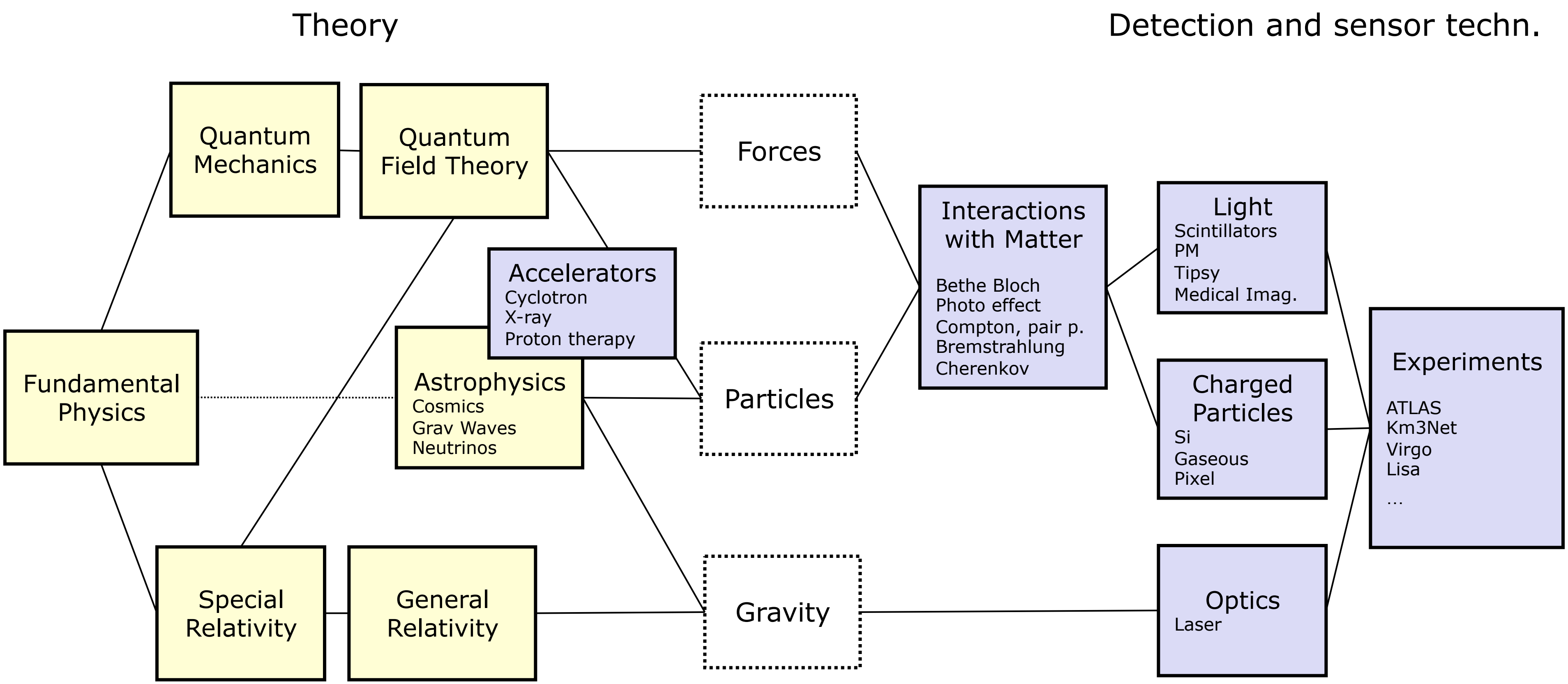
"Elementary Particles"
Theory Lecture 6

Niels Tuning

Thanks

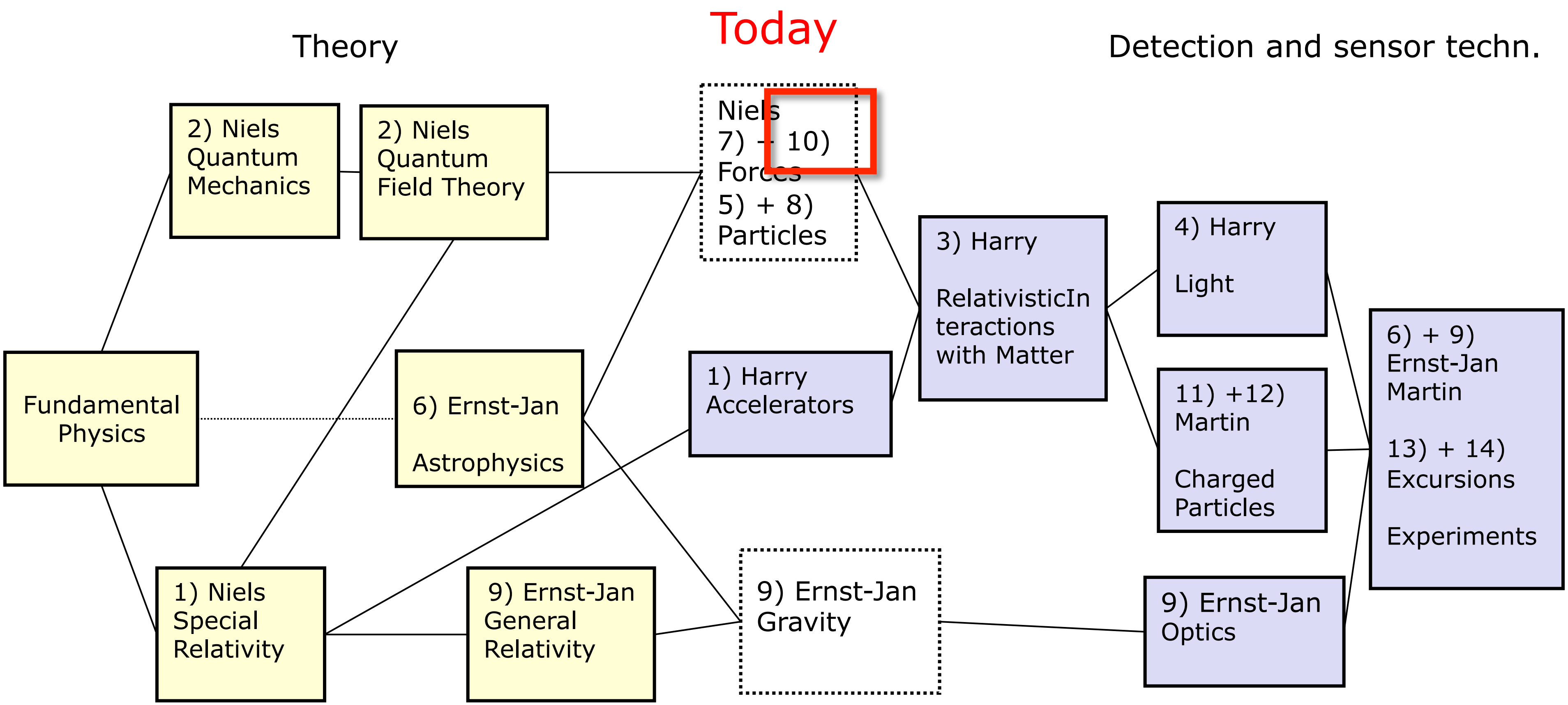
- Ik ben schatplichtig aan:
  - Dr. Ivo van Vulpen (UvA)
  - Prof. dr. ir. Bob van Eijk (UT)
  - Prof. dr. Marcel Merk (VU)

Plan



Theory
Detection and sensor techn.
Quantum Mechanics
Quantum Field Theory
Forces
Interactions with Matter
Bethe Bloch
Photo effect
Compton, pair p.
Bremstrahlung
Cherenkov
Light
Scintillators
PM
Tipsy
Medical Imag.
Accelerators
Cyclotron
X-ray
Proton therapy
Fundamental Physics
Astrophysics
Cosmics
Grav Waves
Neutrinos
Particles
Charged Particles
Si
Gaseous
Pixel
Experiments
ATLAS
Km3Net
Virgo
Lisa
...
Special Relativity
General Relativity
Gravity
Optics
Laser

Plan



Theory
Today
Detection and sensor techn.
2) Niels Quantum Mechanics
2) Niels Quantum Field Theory
Niels 7) + 10) Forces 5) + 8) Particles
3) Harry RelativisticInteractions with Matter
4) Harry Light
Fundamental Physics
6) Ernst-Jan Astrophysics
1) Harry Accelerators
11) +12) Martin Charged Particles
6) + 9) Ernst-Jan Martin
13) + 14) Excursions
Experiments
1) Niels Special Relativity
9) Ernst-Jan General Relativity
9) Ernst-Jan Gravity
9) Ernst-Jan Optics

Schedule

1) 11 Feb: Accelerators (Harry vd Graaf) + Special relativity (Niels Tuning)
2) 18 Feb: Quantum Mechanics (Niels Tuning)
3) 25 Feb: Interactions with Matter (Harry vd Graaf)
4) 3 Mar: Light detection (Harry vd Graaf)
5) 10 Mar: Particles and cosmics (Niels Tuning)
6) 17 Mar: Forces (Niels Tuning)
7) 24 Mar: Astrophysics and Dark Matter (Ernst-Jan Buis)

break

8) 21 Apr: e^+e^- and ep scattering (Niels Tuning)
9) 28 Apr: Gravitational Waves (Ernst-Jan Buis)
10) 12 May: Higgs and big picture (Niels Tuning)
11) 19 May: Charged particle detection (Martin Franse)
12) 26 May: Applications: experiments and medical (Martin Franse)

13) 2 Jun: Nikhef excursie
14) 8 Jun: CERN excursie CANCELLED → We try to organize special lecture(s) Tue 9 June

Plan

	1) Intro: Standard Model & Relativity	***11 Feb***
1900-1940	2) Basis	***18 Feb***
	1) Atom model, strong and weak force	
	2) Scattering theory	
1945-1965	3) Hadrons	***10 Mar***
	1) Isospin, strangeness	
	2) Quark model, GIM	
1965-1975	4) Standard Model	***17 Mar***
	1) QED	
	2) Parity, neutrinos, weak inteaction	
	3) QCD	
1975-2000	5) e^+e^- and DIS	***21 Apr***
2000-2015	6) Higgs and CKM	***12 May***

Homework

Homework Lecture 5

1 The J/ψ meson

The J/ψ meson is the lightest ($c\bar{c}$) bound state, and was discovered in Novemeber 1974, independently and almost at the same time, at the Stanford Linear Accelerator Center (SLAC, close to San Francisco) and at Brookhaven National Laboratories (BNL, close to New York).

a) At SLAC, the J/ψ was created in 6 + 6 GeV e^+e^- collissions, using the SPEAR accelerator. Draw a (Feynman) diagram of the production of the J/ψ in e^+e^- collissions.

b) (EXTRA, not easy...) At Brookhaven, the J/ψ was created by shooting 30 GeV protons from the AGS accelerator, on a Be target. Given the large energy of the protons, the production of the J/ψ in fact occurred through quark-quark collissions. Draw a (Feynman) diagram of the production of the J/ψ in $p+p$ collissions.

c) The J/ψ decays for about 88% to hadrons, 6% to e^+e^-, and 6% to $\mu^+\mu^-$. The decay to two leptons has the cleanest experimental signature, and that is how the J/ψ was discovered at both laboratories. Draw a (Feynman) diagram of the decay of $J/\psi \to \mu^+\mu^-$.

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a) $e^+e^- \to \gamma^* \to c\bar{c} \to J/\psi$

b) $q\bar{q} \to ggg \to c\bar{c} \to J/\psi$

c) $J/\psi \to \gamma^* \to \mu^+\mu^-$

Homework Lecture 5

d) Why does the J/ψ not decay to two τ-leptons?

e) Since the strong interactions is so much stronger than the electro-magnetic interaction, it is surprising that the branching fraction to leptons is still as large as 12%. Let's see why. What is the spin and color of the J/ψ meson?

f) e) What is the spin and color of a gluon?

g) f) Through the exchange of how many gluons does the hadronic decay of J/ψ mesons occur? (Now you know why question b) was difficult...)

d) The decay to two τ-leptons is kinematically not allowed, because the mass of two τ-leptons is larger than the mass of the J/ψ meson.

e) $S_{J/\psi} = 1$, and all hadrons are colorless.

f) $S_{gluon} = 1$, and all gluons have color.

g) $J/\psi \rightarrow 1g$ violates color, 2 gluons add up to spin 0, or spin-2, so the minimum is 3 gluons...

Homework Lecture 5

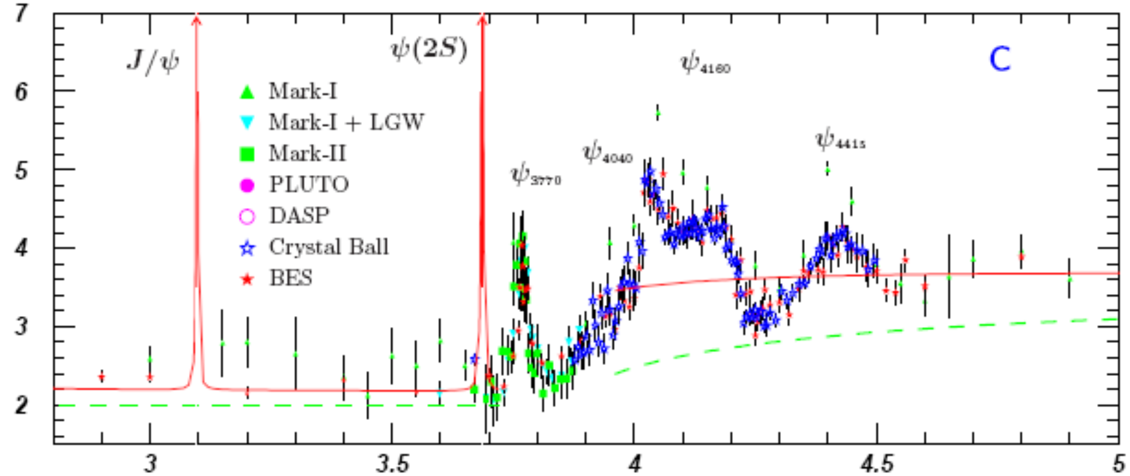
2 R

a) Explain why the jump in R does not seem to happen at $c\bar{c}$ threshold of 3.1 GeV, but higher.

b) Predict R for a center-of-mass energy > 10 GeV.

a) Open charm threshold starts at twice the D^0 mass, 2×1860 MeV.

b) b-quarks contribute: $R = N_c(q_u^2 + q_d^2 + q_s^2 + q_c^2 + q_b^2) = 3(2 \times 4/9 + 3 \times 1/9) = 33/9$



3.3 → 3.7

Homework Lecture 5

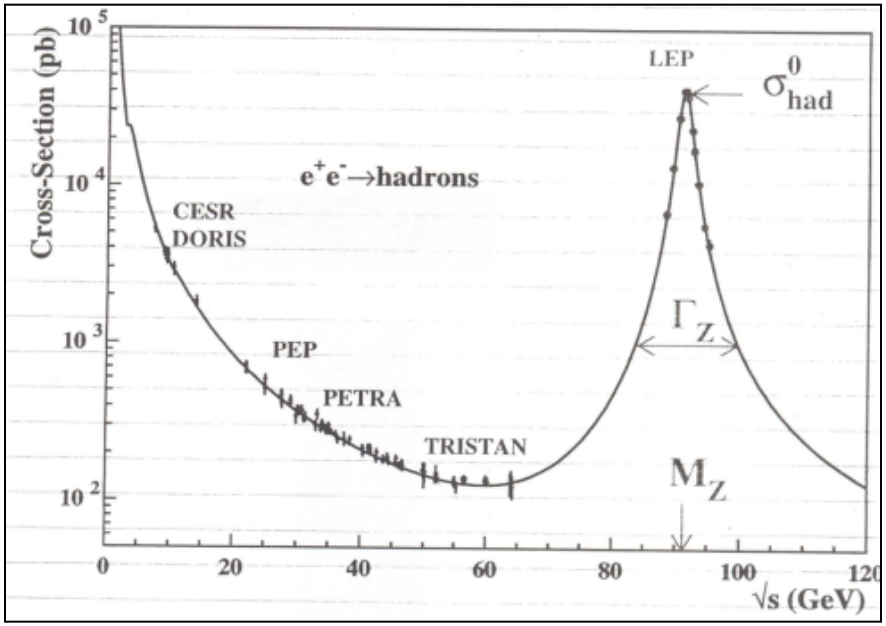
c) Explain why you see a jump above 4 GeV, but no jump above 10 GeV, on Fig.46.6
`http://pdg.lbl.gov/2012/reviews/rpp2012-rev-cross-section-plots.pdf`

d) What is the value of R above the $t\bar{t}$ threshold? To what value of the center-of-mass does the $t\bar{t}$ threshold correspond?

c) R changes from $3(4/9 + 1/9 + 1/9) = 18/9$ to $30/9$ at the charm threshold, whereas it only changes from $30/9$ to $33/9$ at the bottom threshold.

d) Above $\sqrt{s} = 2m_t = 360\,\mathrm{GeV}$ t-quarks contribute: $R = N_c(q_u^2 + q_d^2 + q_s^2 + q_c^2 + q_b^2 + q_t^2) = 3(3 \times 4/9 + 3 \times 1/9) = 45/9 = 5$

Homework Lecture 5



3 Three generations

a) What are the possible final states of the decay of the Z-boson? Hint 1: the Z decays only to fermions; not to photons, as that couples to electric charge, and not to two W particles as they are too heavy. Hint 2: remember that electric charge is conserved.

b) Write down the total decay width, as the sum of partial widths. Remember that quarks come in three types (ie. three colors)! We call the partial width to neutrinos the invisible partial width Γ_{inv}, as the neutrinos escape the detector undetected.

c) The total width of the Z is determined by measuring the total cross section of e^+e^- scattering for various value of the center-of-mass, around the Z-mass. This is called the Z lineshape. The total width is measured as $\Gamma_Z = 2495$ MeV. The partial width to one lepton pair is measured as $\Gamma_{l^+l^-} = 84$ MeV. The partial width to all hadrons is measured as $\Gamma_{had} = 1744$ MeV. Calculate the invisible partial width Γ_{inv}.

d) Write an expression for the number of neutrino types in terms of the total decay width, and the partial widths.

e) Estimate the number of neutrino types in the Standard Model, assuming the following input from the Standard Model, $\Gamma_\nu/\Gamma_{chargedlepton} = 1.991$.

a) The Z can decay to e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$, $u\bar{u}$, dd, $s\bar{s}$, $c\bar{c}$, bb, and to $\nu\bar{\nu}$.

b) $\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + 3(\Gamma_{uu} + \Gamma_{dd} + \Gamma_{ss} + \Gamma_{cc} + \Gamma_{bb}) + \Gamma_{inv}$.

c)

$$\Gamma_{inv} = \Gamma_Z - \Gamma_{had} - 3\Gamma_{l^+l^-} = 2495 - 1744 - 252 = 499\ MeV$$

d)

$$N_\nu = \frac{\Gamma_Z - \Gamma_{had} - 3\Gamma_{l^+l^-}}{\Gamma_\nu} = \frac{\Gamma_{inv}}{\Gamma_\nu}$$

e)

$$N_\nu = 499/(1.991 \times 84) = 2.984$$

Outline for today:

1) Higgs mechanism
2) Higgs discovery at ATLAS

3) CKM-mechanism
4) CP violations at LHCb

Summary Lects. 1-5

Lecture 1: Relativity

- Theory of relativity
  - Lorentz transformations (“boost”)
  - Calculate energy in collissions

$$
\begin{aligned}
x'^0 &= \gamma\left(x^0 - \beta x^1\right) \\
x'^1 &= \gamma\left(x^1 - \beta x^0\right) \\
x'^2 &= x^2 \\
x'^3 &= x^3
\end{aligned}
\qquad \textbf{met} \qquad
\begin{aligned}
\beta &\equiv \frac{v}{c} \\
\gamma &\equiv \frac{1}{\sqrt{1-\beta^2}}
\end{aligned}
$$

- 4-vector calculus

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \quad \left(\mu = 0, 1, 2, 3\right)$$

$$p_\mu p^\mu = (E/c)^2 - |\vec{p}|^2 = (E^2 - c^2|\vec{p}|^2)/c^2 = (m_0 c^4)/c^2$$

- High energies needed to make (new) particles

$$
\begin{aligned}
s &= \left(p_1 + p_2\right)^2 = 2m^2 + 2\left(E^2 + \vec{p}^2\right) \\
&= 2m^2 + 2E^2 + 2\left(E^2 - m^2\right) = 4E^2
\end{aligned}
$$

Lecture 1: 4-vector examples

- 4-vectors:
  - Space-time x^μ
  - Energie-momentum p^μ
  - 4-potential A^μ
  - Derivative ∂^μ
  - Covariant derivative D^μ
  - Gamma matrices γ^μ
- Tensors
  - Metric $g^{\mu\nu}$
  - Electromagnetic tensor $F^{\mu\nu}$

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \quad (\mu = 0, 1, 2, 3)$$

$$p_\mu p^\mu = (E/c)^2 - |\vec{p}|^2 = (E^2 - c^2|\vec{p}|^2)/c^2 = (m_0 c^4)/c^2$$

$$\partial_\mu \to D_\mu \equiv \partial_\mu + iqA_\mu$$

$$\partial_\mu \left(\partial^\mu A^\nu - \partial^\nu A^\mu \right) = j^\nu$$

$$\left(i\gamma^\mu \partial_\mu - m \right) \psi = 0$$

$$x_\mu = g_{\mu\nu} x^\nu$$

$$x_0 = x^0, \, x_1 = -x^1, \, x_2 = -x^2, \, x_3 = -x^3$$

$$g_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$F^{\mu\nu} = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x) = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Lecture 2: Quantum Mechanics & Scattering

- Schrödinger equation
  - Time-dependence of wave function

$$E = \frac{\vec{p}^2}{2m}$$

$$i\frac{\partial}{\partial t}\psi = \frac{-1}{2m}\nabla^2\psi$$

- Klein-Gordon equation
  - Relativistic equation of motion of scalar particles

$$E^2 = \vec{p}^2 + m^2$$

$$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\,\phi$$

- Dirac equation
  - Relativistically correct, and linear
  - Equation of motion for spin-1/2 particles
  - Described by 4-component spinors
  - Prediction of anti-matter

$$(i\gamma^\mu\partial_\mu - m)\,\psi \;=\; 0$$

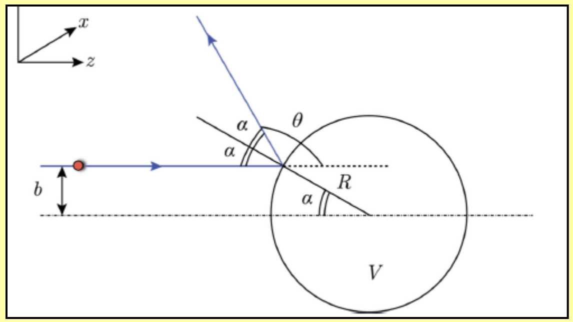
$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

Lecture 2: Quantum Mechanics & Scattering

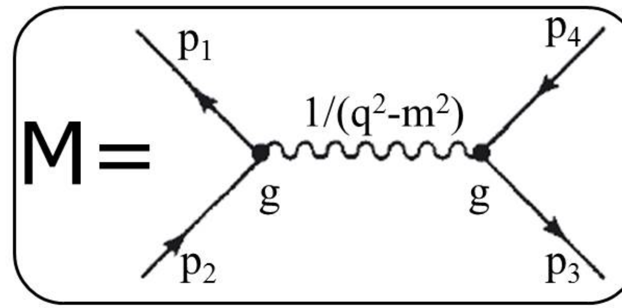
- Scattering Theory
  - (Relative) probability for certain process to happen
  - Cross section

$$\frac{d\sigma}{d\Omega} = D(\theta, \varphi)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 \alpha}{m v_0^2}\right)^2 \frac{1}{4\sin^4\frac{\theta}{2}} \qquad \frac{d\sigma}{d\Omega} = \left(\frac{2m Z_1 Z_2 \alpha}{q^2}\right)^2$$



Classic



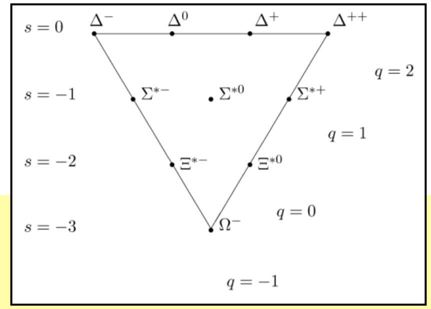
Scattering amplitude in Quantum Field Theory

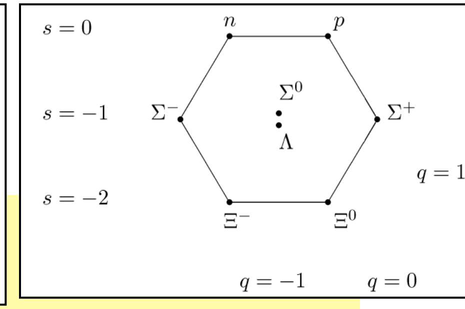
- Fermi's Golden Rule

$$\text{transition rate} = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \times (\text{phase space})$$

  - Decay: "decay width" Γ $\quad a \to b + c$
  - Scattering: "cross section" σ $\quad a + b \to c + d$

Lecture 3: Quarkmodel & Isospin





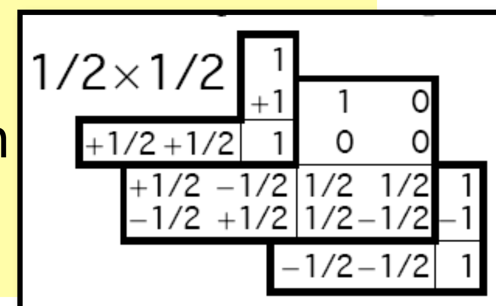
- "Partice Zoo" not elegant

- Hadrons consist of quarks

	d	u	s
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0
S – strangeness	0	0	-1

➢ Observed symmetries
  - Same mass of hadrons: isospin
  - Slow decay of K, Λ: strangeness
  - Fermi-Dirac statistics Δ^{++}, Ω: color

- Combining/decaying particles with (iso)spin
  - Clebsch-Gordan coefficients



Lecture 4: Gauge symmetry and Interactions

- Arbitrary "gauge"
  - Physics invariant
  - Introduce "gauge" fields in derivative

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{q}\partial_\mu \alpha(x)$$

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + iqA_\mu$$

➢ Interactions!

