

Exercise 2 (Nov. 4, 2011)

Interpolate the function

$$f(x) = \frac{x^2 e^{-(x-4.75)^{-2}}}{x - 4.75}$$

in the interval $[0,10]$. For the grid points use $x_i = 0.3i$, $i = 0 \dots 30$, and calculate the interpolated values at intervals of 0.05. Try this with linear interpolation, a natural cubic spline, a third-order polynomial, and a 10th-order polynomial. Compare the accuracy of the methods : give the error of each method in a table (a list), or plot the absolute value of the error in the interpolated result and the function value in a logarithmic plot.

The aim of this exercise is to gain insight in the difference in cubic spline (which is stiff) and cubic interpolation, as well as to see where a higher-order interpolation scheme does well and where it fails. Note, that the derivatives in this exercise become very flat instead of very steep, and that this causes problems too.