Detecting CP violation with B decays

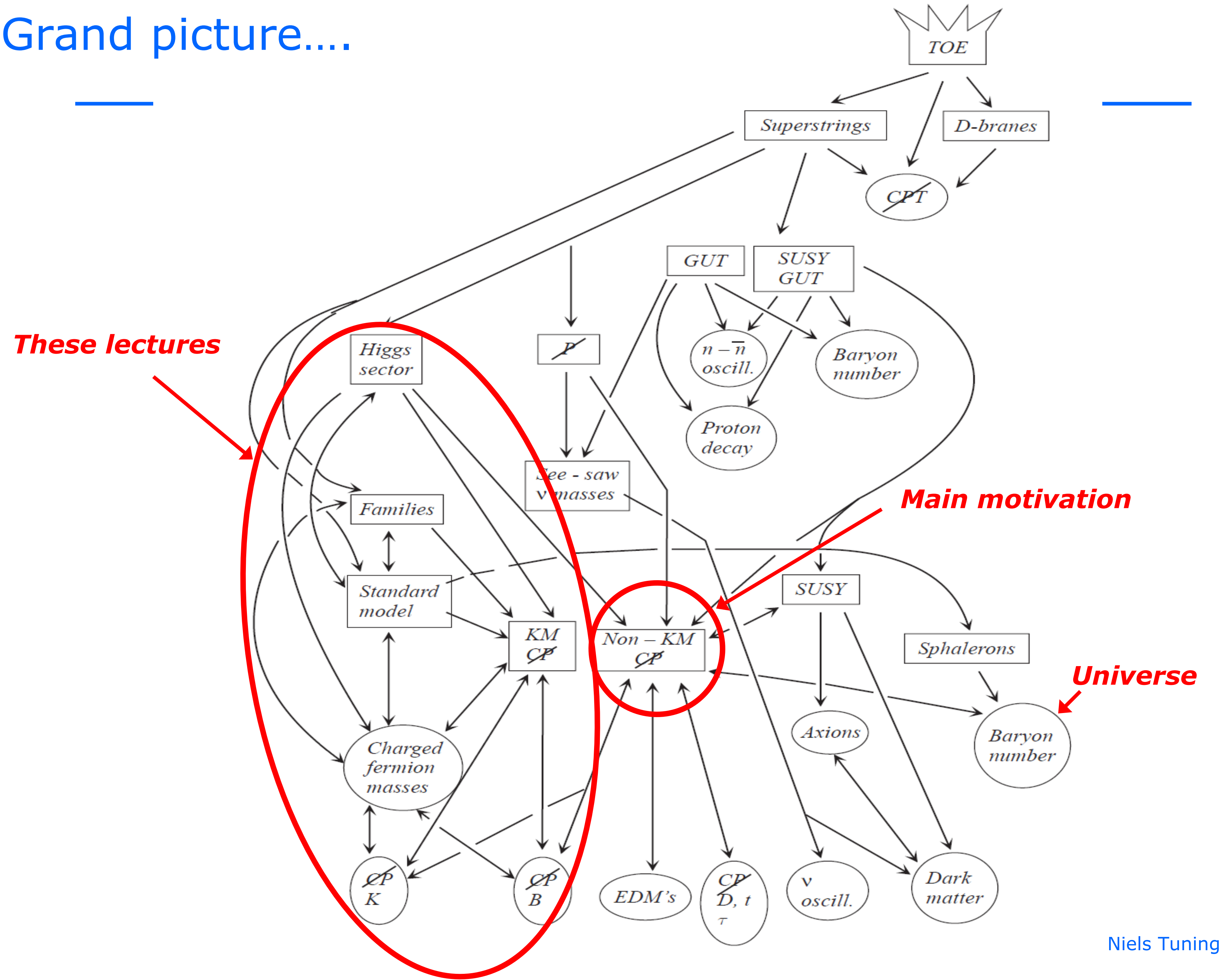
Lecture 1: CP violation

N. Tuning

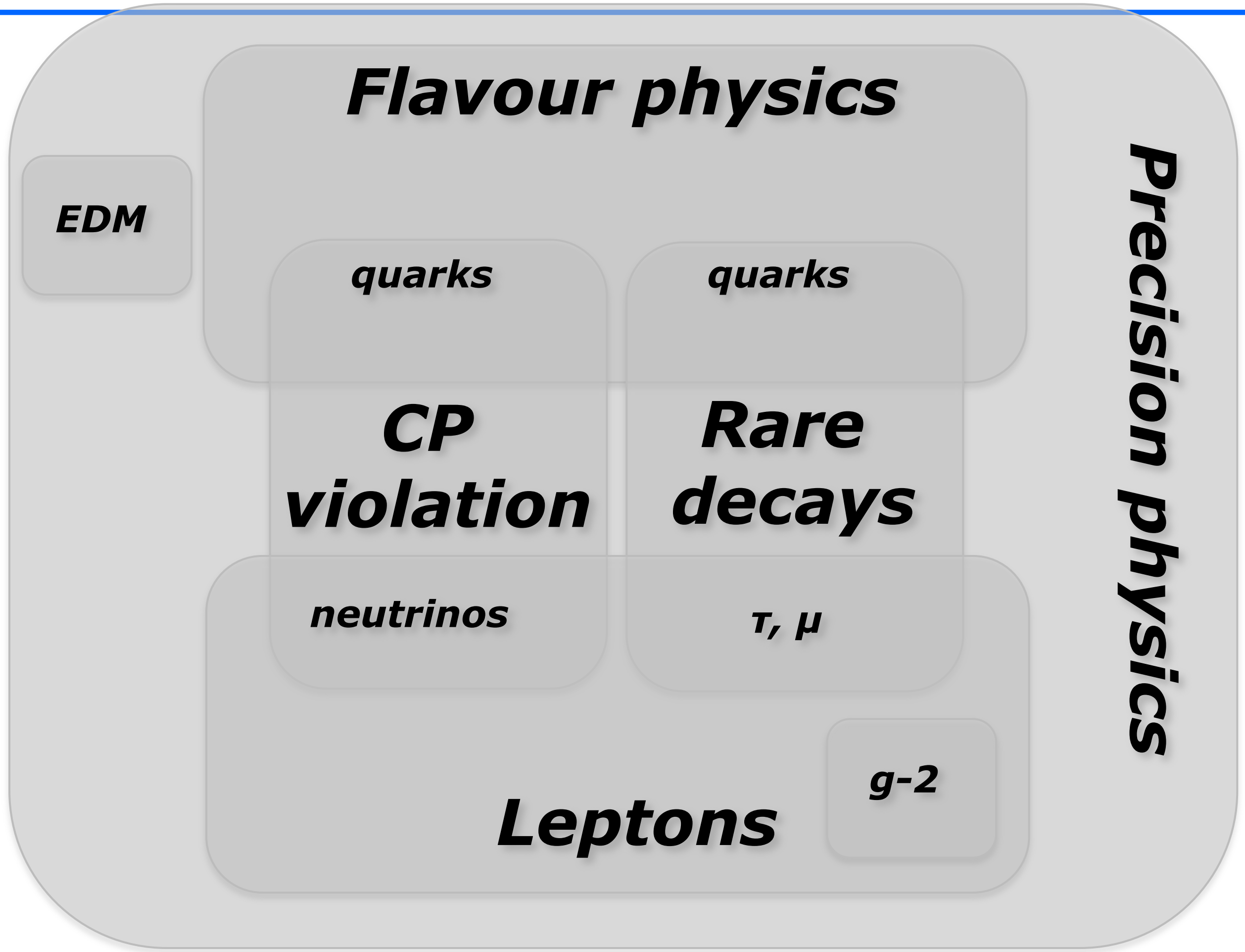
Detecting CP violation with B decays

1) CP violation: CKM and the SM
2) Detecting: Detector requirements
3) B-decays: $\sin 2\beta$, ϕ_s, $B_s^0 \rightarrow D_s^+ K^-$

Grand picture….

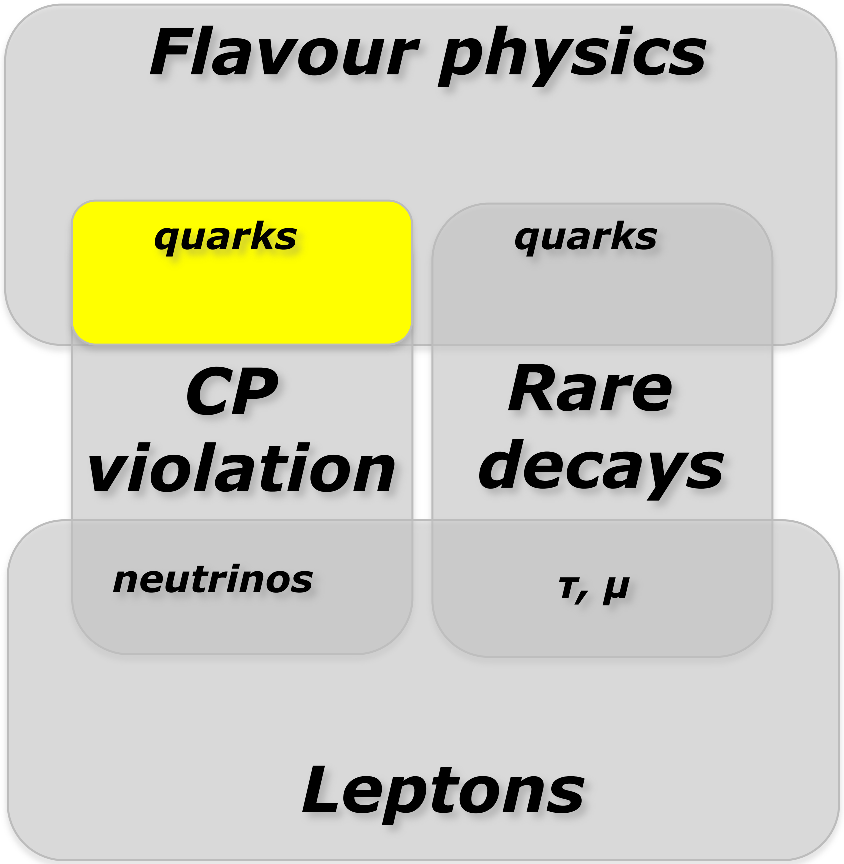


Jargon



Flavour physics
EDM
quarks
quarks
CP violation
Rare decays
neutrinos
τ, μ
g-2
Leptons
Precision physics

Today


Flavour physics
quarks
quarks
CP violation
Rare decays
neutrinos
τ, μ
Leptons

Flavour physics has a track record…

GIM mechanism in $K^0 \to \mu\mu$

Weak Interactions with Lepton-Hadron Symmetry*

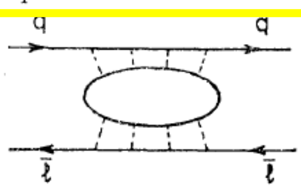
S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Mills theory is discussed.

splitting, beginning at order $G(G\Lambda^2)$, as well as contributions to such unobserved decay modes as $K_2 \to \mu^+ + \mu^-$, $K^+ \to \pi^+ + l + \bar{l}$, etc., involving neutral lepton

We wish to propose a simple model in which the divergences are properly ordered. Our model is founded in a quark model, but one involving four, not three, fundamental fermions; the weak interactions are medi-

new quantum number ℭ for charm.



