Lecture 2: Discussion Topics

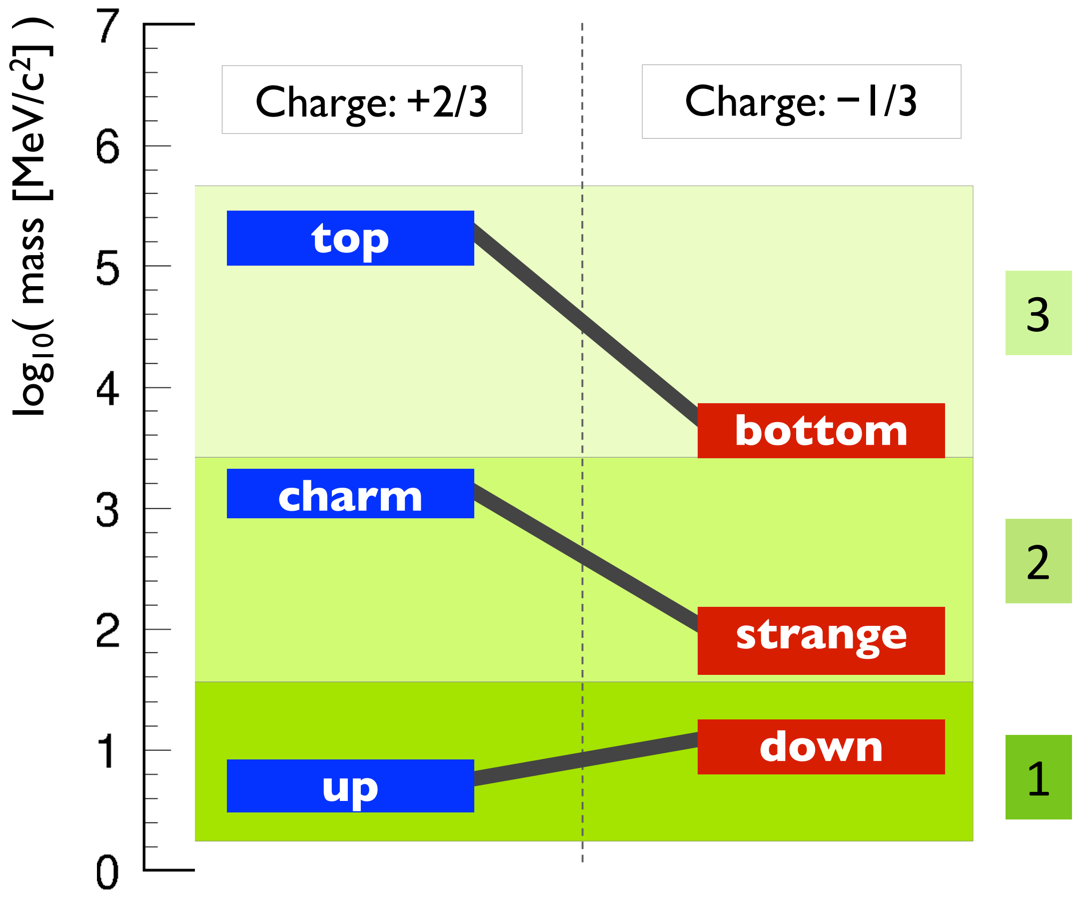
Discussions Topics belonging to Lecture 2

When you are assigned a topic, prepare to lead a discussion on the subject with the tutor group.
You are expected to introduce the topic, prepare a few slides or write on the board,
and be somewhat of an expert.
At the same time you do not have to know everything. You may also address questions to the tutor group.

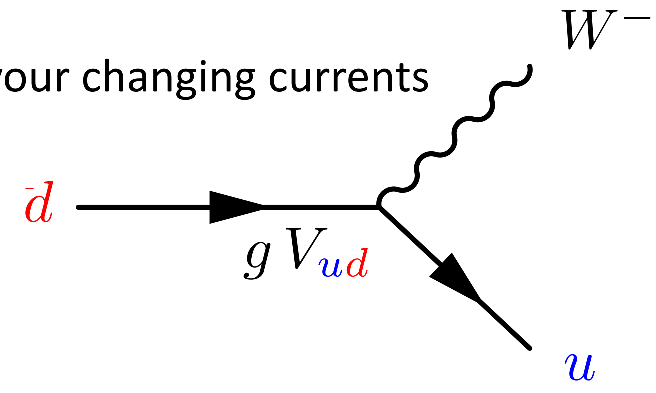
Topic-4: Flavour changing Weak interaction

- What are neutral currents?; what are charged currents?
- Which transitions are allowed by the charged current?
- What is the consequence for stability of 2nd and 3rd generation particles?

