Lecture 1: Discussion Topics

Discussions Topics belonging to Lecture 1

When you are assigned a topic, prepare to lead a discussion on the subject with the tutor group.
You are expected to introduce the topic, prepare a few slides or write on the board,
and be somewhat of an expert.
At the same time you do not have to know everything. You may also address questions to the tutor group.

Topic-1: Natural Units:

Griffiths: p. 9-10

- Redefine the unit $\hbar = \frac{h}{2\pi} \approx 1.055 \times 10^{-34}$ Js to be: $\hbar \equiv 1$
- Redefine the unit $c = 2.998 \times 10^{8}$ m/s to be: $c \equiv 1$
- Explain how Energy, mass, distance, time can then be expressed in the unit GeV
- How can you get answers that can be compared with measurements?
- What are the advantages of doing this?

Topic-1: Natural Units:

Griffiths: p. 9-10

quantity	symbol in natural units	equivalent symbol in ordinary units
space	x	$x/\hbar c$
time	t	$t/\hbar$
mass	m	mc^2
momentum	p	pc
energy	E	E
positron charge	e	$e\sqrt{\hbar c/\epsilon_0}$

Conversion of basic quantities between natural and ordinary units.

quantity	conversion factor	natural unit	normal unit
mass	$1 \text{ kg} = 5.61 \times 10^{26} \text{ GeV}$	GeV	GeV/c^2
length	$1 \text{ m} = 5.07 \times 10^{15} \text{ GeV}^{-1}$	GeV^{-1}	$\hbar c/ \text{ GeV}$
time	$1 \text{ s} = 1.52 \times 10^{24} \text{ GeV}^{-1}$	GeV^{-1}	$\hbar/ \text{ GeV}$

Conversion factors from natural units to ordinary units.

Topic-2: The Lorentz Transformation

- Why are space and time coordinates not universal (ie not the same for each observer)?
- Explain the Lorentz transformation
- When does this effect become noticeable?

Topic-2: The Lorentz Transformation

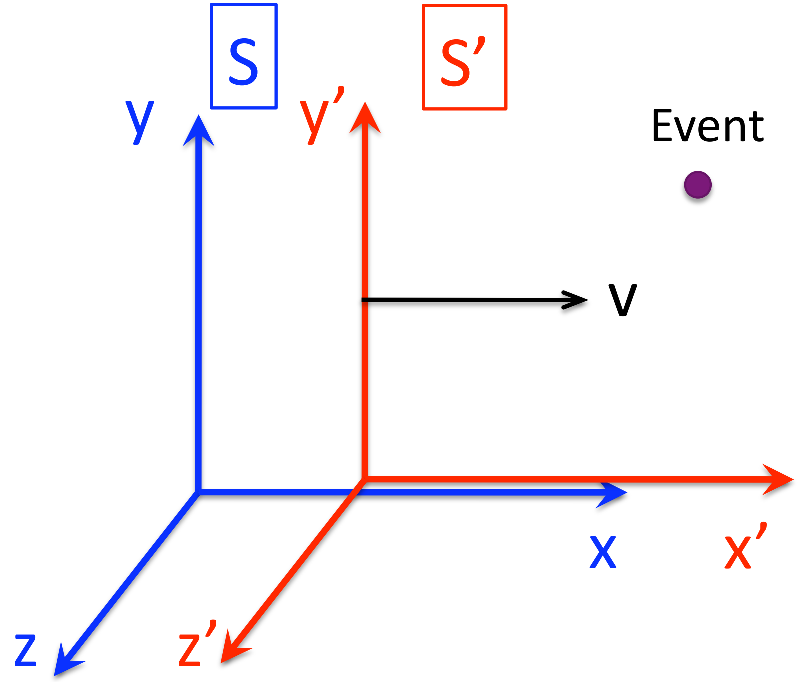
A reference system or coordinate system is used to determine the time and position of an event.

Reference system S is linked to observer Alice at position (x,y,z) = (0,0,0)
An event is fully specified by giving its coordinates and time: (t, x, y, z)

Reference system S' is linked to observer Bob who is moving with velocity v with respect to Alice.
The event has: (t', x', y', z')

How are the coordinates of an event, say a lightning strike in a tree, expressed in coordinates for Alice and for Bob?

