

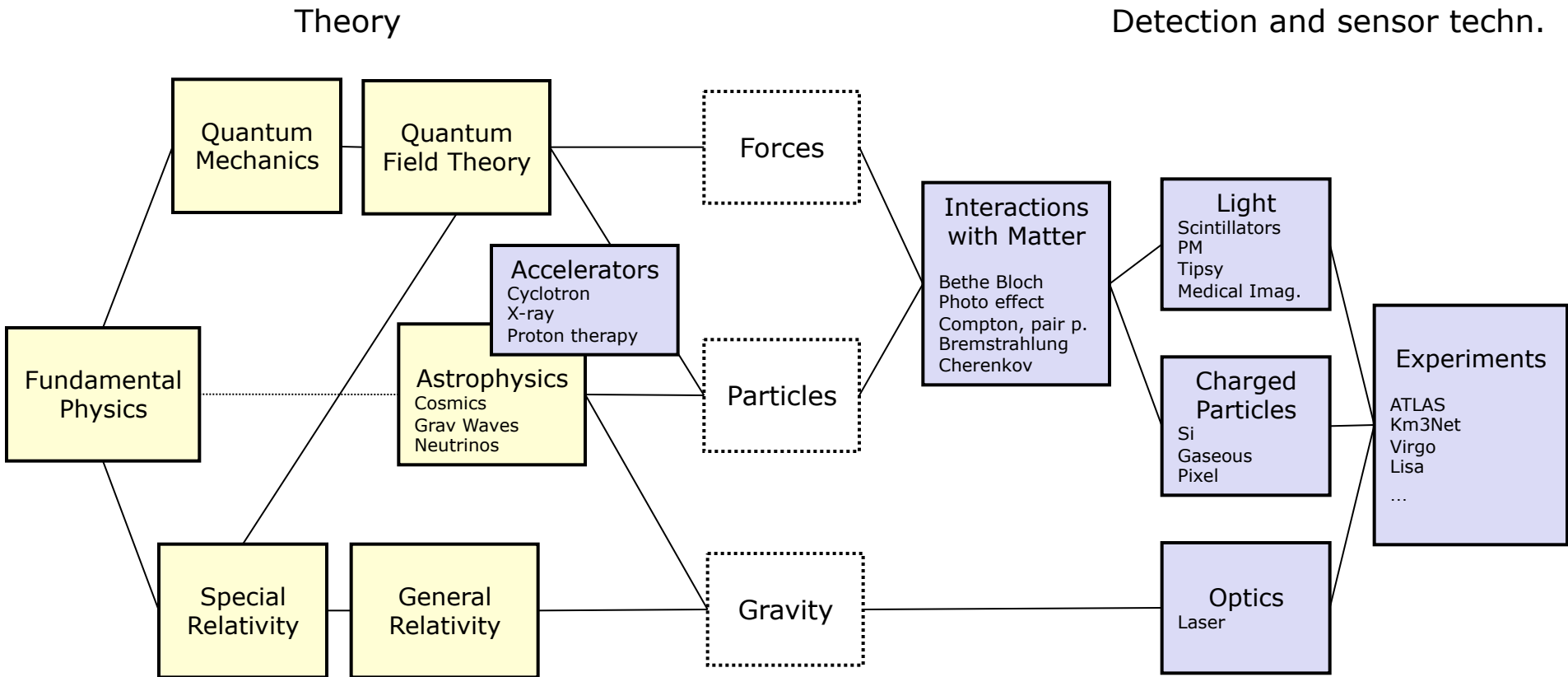
“Elementary Particles”
Lecture 5

Niels Tuning
Harry van der Graaf

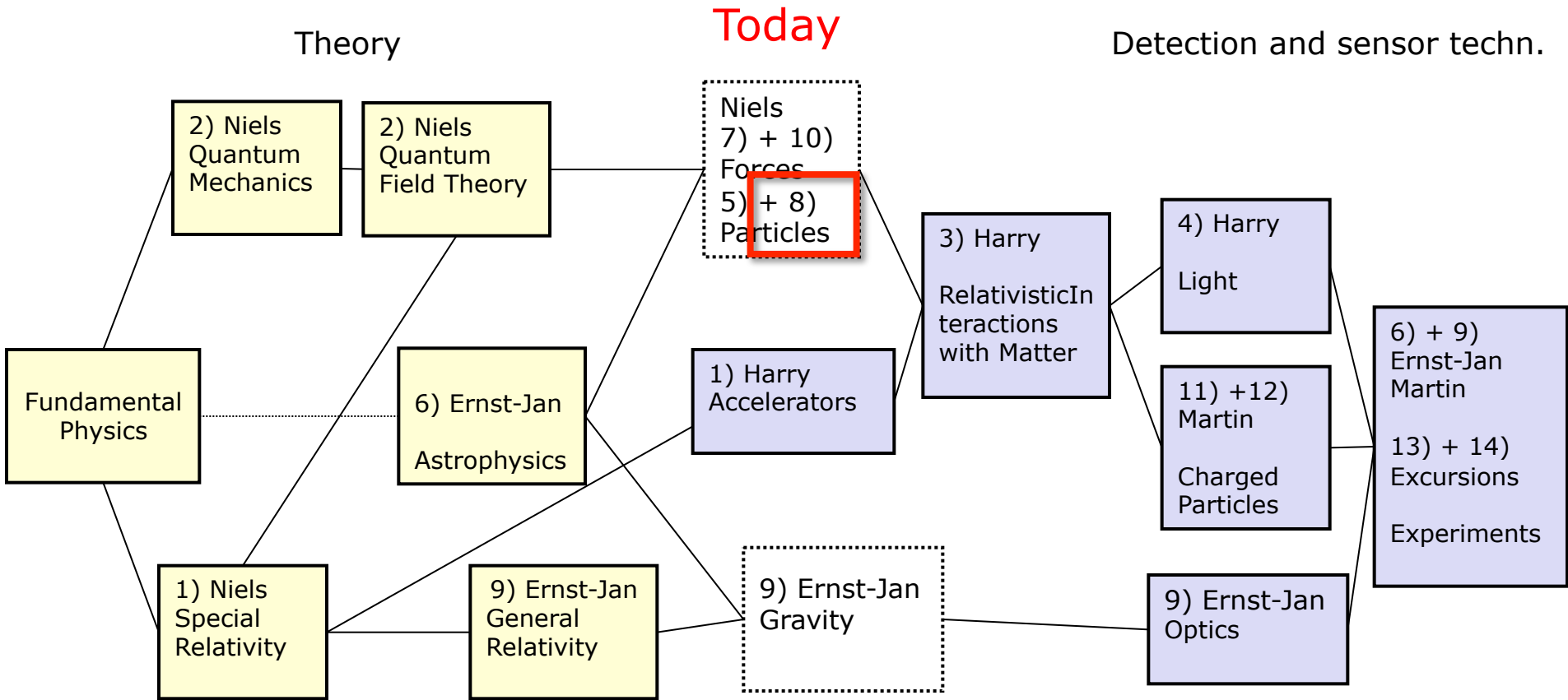
Thanks

- Ik ben schatplichtig aan:
 - Dr. Ivo van Vulpen (UvA)
 - Prof. dr. ir. Bob van Eijk (UT)
 - Prof. dr. Marcel Merk (VU)

Plan



Plan



Schedule

- 1) 11 Feb: Accelerators (Harry vd Graaf) + Special relativity (Niels Tuning)
- 2) 18 Feb: Quantum Mechanics (Niels Tuning)
- 3) 25 Feb: Interactions with Matter (Harry vd Graaf)
- 4) 3 Mar: Light detection (Harry vd Graaf)
- 5) 10 Mar: Particles and cosmics (Niels Tuning)
- 6) 17 Mar: Forces (Niels Tuning)
- 7) 24 Mar: Astrophysics and Dark Matter (Ernst-Jan Buis)
- break
- 8) 21 Apr: e^+e^- and ep scattering (Niels Tuning)
- 9) 28 Apr: Gravitational Waves (Ernst-Jan Buis)
- 10) 12 May: Higgs and big picture (Niels Tuning)
- 11) 19 May: Charged particle detection (Martin Franse)
- 12) 26 May: Applications: experiments and medical (Martin Franse)

- 13) 2 Jun: Nikhef excursie
- 14) 8 Jun: CERN excursie

Plan

	1) Intro: Standard Model & Relativity	11 Feb
1900-1940	2) Basis	18 Feb
	1) Atom model, strong and weak force	
	2) Scattering theory	
1945-1965	3) Hadrons	10 Mar
	1) Isospin, strangeness	
	2) Quark model, GIM	
1965-1975	4) Standard Model	17 Mar
	1) QED	
	2) Parity, neutrinos, weak inteaction	
	3) QCD	
1975-2000	5) e^+e^- and DIS	21 Apr
2000-2015	6) Higgs and CKM	12 May

Homework 3

Exercises Lecture 3:

1 Completing the decuplet

Murray Gell-Mann proposed the quark-model to explain the large number of observed particles. From his scheme, he concluded that there must be a particle with quark content (s, s, s) , the Ω^- particle. Let's have a look at the decay chain.