Topic-4: Flavour Changing Quark Interactions

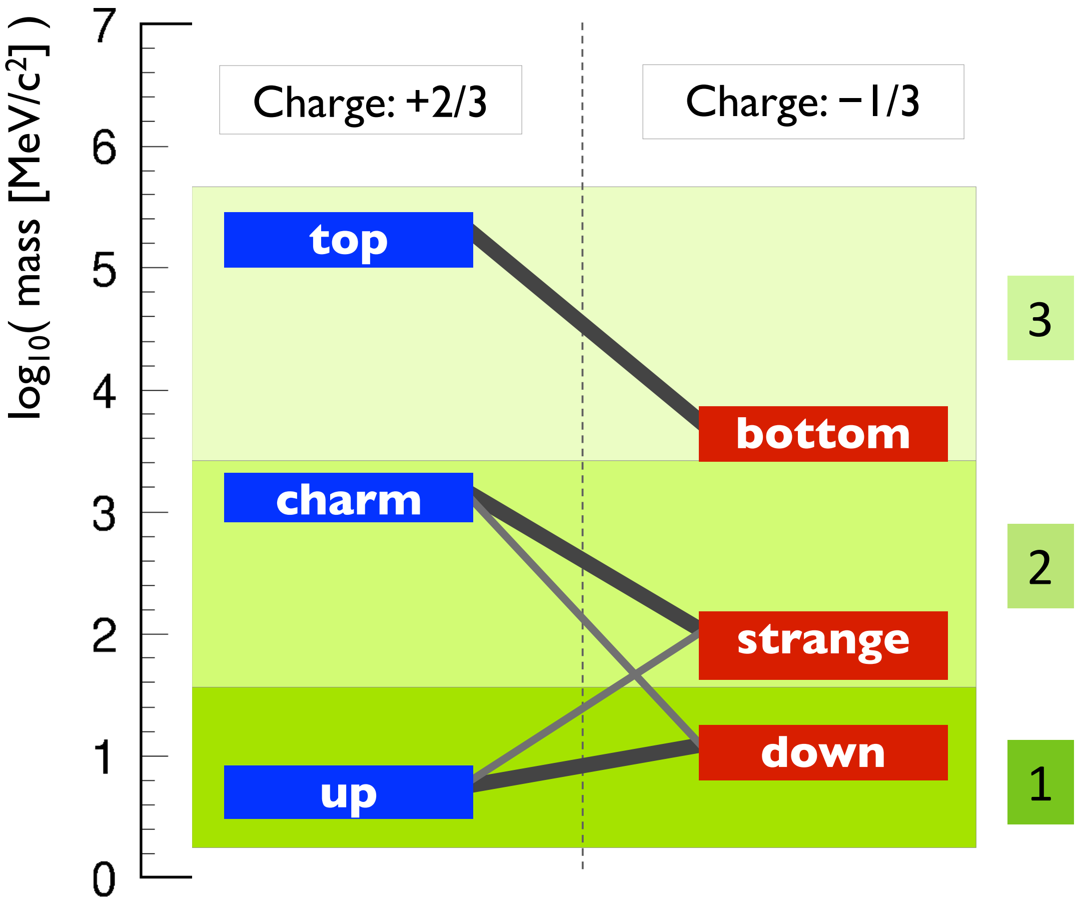


Flavour changing currents

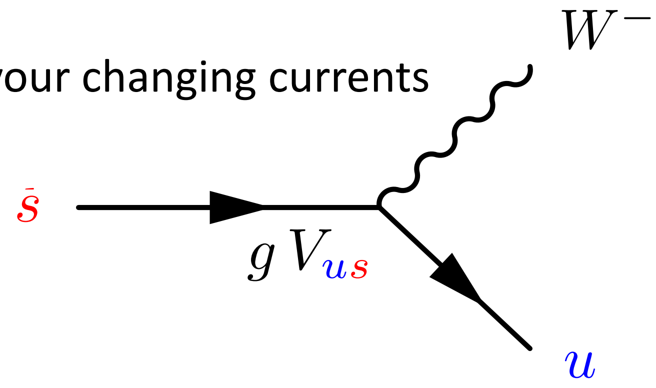


$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & & \\ & V_{cs} & \\ & & V_{tb} \end{pmatrix}$$