Glashow, Iliopoulos, Maiani, Phys.Rev. D2 (1970) 1285

CP violation, $K_L^0 \to \pi\pi$

27 July 1964

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

This Letter reports the results of experimental studies designed to search for the 2π decay of the K_2^0 meson. Several previous experiments have

three-body decays of the K_2^0. The presence of a two-pion decay mode implies that the K_2^0 meson is not a pure eigenstate of CP. Expressed as $K_2^0 = 2^{-1/2}[(K_0 - \bar{K}_0) + \epsilon(K_0 + \bar{K}_0)]$ then $|\epsilon|^2 \cong R_T \tau_1 \tau_2$

Christenson, Cronin, Fitch, Turlay, Phys.Rev.Lett. 13 (1964) 138-140

$B^0 \leftrightarrow \bar{B}^0$ mixing

DESY 87-029
April 1987

OBSERVATION OF B^0-$\bar{B}^0$ MIXING

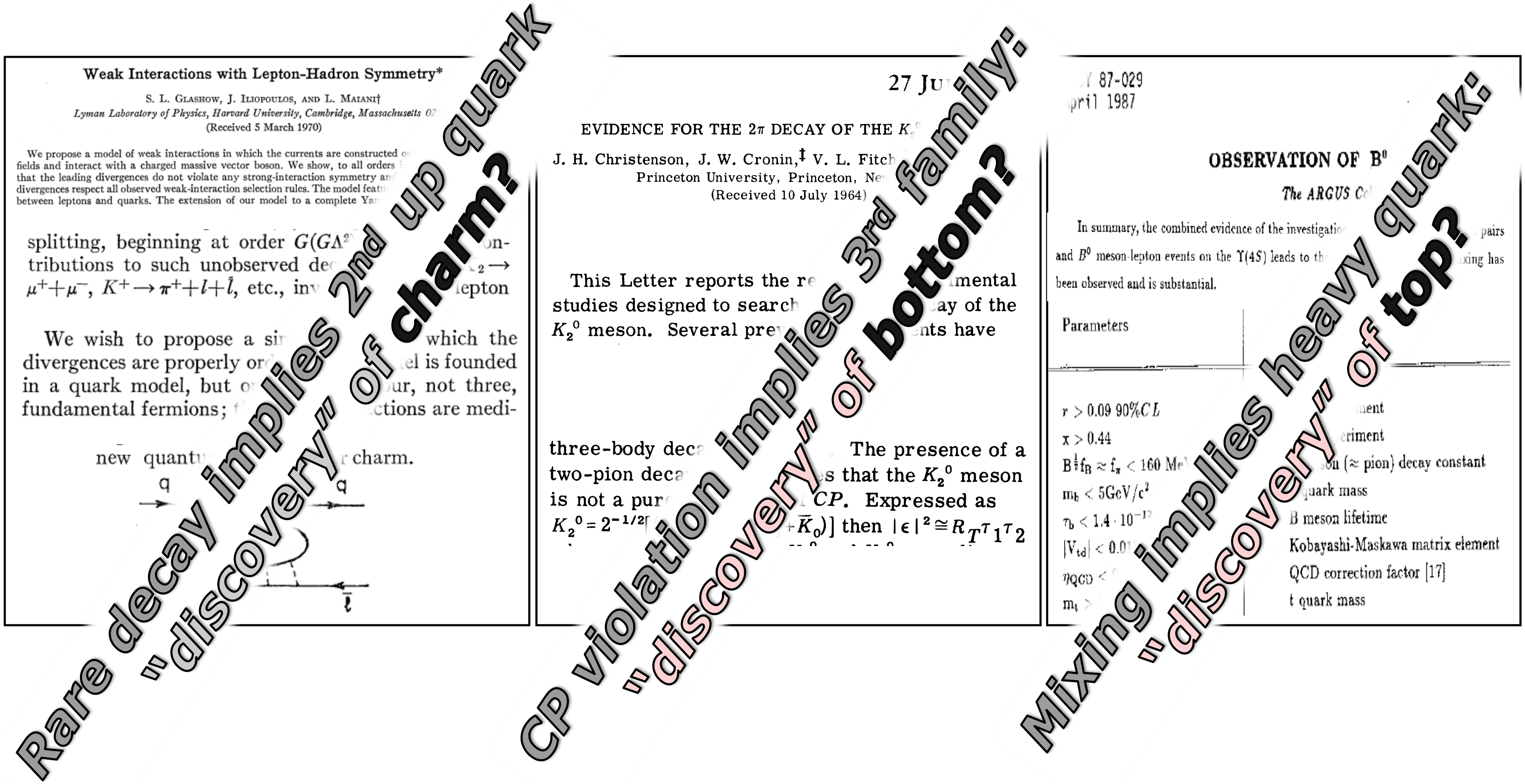
The ARGUS Collaboration

In summary, the combined evidence of the investigation of B^0 meson pairs, lepton pairs and B^0 meson-lepton events on the $\Upsilon(4S)$ leads to the conclusion that B^0-$\bar{B}^0$ mixing has been observed and is substantial.

Parameters	Comments
$r > 0.09$ 90%CL	This experiment
$x > 0.44$	This experiment
$B^{\frac{1}{2}} f_B \approx f_\pi < 160$ MeV	B meson (≈ pion) decay constant
$m_b < 5\,\mathrm{GeV}/c^2$	b-quark mass
$\tau_b < 1.4 \cdot 10^{-12}$s	B meson lifetime
$\lvert V_{td} \rvert < 0.018$	Kobayashi-Maskawa matrix element
$\eta_{QCD} < 0.86$	QCD correction factor [17]
$m_t > 50\,\mathrm{GeV}/c^2$	t quark mass

ARGUS Coll. Phys.Lett.B192:245,1987

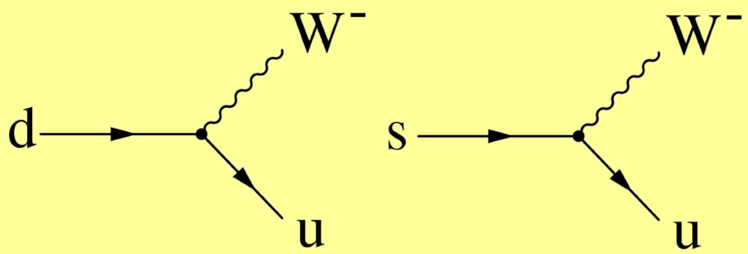
Flavour physics has a track record…



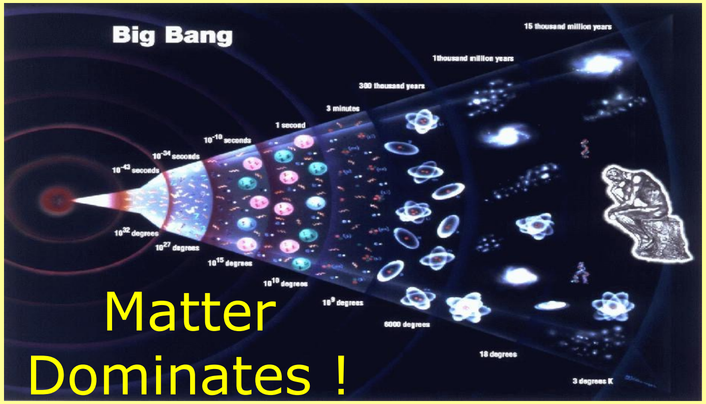
Motivation

- CP-violation (or flavour physics) is about charged current interactions

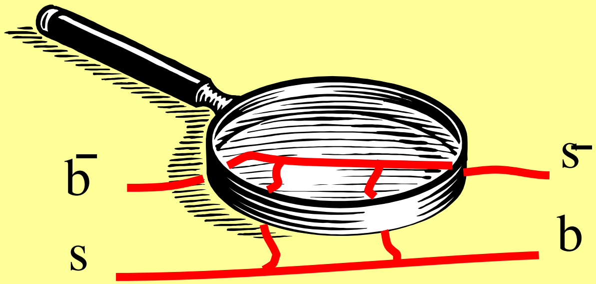
- Interesting because:
  1) Standard Model:
     in the heart of quark interactions



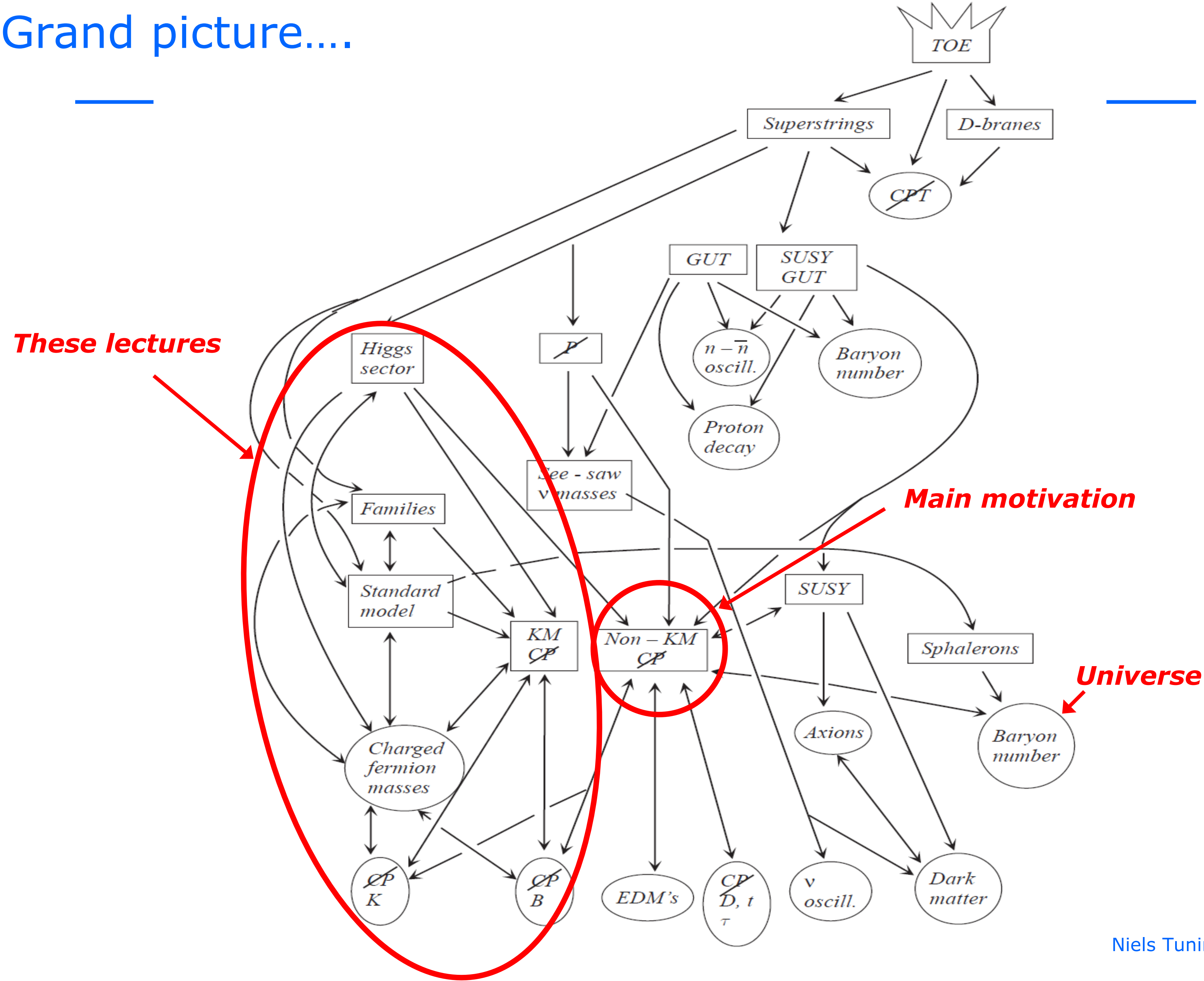
  2) Cosmology:
     related to matter – anti-matter asymetry



