

# Detecting CP violation with B decays

## *Lecture 3: B decays*

N. Tuning

# Detecting CP violation with B decays

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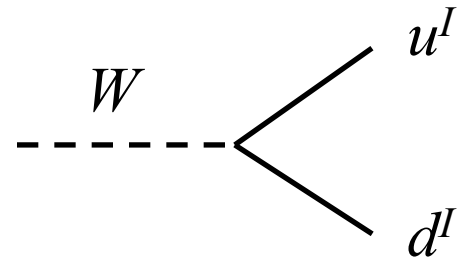
- 1) CP violation: CKM and the SM
- 2) Detecting: Detector requirements
- 3) B-decays:  $\sin 2\beta$ ,  $\phi_s$ ,  $B_s^0 \rightarrow D_s^+ K^-$

# Recap

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$$-\mathcal{L}_{Yuk} = Y_{ij}^d (\bar{u}_L^I, \bar{d}_L^I)_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^I + \dots$$

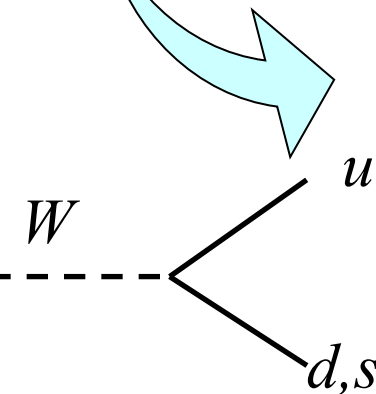
$$\mathcal{L}_{Kinetic} = \frac{g}{\sqrt{2}} \bar{u}_{Li}^I \gamma^\mu W_\mu^- d_{Li}^I + \frac{g}{\sqrt{2}} \bar{d}_{Li}^I \gamma^\mu W_\mu^+ u_{Li}^I + \dots$$



Diagonalize Yukawa matrix  $Y_{ij}$

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



$$-\mathcal{L}_{Mass} = (\bar{d}, \bar{s}, \bar{b})_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + (\bar{u}, \bar{c}, \bar{t})_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \dots$$

$$\mathcal{L}_{CKM} = \frac{g}{\sqrt{2}} \bar{u}_i \gamma^\mu W_\mu^- V_{ij} (1 - \gamma^5) d_j + \frac{g}{\sqrt{2}} \bar{d}_j \gamma^\mu W_\mu^+ V_{ij}^* (1 - \gamma^5) u_i + \dots$$

$$\mathcal{L}_{SM} = \mathcal{L}_{CKM} + \mathcal{L}_{Higgs} + \mathcal{L}_{Mass}$$

# Charged Currents

The charged current term reads:

$$\begin{aligned}
 L_{CC} &= \frac{g}{\sqrt{2}} \bar{u}_{Li}^I \gamma^\mu W_\mu^- d_{Li}^I + \frac{g}{\sqrt{2}} \bar{d}_{Li}^I \gamma^\mu W_\mu^+ u_{Li}^I = J_{CC}^{\mu-} W_\mu^- + J_{CC}^{\mu+} W_\mu^+ \\
 &= \frac{g}{\sqrt{2}} \bar{u}_i \left( \frac{1-\gamma^5}{2} \right) \gamma^\mu W_\mu^- V_{ij} \left( \frac{1-\gamma^5}{2} \right) d_j + \frac{g}{\sqrt{2}} \bar{d}_j \left( \frac{1-\gamma^5}{2} \right) \gamma^\mu W_\mu^+ V_{ji}^\dagger \left( \frac{1-\gamma^5}{2} \right) u_i \\
 &= \frac{g}{\sqrt{2}} \bar{u}_i \gamma^\mu W_\mu^- V_{ij} (1-\gamma^5) d_j + \frac{g}{\sqrt{2}} \bar{d}_j \gamma^\mu W_\mu^+ V_{ij}^* (1-\gamma^5) u_i
 \end{aligned}$$

Under the CP operator this gives:

(Together with (x,t) → (-x,t))

$$L_{CC} \xrightarrow{CP} \frac{g}{\sqrt{2}} \bar{d}_j \gamma^\mu W_\mu^+ V_{ij} (1-\gamma^5) u_i + \frac{g}{\sqrt{2}} \bar{u}_i \gamma^\mu W_\mu^- V_{ij}^* (1-\gamma^5) d_j$$

A comparison shows that CP is conserved only if  $V_{ij} = V_{ij}^*$

In general the charged current term is CP violating

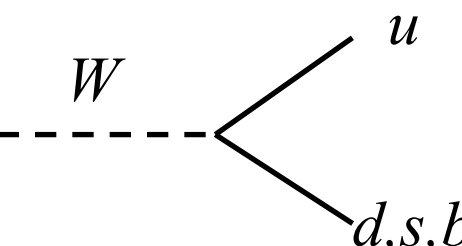
# CKM-matrix: where are the phases?

- Possibility 1: simply 3 ‘rotations’, and put phase on smallest:

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

- Possibility 2: parameterize according to magnitude, in  $O(\lambda)$ :

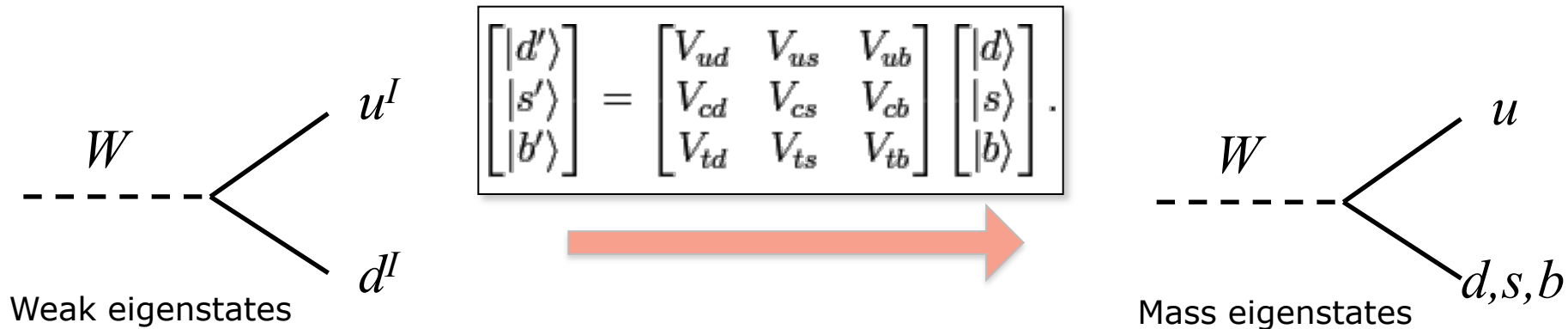


A Feynman diagram on the left shows a dashed line representing a  $W$  boson entering from the left. It splits into two solid lines: one going up and right to a  $u$  quark, and one going down and right to a  $d, s, b$  quark.

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

# (CKM: a quick reminder)

- 1) Matrix to transform weak- and mass-eigenstates:



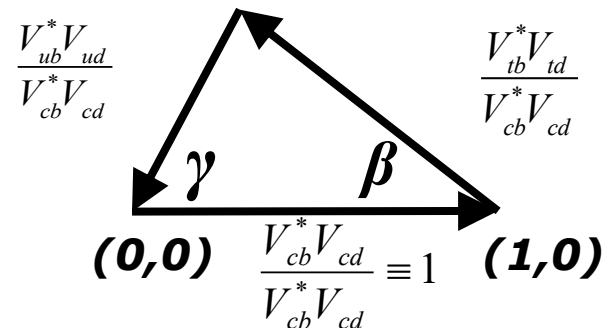
- 2) Matrix has imaginary numbers:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$

- 3) Matrix is unitary:

$$V^+V = \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{ub}^*V_{ud} + V_{cb}^*V_{cd} + V_{tb}^*V_{td} = 0$$



# Summary

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- $p, q$ :  $|B_H\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$

$$|B_L\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

- $\Delta m, \Delta\Gamma$ :  $\Delta m = 2\Re\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$

$$\Delta\Gamma = 4\Im\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$$

$q, p, M_{ij}, \Gamma_{ij}$  related through:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^* / 2}{M_{12} - i\Gamma_{12} / 2}}$$

- $x, y$ : mixing often quoted in *scaled* parameters:

$$x = \frac{\Delta m}{\Gamma} \quad y = \frac{\Delta\Gamma}{2\Gamma}$$

$$\cos(\Delta m t) = \cos\left(\frac{\Delta m}{\Gamma} \frac{t}{\tau}\right) = \cos\left(x \frac{t}{\tau}\right)$$

## Time dependence (if $\Delta\Gamma \sim 0$ , like for $B^0$ ):

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$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle$$

with

$$g_+(t) = e^{-imt} e^{-\Gamma t/2} \cos\frac{\Delta m t}{2}$$

$$|\bar{B}^0(t)\rangle = g_+(t)|\bar{B}^0\rangle + \frac{p}{q}g_-(t)|B^0\rangle$$

$$g_-(t) = e^{-imt} e^{-\Gamma t/2} i \sin\frac{\Delta m t}{2}$$

# Meson Decays

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- Formalism of meson oscillations:

$$|P^0(t)\rangle = \frac{1}{2} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle$$

$$|\langle \bar{P}^0(t) | P^0 \rangle|^2 = |g_-(t)|^2 \left( \frac{p}{q} \right)^2$$

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{1}{2} \Delta\Gamma t \pm \cos \Delta m t \right)$$

- Subsequent: decay

$$P^0 \rightarrow f$$



## Notation: Define $A_f$ and $\lambda_f$

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$$\begin{aligned} A(f) &= \langle f|T|P^0\rangle & \bar{A}(f) &= \langle f|T|\bar{P}^0\rangle \\ A(\bar{f}) &= \langle \bar{f}|T|P^0\rangle & \bar{A}(\bar{f}) &= \langle \bar{f}|T|\bar{P}^0\rangle \end{aligned}$$

and define the complex parameter  $\lambda_f$  (not be confused with the Wolfenstein parameter  $\lambda$  !):

$$\lambda_f = \frac{q \bar{A}_f}{p A_f}, \quad \bar{\lambda}_f = \frac{1}{\lambda_f}, \quad \lambda_{\bar{f}} = \frac{q \bar{A}_{\bar{f}}}{p A_{\bar{f}}}, \quad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}} \quad (3.14)$$

The general expression for the time dependent decay rates,  $\Gamma_{P^0 \rightarrow f}(t) = |\langle f|T|P^0(t)\rangle|^2$ ,

# Some algebra for the decay $P^0 \rightarrow f$

$$|P^0(t)\rangle = \frac{1}{2} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle$$

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \left( |g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)] \right)$$

$$A(f) = \langle f|T|P^0\rangle$$

$$\bar{A}(f) = \langle f|T|\bar{P}^0\rangle$$

$$\lambda_f = \frac{q \bar{A}_f}{p A_f}$$

**Interference**

—  $P^0 \rightarrow f$

—  $P^0 \rightarrow \bar{P}^0 \rightarrow f$

## Some algebra for the decay $P^0 \rightarrow f$

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$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 \left( |g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)] \right) \\
 \Gamma_{P^0 \rightarrow \bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left| \frac{q}{p} \right|^2 \left( |g_-(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2 |g_+(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+(t) g_-^*(t)] \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 \left( |g_-(t)|^2 + |\lambda_f|^2 |g_+(t)|^2 + 2\Re[\lambda_f g_+(t) g_-^*(t)] \right) \\
 \Gamma_{\bar{P}^0 \rightarrow \bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left( |g_+(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2 |g_-(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+^*(t) g_-(t)] \right) \quad (3.15)
 \end{aligned}$$

$$\begin{aligned}
 |g_{\pm}(t)|^2 &= \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{1}{2} \Delta\Gamma t \pm \cos \Delta m t \right) \\
 g_+^*(t) g_-(t) &= \frac{e^{-\Gamma t}}{2} \left( \sinh \frac{1}{2} \Delta\Gamma t + i \sin \Delta m t \right) \\
 g_+(t) g_-^*(t) &= \frac{e^{-\Gamma t}}{2} \left( \sinh \frac{1}{2} \Delta\Gamma t - i \sin \Delta m t \right) \quad (3.16)
 \end{aligned}$$

# The 'master equations'

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 \frac{e^{-\Gamma t}}{2} \left( (1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t + (1 - |\lambda_f|^2) \cos \Delta mt - 2\Im\lambda_f \sin \Delta mt \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 \frac{e^{-\Gamma t}}{2} \left( (1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t - (1 - |\lambda_f|^2) \cos \Delta mt + 2\Im\lambda_f \sin \Delta mt \right)
 \end{aligned}
 \tag{3.17}$$

The diagram highlights the terms in the equations:
 

- (direct) Decay** (blue text and circles):  $(1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t$  and  $(1 - |\lambda_f|^2) \cos \Delta mt$
- Interference** (green text and circles):  $2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t$  and  $2\Im\lambda_f \sin \Delta mt$

The sinh- and sin-terms are associated to the interference between the decays with and without oscillation. Commonly, the master equations are expressed as:

# The 'master equations'

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 \frac{e^{-\Gamma t}}{2} \left( (1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t + (1 - |\lambda_f|^2) \cos \Delta mt - 2\Im\lambda_f \sin \Delta mt \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 \frac{e^{-\Gamma t}}{2} \left( (1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t - (1 - |\lambda_f|^2) \cos \Delta mt + 2\Im\lambda_f \sin \Delta mt \right)
 \end{aligned} \tag{3.17}$$

('direct') Decay      Interference

The sinh- and sin-terms are associated to the interference between the decays with and without oscillation. Commonly, the master equations are expressed as:

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{1}{2} \Delta\Gamma t + D_f \sinh \frac{1}{2} \Delta\Gamma t + C_f \cos \Delta mt - S_f \sin \Delta mt \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{1}{2} \Delta\Gamma t + D_f \sinh \frac{1}{2} \Delta\Gamma t - C_f \cos \Delta mt + S_f \sin \Delta mt \right)
 \end{aligned} \tag{3.18}$$

with

$$D_f = \frac{2\Re\lambda_f}{1 + |\lambda_f|^2} \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad S_f = \frac{2\Im\lambda_f}{1 + |\lambda_f|^2}. \tag{3.19}$$

# Classification of CP Violating effects

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1. CP violation in decay

$$\Gamma(P^0 \rightarrow f) \neq \Gamma(\bar{P}^0 \rightarrow \bar{f})$$

This is obviously satisfied (see Eq. (3.15)) when

$$\left| \frac{\bar{A}_f}{A_f} \right| \neq 1.$$

2. CP violation in mixing

$$\text{Prob}(P^0 \rightarrow \bar{P}^0) \neq \text{Prob}(\bar{P}^0 \rightarrow P^0)$$

$$\left| \frac{q}{p} \right| \neq 1.$$

3. CP violation in interference

$$\Gamma(P^0(\rightsquigarrow \bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

$$\Im \lambda_f = \Im \left( \frac{q \bar{A}_f}{p A_f} \right) \neq 0$$

# Meson Decays

- Formalism of meson *oscillations*:

$$|P^0(t)\rangle = \frac{1}{2} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle$$

- Subsequent: *decay*

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \left( |g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)] \right)$$

—  $P^0 \rightarrow f$

—  $P^0 \rightarrow \bar{P}^0 \rightarrow f$

$$A(f) = \langle f|T|P^0\rangle$$

$$\bar{A}(f) = \langle f|T|\bar{P}^0\rangle$$

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$$

**Interference**

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \frac{e^{-\Gamma t}}{2} \left( (1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t + (1 - |\lambda_f|^2) \cos \Delta m t - 2\Im\lambda_f \sin \Delta m t \right)$$

( 'direct' ) Decay
Interference

# CP violation: type 3

$$\Im \lambda_f = \Im \left( \frac{q \bar{A}_f}{p A_f} \right) \neq 0$$

$$\Gamma(P^0(\rightsquigarrow \bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

$$A_{CP}(t) = \frac{\Gamma_{P^0(t) \rightarrow f} - \Gamma_{\bar{P}^0(t) \rightarrow f}}{\Gamma_{P^0(t) \rightarrow f} + \Gamma_{\bar{P}^0(t) \rightarrow f}}$$

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{1}{2} \Delta \Gamma t + D_f \sinh \frac{1}{2} \Delta \Gamma t + C_f \cos \Delta m t - S_f \sin \Delta m t \right)$$

$$\Gamma_{\bar{P}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{1}{2} \Delta \Gamma t + D_f \sinh \frac{1}{2} \Delta \Gamma t - C_f \cos \Delta m t + S_f \sin \Delta m t \right)$$

$$D_f = \frac{2\Re \lambda_f}{1 + |\lambda_f|^2} \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad S_f = \frac{2\Im \lambda_f}{1 + |\lambda_f|^2}$$

$$A_{CP}(t) = \frac{\Gamma_{P^0(t) \rightarrow f} - \Gamma_{\bar{P}^0(t) \rightarrow f}}{\Gamma_{P^0(t) \rightarrow f} + \Gamma_{\bar{P}^0(t) \rightarrow f}} = \frac{2C_f \cos \Delta m t - 2S_f \sin \Delta m t}{2 \cosh \frac{1}{2} \Delta \Gamma t + 2D_f \sinh \frac{1}{2} \Delta \Gamma t}$$