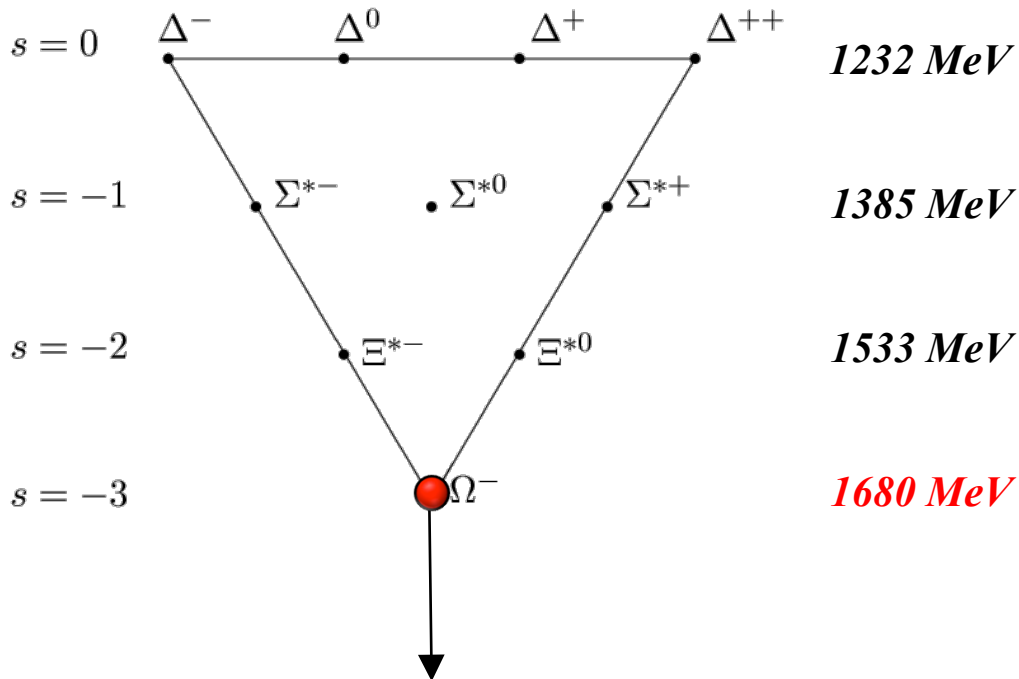
- Gell-Mann predicted the mass of the Ω^- , by inspecting the masses in the baryon decuplet (containing the spin- $\frac{3}{2}$ baryons). What would be your best guess for the mass of the Ω^- , using $m_\Delta = 1232\text{MeV}$, $m_{\Sigma^*} = 1385\text{MeV}$ and $m_{\Xi^*} = 1532\text{MeV}$? How close is your prediction to the observed value? (Browse to the webpage of the Particle Data Group, <http://pdglive.lbl.gov>.)
- Next, look for the π^0 particle on the webpage of the Particle Data Group. How does it decay? What is its lifetime? If the π^0 has an energy of 1.4 GeV, how far will it fly? What is the signature of a π^0 in a bubble-chamber picture?

a) $m_{\Sigma^*} - m_\Delta = 153\text{MeV}$ and $m_{\Xi^*} - m_{\Sigma^*} = 147\text{MeV}$
 $\Rightarrow m_{\Omega^-} \approx m_{\Xi^*} + 150 = 1682\text{MeV}$.
 $m_{\Omega^-} = 1672\text{ MeV}$, so Gell-Mann's estimate was only 10 MeV high...

b) $BR(\pi^0 \rightarrow \gamma\gamma) = 99.8\%$
 $\tau_{\pi^0} = 8.5 \times 10^{-17}\text{s}$
 $1.4\text{ GeV} \Rightarrow t = \gamma \times \tau = (E/m) \times \tau = 10 \times 8.5 \times 10^{-17}\text{s}$
Flight distance: $d = ct = 3 \times 10^8 \times 8.5 \times 10^{-16} = 2.5 \times 10^{-7}\text{m} = 0.25\mu\text{m}$.
The π^0 decays "instantaneously" to two photons.
They can be seen after they converted to an electron-positron pair: $\gamma \rightarrow e^+e^-$.

Exercises Lecture 3: mass of Ω and decay of π^0

- a) Gell-Mann predicted the mass of the Ω^- , by inspecting the masses in the baryon decuplet (containing the spin- $\frac{3}{2}$ baryons). What would be your best guess for the mass of the Ω^- , using $m_{\Delta^-} = 1232\text{MeV}$, $m_{\Sigma^*} = 1385\text{MeV}$ and $m_{\Xi^*} = 1532\text{MeV}$? How close is your prediction to the observed value? (Browse to the webpage of the Particle Data Group, <http://pdglive.lbl.gov>.)
- b) Next, look for the π^0 particle on the webpage of the Particle Data Group. How does it decay? What is its lifetime? If the π^0 has an energy of 1.4 GeV, how far will it fly? What is the signature of a π^0 in a bubble-chamber picture?



Gell-Mann and Zweig predicted the Ω^-



Exercises Lecture 3:

c) Look at the bubble-chamber picture which led to the discovery of the Ω^- at Brookhaven. How does the Ω^- decay? Also, specify the quark content of all particles involved.

d) Subsequently, how does the baryon with $S = 2$ decay? Also, specify the quark content of all particles involved.

$S = -2$

e) Subsequently, how does the baryon with $S = 1$ decay? Also, specify the quark content of all particles involved.

$S = -1$

c) $\Omega^- \rightarrow \Xi^0 \pi^-$
 $(sss) \rightarrow (ssu) + (\bar{u}d)$, (so: $s \rightarrow u(\bar{u}d)$)

d) $\Xi^0 \rightarrow \Lambda^0 \pi^0$
 $(ssu) \rightarrow (sdu) + (\bar{u}u - \bar{d}d)$

e) $\Lambda^0 \rightarrow p \pi^-$
 $(sdu) \rightarrow (udu) + (\bar{u}d)$

Exercises Lecture 3:

2 $\pi^\pm p$ Scattering

We will inspect the cross-section for π^-p and π^+p scattering, as a function of the center-of-mass energy of the $\pi^\pm p$ -system. If we consider π^-p scattering, the elastic process $\pi^-p \rightarrow \pi^-p$ is one of the possibilities. However, if (in the Yukawa picture) a charged pion is exchanged instead of a neutral pion, the quasi-elastic process $\pi^-p \rightarrow \pi^0n$ can occur.

Let's compare the following processes:

- (a) $\pi^+p \rightarrow \pi^+p$
- (b) $\pi^-p \rightarrow \pi^-p$
- (c) $\pi^-p \rightarrow \pi^0n$

- a) Decompose (π^+p) , (π^-p) and (π^0n) in $I = 3/2$ and $I = 1/2$ components, using the Clebsch-Gordan coefficients.
- b) Let's compare the transition amplitudes (or "matrix element" \mathcal{M}) of the three processes (a), (b) and (c), in terms of the $I = 3/2$ and $I = 1/2$ components:

$$(a) \mathcal{M}(\pi^+p \rightarrow \pi^+p) = \langle \pi^+p | \pi^+p \rangle = \mathcal{M}_{3/2}$$

$$(b) \mathcal{M}(\pi^-p \rightarrow \pi^-p) = \langle \pi^-p | \pi^-p \rangle = \frac{1}{3} \mathcal{M}_{3/2} + \frac{2}{3} \mathcal{M}_{1/2}$$

Write the equivalent decomposition for process (c).

a)

$$\pi^+p: |1, +1\rangle = \left| \frac{1}{2}, +\frac{1}{2} \right\rangle = \left| \frac{3}{2}, +\frac{3}{2} \right\rangle \quad (1)$$

$$\pi^-p: |1, -1\rangle = \left| \frac{1}{2}, +\frac{1}{2} \right\rangle = \sqrt{\frac{1}{3}} \left| \frac{3}{2}, -\frac{1}{2} \right\rangle - \sqrt{\frac{2}{3}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \quad (2)$$

$$\pi^0n: |1, 0\rangle = \left| \frac{1}{2}, -\frac{1}{2} \right\rangle = \sqrt{\frac{2}{3}} \left| \frac{3}{2}, -\frac{1}{2} \right\rangle + \sqrt{\frac{1}{3}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \quad (3)$$

b)

$$(c) \mathcal{M}(\pi^-p \rightarrow \pi^0n) = \langle \pi^0n | \pi^-p \rangle = \frac{\sqrt{2}}{3} \mathcal{M}_{3/2} - \frac{\sqrt{2}}{3} \mathcal{M}_{1/2}$$