  3) Beyond Standard Model:
     measurements are sensitive to new particles



Grand picture….



Personal impression:

- People think it is a complicated part of the Standard Model (me too:-). Why?

1) Non-intuitive concepts?

- *Imaginary phase* in transition amplitude, $T \sim e^{i\varphi}$
- *Different bases* to express quark states, d' =0.97 d + 0.22 s + 0.003 b
- *Oscillations* (mixing) of mesons: $|K^0\rangle \leftrightarrow |\bar{K}^0\rangle$

2) Complicated calculations?

$$\Gamma\left(B^0 \to f\right) \propto \left|A_f\right|^2 \left[\left|g_+(t)\right|^2 + \left|\lambda\right|^2 \left|g_-(t)\right|^2 + 2\Re\left(\lambda g_+^*(t) g_-(t)\right)\right]$$

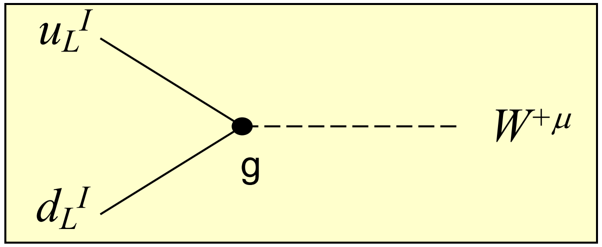
$$\Gamma\left(\bar{B}^0 \to f\right) \propto \left|\bar{A}_f\right|^2 \left[\left|g_+(t)\right|^2 + \frac{1}{\left|\lambda\right|^2} \left|g_-(t)\right|^2 + \frac{2}{\left|\lambda\right|^2} \Re\left(\lambda^* g_+^*(t) g_-(t)\right)\right]$$

3) Many decay modes? *"Beetopaipaigamma…"*

- PDG reports 347 decay modes of the B^0-meson:
  - Γ_1 $l^+ \nu_l$ *anything* (10.33 ± 0.28) × 10^{-2}
  - Γ_{347} $\nu \nu \gamma$ <4.7 × 10^{-5} *CL=90%*
- And for one decay there are often more than one decay *amplitudes*…

CP violation in the SM Lagrangian

- Focus on charged current interaction ($W^{\pm}$): let's trace it



The Standard Model Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- $\mathcal{L}_{Kinetic}$: • Introduce the massless fermion fields
  • Require local gauge invariance ➔ gives rise to existence of gauge bosons

- $\mathcal{L}_{Higgs}$: • Introduce Higgs potential with $<\phi> \neq 0$
  • Spontaneous symmetry breaking

  $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$

  The W^+, W^-, Z^0 bosons acquire a mass

- $\mathcal{L}_{Yukawa}$: • Ad hoc interactions between Higgs field & fermions

Fields: Notation

Y = Q - T_3

Fermions: $\psi_L = \left(\frac{1-\gamma_5}{2}\right)\psi$; $\psi_R = \left(\frac{1+\gamma_5}{2}\right)\psi$ with $\psi = Q_L, u_R, d_R, L_L, l_R, \nu_R$

Quarks:

Under SU2:
Left handed doublets
Right hander singlets

- $\begin{pmatrix} u^I(3,2,1/6) \\ d^I(3,2,1/6) \end{pmatrix}_{Li} \equiv Q^I_{Li}(3,2,1/6)$

Interaction rep.
Left-handed
generation index
$SU(3)_C$
$SU(2)_L$
Hypercharge Y
(=avg el.charge in multiplet)

- $u^I_{Ri}(3,1,2/3)$
- $d^I_{Ri}(3,1,-1/3)$

Leptons:

- $\begin{pmatrix} \nu^I(1,2,-1/2) \\ l^I(1,2,-1/2) \end{pmatrix}_{Li} \equiv L^I_{Li}(1,2,-1/2)$
- $l^I_{Ri}(1,1,-1)$
- $\left(\nu^I_{Ri}\right)$

Scalar field:

- $\phi(1,2,1/2) = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$

Note:
Interaction representation: standard model interaction is independent of generation number

Fields: Notation

Y = Q - T_3

Explicitly:

- The left handed quark doublet :

$$Q^I_{Li}(3,2,1/6) = \begin{pmatrix} u^I_r, u^I_g, u^I_b \\ d^I_r, d^I_g, d^I_b \end{pmatrix}_L, \begin{pmatrix} c^I_r, c^I_g, c^I_b \\ s^I_r, s^I_g, s^I_b \end{pmatrix}_L, \begin{pmatrix} t^I_r, t^I_g, t^I_b \\ b^I_r, b^I_g, b^I_b \end{pmatrix}_L \quad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad (Y = 1/6)$$

- Similarly for the quark singlets:

$$u^I_{Ri}(3,1,\ 2/3) = \left(u^I_r, u^I_r, u^I_r\right)_R, \left(c^I_r, c^I_r, c^I_r\right)_R, \left(t^I_r, t^I_r, t^I_r\right)_R \qquad (Y = 2/3)$$

$$d^I_{Ri}(3,1,-1/3) = \left(d^I_r, d^I_r, d^I_r\right)_R, \left(s^I_r, s^I_r, s^I_r\right)_R, \left(b^I_r, b^I_r, b^I_r\right)_R \qquad (Y = -1/3)$$

- The left handed leptons: $L^I_{Li}(1,2,-1/2) = \begin{pmatrix} \nu^I_e \\ e^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\mu \\ \mu^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\tau \\ \tau^I \end{pmatrix}_L \quad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad (Y = -1/2)$

- And similarly the (charged) singlets: $l^I_{Ri}(1,1,-1) = e^I_R, \mu^I_R, \tau^I_R \qquad (Y = -1)$

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

:The Kinetic Part

$\mathcal{L}_{Kinetic}$: Fermions + gauge bosons + interactions

Procedure:
Introduce the Fermion fields and __demand__ that the theory is local gauge invariant under $SU(3)_C xSU(2)_L xU(1)_Y$ transformations.

Start with the Dirac Lagrangian: $\mathcal{L} = i\bar{\psi}(\partial^\mu \gamma_\mu)\psi$

Replace: $\partial^\mu \rightarrow D^\mu \equiv \partial^\mu + ig_s G_a^\mu L_a + igW_b^\mu T_b + ig'B^\mu Y$

Fields:
- G_a^μ : 8 gluons
- W_b^μ : weak bosons: W_1, W_2, W_3
- B^μ : hypercharge boson

Generators:

L_a : Gell-Mann matrices:	½ λ_a	(3x3)	$SU(3)_C$
T_b : Pauli Matrices:	½ τ_b	(2x2)	$SU(2)_L$
Y : Hypercharge:			$U(1)_Y$

For the remainder we only consider Electroweak: $SU(2)_L$ x $U(1)_Y$

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Kinetic Part

$$L_{kinetic} : i\overline{\psi}(\partial^{\mu}\gamma_{\mu})\psi \rightarrow i\overline{\psi}(D^{\mu}\gamma_{\mu})\psi$$

$$with \quad \psi = Q_{Li}^{I}, \quad u_{Ri}^{I}, \quad d_{Ri}^{I}, \quad L_{Li}^{I}, \quad l_{Ri}^{I}$$

For example, the term with Q_{Li}^{I} becomes:

$$L_{kinetic}(Q_{Li}^{I}) = i\overline{Q_{Li}^{I}}\gamma_{\mu}D^{\mu}Q_{Li}^{I}$$

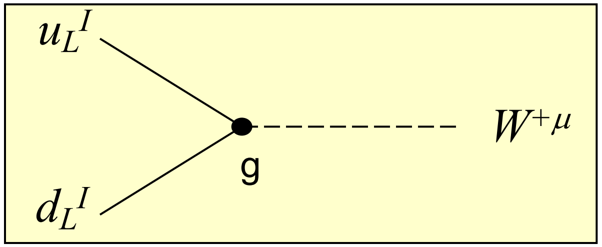
$$= i\overline{Q_{Li}^{I}}\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}g_{s}G_{a}^{\mu}\lambda_{a} + \frac{i}{2}gW_{b}^{\mu}\tau_{b} + \frac{i}{6}g'B^{\mu}\right)Q_{Li}^{I}$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Writing out only the weak part for the quarks:

$$L_{kinetic}^{Weak}(u,d)_{L}^{I} = i\overline{(u,d)_{L}^{I}}\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}g\left(W_{1}^{\mu}\tau_{1} + W_{2}^{\mu}\tau_{2} + W_{3}^{\mu}\tau_{3}\right)\right)\begin{pmatrix} u \\ d \end{pmatrix}_{L}^{I}$$

$$= i\overline{u_{L}^{I}}\gamma_{\mu}\partial^{\mu}u_{L}^{I} + i\overline{d_{L}^{I}}\gamma_{\mu}\partial^{\mu}d_{L}^{I} - \frac{g}{\sqrt{2}}\overline{u_{L}^{I}}\gamma_{\mu}W^{-\mu}d_{L}^{I} - \frac{g}{\sqrt{2}}\overline{d_{L}^{I}}\gamma_{\mu}W^{+\mu}u_{L}^{I} - \ldots$$