## Classification of CP Violating effects - Nr. 3:

Consider  $f = \bar{f}$ :

$$A_{CP}(t) = \frac{\Gamma_{P^0(t) \rightarrow f} - \Gamma_{\bar{P}^0(t) \rightarrow f}}{\Gamma_{P^0(t) \rightarrow f} + \Gamma_{\bar{P}^0(t) \rightarrow f}} = \frac{2C_f \cos \Delta mt - 2S_f \sin \Delta mt}{2 \cosh \frac{1}{2} \Delta \Gamma t + 2D_f \sinh \frac{1}{2} \Delta \Gamma t}$$

If one amplitude dominates the decay, then  $A_f = \bar{A}_f$

$$A_{CP}(t) = \frac{-\Im \lambda_f \sin \Delta mt}{\cosh \frac{1}{2} \Delta \Gamma t + \Re \lambda_f \sinh \frac{1}{2} \Delta \Gamma t}$$

### 3. CP violation in interference

$$\Gamma(P^0(\rightsquigarrow \bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

$$\Im \lambda_f = \Im \left( \frac{q \bar{A}_f}{p A_f} \right) \neq 0$$

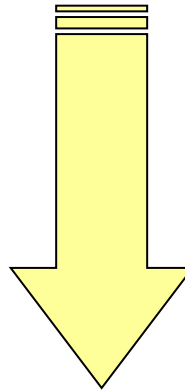
## Relax: $B^0 \rightarrow J/\psi K_S$ simplifies...

$$D_f = \frac{2\Re\lambda_f}{1 + |\lambda_f|^2} \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad S_f = \frac{2\Im\lambda_f}{1 + |\lambda_f|^2}.$$

$$A_{CP}(t) = \frac{\Gamma_{P^0(t) \rightarrow f} - \Gamma_{\bar{P}^0(t) \rightarrow f}}{\Gamma_{P^0(t) \rightarrow f} + \Gamma_{\bar{P}^0(t) \rightarrow f}} = \frac{2C_f \cos \Delta m t - 2S_f \sin \Delta m t}{2 \cosh \frac{1}{2} \Delta \Gamma t + 2D_f \sinh \frac{1}{2} \Delta \Gamma t}$$

$$|\lambda_f|=1$$

$$\Delta\Gamma=0$$

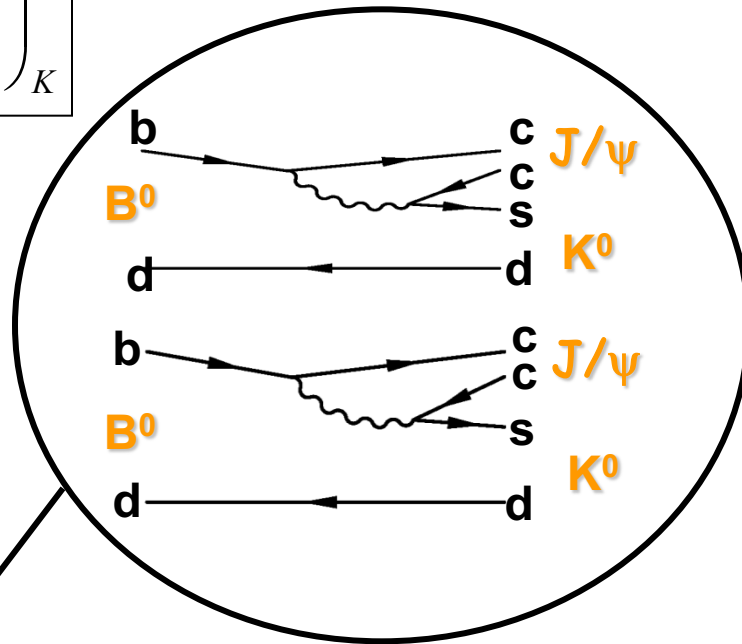
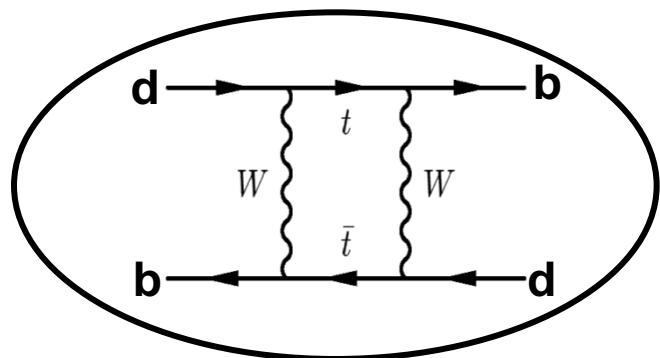


$$A_{CP}(t) = -\Im\lambda_f \sin(\Delta m t)$$

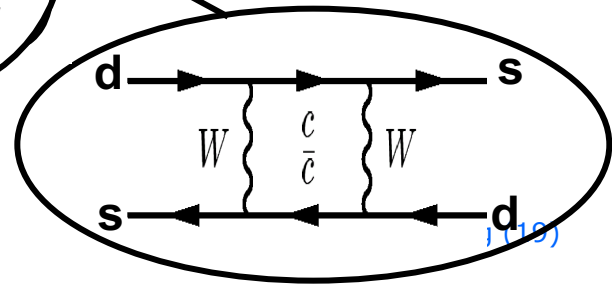
# $\lambda_f$ for $B^0 \rightarrow J/\psi K^0_s$

$$\lambda_f = \frac{q \bar{A}_f}{p A_f}$$

$$\lambda_{J/\psi K_s} = \left( \frac{q}{p} \right)_{B^0} \frac{\bar{A}_{J/\psi K_s}}{A_{J/\psi K_s}} = \left( \frac{q}{p} \right)_{B^0} \frac{\bar{A}_{J/\psi K^0}}{A_{J/\psi K^0}} \left( \frac{p}{q} \right)_K$$



$$\lambda_{J/\psi K_s} = \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left( \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \left( \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \right)$$



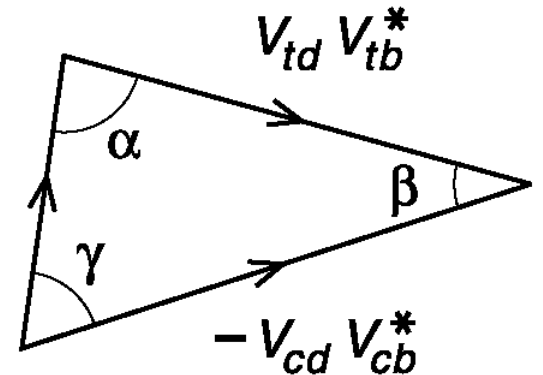
# $\lambda_f$ for $B^0 \rightarrow J/\psi K^0_S$

$$\lambda_{J/\psi K_S} = - \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left( \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \left( \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \right)$$
$$= -e^{-2i\beta}$$

## Time-dependent $CP$ asymmetry

$$A_{CP}(t) = -\sin 2\beta \sin(\Delta mt)$$

- Theoretically clean way to measure  $\beta$
- Clean experimental signature
- Branching fraction:  $O(10^{-4})$ 
  - “Large” compared to other  $CP$  modes!



# CP eigenvalue of final state $J/\psi K^0_S$

- CP  $|J/\psi\rangle = +1 |J/\psi\rangle$
- CP  $|K^0_S\rangle = +1 |K^0_S\rangle$
- CP  $|J/\psi K^0_S\rangle = (-1)^1 |J/\psi K^0_S\rangle$

( S(B)=0  $\rightarrow$  L(J/ $\psi$ K $_S^0$ )=1 !)

$$\lambda_{J/\psi K_S} = - \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left( \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \left( \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \right)$$

$$= -e^{-2i\beta}$$

**Relative minus-sign between state and CP-conjugated state:**

$$A_{CP}(t) = \frac{\Gamma_{\bar{B} \rightarrow f}(t) - \Gamma_{B \rightarrow f}(t)}{\Gamma_{\bar{B} \rightarrow f}(t) + \Gamma_{B \rightarrow f}(t)} = \text{Im}(\lambda_f) \sin \Delta m t$$

$$A_{CP}(t) = -\sin 2\beta \sin(\Delta m t)$$

# Sum of 2 amplitudes: sensitivity to phase

- Now also look at CP-conjugate process
- Investigate situation at time  $t$ , such that  $|A_1| = |A_2|$  :

$\Gamma(B \rightarrow f) =$

$\Gamma(\bar{B} \rightarrow f) =$

$N(B^0 \rightarrow f) \propto |A|^2 \propto (1 - \cos\phi)^2 + \sin^2\phi$   
 $= 1 - 2\cos\phi + \cos^2\phi + \sin^2\phi$   
 $= 2 - 2\cos(\pi/2 - 2\beta)$   
 $\propto 1 - \sin(2\beta)$

$N(\bar{B}^0 \rightarrow f) \propto (1 + \cos\phi)^2 + \sin^2\phi$   
 $= 2 + 2\cos(\pi/2 - 2\beta)$   
 $\propto 1 + \sin(2\beta)$

**CP**

$$A_{CP} = \frac{N_{\bar{B}^0 \rightarrow f} - N_{B^0 \rightarrow f}}{N_{\bar{B}^0 \rightarrow f} + N_{B^0 \rightarrow f}} = \sin(2\beta)$$

- Directly observable result (essentially just from counting) measure CKM phase  $\beta$  directly!

$$A_{CP}(t = \pi / 2\Delta m) = \frac{N_{\bar{B}^0 \rightarrow f} - N_{B^0 \rightarrow f}}{N_{\bar{B}^0 \rightarrow f} + N_{B^0 \rightarrow f}} = \sin(2\beta)$$

# Remember!

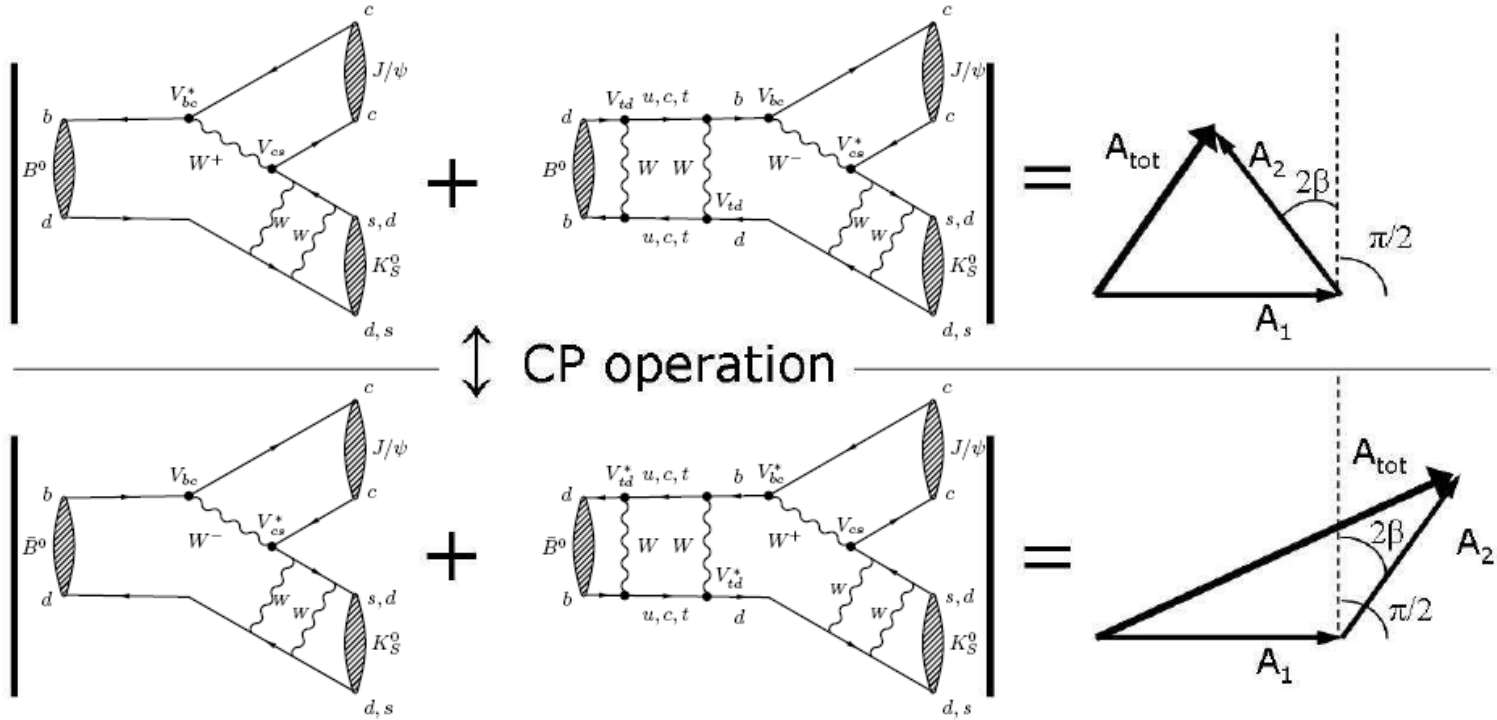
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## Necessary ingredients for CP violation:

- 1) Two (interfering) amplitudes
- 2) Phase difference between amplitudes
  - one CP conserving phase (‘strong’ phase)
  - one CP violating phase (‘weak’ phase)

*2 amplitudes*  
*2 phases*

# Remember!



*2 amplitudes*  
*2 phases*

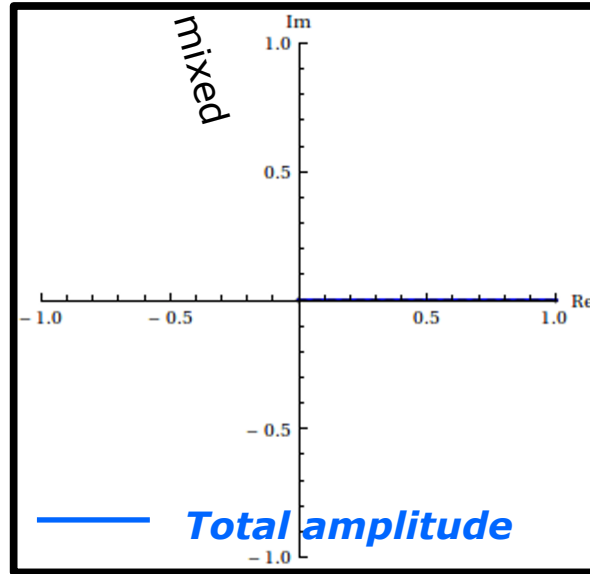
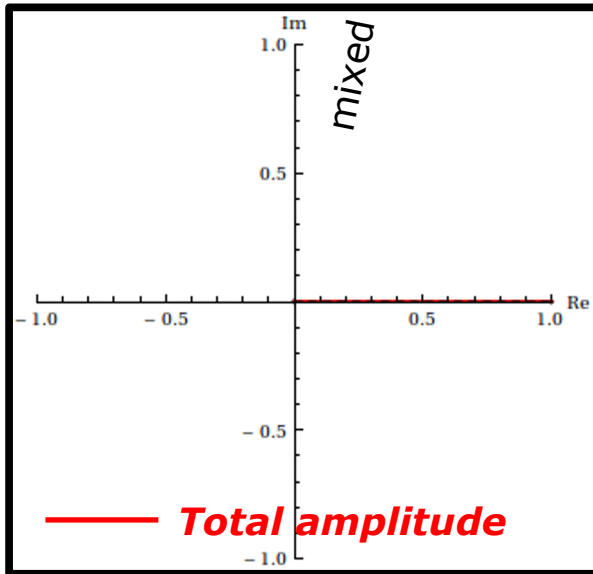


# Time dependent CP violation

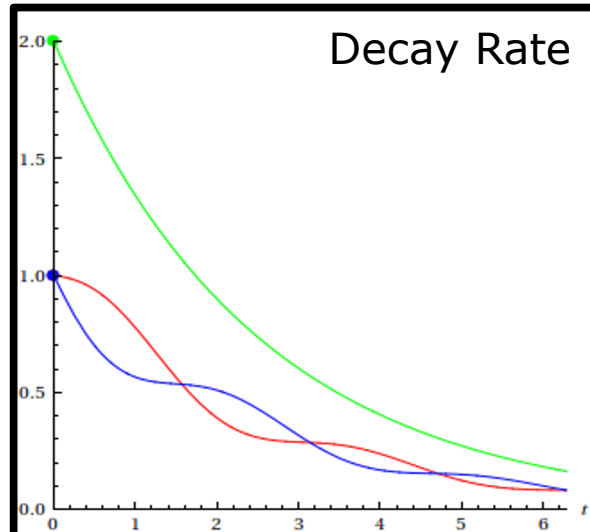
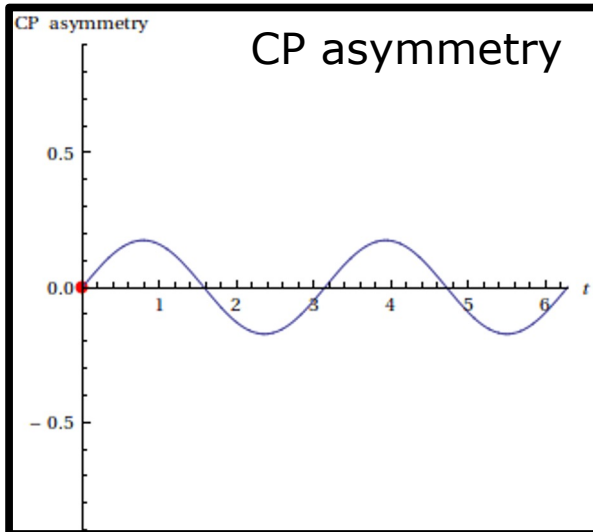
$B^0$  tag