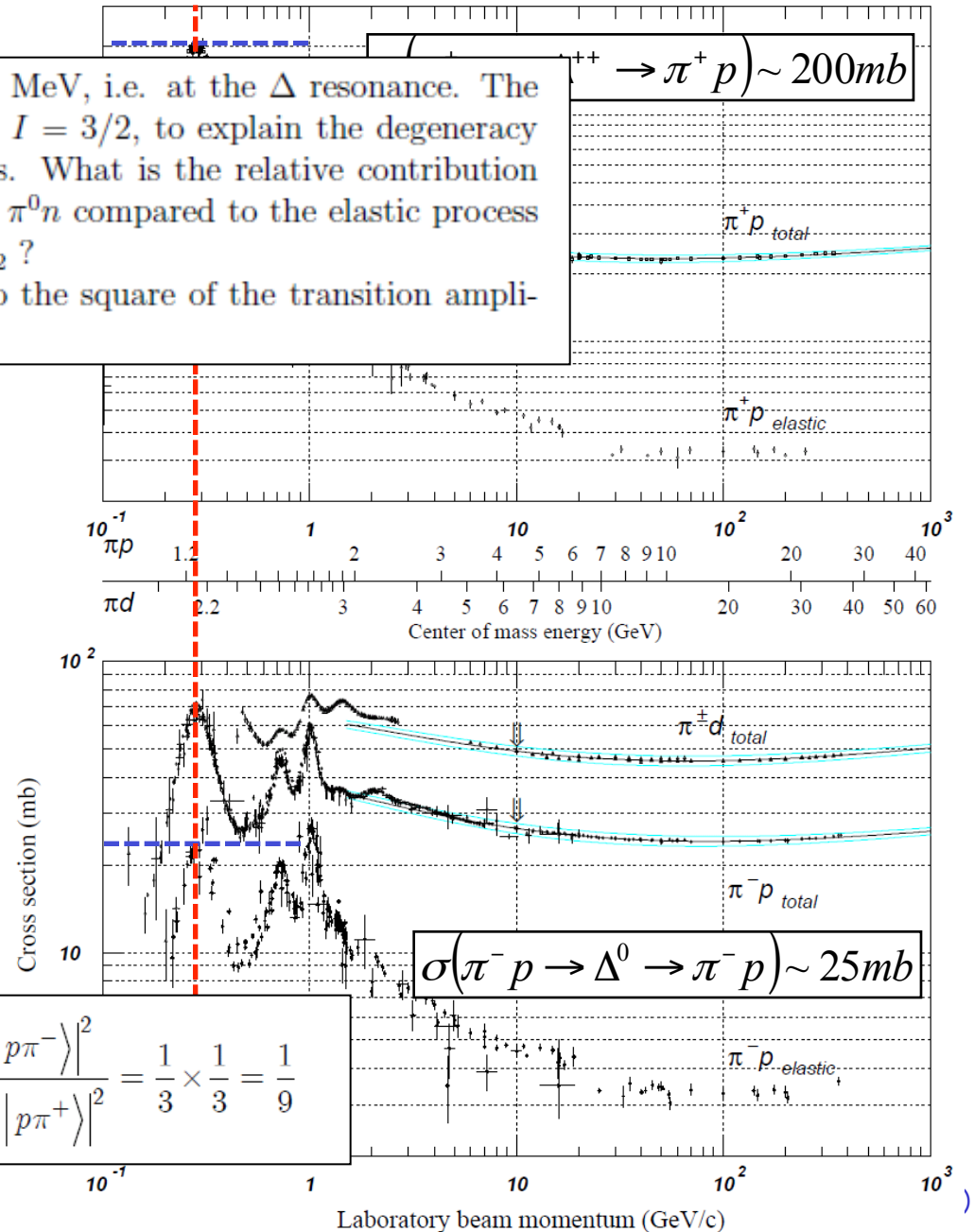
$1 \times 1/2$	$3/2$	$3/2$	$1/2$
$+3/2$	$3/2$	$1/2$	
$+1$	$+1/2$	1	$+1/2$
$+1$	$-1/2$	$1/3$	$2/3$
0	$+1/2$	$2/3$	$-1/3$
		$3/2$	$1/2$
		$-1/2$	$-1/2$
		0	$-1/2$
		-1	$+1/2$
		$2/3$	$1/3$
		$1/3$	$-2/3$

Exercises Lecture 3:

c) Let's compare the cross section at $\sqrt{s} = 1232$ MeV, i.e. at the Δ resonance. The Δ particles (or "resonances") are *isospin-3/2*, $I = 3/2$, to explain the degeneracy of the four particles with (almost) equal mass. What is the relative contribution of the quasi-elastic contribution $\pi^- p \rightarrow \Delta^0 \rightarrow \pi^0 n$ compared to the elastic process $\pi^- p \rightarrow \Delta^0 \rightarrow \pi^- p$, in terms of $\mathcal{M}_{3/2}$ and $\mathcal{M}_{1/2}$? (Note that the cross section is proportional to the square of the transition amplitude.)

Compare Δ resonance in elastic scattering:

- 1) $\pi^+ p \rightarrow \pi^+ p$
- 2) $\pi^- p \rightarrow \pi^- p$



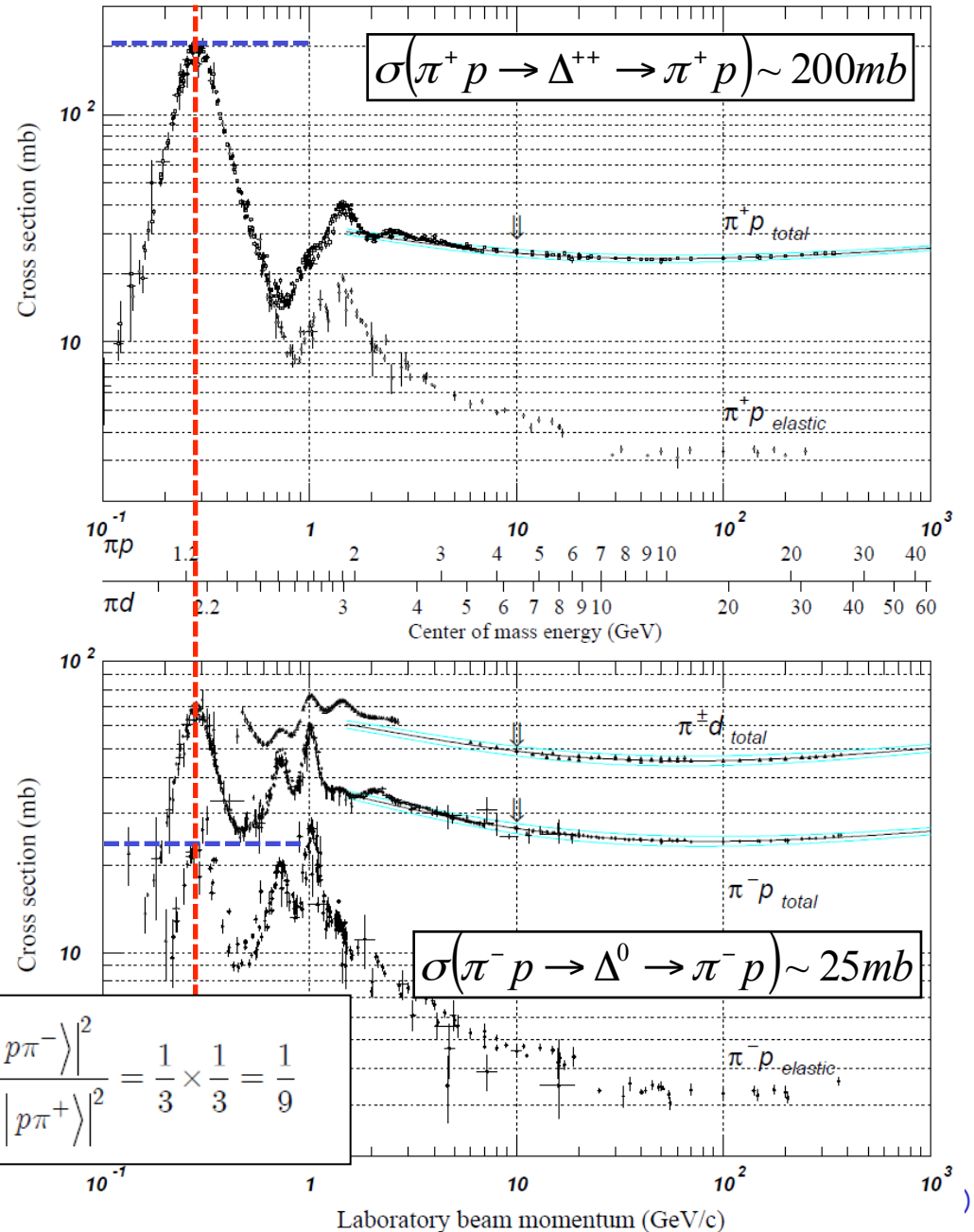
$$\frac{\sigma(p\pi^- \rightarrow \Delta^0 \rightarrow p\pi^-)}{\sigma(p\pi^+ \rightarrow \Delta^{++} \rightarrow p\pi^+)} = \frac{|\langle p\pi^- | \Delta^0 \rangle|^2}{|\langle p\pi^+ | \Delta^{++} \rangle|^2} \times \frac{|\langle \Delta^0 | p\pi^- \rangle|^2}{|\langle \Delta^{++} | p\pi^+ \rangle|^2} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

Exercises Lecture 3:

Compare Δ resonance in elastic scattering:

1) $\pi^+ p \rightarrow \pi^+ p$

2) $\pi^- p \rightarrow \pi^- p$



$$\frac{\sigma(p\pi^- \rightarrow \Delta^0 \rightarrow p\pi^-)}{\sigma(p\pi^+ \rightarrow \Delta^{++} \rightarrow p\pi^+)} = \frac{|\langle p\pi^- | \Delta^0 \rangle|^2}{|\langle p\pi^+ | \Delta^{++} \rangle|^2} \times \frac{|\langle \Delta^0 | p\pi^- \rangle|^2}{|\langle \Delta^{++} | p\pi^+ \rangle|^2} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

Exercises Lecture 3:

- c) Let's compare the cross section at $\sqrt{s} = 1232$ MeV, i.e. at the Δ resonance. The Δ particles (or "resonances") are *isospin-3/2*, $I = 3/2$, to explain the degeneracy of the four particles with (almost) equal mass. What is the relative contribution of the quasi-elastic contribution $\pi^- p \rightarrow \Delta^0 \rightarrow \pi^0 n$ compared to the elastic process $\pi^- p \rightarrow \Delta^0 \rightarrow \pi^- p$, in terms of $\mathcal{M}_{3/2}$ and $\mathcal{M}_{1/2}$? (Note that the cross section is proportional to the square of the transition amplitude.)
- d) So, how does the total $\pi^+ p$ cross section at the Δ resonance compare to the total $\pi^- p$ cross section?

