Lecture 1: Exercises

Exercises belonging to Lecture 1

Exercise-1: Natural Units

Griffiths: p. 9-10, PP1: p 12-13

- In particle physics we make often use of natural units
  - Very confusing at first but very convenient when you are used to it (“sloppy”)
    Set $c = 2.998 \times 10^{8}\, m/s \equiv 1$ and $\hbar = 1.055 \times 10^{-34}\, Js \equiv 1$
    (Just leave them out and put them back at very end of any calculation)
  - Consequence: there is only one basic unit for length, time, mass and energy: GeV

- Exercise: derive the numbers on the conversion table on the next page

Natural Units: conversion factors

Griffiths: p. 9-10, PP1: p 12-13

quantity	symbol in natural units	equivalent symbol in ordinary units
space	x	$x/\hbar c$
time	t	$t/\hbar$
mass	m	mc^2
momentum	p	pc
energy	E	E
positron charge	e	$e\sqrt{\hbar c/\epsilon_0}$

Conversion of basic quantities between natural and ordinary units.

quantity	conversion factor	natural unit	normal unit
mass	$1\text{ kg} = 5.61 \times 10^{26}\text{ GeV}$	GeV	GeV/c^2
length	$1\text{ m} = 5.07 \times 10^{15}\text{ GeV}^{-1}$	GeV^{-1}	$\hbar c/\text{ GeV}$
time	$1\text{ s} = 1.52 \times 10^{24}\text{ GeV}^{-1}$	GeV^{-1}	$\hbar/\text{ GeV}$

Conversion factors from natural units to ordinary units.

Exercise-2 : The Yukawa Potential

- The electric force is transmitted by a photon with $m = 0$. The wave equation for a static electric field caused by pointlike charge e is: $\nabla^2 V(r) = 0$ (Laplace equation)

a) Show that the Coulomb potential $V(r) = -e^2 \frac{1}{r}$ fulfills this equation.

- Note that the potential is ***spherical symmetric,*** ie. use spherical coordinates.

- The nuclear force is transmitted by a pi-meson with $m = m_\pi$. The wave equation for a static nuclear field caused by a pointlike color charge g is: $\nabla^2 U(r) = m^2 U(r)$ (Klein-Gordon equation)

b) Show that the Yukawa potential $U(r) = -g^2 \frac{e^{-r/R}}{r}$ fulfills this equation for a certain value of R, the ***range*** of the force. What is the relation between R and m_π?

- Again note that the potential is ***spherical symmetric.***
- This value is between the electron and proton mass, hence the particle was called a pi-meson or pion.

c) Calculate the range of the force from Heisenberg's uncertainty relation, using $R = c\Delta t$ and $\Delta E \Delta t \leq \frac{\hbar}{2}$ and $\Delta E = mc^2$.

d) The weak force is mediated by W (80 GeV) and Z (91 GeV) bosons. What is the estimated range of the weak force?

Exercise-3: Kinematics: Z-boson production

- The Z-boson particle is a carrier of the weak force. It has a mass of 91.1 GeV. It can be produced experimentally by annihilation of an electron and a positron. The mass of an electron, as well as that of a positron, is 0.511 MeV.
  a) Assume that an electron and a positron are accelerated in opposite directions and collide head-on to produce a Z-boson in the lab frame. Calculate the minimal beam energy required for the electron and the positron in order to produce a Z-boson.
  b) Assume that a beam of positron particles is shot on a target containing electrons. Calculate the beam energy required for the positron beam in order to produce Z-bosons.
  c) This experiment was carried out in the 1990's. Which method do you think was used? Why?