$\overline{B}^0$  tag

unmixed



unmixed

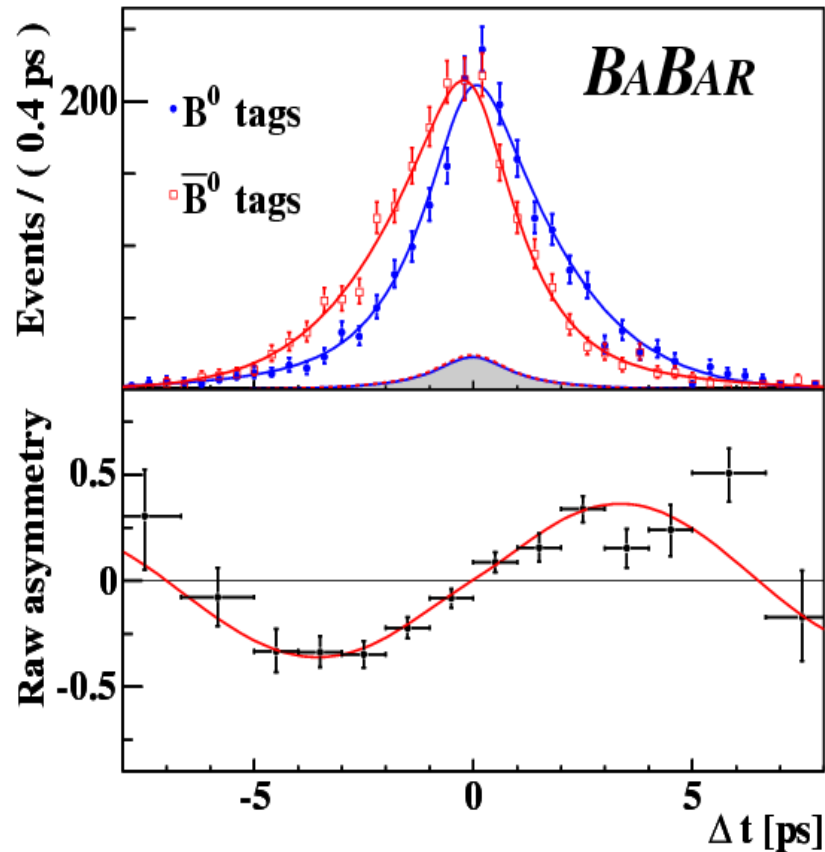


$\phi = 10$  deg  
 $\Gamma/\Delta m = 1.3$

# B-system - Time-dependent CP asymmetry

$$B^0 \rightarrow J/\psi K_S$$

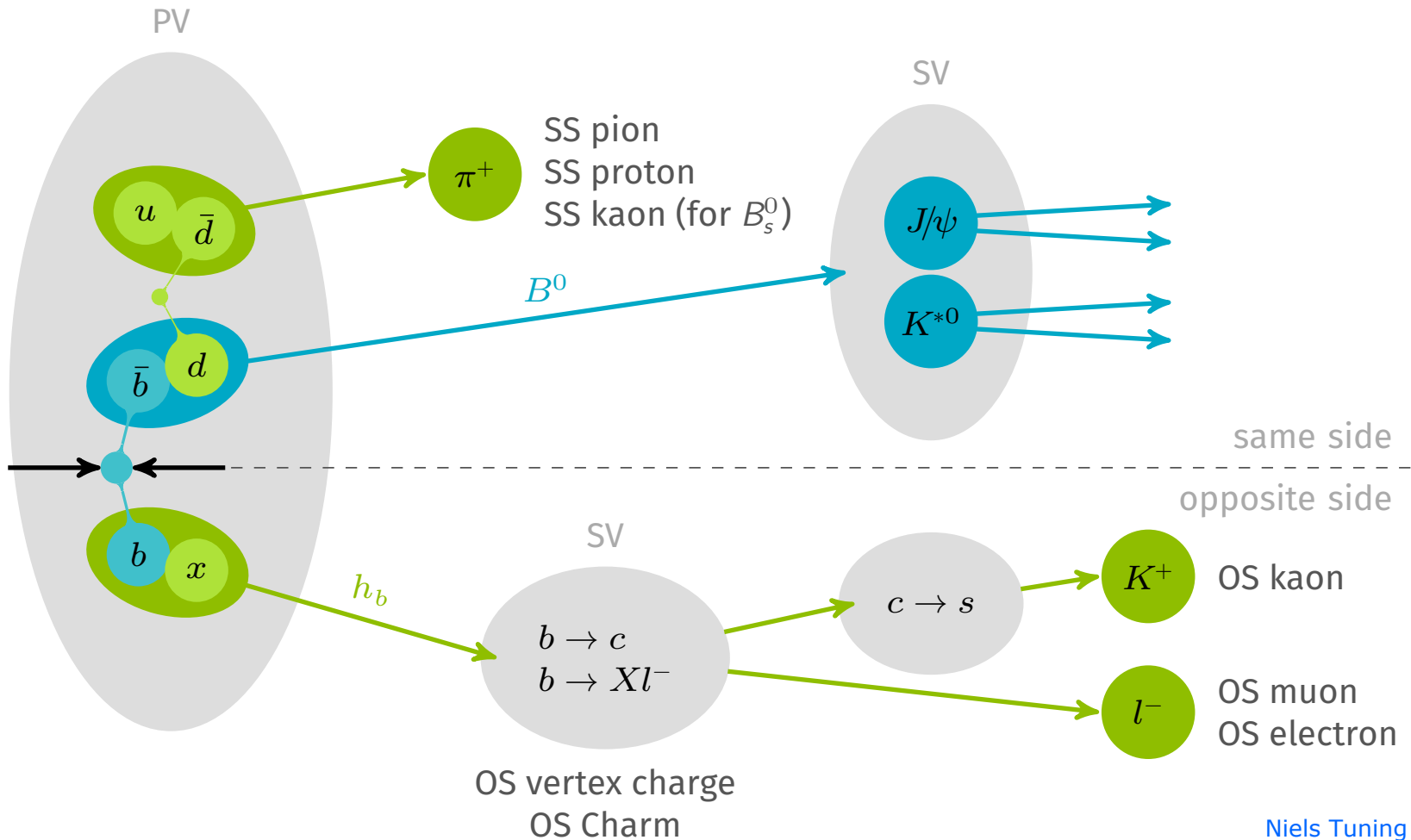
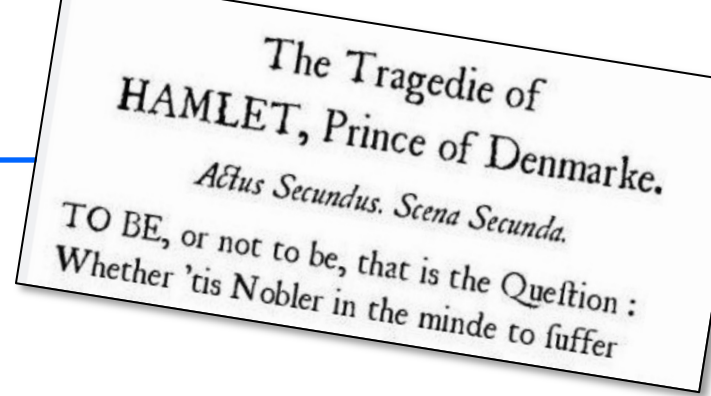
$$A_{CP}(t) = \frac{N_{\bar{B}^0 \rightarrow f} - N_{B^0 \rightarrow f}}{N_{\bar{B}^0 \rightarrow f} + N_{B^0 \rightarrow f}} = \sin(2\beta) \sin(\Delta m t)$$



BaBar (2002)

# sin2β in LHCb

- Flavour tagging: *to "B<sup>0</sup>" or not to "B<sup>0</sup>" ?*



# sin2 $\beta$ in LHCb

---

- Flavour tagging: *to "B<sup>0</sup>" or not to "B<sup>0</sup>" ?*
- Various algorithms
  - Not perfect event-by-event, but statistically useful!
    - **Key parameters: efficiency and wrong-tag fraction**  $\rightarrow \epsilon(1-2\omega)^2$
  - Measure performance with  $B^0 \rightarrow J/\psi K^{*0}$ ,  $B^+ \rightarrow J/\psi K^+$ ,  $B_s^0 \rightarrow D_s^+ \pi^-$

Tagger	$\epsilon$ [%]	$\omega$ [%]	$\epsilon \langle D^2 \rangle = \epsilon(1-2\omega)^2$ [%]
OS $\mu$	$0.915 \pm 0.053$	$30.713 \pm 0.434$	$1.361 \pm 0.062$

# sin2β in LHCb

- Flavour tagging: *to "B<sup>0</sup>" or not to "B<sup>0</sup>" ?*
- Various algorithms
  - Not perfect event-by-event, but statistically useful!
    - **Key parameters: efficiency and wrong-tag fraction** →  $\epsilon(1-2\omega)^2$
  - Measure performance with  $B^0 \rightarrow J/\psi K^{*0}$ ,  $B^+ \rightarrow J/\psi K^+$ ,  $B_s^0 \rightarrow D_s^+ \pi^-$

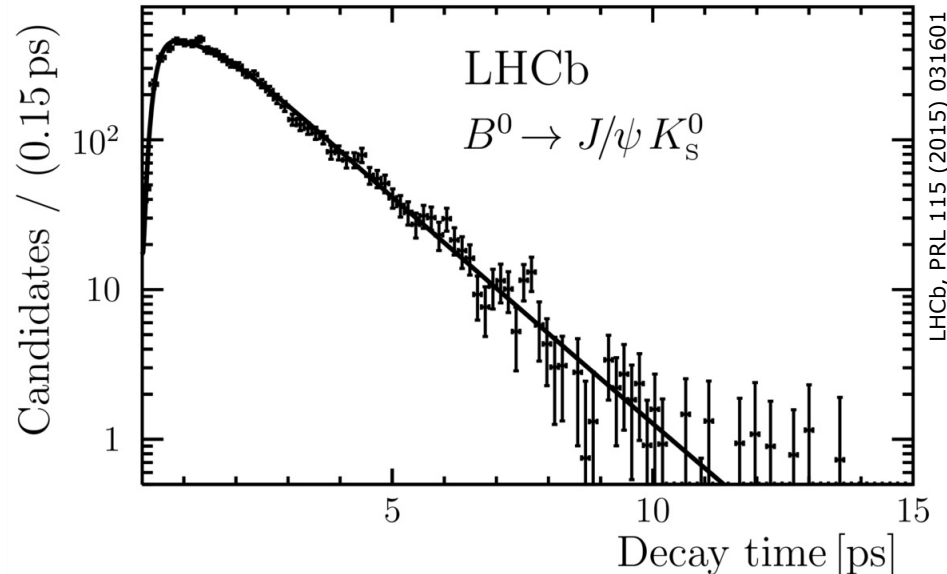
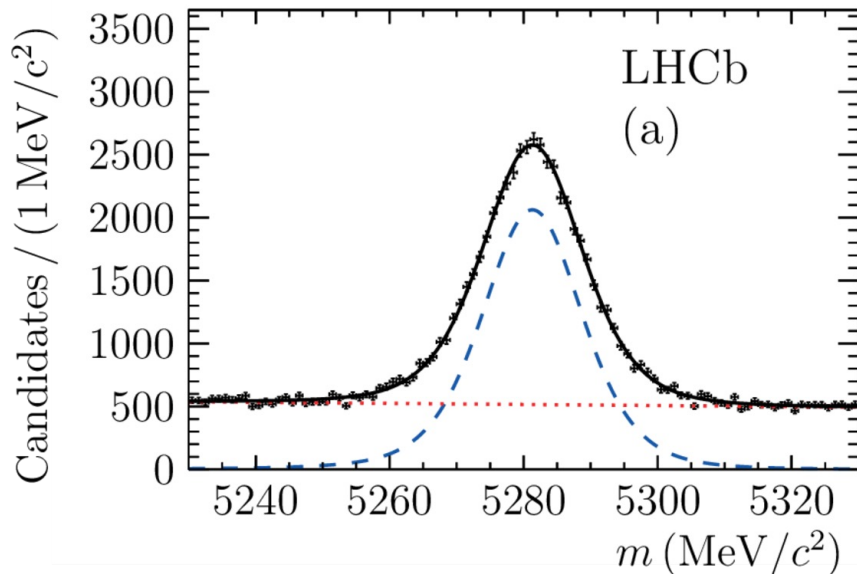
Tagger	$\epsilon$ [%]	$\omega$ [%]	$\epsilon \langle D^2 \rangle = \epsilon(1-2\omega)^2$ [%]
OS $\mu$	$0.915 \pm 0.053$	$30.713 \pm 0.434$	$1.361 \pm 0.062$
OS $e$	$4.451 \pm 0.038$	$34.038 \pm 0.604$	$0.454 \pm 0.035$
OSK	$19.600 \pm 0.073$	$37.557 \pm 0.315$	$1.214 \pm 0.061$
OSVtx	$20.834 \pm 0.075$	$36.994 \pm 0.308$	$1.410 \pm 0.067$
OSc	$5.025 \pm 0.040$	$34.062 \pm 0.620$	$0.511 \pm 0.040$
OScomb	$40.154 \pm 0.090$	$35.123 \pm 0.211$	$3.555 \pm 0.101$
SSK	$68.190 \pm 0.177$	$39.667 \pm 0.507$	$2.912 \pm 0.286$
SS $\pi$	$83.486 \pm 0.068$	$42.561 \pm 0.145$	$1.848 \pm 0.072$
SSp	$37.767 \pm 0.089$	$43.645 \pm 0.221$	$0.610 \pm 0.042$
SScomb	$87.590 \pm 0.061$	$41.787 \pm 0.142$	$2.364 \pm 0.081$

# sin2β

$$\begin{aligned} \mathcal{A}_{[c\bar{c}]K_S^0}(t) &\equiv \frac{\Gamma(\bar{B}^0(t) \rightarrow [c\bar{c}]K_S^0) - \Gamma(B^0(t) \rightarrow [c\bar{c}]K_S^0)}{\Gamma(\bar{B}^0(t) \rightarrow [c\bar{c}]K_S^0) + \Gamma(B^0(t) \rightarrow [c\bar{c}]K_S^0)} \\ &= \frac{S \sin(\Delta m t) - C \cos(\Delta m t)}{\cosh(\Delta\Gamma t/2) + A_{\Delta\Gamma} \sinh(\Delta\Gamma t/2)} \approx S \sin(\Delta m t) \end{aligned}$$

- Flavour tagging essential

- Which  $B^0$  was a  $\bar{B}^0$  ?



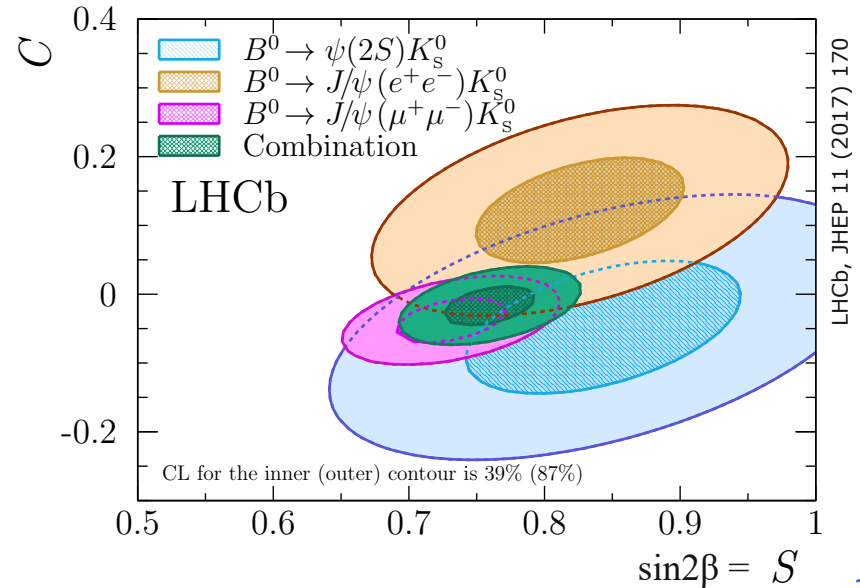
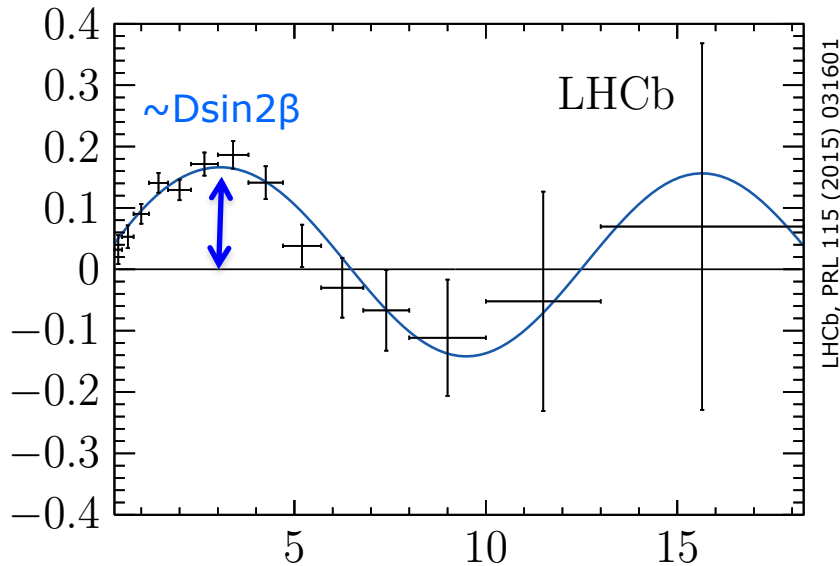
# sin2β

$$\begin{aligned} \mathcal{A}_{[c\bar{c}]K_S^0}(t) &\equiv \frac{\Gamma(\bar{B}^0(t) \rightarrow [c\bar{c}]K_S^0) - \Gamma(B^0(t) \rightarrow [c\bar{c}]K_S^0)}{\Gamma(\bar{B}^0(t) \rightarrow [c\bar{c}]K_S^0) + \Gamma(B^0(t) \rightarrow [c\bar{c}]K_S^0)} \\ &= \frac{S \sin(\Delta m t) - C \cos(\Delta m t)}{\cosh(\Delta\Gamma t/2) + A_{\Delta\Gamma} \sinh(\Delta\Gamma t/2)} \approx S \sin(\Delta m t) \end{aligned}$$