- QED

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

1 photon

- Weak interactions

$$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$

3 weak bosons

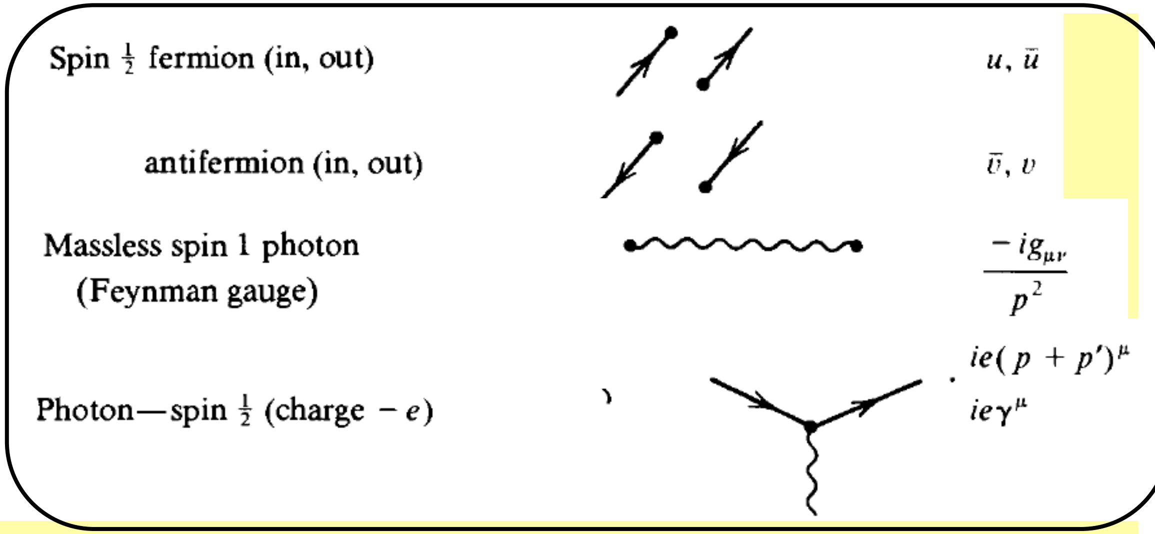
- QCD

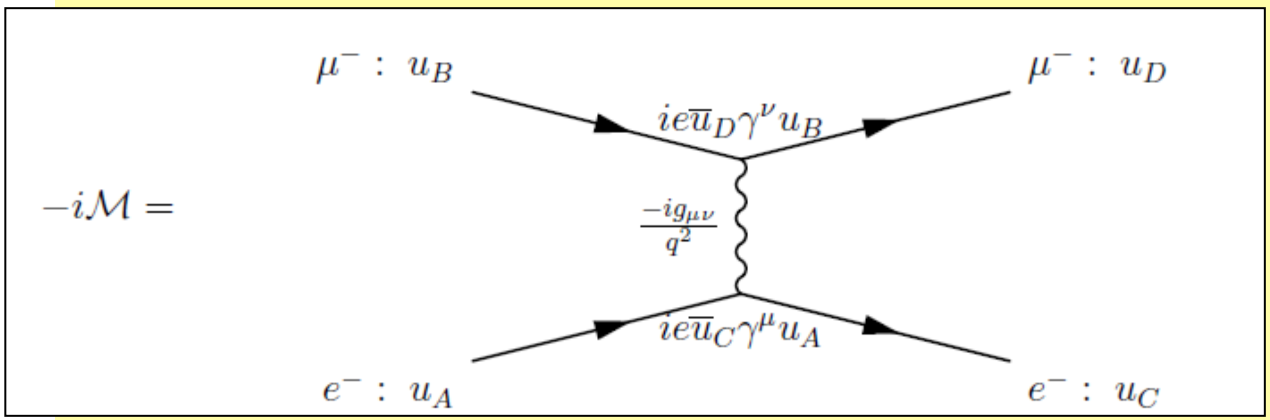
$$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8}\frac{i}{2}\theta_a(x)\lambda_a\right)\psi$$

8 gluons

Feynman rules: Example

- Process: $e^- \mu^- \rightarrow \mu^- e^-$





$$-i\mathcal{M} = -e^2\,\overline{u}_C\gamma^\mu u_A\,\frac{-i}{q^2}\,\overline{u}_D\gamma_\mu u_B$$

$$|\mathcal{M}|^2 = e^4\left[(\overline{u}_C\gamma^\mu u_A)\,\frac{1}{q^2}\,(\overline{u}_D\gamma_\mu u_B)\right]\left[(\overline{u}_C\gamma^\nu u_A)\,\frac{1}{q^2}\,(\overline{u}_D\gamma_\nu u_B)\right]^*$$

Remember the 4-component spinors in Dirac-space:

$$\underbrace{\left[\left(\ \overline{u}\ \right)\left(\ \gamma^\mu\ \right)\left(\ u\ \right)\right]}_{\text{a number}}$$

QED and QCD

QED

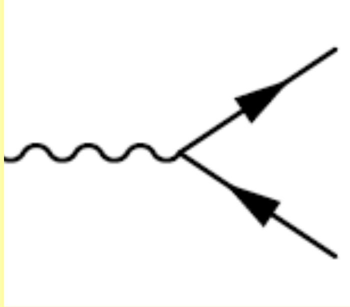
- Local U(1) gauge transformation

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

- Introduce 1 A_μ gauge field
- "Abelian" theory,

$$F^{\mu\nu} = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x)$$

- No self-interacting photon
  - Photons do not have (electric) charge
- Different "running"



QCD

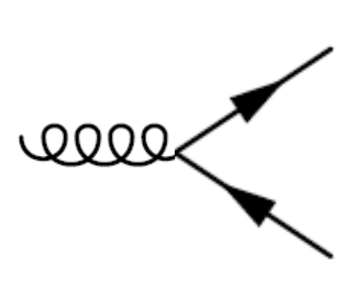
- Local SU(3) gauge transformation

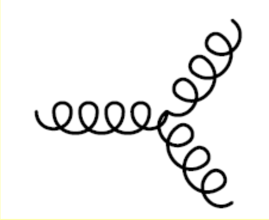
$$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8} \frac{i}{2}\theta_a(x)\lambda_a\right)\psi$$

- Introduce 8 $A_\mu{}^a$ gauge fields
- Non-"Abelian" theory,

$$G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g\, f_{abc}\, A^b_\mu(x)\, A^c_\nu(x)$$

- Self-interacting gluons
  - Gluons have (color) charge
- Different "running"

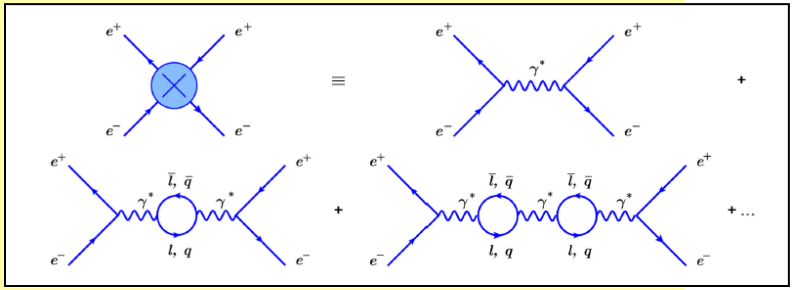




Lecture 5: Running couplings

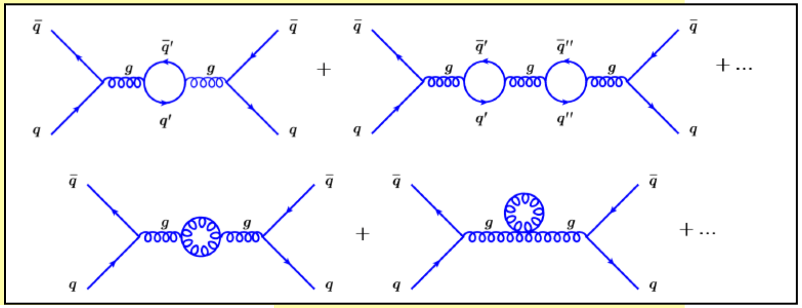
- EM coupling α

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{Q^2}{\mu^2}\right)}$$

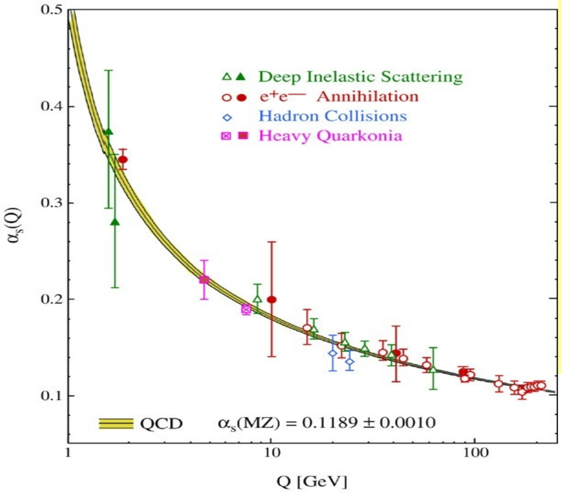


- Strong coupling α_S

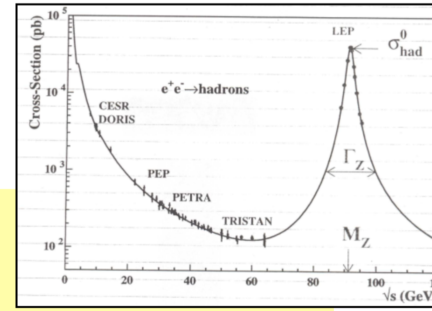
$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\log(Q^2/\mu^2)}$$



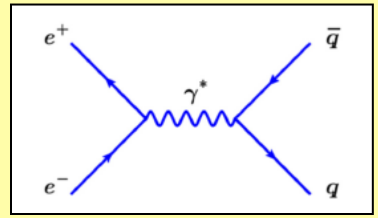
  - Confinement
  - Asymptotic freedom

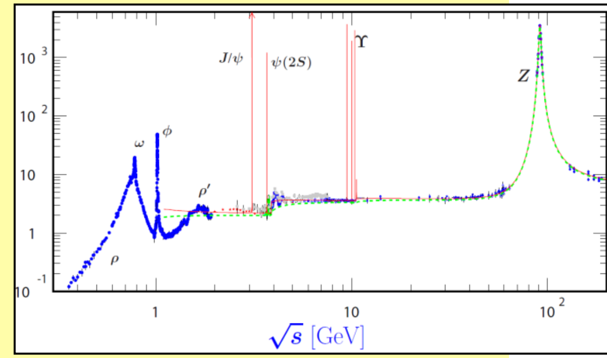


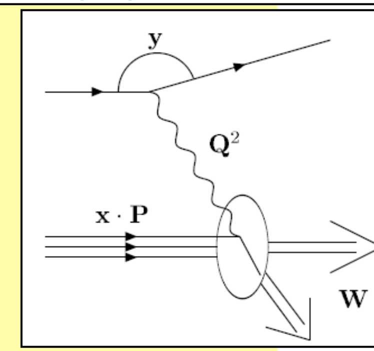
Lecture 5: e^+e^- scattering and DIS



- <u>e^+e^- scattering:</u> QED at work: **R**
  - $e^+e^-\rightarrow\mu^+\mu^-$
  - $e^+e^-\rightarrow cc$
  - $e^+e^-\rightarrow qq\ g$
  - $e^+e^-\rightarrow Z$
  - $e^+e^-\rightarrow WW$

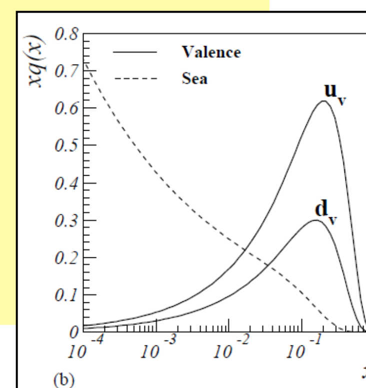






- <u>e^+p scattering:</u> QCD at work: $\mathbf{F_2}$
  - Quarkmodel: do quarks exist??
  - Substructure
  - Bjorken-x, sum rules
  - Scaling
  - 'Parton density functions' (pdf) and 'structure functions'
  - Scaling violations: more quarks at higher Q^2 due to QCD

$$F_2(x) = \sum_q e_q^2 (xq(x) + x\bar{q}(x))$$



Deep Inelastic Scattering

Lepton – proton scattering

or:

Hitting something big, using something small

Scattering

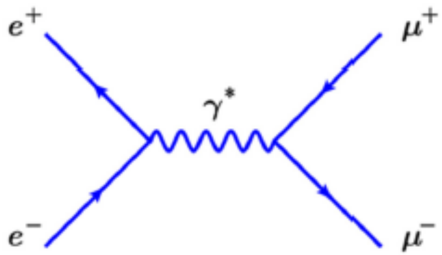
- Rutherford scattering

  (scattering off static point charge)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\frac{1}{2}\theta)}$$

- $e^+e^- \rightarrow \mu^+\mu^-$ scattering

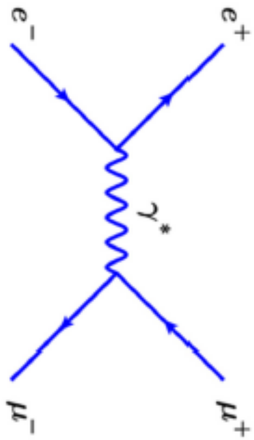
  (s-channel)



$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{\alpha^2}{4s}(1 + \cos^2\theta)$$

- $e^-\mu^+ \rightarrow e^-\mu^+$ scattering

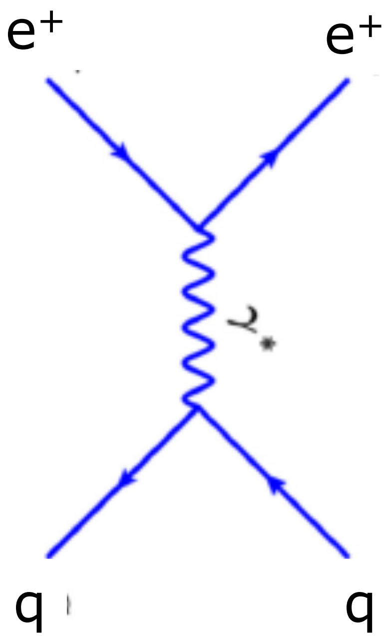
  (t-channel)



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} e^2 \frac{4 + (1+\cos\vartheta)^2}{(1-\cos\vartheta)^2}$$

$e^+q \rightarrow e^+q$

- Point cross section



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} e_q^2 \frac{4 + (1+\cos\vartheta)^2}{(1-\cos\vartheta)^2}$$

$$Q^2 = 2E_e^2(1-\cos\theta)$$

$$y = \sin^2\frac{\theta}{2}$$

$$\frac{d\sigma^{eq\rightarrow eq}}{dQ^2} = \frac{2\pi\alpha^2}{Q^4} e_q^2 \left[2(1-y) + y^2\right]$$

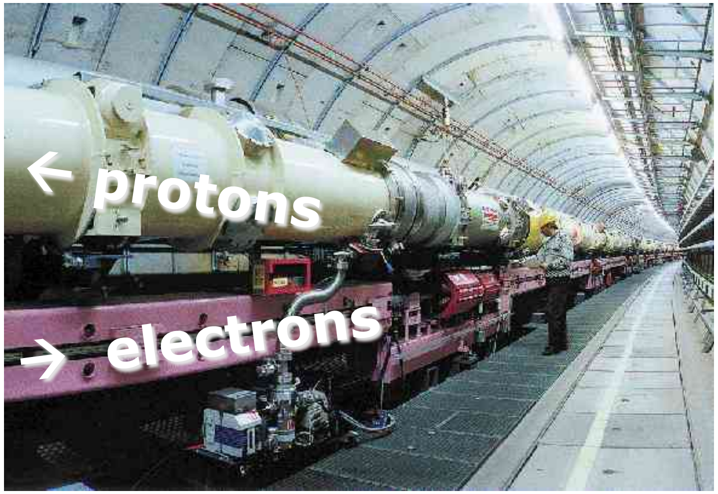
DIS experiments

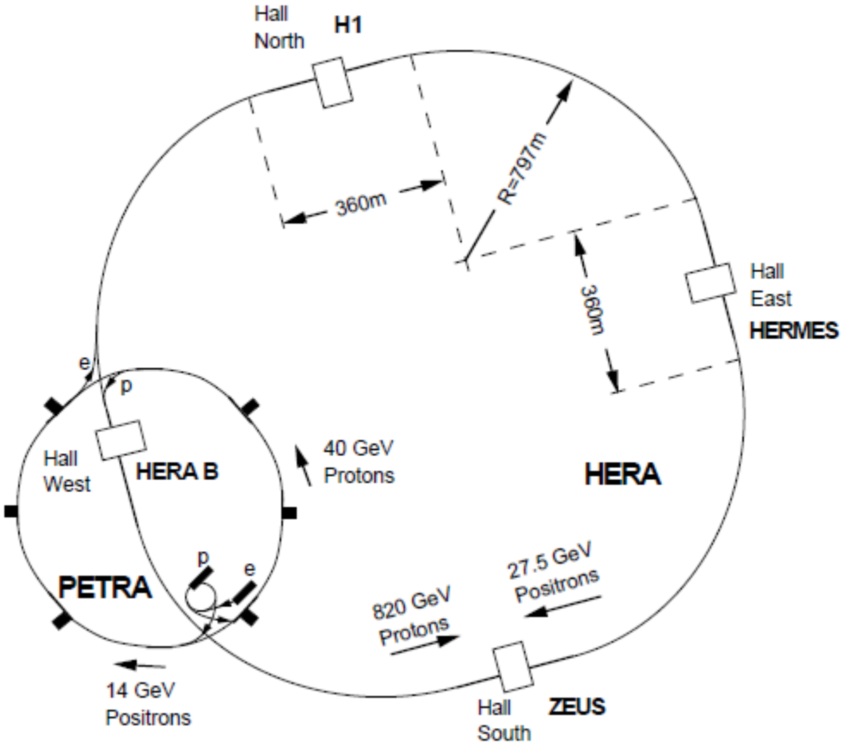
Experiment	Accel	Lab	lepton	E_{lep}	E_{had}	Year
SLAC-MIT		SLAC	e	20	fixed	1967-1973
Gargamelle		CERN	ν		fixed	
E80 -	SLC	SLAC			fixed	
CHORUS	SPS	CERN	ν	10-200	fixed	1998
CCFR	Tevatron	Fermilab	ν	30-360	fixed	
NMC	SPS	CERN	μ	90-280	fixed	1986-1989
EMC/SMC	SPS	CERN	μ	100-190	fixed	1984-1994
BCDMS	SPS	CERN	μ	100-280	fixed	
ZEUS, H1	HERA	DESY	e	27.5	920	1992-2007

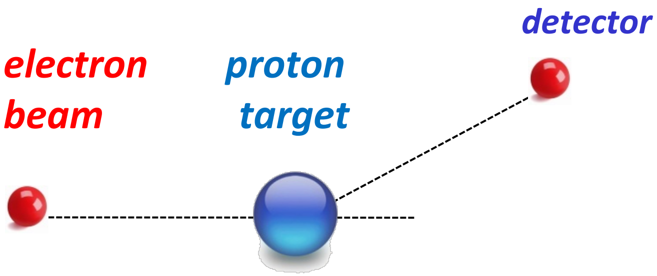
NB: Table not complete

- Easiest: fixed target
  - ep scattering
  - μp scattering
  - νp scattering

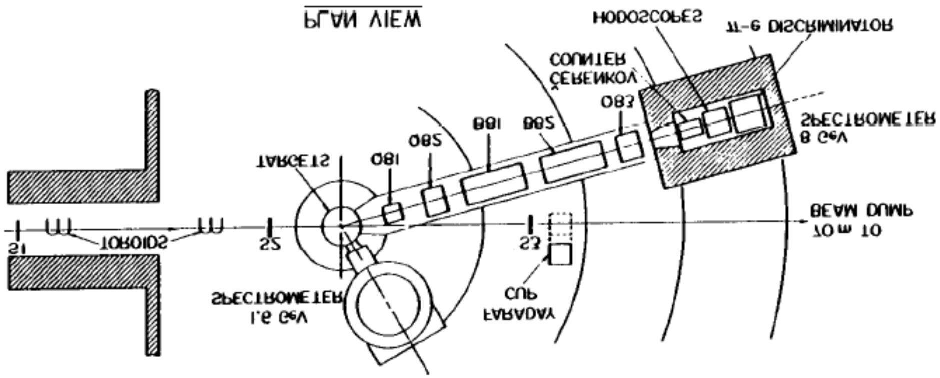
- 1990's: ep collider

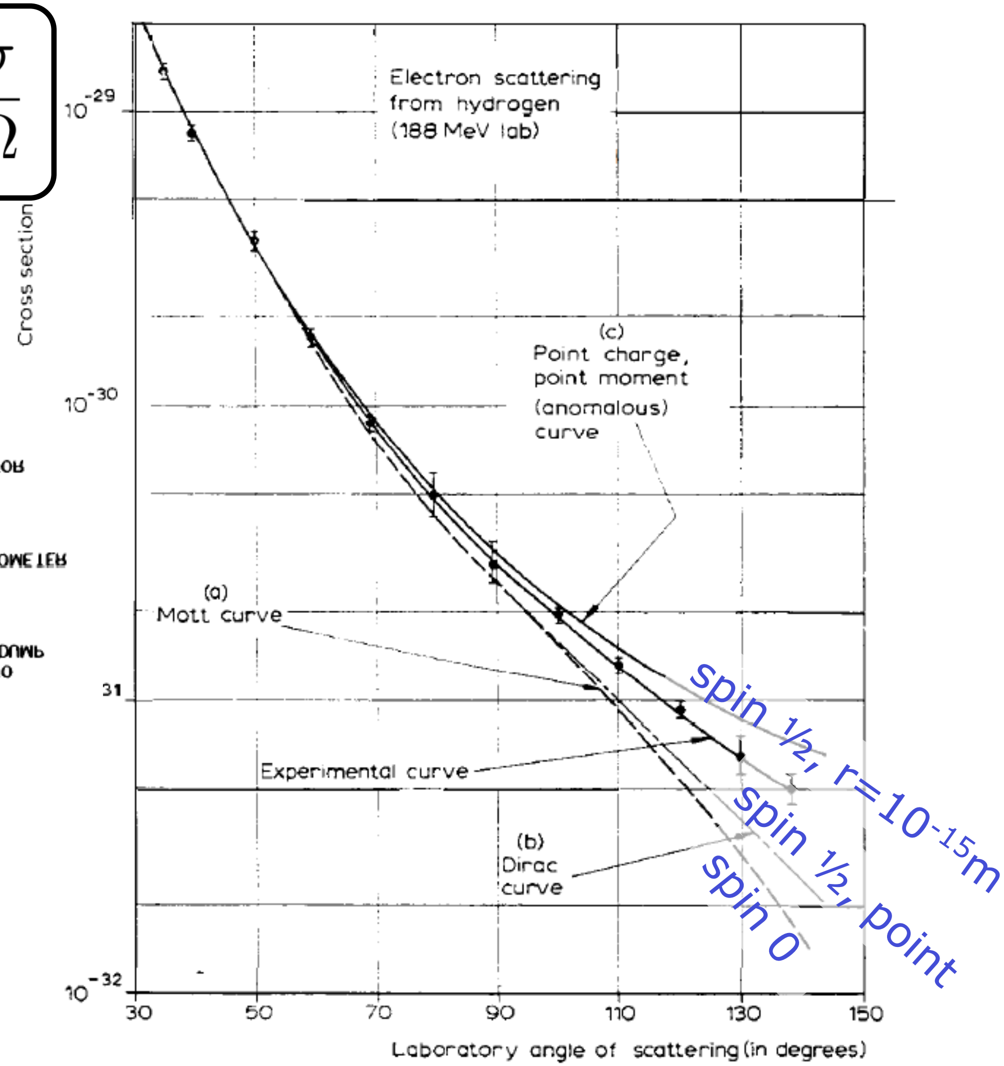






$$\frac{d\sigma}{d\Omega}$$



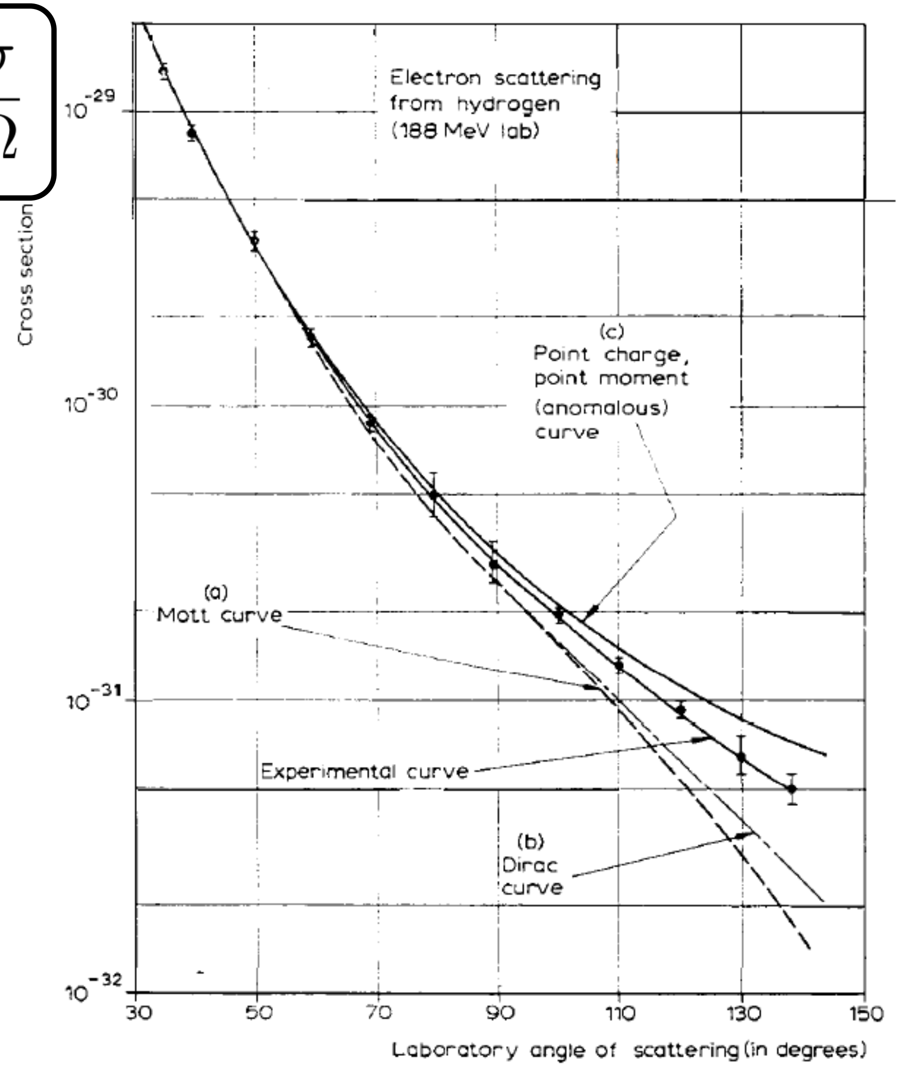


Robert Hofstadter

Fig. 9. Electron scattering from the proton at an incident energy of 188 MeV. *Curve (a)* shows the theoretical Mott curve for a spinless point proton. *Curve (b)* shows the theoretical curve for a point proton with a Dirac magnetic moment alone. *Curve* (c) shows the theoretical behavior of a point proton having the anomalous Pauli contribution in addition to the Dirac value of the magnetic moment. The deviation of the experimental curve from the Curve (c) represents the effect of form factors for the proton and indicates structure within the proton. The best fit in this figure indicates an rms radius close to $0.7 \cdot 10^{-13}$ cm.

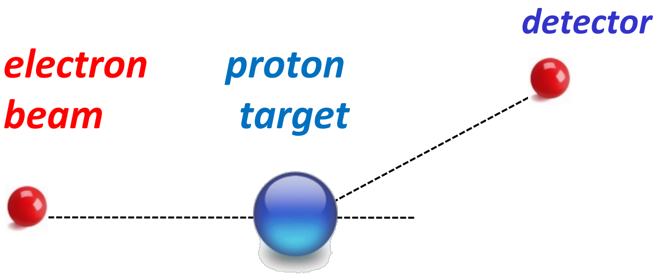
Sub-structure

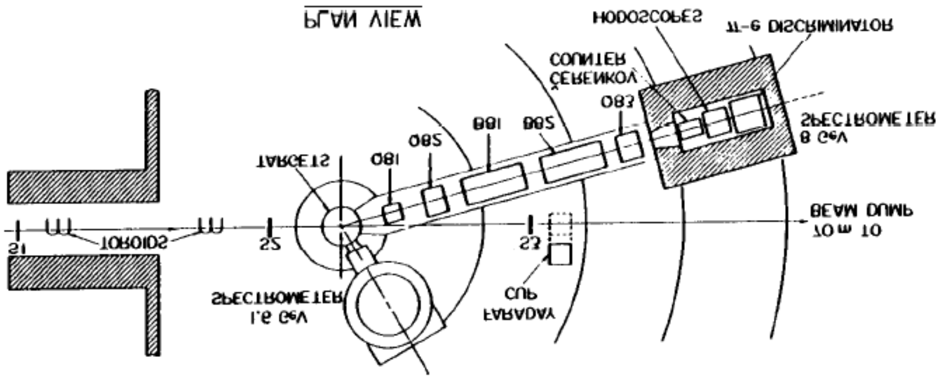
$$\frac{d\sigma}{d\Omega}$$

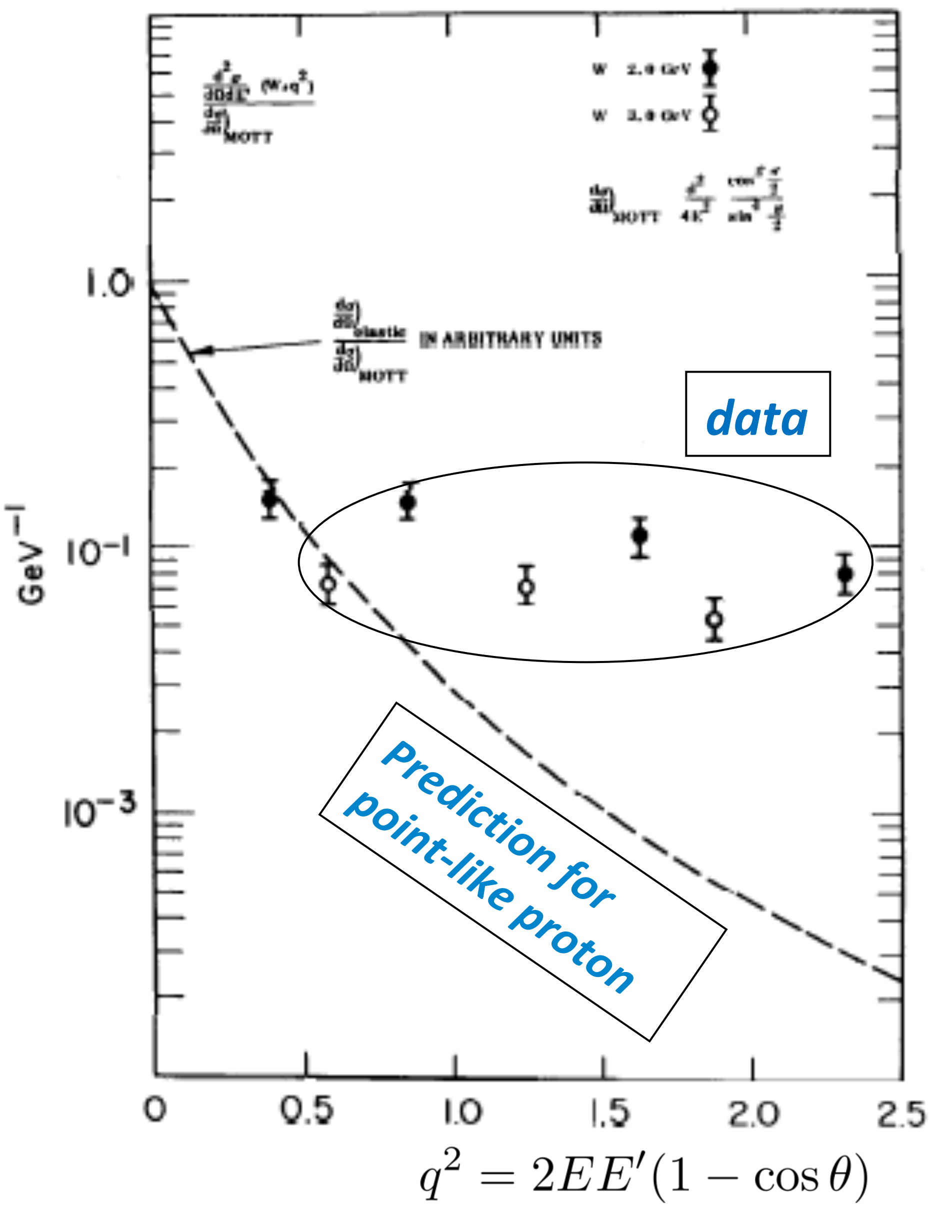


- Remember Rutherford
  - Back-scatter of α from nucleus
- Now:
  - Back-scatter of e from quarks

$$\frac{d\sigma}{d\Omega}$$







Scaling

J.D. Bjorken "scaling hypothesis" (1967):

- If scattering is caused by pointlike constituents structure functions must be independent of Q^2

➢ *Would you expect a Q^2 dependence?*

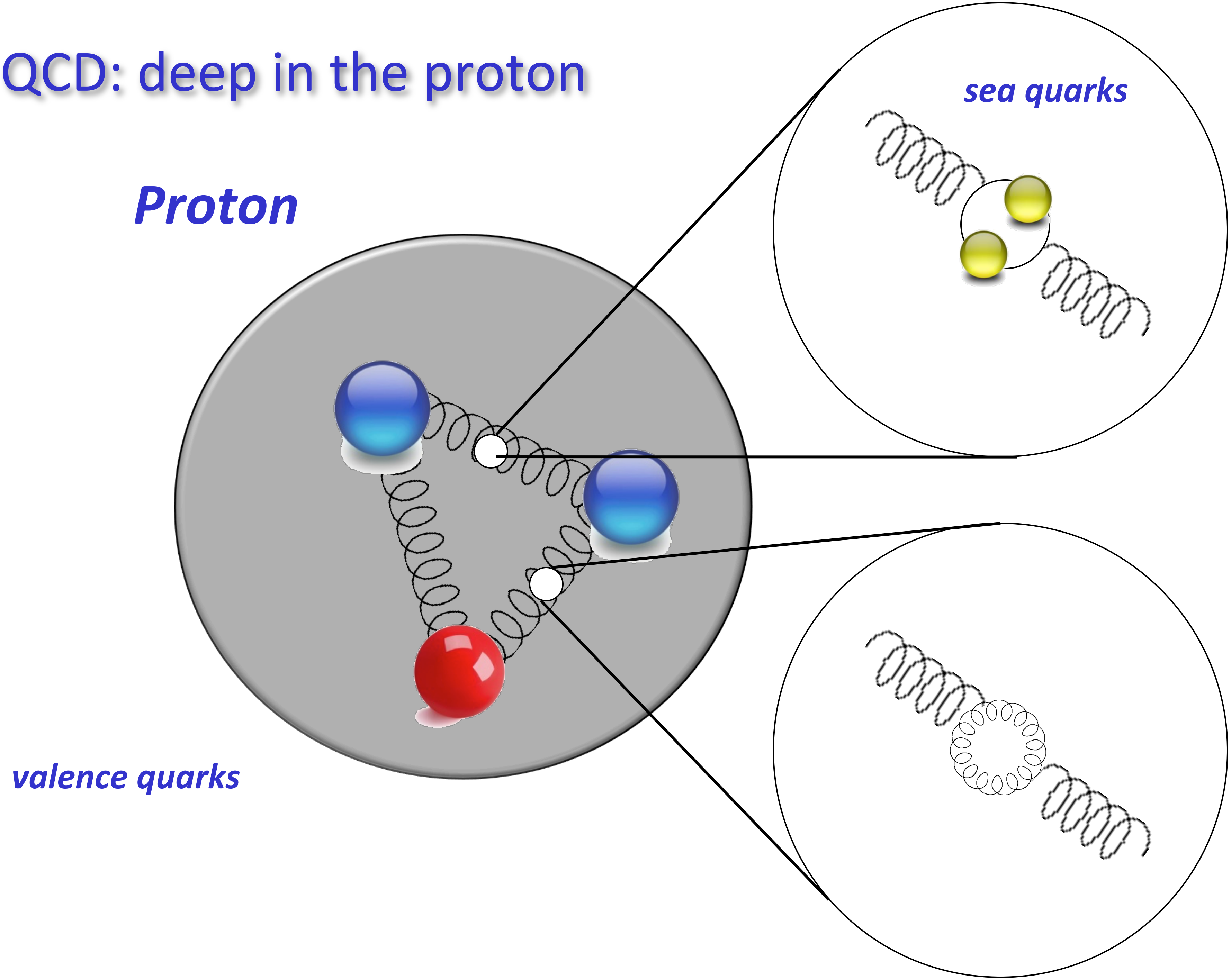
R. Feynmans "parton model" (1969):