Topic-4: Flavour Changing Quark Interactions

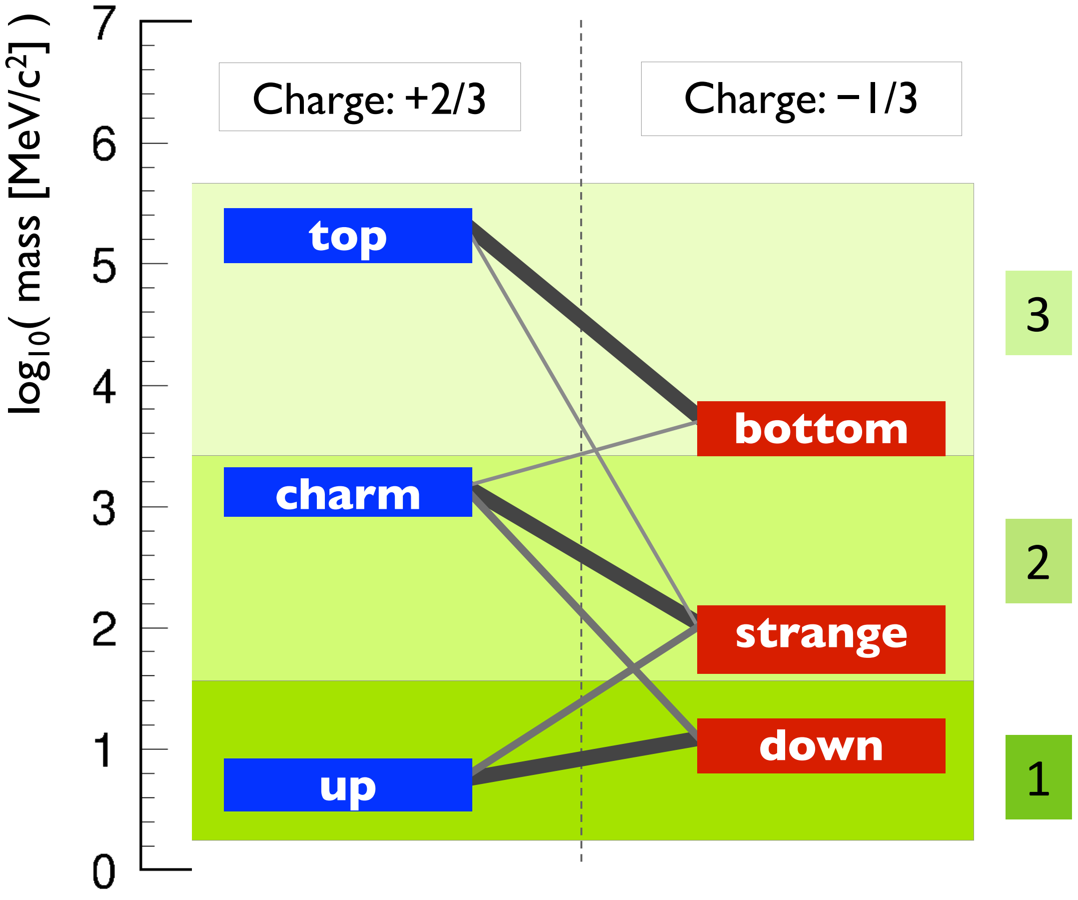


Flavour changing currents

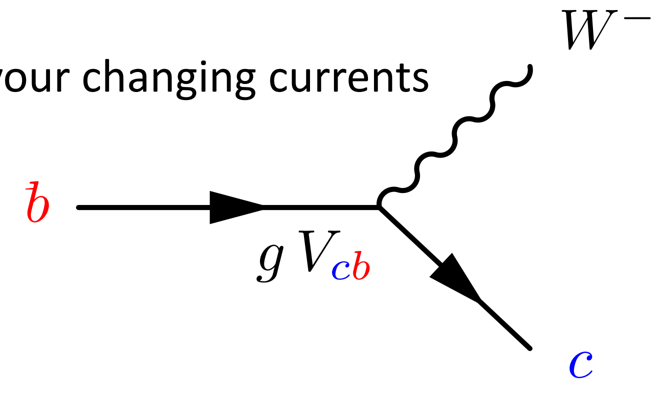


$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & \\ V_{cd} & V_{cs} & \\ & & V_{tb} \end{pmatrix}$$