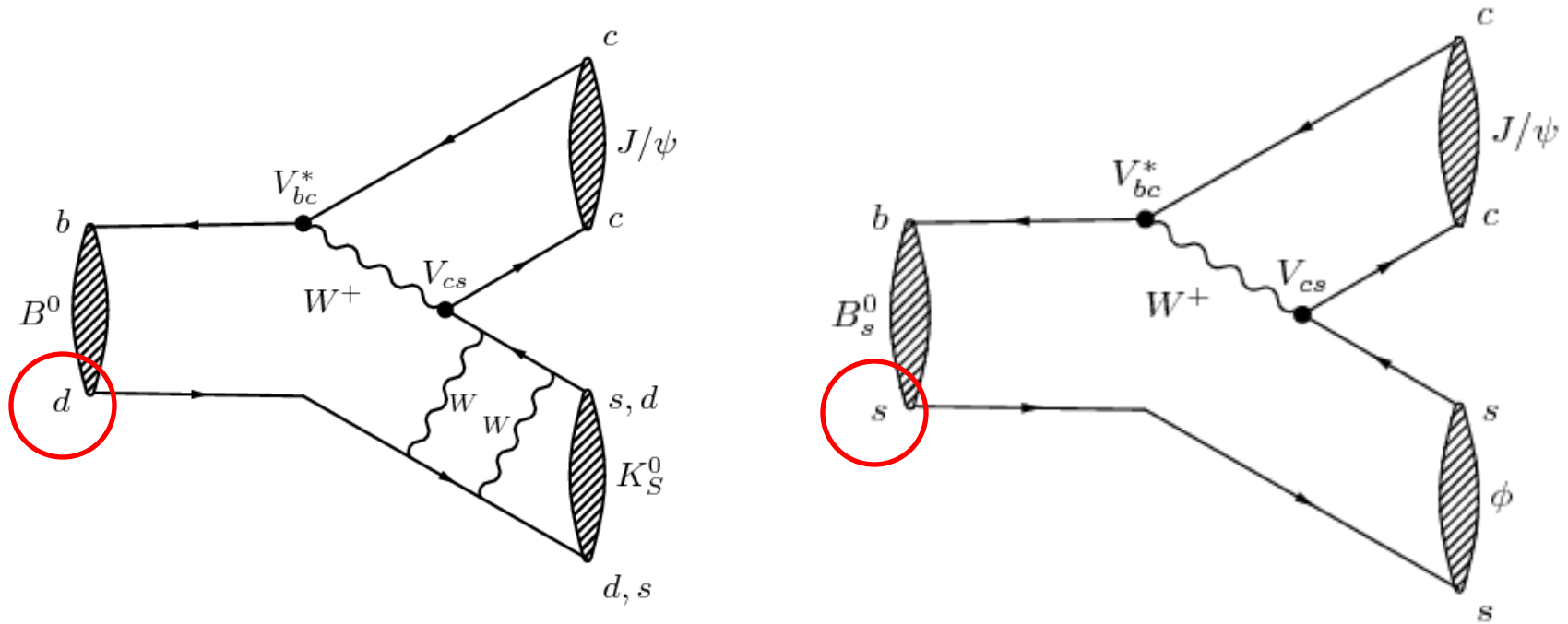
- Flavour tagging essential

- Wrong tag fraction  $w \sim 35\%$
- $D = (1 - 2w) \sim 0.3$

$$\mathbf{A}_{CP}(t) = \mathbf{D} \sin(2\beta) \sin(\Delta m t)$$



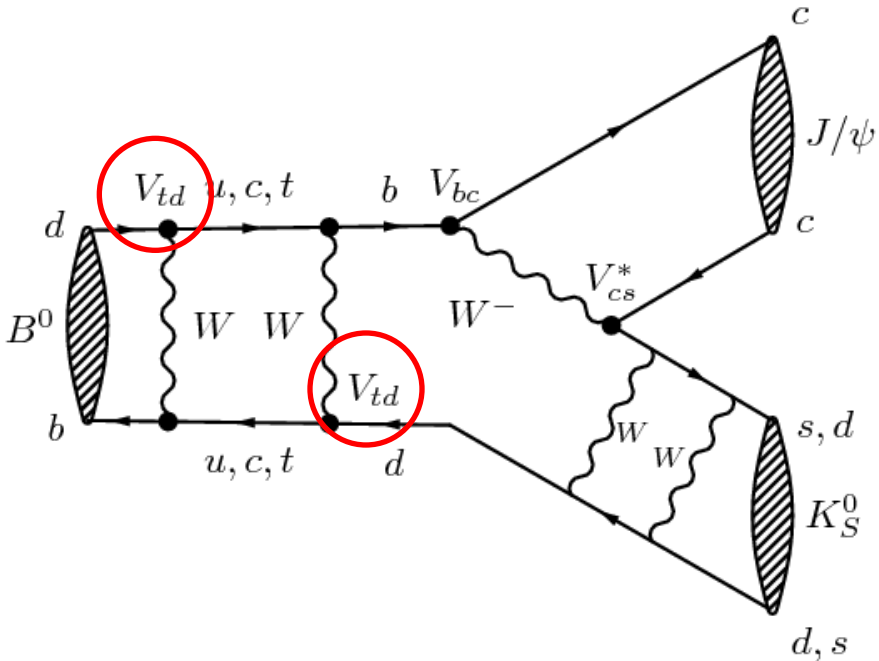
$\beta_s$ :  $B_s^0 \rightarrow J/\psi\phi$  :  $B_s^0$  analogue of  $B^0 \rightarrow J/\psi K_S^0$



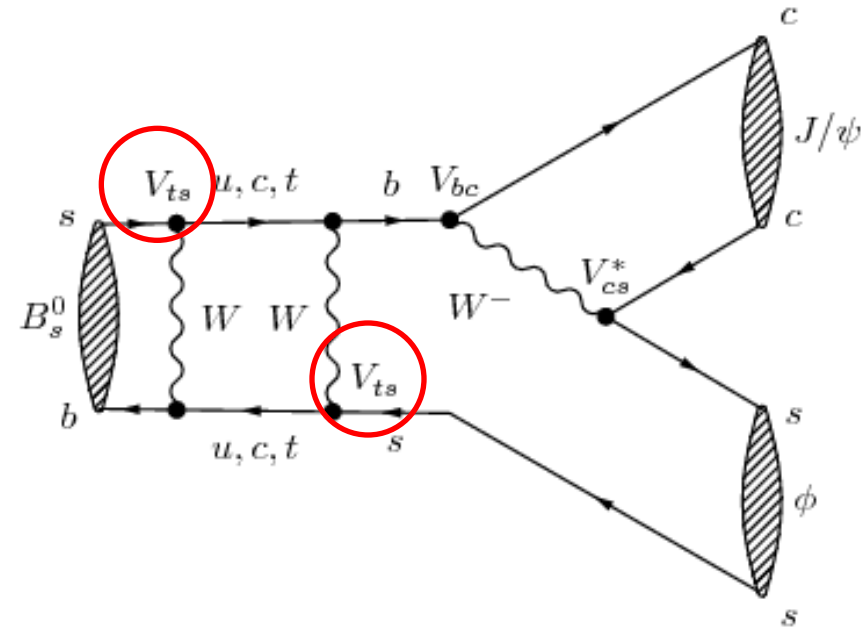
- Replace spectator quark  $d \rightarrow s$



# $\beta_s$ : $B_s^0 \rightarrow J/\psi\phi$ : $B_s^0$ analogue of $B^0 \rightarrow J/\psi K^0_S$



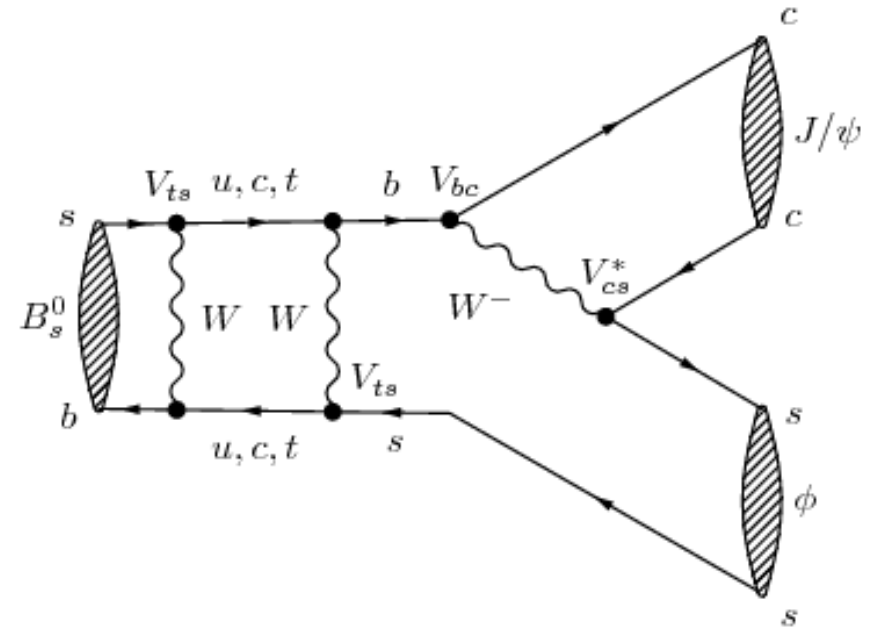
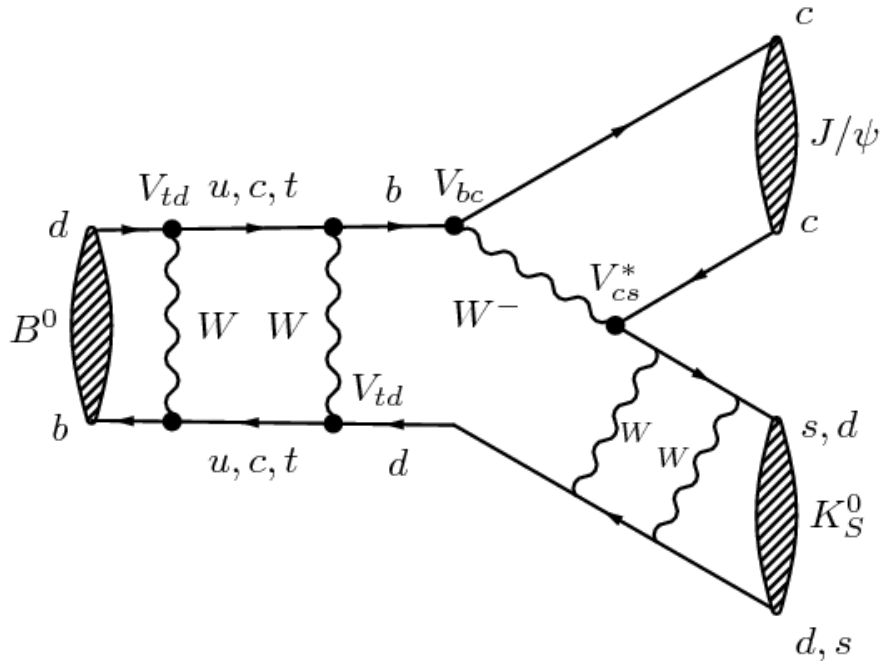
$$\beta \equiv \arg \left[ -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right]$$



$$\beta_s \equiv \arg \left[ -\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right]$$

$$V_{CKM, \text{Wolfenstein}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

# $\beta_S$ : $B_s^0 \rightarrow J/\psi\phi$ : $B_s^0$ analogue of $B^0 \rightarrow J/\psi K^0_S$



Differences:

	$B^0$	$B_s^0$
CKM	$V_{td}$	$V_{ts}$
$\Delta\Gamma$	$\sim 0$	$\sim 0.1$
Final state (spin)	$K^0 : s=0$	$\phi : s=1$
Final state (K)	$K^0$ mixing	-

# $\beta_s: B_s^0 \rightarrow J/\psi\phi$

$$A_{CP}(t) = \frac{\Gamma_{B_s^0(t) \rightarrow J/\psi\phi} - \Gamma_{\bar{B}_s^0(t) \rightarrow J/\psi\phi}}{\Gamma_{B_s^0(t) \rightarrow J/\psi\phi} + \Gamma_{\bar{B}_s^0(t) \rightarrow J/\psi\phi}} = \frac{\Im\lambda_{J/\psi\phi} \sin \Delta mt}{\cosh \frac{1}{2} \Delta\Gamma t + \Re\lambda_{J/\psi\phi} \sinh \frac{1}{2} \Delta\Gamma t}$$

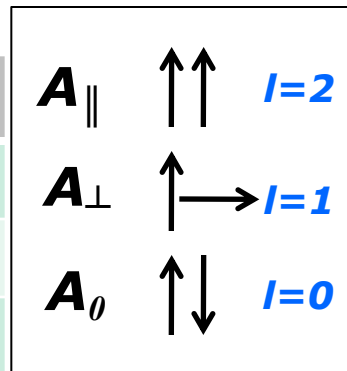
$$\lambda_{J/\psi\phi} = \left(\frac{q}{p}\right)_{B_s^0} \left(\eta_{J/\psi\phi} \frac{\bar{A}_{J/\psi\phi}}{A_{J/\psi\phi}}\right) = (-1)^l \left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right) \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right)$$

$$\Im\lambda_{J/\psi\phi} = (-1)^l \sin(-2\beta_s)$$

$$CP|J/\psi\phi\rangle_l = (-1)^l |J/\psi\phi\rangle_l$$

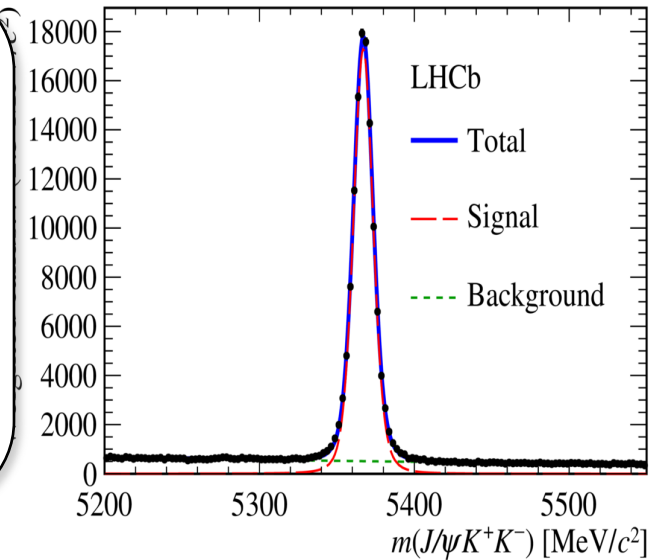
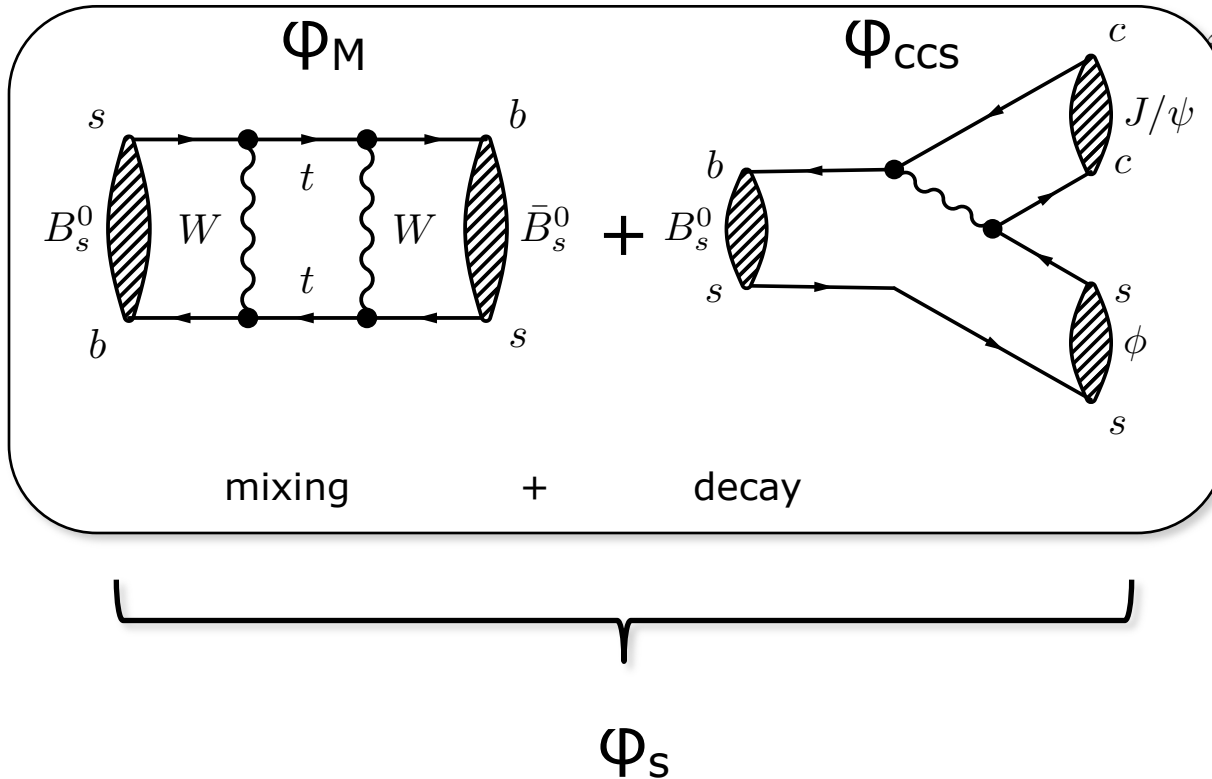
*$V_{ts}$  large, oscillations fast,  
need good vertex detector  
3 amplitudes*