- Proton consists of 'constituents'
- *"Physicists were reluctant to identify these objects with quarks at the time, instead calling them "partons" – a term coined by Richard Feynman."*

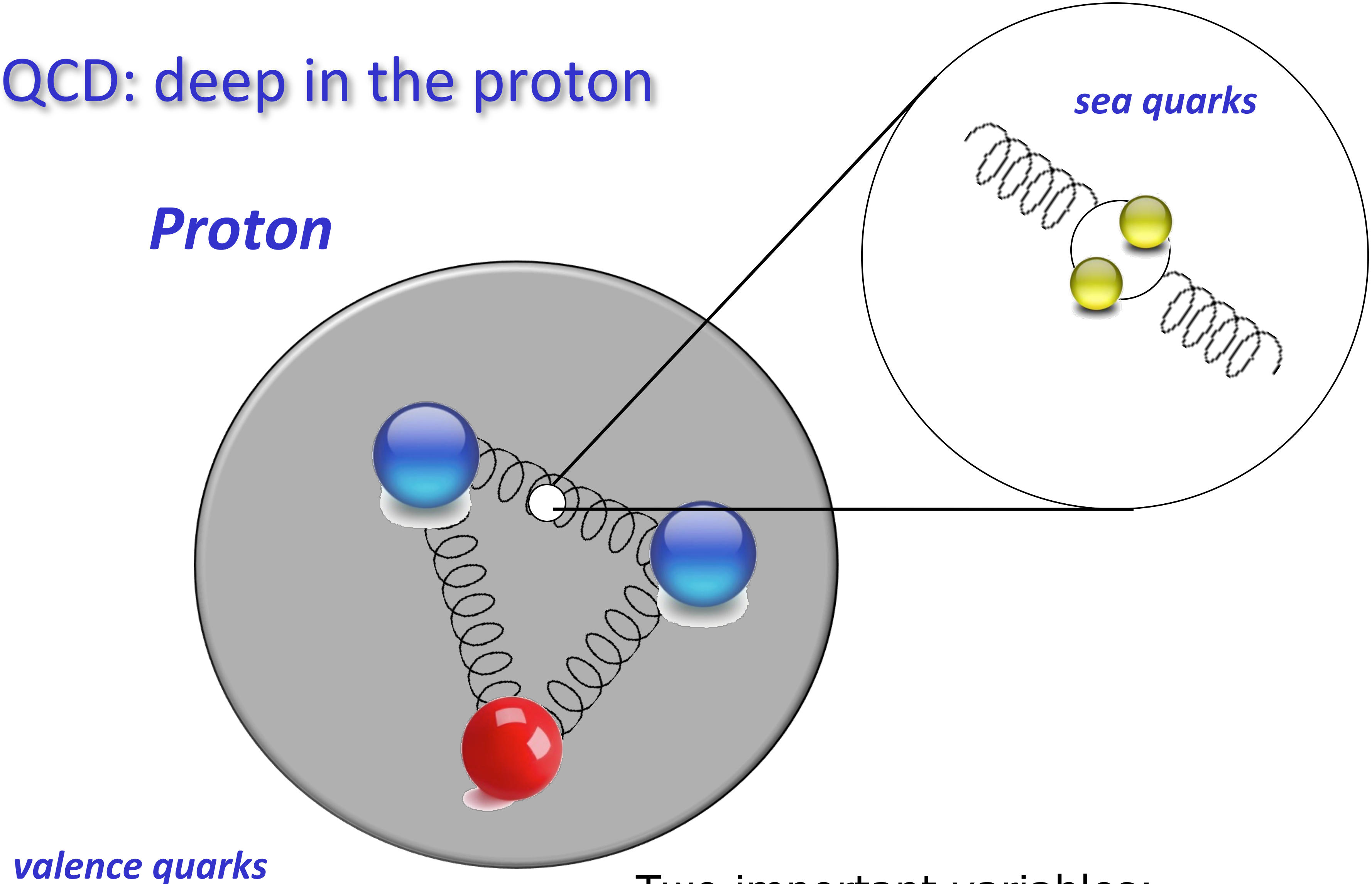
A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks"[6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

Figure 1.1: *Murray Gell-Mann suggested in 1964 that the proton consists of three "quarks"* [6] *[1].*

QCD: deep in the proton



QCD: deep in the proton



Two important variables:

- **Q^2**: 4-mom. transfer, scale
- **x**: fractional momentum of quark

Deep Inelastic Scattering

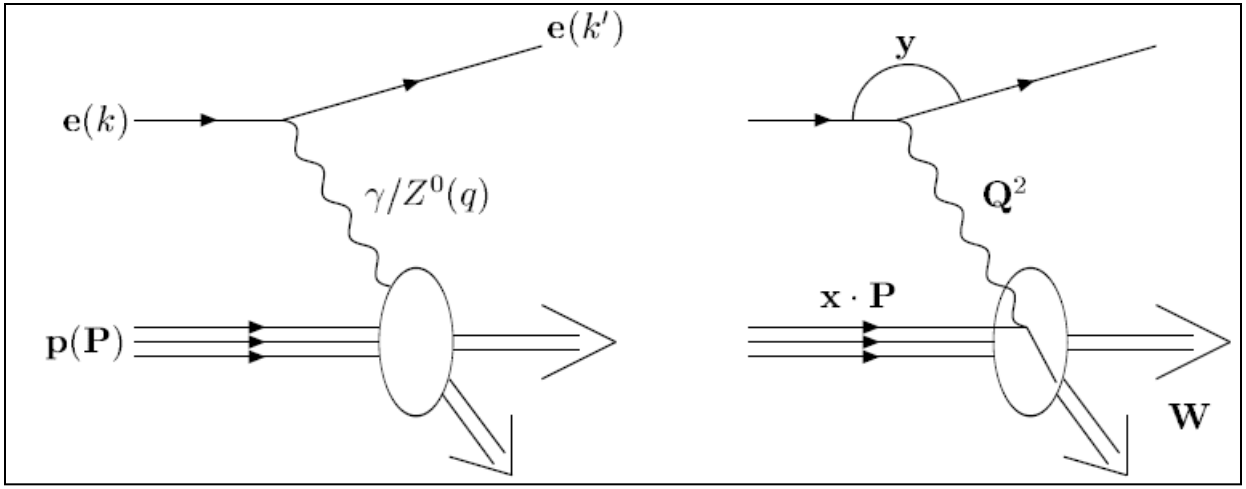
- eq scattering:

$$\frac{d\sigma^{eq\to eq}}{dQ^2} = \frac{2\pi\alpha^2}{Q^4} e_q^2 \left[2(1-y) + y^2\right]$$

- ep scattering:

$$\frac{d^2\sigma^{ep\to eX}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} (1 + (1-y)^2) F_2(x)$$

Form factor



Q^2	$\equiv$	$-q^2 = (k-k')^2$	: Virtuality of the photon
x	$\equiv$	$\frac{-q^2}{2P\cdot q}$	: 4-Momentum fraction carried by the struck quark
y	$\equiv$	$\frac{P\cdot q}{P\cdot k}$	: Inelasticity
W^2	$\equiv$	$(P+q)^2$	: Square of the invariant mass of the hadronic final state

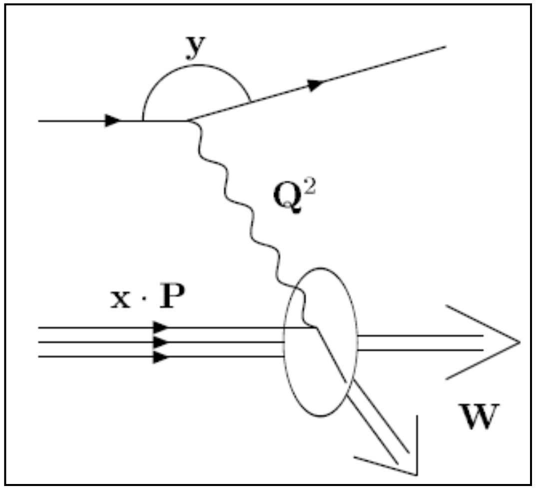
(From: PhD thesis N.Tuning)

Deep Inelastic Scattering

- ep scattering:

$$\frac{d^2\sigma^{ep\to eX}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}(1+(1-y)^2)F_2(x)$$

- $F_2(x)$: proton structure function
- $q(x)$: parton density function



$$F_2(x) = \sum_q e_q^2 (xq(x) + x\bar{q}(x))$$

Parton Densities

- ep scattering:

$$\frac{\mathrm{d}^2\sigma^{ep\to eX}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{2\pi\alpha^2}{xQ^4}(1+(1-y)^2)F_2(x)$$

- $F_2(x)$: proton structure function
- $q(x)$: parton density function

$$F_2(x) = \sum_q e_q^2(xq(x) + x\bar{q}(x))$$

- But… the proton had 3 quarks?!
- Sum rules:

$$\int_0^1 (u(x) - \bar{u}(x))\ \mathrm{d}x = 2;$$

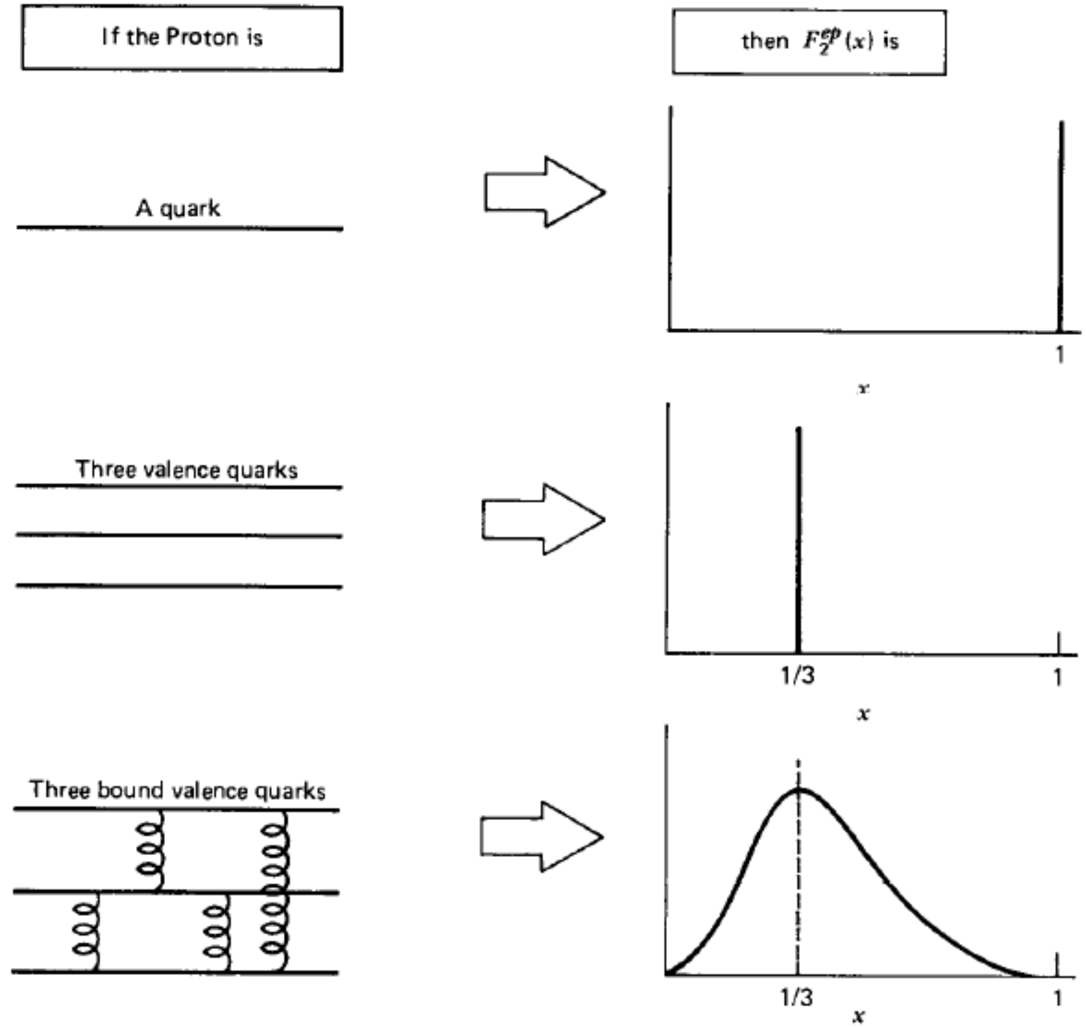
$$\int_0^1 (d(x) - \bar{d}(x))\ \mathrm{d}x = 1;$$

$$\int_0^1 (s(x) - \bar{s}(x))\ \mathrm{d}x = 0,$$

Proton: x

- What is 'momentum fraction' distribution of quarks??

- Quarks:
  - "Valence"
  - "Sea"



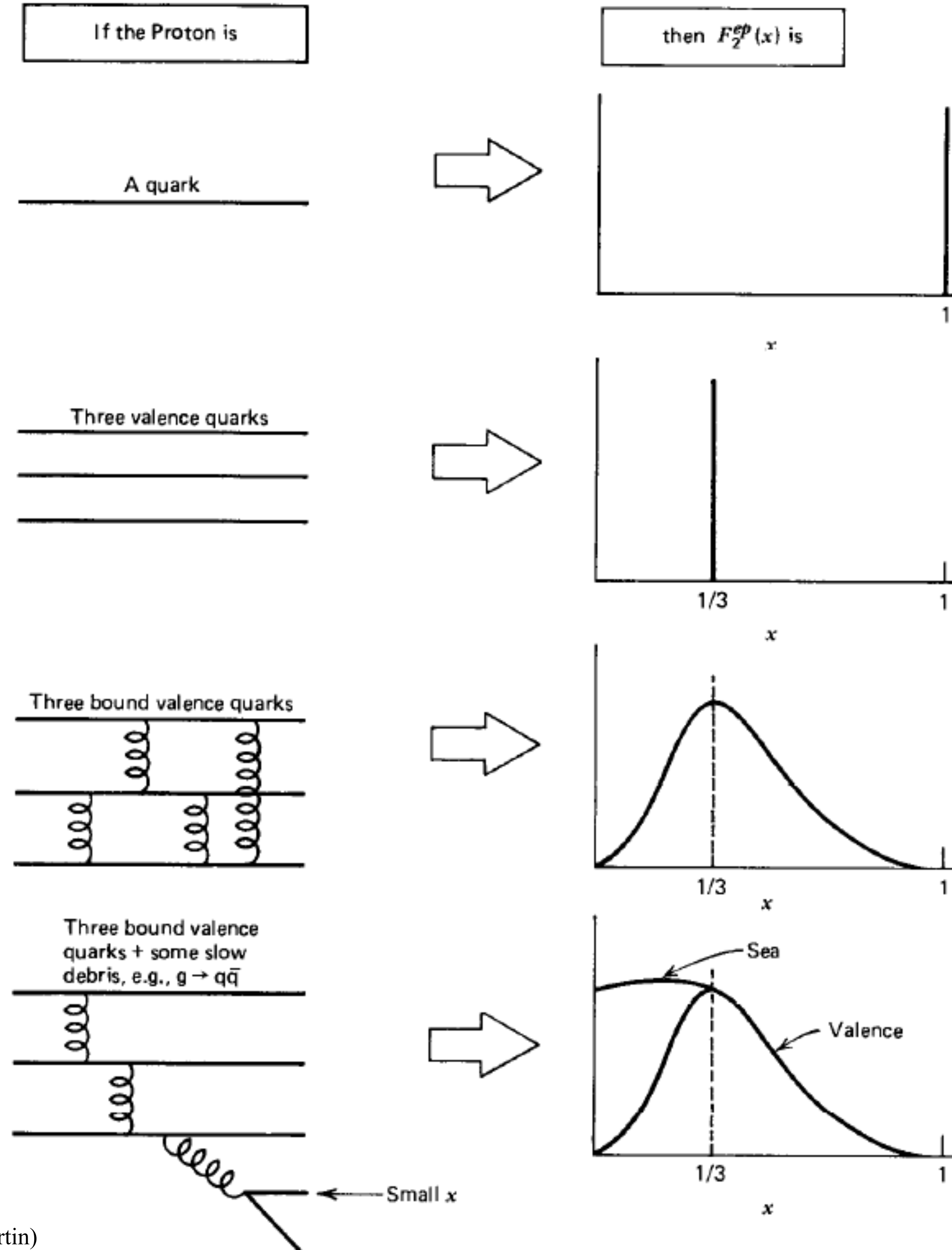
(From: Halzen & Martin)

Proton: x

- What is 'momentum fraction' distribution of quarks??

- Quarks:
  - "Valence"
  - "Sea"

- Dynamic, QCD !



(From: Halzen & Martin)

Proton: x

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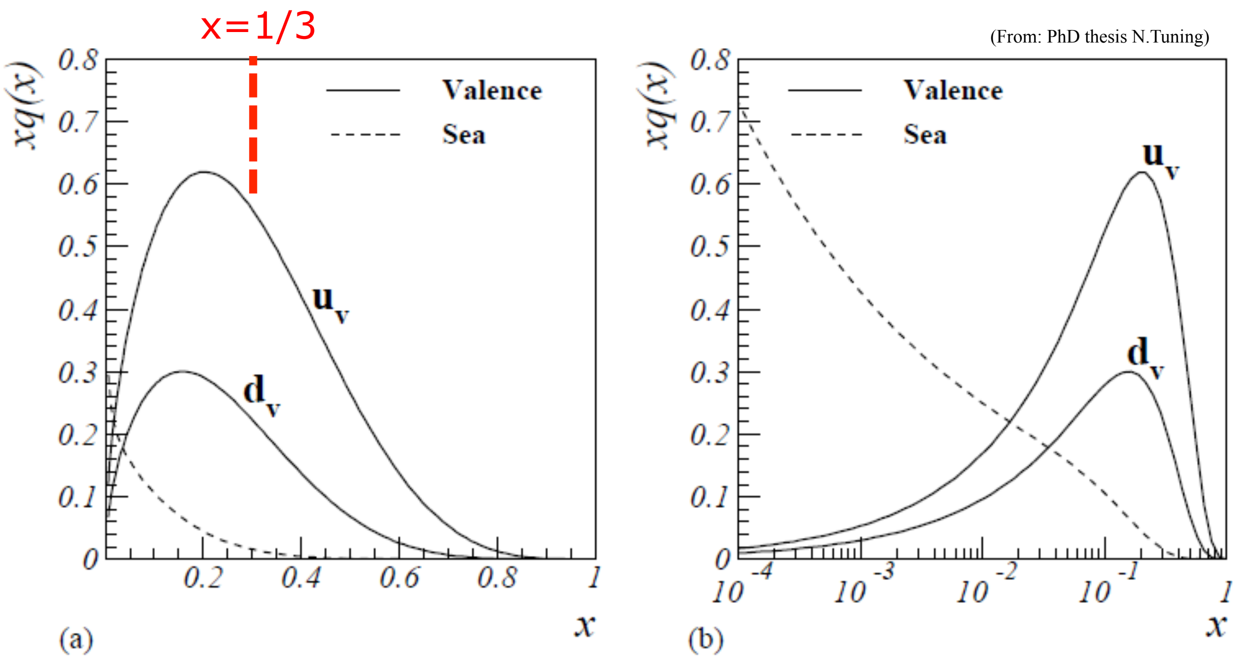
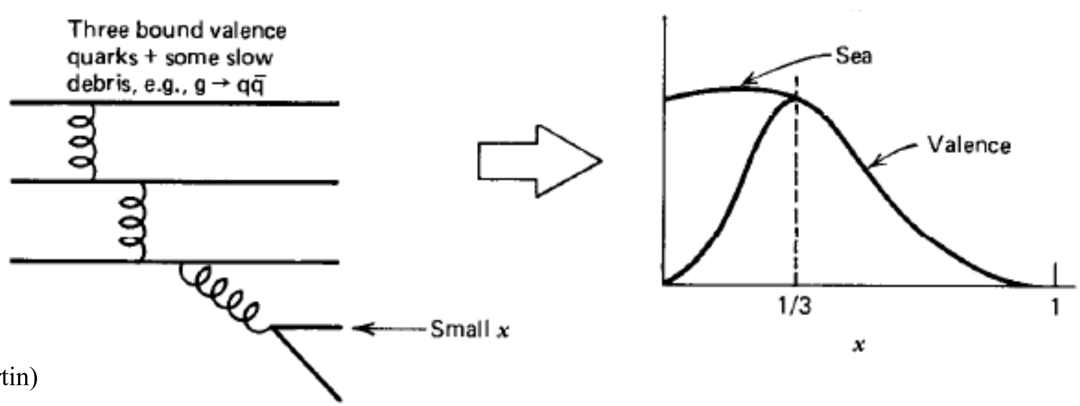
(From: PhD thesis N.Tuning)

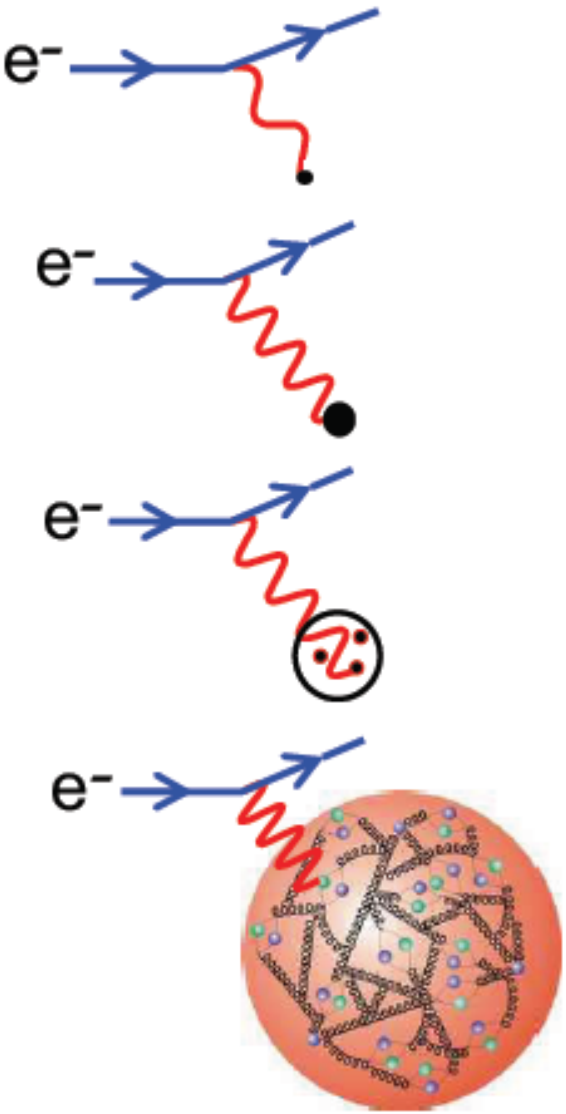




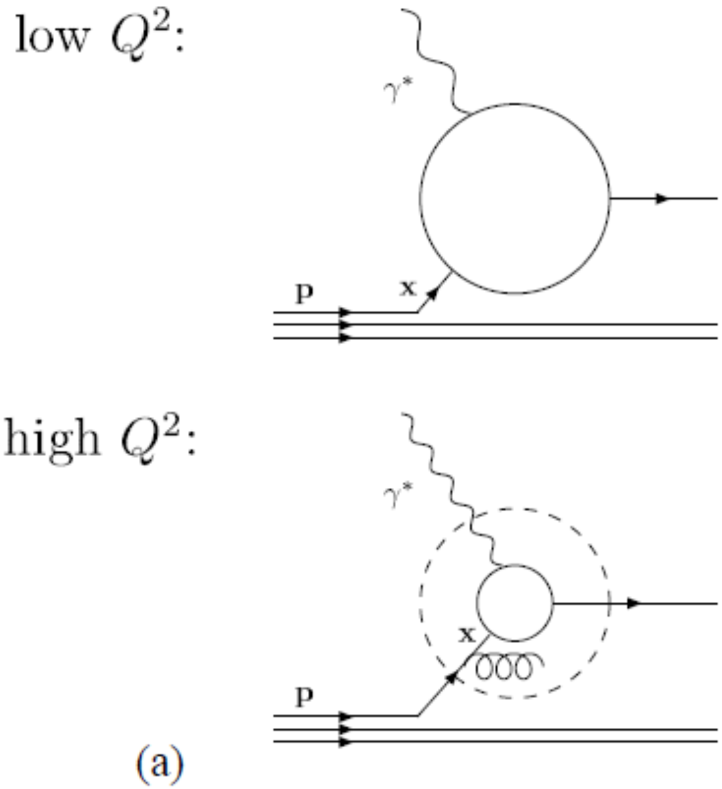
(From: Halzen & Martin)

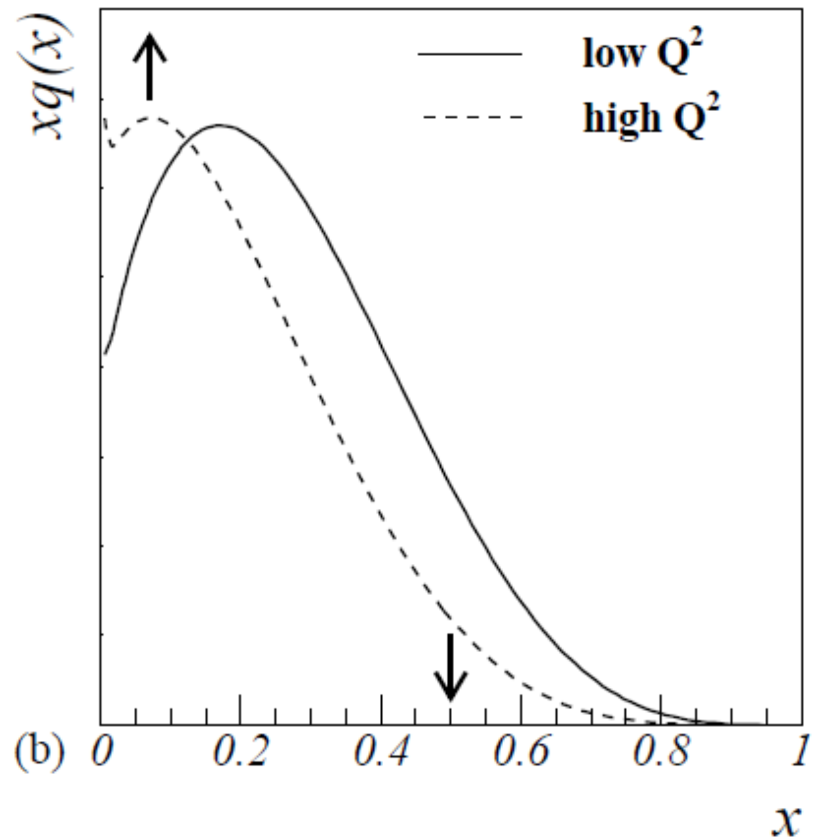
Proton: Q^2

- The “deeper” one looks into the proton, the more quarks and gluons



Proton: x, Q^2

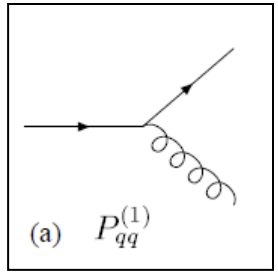




(From: PhD thesis N.Tuning)

Proton: x, Q^2

- The "deeper" one looks into the proton, the more quarks and gluons
- "QCD evolution"
- Describes quark-gluon splitting



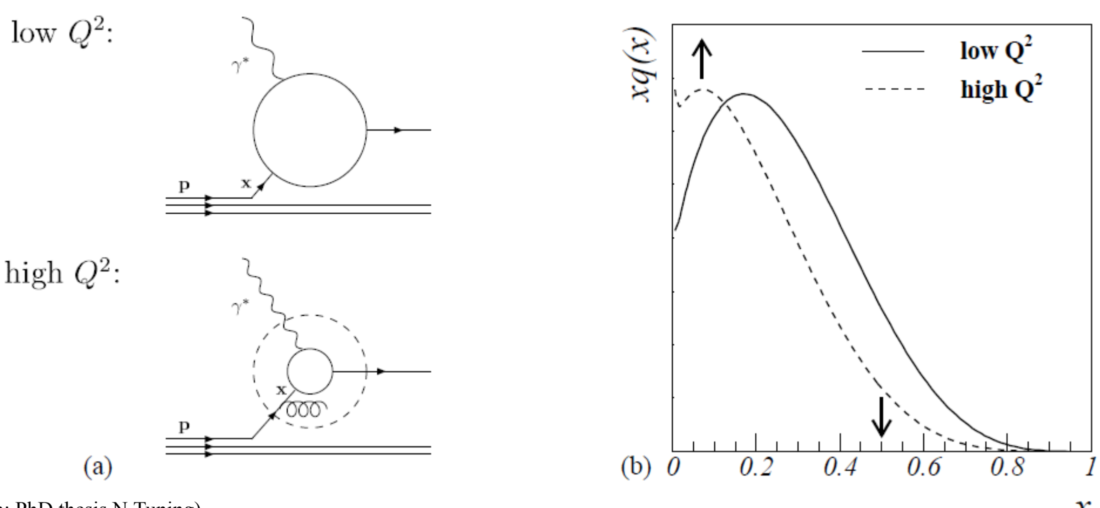
$$\frac{d\sigma^{\gamma^* q \to qg}}{dp_T^2} = \frac{4\pi\alpha^2}{s} e_q^2 \frac{1}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z)$$

$$\sigma^{\gamma^* q \to qg} = \frac{4\pi\alpha^2}{s} e_q^2 \frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2}$$