$L = J_{\mu}W^{\mu}$

$W^{+} = (1/\sqrt{2})\,(W_1 + i\,W_2)$

$W^{-} = (1/\sqrt{2})\,(W_1 - i\,W_2)$

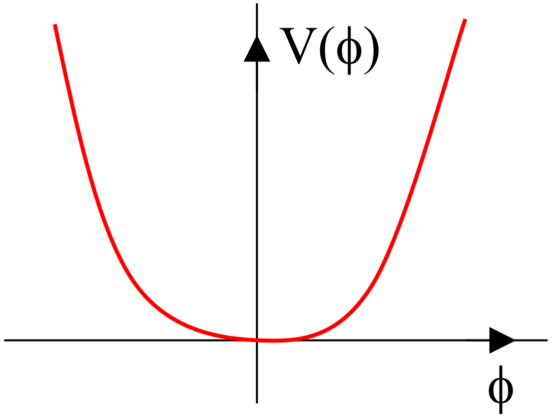
$\mathsf{L}_{SM} = \mathsf{L}_{Kinetic} + \mathsf{L}_{Higgs} + \mathsf{L}_{Yukawa}$: The Higgs Potential

$$\mathsf{L}_{Higgs} = D_\mu \phi^\dagger D^\mu \phi - V_{Higgs} \qquad V_{Higgs} = \frac{1}{2}\mu^2\left(\phi^\dagger\phi\right) + |\lambda|\left(\phi^\dagger\phi\right)^2$$

Symmetry

$\mu^2 > 0$:

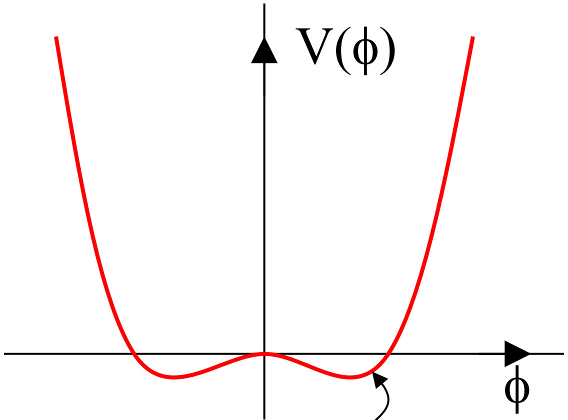
$<\varphi> = 0$



Broken Symmetry

$\mu^2 < 0$:

$$<\varphi> = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$



$v = \sqrt{-\mu^2/|\lambda|} \sim 246$ GeV

Spontaneous Symmetry Breaking: The Higgs field adopts a non-zero vacuum expectation value

Procedure: $\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \Re e\,\varphi^+ + i\Im m\,\phi^+ \\ \Re e\,\varphi^0 + i\Im m\,\phi^0 \end{pmatrix}$

Substitute: $\Re e\,\varphi^0 = \dfrac{v + H^0}{\sqrt{2}}$

And rewrite the Lagrangian (tedious):

(The other 3 Higgs fields are "eaten" by the W, Z bosons)

1. $G_{SM} : \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right) \rightarrow \left(SU(3)_C \times U(1)_{EM}\right)$
2. The W^+, W^-, Z^0 bosons acquire mass
3. The Higgs boson *H* appears

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

: The Yukawa Part

Since we have a Higgs field we can (should?) add (ad-hoc) interactions between ϕ and the fermions in a gauge invariant way.

The result is:

doublets — singlet

$$-\mathcal{L}_{Yukawa} = Y_{ij}\left(\overline{\psi}_{Li}\,\phi\right)\psi_{Rj} + h.c.$$

$$= Y_{ij}^{d}\left(\overline{Q_{Li}^{I}}\,\phi\right)d_{Rj}^{I} + Y_{ij}^{u}\left(\overline{Q_{Li}^{I}}\,\tilde{\phi}\right)u_{Rj}^{I} + Y_{ij}^{l}\left(\overline{L_{Li}^{I}}\,\phi\right)l_{Rj}^{I} + h.c.$$

i, j : indices for the 3 generations!

With: $\tilde{\phi} = i\sigma_2\,\phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}$

(The CP conjugate of ϕ
To be manifestly invariant under SU(2))

Y_{ij}^{d} , Y_{ij}^{u} , Y_{ij}^{l} are arbitrary complex matrices which operate in family space (3x3)

➔ Flavour physics!

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Yukawa Part

Writing the first term explicitly:

$$Y^d_{ij}\,(\overline{u^I_L}, \overline{d^I_L})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d^I_{Rj} \quad =$$

$$\begin{pmatrix} Y^d_{11}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{12}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{13}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \\ Y^d_{21}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{22}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{13}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \\ Y^d_{31}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{32}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{33}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \end{pmatrix} \bullet \begin{pmatrix} d^I_R \\ s^I_R \\ b^I_R \end{pmatrix}$$

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

: The Yukawa Part

There are 3 Yukawa matrices (in the case of massless neutrino's):

$$Y_{ij}^{d} \quad , \quad Y_{ij}^{u} \quad , \quad Y_{ij}^{l}$$

Each matrix is 3x3 complex:

- 27 real parameters
- 27 imaginary parameters ("phases")

➢ many of the parameters are equivalent, since the physics described by one set of couplings is the same as another

➢ It can be shown (see ref. [Nir]) that the independent parameters are:

  - 12 real parameters
  - 1 imaginary phase

➢ This single phase is the source of all CP violation in the Standard Model

……Revisit later

$L_{Yukawa} \xrightarrow{S.S.B} L_{Mass}$: The Fermion Masses

Start with the Yukawa Lagrangian

$$-L_{Yuk} = Y_{ij}^{d}\,(\overline{u_L^I}, \overline{d_L^I})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^I + Y_{ij}^{u}(\ldots) + Y_{ij}^{l}(\ldots)$$

$$S.S.B. \;:\; \Re e\left(\varphi^0\right) \rightarrow \frac{v+H}{\sqrt{2}}$$

After which the following mass term emerges:

$$-L_{Yuk} \rightarrow -L_{Mass} = \overline{d_{Li}^I}\, M_{ij}^d\, d_{Rj}^I + \overline{u_{Li}^I}\, M_{ij}^u\, u_{Rj}^I + \overline{l_{Li}^I}\, M_{ij}^l\, l_{Rj}^I + h.c.$$

with $M_{ij}^d \equiv \frac{v}{\sqrt{2}} Y_{ij}^d \;,\; M_{ij}^u \equiv \frac{v}{\sqrt{2}} Y_{ij}^u \;,\; M_{ij}^l \equiv \frac{v}{\sqrt{2}} Y_{ij}^l$

L_{Mass} is CP violating in a similar way as L_{Yuk}

$L_{Yukawa} \xrightarrow{\text{S.S.B}} L_{Mass}$: The Fermion Masses

Writing in an explicit form:

$$-L_{Mass} = \left(\overline{d^I},\overline{s^I},\overline{b^I}\right)_L \cdot \left(M^d\right) \cdot \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R + \left(\overline{u^I},\overline{c^I},\overline{t^I}\right)_L \cdot \left(M^u\right) \cdot \begin{pmatrix} u^I \\ c^I \\ t^I \end{pmatrix}_R + \left(\overline{e^I},\overline{\mu^I},\overline{\tau^I}\right)_L \cdot \left(M^l\right) \cdot \begin{pmatrix} e^I \\ \mu^I \\ \tau^I \end{pmatrix}_R + h.c.$$

The matrices M can always be diagonalised by unitary matrices V_L^f and V_R^f such that:

$$V_L^f \; M^f \; V_R^{f\dagger} = M^f_{diagonal}$$

$$\left[\left(\overline{d^I},\overline{s^I},\overline{b^I}\right)_L V_L^{f\dagger} \, V_L^f \; M^f \; V_R^{f\dagger} \, V_R^f \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R\right]$$

Then the real fermion mass eigenstates are given by:

$$d_{Li} = \left(V_L^d\right)_{ij} \cdot d^I_{Lj} \qquad d_{Ri} = \left(V_R^d\right)_{ij} \cdot d^I_{Rj}$$

$$u_{Li} = \left(V_L^u\right)_{ij} \cdot u^I_{Lj} \qquad u_{Ri} = \left(V_R^u\right)_{ij} \cdot u^I_{Rj}$$

$$l_{Li} = \left(V_L^l\right)_{ij} \cdot l^I_{Lj} \qquad l_{Ri} = \left(V_R^l\right)_{ij} \cdot l^I_{Rj}$$

d_L^I, u_L^I, l_L^I are the weak interaction eigenstates

d_L, u_L, l_L are the mass eigenstates (“physical particles”)

$L_{Yukawa} \xrightarrow{S.S.B} L_{Mass}$: The Fermion Masses

In terms of the mass eigenstates:

$$-L_{Mass} = \left(\bar{d},\bar{s},\bar{b}\right)_L \cdot \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\bar{u},\bar{c},\bar{t}\right)_L \cdot \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \cdot \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$

$$+ \left(\bar{e},\bar{\mu},\bar{\tau}\right)_L \cdot \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} \cdot \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R + h.c.$$

$$\begin{aligned} -L_{Mass} = \; & m_u \bar{u}u + m_c \bar{c}c + m_t \bar{t}t \\ & + m_d \bar{d}d + m_s \bar{s}s + m_b \bar{b}b \\ & + m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau \end{aligned}$$

In flavour space one can choose:

Weak basis: The gauge currents are diagonal in flavour space, but the flavour mass matrices are non-diagonal

Mass basis: The fermion masses are diagonal, but some gauge currents (charged weak interactions) are not diagonal in flavour space

In the weak basis: L_{Yukawa} = CP violating

In the mass basis: $L_{Yukawa} \rightarrow L_{Mass}$ = CP conserving

➔ What happened to the charged current interactions (in $L_{Kinetic}$) ?