Topic-4: Flavour Changing Quark Interactions

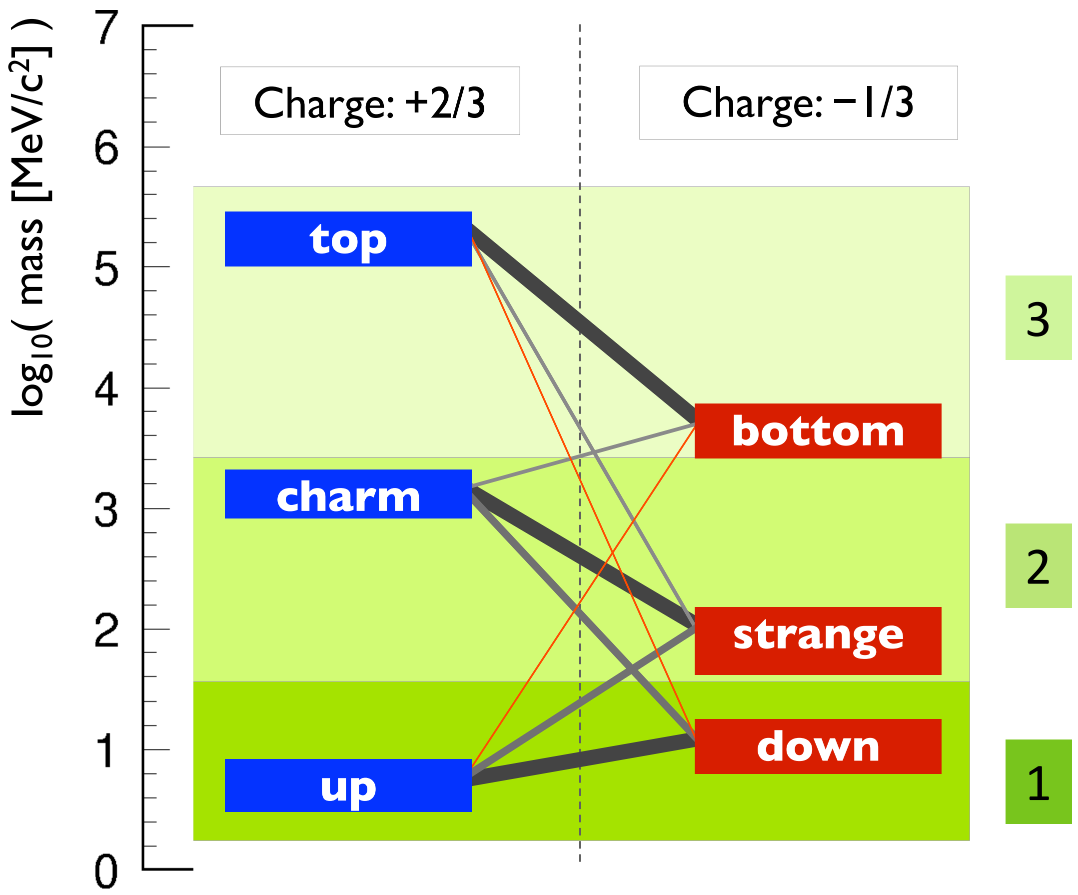


Flavour changing currents

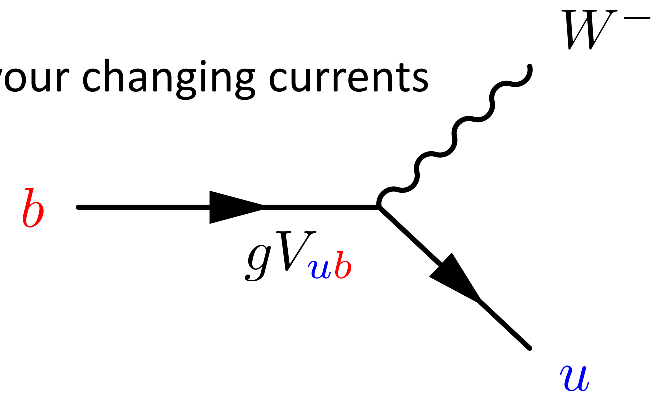


$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & \\ V_{cd} & V_{cs} & V_{cb} \\ & V_{ts} & V_{tb} \end{pmatrix}$$