➢ DGLAP evolution eqs:

$$\frac{dq(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left(q(y,Q^2) P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2) P_{qg}\left(\frac{x}{y}\right) \right)$$

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left(\sum_q q(y,Q^2) P_{gq}\left(\frac{x}{y}\right) + g(y,Q^2) P_{gg}\left(\frac{x}{y}\right) \right)$$



(From: PhD thesis N.Tuning)

André David
@DrAndreDavid

At high-enough energies, protons are gggugggggggggggggggggggggggggggggggggggggggggg ggggggggggggggggggggggggggggggggggggggguggggg gggggggggggggggggggggggggggggggggggggggggggggg ggggggggggggggggggggggggggggggggggggggggdggggg gggggggggggggggggggggggggggggggggggggggggggggg ddggggggggggggggggggggggssgg

> **r/particlephysics** @rparticles · May 10
> Can I write proton as udu instead of uud? If not then why? dlvr.it/RWLqwY

7:59 AM · May 10, 2020 · Twitter for iPhone

Scaling violations

J.D. Bjorken "scaling hypothesis" (1967):

- If scattering is caused by pointlike constituents structure functions must be independent of Q^2
- *Would you expect a Q^2 dependence?*

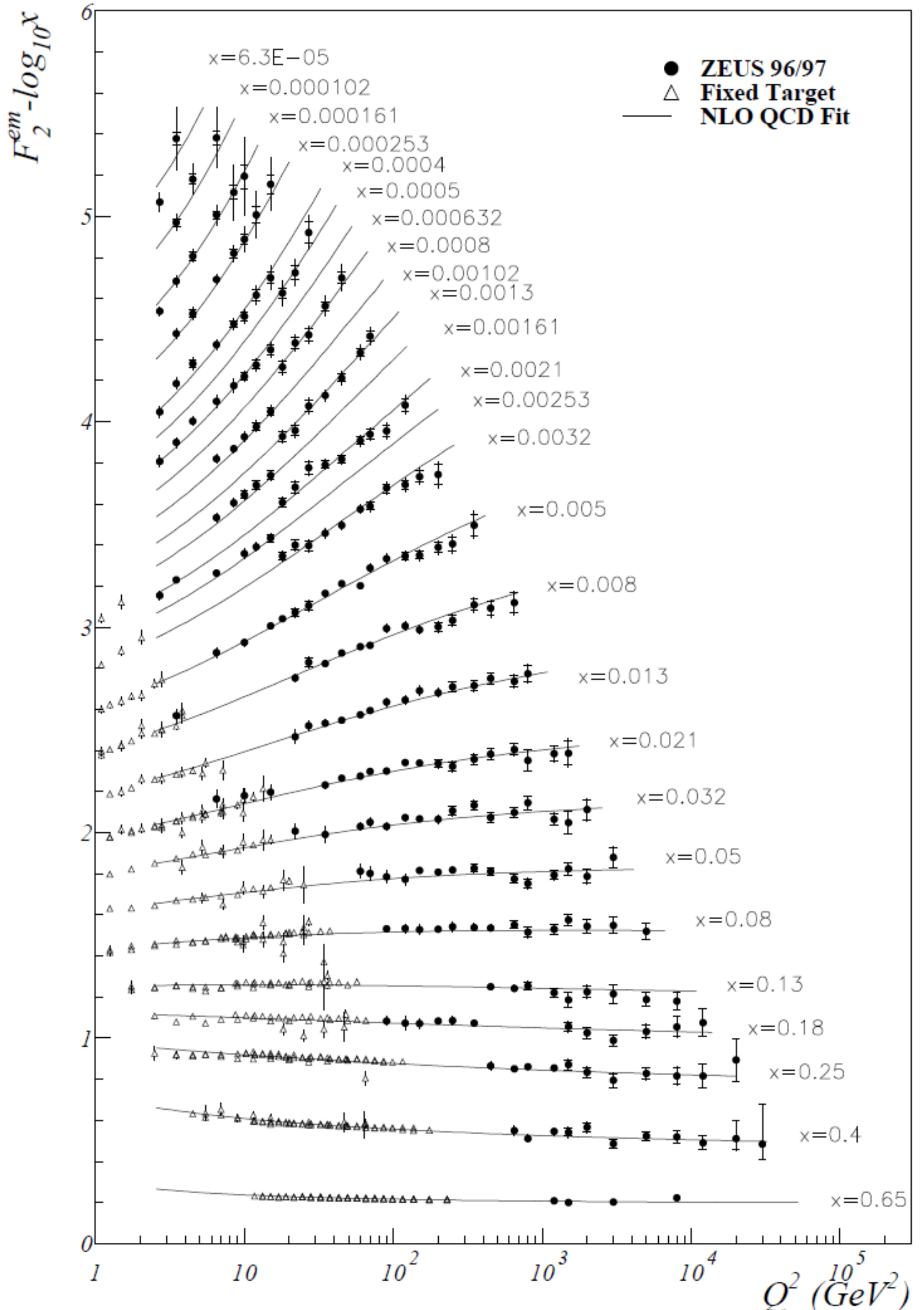
➢ Yes, due to QCD, ie. quark/gluon splitting !

- Matured in mid '70s
- The proton is "dynamic" !

➢ Measurement of $F_2(x,Q^2)$ very accurate test of QCD

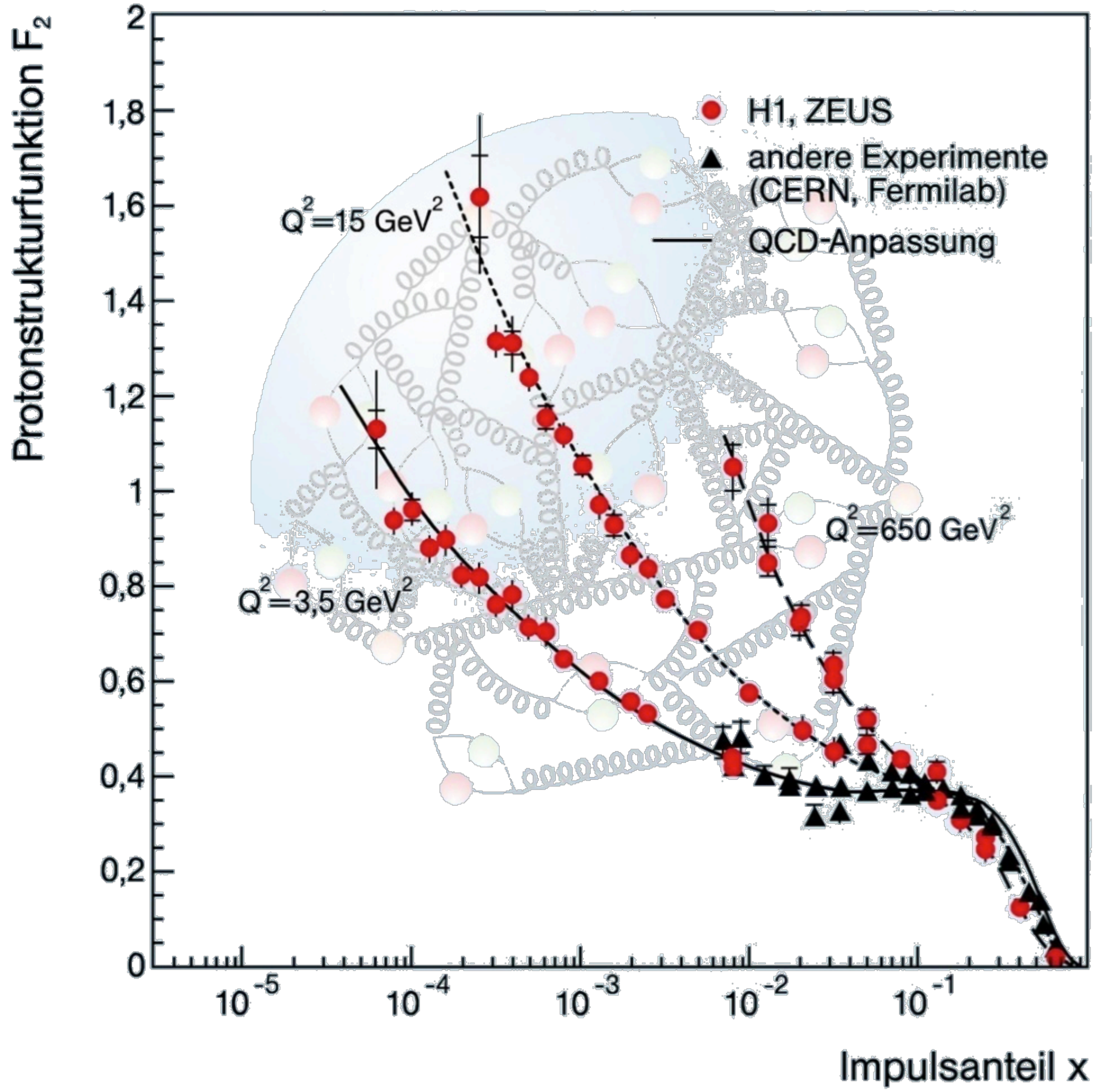
Scaling violations

- Measurement of $F_2(x,Q^2)$ very accurate test of QCD



Proton Structure

- The deeper you look, the more low-x quarks



Proton Structure

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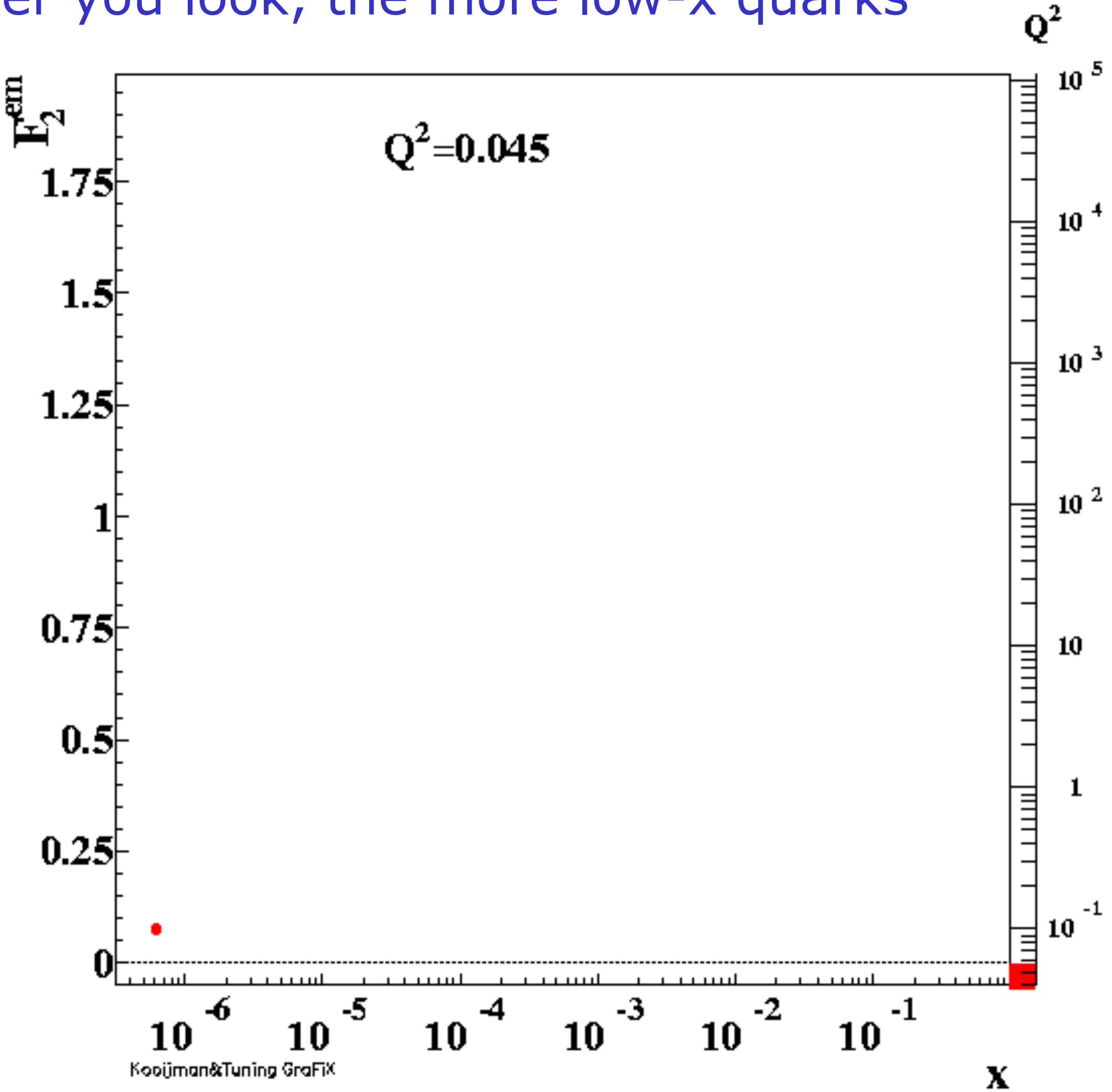
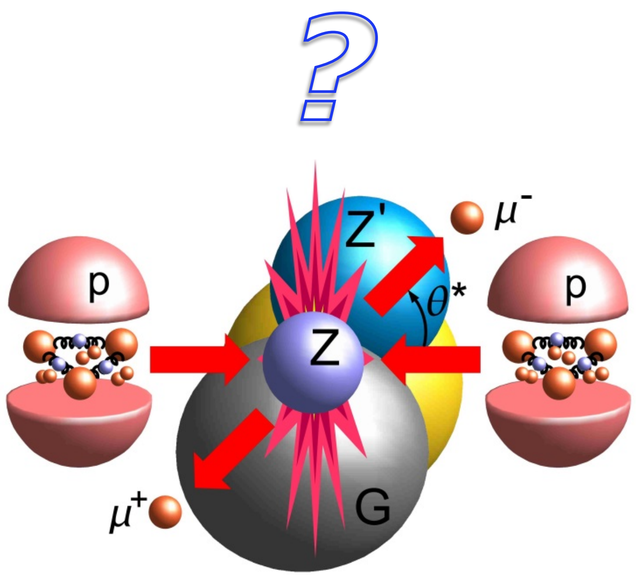
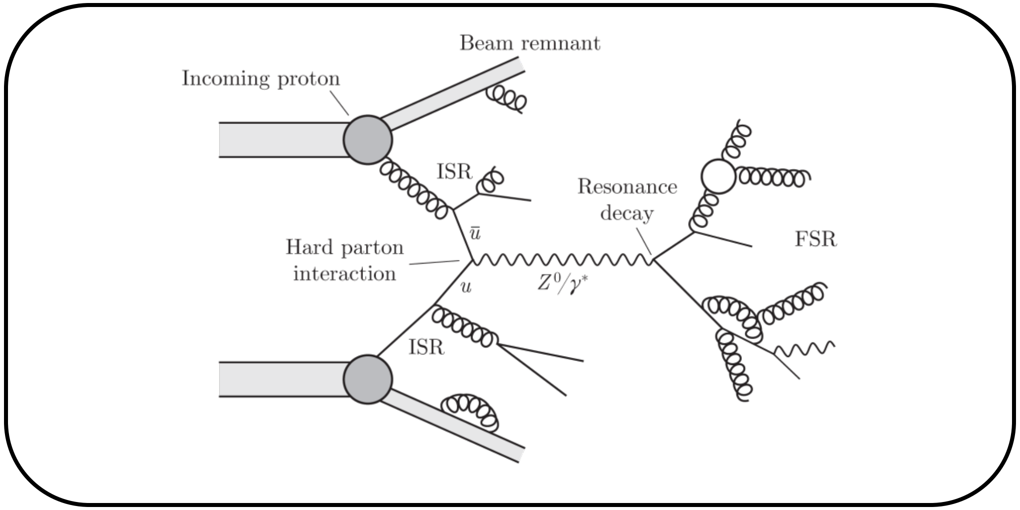
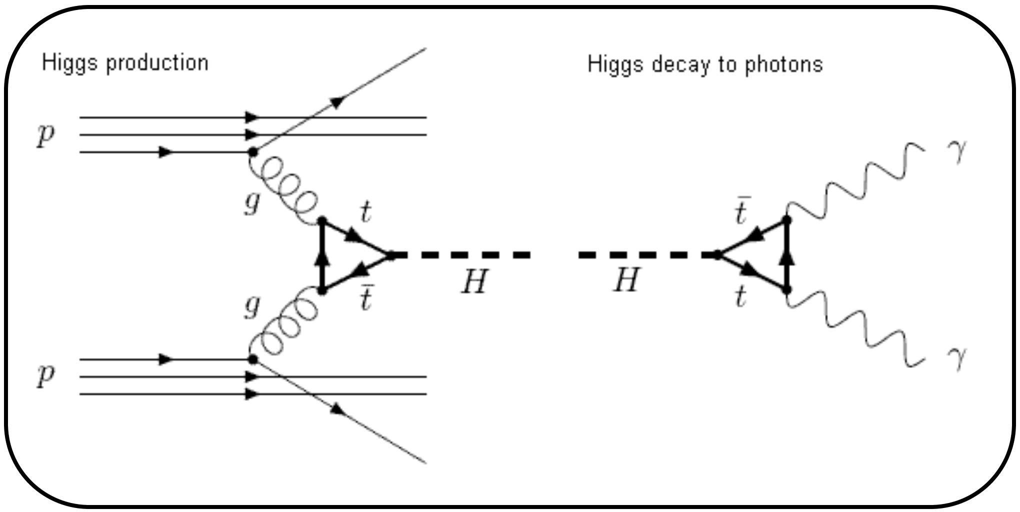


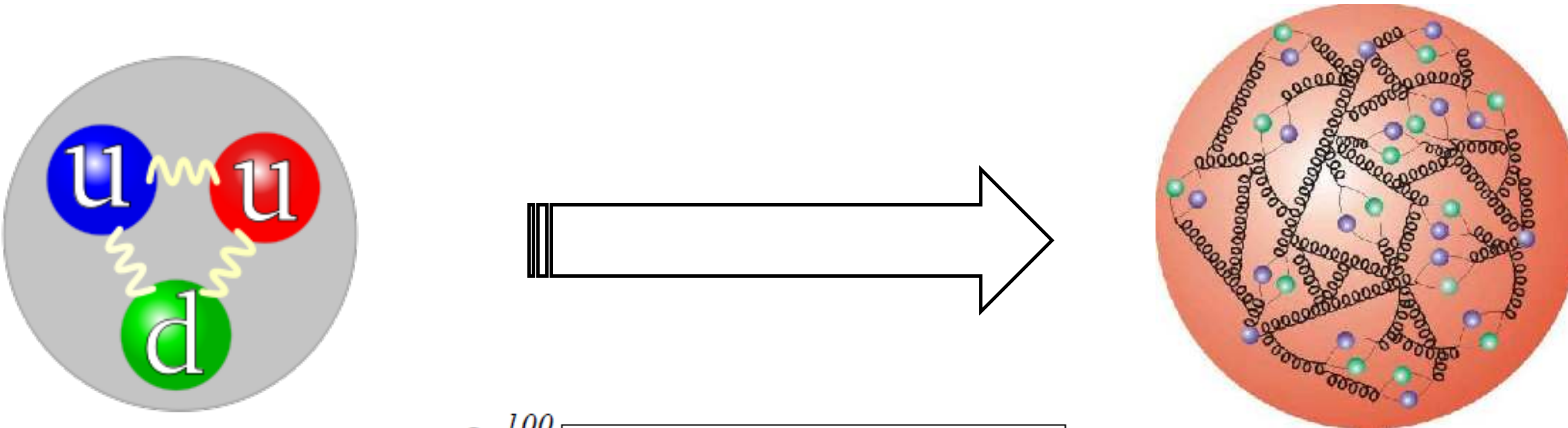
Proton Structure: knowledge needed for predictions

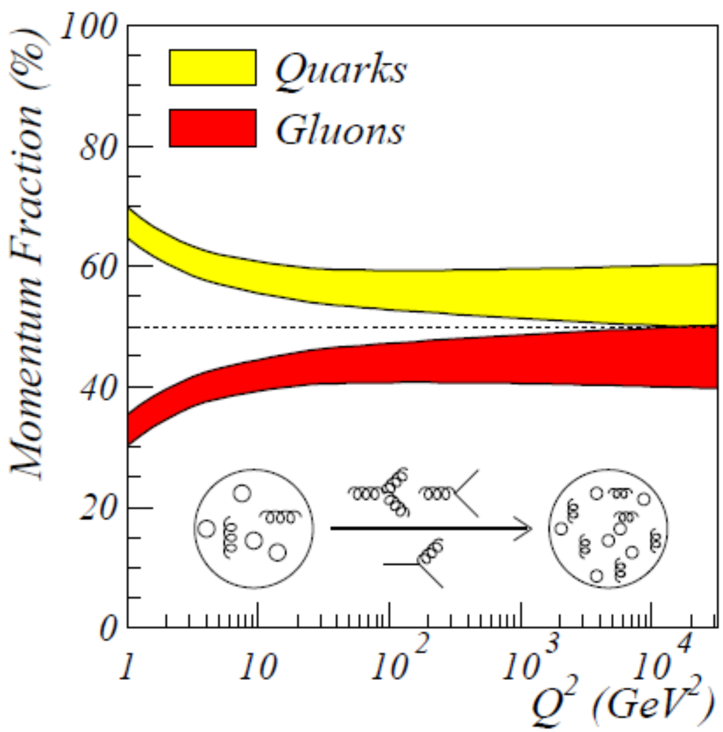






Proton Structure





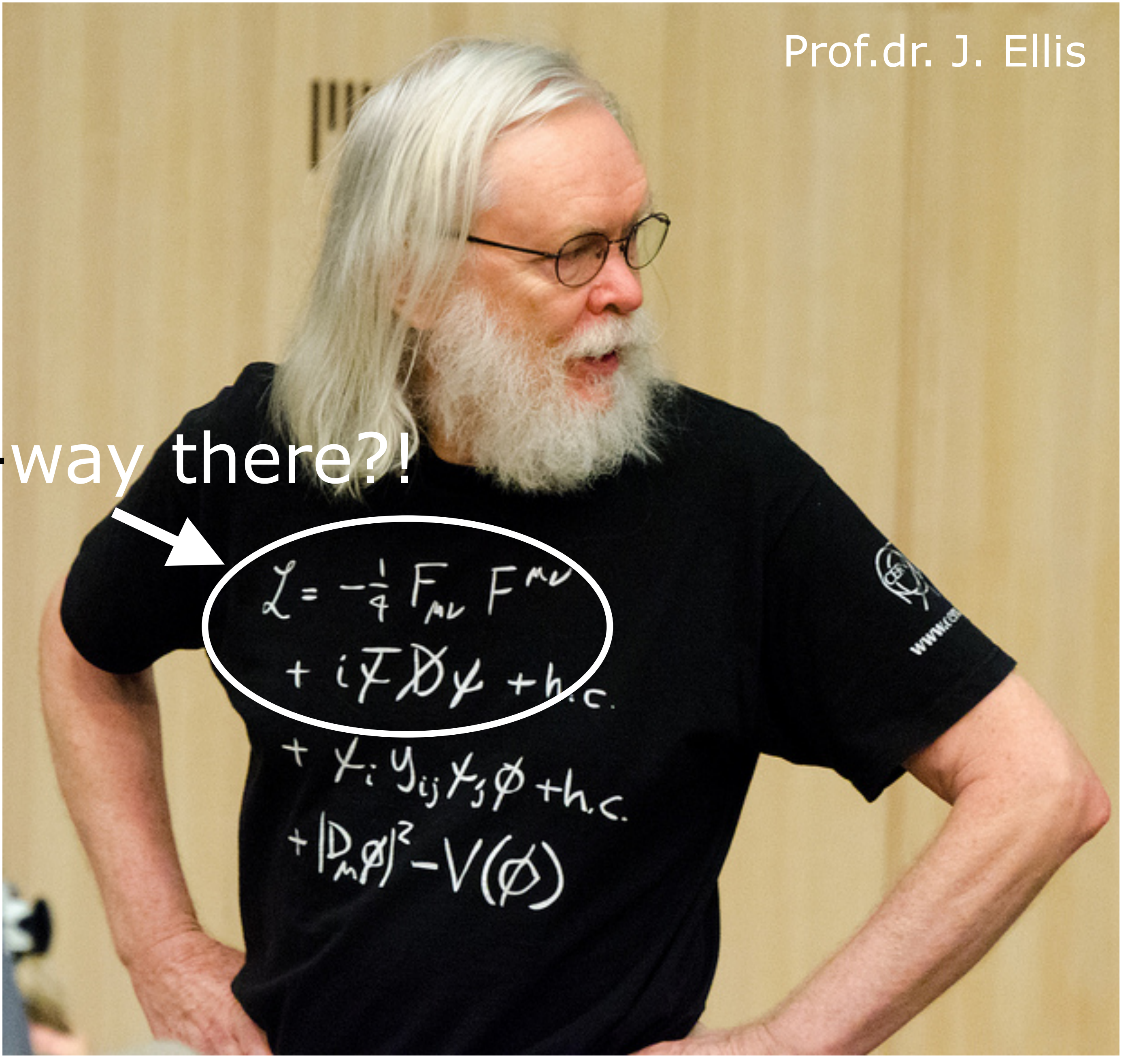
Standard Model

$$\mathcal{L} = \overline{\psi}\left(i\gamma_{\mu}D^{\mu} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Todo-list:

- No masses for W, Z !?
  - (LHC/ATLAS) Higgs mechanism, Yukawa couplings
- Interactions between the three families !?
  - (LHC/LHCb) CKM-mechanism, CP violation

Half-way there?!



Higgs mechanism

Higgs mechanism

- Let's give the *photon* a mass!
  - Not realized in Nature
  - But is a simpler example

Higgs mechanism

- Let's give the *photon* a mass!

- Introduce a complex scalar field:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right) - V(\phi)$$

  - with:

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

  - and the Lagrangian is invariant under:

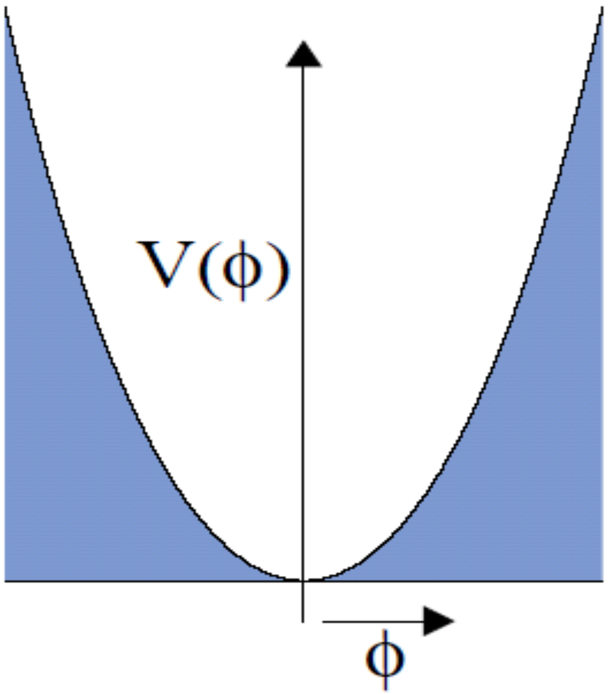
$$A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu\eta(x),$$
$$\phi(x) \rightarrow e^{ie\eta(x)}\phi(x).$$

Scalar potential V(φ)

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\mu^2 < 0:$$

- Question: what is on the x- and y-axis…?

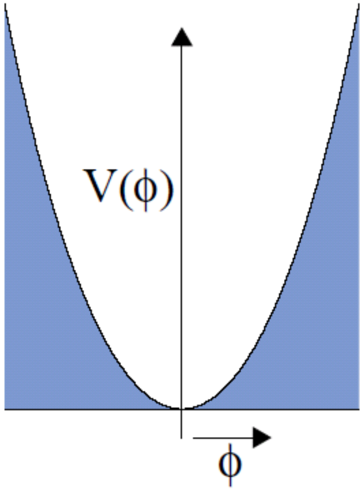


Scalar potential V(φ)

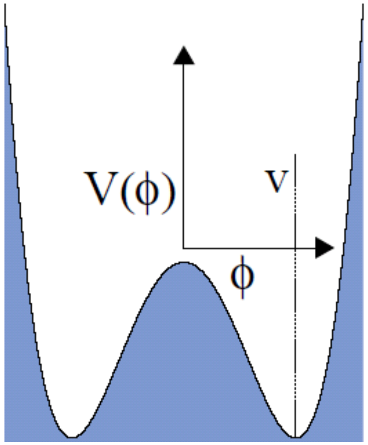
What if $\mu^2 > 0$??

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$\mu^2 < 0:$



$\mu^2 > 0:$



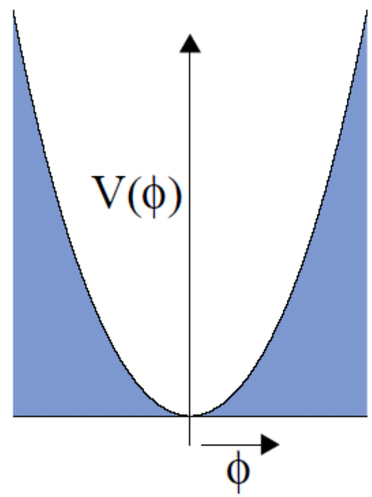
Scalar potential V(φ)

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

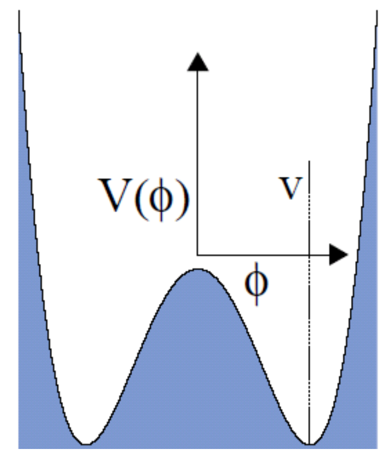
If $\mu^2 > 0$:

- φ will acquire a vaccum expectation value v,
- "spontaneously" !
- System not any more
  "spherical" symmetric

$$\mu^2 < 0:$$



$$\mu^2 > 0:$$



vev

SSB

➢ Spontaneous Symmetry Breaking

$$\langle\phi\rangle = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

Complex scalar field φ

If $\mu^2 > 0$:

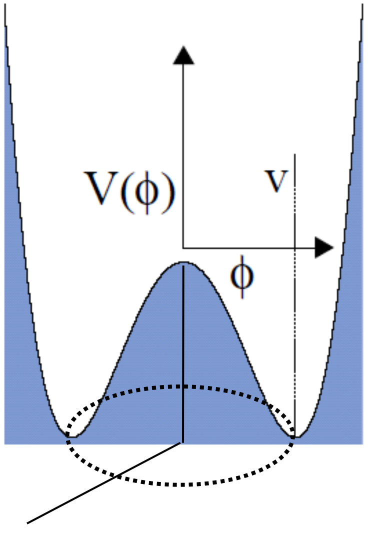
- φ will acquire a vaccum expectation value v

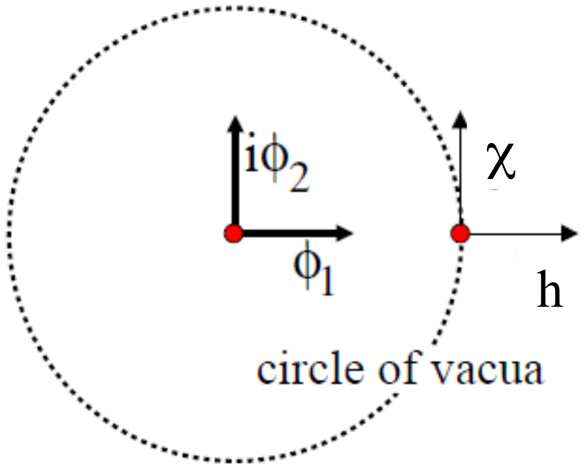
- Parameterize φ as:
  - h: Higgs boson
  - χ: Goldstone boson
  - Both *real* scalar fields

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\phi = \frac{v+h}{\sqrt{2}} e^{i\chi/v}$$

$$\mu^2 > 0:$$





$$\langle\phi\rangle = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

Higgs mechanism

- Let's give the photon a mass!

