Lecture 3: Discussion Topics

Discussions Topics belonging to Lecture 3

When you are assigned a topic, prepare to lead a discussion on the subject with the tutor group.
You are expected to introduce the topic, prepare a few slides or write on the board,
and be somewhat of an expert.
At the same time you do not have to know everything. You may also address questions to the tutor group.

Topic-7: Lorentz and Dirac indices

Griffiths §4.2 – §4.3

- Explain the difference between Lorentz indices and Dirac indices
- Is γ^{μ} a four-vector? Why (not)?
- Is $j^{\mu} = \bar{\psi}\gamma^{\mu}\psi$ a four-vector? Why (not)?

Topic-8: Helicity and Chirality

Griffiths §4.2 – §4.3

- Explain the difference between helicity and chirality
  - How is each one defined?
- Which of the two is Lorentz invariant?
- Which one of the two do we refer to when talking about the left or right *handedness* of a particle?

Topic-8: Helicity vs Chirality

a) Write out the chirality operator γ^5 in the Dirac-Pauli representation.

b) The helicity operator is defined as $\lambda = \frac{1}{2}\vec{\Sigma}\cdot\hat{p}$. Show that helicity operator and the chirality operator have the same effect on a spinor solution, i.e.

$$\gamma^5\psi = \gamma^5\begin{pmatrix}\chi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)}\end{pmatrix} \approx \lambda\begin{pmatrix}\chi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)}\end{pmatrix} = \lambda\psi$$

with: $\chi^{(1)} = \binom{1}{0}$; $\chi^{(2)} = \binom{0}{1}$

in the relativistic limit where $E \gg m$

c) Show explicitly that for a Dirac spinor:

$\bar{\psi}\gamma^\mu\psi = \overline{\psi_L}\gamma^\mu\psi_L + \overline{\psi_R}\gamma^\mu\psi_R$ making use of $\psi = \psi_L + \psi_R$ and the projection operators: $\psi_L = \frac{1}{2}(1-\gamma^5)$ and $\psi_R = \frac{1}{2}(1+\gamma^5)$

d) Explain why the weak interaction is called left-handed.

Wolfgang Pauli

"I cannot believe God is a weak left-hander."

Topic-9: Maxwell's equation

- Maxwell's equations can be described relativistically with the 4-vector field A^μ.
- Show how you get E and B fields from A^μ
- Explain the concept of gauge invariance.
- Is the A^μ field physical or not?
- The photon is a spin-1 quantum, but why can it not have a spin-0 component?

Topic-9: The photon field and gauge invariance

- Field A^μ is just introduced as a mathematical tool
  - Gauge freedom: you are free to choose any A^μ as long as $\vec{E}$ and $\vec{B}$ fields don't change:

$$A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu \lambda$$

$$V \rightarrow V' = V + \frac{\partial \lambda}{\partial t}$$

$$\vec{A} \rightarrow \overrightarrow{A'} = \vec{A} - \vec{\nabla}\lambda$$

- Choose the Lorentz gauge condition: $\partial_\mu A^\mu = 0$
- Maxwell equation in Lorentz gauge becomes:

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = j^\nu \quad \rightarrow \quad \partial_\mu \partial^\mu A^\nu = j^\nu$$

  - Very similar to Klein-Gordon equation $\partial_\mu \partial^\mu \phi + m^2 \phi = 0$
    - But now 4-equations → 4 polarizations states of the photon field??
- Photon field solutions: $A^\mu(x) = N\varepsilon^\mu(p)e^{-ip_\nu x^\nu}$
  - A gauge transformation implies: $\varepsilon^\mu \rightarrow \varepsilon'^\mu = \varepsilon^\mu + ap^\mu$
  - Different polarization vectors which differ by multiple of p^μ describe same photon
    - Only 3 degrees of freedom remain → 3 polarization states: spin: -1, 0, 1 → choose $\varepsilon^0 = 0$
- Mass of the photon is zero:
  - Thus $p^\mu p_\mu = 0 \rightarrow \varepsilon^\mu p_\mu \rightarrow \vec{\varepsilon} \cdot \vec{p} = 0$
  - Now only two transverse polarization states remain: Chose $\vec{p} = (0,0,p) \rightarrow \vec{\varepsilon}^1 = (1,0,0)$ and $\vec{\varepsilon}^2 = (0,1,0)$