Topic-4: Flavour Changing Quark Interactions

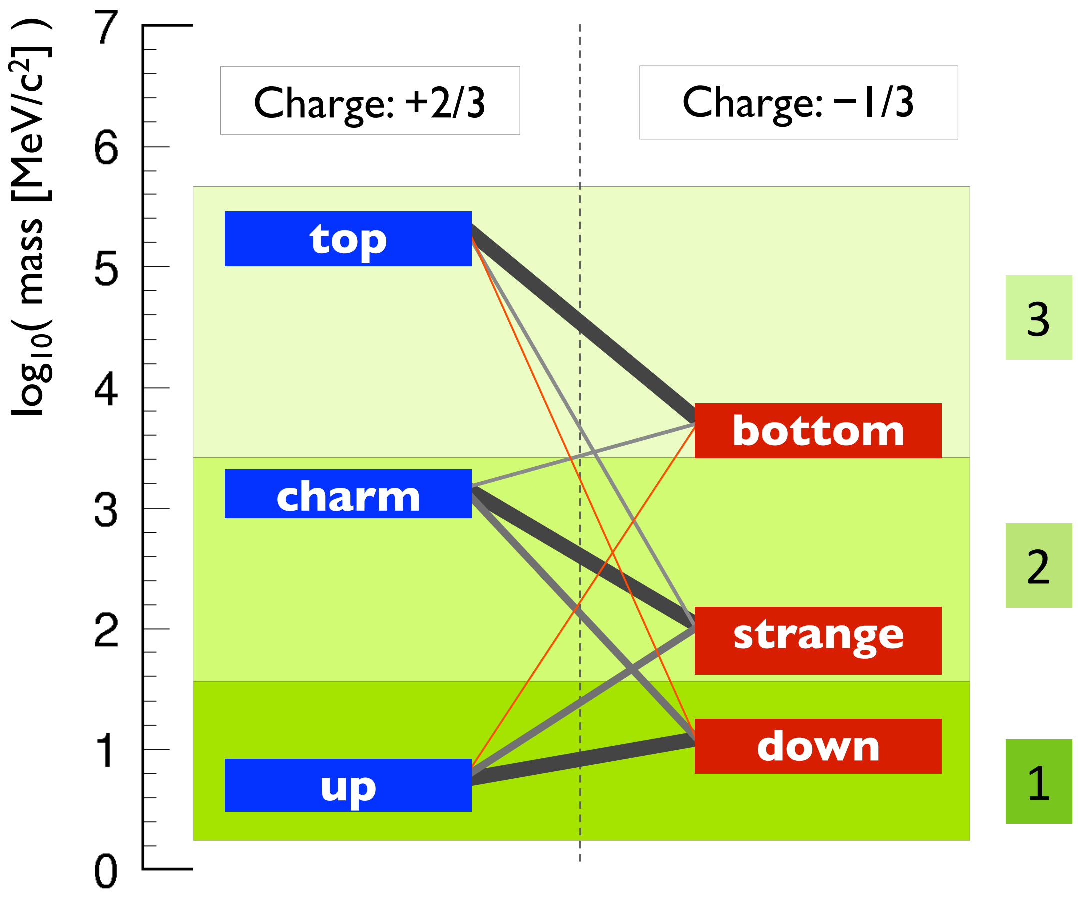


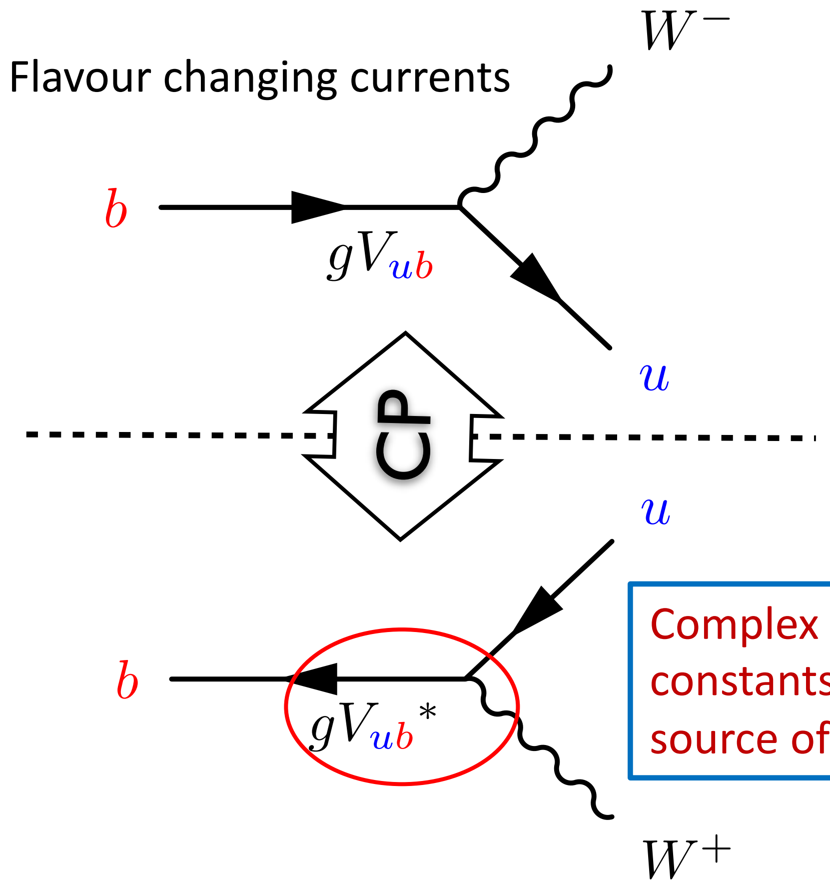
Flavour changing currents



$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Topic-4: Flavour Changing Quark Interactions – CP Violation





Complex coupling constants are the source of CP viol.

- Particles and antiparticles have complex conjugated coupling constants
  - This leads to CP violation
  - Matter dominated universe

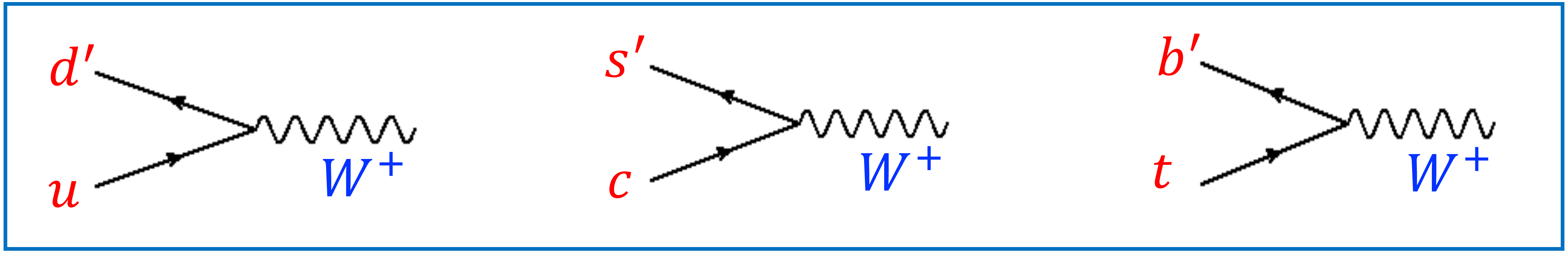
$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Topic-5: The CKM and PMNS matrices

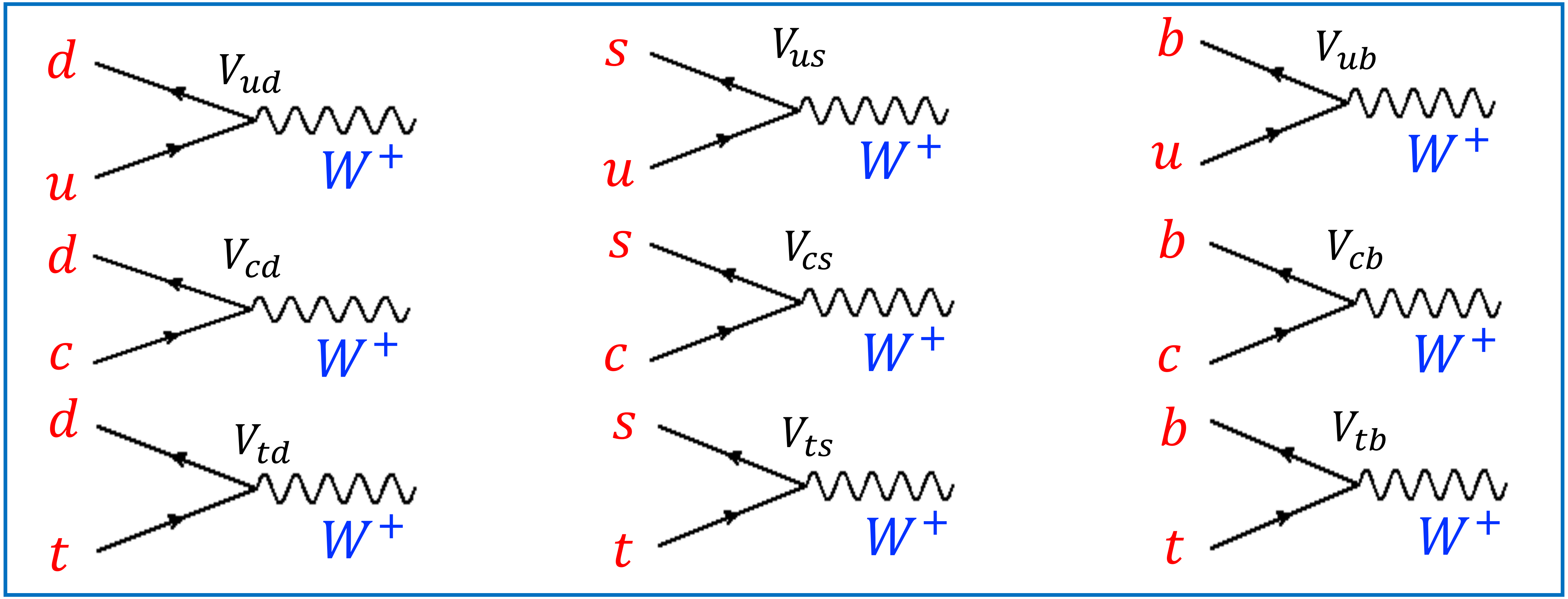
- What is different and what is the same for quarks and lepton in the charged current weak interaction?
- Explain how possibly a matter – antimatter asymmetry can be implemented?
  - Think of complex coupling constants