	$B^0$	$B_s^0$
CKM	$V_{td}$	$V_{ts}$
$\Delta\Gamma$	$\sim 0$	$\sim 0.1$
Final state (spin)	$K^0 : s=0$	$\phi : s=1$
Final state (K)	$K^0$ mixing	-



# $\varphi_s$ with $B_s^0 \rightarrow J/\psi \phi$

("the  $\sin 2\beta$  of the  $B_s^0$  system")



# $\phi_s$ with $B_s^0 \rightarrow J/\psi \phi$

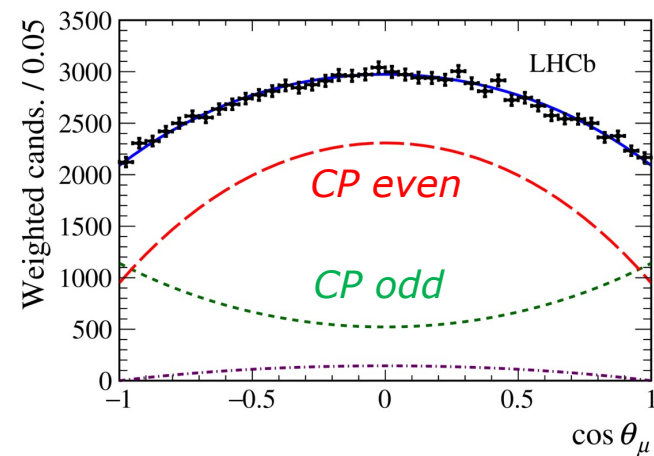
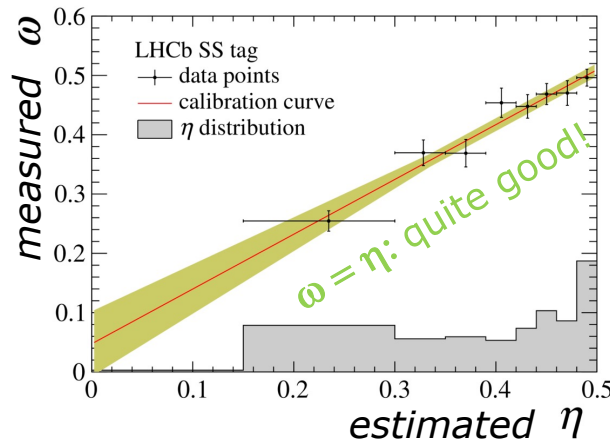
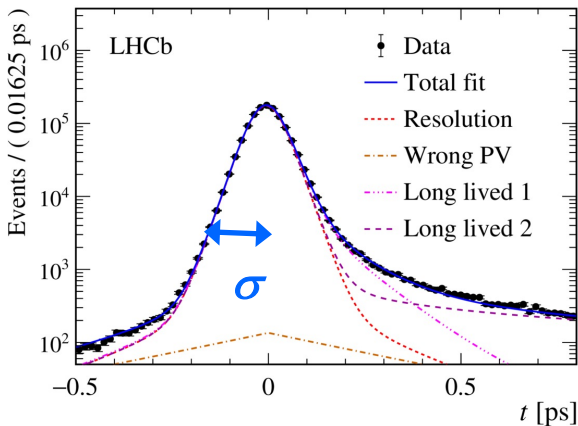
- Some challenges:

- 1) Rapid  $B_s^0$  oscillations: decay time resolution
- 2) "Same side" kaon-tagging: calibration with hadronic final state
- 3) Mix of CP eigenstates: angular analysis

1) Decay time resolution from prompt  $J/\psi$  :

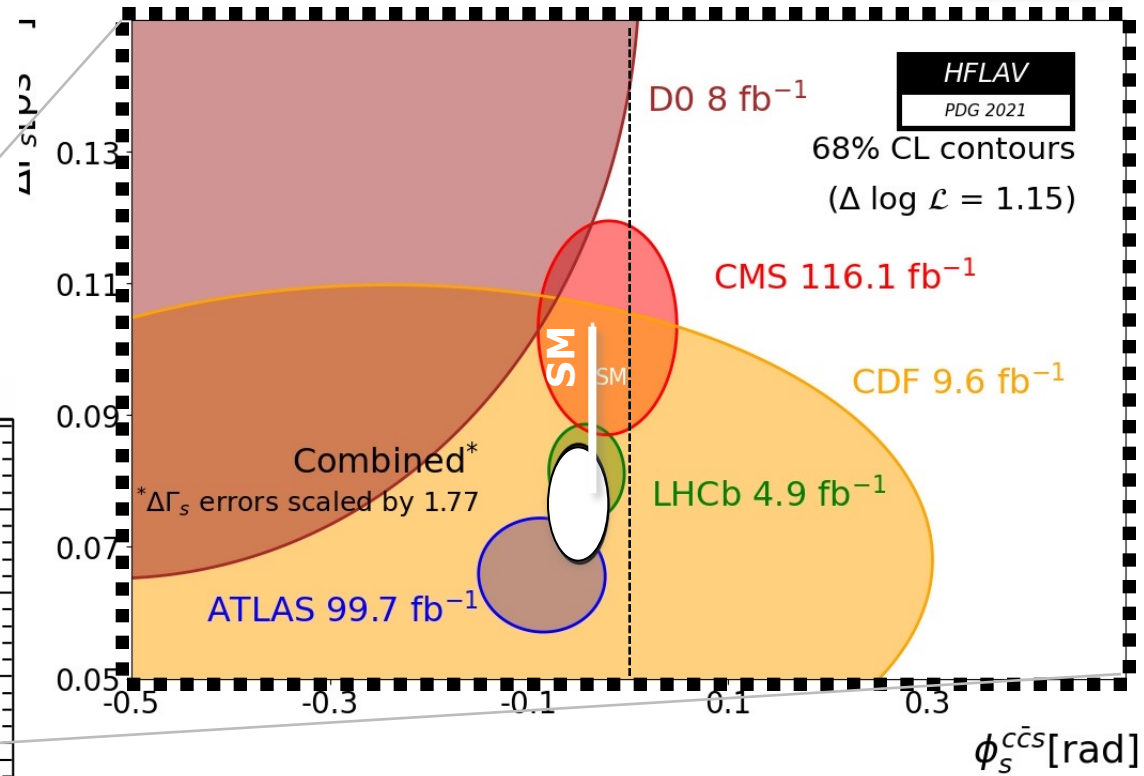
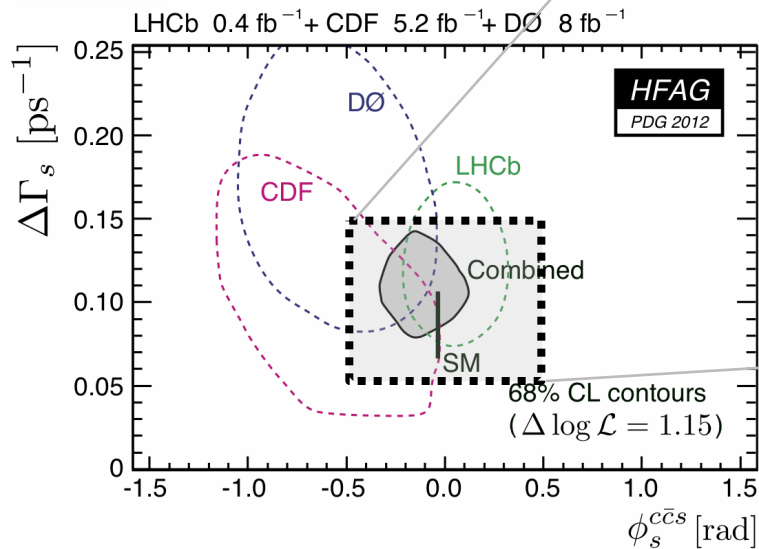
2) Tagging calibration from  $B_s^0 \rightarrow D_s \pi$

3) Angular analysis to disentangle CP + and CP -



- LHCb 2011-2016

2012



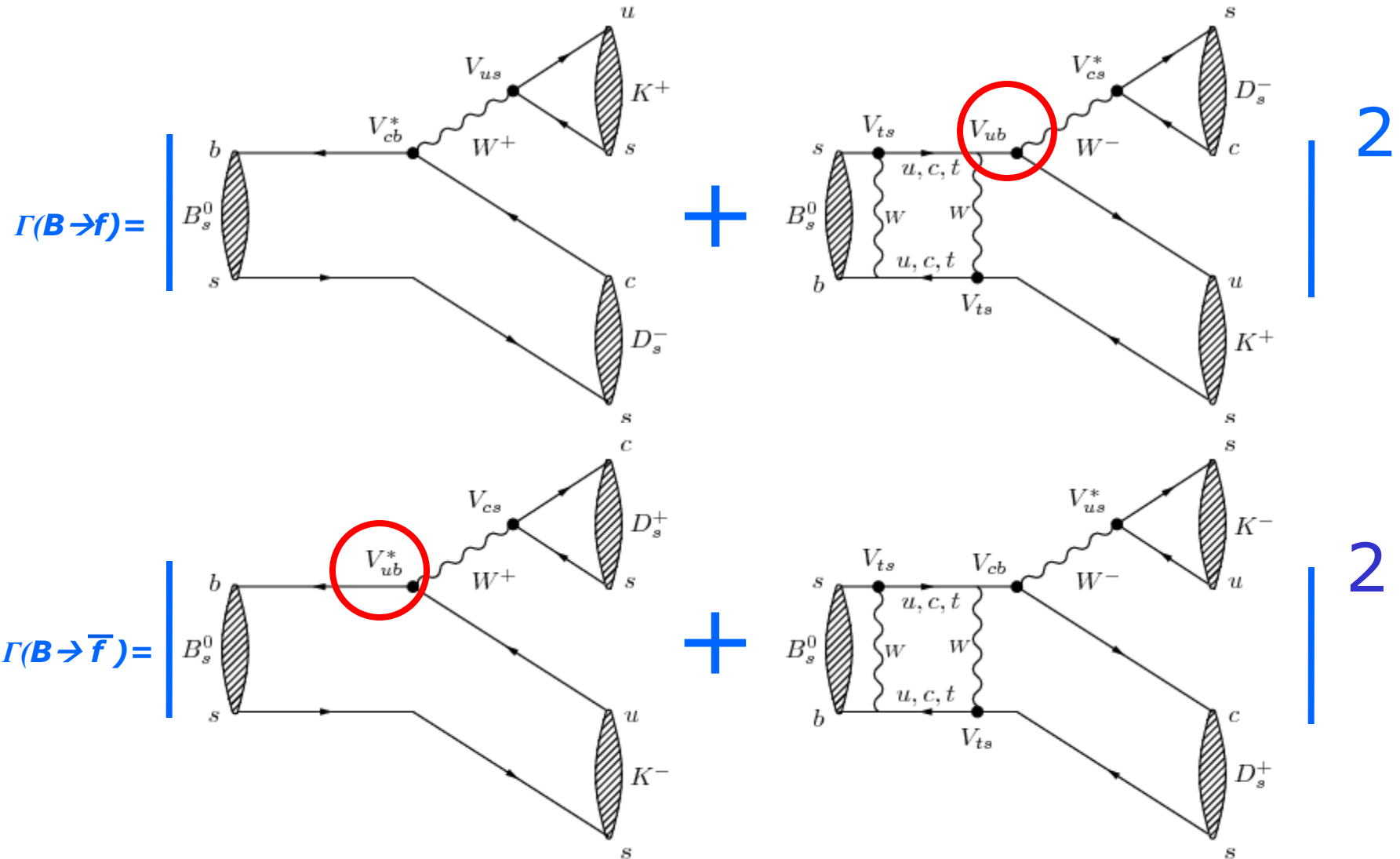
$$\phi_s = -50 \pm 19 \text{ mrad (HFLAV)}$$

$$\phi_s = -42 \pm 25 \text{ mrad (LHCb)}$$

$$\phi_s = -37 \pm 1 \text{ mrad (SM)}$$

CKMfitter,  
Phys. Rev. D84, 033005 (2011),  
updated with Summer 2019 results

Measure  $\gamma$ :  $B_s^0 \rightarrow D_s^\pm K^{-/+}$  : both  $\lambda_f$  and  $\lambda_{\bar{f}}$



NB: In addition  $\bar{B}_s \rightarrow D_s^\pm K^{-/+}$  : both  $\bar{\lambda}_f$  and  $\bar{\lambda}_{\bar{f}}$

# Formalism: $B_s^0 \rightarrow D_s^+ K^-$

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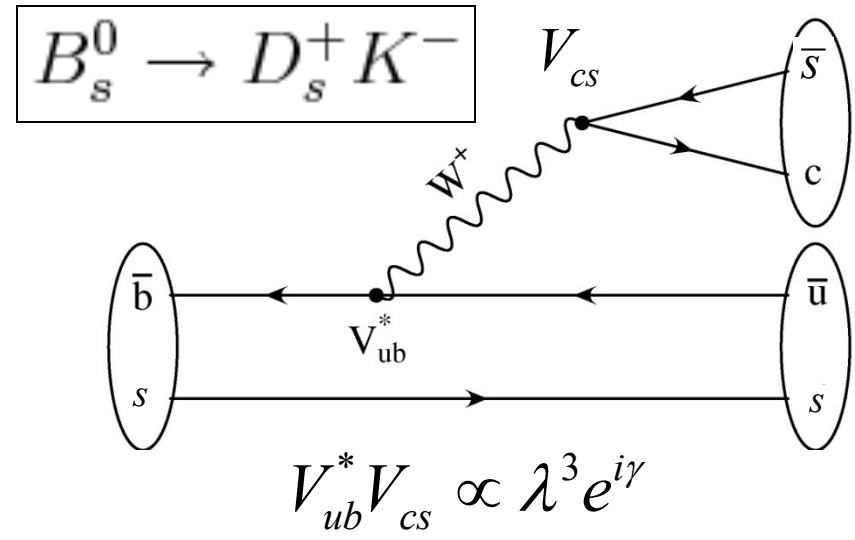
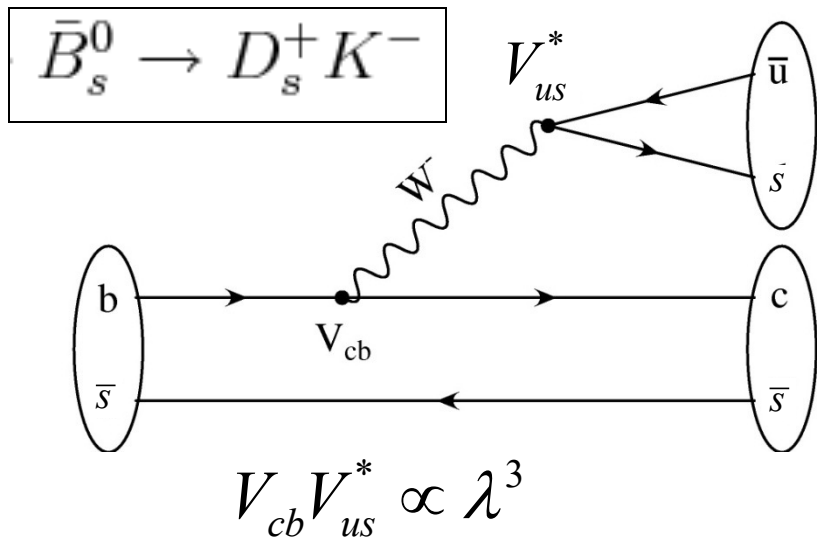
- Time-dependent decay rates:

$$\begin{aligned} \frac{d\Gamma_{B_s^0 \rightarrow f}(t)}{dt} &= \frac{1}{2} |A_f|^2 (1 + |\lambda_f|^2) e^{-\Gamma_s t} \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + A_f^{\Delta\Gamma} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) \right. \\ &\quad \left. + C_f \cos(\Delta m_s t) - S_f \sin(\Delta m_s t) \right], \\ \frac{d\Gamma_{\bar{B}_s^0 \rightarrow f}(t)}{dt} &= \frac{1}{2} |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) e^{-\Gamma_s t} \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + A_f^{\Delta\Gamma} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) \right. \\ &\quad \left. - C_f \cos(\Delta m_s t) + S_f \sin(\Delta m_s t) \right], \end{aligned}$$

$$\begin{aligned} C_f &= \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} = -C_{\bar{f}} = -\frac{1 - |\lambda_{\bar{f}}|^2}{1 + |\lambda_{\bar{f}}|^2} \\ S_f &= \frac{2\mathcal{I}m(\lambda_f)}{1 + |\lambda_f|^2}, \quad A_f^{\Delta\Gamma} = \frac{-2\mathcal{R}e(\lambda_f)}{1 + |\lambda_f|^2} \\ S_{\bar{f}} &= \frac{2\mathcal{I}m(\lambda_{\bar{f}})}{1 + |\lambda_{\bar{f}}|^2}, \quad A_{\bar{f}}^{\Delta\Gamma} = \frac{-2\mathcal{R}e(\lambda_{\bar{f}})}{1 + |\lambda_{\bar{f}}|^2} \end{aligned}$$