- Introduce a complex scalar field:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger (D^\mu\phi) - V(\phi)$$

  - with: $V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$

  - and: $\phi = \frac{v+h}{\sqrt{2}} e^{i\chi/v}$

➢ Then:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - evA_\mu\partial^\mu\chi + \frac{e^2v^2}{2}A_\mu A^\mu + \frac{1}{2}\left(\partial_\mu h\partial^\mu h - 2\mu^2h^2\right) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + (h,\chi)\,\mathrm{int.}$$

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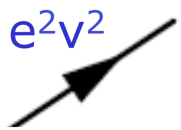
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - evA_\mu \partial^\mu \chi + \frac{e^2 v^2}{2} A_\mu A^\mu + \frac{1}{2}\left(\partial_\mu h \partial^\mu h - 2\mu^2 h^2\right) + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + (h,\chi)\,\text{int.}$$

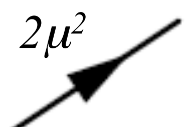
Higgs mechanism

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - evA_\mu\partial^\mu\chi + \frac{e^2v^2}{2}A_\mu A^\mu + \frac{1}{2}\left(\partial_\mu h\partial^\mu h - 2\mu^2h^2\right) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + (h,\chi)\text{ int.}$$

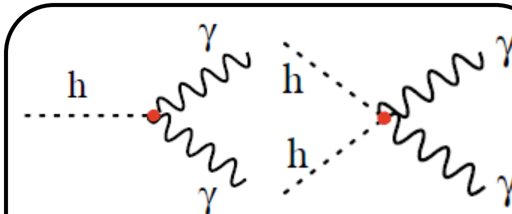
Photon field A^μ



Photon A with mass e^2v^2



Higgs h with mass $2\mu^2$

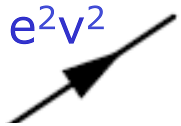


Interactions

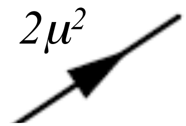
Higgs mechanism

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - evA_\mu\partial^\mu\chi + \frac{e^2v^2}{2}A_\mu A^\mu + \frac{1}{2}\left(\partial_\mu h\partial^\mu h - 2\mu^2h^2\right) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + (h,\chi)\,\text{int.}$$

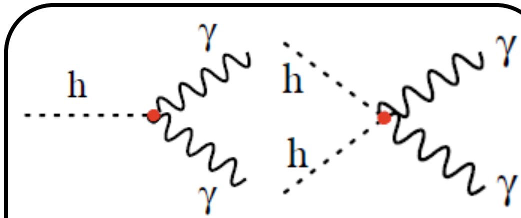
Photon field A^μ



Photon A with mass e^2v^2



Higgs h with mass $2\mu^2$



Interactions

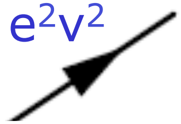
- What about this field χ ?

Higgs mechanism

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad + \frac{e^2v^2}{2}A_\mu A^\mu + \frac{1}{2}\left(\partial_\mu h \partial^\mu h - 2\mu^2 h^2\right) + \qquad (h \quad) \text{ int.}$$

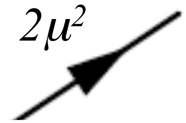
Photon field A^μ

e^2v^2

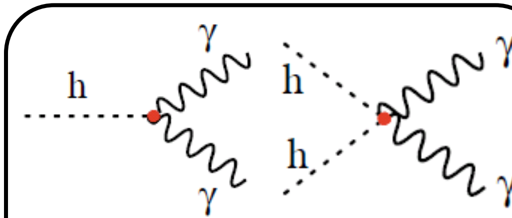


Photon A with mass e^2v^2

$2\mu^2$



Higgs h with mass $2\mu^2$



Interactions

- Unitary gauge:

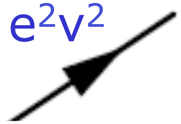
$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{ev}\partial_\mu \chi$$

➢ Goldstone boson has been “eaten” by the photon mass

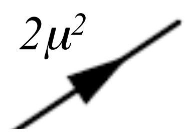
Higgs mechanism

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad + \frac{e^2v^2}{2}A_\mu A^\mu + \frac{1}{2}\left(\partial_\mu h \partial^\mu h - 2\mu^2 h^2\right) + \qquad (h \quad) \text{ int.}$$

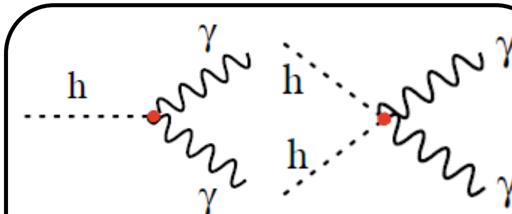
Photon field A^μ



Photon A with mass e^2v^2



Higgs h with mass $2\mu^2$



Interactions

- Unitary gauge:

$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{ev}\partial_\mu \chi$$

- Degrees of freedom
  - Before: massless photon: 2, complex scalar field φ: 2 → Total: 4
  - After: massive photon: 3, one real scalar field h: 1 → Total: 4

➢ Goldstone boson has been "eaten" by the photon mass

Higgs mechanism

- Let's give the *photon* a mass?
  - Not realized in Nature

Higgs mechanism in the Standard Model

- Let's give the W,Z a mass!
- Introduce a doublet of complex scalar fields:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\mathcal{L}_{Higgs} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi)$$

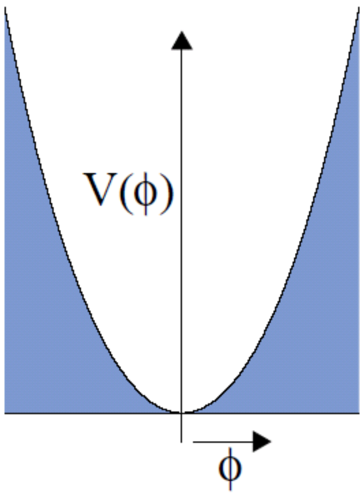
$$D_\mu \phi = \left(\partial_\mu + ig\, T^i W^i_\mu + i \frac{1}{2} g' B_\mu \right) \phi$$

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi \right)^2$$

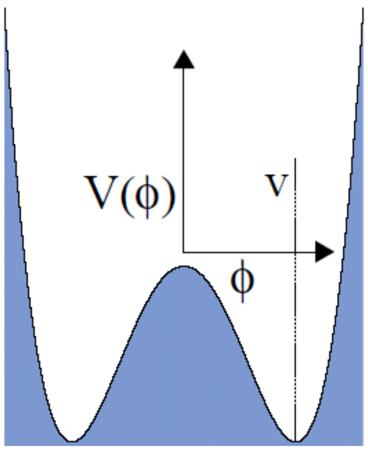
Spontaneous symmetry breaking

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \Rightarrow \quad \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$\mu^2 < 0:$



$\mu^2 > 0:$



Spontaneous symmetry breaking

$$
\begin{aligned}
(D^\mu\phi)^\dagger (D_\mu\phi) &= \left| \left(\partial_\mu + \frac{i}{2} g \tau^k W_\mu^k + \frac{i}{2} g' B_\mu\right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\
&= \frac{v^2}{8} \left| \left(g \tau^k W_\mu^k + g' B_\mu\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2 \\
&= \frac{v^2}{8} \left| \begin{pmatrix} g W_\mu^1 - i g W_\mu^2 \\ -g W_\mu^3 + g' B_\mu \end{pmatrix} \right|^2 \\
&= \frac{v^2}{8} \left[g^2 \left((W_\mu^1)^2 + (W_\mu^2)^2 \right) + (g W_\mu^3 - g' B_\mu)^2 \right]
\end{aligned}
$$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

τ^k: Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- ➢ Mass terms!
- How about the physical fields?

Rewriting in terms of physical gauge bosons

$$\underbrace{W_1 \quad W_2}_{\text{W}^+ \text{ and W}^- \text{ bosons}} \qquad \underbrace{W_3 \quad B}_{\text{Z-boson and } \gamma}$$

1) W_1, W_2 :

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right)$$

2) W_3, B:

$$(-gW_3 + g' \quad B_\mu)^2$$

- Let's do a 'trick' and 'rotate' the W_3 and B fields to get the Z and A fields

Rewriting in terms of physical gauge bosons

$$\underbrace{W_1 \quad W_2}_{\text{W}^+ \text{ and W}^- \text{ bosons}} \qquad \underbrace{W_3 \quad B}_{\text{Z-boson and } \gamma}$$

1) W_1, W_2 :

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$$

2) W_3, B:

$$(-gW_3 + g' \quad B_\mu)^2 = (W_3, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_3 \\ B_\mu \end{pmatrix}$$

Diagonalize the matrix $\Leftrightarrow$ find eigenstates + eigenvalues

$$(-gW_3 + g' \quad B_\mu)^2 = (g^2 + g'^2)Z_\mu^2 + 0 \cdot A_\mu^2$$

Rewriting in terms of physical gauge bosons

$$\underbrace{W_1 \quad W_2}_{\text{W}^+ \text{ and W}^- \text{ bosons}} \qquad \underbrace{W_3 \quad B}_{\text{Z-boson and } \gamma}$$

1) W_1, W_2 : $$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$$

2) W_3, B: $$(-gW_3 + g' \quad B_\mu)^2 = (W_3, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_3 \\ B_\mu \end{pmatrix}$$

eigenvalue *eigenvector*

$$\lambda = 0 \quad \rightarrow \quad \frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} g' \\ g \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}}(g'W_3 + gB_\mu) = A_\mu \quad \text{photon}(\gamma)$$

$$\lambda = (g^2+g'^2) \quad \rightarrow \quad \frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} g \\ -g' \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}}(gW_3 - g'B_\mu) = Z_\mu \quad \text{Z-boson (Z)}$$

Rewriting in terms of physical gauge bosons

$$\underbrace{W_1 \quad W_2}_{\text{W}^+ \text{ and W}^- \text{ bosons}} \qquad \underbrace{W_3 \quad B}_{\text{Z-boson and } \gamma}$$

1) W_1, W_2 :

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

2) W_3, B:

$$(-gW_3 + g' \quad B_\mu)^2 = (W_3, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_3 \\ B_\mu \end{pmatrix}$$

eigenvalue — *eigenvector*

$$\lambda = 0 \quad \rightarrow \quad \frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} g' \\ g \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}}\left(g'W_\mu^3 + gB_\mu\right) = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu = A_\mu \quad \text{(photon)}$$

$$\lambda = (g^2+g'^2) \quad \rightarrow \quad \frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} g \\ -g' \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}}\left(gW_\mu^3 - g'B_\mu\right) = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu = Z_\mu \quad \text{(Z-boson)}$$

Weak mixing angle (or Weinberg angle): θ_W

Rewriting in terms of physical gauge bosons

$$\underbrace{W_1 \quad W_2}_{\text{W}^+ \text{ and W}^- \text{ bosons}} \qquad \underbrace{W_3 \quad B}_{\text{Z-boson and } \gamma}$$

1) W_1, W_2 :

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right)$$

2) W_3, B:

$$(-gW_3 + g' \quad B_\mu)^2 = (W_3, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_3 \\ B_\mu \end{pmatrix}$$

⇓

$$(-gW_3 + g' \quad B_\mu)^2 = (g^2 + g'^2)Z_\mu^2 + 0 \cdot A_\mu^2$$

Electro-weak unification

➢ Electromagnetic and weak forces intricately connected!

$$(-gW_3 + g' \quad B_\mu)^2 = (g^2 + {g'}^2)Z_\mu^2 + 0 \cdot A_\mu^2$$

Spontaneous symmetry breaking

(Keep the vacuum neutral)

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\begin{aligned}
(D^\mu\phi)^\dagger(D_\mu\phi) &= \left|\left(\partial_\mu + \frac{i}{2}g\tau^k W_\mu^k + \frac{i}{2}g' B_\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2 \\
&= \frac{v^2}{8}\left|\left(g\tau^k W_\mu^k + g' B_\mu\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|^2 \\
&= \frac{v^2}{8}\left|\begin{pmatrix} gW_\mu^1 - igW_\mu^2 \\ -gW_\mu^3 + g' B_\mu \end{pmatrix}\right|^2 \\
&= \frac{v^2}{8}\left[g^2\left(\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2\right) + \left(gW_\mu^3 - g' B_\mu\right)^2\right]
\end{aligned}$$

- Mass terms!
- How about the physical fields?

$$(D^\mu\phi)^\dagger(D_\mu\phi) = \frac{1}{8}v^2[g^2(W^+)^2 + g^2(W^-)^2 + (g^2 + g'^2)Z_\mu^2 + 0\cdot A_\mu^2]$$

Spontaneous symmetry breaking

$$(D^\mu\phi)^\dagger(D_\mu\phi) = \left|\left(\partial_\mu + \frac{i}{2}g\tau^k W^k_\mu + \frac{i}{2}g' B_\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}\right|^2$$

$$= \frac{v^2}{8}\left|\left(g\tau^k W^k_\mu + g' B_\mu\right)\begin{pmatrix}0\\1\end{pmatrix}\right|^2$$

$$= \frac{v^2}{8}\left|\begin{pmatrix} gW^1_\mu - igW^2_\mu \\ -gW^3_\mu + g'B_\mu\end{pmatrix}\right|^2$$

$$= \frac{v^2}{8}\left[g^2\left((W^1_\mu)^2 + (W^2_\mu)^2\right) + (gW^3_\mu - g'B_\mu)^2\right]$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v+h\end{pmatrix}$$

Physical fields:	Mass term	Mass
$W^\pm_\mu \equiv \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$	$\frac{1}{2}\left(\frac{g\ v}{2}\right)^2 W^\dagger_\mu W^\mu$	$m_W = \frac{g\ v}{2}$
$(W_3, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2\end{pmatrix}\begin{pmatrix}W_3\\B_\mu\end{pmatrix}$	$(g^2+g'^2)Z^2_\mu + 0\cdot A^2_\mu$	$M_Z = \frac{1}{2}v\sqrt{(g^2+g'^2)}$

Summary:

1) Introduce doublet of scalar fields:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

2) With potential:

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

3) S.S.B.:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

4) Mass terms for gauge fields:

$$D_\mu \phi = \left(\partial_\mu + ig\, T^i W_\mu^i + i\frac{1}{2} g' B_\mu\right) \phi$$

$$\left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) = \frac{1}{8} v^2 [g^2 (W^+)^2 + g^2 (W^-)^2 + (g^2 + g'^2) Z_\mu^2 + 0 \cdot A_\mu^2]$$

Value of boson masses

$$A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu \quad \text{(photon)}$$
$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu \quad \text{(Z-boson)}$$

- Photon couples to e:

$$e = g\sin(\theta_W) = g'\cos(\theta_W)$$

- Prediction for ratio of masses:

$$\frac{M_W}{M_Z} = \frac{\frac{1}{2}vg}{\frac{1}{2}v\sqrt{g^2+g'^2}} = \cos(\theta_W)$$

- Veltman parameter:

$$\rho = \frac{M_W^2}{M_Z^2\cos^2(\theta_W)} = 1$$

- Higgs mass:

$$m_h = \sqrt{2\lambda v^2}$$

Muon decay: $\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}} \rightarrow$ $v = 1/\sqrt{2G_F}$

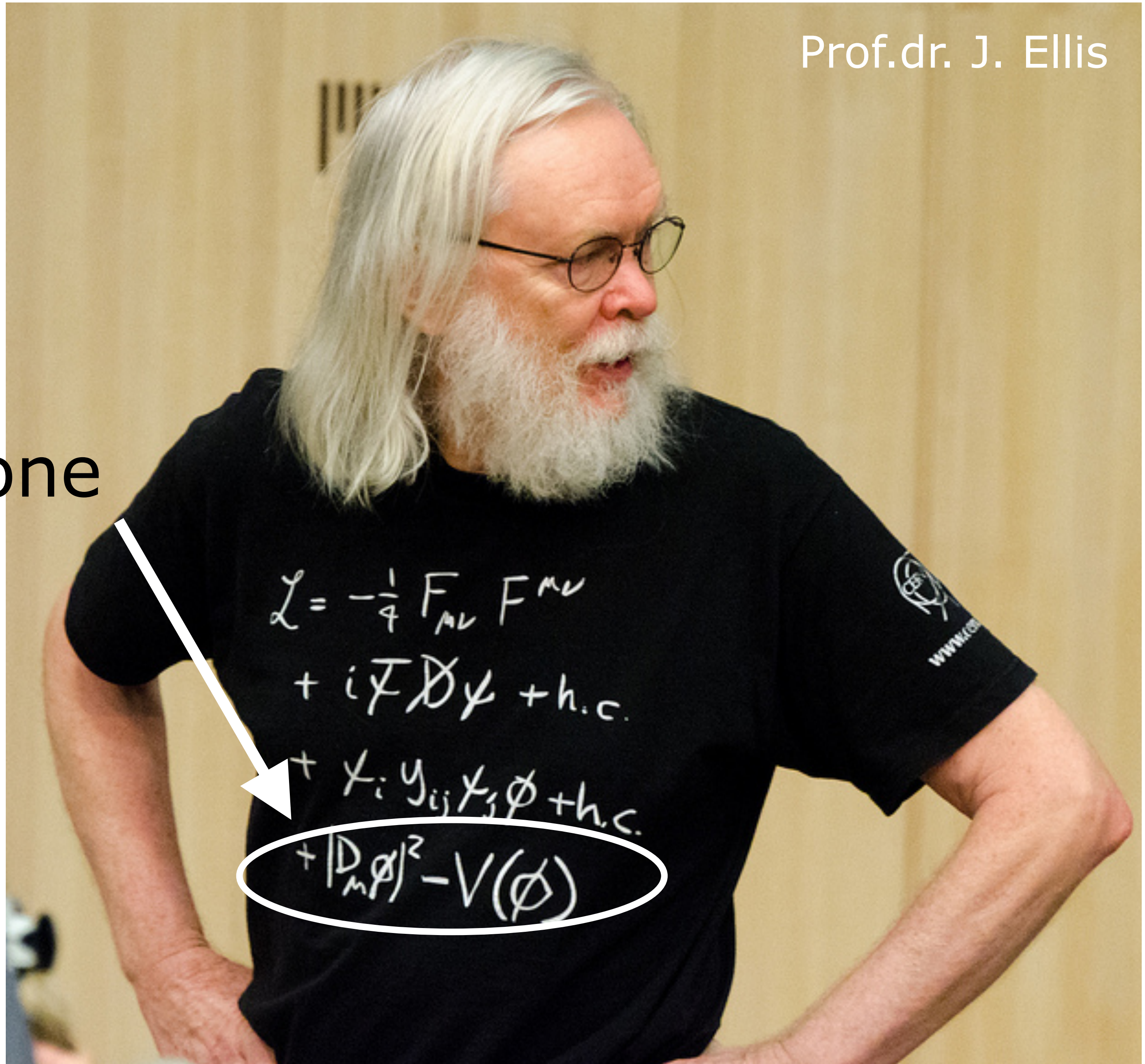
$$v = 246\ \text{GeV}$$

Fermion masses?

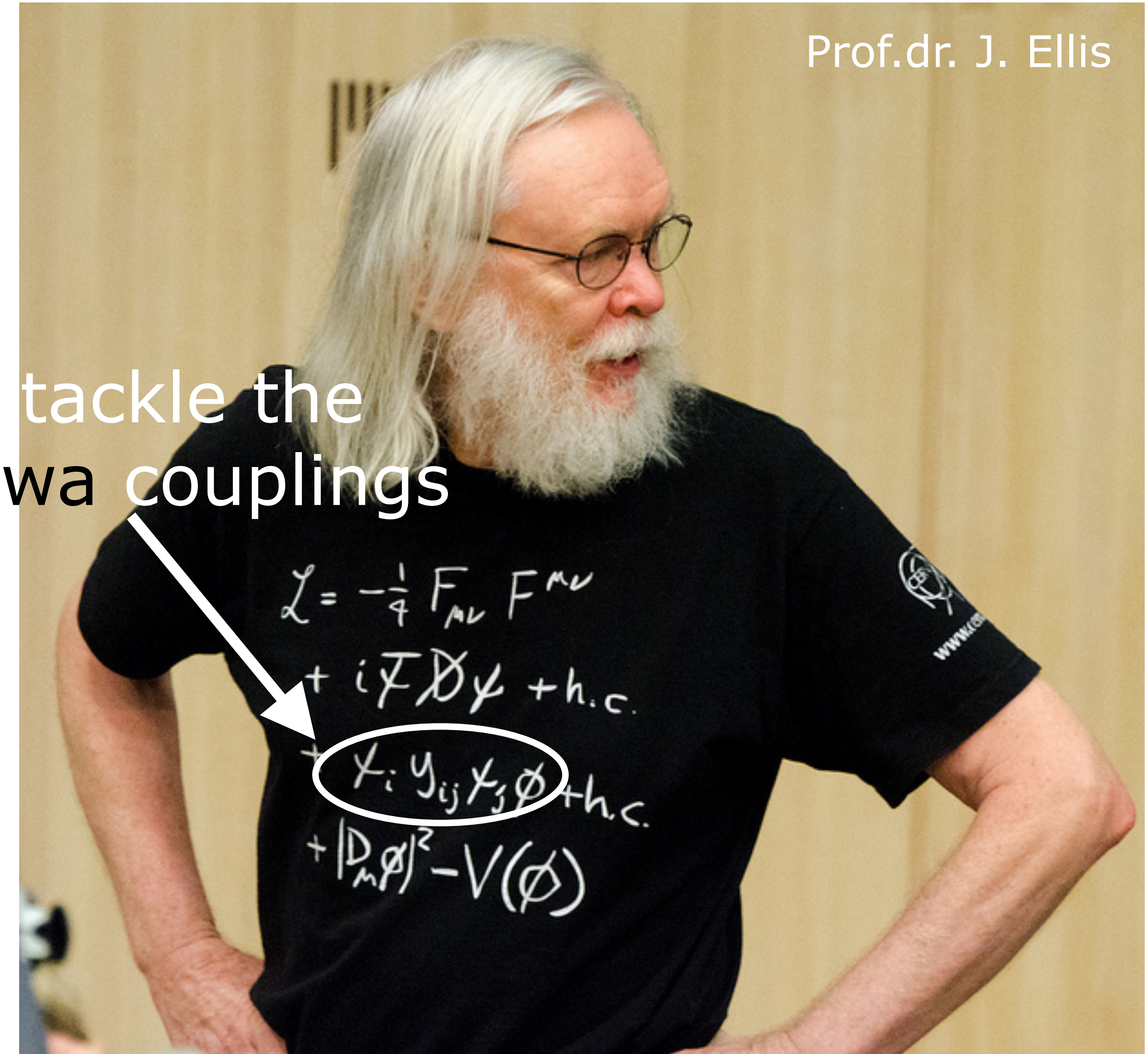
- Add ad-hoc (!?) term to Lagrangian:

$$\mathcal{L}_{\text{fermion-mass}} = -\lambda_f [\bar{\psi}_L \phi \psi_R + \bar{\psi}_R \bar{\phi} \psi_L]$$

Done


Prof.dr. J. Ellis
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

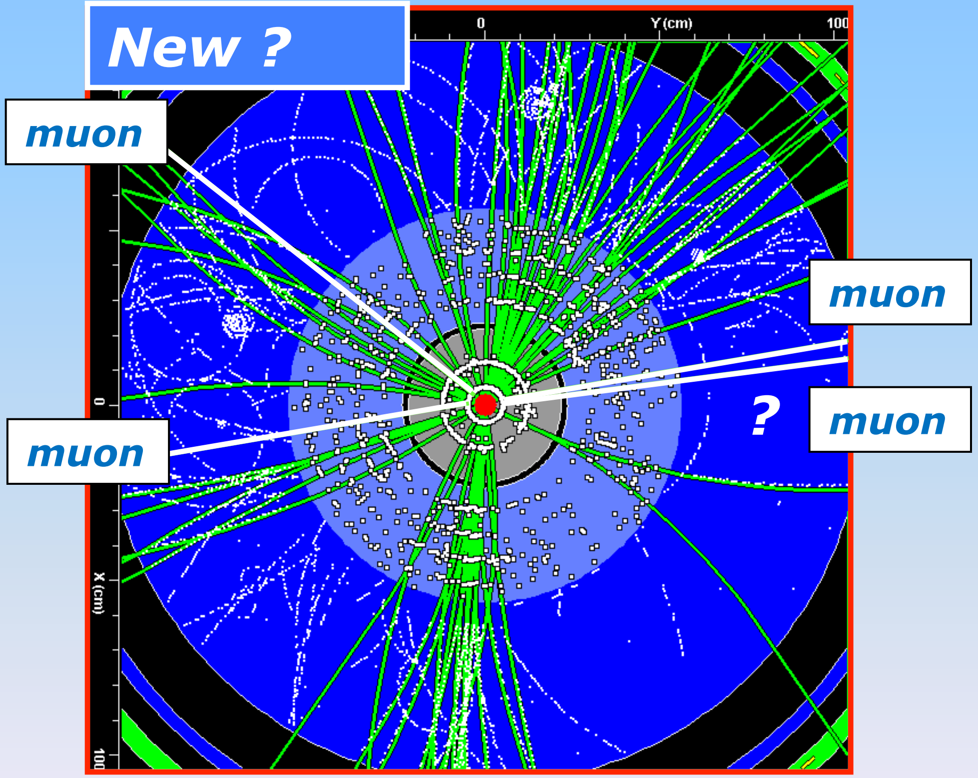
Let's tackle the Yukawa couplings

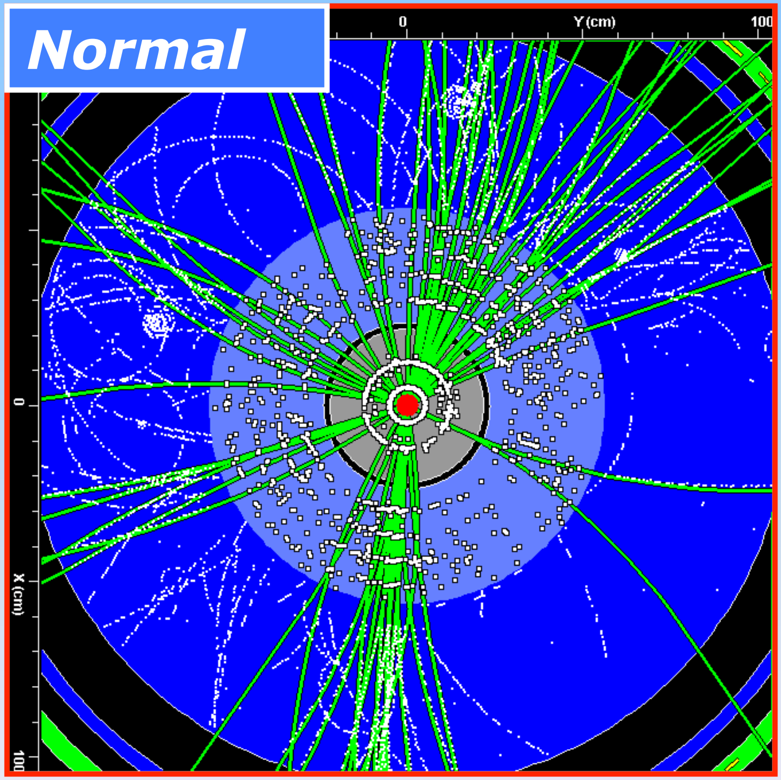


First:
Higgs discovery


LHCb
ATLAS
CMS
ALICE

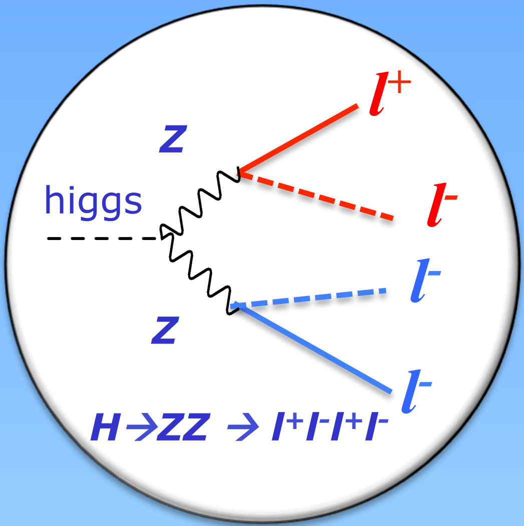
How are discoveries made?