$L_W \rightarrow L_{CKM}$: The Charged Current

The charged current interaction for quarks in the ***interaction*** basis is:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \overline{u_{Li}^I} \gamma^\mu d_{Li}^I W_\mu^+$$

The charged current interaction for quarks in the ***mass*** basis is:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \overline{u_{Li}} V_L^u \gamma^\mu V_L^{d\dagger} d_{Li} W_\mu^+$$

The unitary matrix: $V_{CKM} = \left(V_L^u \cdot V_L^{d\dagger}\right)$ With: $V_{CKM} \cdot V_{CKM}^\dagger = 1$

is the Cabibbo Kobayashi Maskawa mixing matrix:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \left(\bar{u}, \bar{c}, \bar{t}\right)_L \left(V_{CKM}\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \gamma^\mu W_\mu^+$$

Lepton sector: similarly $V_{MNS} = \left(V_L^\nu \cdot V_L^{l\dagger}\right)$

However, for massless neutrino's: $V_L{}^\nu$ = arbitrary. Choose it such that $V_{MNS} = 1$

➔ There is no mixing in the lepton sector

Charged Currents

The charged current term reads:

$$L_{CC} = \frac{g}{\sqrt{2}} \overline{u_{Li}^{I}} \gamma^{\mu} W_{\mu}^{-} d_{Li}^{I} + \frac{g}{\sqrt{2}} \overline{d_{Li}^{I}} \gamma^{\mu} W_{\mu}^{+} u_{Li}^{I} = J_{CC}^{\mu -} W_{\mu}^{-} + J_{CC}^{\mu +} W_{\mu}^{+}$$

$$= \frac{g}{\sqrt{2}} \overline{u_i} \left(\frac{1-\gamma^5}{2} \right) \gamma^{\mu} W_{\mu}^{-} V_{ij} \left(\frac{1-\gamma^5}{2} \right) d_j + \frac{g}{\sqrt{2}} \overline{d_j} \left(\frac{1-\gamma^5}{2} \right) \gamma^{\mu} W_{\mu}^{+} V_{ji}^{\dagger} \left(\frac{1-\gamma^5}{2} \right) u_i$$

$$= \frac{g}{\sqrt{2}} \overline{u_i} \gamma^{\mu} W_{\mu}^{-} V_{ij} \left(1-\gamma^5\right) d_j + \frac{g}{\sqrt{2}} \overline{d_j} \gamma^{\mu} W_{\mu}^{+} V_{ij}^{*} \left(1-\gamma^5\right) u_i$$

Under the CP operator this gives:

(Together with (x,t) -> (-x,t))

$$L_{CC} \xrightarrow{CP} \frac{g}{\sqrt{2}} \overline{d_j} \gamma^{\mu} W_{\mu}^{+} V_{ij} \left(1-\gamma^5\right) u_i + \frac{g}{\sqrt{2}} \overline{u_i} \gamma^{\mu} W_{\mu}^{i} V_{ij}^{*} \left(1-\gamma^5\right) d_j$$

A comparison shows that CP is conserved only if $V_{ij} = V_{ij}^{*}$

In general the charged current term is CP violating

The Standard Model Lagrangian (recap)

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- $\mathcal{L}_{Kinetic}$: •Introduce the massless fermion fields
  •Require local gauge invariance → gives rise to existence of gauge bosons
  ➔ CP Conserving

- $\mathcal{L}_{Higgs}$: •Introduce Higgs potential with $<\phi> \neq 0$
  •Spontaneous symmetry breaking
  ➔ CP Conserving

  $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$

  The W^+, W^-, Z^0 bosons acquire a mass

- $\mathcal{L}_{Yukawa}$: •Ad hoc interactions between Higgs field & fermions
  ➔ CP violating with a single phase

- $\mathcal{L}_{Yukawa} \rightarrow \mathcal{L}_{mass}$:
  - fermion weak eigenstates:
    - mass matrix is (3x3) non-diagonal

    ➔ CP-violating
  - fermion mass eigenstates:
    - mass matrix is (3x3) diagonal

    ➔ CP-conserving!

- $\mathcal{L}_{Kinetic}$ in mass eigenstates: CKM – matrix ➔ CP violating with a single phase

Recap

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

$$-L_{Yuk} = Y_{ij}^{d}\,(\overline{u_L^I}, \overline{d_L^I})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^I + \ldots$$

$$L_{Kinetic} = \frac{g}{\sqrt{2}} \overline{u_{Li}^I} \gamma^\mu W_\mu^- d_{Li}^I + \frac{g}{\sqrt{2}} \overline{d_{Li}^I} \gamma^\mu W_\mu^+ u_{Li}^I + \ldots$$

Diagonalize Yukawa matrix Y_{ij}

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$-L_{Mass} = \left(\bar{d}, \bar{s}, \bar{b}\right)_L \cdot \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\bar{u}, \bar{c}, \bar{t}\right)_L \cdot \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \cdot \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \ldots$$

$$L_{CKM} = \frac{g}{\sqrt{2}} \overline{u_i} \gamma^\mu W_\mu^- V_{ij} \left(1 - \gamma^5\right) d_j + \frac{g}{\sqrt{2}} \overline{d_j} \gamma^\mu W_\mu^+ V_{ij}^* \left(1 - \gamma^5\right) u_i + \ldots$$

$$L_{SM} = L_{CKM} + L_{Higgs} + L_{Mass}$$

Ok…. We've got the CKM matrix, now what?

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- It's *unitary*
  - *"probabilities add up to 1":*
  - d'=0.97 d + 0.22 s + 0.003 b $(0.97^2+0.22^2+0.003^2=1)$
- How many free parameters?
  - How many real/complex?
- How do we normally visualize these parameters?

What do we know about the CKM matrix?

- Magnitudes of elements have been measured over time
  - Result of a *large* number of measurements and calculations

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.04214 \\ 0.00896 & 0.04133 & 0.99911 \end{pmatrix} \pm \begin{pmatrix} 0.00010 & 0.00044 & 0.00012 \\ 0.00044 & 0.00011 & 0.00076 \\ 0.00024 & 0.00974 & 0.00003 \end{pmatrix}$$

Magnitude of elements shown only, no information of phase

What do we know about the CKM matrix?

- Magnitudes of elements have been measured over time
  - Result of a *large* number of measurements and calculations

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\lambda \approx \sin\theta_C = \sin\theta_{12} \approx 0.24$$

Magnitude of elements shown only, no information of phase

Intermezzo: How about the leptons?

- We now know that neutrinos also have flavour oscillations
  - Neutrinos have mass
  - Diagonalizing Y^l_{ij} doesn't come for free any longer

$$\mathcal{L}_{Yukawa} = Y_{ij}\overline{\psi_{Li}}\,\phi\,\psi_{Rj} + h.c.$$
$$= Y^d_{ij}\overline{Q^I_{Li}}\,\phi\,d^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\,\tilde{\phi}\,u^I_{Rj} + Y^l_{ij}\overline{L^I_{Li}}\,\phi\,l^I_{Rj}$$

- thus there is the equivalent of a CKM matrix for them:
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \begin{bmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

vs

$$\begin{bmatrix}|d'\rangle\\ |s'\rangle\\ |b'\rangle\end{bmatrix} = \begin{bmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{bmatrix}\begin{bmatrix}|d\rangle\\ |s\rangle\\ |b\rangle\end{bmatrix}.$$

Intermezzo: How about the leptons?

- the equivalent of the CKM matrix
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

vs

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}.$$

- a completely different hierarchy!

$$U_{MNSP} \approx \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.37 & 0.57 & 0.70 \\ 0.39 & 0.59 & 0.69 \end{pmatrix}$$

$$V_{CKM} = \begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.04214 \\ 0.00896 & 0.04133 & 0.99911 \end{pmatrix}$$

From 2 to 3 generations

- 2 generations: d'=0.97 d + 0.22 s   (θ_c=13°)

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- 3 generations: d'=0.97 d + 0.22 s + 0.003 b

Parameterization used by Particle Data Group (3 Euler angles, 1 phase):

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

Wolfenstein parameterization

$$\sin\theta_{12} = \lambda \quad (2.7)$$

$$\sin\theta_{23} = A\lambda^2 \quad (2.8)$$

$$\sin\theta_{13} e^{-i\delta_{13}} = A\lambda^3(\rho - i\eta) \quad (2.9)$$

where A, ρ and η are numbers of order unity. The CKM matrix then becomes $\mathcal{O}(\lambda^3)$:

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \delta V \quad (2.10)$$

3 real parameters: A, λ, ρ

1 imaginary parameter: η

Wolfenstein parameterization

$$\sin\theta_{12} = \lambda \quad (2.7)$$

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The higher order terms in the Wolfenstein parametrization are of particular importance for the B_s-system, as we will see in chapter 4, because the phase in $|V_{ts}|$ is only apparent at $\mathcal{O}(\lambda^4)$:

$$\delta V = \begin{pmatrix} -\frac{1}{8}\lambda^4 & 0 & 0 \\ \frac{1}{2}A^2\lambda^5(1-2(\rho + i\eta)) & -\frac{1}{8}\lambda^4(1+4A^2) & 0 \\ \frac{1}{2}A\lambda^5(\rho + i\eta) & \frac{1}{2}A\lambda^4(1-2(\rho + i\eta)) & -\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6) \quad (2.11)$$

3 real parameters: A, λ, ρ

1 imaginary parameter: η

Deriving the triangle interpretation

- Starting point: the 9 unitarity constraints on the CKM matrix

$$V^{+}V = \begin{pmatrix} V^{*}_{ud} & V^{*}_{cd} & V^{*}_{td} \\ V^{*}_{us} & V^{*}_{cs} & V^{*}_{ts} \\ V^{*}_{ub} & V^{*}_{cb} & V^{*}_{tb} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Pick (arbitrarily) orthogonality condition with (i,j)=(3,1)

$$V^{*}_{ub}V_{ud} + V^{*}_{cb}V_{cd} + V^{*}_{tb}V_{td} = 0$$

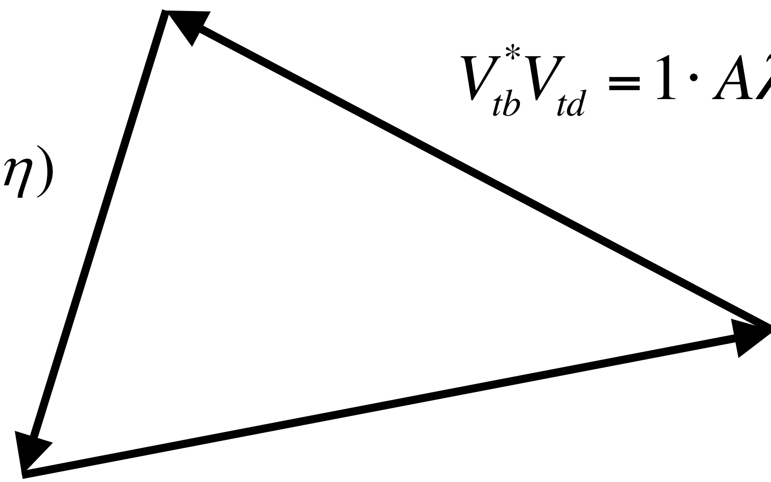
Visualizing the unitarity constraint

- Sum of three complex vectors is zero → Form triangle when put head to tail

(Wolfenstein params to order λ⁴)