- c) The magnitude of the cross sections for the three processes are related as follows:

$$\sigma_{(a)} : \sigma_{(b)} : \sigma_{(c)} = 9|\mathcal{M}_{3/2}|^2 : |\mathcal{M}_{3/2} + 2\mathcal{M}_{1/2}|^2 : |\sqrt{2}\mathcal{M}_{3/2} - \sqrt{2}\mathcal{M}_{1/2}|^2$$

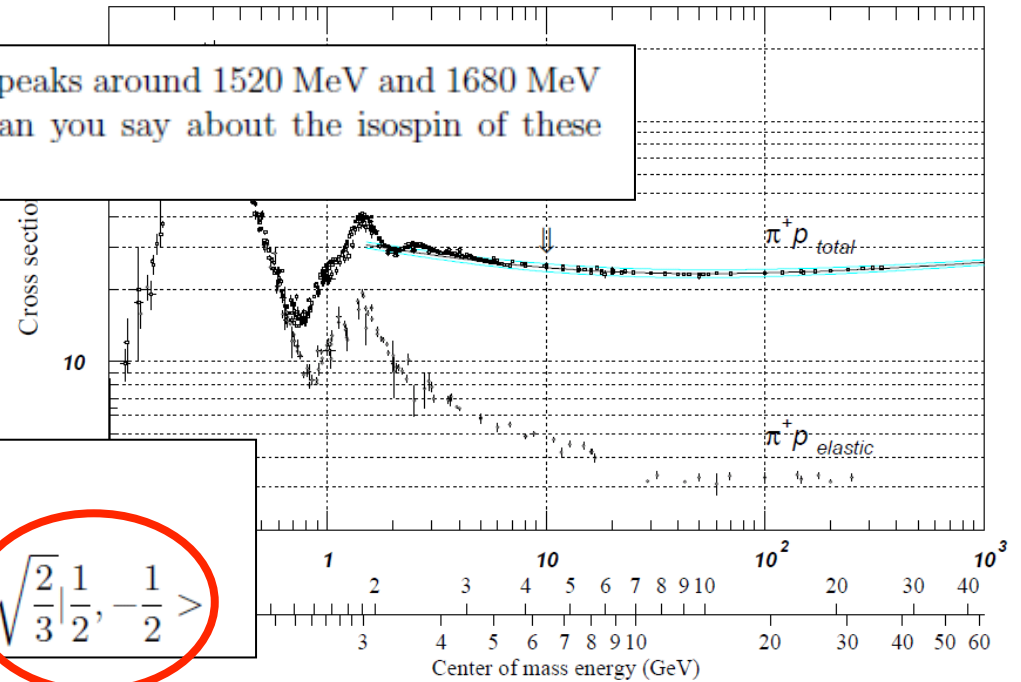
- d) So, the contribution to the cross section at the Δ resonance ($I = 3/2$) is

$$\sigma_{(B),3/2} : \sigma_{(C),3/2} = 1 : 2,$$

so the quasi-inelastic process (C) contributes twice as much as the elastic process (B). The elastic $\pi^- p$ cross section was $9\times$ smaller compared to $\pi^+ p$. The total $\pi^- p$ cross section is $3\times$ smaller compared to $\pi^+ p$.

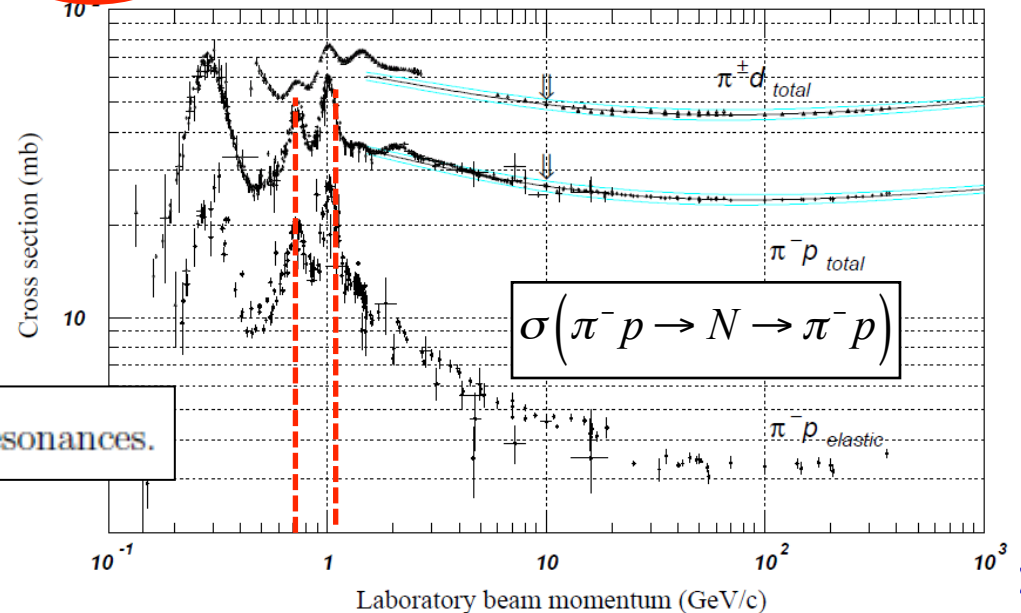
Exercises Lecture 3:

e) In the spectrum for π^-p scattering there are peaks around 1520 MeV and 1680 MeV that are absent in π^+p scattering. What can you say about the isospin of these resonances?



$$\pi^+p : |1, +1\rangle \left| \frac{1}{2}, +\frac{1}{2} \right\rangle = \left| \frac{3}{2}, +\frac{3}{2} \right\rangle$$

$$\pi^-p : |1, -1\rangle \left| \frac{1}{2}, +\frac{1}{2} \right\rangle = \sqrt{\frac{1}{3}} \left| \frac{3}{2}, -\frac{1}{2} \right\rangle + \sqrt{\frac{2}{3}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$$



e) The peaks in π^-p scattering are $I = 1/2$ resonances.

$$\sigma(\pi^-p \rightarrow N \rightarrow \pi^-p)$$

Exercises Lecture 3:

3 Decay rates

We will use the Clebsch-Gordan tables to predict some decay rates, see <http://pdg.lbl.gov/2012/reviews/rpp2012-rev-clebsch-gordan-coefs.pdf>.

The fractional decay rate to a specific final state is called “branching fraction” or “branching ratio”. For a given particle, the sum of all branching fractions add up to 100%.

- Let's consider the ρ particle. This is a meson triplet with the same quark content as the pion, and also manifests itself as an isospin triplet. (The difference with the pion is that the ρ is heavier, about 770 MeV instead of 140 MeV, and that it has spin-1, and not spin-0 as the pion.) Decompose the ρ^+ ($|1, +1\rangle$) in $I = 1$ isospin components (ie. look at the table 1×1).
- The ρ decays as $\rho \rightarrow \pi\pi$. What is the branching ratio for $\rho^+ \rightarrow \pi^+\pi^0$?
- Decompose the ρ^0 ($|1, 0\rangle$) in $I = 1$ iso-spin components (ie. look at the table 1×1). What is the branching ratio for $\rho^0 \rightarrow \pi^0\pi^0$?

1×1		(a)				
	2	2	1			
	+2	1	+1			
+1	+1	1	+1			
	0	1/2	1/2	2	1	0
0	+1	1/2	-1/2	0	0	0
	-1	1/6	1/2	1/3		
	0	2/3	0	-1/3		
	+1	1/6	-1/2	1/3		

- Decompose $|1, +1\rangle$ in $I = 1$ components, so look at table 1×1 :

$$\rho^+ : |1, +1\rangle = \sqrt{\frac{1}{2}}|1, +1\rangle |1, 0\rangle - \sqrt{\frac{1}{2}}|1, 0\rangle |1, +1\rangle$$

- The ρ^+ decays purely to $\pi^+\pi^0$. (The relative minus sign when interchanging the π^+ and π^0 come from the fact that the ρ has spin-1, and the pions have spin-0. The total orbital momentum is conserved, and so the final state must have a relative *orbital* momentum $L = 1$. This gives a factor $(-1)^L$ when swapping the pions...)

-

$$|1, 0\rangle = \sqrt{\frac{1}{2}}|1, -1\rangle |1, +1\rangle + 0|1, 0\rangle |1, 0\rangle - \sqrt{\frac{1}{2}}|1, +1\rangle |1, -1\rangle$$

The ρ^0 cannot decay to $\pi^0\pi^0$!