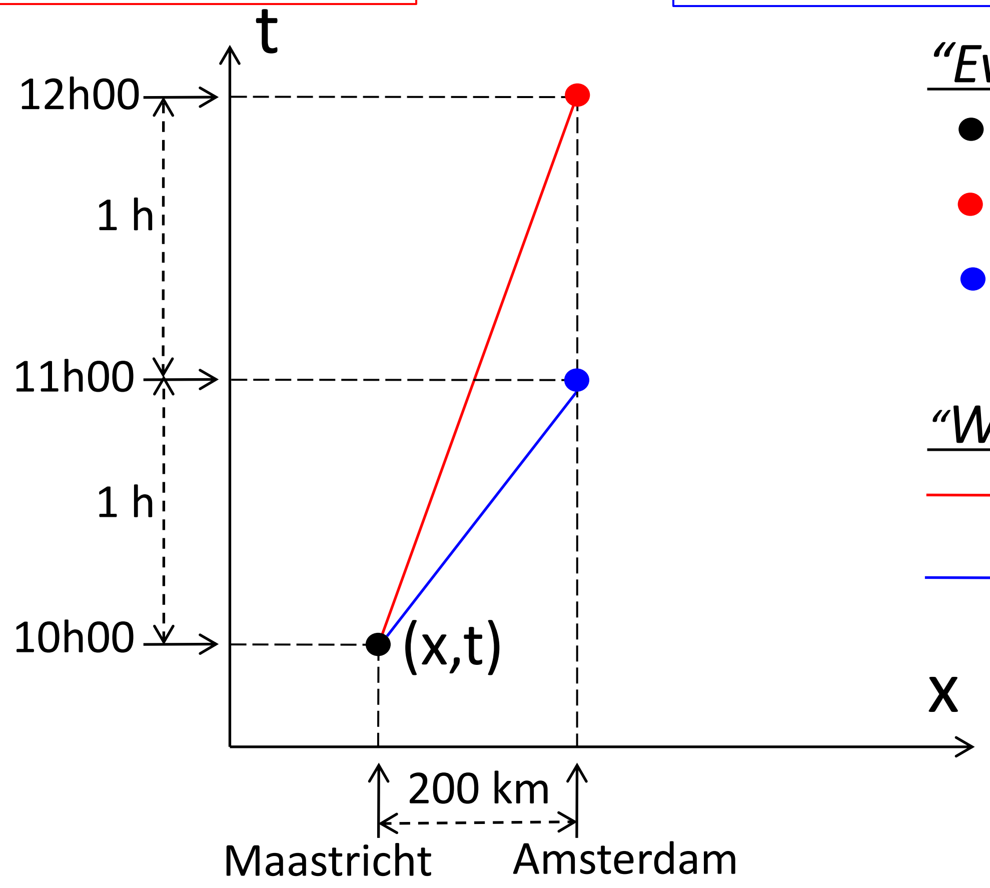
(t, x, y, z) → (t', x', y', z')



Topic-2: Space-time diagram

Bob drives from Maastricht to Amsterdam with 100 km/h.

Alice drives from Maastricht to Amsterdam with 200 km/h.



"Events":