$$V_{tb}^{*}V_{td} = 1 \cdot A\lambda^{3}(1-\rho-i\eta)$$

$$V_{ub}^{*}V_{ud} = A\lambda^{3}(\rho+i\eta)$$

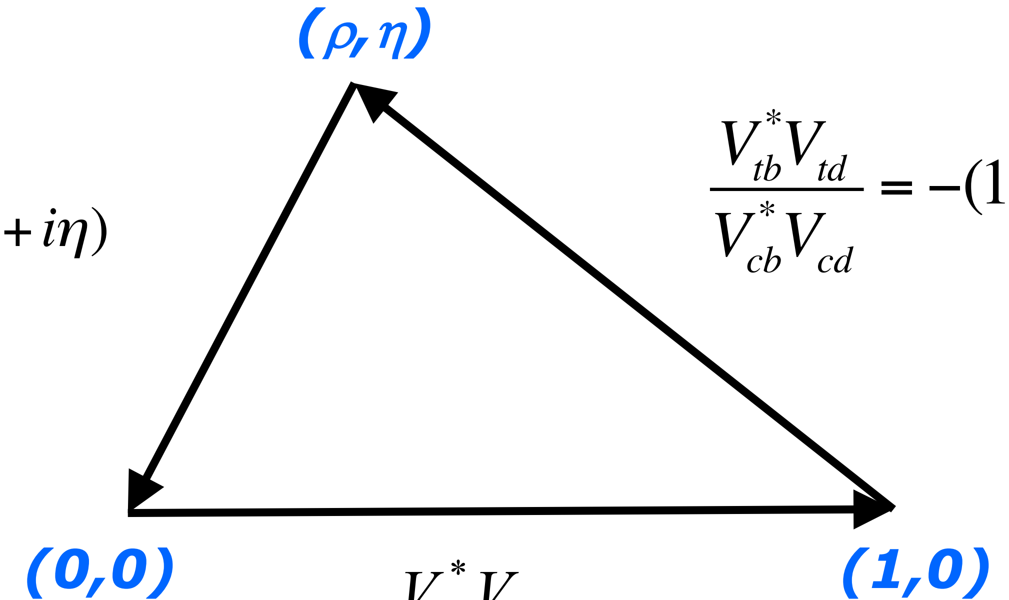


$$V_{cb}^{*}V_{cd} = A\lambda^{2}\cdot(-\lambda)$$

Visualizing the unitarity constraint

- Divide all sides by length of base

$$\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = -(\rho + i\eta)$$



$$\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = -(1 - \rho - i\eta)$$

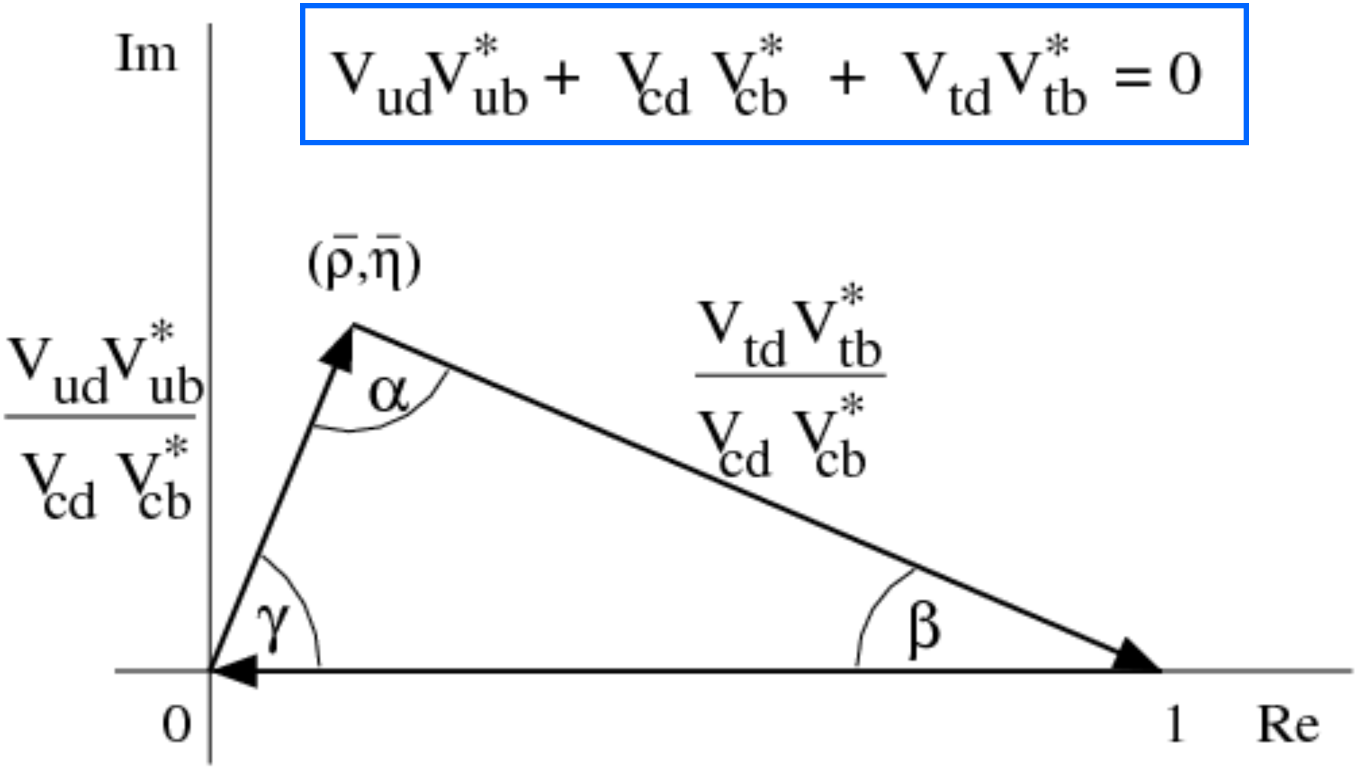
$$\frac{V_{cb}^* V_{cd}}{V_{cb}^* V_{cd}} \equiv 1$$

- Constructed a triangle with apex (ρ,η)

"The" Unitarity triangle

- We can visualize the CKM-constraints in (ρ,η) plane

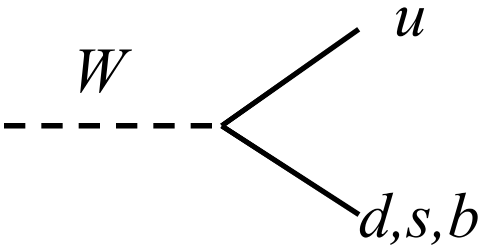
$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$



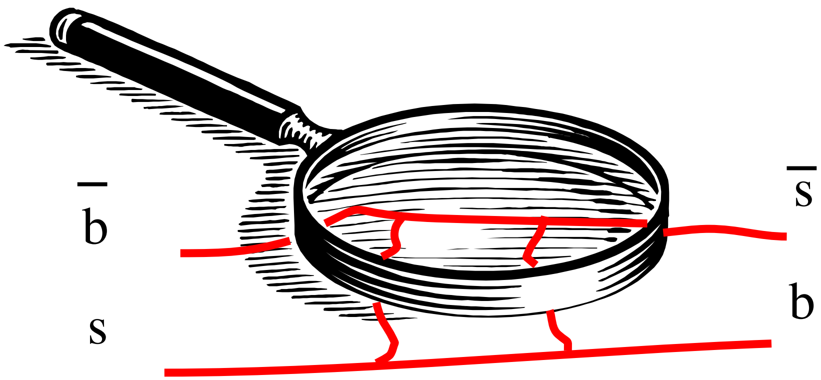
Quarks → Mesons

- Quarks:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



- Mesons
  - "Oscillations" important ingredient!



Dynamics of Neutral B (or K) mesons…

Time evolution of B^0 and $\overline{B}^0$ can be described by an *effective* Hamiltonian:

$$i\frac{\partial}{\partial t}\Psi = H\Psi \qquad \Psi(t) = a(t)\left|B^0\right\rangle + b(t)\left|\overline{B}^0\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}}$$

No mixing, no decay…

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

No mixing, but with decays…
(i.e.: H is not Hermitian!)

➔With decays included, probability of observing *either* B^0 *or* B^0 must go down as time goes by:

$$\frac{d}{dt}\left(\left|a(t)\right|^2 + \left|b(t)\right|^2\right) = -\begin{pmatrix} a(t)^* & b(t)^* \end{pmatrix}\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \qquad \Rightarrow \Gamma > 0$$

Describing Mixing…

Time evolution of B^0 and $\overline{B}^0$ can be described by an *effective* Hamiltonian:

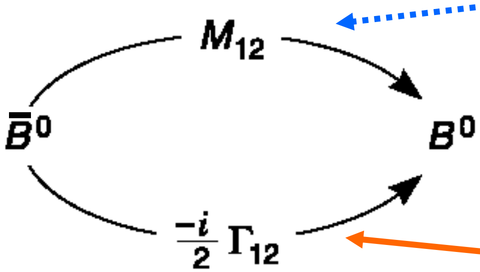
$$i\frac{\partial}{\partial t}\Psi = H\Psi \qquad \Psi(t) = a(t)\left|B^0\right\rangle + b(t)\left|\overline{B}^0\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

Where to put the mixing term?

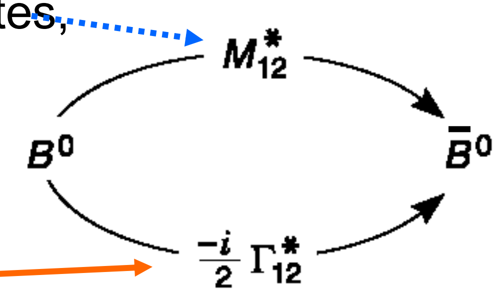
$$H = \underbrace{\begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

Now with mixing – but what is the difference between M_{12} and Γ_{12}?