Homework 4

Exercises Lecture 4:

Adjoint spinor:

1 Dirac equation from the Lagrangian

The Dirac equation was found by Paul Dirac by constructing the equation of motion that is both relativistically correct (like the Klein-Gordon equation), and linear in d/dt (like the Schrödinger equation) to avoid negative-energy solutions,

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0.$$

Hamilton's principle of stationary (or "least") action says that the "path" taken by the system between times t_1 and t_2 , is the one for which the change in action is minimal. (The action S is obtained from the time-integral of the Lagrangian, i.e. by integrating the difference of kinetic and potential energy of the system over time.) This requirement is equivalent to the Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial \psi(x)} = \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi(x))}.$$

Show that the Euler-Lagrange equation of the Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi$$

leads to the Dirac equation, and its adjoint, $(i\gamma^\mu \partial_\mu + m)\bar{\psi}(x) = 0$. Note that you need to consider ψ and $\bar{\psi}$ as independent fields.

(NB: The Dirac equation for the adjoint spinor is obtained as follows:

$$i\gamma^0 \frac{\partial \psi}{\partial t} + i\gamma^k \frac{\partial \psi}{\partial x^k} - m\psi = 0$$

Hermitean conjugate:

$$-i \frac{\partial \psi^\dagger}{\partial t} \gamma^0 - i \frac{\partial \psi^\dagger}{\partial x^k} (-\gamma^k) - m\psi^\dagger = 0$$

However, the minus sign in $-\gamma^k$ disturbs the Lorentz invariant form. This can be restored by multiplying from the right by γ^0 . This is the reason the adjoint spinor is introduced: $\bar{\psi} = \psi^\dagger \gamma^0$.

$$-i \frac{\partial \bar{\psi}}{\partial t} \gamma^0 - i \frac{\partial \bar{\psi}}{\partial x^k} \gamma^k - m\bar{\psi} = 0 \implies i\gamma_\mu \partial^\mu \bar{\psi} + m\bar{\psi} = 0$$

Exercises Lecture 4:

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Consider ψ and $\bar{\psi}$ as independent fields, ie. $\partial\psi/d\bar{\psi} = 0$.

$$\frac{\partial \mathcal{L}}{\partial \psi(x)} = \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi(x))}.$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{\psi}(x)} &= i\gamma_\mu \partial^\mu \psi - m\psi \\ \frac{\partial \mathcal{L}}{\partial (\partial_\mu \bar{\psi}(x))} &= i\gamma^\mu \Rightarrow \partial_\mu (i\gamma^\mu) = 0 \\ &\Rightarrow i\gamma_\mu \partial^\mu \psi - m\psi = 0 \end{aligned} \tag{1}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \psi(x)} &= -m\bar{\psi} \\ \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi(x))} &= i\bar{\psi}\gamma_\mu \\ &\Rightarrow i\gamma_\mu \partial^\mu \bar{\psi} + m\bar{\psi} = 0 \end{aligned} \tag{2}$$

Exercises Lecture 4:

2 Massless gauge bosons

a) The Lagrangian that describes the fermions in QED is

$$\mathcal{L}_{\text{QED,fermion}} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi.$$

Show that the Lagrangian is invariant under the local gauge transformation

$$\begin{aligned} \psi &\rightarrow \psi' = e^{ie\alpha(x)}\psi, \\ \text{with } A^\mu &\rightarrow A'^\mu = A^\mu - \partial^\mu\alpha(x). \end{aligned} \quad (1)$$

a)

$$\begin{aligned} \mathcal{L}'_{\text{QED,fermion}} &= \bar{\psi}'(i\gamma^\mu D'_\mu - m)\psi' \\ &= \bar{\psi}'(i\gamma^\mu(\partial_\mu + ieA'_\mu) - m)\psi' \\ &= e^{-ie\alpha(x)}\bar{\psi}\left(i\gamma^\mu(\partial_\mu + ieA_\mu - ie(\partial_\mu\alpha(x))) - m\right)e^{ie\alpha(x)}\psi \\ \text{(Use: } \partial_\mu(e^{ie\alpha(x)}\psi) &= e^{ie\alpha(x)}(ie(\partial_\mu\alpha(x))\psi + \partial_\mu\psi)) \\ &= \bar{\psi}\left(i\gamma^\mu\left(ie(\partial_\mu\alpha(x)) + \partial_\mu + ieA_\mu - ie(\partial_\mu\alpha(x))\right) - m\right)\psi \\ &= \bar{\psi}\left(i\gamma^\mu(\partial_\mu - ieA_\mu - m)\right)\psi \\ &= \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \\ &= \mathcal{L}_{\text{QED,fermion}} \end{aligned} \quad (2)$$

Exercises Lecture 4:

- b) Adding the term that describes the free photons (which “by the way” lead to the Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$), gives

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

If the photon would have a mass, the corresponding mass term would be $\mathcal{L}_{\gamma\text{mass}} = \frac{1}{2}m^2 A^\mu A_\mu$. Local gauge invariance implies that the Lagrangian remains unchanged under the transformation

$$A^\mu \rightarrow A'^\mu = A^\mu - \partial^\mu \alpha(x)$$

Show that the mass term of the photon violates local gauge invariance.

b)

$$\begin{aligned} A^\mu A_\mu \rightarrow A'^\mu A'_\mu &= (A^\mu - \partial^\mu \alpha(x))(A_\mu - \partial_\mu \alpha(x)) \\ &= A^\mu A_\mu - A^\mu \partial_\mu \alpha(x) - \partial^\mu \alpha(x) A_\mu + (\partial^\mu \alpha(x))(\partial_\mu \alpha(x)) \\ &\neq A^\mu A_\mu \end{aligned} \tag{3}$$

That is why the photon must be massless!

Exercises Lecture 4:

Of course... The W and Z are massive...

How is that then possible?!

That was the enigma in 60's and the tric was a very weird phenomenon...

The Higgs mechanism: we will discuss this in two weeks

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That is why the photon must be massless!

Exercises Lecture 4:

3 Self-interacting gauge bosons

Instead of the “simple” phase factor in QED, see Eq. 1, we will now consider a rotation in isospin space

$$\psi \rightarrow \psi' = e^{\frac{i}{2}\vec{\tau}\cdot\vec{\alpha}(x)}\psi, \quad (2)$$

with ψ a two-component object in isospin space.

- a) In order to keep the Lagrangian invariant under this gauge transformation, the covariant derivative $D_\mu = \mathbb{1}\partial_\mu + igB_\mu$ is introduced, with B_μ a (2×2) matrix. It can be expressed in terms of the three gauge fields $\vec{b}_\mu(x) = (b_{\mu,1}(x), b_{\mu,2}(x), b_{\mu,3}(x))$. Write B_μ as a (2×2) matrix, using the Pauli matrices $\vec{\tau}$, starting from $B_\mu = \frac{1}{2}\vec{\tau}\cdot\vec{b}_\mu(x)$.

- a) Use the Pauli matrices:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} B_\mu &= \frac{1}{2}\vec{\tau}\cdot\vec{b}_\mu(x) \\ &= \frac{1}{2}(\tau_1 b_{\mu,1} + \tau_2 b_{\mu,2} + \tau_3 b_{\mu,3}) \\ &= \frac{1}{2} \begin{pmatrix} b_{\mu,3} & b_{\mu,1} - ib_{\mu,2} \\ b_{\mu,1} + ib_{\mu,2} & -b_{\mu,3} \end{pmatrix} \end{aligned}$$

Exercises Lecture 4:

- b) Again, we wish the Lagrangian to stay invariant under the gauge transformation. Let's investigate again what happens with the Lagrangian under the gauge transformation

$$\psi \rightarrow \psi' = e^{\frac{i}{2}\vec{\tau}\cdot\vec{\alpha}(x)}\psi, \quad (3)$$

We wish that again the derivative behaves like:

$$D_\mu\psi \rightarrow D'_\mu\psi' = e^{\frac{i}{2}\vec{\tau}\cdot\vec{\alpha}(x)}(D_\mu\psi),$$

(i.e. that you can “pull the exponent through” the derivative), such that $\mathcal{L}' = \mathcal{L}$. We will find what then the transformation of the B_μ field should be.

Write out $D'_\mu\psi'$ (using $D_\mu = \mathbb{1}\partial_\mu + igB_\mu$ and $\psi' = e^{\frac{i}{2}\vec{\tau}\cdot\vec{\alpha}(x)}\psi \equiv G\psi$) in terms of B'_μ and G .

b)

$$D'_\mu\psi' = (\partial_\mu + igB'_\mu)G\psi = G(\partial_\mu\psi) + (\partial_\mu G)\psi + igB'_\mu(G\psi)$$

Exercises Lecture 4:

b)

$$D'_\mu \psi' = (\partial_\mu + igB'_\mu)G\psi = \underline{G(\partial_\mu \psi) + (\partial_\mu G)\psi + igB'_\mu(G\psi)}$$

c) If you compare your answer to the desired result

$$D'_\mu \psi' = G(D_\mu \psi),$$

show that you then find the following gauge field transformation for B_μ :

$$B'_\mu = G(B_\mu)G^{-1} + \frac{i}{g}(\partial_\mu G)G^{-1}$$

c)

$$D'_\mu \psi' = G(D_\mu \psi) = G(\partial_\mu + igB_\mu)\psi = \underline{G(\partial_\mu \psi) + igG(B_\mu \psi)}$$

Equalizing this result to b) gives:

$$\cancel{G(\partial_\mu \psi)} + igG(B_\mu \psi) = \cancel{G(\partial_\mu \psi)} + (\partial_\mu G)\psi + igB'_\mu(G\psi)$$

$$\Rightarrow (\partial_\mu G)\psi + igB'_\mu(G\psi) = igG(B_\mu \psi)$$

$$\Rightarrow igB'_\mu(G\psi) = igG(B_\mu \psi) - (\partial_\mu G)\psi$$

$$\Rightarrow B'_\mu(G\psi) = G(B_\mu \psi) + \frac{i}{g}(\partial_\mu G)\psi$$

$$\Rightarrow B'_\mu = G(B_\mu)G^{-1} + \frac{i}{g}(\partial_\mu G)G^{-1}$$

Exercises Lecture 4:

- d) (EXTRA) If the gauge transformation is “very small”, we can use the approximation (Taylor expansion) $e^{ix} \approx 1 + ix$,

$$e^{\frac{i}{2}\vec{\tau}\cdot\vec{\alpha}(x)} = G \approx \mathbb{1} + \frac{i}{2}\vec{\tau}\cdot\vec{\alpha}(x),$$

to demonstrate that the three \vec{b}_μ fields transform as

$$\vec{b}'_\mu = \vec{b}_\mu - \vec{\alpha} \times \vec{b}_\mu - \frac{1}{g}\partial_\mu\vec{\alpha}.$$

In other words, the transformation of each of the three \vec{b}_μ fields, involve the other \vec{b}_μ fields.