- Departure Alice & Bob
- Arrival Bob
- Arrival Alice

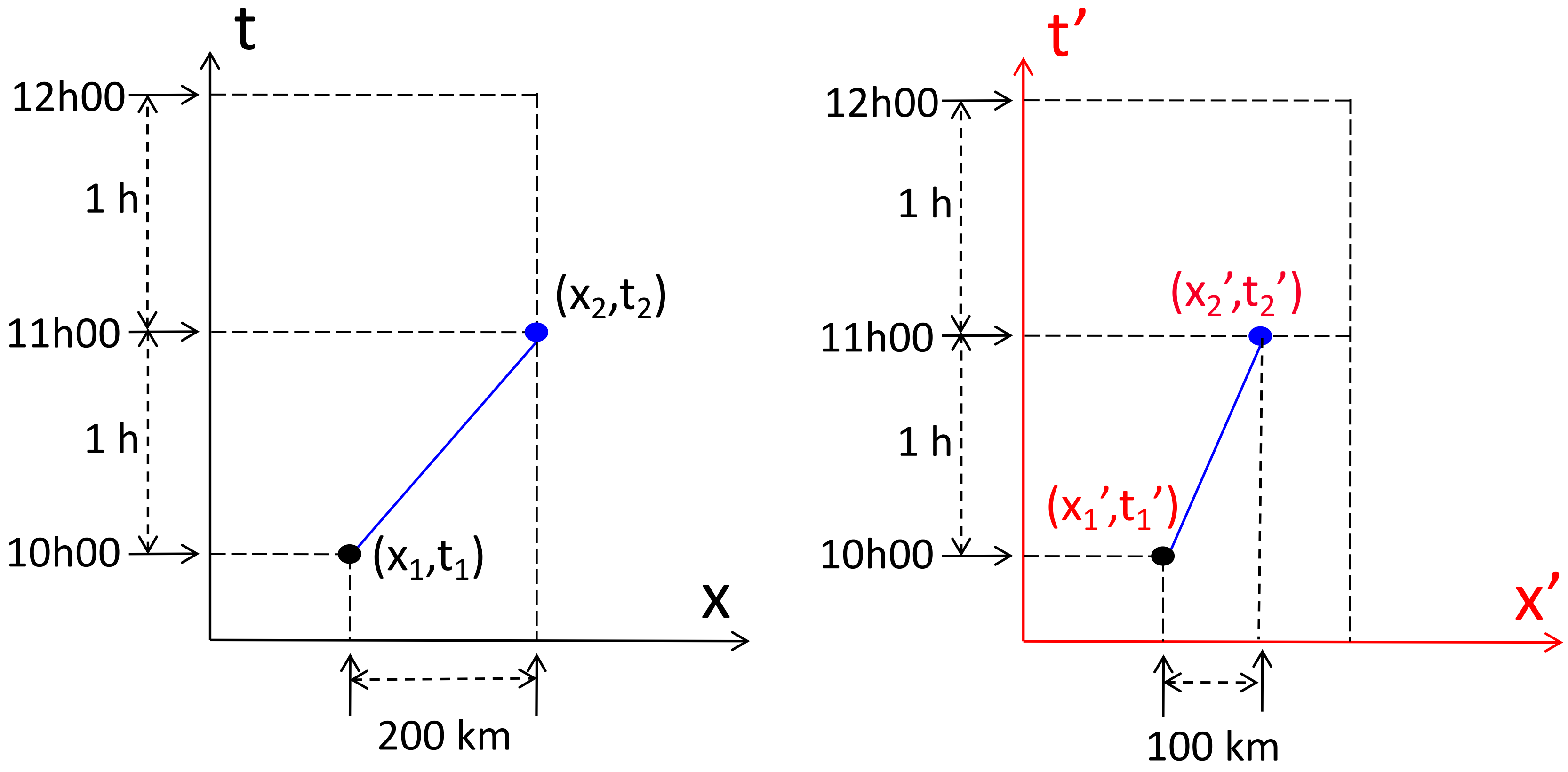
"World lines":

- Bob's world-line
- Alice's world-line

Events with space-time coordinates: (x,t)
More general: it is a 4-dimensional space: (x,y,z,t)

Topic-2: Coordinate transformation

How does Alice's trip look like in the coordinates of the reference system of Bob?