Measure  $\gamma$ :  $B_s \rightarrow D_s^\pm K^{-/+}$  --- first one  $f$ :  $D_s^+ K^-$



- This time  $|A_f| \neq |\bar{A}_f|$ , so  $|\lambda| \neq 1$ !

$$\left( \frac{\bar{A}_{D_s^- K^+}}{A_{D_s^- K^+}} \right) = \left( \frac{V_{ub} V_{cs}^*}{V_{cb}^* V_{us}} \right) \left( \frac{A_2}{A_1} \right) \quad \lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$$

- In fact, not only magnitude, but also phase difference:

$$\frac{A_{D_s^- K^+}}{\bar{A}_{D_s^- K^+}} = \frac{|A_{D_s^- K^+}|}{|\bar{A}_{D_s^- K^+}|} e^{i(\delta_s - \gamma)}$$

## Measure $\gamma$ : $B_s \rightarrow D_s^\pm K^{-/+}$

---

- $B_s^0 \rightarrow D_s^- K^+$  has phase difference  $(\delta - \gamma)$ :

$$\frac{A_{D_s^- K^+}}{\bar{A}_{D_s^- K^+}} = \frac{|A_{D_s^- K^+}|}{|\bar{A}_{D_s^- K^+}|} e^{i(\delta_s - \gamma)}$$

- Need  $B_s^0 \rightarrow D_s^+ K^-$  to disentangle  $\delta$  and  $\gamma$ :

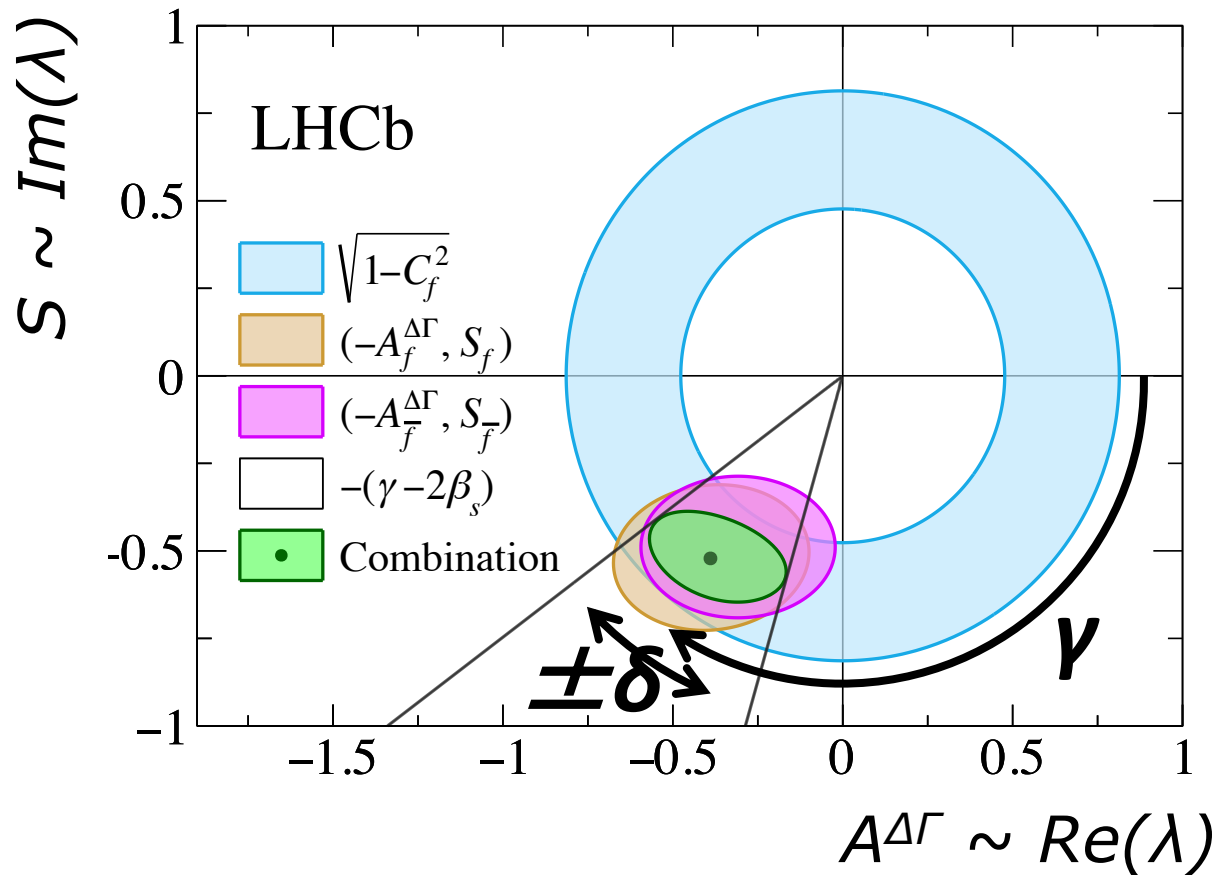
$$\lambda_{D_s^- K^+} = \left(\frac{q}{p}\right)_{B_s^0} \left(\frac{\bar{A}_{D_s^- K^+}}{A_{D_s^- K^+}}\right) = \left|\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right| \left|\frac{V_{ub} V_{cs}^*}{V_{cb}^* V_{us}}\right| \left|\frac{A_2}{A_1}\right| e^{i(-2\beta_s - \gamma + \delta_s)}$$

$$\lambda_{D_s^+ K^-} = \left(\frac{q}{p}\right)_{B_s^0} \left(\frac{\bar{A}_{D_s^+ K^-}}{A_{D_s^+ K^-}}\right) = \left|\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right| \left|\frac{V_{us}^* V_{cb}}{V_{cs} V_{ub}^*}\right| \left|\frac{A_1}{A_2}\right| e^{i(-2\beta_s - \gamma - \delta_s)}$$

# Formalism

- Polar:  $|\lambda|$  and  $\gamma$
- Cartesian:  $A^{\Delta\Gamma}$  and  $S$

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$$



A note on conventions:

1) We use:  $\Delta\Gamma_s = \Gamma_L - \Gamma_H > 0$

2) Opposite convention is equivalent if at the same time  $A^{\Delta\Gamma} \rightarrow -A^{\Delta\Gamma}$

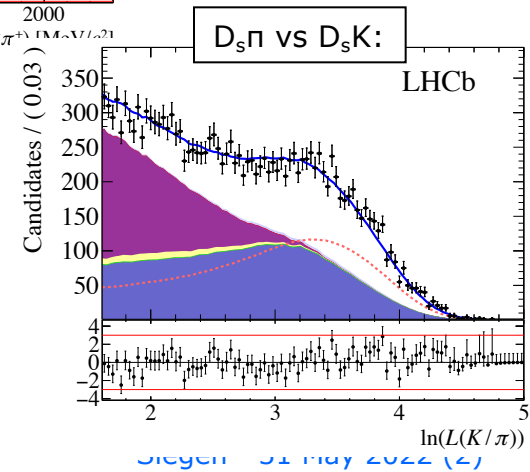
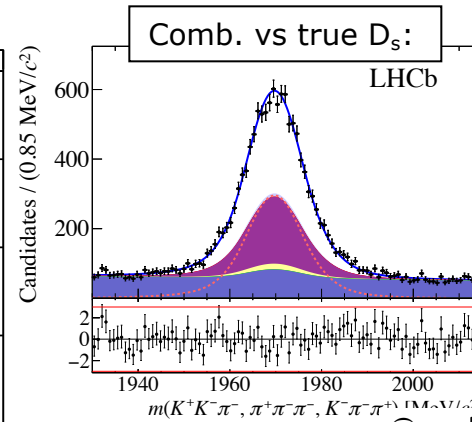
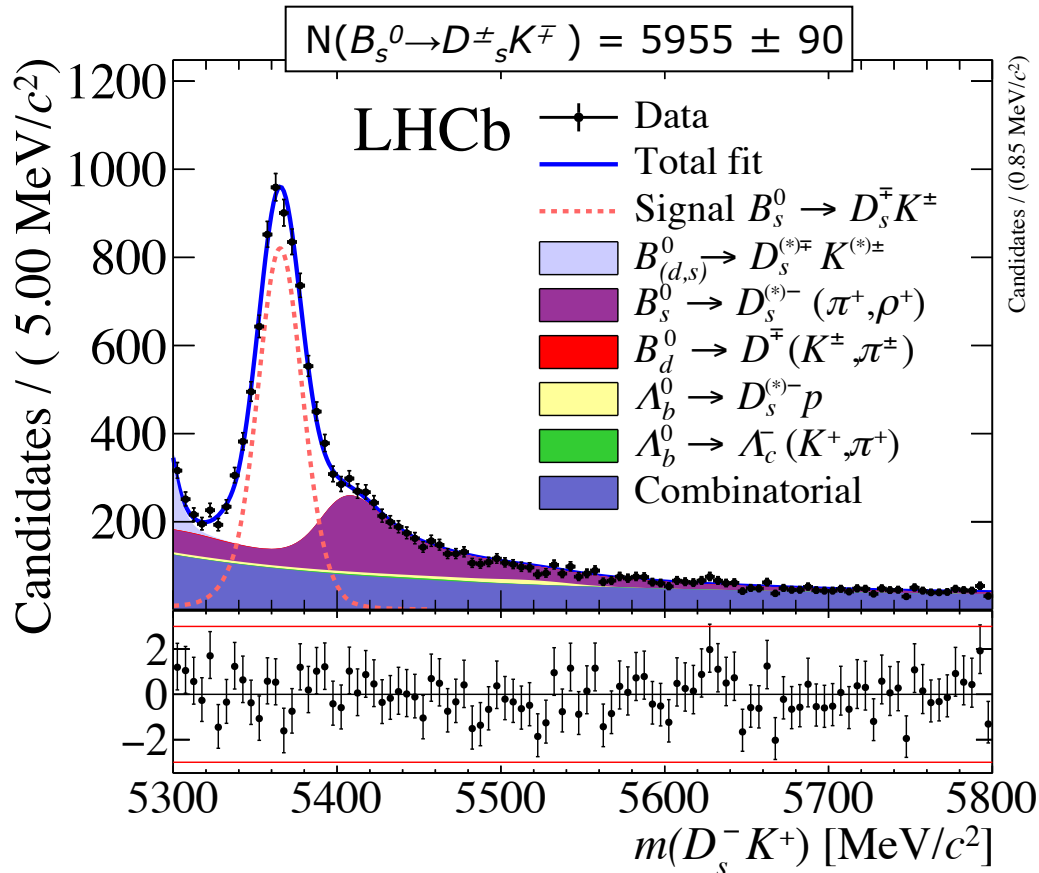
# $B_s^0 \rightarrow D^\pm K^\mp$ Analysis

---

- Obtain  $B_s^0$  signal sample: 3D fit to ( $m_B$ ,  $m_{D_s}$ , PID)
- $B_s^0$  or  $\bar{B}_s^0$ : Flavour Tagging
- Decay time: Resolution & acceptance
- Result: Decay time fit

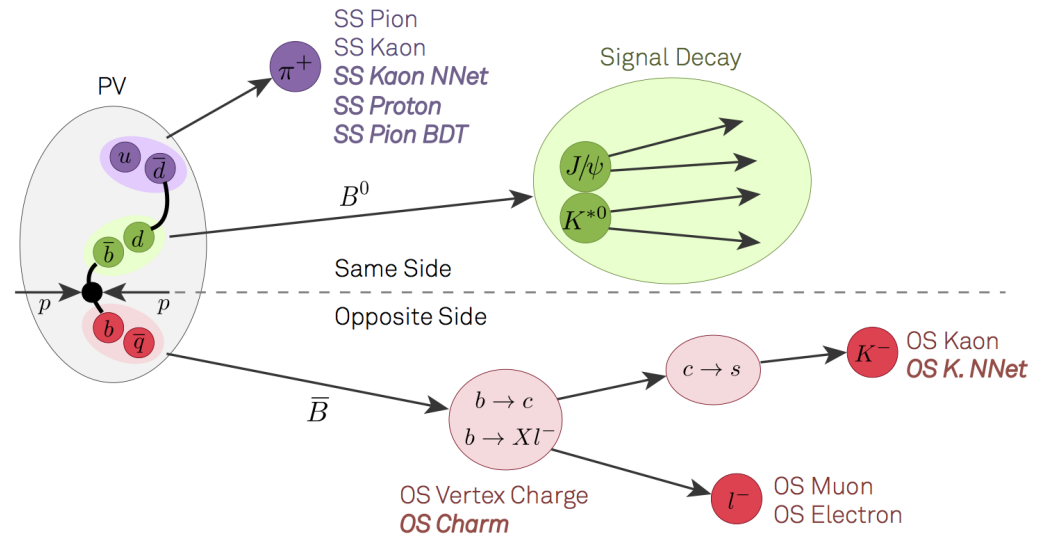
# $B_s^0 \rightarrow D_s^\pm K^\mp$ Analysis: mass fit

- Need to (statistically) separate signal from background
- Backgrounds:
  - Combinatorial
  - Partially reconstructed background ( $B_s^0 \rightarrow D_s^* K^\mp$ , etc)
  - Misidentified background ( $B_s^0 \rightarrow D_s^\pm \pi^\mp$ )



# $B_s^0 \rightarrow D_s^\pm K^\mp$ Analysis: Flavour Tagging

- To  $B_s^0$  or not to  $B_s^0$  :



sketch: Julian Wishahi

- Use  $B_s^0 \rightarrow D_s^+ \pi^-$  to calibrate!

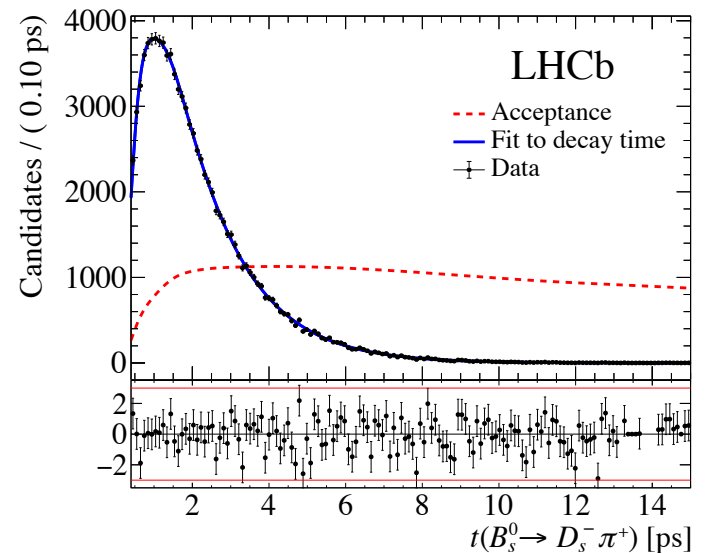
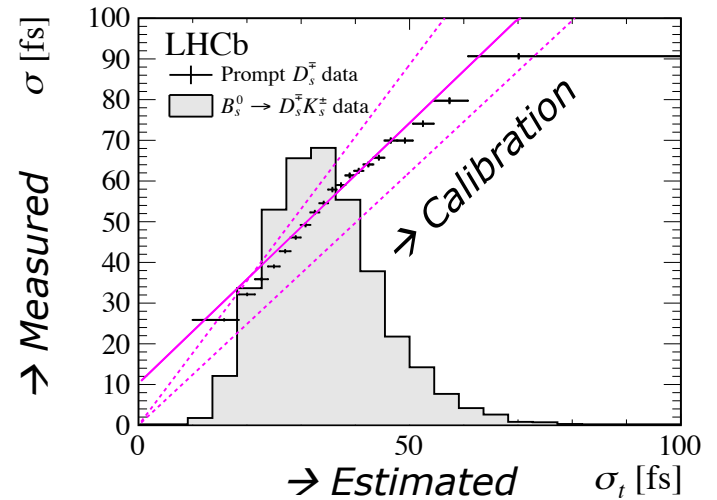
$B_s^0 \rightarrow D_s^- \pi^+$	$\epsilon_{\text{tag}} [\%]$	$\epsilon_{\text{eff}} [\%]$
OS only	$12.94 \pm 0.11$	$1.41 \pm 0.11$
SS only	$39.70 \pm 0.16$	$1.29 \pm 0.13$
Both OS and SS	$24.21 \pm 0.14$	$3.10 \pm 0.18$
Total	$76.85 \pm 0.24$	$5.80 \pm 0.25$

# $B_s^0 \rightarrow D_s^\pm K^\mp$ Analysis: Decay time

Use  $B_s^0 \rightarrow D_s^+ \pi^-$  to calibrate!