What is the consequence of this for the phenomenology (behaviour) of the gauge fields?

- d) Self-interacting \vec{b}_μ fields!

Plan

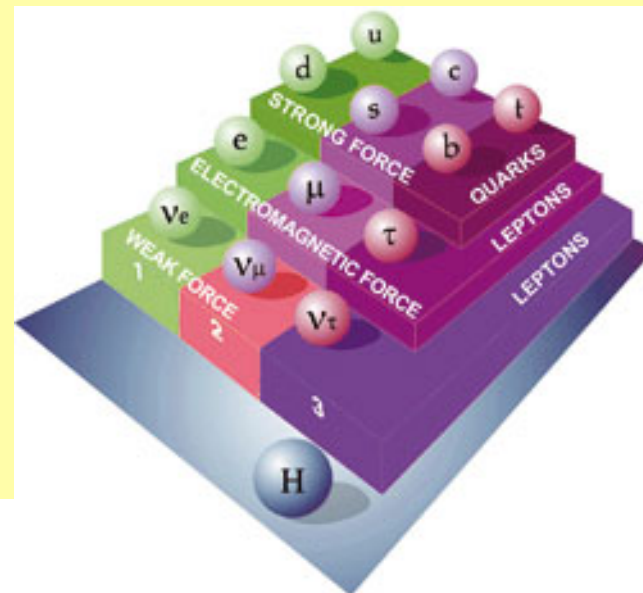
	1) Intro: Standard Model & Relativity	12 Feb
1900-1940	2) Basis	19 Feb
	1) Atom model, strong and weak force	
	2) Scattering theory	
1945-1965	3) Hadrons	12 Mar
	1) Isospin, strangeness	
	2) Quark model, GIM	
1965-1975	4) Standard Model	19 Mar
	1) QED	
	2) Parity, neutrinos, weak inteaction	
	3) QCD	
1975-2000	5) e^+e^- and DIS	7 May
2000-2015	6) Higgs and CKM	21 May

Summary Lects. 1-4

Lecture 1: Standard Model & Relativity

- Standard Model Lagrangian
- Standard Model Particles

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i\bar{\psi}\not{D}\psi + \text{h.c.} \\ & + \chi_i y_{ij} \chi_j \phi + \text{h.c.} \\ & + |D_\mu \phi|^2 - V(\phi)\end{aligned}$$



Lecture 1: Relativity

- Theory of relativity
 - Lorentz transformations ("boost")
 - Calculate energy in collisions

$$\begin{aligned}x^{10} &= \gamma(x^0 - \beta x^1) \\x^{11} &= \gamma(x^1 - \beta x^0) \\x^{12} &= x^2 \\x^{13} &= x^3\end{aligned} \quad \text{met} \quad \begin{aligned}\beta &\equiv \frac{v}{c} \\ \gamma &\equiv \frac{1}{\sqrt{1 - \beta^2}}\end{aligned}$$

- 4-vector calculus

$$p_\mu p^\mu = (E/c)^2 - |\vec{p}|^2 = (E^2 - c^2|\vec{p}|^2)/c^2 = (m_0 c^4)/c^2$$

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \quad (\mu = 0, 1, 2, 3)$$

- High energies needed to make (new) particles



$$\begin{aligned}s &= (p_1 + p_2)^2 = 2m^2 + 2(E^2 + \vec{p}^2) \\ &= 2m^2 + 2E^2 + 2(E^2 - m^2) = 4E^2\end{aligned}$$

Lecture 1: 4-vector examples

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \quad (\mu = 0, 1, 2, 3)$$

- 4-vectors:

- Space-time x^μ
- Energie-momentum p^μ
- 4-potential A^μ
- Derivative ∂^μ
- Covariant derivative D^μ
- Gamma matrices γ^μ

$$p_\mu p^\mu = (E/c)^2 - |\vec{p}|^2 = (E^2 - c^2|\vec{p}|^2)/c^2 = (m_0c^4)/c^2$$

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + iqA_\mu$$

$$\partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = j^\nu$$

$$(i\gamma^\mu \partial_\mu - m) \psi = 0$$

- Tensors

- Metric $g^{\mu\nu}$
- Electromagnetic tensor $F^{\mu\nu}$

$$x_\mu = g_{\mu\nu} x^\nu$$

$$x_0 = x^0, x_1 = -x^1, x_2 = -x^2, x_3 = -x^3$$

$$g_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$F^{\mu\nu} = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x) = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Lecture 2: Quantum Mechanics & Scattering

- Schrödinger equation

- Time-dependence of wave function

$$E = \frac{\vec{p}^2}{2m}$$

$$i\frac{\partial}{\partial t}\psi = \frac{-1}{2m}\nabla^2\psi$$

- Klein-Gordon equation

- Relativistic equation of motion of scalar particles

$$E^2 = \vec{p}^2 + m^2$$

$$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\phi$$

- Dirac equation

- Relativistically correct, and linear
- Equation of motion for spin-1/2 particles
- Described by 4-component spinors
- Prediction of anti-matter

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$



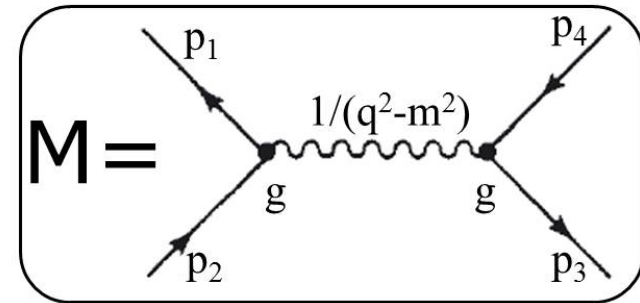
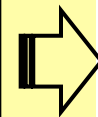
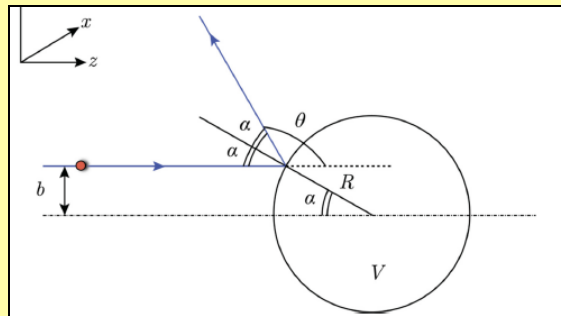
$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

Lecture 2: Quantum Mechanics & Scattering

- Scattering Theory

- (Relative) probability for certain process to happen
- Cross section

$$\frac{d\sigma}{d\Omega} = D(\theta, \varphi)$$



Scattering amplitude in Quantum Field Theory

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 \alpha}{mv_0^2} \right)^2 \frac{1}{4 \sin^4 \frac{\theta}{2}}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2mZ_1 Z_2 \alpha}{q^2} \right)^2$$

Classic

- Fermi's Golden Rule

$$\text{transition rate} = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \times (\text{phase space})$$

- Decay:

"decay width"

Γ

$$a \rightarrow b + c$$

- Scattering:

"cross section"

σ

$$a + b \rightarrow c + d$$

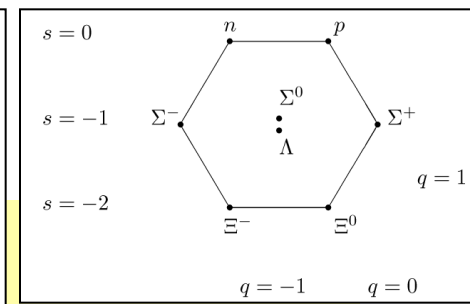
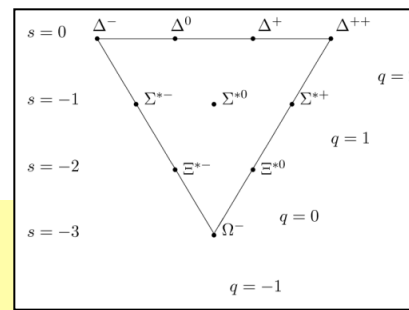
Lecture 3: Quarkmodel & Isospin

- "Partice Zoo" not elegant

- Hadrons consist of quarks

➤ Observed symmetries

- Same mass of hadrons:
- Slow decay of K, Λ :
- Fermi-Dirac statistics Δ^{++}, Ω :



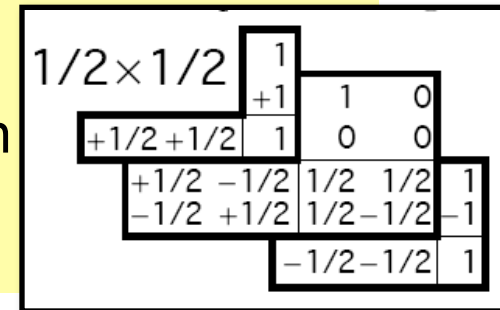
	d	u	s
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0
S – strangeness	0	0	-1

isospin

strangeness

color

- Combining/decaying particles with (iso)spin
 - Clebsch-Gordan coefficients



Lecture 4: Gauge symmetry and Interactions

- Arbitrary “gauge”

- Physics invariant
- Introduce “gauge” fields in derivative

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{q} \partial_\mu \alpha(x)$$

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + iqA_\mu$$

➤ Interactions!

