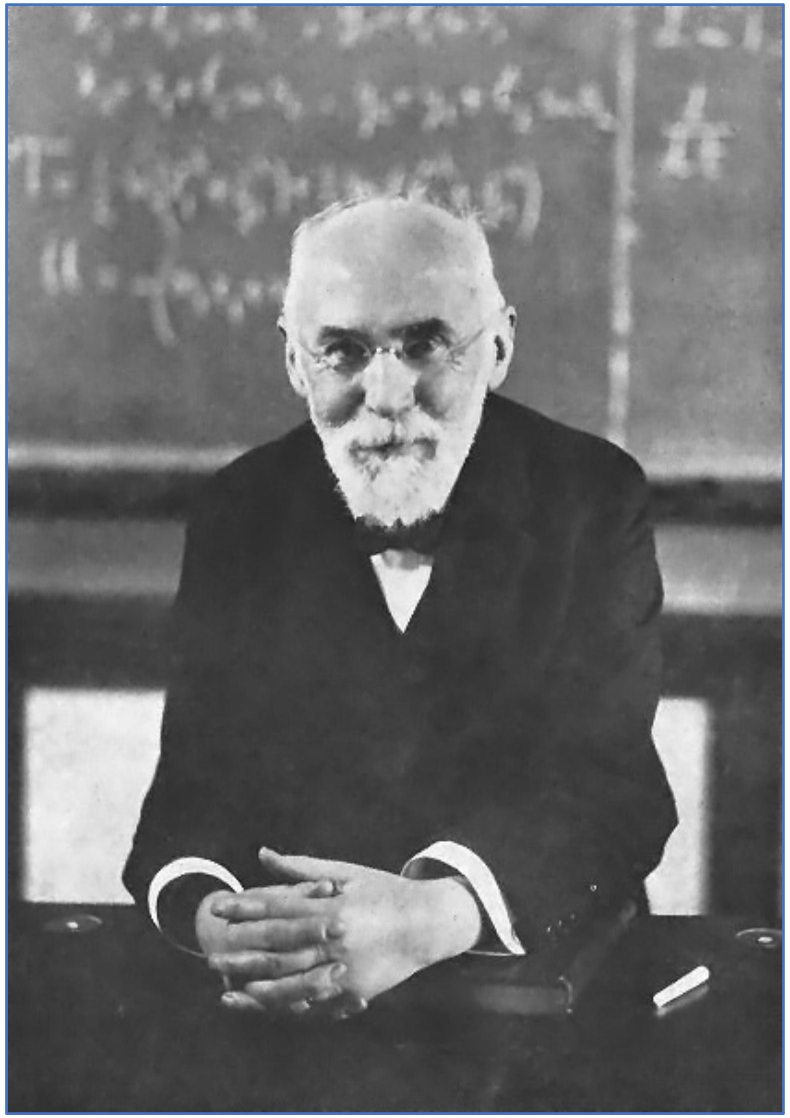
Topic-2: Lorentz Transformations

Hendrik Anton Lorentz (1853 – 1928)

Dutch Physicist in Leiden
(Nobelprize 1902 with Pieter Zeeman)

To explain the Michelson-Morley experiment he assumed that bodies contracted due to intermolecular forces as they were moving through the aether.
(He believed in the existence if the aether)

Einstein derived it from the relativity principle and also saw that time has to be modified.



Topic-2: Let's go crazy and derive them...

Start with classical **Galilei Transformation**:

$$\boxed{x' = x - vt}$$

$$x = x' + vt'$$

Let's try a modification by including a factor:

$$x' = f(x - vt)$$

$$x = f(x' + vt')$$

For light: $x = ct$ and $x' = ct'$, so:

$$ct' = f(ct - vt)$$

$$ct = f(ct' + vt')$$

Then: $$t' = f\left(\frac{c-v}{c}\right)t$$

$$t = f\left(\frac{c+v}{c}\right)t'$$

Substitute first into second:

$$t = f\left(\frac{c+v}{c}\right) f\left(\frac{c-v}{c}\right) t$$

Divide by t: $$1 = \left(\frac{c+v}{c}\right)\left(\frac{c-v}{c}\right) f^2 = \left(\frac{c^2-v^2}{c^2}\right) f^2$$

It follows then that: $$f^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - v^2/c^2}$$

So that we find: $$f = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$$

Therefor we have derived the **Lorentz transformation**: $$\boxed{x' = \gamma(x - vt)}$$

Similarly we find the Lorentz transformation for time:
(see lecture notes)

$$\boxed{t' = \gamma\left(t - \frac{v}{c^2}x\right)}$$

whereas the **Galilei translation** was:

$$\boxed{t' = t}$$

Topic-3: Four vectors & co- and contra-variance

- Explain the so-called covariant (ie 4-vector) notation.
- What is the difference between contra-variant and co-variant?
- Explain Einstein’s summation convention for indices

Topic-3: Four vectors & co- and contra-variance

See Griffiths: chapter 3

- Four vector: $x^{\mu} = (x^0, x^1, x^2, x^3)$ with $x^0 = ct \;\; \Rightarrow \; x^0 = t$ (use $c \equiv 1$ convention)
- We call this a ***contravariant*** vector and: $x^{\mu} = (x^0, \vec{x}\,)$
- Lorentz transformation along x^1axis using $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$ is:

$$x^{0\prime} = \gamma(x^0 - \beta x^1)$$
$$x^{1\prime} = \gamma(x^1 - \beta x^0)$$
$$x^{2\prime} = x^2$$
$$x^{3\prime} = x^3$$

Write it in Matrix notation:

$$x^{\mu\prime} = \Lambda^{\mu}_{\nu} x^{\nu} \;\; ; \;\; \Lambda^{\mu}_{\nu} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Lorentz transformations leave the "length" s invariant $\mathrm{s} = x \cdot x = |x|^2 = {x^0}^2 - |\vec{x}|^2$
  - Explicitly: $(ct)^2 - |\vec{x}|^2 = (ct')^2 - |\vec{x}'|^2 = s$ is invariant.