- Resolution

- Acceptance



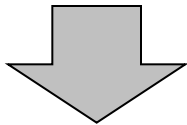
# $B_s^0 \rightarrow D_s^\pm K^\mp$ Analysis: Result from fit

- Decay time fit:
  - Fix some parameters

$$\left\{ \begin{array}{l} \Delta m_s = (17.757 \pm 0.021) \text{ ps}^{-1}, \\ \Gamma_s = (0.6643 \pm 0.0020) \text{ ps}^{-1}, \\ \Delta \Gamma_s = (0.083 \pm 0.006) \text{ ps}^{-1}, \\ \rho(\Gamma_s, \Delta \Gamma_s) = -0.239, \\ A_{\text{prod}} = (1.1 \pm 2.7)\%, \\ A_{\text{det}} = (1 \pm 1)\% \end{array} \right.$$

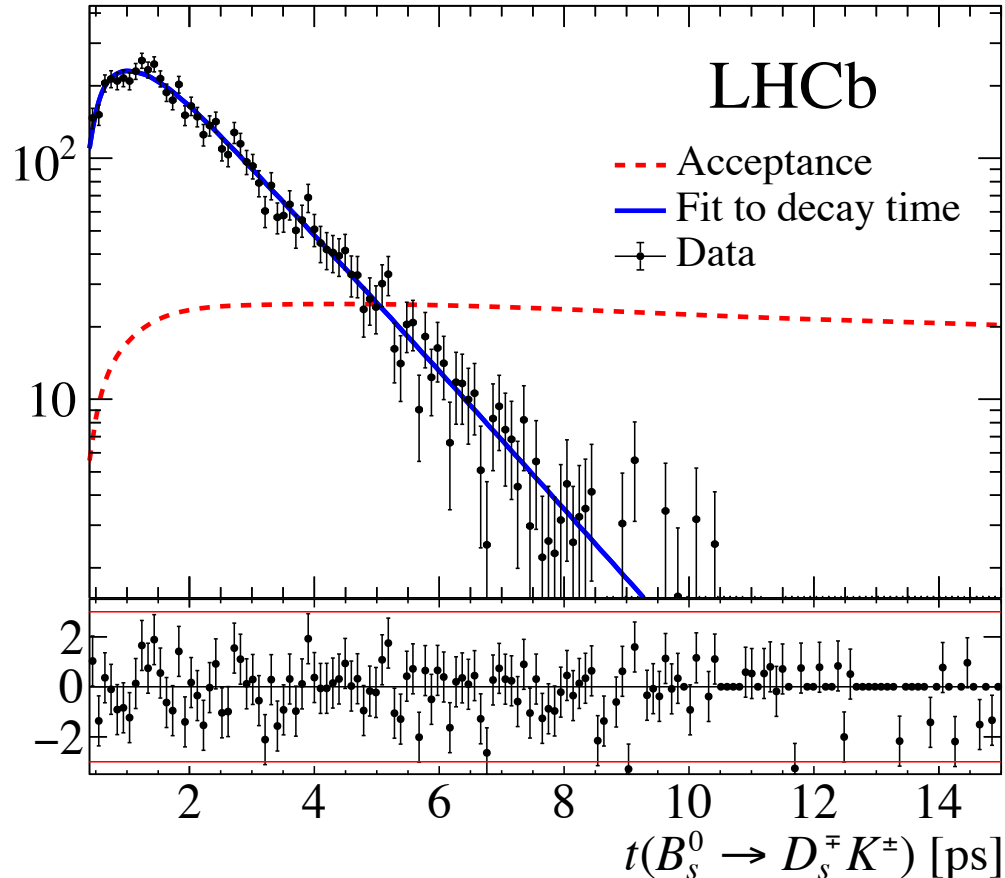
$$\frac{d\Gamma_{B_s^0 \rightarrow f}(t)}{dt} = \frac{1}{2} |A_f|^2 (1 + |\lambda_f|^2) e^{-\Gamma_s t} \left[ \cosh\left(\frac{\Delta \Gamma_s t}{2}\right) + A_f^{\Delta \Gamma} \sinh\left(\frac{\Delta \Gamma_s t}{2}\right) + C_f \cos(\Delta m_s t) - S_f \sin(\Delta m_s t) \right],$$

$$\frac{d\Gamma_{\bar{B}_s^0 \rightarrow f}(t)}{dt} = \frac{1}{2} |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) e^{-\Gamma_s t} \left[ \cosh\left(\frac{\Delta \Gamma_s t}{2}\right) + A_f^{\Delta \Gamma} \sinh\left(\frac{\Delta \Gamma_s t}{2}\right) - C_f \cos(\Delta m_s t) + S_f \sin(\Delta m_s t) \right],$$



Parameter	Value
$C_f$	$0.730 \pm 0.142 \pm 0.045$
$A_f^{\Delta \Gamma}$	$0.387 \pm 0.277 \pm 0.153$
$A_{\bar{f}}^{\Delta \Gamma}$	$0.308 \pm 0.275 \pm 0.152$
$S_f$	$-0.519 \pm 0.202 \pm 0.070$
$S_{\bar{f}}$	$-0.489 \pm 0.196 \pm 0.068$

Candidates / (0.10 ps)





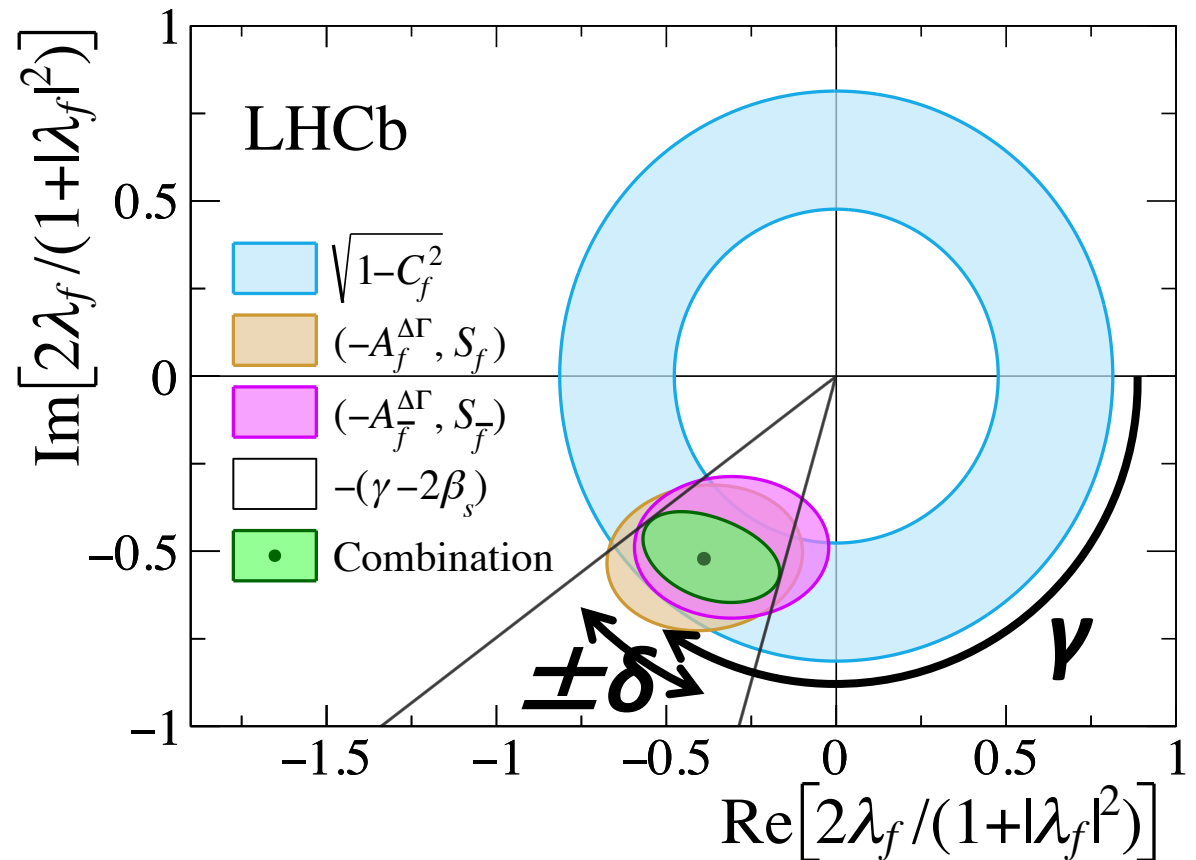
# $B_s^0 \rightarrow D_s^\pm K^\mp$ Analysis: Result

- From  $A, S, C$  to  $\gamma, \delta, r$

$$\gamma = (128^{+17}_{-22})^\circ,$$

$$\delta = (358^{+13}_{-14})^\circ,$$

$$r_{D_s K} = 0.37^{+0.10}_{-0.09},$$



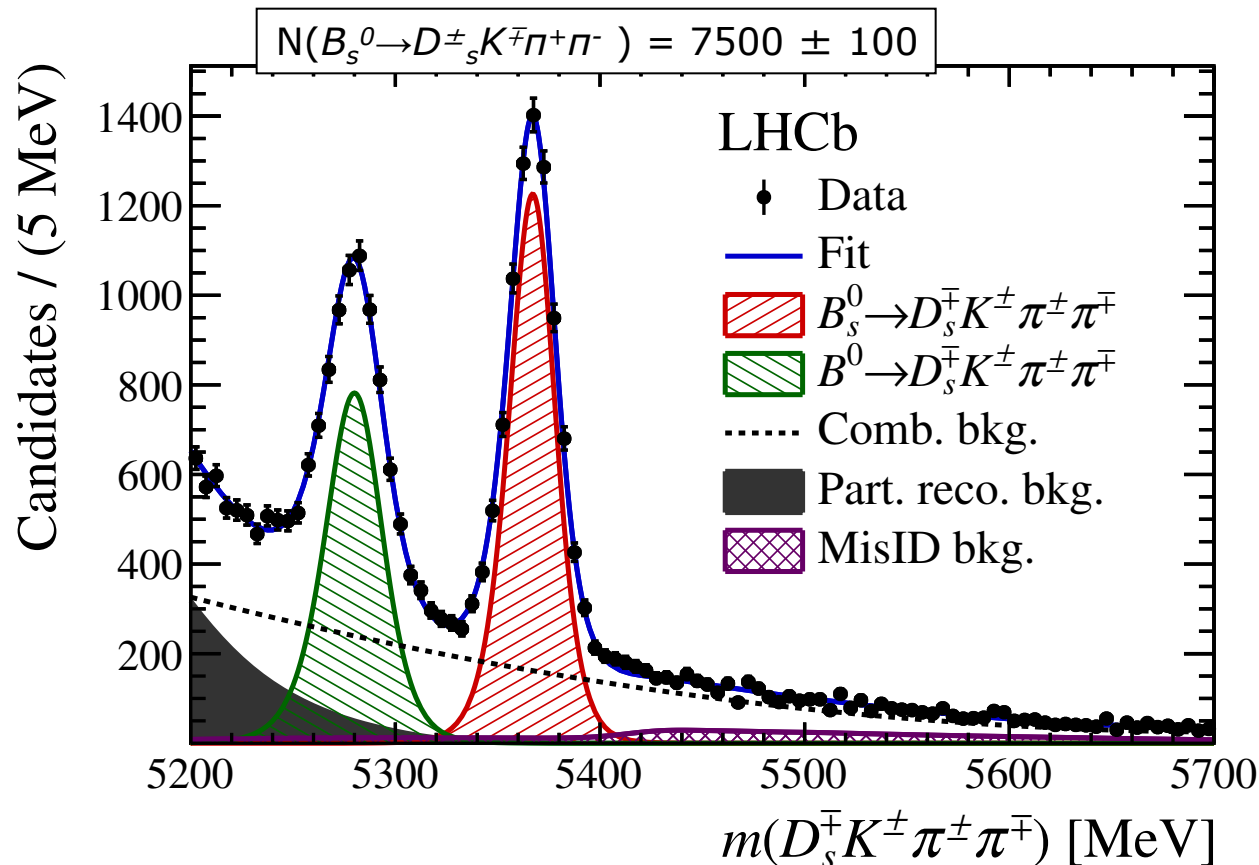
## $B_s^0 \rightarrow D^\pm K^\mp \pi^+ \pi^-$ Analysis:

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- Obtain  $B_s^0$  signal sample: Fit to  $m_B$
- $B_s^0$  or  $\bar{B}_s^0$ : Flavour Tagging
- Decay time: Resolution & acceptance
- Result: Decay time fit
  
- + Amplitude analysis

# $B_s^0 \rightarrow D_s^\pm K^\mp \pi^+ \pi^-$ Analysis: mass fit

- Need to (statistically) separate signal from background
- Backgrounds:
  - Combinatorial
  - Partially reconstructed background ( $B_s^0 \rightarrow D_s^* K^\mp$ , etc)
  - Misidentified background ( $B_s^0 \rightarrow D_s^\pm \pi^\mp$ )

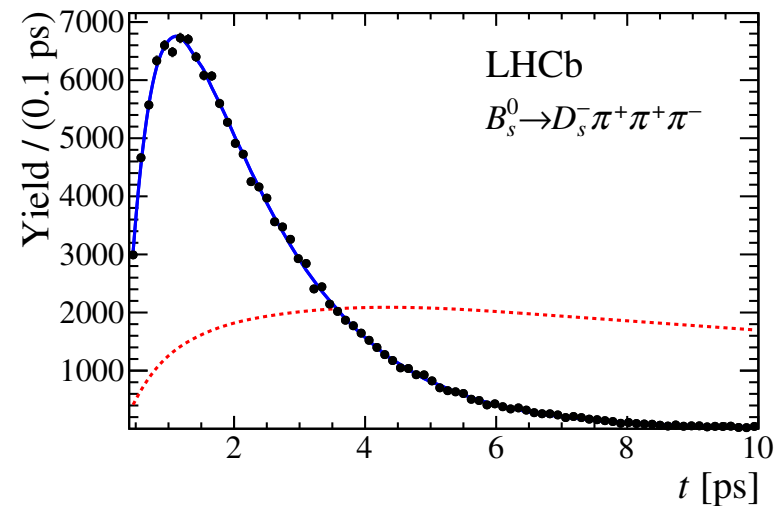


# $B_s^0 \rightarrow D_s^\pm K^\mp \pi^+ \pi^-$ Analysis:

- Flavour Tagging
- Decay time acceptance

(b) Run 2 data.

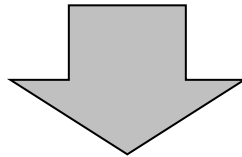
	$\epsilon_{\text{tag}}[\%]$	$\langle \omega \rangle[\%]$	$\epsilon_{\text{eff}}[\%]$
Only OS	$11.91 \pm 0.04$	$37.33 \pm 0.41$	$1.11 \pm 0.05$
Only SS	$40.95 \pm 0.08$	$42.41 \pm 0.29$	$1.81 \pm 0.10$
Both OS-SS	$28.96 \pm 0.12$	$35.51 \pm 0.32$	$3.61 \pm 0.13$
Combined	$81.82 \pm 0.15$	$39.23 \pm 0.32$	$6.52 \pm 0.17$



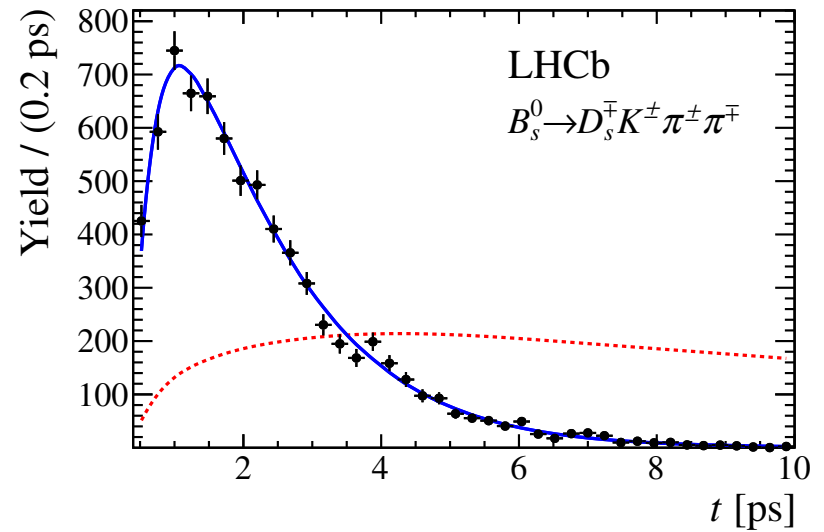
# $B_s^0 \rightarrow D_s^\pm K^\mp \pi^+ \pi^-$ Analysis: Result

- Time fit:

Fit parameter	Value
$C_f$	$0.631 \pm 0.096 \pm 0.032$
$A_f^{\Delta\Gamma}$	$-0.334 \pm 0.232 \pm 0.097$
$A_{\bar{f}}^{\Delta\Gamma}$	$-0.695 \pm 0.215 \pm 0.081$
$S_f$	$-0.424 \pm 0.135 \pm 0.033$
$S_{\bar{f}}$	$-0.463 \pm 0.134 \pm 0.031$