Alice as seen from Maastricht
S = fixed reference system in Maastricht

Alice as seen from Bob
S' = fixed reference to Bob

Bob's frame moves with velocity v (100km/h) with respect to Maastricht

Topic-2: Coordinate transformation

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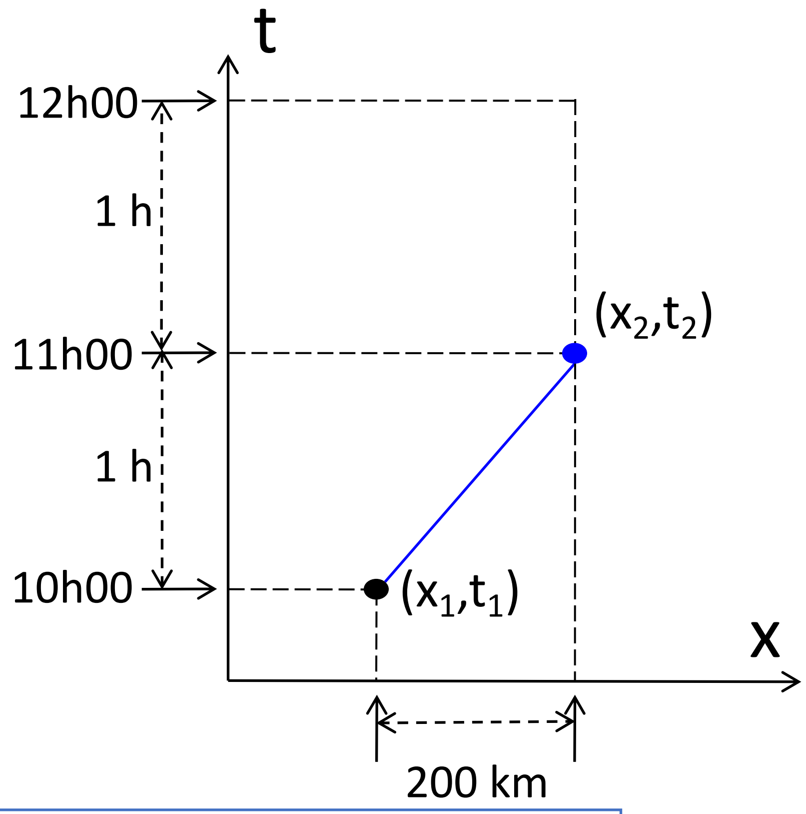
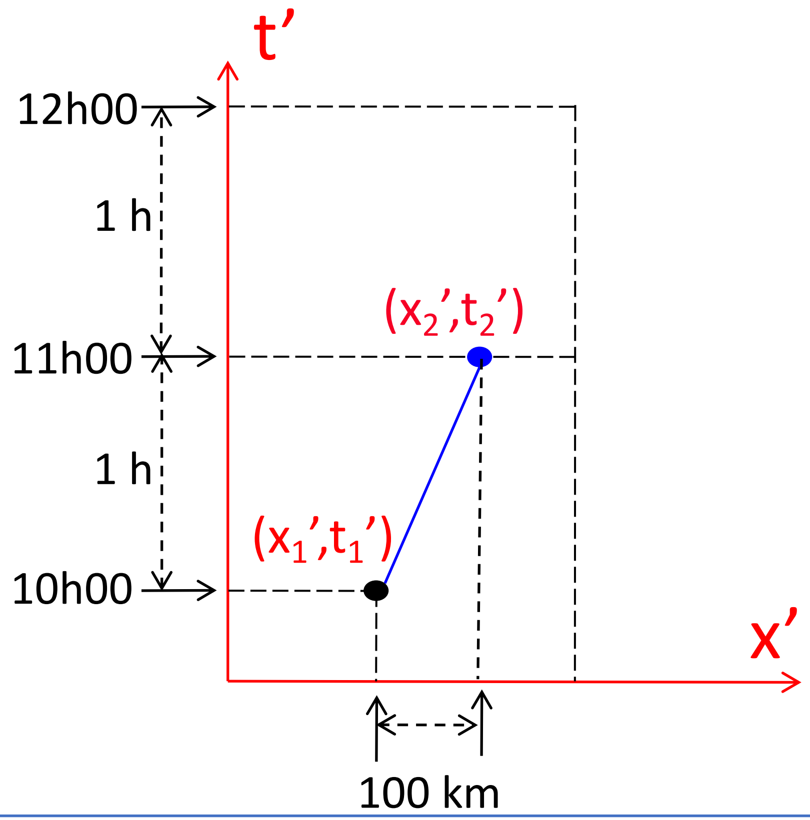




