Lecture 4: Exercises

Exercises belonging to Lecture 4

Exercise – 13 : Charge Current

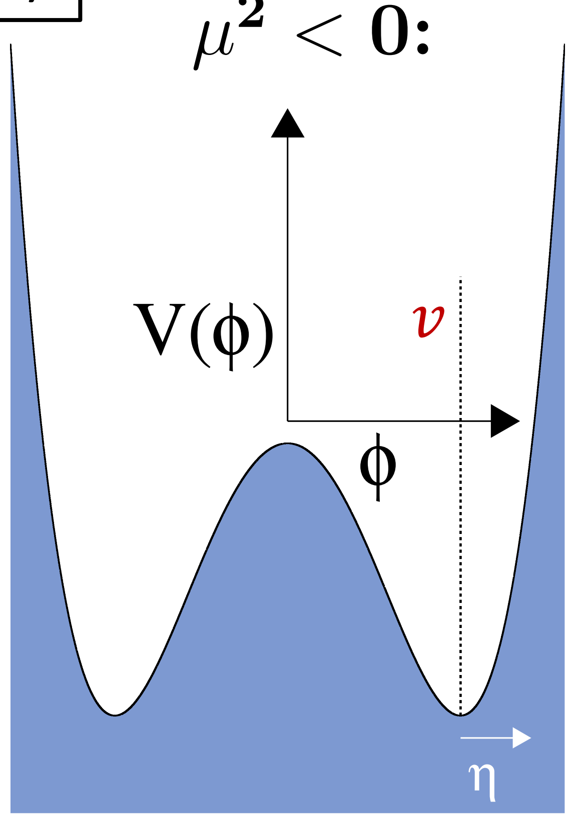
- Show that the definition $W_\mu^\pm = \frac{b_\mu^1 \mp i b_\mu^2}{\sqrt{2}}$ leads to the charged current:

$\mathcal{L} = -W_\mu^+ J^{\mu^+} - W_\mu^- J^{\mu^-}$ with $J^{\mu^+} = \frac{g}{\sqrt{2}} \overline{\Psi} \gamma_\mu \tau^+ \Psi$ and $J^{\mu^-} = \frac{g}{\sqrt{2}} \overline{\Psi} \gamma_\mu \tau^- \Psi$

Exercise – 14 : Symmetry breaking

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

Case B)



- Redefine coordinates: $\eta \equiv \phi - v$
- Exercise: re-write the Lagrangian in η and v to show:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda v^4$$

- Ignore the constant term $\frac{1}{4}\lambda v^4$ and neglect higher order η^3:

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2\eta^2$$

- This describes a new scalar field η with a mass $\frac{1}{2}m_\eta^2 = \lambda v^2 \Rightarrow m_\eta = \sqrt{2\lambda v^2}$ $(= \sqrt{-2\mu^2})$
- Price to pay: Lagrangian is no longer symmetric under $\eta \to -\eta$ in the new field.

Exercise – 15 : Mass of the proton

Besides giving mass to the weak vector bosons, it was briefly flashed that the same Higgs mechanism is responsible for giving mass to the fermion masses in the Standard Model, through ad-hoc Yukawa couplings. The mass of a 'naked' quark can be estimated through models of soft QCD, where it enters as a parameter for e.g. the binding energy of a meson. For up and down, they are found to be roughly 2 resp. 5 MeV/c .

a) What fraction of the proton mass is due to the Higgs mass of the constituent quarks?

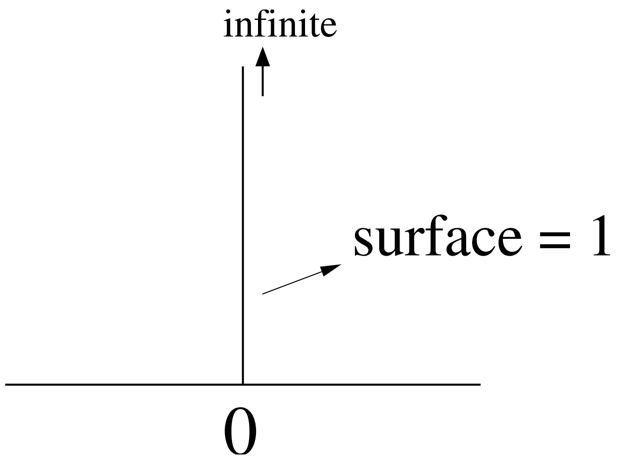
b) Can you find out where the other part of the proton mass comes from?

Exercise – 16: Dirac delta function (1)

See Griffiths Appendix A

- Consider a function defined by the following prescription:

$$\delta(x) = \lim_{\Delta \to 0} \begin{cases} 1/\Delta & \text{for } |x| < \Delta/2 \\ 0 & \text{otherwise} \end{cases}$$



- The integral of this function is normalized: $\int_{-\infty}^{\infty} \delta(x)\, \mathrm{d}x = 1$
- For a function $f(x)$ we have: $f(x)\delta(x) = f(0)\delta(x)$

…and therefore: $\int_{-\infty}^{\infty} f(x)\delta(x)\, \mathrm{d}x = f(0) \int_{-\infty}^{\infty} \delta(x)\, \mathrm{d}x = f(0)$

- Exercise:
  a) Prove that: $\delta(kx) = \frac{1}{|k|}\delta(x)$
  b) Prove that: $\delta\big(g(x)\big) = \sum_{i=1}^{n} \frac{1}{|g'(x_i)|}\delta(x - x_i)$, where $g(x_i) = 0$ are the zero-points
     - Hint: make a Taylor expansion of g around the 0-points.

Exercise – 16: Dirac delta function (2)

- The delta function has many forms. One of them is: $\delta(x) = \lim_{\alpha\to\infty} \frac{1}{\pi}\frac{\sin^2 \alpha x}{\alpha x^2}$

c) Make this plausible by sketching the function $\sin^2(\alpha x)\,/(\pi\alpha x^2)$ for two relevant values of α

- Remember the Fourier transform:

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(k)\, e^{ikx}\, \mathrm{d}k$$

$$g(k) = \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, \mathrm{d}x$$

d) Use this to show that another (important!) representation of the Dirac delta-function is given by:

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}\, \mathrm{d}k$$

🡐 We will use this later in the lecture!