Topic-5: A story of eigenstates

- *Mass* eigenstates $|q\rangle$ are the eigenstate solutions of the free Hamiltonian
- *Weak* or *flavour* eigenstates $|q'\rangle$ are the eigenstate solutions of the weak interaction Hamiltonian.
    - They are unitary linear combination, or "rotation" of mass eigenstates.
- The the weak interaction can be written as:



Topic-5: Flavour eigenstates and Mass eigenstates



Topic-5: Why a matrix notation?

- Model:
  - Charged weak current does ***not*** couple to

$$\binom{u}{d}, \binom{c}{s}, \binom{t}{b}$$

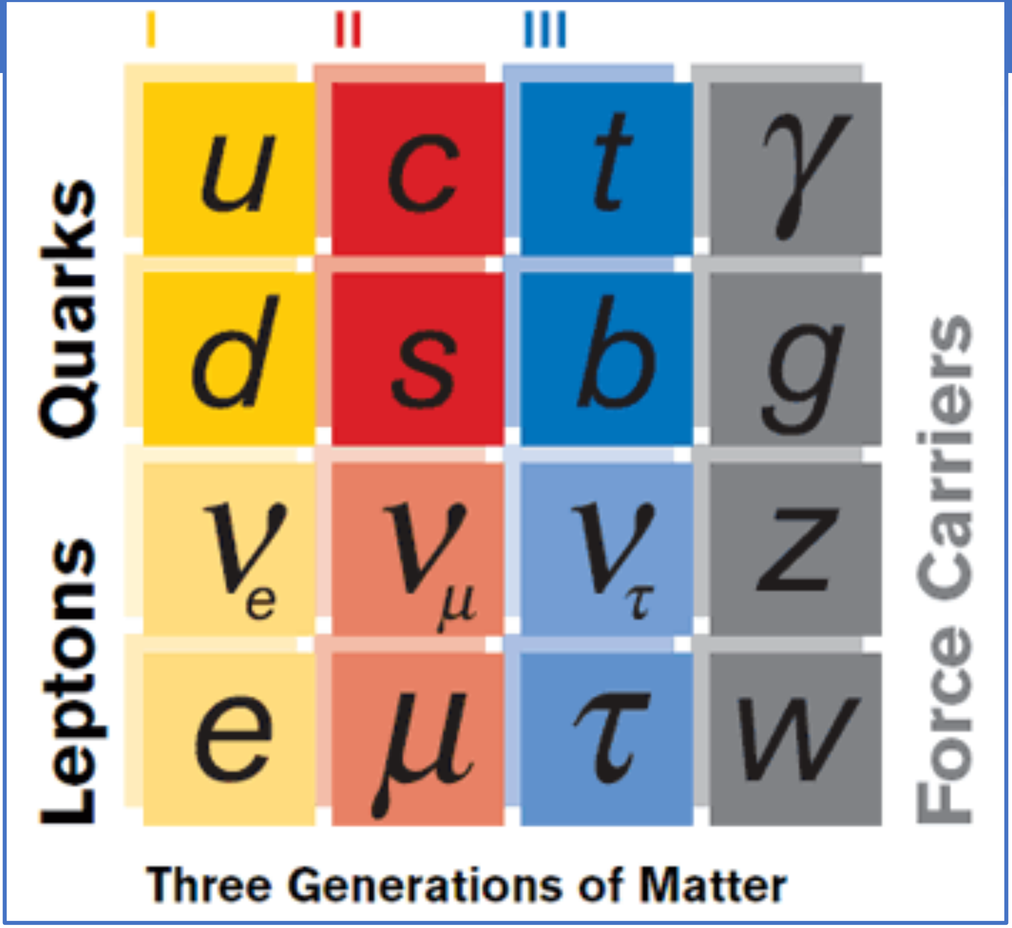
  - but instead to

$$\binom{u}{d'}, \binom{c}{s'}, \binom{t}{b'}$$

  - Where

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM matrix:
Cabibbo,
Kobayashi
Maskawa



Topic-5: The CKM matrix V_{CKM} - 3 vs 2 Generations

$$V_{CKM}: \quad \begin{array}{c|ccc} & d & s & b \\ \hline u & V_{ud} & V_{us} & V_{ub} \\ c & V_{cd} & V_{cs} & V_{cb} \\ t & V_{td} & V_{ts} & V_{tb} \end{array}$$

- Wolfenstein parametrization: V_{CKM} =

$$\begin{pmatrix} 1-{}^{1}/_{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-{}^{1}/_{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

➔ 1 CP violating phase

$$V_{CKM}: \quad \begin{array}{c|cc} & d & s \\ \hline u & V_{ud} & V_{us} \\ c & V_{cd} & V_{cs} \end{array}$$