Higgs → ZZ → 4 leptons

small number of beautiful events



120.000 Higgs bosons

⇩

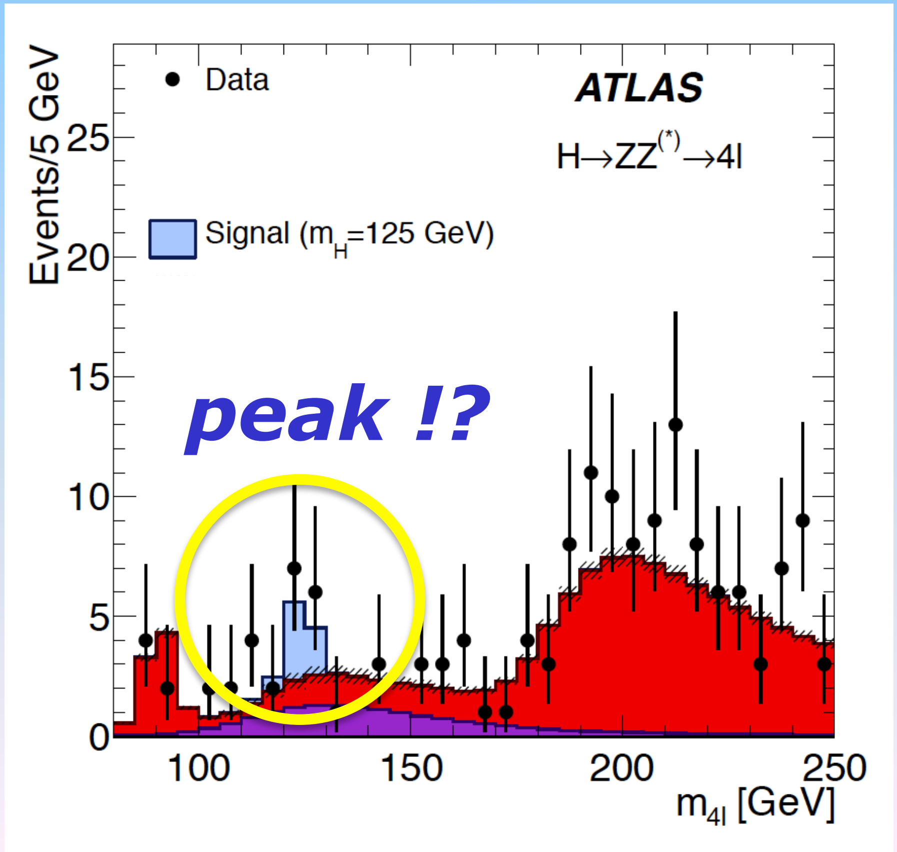
Only 1 in 1000 Higgs bosons
decays to 4 leptons

50% chance that ATLAS detector finds them

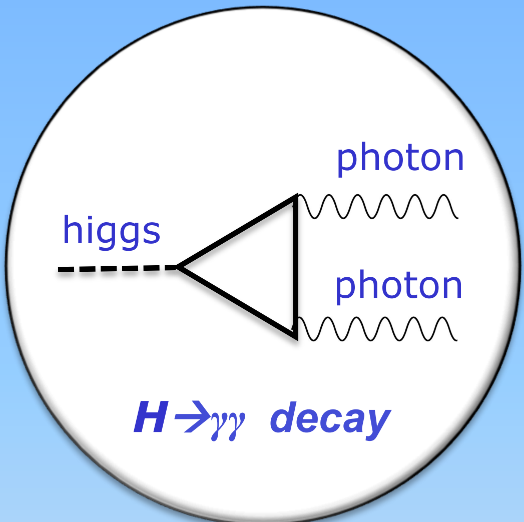
⇩

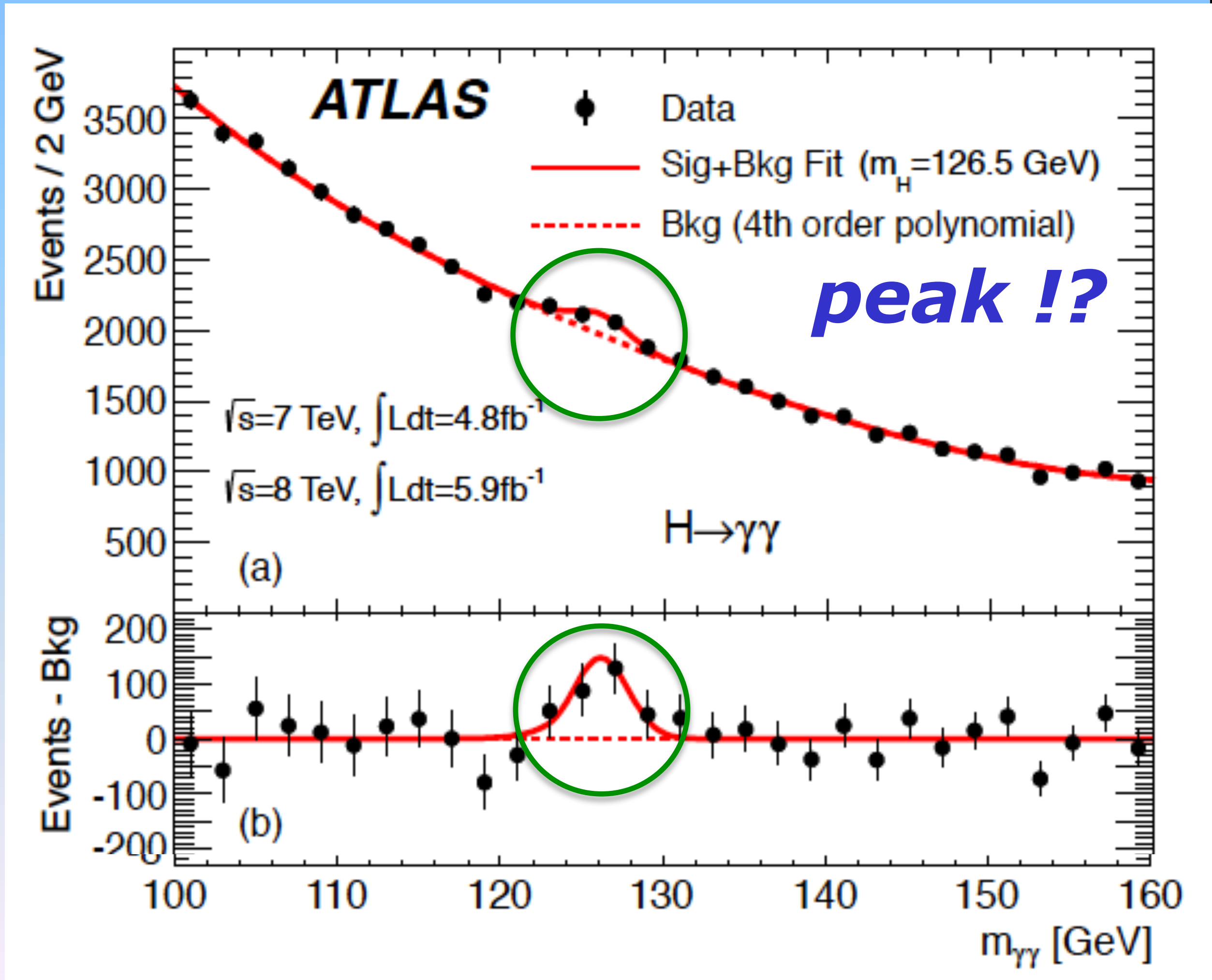
60 (Higgs → 4 lepton) events

'other'	52 events
with Higgs	68 events



Higgs → 2 photons





Interpretation of excess

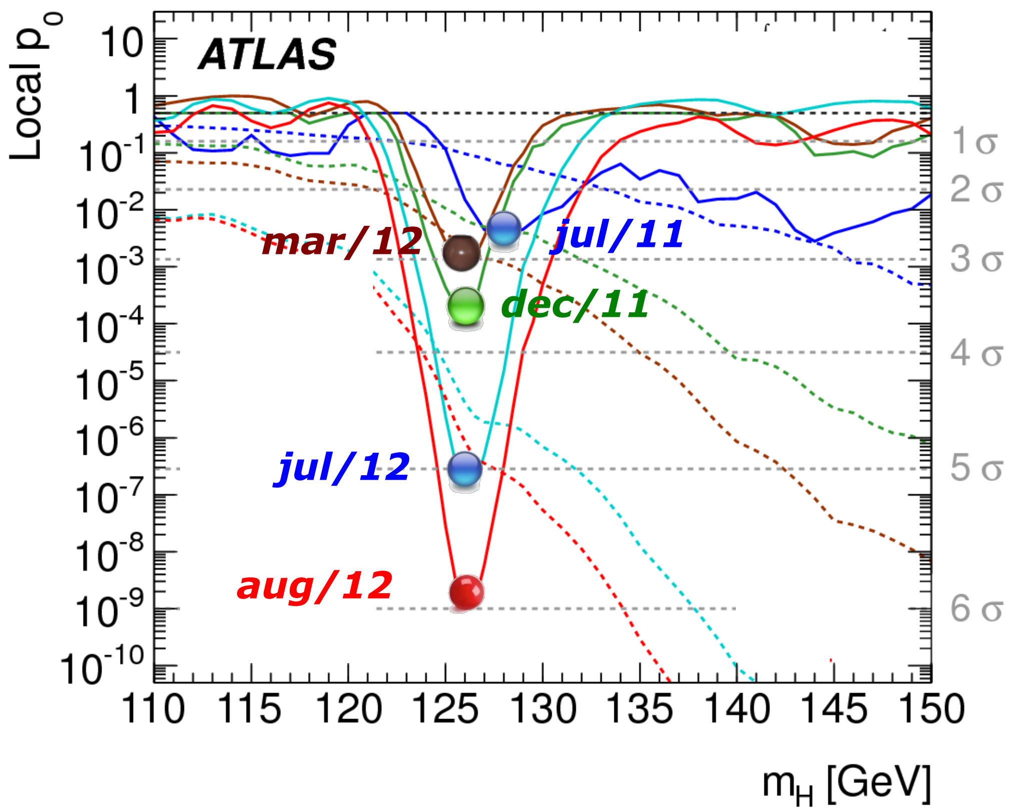
Claim discovery if:

Probability of observing excess smaller than 1 in 1 milion

Throwing 8 times 6 in a row

Discovery in slow-motion

Time-line higgs discovery



EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH (CERN)

CERN-PH-EP-2012-218

Accepted by: Physics Letters B

Observation of a New Particle in the Search for the Standard Model Higgs Boson with the ATLAS Detector at the LHC

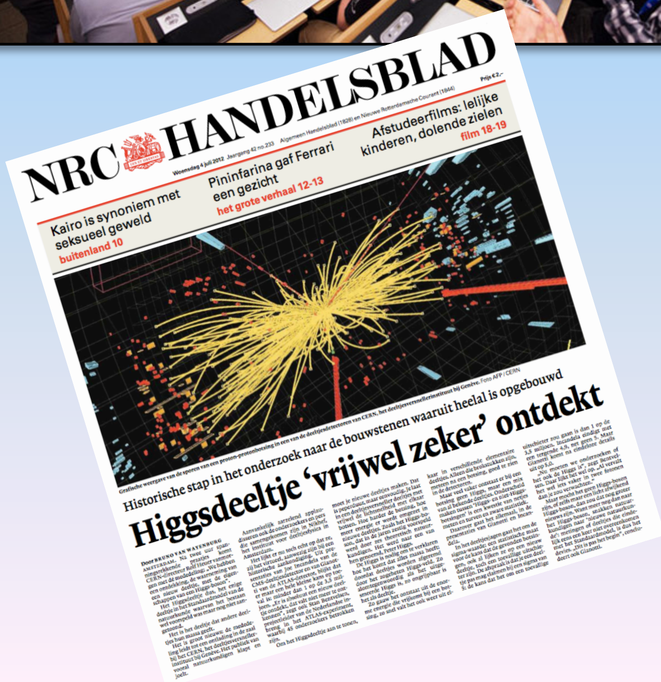
The ATLAS Collaboration

This paper is dedicated to the memory of our ATLAS colleagues who did not live to see the full impact and significance of their contributions to the experiment.

Abstract

A search for the Standard Model Higgs boson in proton-proton collisions with the ATLAS detector at the LHC is presented. The datasets used correspond to integrated luminosities of approximately 4.8 fb^{-1} collected at $\sqrt{s} = 7$ TeV in 2011 and 5.8 fb^{-1} at $\sqrt{s} = 8$ TeV in 2012. Individual searches in the channels $H \to ZZ^{(*)} \to 4\ell$, $H \to \gamma\gamma$ and $H \to WW^{(*)} \to e\nu\mu\nu$ in the 8 TeV data are combined with previously published results of searches for $H \to ZZ^{(*)}$, $WW^{(*)}$, $b\bar{b}$ and $\tau^+\tau^-$ in the 7 TeV data and results from improved analyses of the $H \to ZZ^{(*)} \to 4\ell$ and $H \to \gamma\gamma$ channels in the 7 TeV data. Clear evidence for the production of a neutral boson with a measured mass of $126.0 \pm 0.4\,(\text{stat}) \pm 0.4\,(\text{sys})$ GeV is presented. This observation, which has a significance of 5.9 standard deviations, corresponding to a background fluctuation probability of 1.7×10^{-9}, is compatible with the production and decay of the Standard Model Higgs boson.

Discovery of Higgs particle on July 4, 2012



NRC HANDELSBLAD

Kairo is synoniem met seksueel geweld
buitenland 10

Pininfarina gaf Ferrari een gezicht
het grote verhaal 12-13

Afstudeerfilms: lelijke kinderen, dolende zielen
film 18-19

Historische stap in het onderzoek naar de bouwstenen waaruit heelal is opgebouwd

Higgsdeeltje 'vrijwel zeker' ontdekt

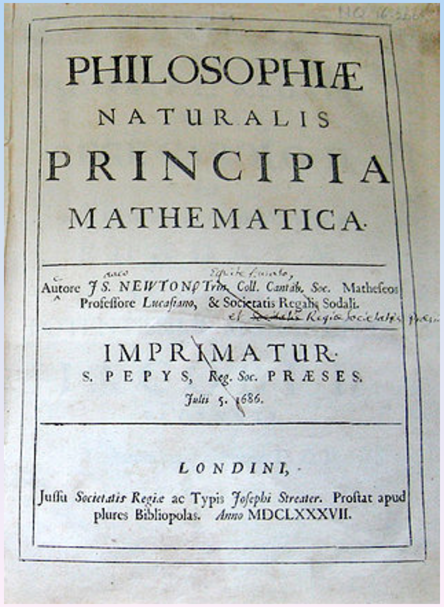
What is mass ?? Anno 1687

Mass is de ‘exchange rate’ between force and acceleration:

Newton

$$F = m \times a$$

Does not describe what mass is ...



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTONO, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

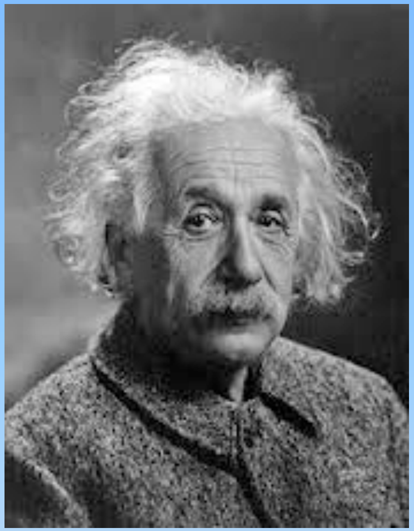
What is mass ?? Anno 1905

Mass is energy

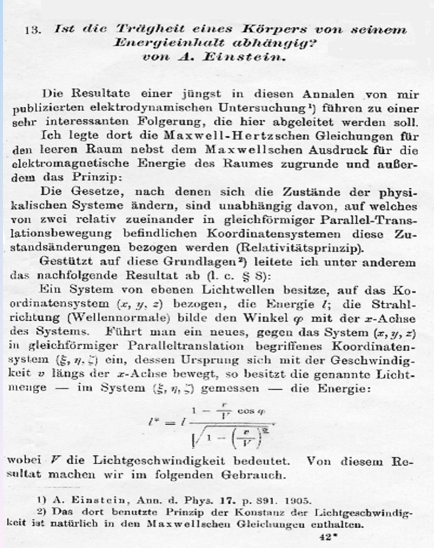
$$E = m \times c^2$$

Describes what mass is !

But not where it comes from …



Einstein



13. *Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?*
von A. Einstein.

Die Resultate einer jüngst in diesen Annalen von mir publizierten elektrodynamischen Untersuchung[1]) führen zu einer sehr interessanten Folgerung, die hier abgeleitet werden soll.

Ich legte dort die Maxwell-Hertzschen Gleichungen für den leeren Raum nebst dem Maxwellschen Ausdruck für die elektromagnetische Energie des Raumes zugrunde und außerdem das Prinzip:

Die Gesetze, nach denen sich die Zustände der physikalischen Systeme ändern, sind unabhängig davon, auf welches von zwei relativ zueinander in gleichförmiger Parallel-Translationsbewegung befindlichen Koordinatensystemen diese Zustandsänderungen bezogen werden (Relativitätsprinzip).

Gestützt auf diese Grundlagen[2]) leitete ich unter anderem das nachfolgende Resultat ab (l. c. § 8):

Ein System von ebenen Lichtwellen besitze, auf das Koordinatensystem (x, y, z) bezogen, die Energie l; die Strahlrichtung (Wellennormale) bilde den Winkel φ mit der x-Achse des Systems. Führt man ein neues, gegen das System (x, y, z) in gleichförmiger Paralleltranslation begriffenes Koordinatensystem (ξ, η, ζ) ein, dessen Ursprung sich mit der Geschwindigkeit v längs der x-Achse bewegt, so besitzt die genannte Lichtmenge — im System (ξ, η, ζ) gemessen — die Energie:

$$l^* = l \frac{1 - \frac{v}{V} \cos \varphi}{\sqrt{1 - \left(\frac{v}{V}\right)^2}},$$

wobei V die Lichtgeschwindigkeit bedeutet. Von diesem Resultat machen wir im folgenden Gebrauch.

1) A. Einstein, Ann. d. Phys. **17**. p. 891. 1905.

2) Das dort benutzte Prinzip der Konstanz der Lichtgeschwindigkeit ist natürlich in den Maxwellschen Gleichungen enthalten.

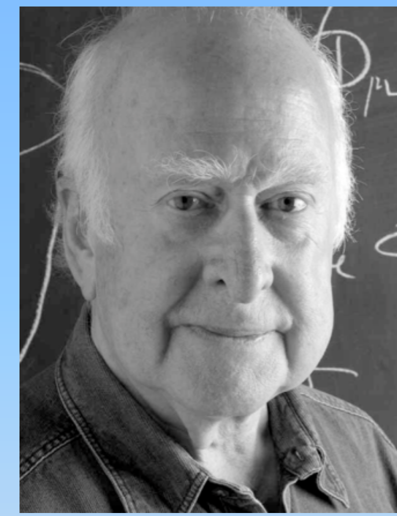
42*

What is mass ?? Anno 1964

Mass of elementary particles is due to

"friction" of ubiquitous 'Higgs field'

$$m: \psi\psi H$$



Higgs

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

P. W. HIGGS

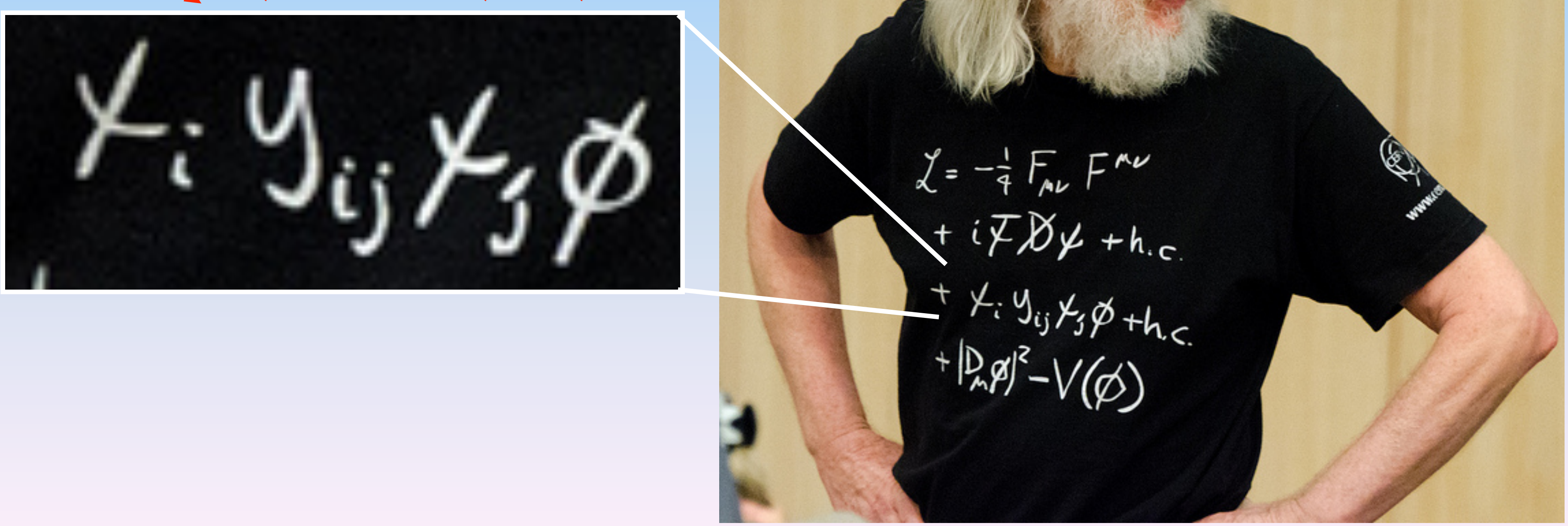
Received 27 July 1964

What is mass ?? Anno 1964

Mass of elementary particles is due to

“friction” of ubiquitous ‘Higgs field’

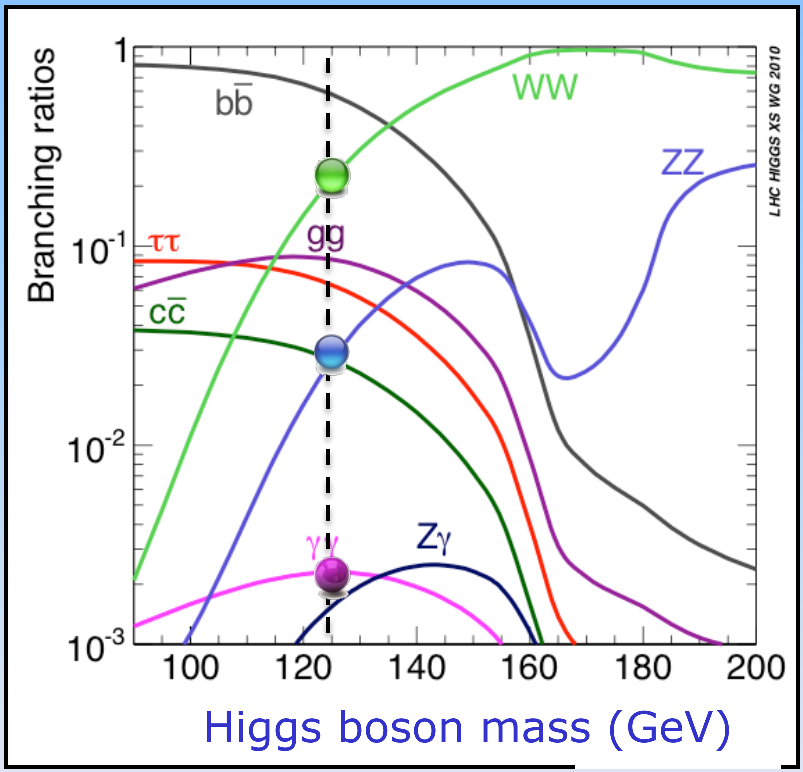
m: ψψH

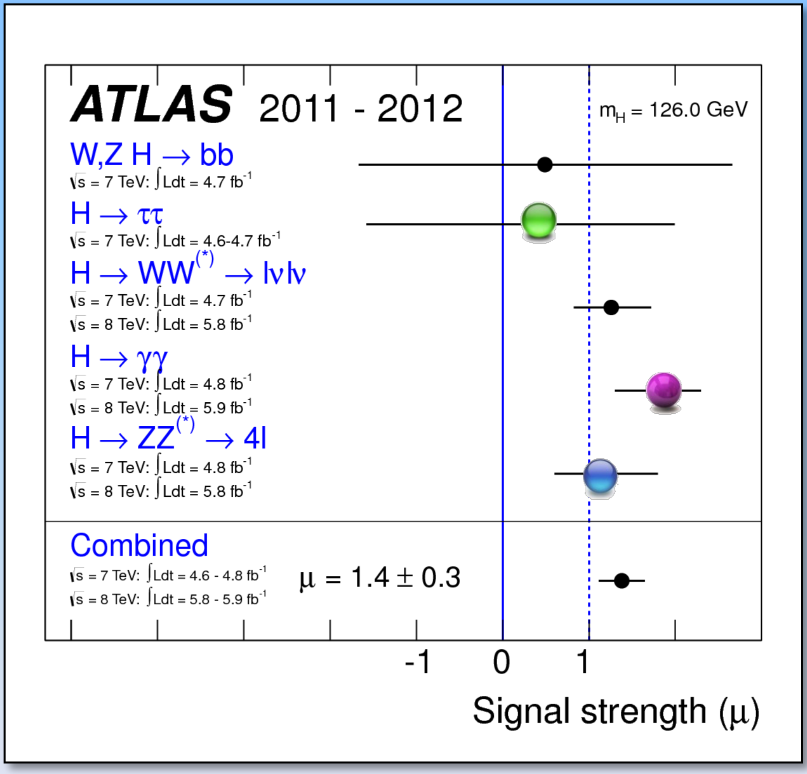


Next: Higgs' properties as expected?

m_h=125 GeV ↓

Standard Model ↓





prediction

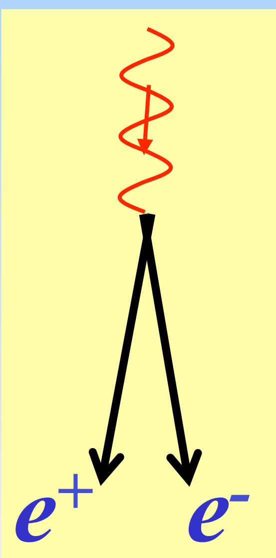
measurement

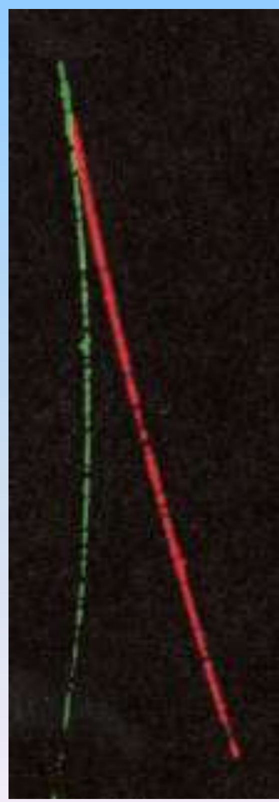
Philosophy?

Higgs: Particle? Field?

Particle

Photon (light particle)



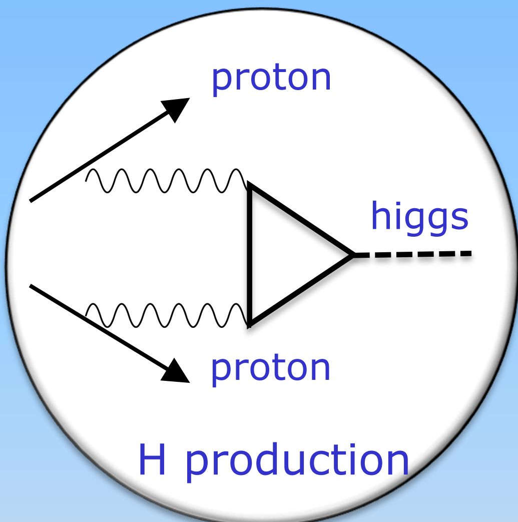


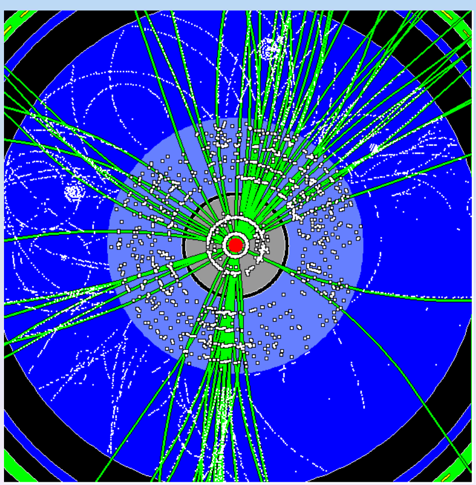
Field

Electrical field

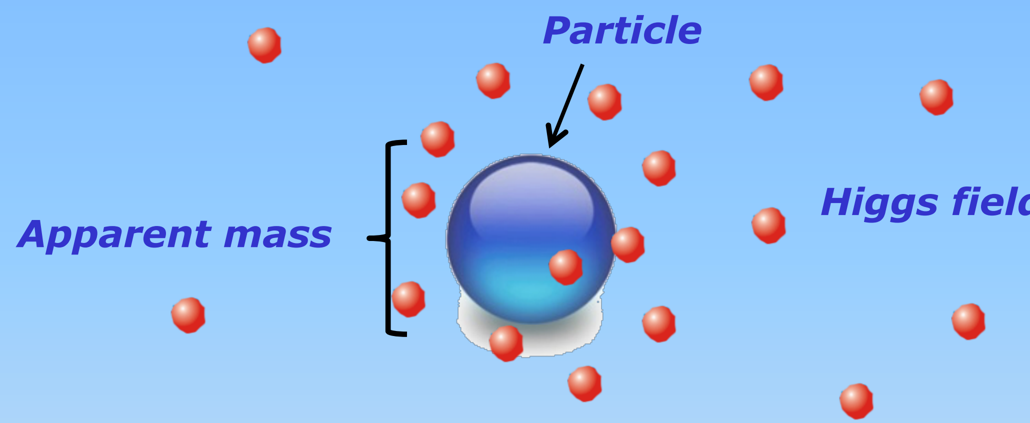
Why is the Higgs particle so special?

Particle





Field



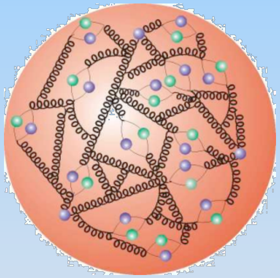
As if the fish discovered the water he's in…

What is mass?

Mass of elementary particles is due to

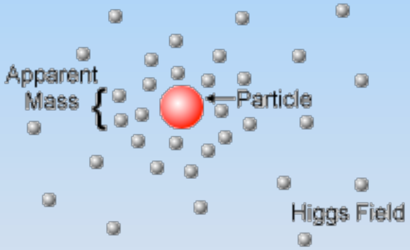
"friction" of ubiquitous 'Higgs field'

Einstein:
proton mass =
binding energy



Elementary particle
in empty space:
no rest-energy=
no mass

Elementary particle
in Higgs field:
rest energy =
interaction with Higgs field
= mass!