Classical (Gallilei Transformation):

$$t' = t$$
$$x' = x - v\,t$$

Relativistic (Lorentz Transformation):

$$t' = \gamma\left(t - \frac{v}{c^2}\,x\right)$$
$$x' = \gamma\,(x - v\,t)$$

with: $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

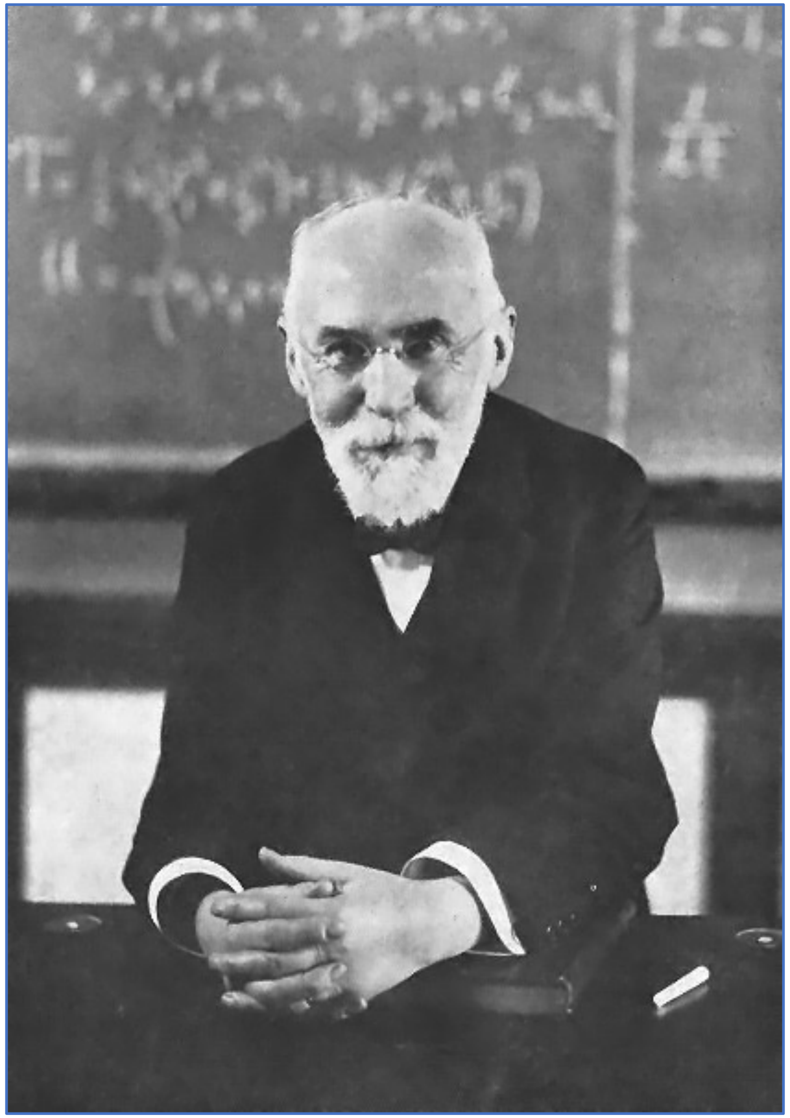
Topic-2: Lorentz Transformations

Hendrik Anton Lorentz (1853 – 1928)

Dutch Physicist in Leiden
(Nobelprize 1902 with Pieter Zeeman)

To explain the Michelson-Morley experiment he assumed that bodies contracted due to intermolecular forces as they were moving through the aether.
(He believed in the existence if the aether)

Einstein derived it from the relativity principle and also saw that time has to be modified.