M_{12} describes $B^0 \leftrightarrow B^0$ via *off-shell* states, e.g. the weak box diagram

Γ_{12} describes $B^0 \leftrightarrow f \leftrightarrow B^0$ via *on-shell* states, eg. $f=\pi^+\pi^-$



Solving the Schrödinger Equation

$$i\frac{\partial}{\partial t}\psi(t)=\begin{pmatrix} M-\frac{i}{2}\Gamma & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*} & M-\frac{i}{2}\Gamma \end{pmatrix}\psi(t)$$

Eigenvalues:

– Mass and lifetime of physical states: mass eigenstates

$$\begin{vmatrix} M-\frac{i}{2}\Gamma-\lambda & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*} & M-\frac{i}{2}\Gamma-\lambda \end{vmatrix}=0$$

$$\text{notation } F=\sqrt{(M_{12}-\tfrac{i}{2}\Gamma_{12})(M_{12}^{*}-\tfrac{i}{2}\Gamma_{12}^{*})}$$

$$\begin{aligned} m_1+\frac{i}{2}\Gamma_1 &= M-\Re F-\frac{i}{2}\Gamma-\Im F \\ m_2+\frac{i}{2}\Gamma_2 &= M+\Re F-\frac{i}{2}\Gamma+\Im F \end{aligned}$$

$$\Delta m = 2\Re\sqrt{\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*}\right)}$$

$$\Delta\Gamma = 4\Im\sqrt{\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*}\right)}$$

Solving the Schrödinger Equation

$$i\frac{\partial}{\partial t}\psi(t)=\begin{pmatrix} M-\frac{i}{2}\Gamma & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*} & M-\frac{i}{2}\Gamma \end{pmatrix}\psi(t)$$

Eigenvectors:

– mass eigenstates

$$\begin{aligned} |P_1\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \\ |P_2\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \end{aligned}$$

find p and q by solving

$$\begin{pmatrix} M-\frac{i}{2}\Gamma & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*} & M-\frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \lambda_{\pm}\begin{pmatrix} p \\ q \end{pmatrix}$$

$$|B_H\rangle = p|B\rangle + q|\overline{B}\rangle$$

$$|B_L\rangle = p|B\rangle - q|\overline{B}\rangle$$

$$q/p=\sqrt{\left(M_{12}^{*}-\frac{i}{2}\Gamma_{12}^{*}\right)\Big/\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)}$$

Time evolution

- With diagonal Hamiltonian, usual time evolution is obtained:

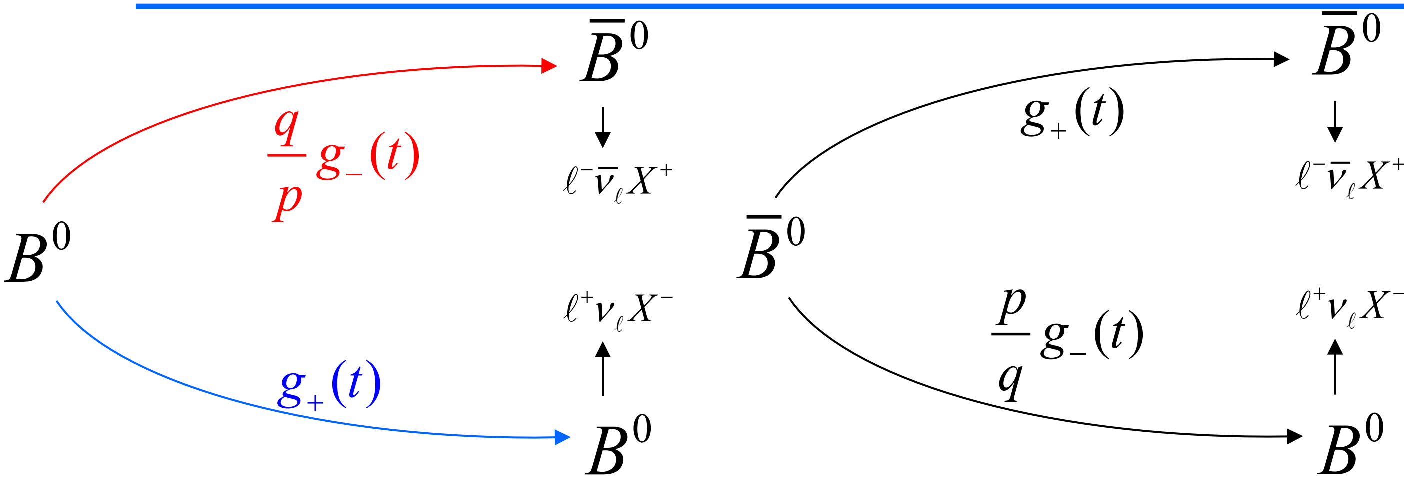
$$
\begin{aligned}
|P_H(t)\rangle &= e^{-im_H t - \frac{1}{2}\Gamma_H t}|P_H(0)\rangle \\
|P_L(t)\rangle &= e^{-im_L t - \frac{1}{2}\Gamma_L t}|P_L(0)\rangle
\end{aligned}
$$

$$
\begin{aligned}
|P^0\rangle &= \frac{1}{2p}\left[|P_H\rangle + |P_L\rangle\right] \\
|\bar{P}^0\rangle &= \frac{1}{2q}\left[|P_H\rangle - |P_L\rangle\right]
\end{aligned}
$$

$$
\begin{aligned}
|P_H\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \\
|P_L\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle
\end{aligned}
$$

$$
\begin{aligned}
|P^0(t)\rangle &= \frac{1}{2p}\left\{e^{-im_H t - \frac{1}{2}\Gamma_H t}|P_H(0)\rangle + e^{-im_L t - \frac{1}{2}\Gamma_L t}|P_L(0)\rangle\right\} \\
&= \frac{1}{2p}\left\{e^{-im_H t - \frac{1}{2}\Gamma_H t}(p|P^0\rangle + q|\bar{P}^0\rangle) + e^{-im_L t - \frac{1}{2}\Gamma_L t}(p|P^0\rangle - q|\bar{P}^0\rangle)\right\} \\
&= \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)|P^0\rangle + \frac{q}{2p}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)|\bar{P}^0\rangle \\
&= g_+(t)|P^0\rangle + \left(\frac{q}{p}\right)g_-(t)|\bar{P}^0\rangle \qquad (3.6)
\end{aligned}
$$

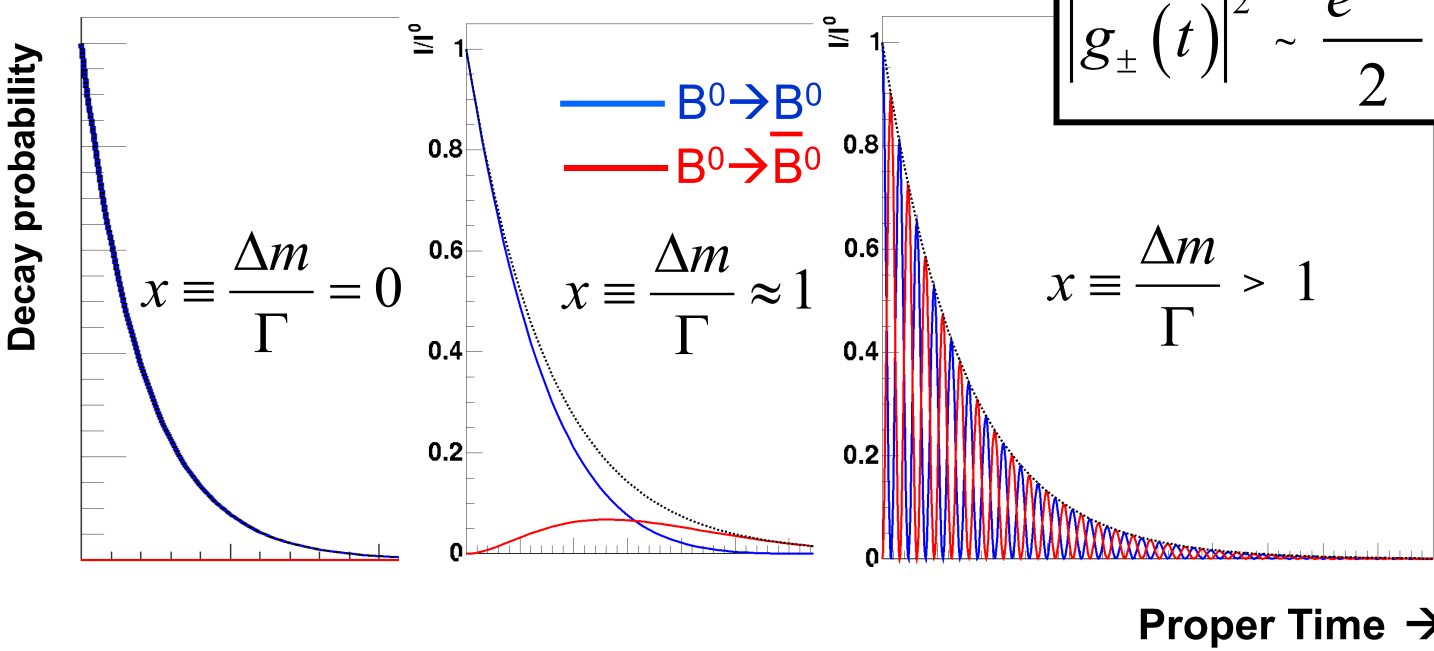
Measuring B Oscillations



For B^0, expect:
$\Delta\Gamma \sim 0$,
$|q/p|=1$

$$\left|g_\pm(t)\right|^2 \sim \frac{e^{-\Gamma t}}{2}\left[1 \pm \cos\left(\Delta m \cdot t\right)\right]$$

Examples:



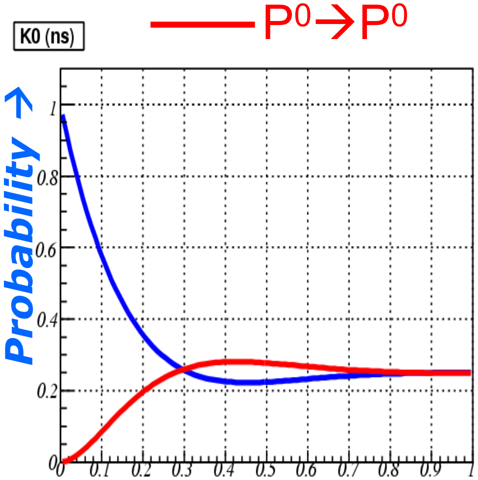
Compare the mesons:

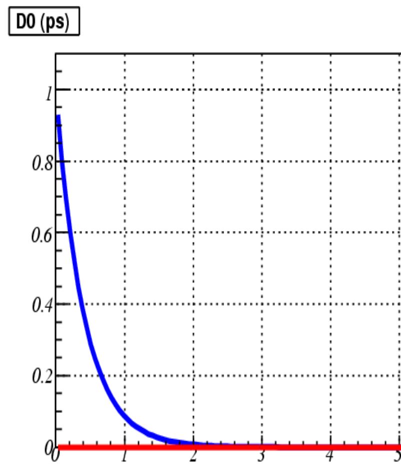
Probability to measure P or $\overline{P}$, when we start with 100% P

$P^0 \rightarrow P^0$ (blue)

$P^0 \rightarrow \overline{P}^0$ (red)