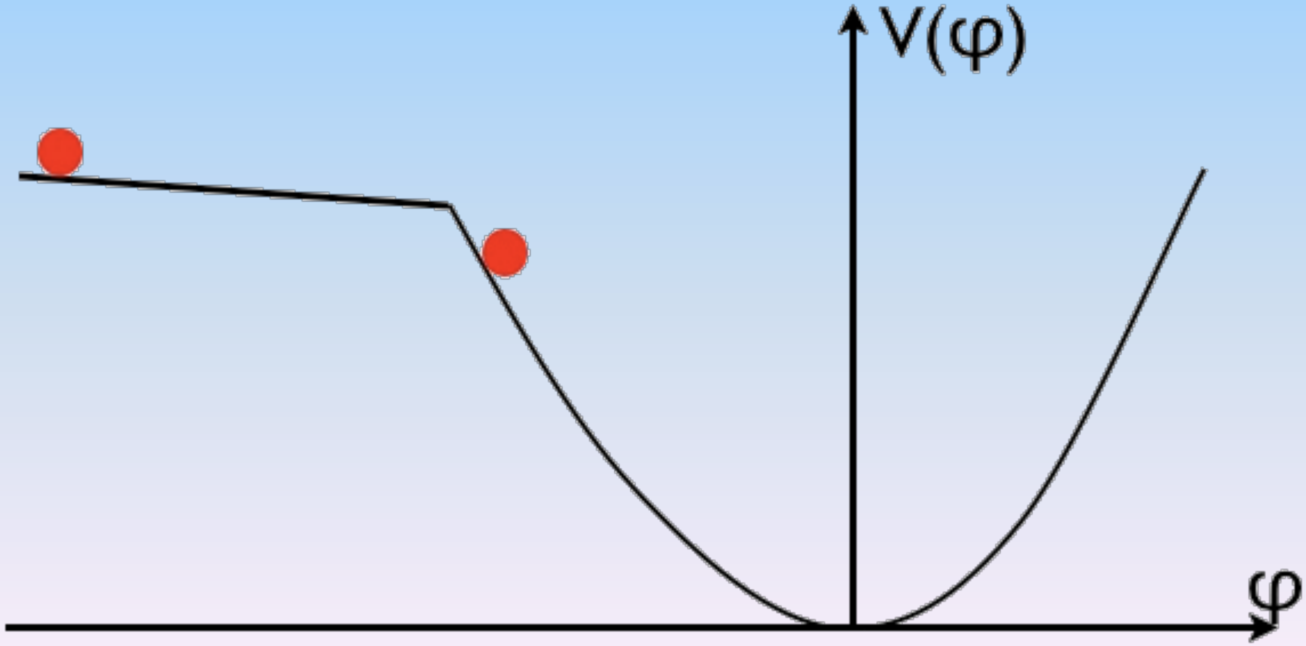
Revolutionary – with spectaculair consequences :

space is not empty, but filled with sort of 'ether'

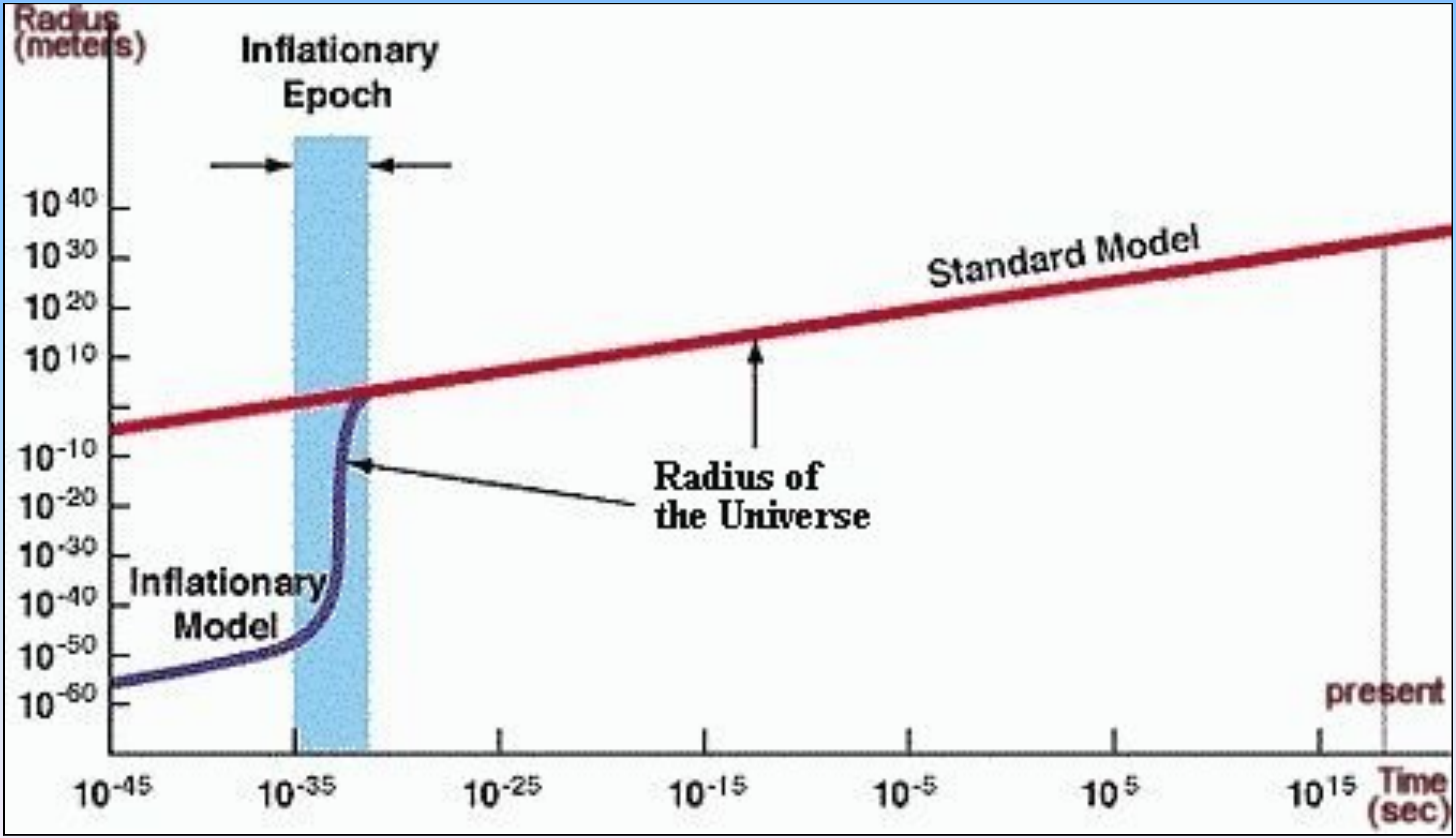
Another field: the Big Bang

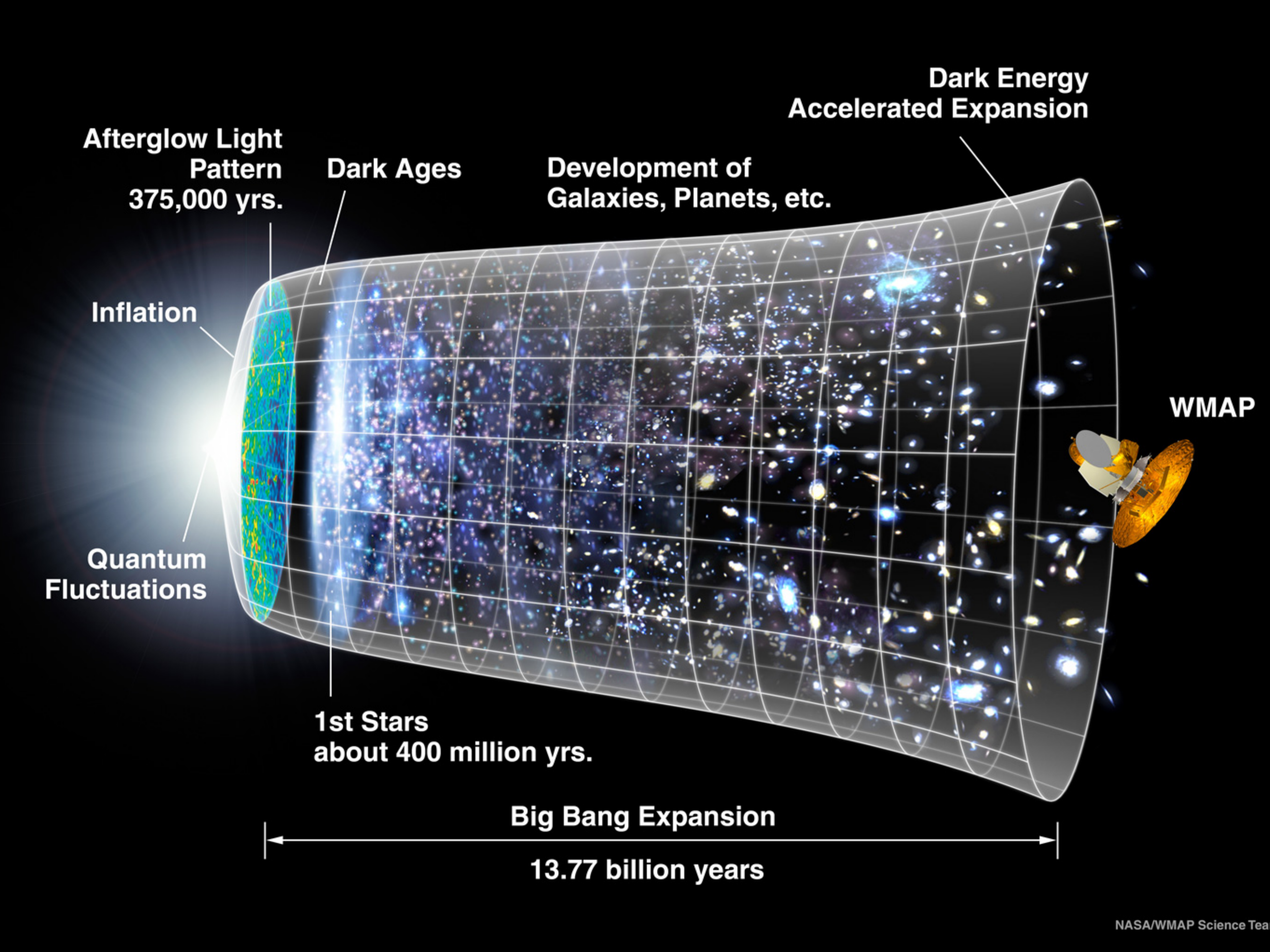
One of Higgs' properties match that of another field…

The inflaton that inflated the Universe between 10^{-33} and 10^{-32} seconds after the Big Bang



Another field: the Big Bang




Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years
NASA/WMAP Science Tea

Couplings across generations: CKM

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm Kinetic} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}$$

$\mathcal{L}_{\rm Kinetic}$ Introduce the massless fermion fields
Require local gauge invariance ➔ existence of gauge bosons

$\mathcal{L}_{\rm Higgs}$ Introduce Higgs potential with $<\varphi> \neq 0$
Spontaneous symmetry breaking

$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$

W^+, W^-, Z^0 bosons acquire a mass

$\mathcal{L}_{\rm Yukawa}$ Ad hoc interactions between Higgs field & fermions

Fields: Notation

Fermions: $\psi_L = \left(\frac{1-\gamma_5}{2}\right)\psi \quad ; \quad \psi_R = \left(\frac{1+\gamma_5}{2}\right)\psi$ with $\psi = Q_L, u_R, d_R, L_L, l_R, \nu_R$

Quarks:

Under SU2:
Left handed doublets
Right handed singlets

$$\begin{pmatrix} u^I(3,2,1/3) \\ d^I(3,2,1/3) \end{pmatrix}_{L\,i} \longrightarrow Q^I_{Li}(3,2,1/3)$$

Interaction rep.
Left-handed
generation index
$SU(3)_C$
$SU(2)_L$
Hypercharge Y

$$Q = I_3 + \frac{Y}{2}$$

$$u^I_{Ri}(3,1,4/3) \qquad d^I_{Ri}(3,1,-2/3)$$

Leptons:

$$\begin{pmatrix} \nu^I(1,2,-1) \\ l^I(1,2,-1) \end{pmatrix}_{L\,i} \longrightarrow L^I_{Li}(1,2,-1)$$

$$l^I_{Ri}(1,1,-2) \qquad \nu^I_{Ri}$$

Scalar field:

$$\phi\ (1,2,1) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Interaction representation: standard model

Fields: explicitly

Explicitly:

- The left handed quark doublet :

$$Q^I_{Li}(3,2,1/3) = \begin{pmatrix} u^I_r, u^I_g, u^I_b \\ d^I_r, d^I_g, d^I_b \end{pmatrix}_L, \begin{pmatrix} c^I_r, c^I_g, c^I_b \\ s^I_r, s^I_g, s^I_b \end{pmatrix}_L, \begin{pmatrix} t^I_r, t^I_g, t^I_b \\ b^I_r, b^I_g, b^I_b \end{pmatrix}_L$$

- Similarly for the quark singlets:

$$u^I_{Ri}(3,1,4/3) = \left(u^I_r, u^I_r, u^I_r\right)_R, \left(c^I_r, c^I_r, c^I_r\right)_R, \left(t^I_r, t^I_r, t^I_r\right)_R$$

$$d^I_{Ri}(3,1,-2/3) = \left(d^I_r, d^I_r, d^I_r\right)_R, \left(s^I_r, s^I_r, s^I_r\right)_R, \left(b^I_r, b^I_r, b^I_r\right)_R$$

- The left handed leptons: $L^I_{Li}(1,2,-1) = \begin{pmatrix} \nu^I_e \\ e^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\mu \\ \mu^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\tau \\ \tau^I \end{pmatrix}_L$

- And similarly the (charged) singlets: $l^I_{Ri}(1,1,-2) = e^I_R, \mu^I_R, \tau^I_R$

$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Kinetic}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$

$\mathcal{L}_{\text{Kinetic}}$: Fermions + gauge bosons + interactions

Procedure: Introduce the fermion fields and ***demand*** *that the theory is local gauge invariant under* $SU(3)_C \times SU(2)_L \times U(1)_Y$ *transformations.*

Start with the Dirac Lagrangian: $L = i\bar{\psi}\left(\partial^\mu \gamma_\mu\right)\psi$

Replace: $$\partial^\mu \rightarrow D^\mu = \partial^\mu + ig_s G_a^\mu L_a + \frac{1}{2} ig W_i^\mu \tau_i + \frac{1}{2} ig' B^\mu Y$$

Fields:
- G_a^μ : 8 gluons
- W_b^μ : weak bosons: W_1, W_2, W_3
- B^μ : hypercharge boson

Generators:

L_a : Gell-Mann matrices:	$\frac{1}{2}\lambda_a$	(3x3)	$SU(3)_C$
σ_b : Pauli Matrices:	$\frac{1}{2}\tau_b$	(2x2)	$SU(2)_L$
Y : Hypercharge:			$U(1)_Y$

For the remainder we only consider Electroweak: $SU(2)_L \times U(1)_Y$

$$\mathcal{L}_{\rm SM} = \boxed{\mathcal{L}_{\rm Kinetic}} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}$$

$$L_{kinetic} : i\overline{\psi}(\partial^{\mu}\gamma_{\mu})\psi \rightarrow i\overline{\psi}(D^{\mu}\gamma_{\mu})\psi$$

$$with \quad \psi = Q_{Li}^{I}, \quad u_{Ri}^{I}, \quad d_{Ri}^{I}, \quad L_{Li}^{I}, \quad l_{Ri}^{I}$$

- Example: the term with Q_{Li}^{I} becomes:

$$L_{kinetic}(Q_{Li}^{I}) = i\overline{Q_{Li}^{I}}\gamma_{\mu}D^{\mu}Q_{Li}^{I}$$

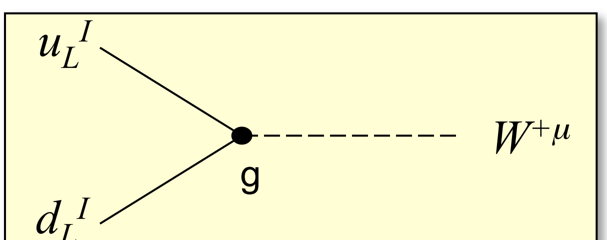
$$= i\overline{Q_{Li}^{I}}\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}gW_{b}^{\mu}\tau_{b} + \frac{i}{6}g'B^{\mu}\right)Q_{Li}^{I}$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Writing out only the weak part for the quarks:

$$L_{kinetic}^{Weak}(u,d)_{L}^{I} = i\overline{(u,d)}_{L}^{I}\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}g\left(W_{1}^{\mu}\tau_{1} + W_{2}^{\mu}\tau_{2} + W_{3}^{\mu}\tau_{3}\right)\right)\begin{pmatrix} u \\ d \end{pmatrix}_{L}^{I}$$

$$= i\overline{u_{L}^{I}}\gamma_{\mu}\partial^{\mu}u_{L}^{I} + i\overline{d_{L}^{I}}\gamma_{\mu}\partial^{\mu}d_{L}^{I} - \frac{g}{\sqrt{2}}\overline{u_{L}^{I}}\gamma_{\mu}W^{-\mu}d_{L}^{I} - \frac{g}{\sqrt{2}}\overline{d_{L}^{I}}\gamma_{\mu}W^{+\mu}u_{L}^{I} -$$



$L = J_{\mu}W^{\mu}$

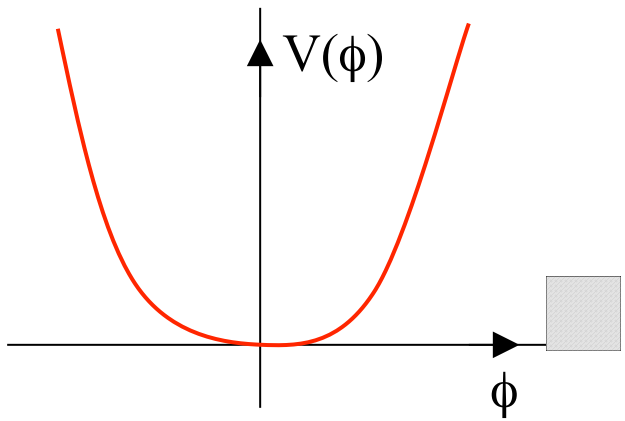
$$W^{\pm} = \frac{1}{\sqrt{2}}(W_1 \mp W_2)$$

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm Kinetic} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}$$

$$\mathcal{L}_{\rm Higgs} = D_\mu \phi^\dagger D^\mu \phi - V_{\rm Higgs} \text{ ,with } V_{\rm Higgs} = \frac{1}{2}\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$$

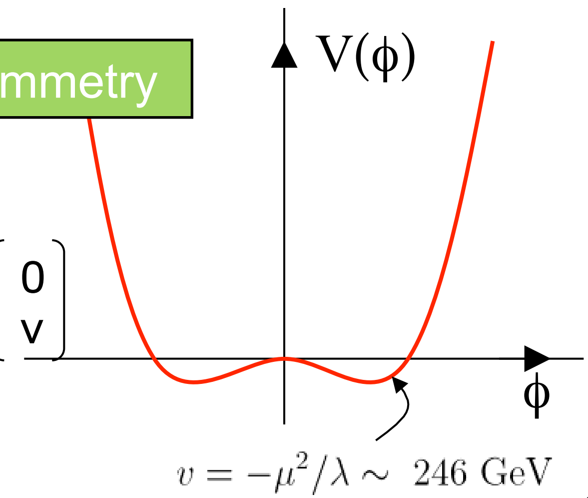
Symmetry

$$\mu^2 > 0$$
$$\langle \phi_0 \rangle = 0$$



Broken symmetry

$$\mu^2 < 0$$
$$\langle \phi_0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$



Spontaneous Symmetry Breaking: The Higgs field adopts a non-zero vacuum expectation value

Procedure: $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \mathcal{R}e(\phi^+) + \mathcal{I}m(\phi^+) \\ \mathcal{R}e(\phi^0) + \mathcal{I}m(\phi^0) \end{pmatrix}$ Substitute: $\mathcal{R}e(\phi^0) = \frac{1}{\sqrt{2}}(v+h)$

And rewrite the Lagrangian (tedious):

1. $G_{SM} : \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right) \rightarrow \left(SU(3)_C \times U(1)_{EM}\right)$
2. The W^+, W^-, Z^0 bosons acquire mass
3. The Higgs boson *H* appears

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm Kinetic} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}$$

Since we have a Higgs field we can add **(ad-hoc)** interactions between Higgs field and the fermions in a gauge invariant way

The result is:

(doublets: $\overline{\psi}_{Li}$, ϕ; singlet: ψ_{Rj})

$$\mathcal{L}_{\rm Yukawa} = Y_{ij}\left(\overline{\psi}_{Li}\,\phi\right)\psi_{Rj} + h.c.$$

$$= Y_{ij}^{d}\left(\overline{Q_{Li}^{I}}\,\phi\right)d_{Rj}^{I} + Y_{ij}^{u}\left(\overline{Q_{Li}^{I}}\,\tilde{\phi}\right)u_{Rj}^{I} + Y_{ij}^{l}\left(\overline{L_{Li}^{I}}\,\phi\right)l_{Rj}^{I} + h.c.$$

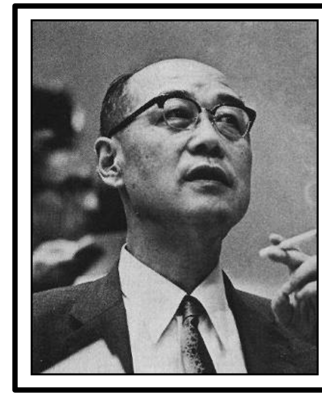
i, j : indices for the 3 generations!

With: $\tilde{\phi} = i\sigma_2\,\phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}$

(The CP conjugate of φ)

Y_{ij}^{d} Y_{ij}^{u} Y_{ij}^{l} are arbitrary complex matrices which operate in family space (3x3) ➔ flavour physics

$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{Kinetic}} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{Yukawa}}$: The Yukawa Part



Writing the first term explicitly:

$$Y^d_{ij}\,(\overline{u^I_L}, \overline{d^I_L})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d^I_{Rj} \quad =$$

$$\begin{pmatrix}
Y^d_{11}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{12}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{13}\left(\overline{u^I_L}, \overline{d^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \\
Y^d_{21}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{22}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{13}\left(\overline{c^I_L}, \overline{s^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \\
Y^d_{31}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{32}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} & Y^d_{33}\left(\overline{t^I_L}, \overline{b^I_L}\right)\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}
\end{pmatrix} \bullet \begin{pmatrix} d^I_R \\ s^I_R \\ b^I_R \end{pmatrix}$$

$\mathcal{L}_{\text{Yukawa}} \overset{\textbf{SSB}}{\longrightarrow} \mathcal{L}_{\text{mass}}$

- Start with the Yukawa Lagrangian

$$\mathcal{L}_{\text{Yukawa}} = Y_{ij}^{d}\,(\overline{u_L^I}, \overline{d_L^I})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^I + Y_{ij}^{u}\,(\ldots) + Y_{ij}^{l}\,(\ldots)$$

Spontaneous symmetry breaking → $\mathcal{R}e(\phi^0) = \frac{1}{\sqrt{2}}(v+h)$

- After which the following mass term emerges:

$$\mathcal{L}_{\text{Yukawa}} \rightarrow \mathcal{L}_{\text{mass}} = \overline{d_{Li}^I} M_{ij}^d d_{Rj}^I + \overline{u_{Li}^I} M_{ij}^u u_{Rj}^I + \overline{l_{Li}^I} M_{ij}^l l_{Rj}^I + h.c.$$

, with $M_{ij}^d = \frac{1}{\sqrt{2}} Y_{ij}^d$, $M_{ij}^u = \frac{1}{\sqrt{2}} Y_{ij}^u$, $M_{ij}^l = \frac{1}{\sqrt{2}} Y_{ij}^l$

$$\mathcal{L}_{\text{Yukawa}} \xrightarrow{SSB} \mathcal{L}_{\text{mass}}$$

Writing in an explicit form:

$$\mathcal{L}_{\text{mass}} = \left(\overline{d^I}, \overline{s^I}, \overline{b^I}\right)_L \cdot \left(M^d\right) \cdot \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R + \left(\overline{u^I}, \overline{c^I}, \overline{t^I}\right)_L \cdot \left(M^u\right) \cdot \begin{pmatrix} u^I \\ c^I \\ t^I \end{pmatrix}_R + \left(\overline{e^I}, \overline{\mu^I}, \overline{\tau^I}\right)_L \cdot \left(M^l\right) \cdot \begin{pmatrix} e^I \\ \mu^I \\ \tau^I \end{pmatrix}_R + h.c.$$

The matrices M can always be diagonalised by unitary matrices V_L^f and V_R^f such that:

$$V_L^f \, M^f \, V_R^{f\dagger} = M^f_{diagonal}$$

$$\left[\left(\overline{d^I}, \overline{s^I}, \overline{b^I}\right)_L V_L^{f\dagger} V_L^f \, M^f \, V_R^{f\dagger} V_R^f \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R\right]$$

Then the **real** fermion mass eigenstates are given by:

$$d_{Li} = \left(V_L^d\right)_{ij} \cdot d_{Lj}^I \qquad d_{Ri} = \left(V_R^d\right)_{ij} \cdot d_{Rj}^I$$

$$u_{Li} = \left(V_L^u\right)_{ij} \cdot u_{Lj}^I \qquad u_{Ri} = \left(V_R^u\right)_{ij} \cdot u_{Rj}^I$$

$$l_{Li} = \left(V_L^l\right)_{ij} \cdot l_{Lj}^I \qquad l_{Ri} = \left(V_R^l\right)_{ij} \cdot l_{Rj}^I$$

d_L^I, u_L^I, l_L^I are the weak interaction eigenstates
d_L, u_L, l_L are the mass eigenstates (“physical particles”)

$$\mathcal{L}_{\text{Yukawa}} \overset{SSB}{\longrightarrow} \mathcal{L}_{\text{mass}}$$

In terms of the mass eigenstates:

$$\mathcal{L}_{\text{mass}} = \left(\bar{d},\bar{s},\bar{b}\right)_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\bar{u},\bar{c},\bar{t}\right)_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$

$$+ \left(\bar{e},\bar{\mu},\bar{\tau}\right)_L \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R + h.c.$$

$$\begin{aligned} \mathcal{L}_{\text{mass}} = \; & m_u \bar{u}u + m_c \bar{c}c + m_t \bar{t}t \\ & + m_d \bar{d}d + m_s \bar{s}s + m_b \bar{b}b \\ & + m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau \end{aligned}$$

In flavour space one can choose:

Weak basis: The gauge currents are diagonal in flavour space, but the flavour mass matrices are non-diagonal

Mass basis: The fermion masses are diagonal, but some gauge currents (charged weak interactions) are not diagonal in flavour space

➔ What happened to the charged current interactions (in $L_{Kinetic}$) ?

$\mathcal{L}_W \rightarrow \mathcal{L}_{CKM}$: The Charged Current

The charged current interaction for quarks in the **interaction** basis is:

$$\mathcal{L}_{W+} = \frac{g}{\sqrt{2}} \overline{u^I_{Li}} \gamma^\mu d^I_{Li} W^+_\mu$$

The charged current interaction for quarks in the **mass** basis is:

$$\mathcal{L}_{W+} = \frac{g}{\sqrt{2}} \overline{u_{Li}} V^u_L \gamma^\mu V^{d\dagger}_L d_{Li} W^+_\mu$$

The unitary matrix: $V_{CKM} = \left(V^u_L \cdot V^{d\dagger}_L\right)$ with: $V_{CKM} \cdot V^\dagger_{CKM} = 1$

is the Cabibbo Kobayashi Maskawa mixing matrix:

$$\mathcal{L}_{W+} = \frac{g}{\sqrt{2}} \left(\bar{u}, \bar{c}, \bar{t}\right)_L \left(V_{CKM}\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \gamma^\mu W^+_\mu$$

Lepton sector: similarly $V_{MNS} = \left(V^\nu_L \cdot V^{l\dagger}_L\right)$

However, for massless neutrino's: $V_L{}^\nu$ = arbitrary.
Choose it such that $V_{MNS} = 1$ ➔ no mixing in the lepton sector

Charged Currents

- The charged current term reads:

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}\overline{u_{Li}^{I}}\gamma^{\mu}W_{\mu}^{-}d_{Li}^{I} \quad + \quad \frac{g}{\sqrt{2}}\overline{d_{Li}^{I}}\gamma^{\mu}W_{\mu}^{+}u_{Li}^{I} \quad = \quad J_{CC}^{\mu-}\,W_{\mu}^{-} + J_{CC}^{\mu+}\,W_{\mu}^{+}$$

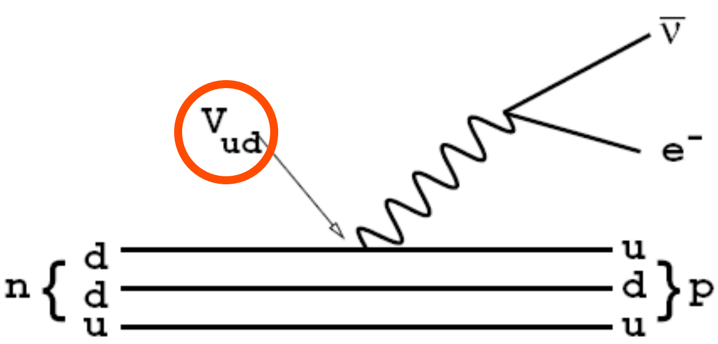
$$= \frac{g}{\sqrt{2}}\overline{u}_{i}\left(\frac{1-\gamma^{5}}{2}\right)\gamma^{\mu}W_{\mu}^{-}V_{ij}\left(\frac{1-\gamma^{5}}{2}\right)d_{j} \quad + \quad \frac{g}{\sqrt{2}}\overline{d}_{j}\left(\frac{1-\gamma^{5}}{2}\right)\gamma^{\mu}W_{\mu}^{+}V_{ji}^{\dagger}\left(\frac{1-\gamma^{5}}{2}\right)u_{i}$$

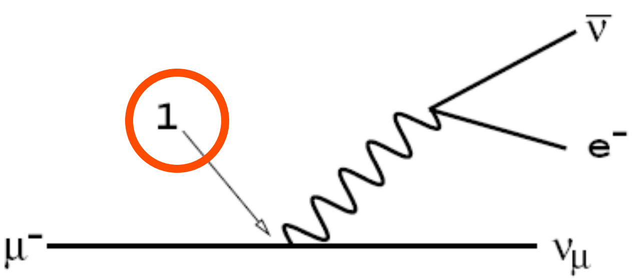
$$= \frac{g}{\sqrt{2}}\overline{u}_{i}\gamma^{\mu}W_{\mu}^{-}V_{ij}\left(1-\gamma^{5}\right)d_{j} \quad + \quad \frac{g}{\sqrt{2}}\overline{d}_{j}\gamma^{\mu}W_{\mu}^{+}V_{ij}^{*}\left(1-\gamma^{5}\right)u_{i}$$

How do you measure those numbers?

- Magnitudes are typically determined from *ratio* of decay rates
- Example 1 – Measurement of V_{ud}
  - Compare decay rates of neutron decay and muon decay
  - Ratio proportional to V_{ud}^2
  - $|V_{ud}| = 0.9735 \pm 0.0008$
  - V_{ud} of order 1

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





What do we know about the CKM matrix?