V_{CKM} =

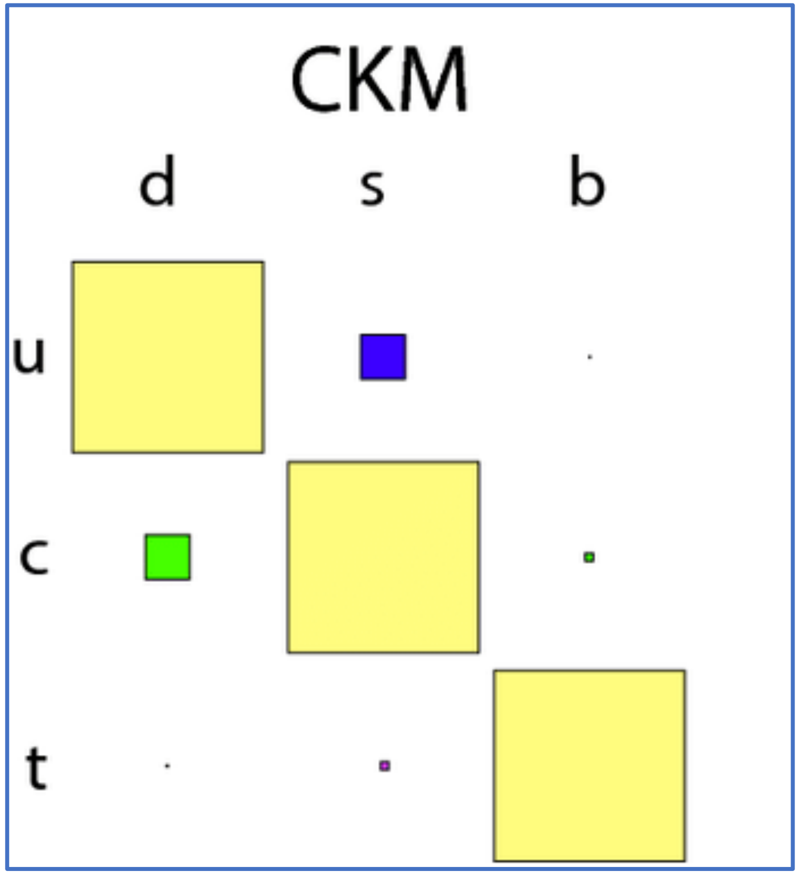
$$\begin{pmatrix} 1-{}^{1}/_{2}\lambda^2 & \lambda \\ -\lambda & 1-{}^{1}/_{2}\lambda^2 \end{pmatrix}$$