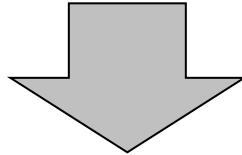
Parameter	Model-independent
$r$	$0.47^{+0.08}_{-0.08} +0.02_{-0.03}$
$\kappa$	$0.88^{+0.12}_{-0.19} +0.04_{-0.07}$
$\delta$ [°]	$-6^{+10}_{-12} +2_{-4}$
$\gamma - 2\beta_s$ [°]	$42^{+19}_{-13} +6_{-2}$



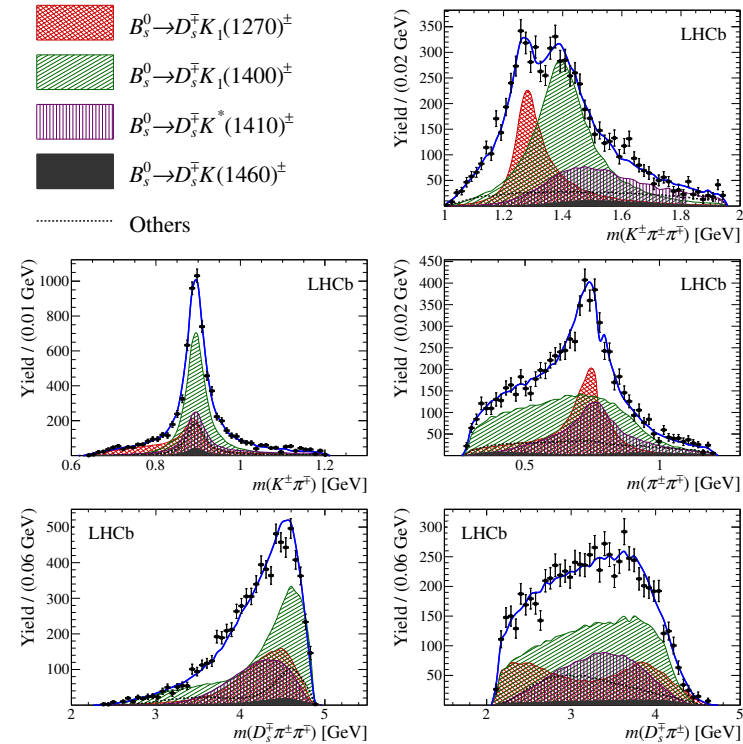
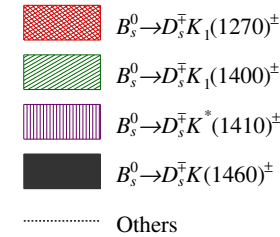
# $B_s^0 \rightarrow D_s^\pm K^\mp \pi^+ \pi^-$ Analysis: Amplitude analysis

- Let's take it one step further:
- Time-dependent amplitude fit

Decay channel	$F_i^c$ [%]	$F_i^u$ [%]
$B_s^0 \rightarrow D_s^\mp (K_1(1270)^\pm \rightarrow K^*(892)^0 \pi^\pm)$	$13.0 \pm 2.4 \pm 2.7 \pm 3.4$	$4.1 \pm 2.2 \pm 2.9 \pm 2.6$
$B_s^0 \rightarrow D_s^\mp (K_1(1270)^\pm \rightarrow K^\pm \rho(770)^0)$	$16.0 \pm 1.4 \pm 1.8 \pm 2.1$	$5.1 \pm 2.2 \pm 3.5 \pm 2.0$
$B_s^0 \rightarrow D_s^\mp (K_1(1270)^\pm \rightarrow K_0^*(1430)^0 \pi^\pm)$	$3.4 \pm 0.5 \pm 1.0 \pm 0.4$	$1.1 \pm 0.5 \pm 0.6 \pm 0.5$
$B_s^0 \rightarrow D_s^\mp (K_1(1400)^\pm \rightarrow K^*(892)^0 \pi^\pm)$	$63.9 \pm 5.1 \pm 7.4 \pm 13.5$	$19.3 \pm 5.2 \pm 8.3 \pm 7.8$
$B_s^0 \rightarrow D_s^\mp (K^*(1410)^\pm \rightarrow K^*(892)^0 \pi^\pm)$	$12.8 \pm 0.8 \pm 1.5 \pm 3.2$	$12.6 \pm 2.0 \pm 2.6 \pm 4.1$
$B_s^0 \rightarrow D_s^\mp (K^*(1410)^\pm \rightarrow K^\pm \rho(770)^0)$	$5.6 \pm 0.4 \pm 0.6 \pm 0.7$	$5.6 \pm 1.0 \pm 1.2 \pm 1.8$
$B_s^0 \rightarrow D_s^\mp (K(1460)^\pm \rightarrow K^*(892)^0 \pi^\pm)$		$11.9 \pm 2.5 \pm 2.9 \pm 3.1$
$B_s^0 \rightarrow (D_s^\mp \pi^\pm)_P K^*(892)^0$	$10.2 \pm 1.6 \pm 1.8 \pm 4.5$	$28.4 \pm 5.6 \pm 6.4 \pm 15.3$
$B_s^0 \rightarrow (D_s^\mp K^\pm)_P \rho(770)^0$	$0.9 \pm 0.4 \pm 0.5 \pm 1.0$	
Sum	$125.7 \pm 6.4 \pm 6.9 \pm 19.9$	$88.1 \pm 7.0 \pm 10.0 \pm 20.9$



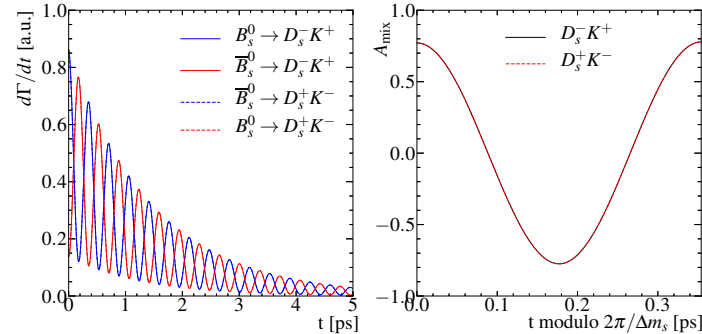
Parameter	Model-independent	Model-dependent
$r$	$0.47^{+0.08}_{-0.08} {}^{+0.02}_{-0.03}$	$0.56 \pm 0.05 \pm 0.04 \pm 0.07$
$\kappa$	$0.88^{+0.12}_{-0.19} {}^{+0.04}_{-0.07}$	$0.72 \pm 0.04 \pm 0.06 \pm 0.04$
$\delta$ [°]	$-6^{+10}_{-12} {}^{+2}_{-4}$	$-14 \pm 10 \pm 4 \pm 5$
$\gamma - 2\beta_s$ [°]	$42^{+19}_{-13} {}^{+6}_{-2}$	$42 \pm 10 \pm 4 \pm 5$



*Gain in precision*

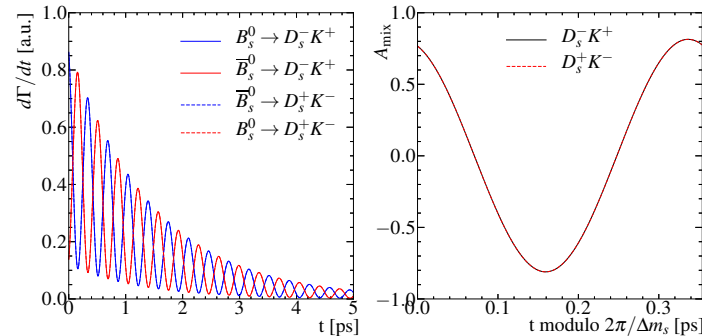
# Comparison: $B_s^0 \rightarrow D_s^\pm K^\mp \pi^+ \pi^-$ vs $B_s^0 \rightarrow D_s^\pm K^\mp$

- Illustration of weak and strong phase:



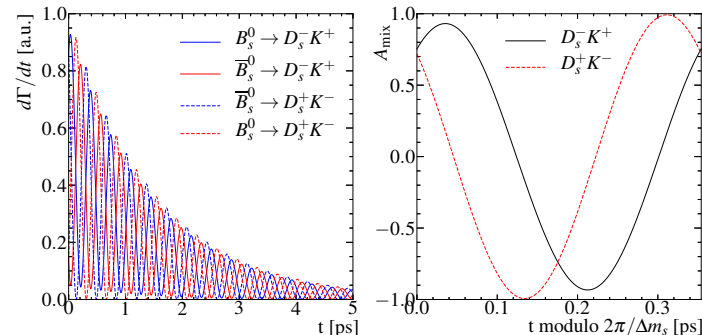
(a) Effect of interference for  $r = 0.4$ ,  $\delta = 0$ ,  $\gamma = 0$ .

$$\delta=0, \gamma=0$$



(b) Effect of the strong phase difference  $r = 0.4$ ,  $\delta = 20^\circ$ ,  $\gamma = 0$ .

$$\delta=20, \gamma=0$$

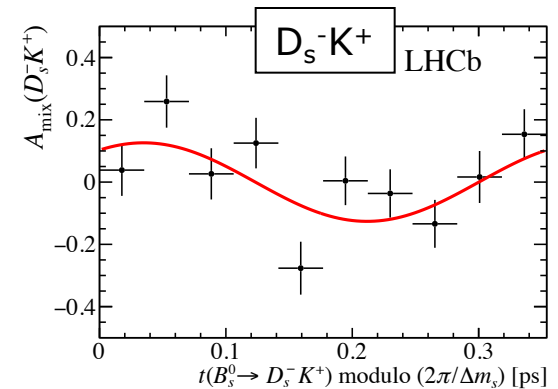
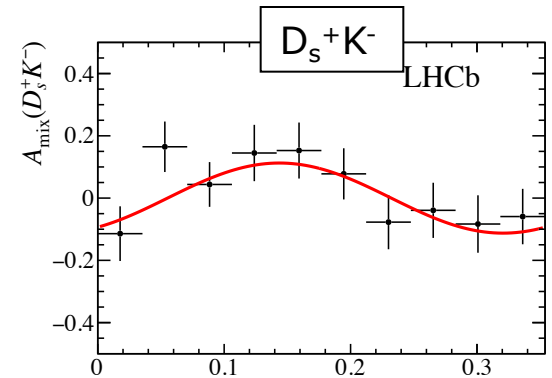
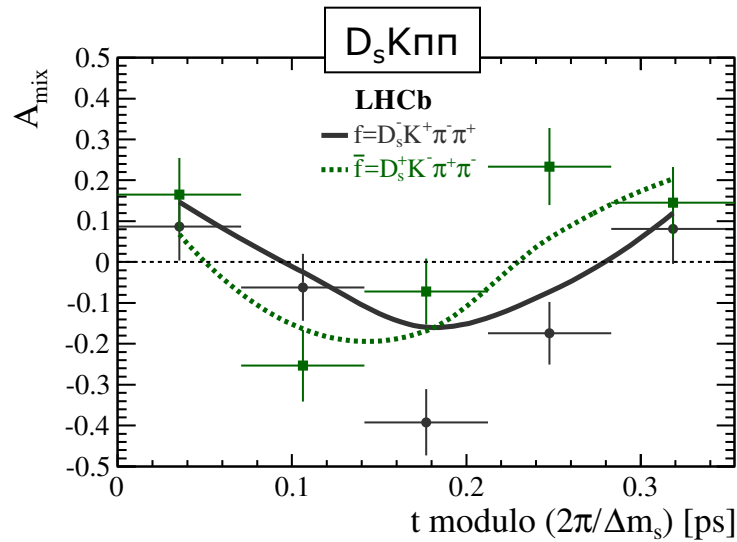


(c) Effect of the weak phase difference  $r = 0.4$ ,  $\delta = 20^\circ$ ,  $\gamma = 70^\circ$ .

$$\delta=20, \gamma=70$$

# Comparison: $B_s^0 \rightarrow D_s^\pm K^\mp \pi^+ \pi^-$ vs $B_s^0 \rightarrow D_s^\pm K^\mp$

- Confirmed results with identical code...



Parameter	Model-independent
$r$	$0.47^{+0.08}_{-0.08} {}^{+0.02}_{-0.03}$
$\kappa$	$0.88^{+0.12}_{-0.19} {}^{+0.04}_{-0.07}$
$\delta$ [°]	$-6^{+10}_{-12} {}^{+2}_{-4}$
$\gamma - 2\beta_s$ [°]	$42^{+19}_{-13} {}^{+6}_{-2}$

$$\gamma = (128^{+17}_{-22})^\circ,$$

$$\delta = (358^{+13}_{-14})^\circ,$$

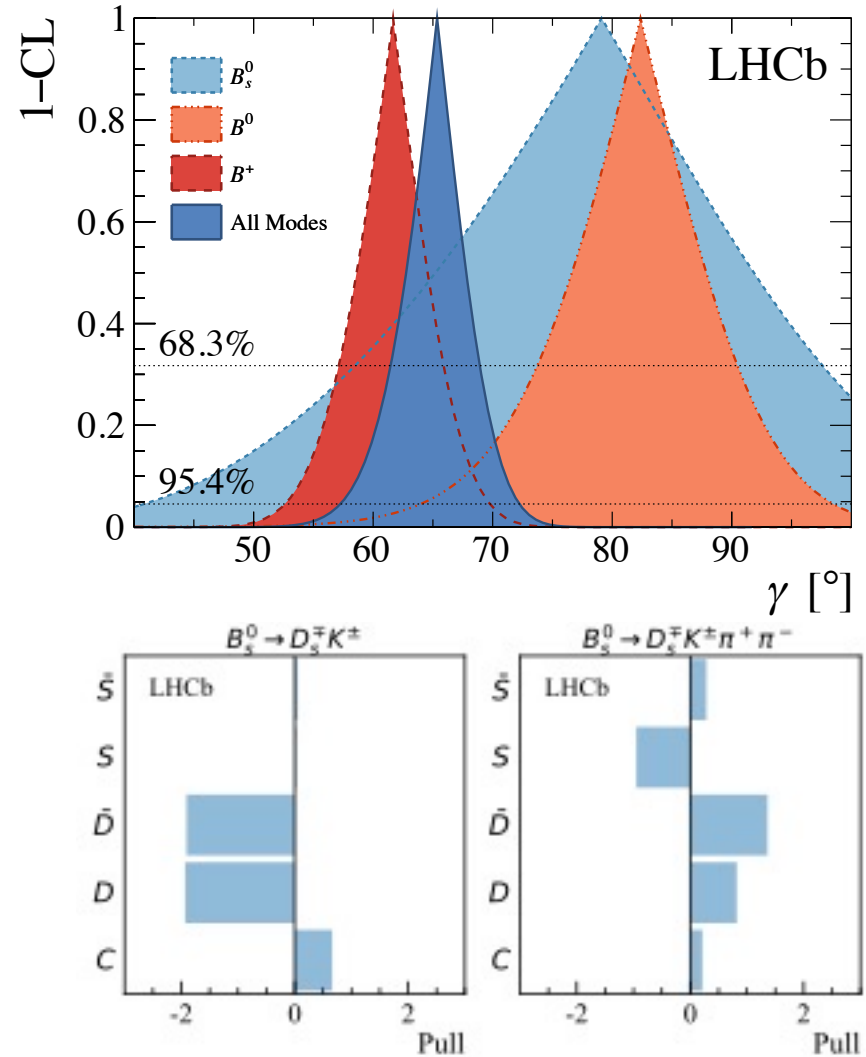
$$r_{D_s K} = 0.37^{+0.10}_{-0.09},$$



# $\gamma$ from $B_s^0 \rightarrow D_s^\pm K^\mp \pi^+ \pi^-$ and $B_s^0 \rightarrow D_s^\pm K^\mp$

- Contribution to  $\gamma$  average

	Measurement	$\chi^2$	No. of obs.
	$B^\pm \rightarrow Dh^\pm, D \rightarrow h^\pm h'^\mp$	2.71	8
	$B^\pm \rightarrow Dh^\pm, D \rightarrow h^\pm \pi^\mp \pi^+ \pi^-$	7.36	8
	$B^\pm \rightarrow Dh^\pm, D \rightarrow h^\pm h'^\mp \pi^0$	7.14	11
	$B^\pm \rightarrow Dh^\pm, D \rightarrow K_S^0 h^+ h^-$	4.67	6
	$B^\pm \rightarrow Dh^\pm, D \rightarrow K_S^0 K^\pm \pi^\mp$	7.57	7
	$B^\pm \rightarrow D^* h^\pm, D \rightarrow h^\pm h'^\mp$	7.31	16
	$B^\pm \rightarrow DK^{*\pm}, D \rightarrow h^\pm h'^\mp (\pi^+ \pi^-)$	3.71	12
	$B^0 \rightarrow DK^{*0}, D \rightarrow h^\pm h'^\mp (\pi^+ \pi^-)$	9.45	12
	$B^0 \rightarrow DK^{*0}, D \rightarrow K_S^0 h^+ h^-$	3.26	4
	$B^\pm \rightarrow Dh^\pm \pi^+ \pi^-, D \rightarrow h^\pm h'^\mp$	1.34	11
	$B_s^0 \rightarrow D_s^\mp K^\pm$	5.71	5
	$B_s^0 \rightarrow D_s^\mp K^\pm \pi^+ \pi^-$	2.88	5
	$B^0 \rightarrow D^\mp \pi^\pm$	0.00	2



# Detecting CP violation with B decays

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- 1) CP violation: CKM and the SM
- 2) Detecting: Detector requirements
- 3) B-decays:  $\sin 2\beta$ ,  $\phi_s$ ,  $B_s^0 \rightarrow D_s^+ K^-$