- Magnitudes of elements have been measured over time
  - Result of a *large* number of measurements and calculations

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

4 parameters
•3 real
•1 phase

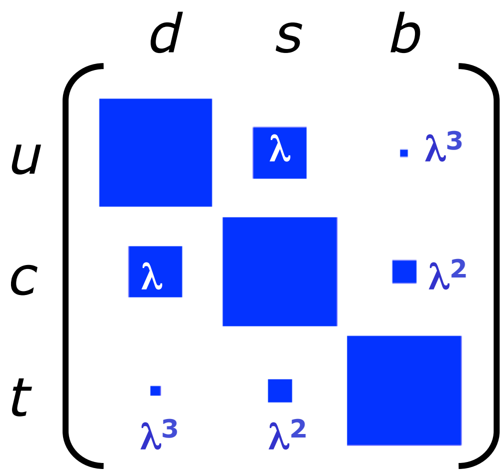
$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.9738 \pm 0.0002 & 0.227 \pm 0.001 & 0.00396 \pm 0.00009 \\ 0.227 \pm 0.001 & 0.9730 \pm 0.0002 & 0.0422 \pm 0.0005 \\ 0.0081 \pm 0.0005 & 0.0416 \pm 0.0005 & 0.99910 \pm 0.00004 \end{pmatrix}$$

Magnitude of elements shown only, no information of phase

Approximately diagonal form

- Values are strongly ranked:
  - Transition within generation favored
  - Transition from 1st to 2nd generation suppressed by $\sin(\theta_c)$
  - Transition from 2nd to 3rd generation suppressed bu $\sin^2(\theta_c)$
  - Transition from 1st to 3rd generation suppressed by $\sin^3(\theta_c)$

CKM magnitudes



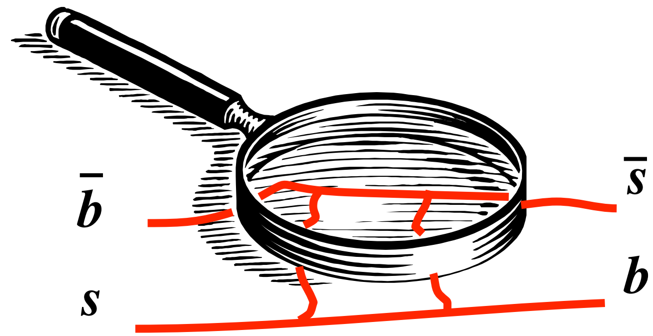
$\lambda = \cos(\theta_c) = 0.23$

Why the ranking?
We don't know (yet)!

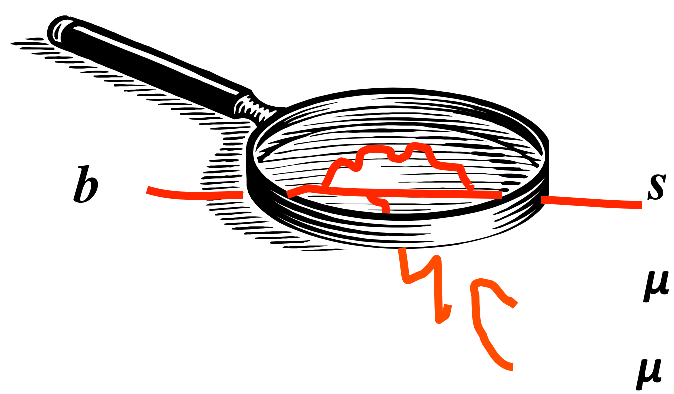
*If you figure this out, you **will** win the nobel prize*

LHCb experiment: study the *B* particle

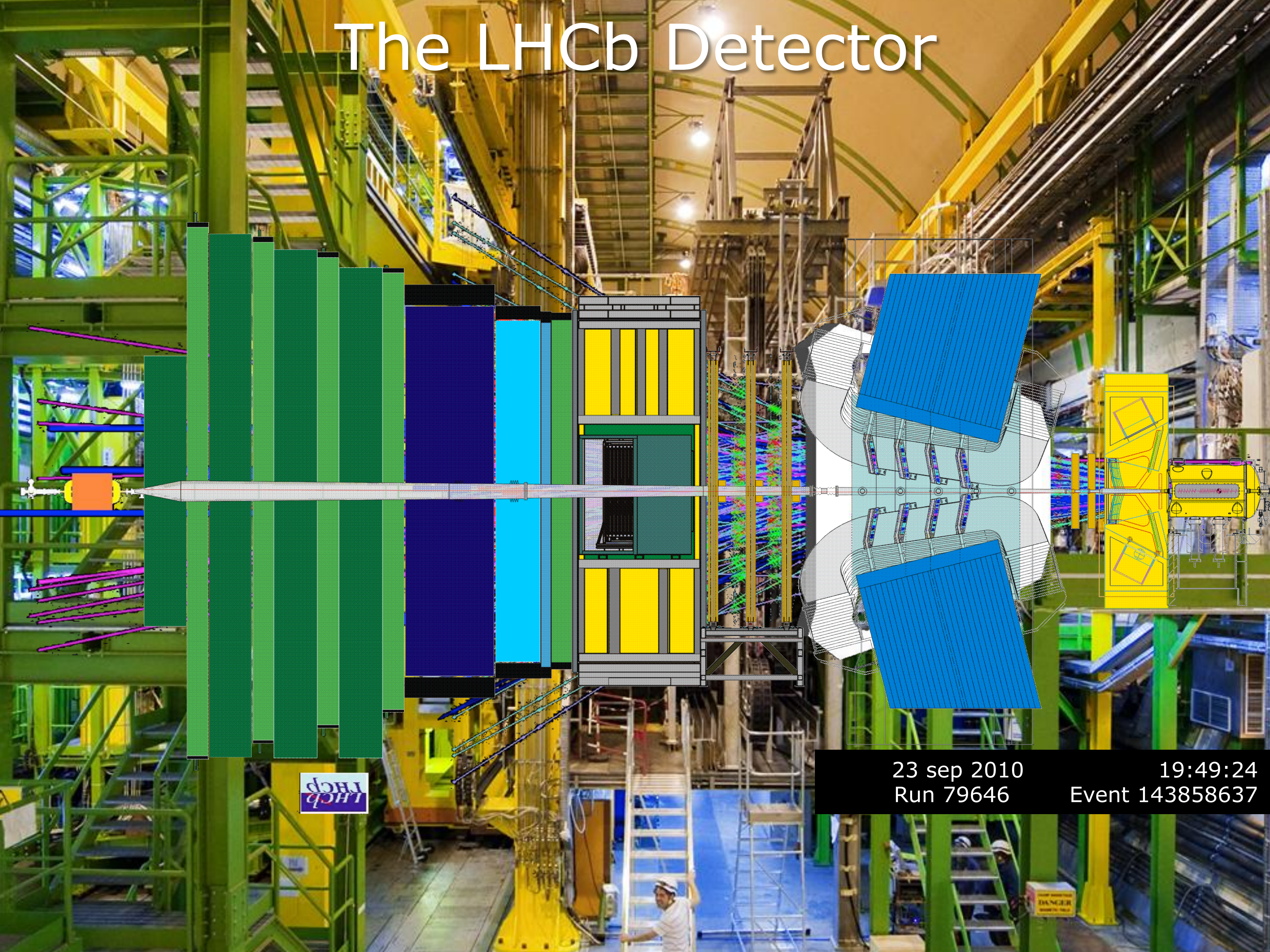
1) Find differences between matter and anti-matter



2) Find new particles

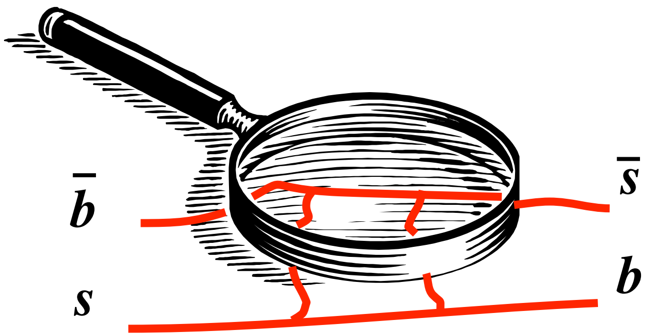


The LHCb Detector



LHCb experiment: study the *B* particle

1) Find differences between matter and anti-matter



CP violation

Final remarks: How about the leptons?

- We now know that neutrinos also have flavour oscillations
  - Neutrinos have mass
  - Diagonalizing Y^l_{ij} doesn't come for free any longer

$$\begin{aligned}\mathcal{L}_{Yukawa} &= Y_{ij}\overline{\psi_{Li}}\,\phi\,\psi_{Rj} + h.c. \\ &= Y^d_{ij}\overline{Q^I_{Li}}\,\phi\,d^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\,\tilde{\phi}\,u^I_{Rj} + Y^l_{ij}\overline{L^I_{Li}}\,\phi\,l^I_{Rj}\end{aligned}$$

- thus there is the equivalent of a CKM matrix for them:
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \begin{bmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} \quad \textbf{\textit{vs}} \quad \begin{bmatrix}|d'\rangle\\ |s'\rangle\\ |b'\rangle\end{bmatrix} = \begin{bmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{bmatrix}\begin{bmatrix}|d\rangle\\ |s\rangle\\ |b\rangle\end{bmatrix}.$$

Final remarks : How about the leptons?

- the equivalent of the CKM matrix
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} \quad \textbf{\textit{vs}} \quad \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}.$$

- a completely different hierarchy!

$$U_{MNSP} \approx \begin{pmatrix} 0.85 & 0.53 & 0 \\ -0.37 & 0.60 & 0.71 \\ -0.37 & 0.60 & -0.71 \end{pmatrix}$$

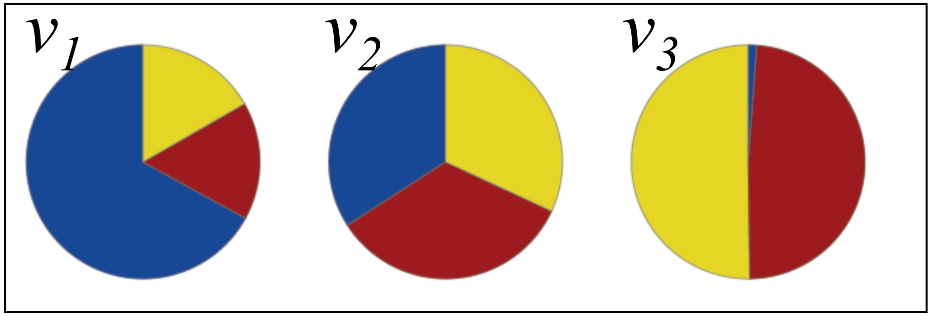
$$V_{CKM} = \begin{pmatrix} 0.97428 & 0.2253 & 0.00347 \\ 0.2252 & 0.97345 & 0.0410 \\ 0.00862 & 0.0403 & 0.999152 \end{pmatrix}$$

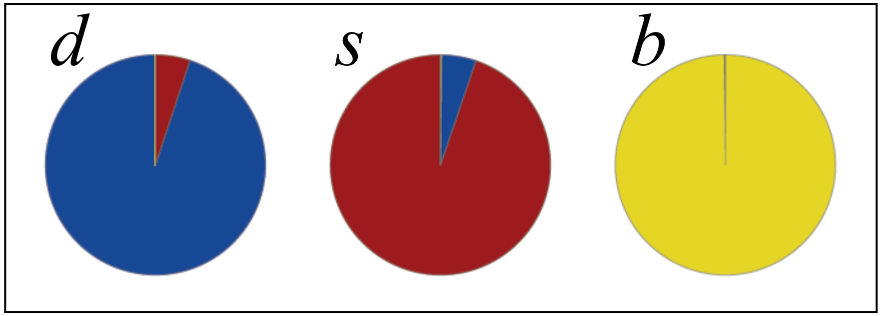
Final remarks: How about the leptons?

- the equivalent of the CKM matrix
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \begin{bmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} \quad \textbf{vs} \quad \begin{bmatrix}|d'\rangle\\ |s'\rangle\\ |b'\rangle\end{bmatrix} = \begin{bmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{bmatrix}\begin{bmatrix}|d\rangle\\ |s\rangle\\ |b\rangle\end{bmatrix}.$$

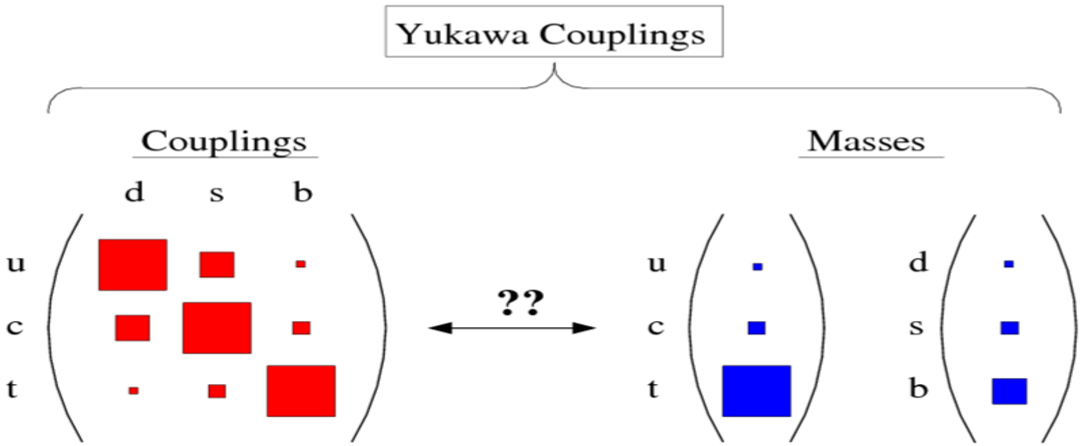
- a completely different $\begin{pmatrix}|U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2\\ |U_{\mu 1}|^2 & |U_{\mu 2}|^2 & |U_{\mu 3}|^2\\ |U_{\tau 1}|^2 & |U_{\tau 2}|^2 & |U_{\tau 3}|^2\end{pmatrix} \approx \begin{pmatrix}\frac{2}{3} & \frac{1}{3} & 0\\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2}\\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2}\end{pmatrix}$





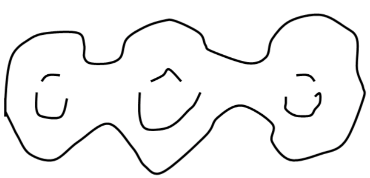
See eg. PhD thesis R. de Adelhart Toorop

What's going on??



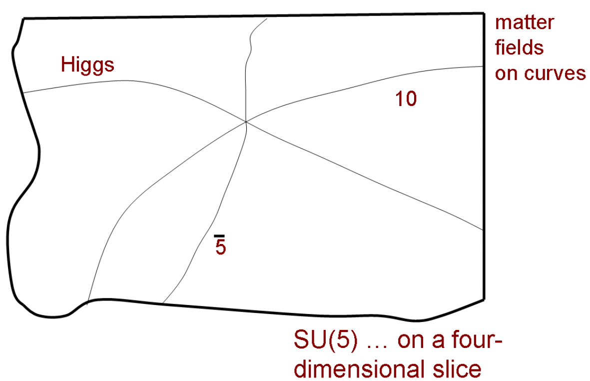
- ??? Edward Witten, 17 Feb 2009…

In this approach, the ordinary Higgs field is a wavefunction on K, as are the quark and lepton fields



Quark and lepton masses and the CKM matrix are determined by the overlaps of these wavefunctions.

The picture is a little like this:



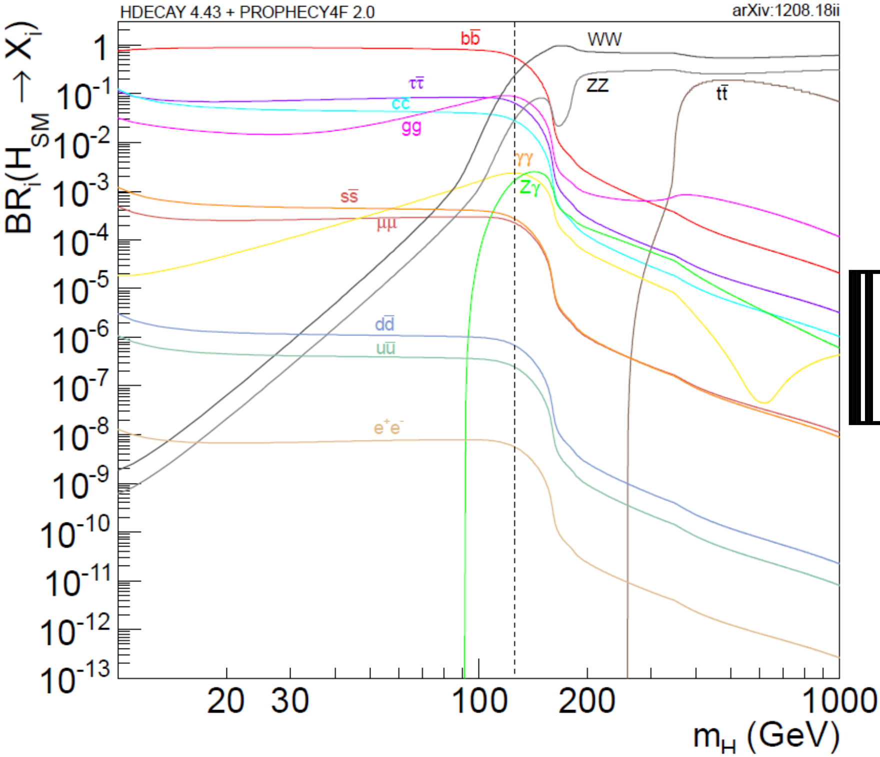
Higgs fields and quarks and leptons are supported on the three curves, and the Yukawa couplings that gives masses to down quarks and charged leptons come from the intersection drawn. (Up quark masses come from a similar intersection.)

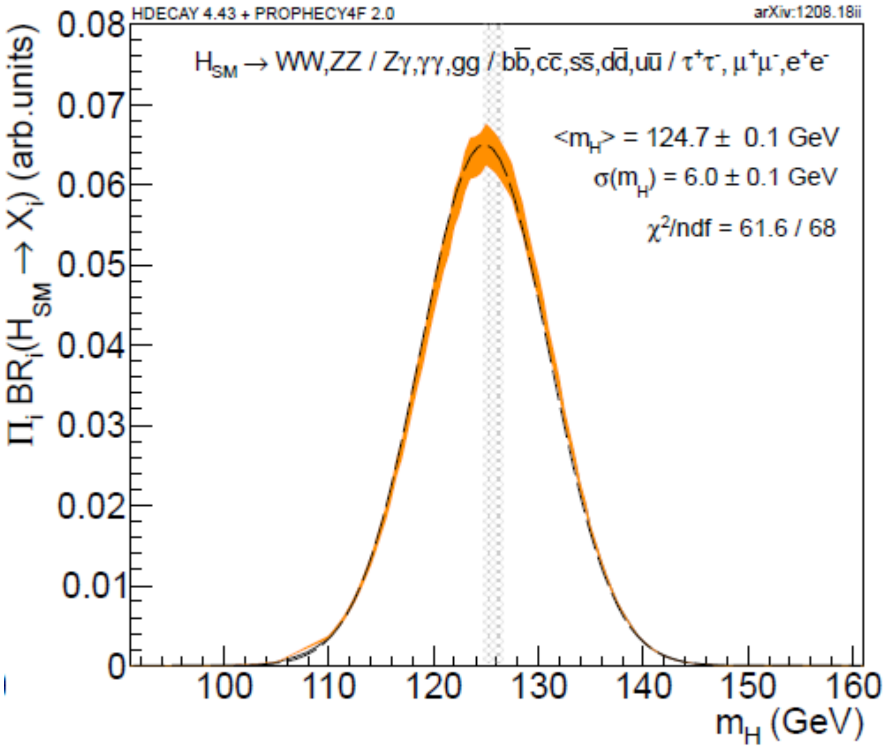
In the leading approximation, only one particle of each type (i.e. the third generation particles – top, bottom, tau) get masses. The others have wavefunctions that vanish at the intersection point.

- See "*From F-Theory GUT's to the LHC*" by Heckman and Vafa (arXiv:0809.3452)

Kabbalah!

- Is 125 GeV coincidental?





David d'Enterria
http://arxiv.org/pdf/1208.1993v1.pdf

Kabbalah?

$$\mathcal{L}_{\text{fermion-mass}} = -\lambda_f[\bar{\psi}_L \phi \psi_R + \bar{\psi}_R \bar{\phi} \psi_L]$$

- More serious stuff:

3.1.1 Lepton masses

$$\begin{aligned}
\mathcal{L}_e &= -\lambda_e \frac{1}{\sqrt{2}} \left[(\bar{\nu}, \bar{e})_L \begin{pmatrix} 0 \\ v+h \end{pmatrix} e_R + \bar{e}_R (0, v+h) \begin{pmatrix} \nu \\ e \end{pmatrix}_L \right] \\
&= -\frac{\lambda_e (v+h)}{\sqrt{2}} [\bar{e}_L e_R + \bar{e}_R e_L] \\
&= -\frac{\lambda_e (v+h)}{\sqrt{2}} \bar{e} e \\
&= - \underbrace{\frac{\lambda_e v}{\sqrt{2}} \bar{e} e}_{\text{electron mass term}} - \underbrace{\frac{\lambda_e}{\sqrt{2}} h \bar{e} e}_{\text{electron-higgs interaction}}
\end{aligned}$$

$$m_e = \frac{\lambda_e v}{\sqrt{2}} \qquad \frac{\lambda_e}{\sqrt{2}} \propto m_e$$

$$m_t = \frac{\lambda_t v}{\sqrt{2}} \quad \Longrightarrow \quad \lambda_t = \frac{m_t \sqrt{2}}{v} = \frac{244.8}{246} = 1.00 \text{ !?}$$

Kabbalah?

- More serious stuff!

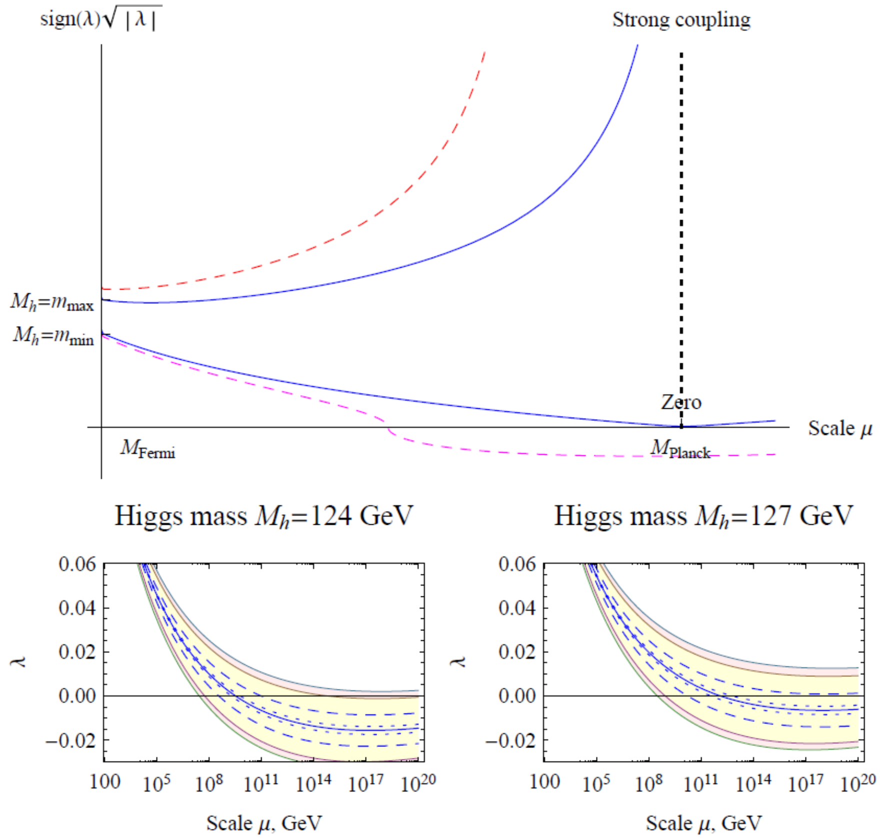


Figure 1. Higgs self-coupling in the SM as a function of the energy scale. The top plot depicts possible behaviors for the whole Higgs boson mass range—Landau pole, stable, or unstable electroweak vacuum. The lower plots show detailed behavior for low Higgs boson masses, with dashed (dotted) line corresponding to the experimental uncertainty in the top mass M_t (strong coupling constant α_s), and the shaded yellow (pink) regions correspond to the total experimental error and theoretical uncertainty, with the latter estimated as 1.2 GeV (2.5 GeV), see section 2 for detailed discussion.

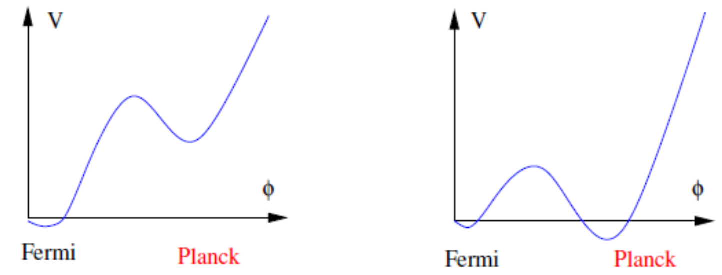


Figure 2. Schematic depiction of the SM effective potential V for the Higgs field for $M_H > M_{min}^{stability}$ (left) and $M_H < M_{min}^{stability}$ (right).

Shaposhnikov et al
http://arxiv.org/pdf/1205.2893.pdf

End

Enough to wonder about…

- Couplings of Higgs to fermions, bosons?
- Why different masses?
- Relation between masses and W-couplings?
- Quark couplings and lepton couplings so different?

(h.c.)

Standard Model Lagrangian (including neutrino mass terms)
From *An Introduction to the Standard Model of Particle Physics, 2nd Edition*,
W. N. Cottingham and D. A. Greenwood, Cambridge University Press, Cambridge, 2007,
Extracted by J.A. Shifflett, updated from Particle Data Group tables at pdg.lbl.gov, 2 Feb 2015.

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}tr(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) && \text{(U(1), SU(2) and SU(3) gauge terms)}\\
& +(\bar\nu_L,\bar e_L)\,\tilde\sigma^\mu iD_\mu \begin{pmatrix}\nu_L\\ e_L\end{pmatrix} + \bar e_R\sigma^\mu iD_\mu e_R + \bar\nu_R\sigma^\mu iD_\mu \nu_R + \text{(h.c.)} && \text{(lepton dynamical term)}\\
& -\frac{\sqrt2}{v}\left[(\bar\nu_L,\bar e_L)\,\phi M^e e_R + \bar e_R \bar M^e \bar\phi \begin{pmatrix}\nu_L\\ e_L\end{pmatrix}\right] && \text{(electron, muon, tauon mass term)}\\
& -\frac{\sqrt2}{v}\left[(-\bar e_L,\bar\nu_L)\,\phi^* M^\nu \nu_R + \bar\nu_R \bar M^\nu \phi^T \begin{pmatrix}-e_L\\ \nu_L\end{pmatrix}\right] && \text{(neutrino mass term)}\\
& +(\bar u_L,\bar d_L)\,\tilde\sigma^\mu iD_\mu \begin{pmatrix}u_L\\ d_L\end{pmatrix} + \bar u_R\sigma^\mu iD_\mu u_R + \bar d_R\sigma^\mu iD_\mu d_R + \text{(h.c.)} && \text{(quark dynamical term)}\\
& -\frac{\sqrt2}{v}\left[(\bar u_L,\bar d_L)\,\phi M^d d_R + \bar d_R \bar M^d \bar\phi \begin{pmatrix}u_L\\ d_L\end{pmatrix}\right] && \text{(down, strange, bottom mass term)}\\
& -\frac{\sqrt2}{v}\left[(-\bar d_L,\bar u_L)\,\phi^* M^u u_R + \bar u_R \bar M^u \phi^T \begin{pmatrix}-d_L\\ u_L\end{pmatrix}\right] && \text{(up, charmed, top mass term)}\\
& +\overline{(D_\mu\phi)}D^\mu\phi - m_h^2[\bar\phi\phi - v^2/2]^2/2v^2. && \text{(Higgs dynamical and mass term)} \quad (1)
\end{aligned}
$$

where (h.c.) means Hermitian conjugate of preceeding terms, $\bar\psi = \text{(h.c.)}\psi = \psi^\dagger = \psi^{*T}$, and the derivative operators are

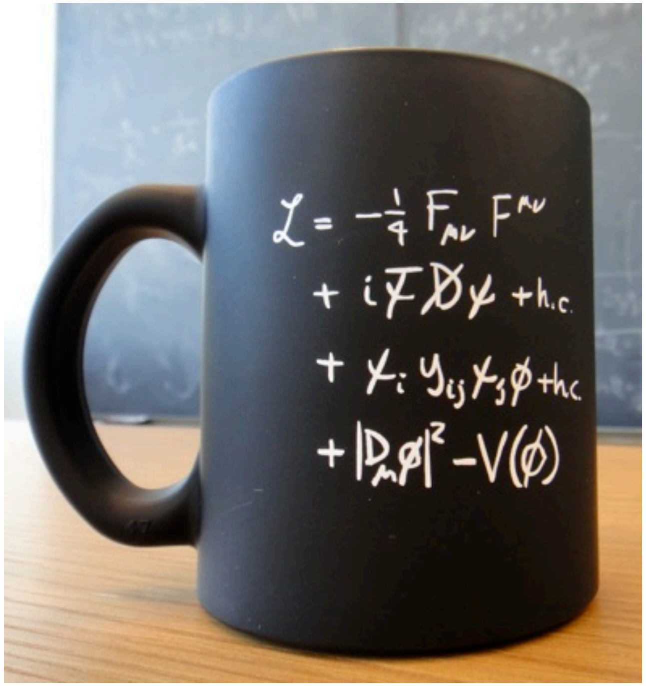
(h.c.)

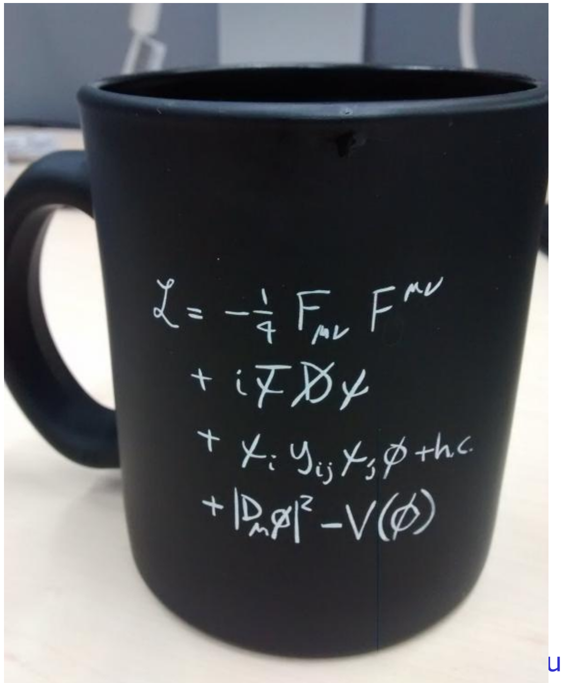
(hot coffee?)

The problem is that the term above,

$$i\bar{\psi}\not{D}\psi$$

already includes its Hermitian conjugate. In physics-speak, we say that the kinetic term is *self-conjugate* (or *Hermitian,* or *self-adjoint*). This just means that there is no additional "+h.c." necessary. In fact, including the "+h.c." means that you are writing the same term twice and the equation is no longer "canonically normalized." This just means that you ought to rescale some of your variables.





From: https://www.quantumdiaries.org/2011/06/26/cern-mug-summarizes-standard-model-but-is-off-by-a-factor-of-2/