- QED

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)} \psi(x)$$

1 photon

- Weak interactions

$$\psi \rightarrow \psi' = \exp\left(i \frac{\vec{\tau} \cdot \vec{\alpha}}{2}\right) \psi$$

3 weak bosons

- QCD

$$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8} \frac{i}{2} \theta_a(x) \lambda_a\right) \psi$$

8 gluons

Three generations
of matter (fermions)

	I	II	III	
mass	$2.4 \text{ MeV}/c^2$	$1.27 \text{ GeV}/c^2$	$171.2 \text{ GeV}/c^2$	0
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name	u up	c charm	t top	γ photon
	$4.8 \text{ MeV}/c^2$	$104 \text{ MeV}/c^2$	$4.2 \text{ GeV}/c^2$	0
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
Quarks	d down	s strange	b bottom	g gluon
	$<2.2 \text{ eV}/c^2$	$<0.17 \text{ MeV}/c^2$	$<15.5 \text{ MeV}/c^2$	$91.2 \text{ GeV}/c^2$
	0	0	0	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z^0 Z boson
	$0.511 \text{ MeV}/c^2$	$105.7 \text{ MeV}/c^2$	$1.777 \text{ GeV}/c^2$	$80.4 \text{ GeV}/c^2$
	-1	-1	-1	± 1
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
Leptons	e electron	μ muon	τ tau	W^\pm W boson
				Gauge bosons

Lecture 4: Forces

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i\bar{\psi} \cancel{D} \psi + \text{h.c.} \\ & + \bar{\psi}_i \gamma_{ij} \psi_j \phi + \text{h.c.} \\ & + |D_\mu \phi|^2 - V(\phi)\end{aligned}$$

Next:

1) Reminder: Gauge invariance, and the Lagrangian

- Electro-magnetic interactions: QED Electric charge
- Weak interactions: “QFT” Weak isospin/Flavour
- Strong interactions: QCD Colour

2) e^+e^- scattering

- $e^+e^- \rightarrow \mu^+\mu^-$
- $e^+e^- \rightarrow cc$ Discovery of charm and colour (quantity “R”)
- $e^+e^- \rightarrow qq g$ Discovery of the gluon
- $e^+e^- \rightarrow Z$ 3 neutrino’ s
- $e^+e^- \rightarrow WW$

3) Deep Inelastic Scattering (DIS) (lepton-proton scattering)

- Quarkmodel: do quarks exist??
- Sub-structure
- Bjorken-x, sum rules
- Scaling (violations)
- ‘Parton density functions’ (pdf) and ‘structure functions’

QED & QCD

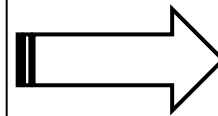
Lagrangian



Equation of motion

- spin-0 particles (Klein-Gordon)

$$\mathcal{L} = \mathcal{L}_{KG}^{free} = \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

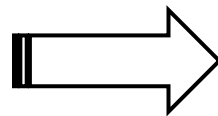


$$(\partial_\mu \partial^\mu + m^2) \phi(x) = 0$$

Klein-Gordon equation

- spin-1/2 fermions (Dirac)

$$\mathcal{L} = \mathcal{L}_{Dirac}^{free} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$$

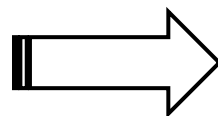


$$(i\gamma^\mu\partial_\mu - m)\psi(x) = 0$$

Dirac equation

- Photons

$$\mathcal{L} = \mathcal{L}_{EM} = -\frac{1}{4} (\partial^\mu A^\nu - \partial^\nu A^\mu) (\partial_\mu A_\nu - \partial_\nu A_\mu) - j^\mu A_\mu = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$



$$\partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = j^\nu$$

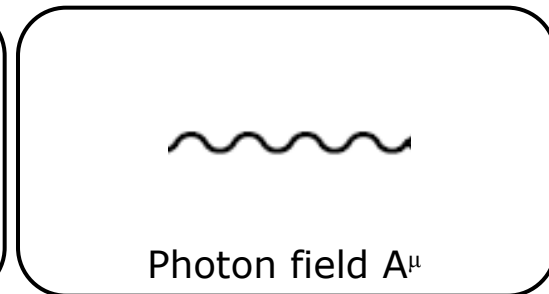
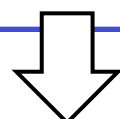
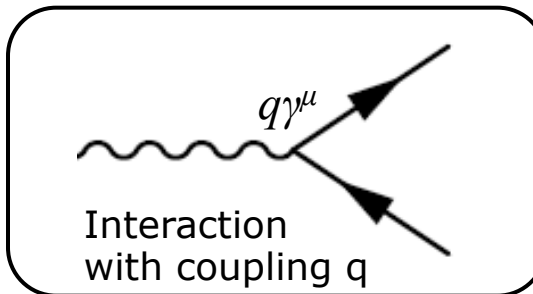
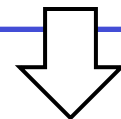
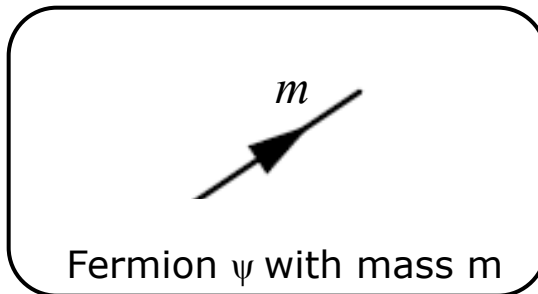
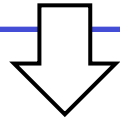
Maxwell equations

Gauge Invariance

- 1) Arbitrary gauge $A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu \Lambda$
- 2) Keep Eqs valid
 - This implies: $\psi \rightarrow \psi' = \psi e^{i\alpha(x)}$
 - And this implies: $\partial^\mu \rightarrow D^\mu \equiv \partial^\mu + iqA^\mu$

Quantum Electro Dynamics - QED

$$\mathcal{L}_{QED} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi - q A_\mu \bar{\psi} \gamma^\mu \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$



Electroweak theory

$$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau} \cdot \vec{\alpha}}{2}\right) \psi$$

➤ We measured that left and right are different!

- Instead of “strong” isospin, switch to “weak” isospin:

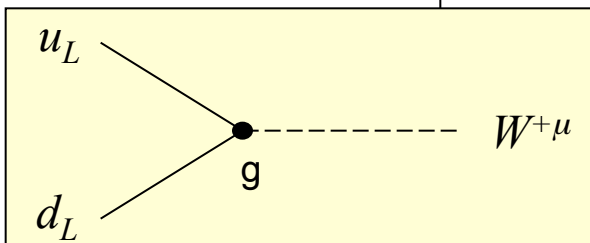
$$\psi = \begin{pmatrix} p \\ n \end{pmatrix} \quad \Rightarrow \quad \psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \text{and} \quad \psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

- Formalism the same

$$L_{kinetic}(\psi_L) = i\psi_L \gamma_\mu D^\mu \psi_L = i\bar{\psi}_L \gamma_\mu \left(\partial^\mu + ig \frac{1}{2} \vec{\tau} \cdot \vec{b}^\mu + iqA^\mu \right) \psi_L$$

$$L_{kinetic}^{weak}(u, d)_L = i(u, d)_L \gamma_\mu \left(\partial^\mu + ig \frac{1}{2} (b_1^\mu \tau_1 + b_2^\mu \tau_2 + b_3^\mu \tau_3) \right) \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$= i\bar{u}_L \gamma_\mu \partial^\mu u_L + i\bar{d}_L \gamma_\mu \partial^\mu d_L - \frac{g}{\sqrt{2}} \bar{u}_L \gamma_\mu W^{-\mu} d_L - \frac{g}{\sqrt{2}} \bar{d}_L \gamma_\mu W^{+\mu} u_L - \dots$$



$$W_\mu^\pm \equiv \frac{b_\mu^1 \mp ib_\mu^2}{\sqrt{2}}$$

Symmetries

- Charge ψ

- Isospin $\psi = \begin{pmatrix} u \\ d \end{pmatrix}_L$

- Color $\psi = \begin{pmatrix} r \\ g \\ b \end{pmatrix}$

More gauge transformations

- We had:

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

U(1) (QED)

- Then:

$$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau} \cdot \vec{\alpha}}{2}\right)\psi$$

SU(2) (Weak)

- How about:

$$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8} \frac{i}{2} \theta_a(x) \lambda_a\right)\psi$$

SU(3) (QCD)

(Why 8...? Group theory: $3 \times 3 = 8 + 1$...)