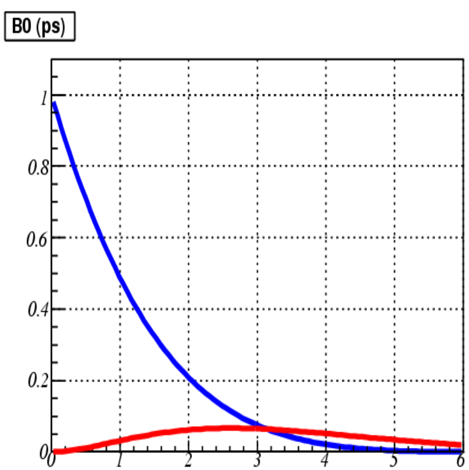
Probability →

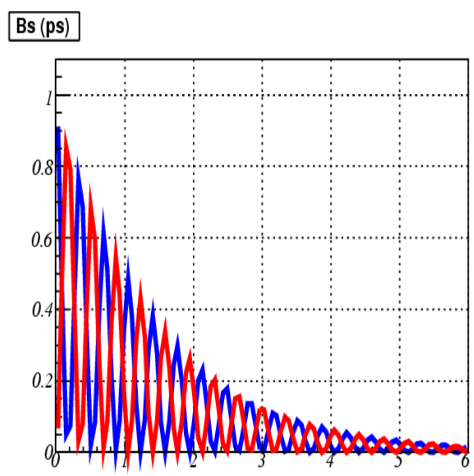




	$<\tau>$	Δm	$x=\Delta m/\Gamma$	$y=\Delta\Gamma/2\Gamma$
K^0	$2.6\ 10^{-8}$ s	$5.29\ ns^{-1}$	$\Delta m/\Gamma_S=0.49$	~ 1
D^0	$0.41\ 10^{-12}$ s	$0.001\ fs^{-1}$	~0	0.01
B^0	$1.53\ 10^{-12}$ s	$0.507\ ps^{-1}$	0.78	~0
B_s^0	$1.47\ 10^{-12}$ s	$17.8\ ps^{-1}$	12.1	~0.05

By the way,
$\hbar=6.58\ 10^{-22}$ MeVs





Time →

$x=\Delta m/\Gamma$: avg nr of oscillations before decay

Oscillations (1)

- Start with Schrodinger equation:

$$i\frac{\partial\psi}{\partial t} = H\psi = (M - \frac{i}{2}\Gamma)\psi = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\psi$$

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

(2-component state in P^0 and $\overline{P^0}$ subspace)

- Find eigenvalue:

$$\begin{vmatrix} M - \frac{i}{2}\Gamma - \lambda & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma - \lambda \end{vmatrix} = 0$$

- Solve eigenstates:

$$\psi_{\pm} = \begin{pmatrix} p \\ \pm q \end{pmatrix}$$

$$\begin{aligned} |P_1\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \\ |P_2\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \end{aligned}$$

we find p and q by solving

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \lambda_{\pm}\begin{pmatrix} p \\ q \end{pmatrix} \longrightarrow \frac{q}{p} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}$$

- Eigenstates have diagonal Hamiltonian: mass eigenstates!

Oscillations (2)

- Two mass eigenstates

$$
\begin{aligned}
|P_H\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \\
|P_L\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle
\end{aligned}
$$

- Time evolution:

$$
\begin{aligned}
|P_H(t)\rangle &= e^{-im_H t - \frac{1}{2}\Gamma_H t}|P_H(0)\rangle \\
|P_L(t)\rangle &= e^{-im_L t - \frac{1}{2}\Gamma_L t}|P_L(0)\rangle
\end{aligned}
$$

$$
\begin{aligned}
|P^0\rangle &= \frac{1}{2p}\left[|P_H\rangle + |P_L\rangle\right] \\
|\bar{P}^0\rangle &= \frac{1}{2q}\left[|P_H\rangle - |P_L\rangle\right]
\end{aligned}
$$

$$
\left|P^0(t)\right\rangle = \frac{1}{2}\left(e^{-im_Ht-\frac{1}{2}\Gamma_Ht} + e^{-im_Lt-\frac{1}{2}\Gamma_Lt}\right)|P^0\rangle + \frac{q}{2p}\left(e^{-im_Ht-\frac{1}{2}\Gamma_Ht} - e^{-im_Lt-\frac{1}{2}\Gamma_Lt}\right)|\bar{P}^0\rangle
$$

- Probability for $|P^0\rangle \rightarrow |\overline{P^0}\rangle$!
- Express in $M=m_H+m_L$ and $\Delta m=m_H-m_L$ → Δm dependence

Oscillations: summary

- p, q: $|B_H\rangle = p|B^0\rangle + q|\overline{B}^0\rangle$

  $|B_L\rangle = p|B^0\rangle - q|\overline{B}^0\rangle$

- Δm, ΔΓ: $\Delta m = 2\Re\sqrt{\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)}$

  $\Delta\Gamma = 4\Im\sqrt{\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)}$

q,p,M_{ij},Γ_{ij} related through:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}$$

- x,y: mixing often quoted in *scaled* parameters: $x = \frac{\Delta m}{\Gamma}$ $\quad y = \frac{\Delta\Gamma}{2\Gamma}$

$$\cos(\Delta m t) = \cos\left(\frac{\Delta m}{\Gamma}\frac{t}{\tau}\right) = \cos\left(x\frac{t}{\tau}\right)$$

Time dependence (if ΔΓ~0, like for B^0):

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\overline{B^0}\rangle$$

$$|\overline{B^0}(t)\rangle = g_+(t)|\overline{B^0}\rangle + \frac{p}{q}g_-(t)|B^0\rangle$$

with

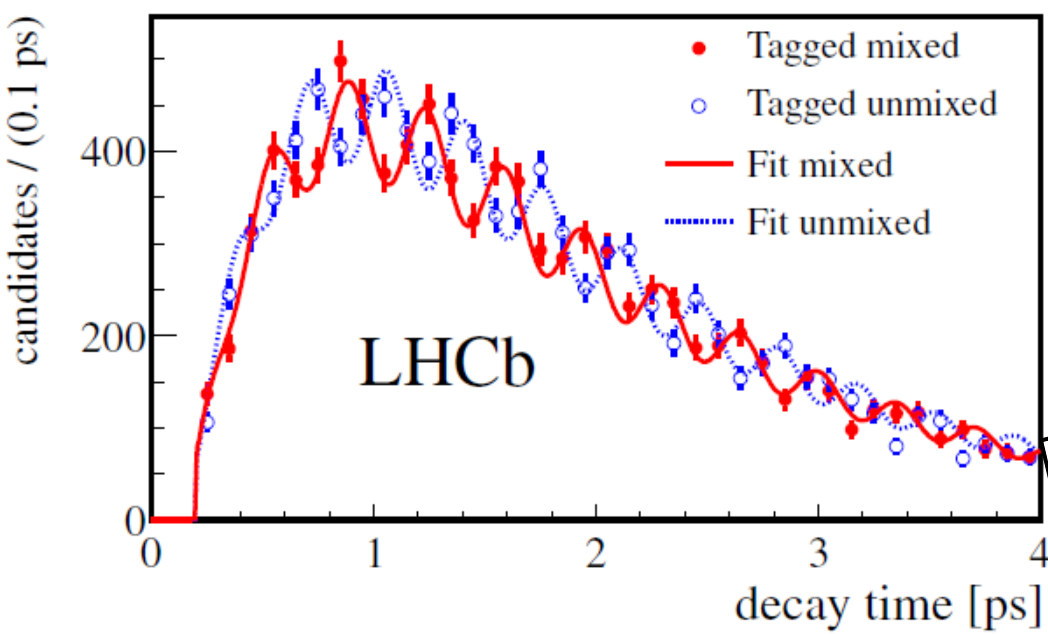
$$g_+(t) = e^{-imt}e^{-\Gamma t/2}\cos\frac{\Delta m t}{2}$$

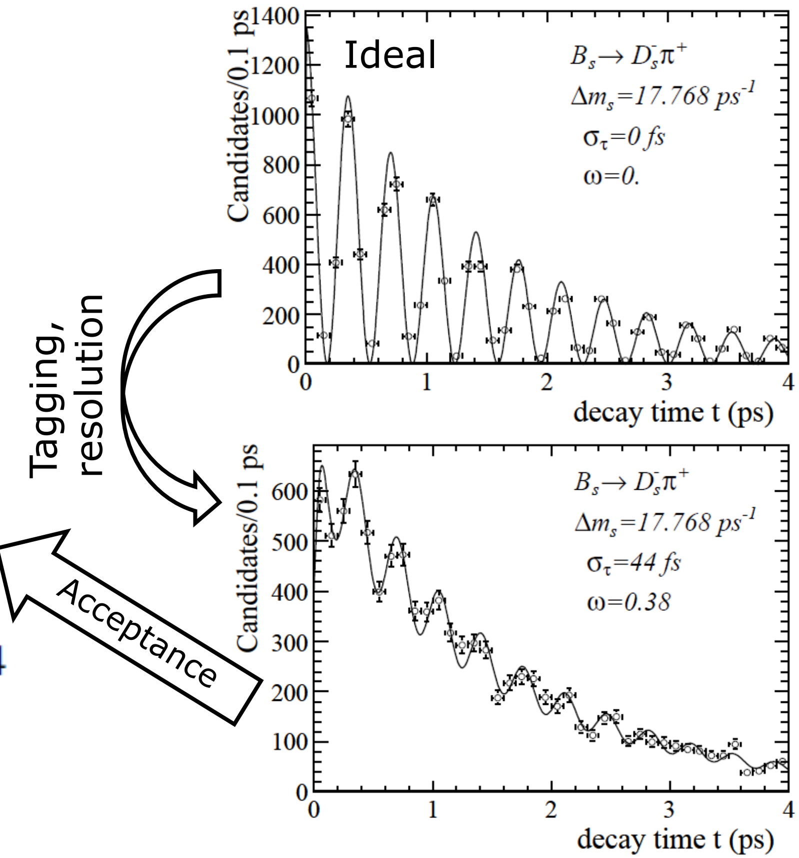
$$g_-(t) = e^{-imt}e^{-\Gamma t/2}\, i\sin\frac{\Delta m t}{2}$$

B_s^0 mixing (Δm_s): New: LHCb

$$\frac{N_{B^0\to B^0}(t) - N_{B^0\to \bar{B}^0}(t)}{N_{B^0\to B^0}(t) + N_{B^0\to \bar{B}^0}(t)} = \cos(\Delta m \cdot t)$$

$$\Delta m_s = 17.768 \pm 0.023\ (\mathrm{stat}) \pm 0.006\ (\mathrm{syst})\ \mathrm{ps}^{-1}$$





LHCb, arXiv:1304.4741

Mixing → CP violation?

- NB: Just mixing is not necessarily CP violation!
- However, by studying certain *decays with and without mixing*, CP violation is observed

- Next: Measuring CP violation…

Detecting CP violation with B decays

1) CP violation: CKM and the SM
2) Detecting: Detector requirements
3) B-decays: $\sin 2\beta$, ϕ_s, $B_s^0 \rightarrow D_s^+ K^-$