"Metric tensor"

- This can be written as:

$$s = (x^0, x^1, x^2, x^3) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} (x^0, x^1, x^2, x^3)$$

Topic-3: Scalar product and co- and contra-variant

- Define ***co-variant*** vectors: $x_\mu = (x^0, -\vec{x}\,)$
- Define the metric tensor: $g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$
- To obtain a co-variant from contra-variant vector:

$$x_\mu = \sum_\mu g_{\mu\nu} x^\nu \ = \ g_{\mu\nu} x^\nu$$

- Then the invariance distance $s = x^\mu x_\mu = \sum_{\mu\nu} x^\mu g_{\mu\nu} x^\nu = {x^0}^2 - |\vec{x}|^2$
- We speak of a scalar product: $A \cdot B = A^\mu B_\mu \ = g_{\mu\nu} A^\mu B^\nu$, where a sum is always implicit over *contravariant* and *covariant* indices.
- The ***scalar product*** or ***inproduct*** of Lorentz 4-vectors is always Lorentz invariant:
  - $I = a_\mu b^\mu = a \cdot b$ for any Lorentz 4-vectors a^μ and b^μ
  - Example are the space-time vectors a^μ , but also the 4-momentum vector $p^\mu = (E, \vec{p})$
  - $E^2 = \vec{p}^2 c^2 + m^2 c^4 \ \Rightarrow \ p^\mu = (E, \vec{p}) \ \Rightarrow \ p_\mu p^\mu = E^2 - \vec{p}^2 = m^2$ ***the invariant mass***

Definition:
A contravariant vector transforms like x_μ and a covariant vector transforms like Note the Einstein summation convention!

Note the Einstein summation convention!

Topic-3: Co- and contravariant derivatives

Griffiths: chapter 3

- Contravariant vector:

$$x^{\mu} = (ct, \vec{x})$$

But covariant derivative:

$$\partial^{\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}\right)$$

- Covariant vector:

$$x_{\mu} = (ct, -\vec{x})$$

But covariant derivative:

$$\partial_{\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

Note that the minus sign is “opposite” to the case of the coordinate four-vectors.

Lecture 1: Exercises

Exercises belonging to Lecture 1

Exercise-1: Natural Units

Griffiths: p. 9-10, PP1: p 12-13

- In particle physics we make often use of natural units
  - Very confusing at first but very convenient when you are used to it ("sloppy")
    Set $c = 2.998\times10^{8}\ m/s \equiv 1$ and $\hbar = 1.055\times10^{-34}\ Js \equiv 1$
    (Just leave them out and put them back at very end of any calculation)
  - Consequence: there is only one basic unit for length, time, mass and energy: GeV

- Exercise: derive the numbers on the conversion table on the next page

Natural Units: conversion factors

Griffiths: p. 9-10, PP1: p 12-13

quantity	symbol in natural units	equivalent symbol in ordinary units
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Conversion of basic quantities between natural and ordinary units.

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time	$1\text{ s} = 1.52 \times 10^{24}\text{ GeV}^{-1}$	GeV^{-1}	$\hbar/\text{ GeV}$

Conversion factors from natural units to ordinary units.

Exercise-2 : The Yukawa Potential

- The electric force is transmitted by a photon with $m = 0$. The wave equation for a static electric field caused by pointlike charge e is: $\nabla^2 V(r) = 0$ (Laplace equation)

a) Show that the Coulomb potential $V(r) = -e^2 \frac{1}{r}$ fulfills this equation.

  - Note that the potential is ***spherical symmetric,*** ie. use spherical coordinates.

- The nuclear force is transmitted by a pi-meson with $m = m_\pi$. The wave equation for a static nuclear field caused by a pointlike color charge g is: $\nabla^2 U(r) = m^2 U(r)$ (Klein-Gordon equation)

b) Show that the Yukawa potential $U(r) = -g^2 \frac{e^{-r/R}}{r}$ fulfills this equation for a certain value of R, the ***range*** of the force. What is the relation between R and m_π?

  - Again note that the potential is ***spherical symmetric.***
  - This value is between the electron and proton mass, hence the particle was called a pi-meson or pion.

c) Calculate the range of the force from Heisenberg's uncertainty relation, using $R = c\Delta t$ and $\Delta E \Delta t \leq \frac{\hbar}{2}$ and $\Delta E = mc^2$.

d) The weak force is mediated by W (80 GeV) and Z (91 GeV) bosons. What is the estimated range of the weak force?

Exercise-3: Kinematics: Z-boson production

- The Z-boson particle is a carrier of the weak force. It has a mass of 91.1 GeV. It can be produced experimentally by annihilation of an electron and a positron. The mass of an electron, as well as that of a positron, is 0.511 MeV.
  a) Assume that an electron and a positron are accelerated in opposite directions and collide head-on to produce a Z-boson in the lab frame. Calculate the minimal beam energy required for the electron and the positron in order to produce a Z-boson.
  b) Assume that a beam of positron particles is shot on a target containing electrons. Calculate the beam energy required for the positron beam in order to produce Z-bosons.
  c) This experiment was carried out in the 1990's. Which method do you think was used? Why?