Topic-2: Let's go crazy and derive them…

Start with classical Galilei Transformation:

$$x' = x - vt$$

$$x = x' + vt'$$

Let's try a modification by including a factor:

$$x' = f(x - vt)$$

$$x = f(x' + vt')$$

For light: $x = ct$ and $x' = ct'$, so:

$$ct' = f(ct - vt)$$

$$ct = f(ct' + vt')$$

Then: $$t' = f\left(\frac{c - v}{c}\right)t$$

$$t = f\left(\frac{c + v}{c}\right)t'$$

Substitute first into second:

$$t = f\left(\frac{c + v}{c}\right) f\left(\frac{c - v}{c}\right) t$$

Divide by t: $$1 = \left(\frac{c+v}{c}\right)\left(\frac{c-v}{c}\right) f^2 = \left(\frac{c^2-v^2}{c^2}\right) f^2$$

It follows then that: $$f^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - v^2/c^2}$$

So that we find: $$f = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$$

Therefor we have derived the Lorentz transformation: $$x' = \gamma(x - vt)$$

Similarly we find the Lorentz transformation for time: (see lecture notes)

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

whereas the Galilei translation was:

$$t' = t$$

Topic-3: Four vectors & co- and contra-variance

- Explain the so-called covariant (ie 4-vector) notation.
- What is the difference between contra-variant and co-variant?
- Explain Einstein's summation convention for indices

Topic-3: Four vectors & co- and contra-variance

See Griffiths: chapter 3

- Four vector: $x^\mu = (x^0, x^1, x^2, x^3)$ with $x^0 = ct \;\Rightarrow\; x^0 = t$ (use $c \equiv 1$ convention)
- We call this a ***contravariant*** vector and: $x^\mu = (x^0, \vec{x}\,)$
- Lorentz transformation along x^1 axis using $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$ is:

$$x^{0\prime} = \gamma(x^0 - \beta x^1)$$
$$x^{1\prime} = \gamma(x^1 - \beta x^0)$$
$$x^{2\prime} = x^2$$
$$x^{3\prime} = x^3$$

Write it in Matrix notation:

$$x^{\mu\prime} = \Lambda^\mu_\nu x^\nu \;\; ; \;\; \Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\beta & 0 & 0 \\ -\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Lorentz transformations leave the "length" s invariant $s = x \cdot x = |x|^2 = {x^0}^2 - |\vec{x}|^2$
  - Explicitly: $(ct)^2 - |\vec{x}|^2 = (ct')^2 - |\vec{x}'|^2 = s$ is invariant.
- This can be written as:

$$s = (x^0, x^1, x^2, x^3) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} (x^0, x^1, x^2, x^3)$$

"Metric tensor"

Topic-3: Scalar product and co- and contra-variant

- Define ***co-variant*** vectors: $x_\mu = (x^0, -\vec{x}\,)$
- Define the metric tensor: $g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

Definition:
A contravariant vector transforms like x_μ and a covariant vector transforms like Note the Einstein summation convention!

- To obtain a co-variant from contra-variant vector:

$$x_\mu = \sum_\mu g_{\mu\nu} x^\nu \;=\; g_{\mu\nu} x^\nu$$

Note the Einstein summation convention!

- Then the invariance distance $s = x^\mu x_\mu = \sum_{\mu\nu} x^\mu g_{\mu\nu} x^\nu = {x^0}^2 - |\vec{x}|^2$
- We speak of a scalar product: $A \cdot B = A^\mu B_\mu \; = g_{\mu\nu} A^\mu B^\nu$, where a sum is always implicit over *contravariant* and *covariant* indices.
- The ***scalar product*** or ***inproduct*** of Lorentz 4-vectors is always Lorentz invariant:
  - $I = a_\mu b^\mu = a \cdot b$ for any Lorentz 4-vectors a^μ and b^μ
  - Example are the space-time vectors a^μ , but also the 4-momentum vector $p^\mu = (E, \vec{p})$
  - $E^2 = \vec{p}^2 c^2 + m^2 c^4 \;\Rightarrow\; p^\mu = (E, \vec{p}) \;\Rightarrow\; p_\mu p^\mu = E^2 - \vec{p}^2 = m^2$ ***the invariant mass***

Topic-3: Co- and contravariant derivatives

Griffiths: chapter 3

- Contravariant vector:

$$x^\mu = (ct, \vec{x})$$

But covariant derivative:

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\vec{\nabla}\right)$$

- Covariant vector:

$$x_\mu = (ct, -\vec{x})$$

But covariant derivative:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

Note that the minus sign is "opposite" to the case of the coordinate four-vectors.