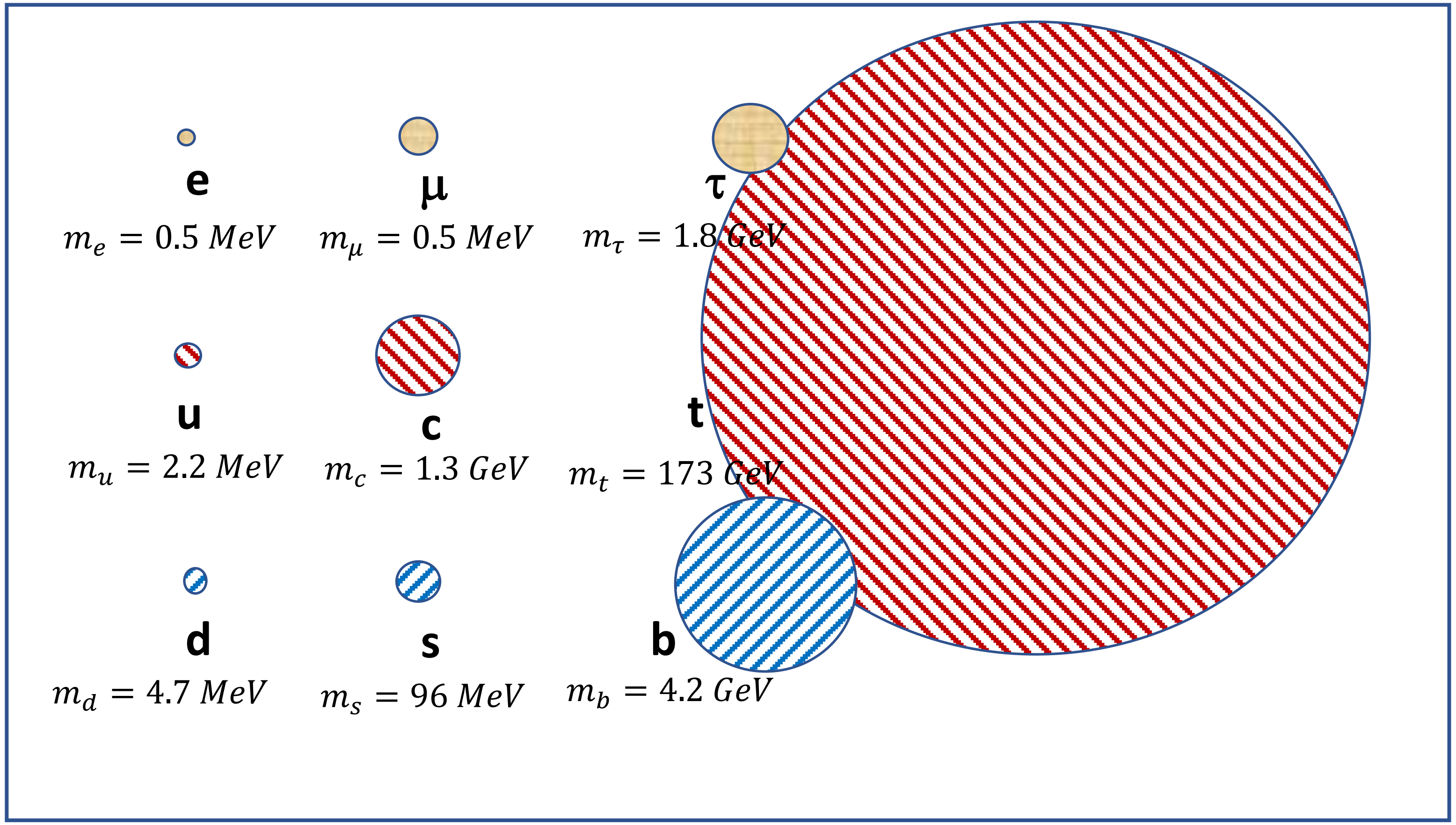
➔ No CP violation

- *3 generations is the minimal particle content to generate CP violation* (In Standard Model).

Topic-5: The Flavour Puzzle

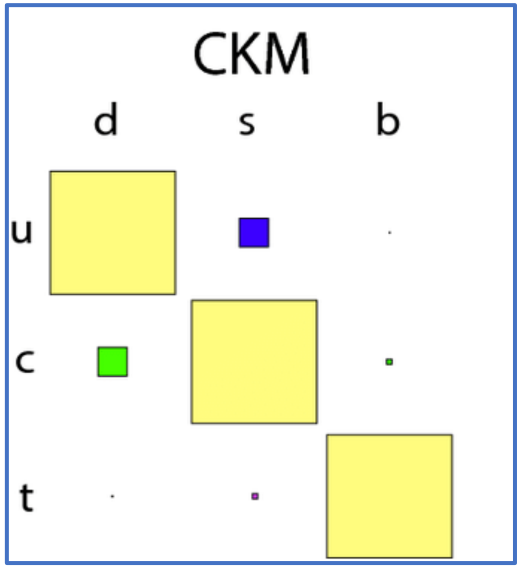
- Why 3?
- Why are the couplings what they are?
- Is there a relation with the masses of the quarks?

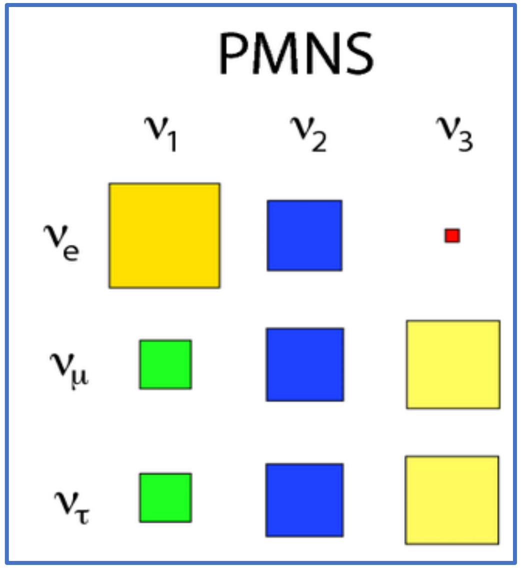




Topic-5: Flavour in the leptons!

- It turns out neutrino's have mass, too!
  - The mass is very tiny
- The generation mixing also occurs for neutrino's
  - Slightly different nomenclature:
  - Quarks (CKM): $\begin{pmatrix} u' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$
  - Leptons (PMNS): $\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} \\ U_{2e} & U_{2\mu} & U_{c\tau} \\ U_{3e} & U_{3\mu} & U_{3\tau} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$
    - U_{PMNS} : Pontecorvo, Maki, Nakagawa, Sakata mixing matrix
- (Difficult) Question: why is lepton mixing not seen in decays?
  - It is only seen in neutrino oscillations





Topic-6: Variational calculus and Lagrangians

- Explain the idea behind variational calculus?
- What is a conservative Force?
- Define a Lagrangian: $\mathcal{L} = T - V = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(x, y, z)$
- Show how Newton's law in one dimension: $F = m\ddot{x}$ leads to the Euler Lagrange equation (in case of a conservative force):

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = \frac{\partial \mathcal{L}}{\partial x}$$

Topic-6: Lagrange Formalism classical

Griffiths §10.1

- Classical Mechanics: The Lagrangian leads to equations of motion
  - $L(q_i, \dot{q}_i) = T - V$ where q_i and $\dot{q}_i$ are the generalized coordinates and velocities.
  - The path of a particle is found from *Hamilton's principle of least action*

$$S = \int_{t_1}^{t_2} dt\, L(q, \dot{q}) = 0 \qquad\qquad \delta S = 0$$

From this the *Euler Lagrange equations* follow and provide the equations of motion:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}$$

See: https://en.wikipedia.org/wiki/Lagrangian_mechanics

- Example: Ball falls from height $y = h$: $q = y\,,\ \dot{q} = dy/dt = v_y$
  - $E_{pot} = V = mgq$
  - $E_{kin} = T = \frac{1}{2}m\dot{q}^2$

$$L = T - V = \frac{1}{2}m\dot{q}^2 - mgq$$

- Euler Lagrange: $\partial L/\partial q = mg$; $\partial L/\partial \dot{q} = m\dot{q}$
  - $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}$ gives $m\ddot{q} = mg \rightarrow \dot{q} = gt + v_0 \rightarrow q = y = \frac{1}{2}gt^2 + v_0 t + y_0$