SU(2) → SU(3)

- We had:

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

U(1) (QED)

- Then:

$$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau} \cdot \vec{\alpha}}{2}\right)\psi$$

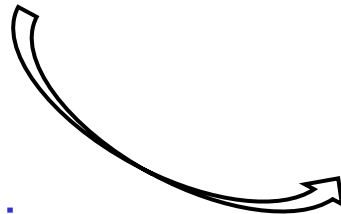
SU(2) (Weak)

- How about:

$$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8} \frac{i}{2} \theta_a(x) \lambda_a\right)\psi$$

SU(3) (QCD)

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



- **Gell-Mann matrices:**
SU(3)-equivalent of
Pauli-matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Symmetries

- Charge ψ

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

U(1) (QED)

- Isospin

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau} \cdot \vec{\alpha}}{2}\right)\psi$$

SU(2) (Weak)

- Color

$$\psi = \begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

$$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8} \frac{i}{2} \theta_a(x) \lambda_a\right)\psi$$

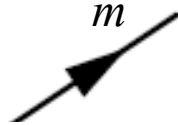
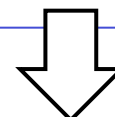
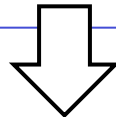
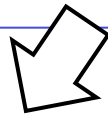
SU(3) (QCD)

(Why 8...? Group theory: 3x3=8+1 ...)

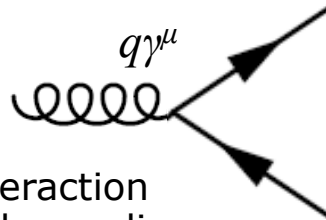
$$\begin{pmatrix} (r\bar{b} + b\bar{r})/\sqrt{2} & -i(r\bar{b} - b\bar{r})/\sqrt{2} \\ (r\bar{g} + g\bar{r})/\sqrt{2} & -i(r\bar{g} - g\bar{r})/\sqrt{2} \\ (b\bar{g} + g\bar{b})/\sqrt{2} & -i(b\bar{g} - g\bar{b})/\sqrt{2} \\ (r\bar{r} - b\bar{b})/\sqrt{2} & (r\bar{r} + g\bar{g} - 2b\bar{b})/\sqrt{6} \end{pmatrix}$$

QCD Lagrangian

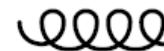
$$\mathcal{L}_{\text{QCD}} = \bar{\psi} (i\gamma_{\mu}(\partial^{\mu} - iA^{\mu}) - m) \psi - \frac{1}{2g^2} \text{tr}\{G_{\mu\nu} G^{\mu\nu}\}$$



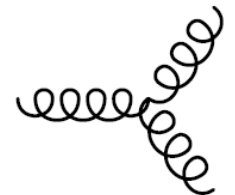
Fermion ψ with mass m



Interaction
with coupling q



8 Gluon fields A^{μ}



Self-interaction



QED and QCD

QED

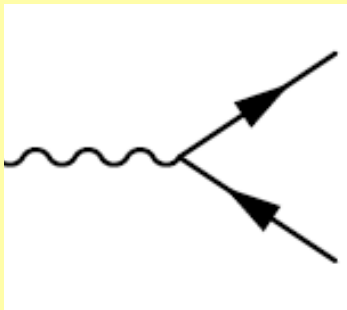
- Local U(1) gauge transformation

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

- Introduce 1 A_μ gauge field
- “Abelian” theory,

$$F^{\mu\nu} = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x)$$

- No self-interacting photon
 - Photons do not have (electric) charge
- Different “running”



QCD

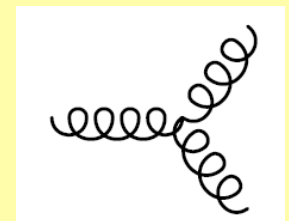
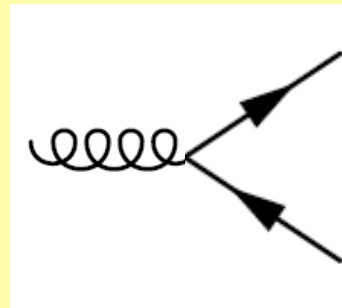
- Local SU(3) gauge transformation

$$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8} \frac{i}{2} \theta_a(x) \lambda_a\right) \psi$$

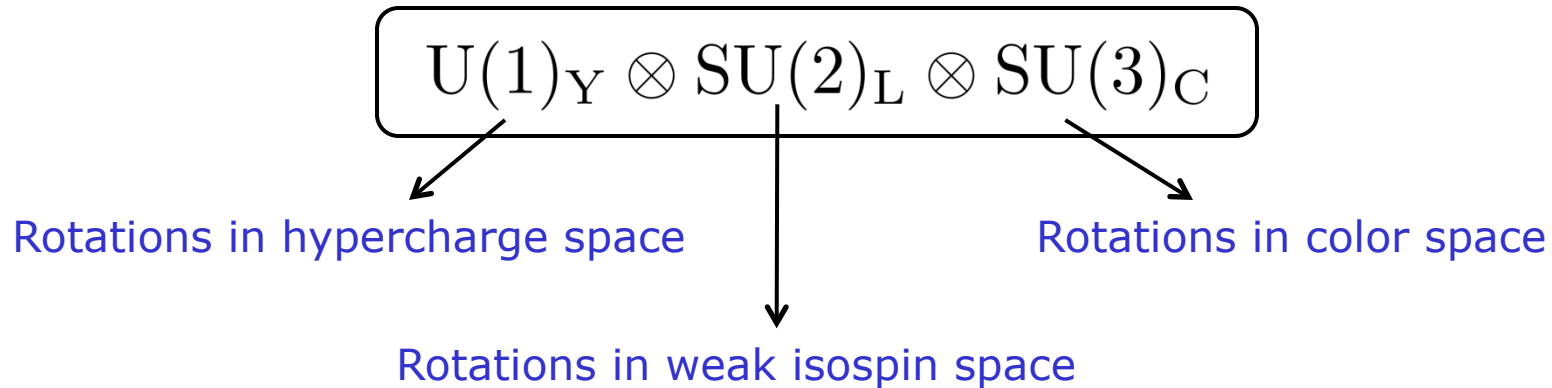
- Introduce 8 A_μ^a gauge fields
- Non-“Abelian” theory,

$$G_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g f_{abc} A_\mu^b(x) A_\nu^c(x)$$

- Self-interacting gluons
 - Gluons have (color) charge
- Different “running”



Which symmetries do we impose ?



For example $SU(2)_L$:

2x2 complex matrices (det=1) → 3 basis-rotations → 3 vector fields

QED:	$U(1)_Y$	→ 1 degree of freedom:	} γ W^+, W^- en Z^0 8 gluons	} All spin-1
Weak Force:	$SU(2)_L$	→ 3 degrees of freedom:		
Strong Force:	$SU(3)_C$	→ 8 degrees of freedom:		

Standard Model now (almost) complete!

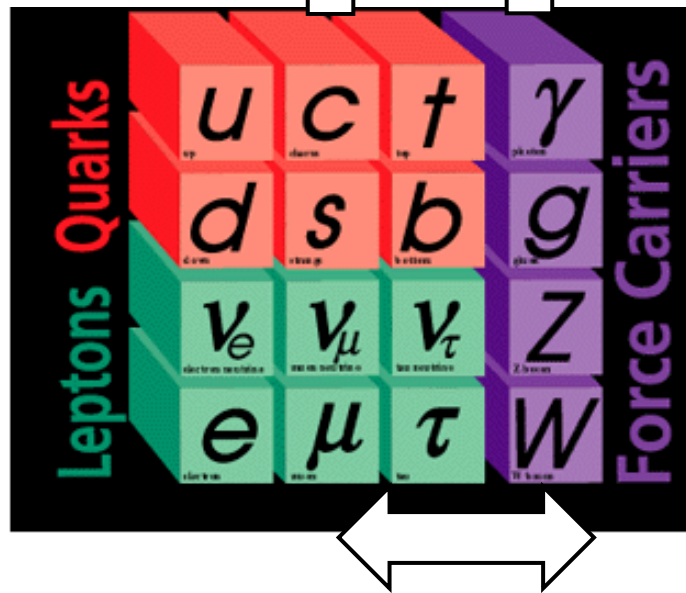
Three generations
of matter (fermions)

	I	II	III	
mass	$2.4 \text{ MeV}/c^2$	$1.27 \text{ GeV}/c^2$	$171.2 \text{ GeV}/c^2$	0
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name	u up	c charm	t top	γ photon
	$4.8 \text{ MeV}/c^2$	$104 \text{ MeV}/c^2$	$4.2 \text{ GeV}/c^2$	0
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
Quarks	d down	s strange	b bottom	g gluon
	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$	$91.2 \text{ GeV}/c^2$
	0	0	0	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z^0 Z boson
	$0.511 \text{ MeV}/c^2$	$105.7 \text{ MeV}/c^2$	$1.777 \text{ GeV}/c^2$	$80.4 \text{ GeV}/c^2$
	-1	-1	-1	± 1
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
Leptons	e electron	μ muon	τ tau	W^\pm W boson
				Gauge bosons

Standard Model

$$\mathcal{L} = \bar{\psi} \left(i\gamma_{\mu} D^{\mu} - m \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

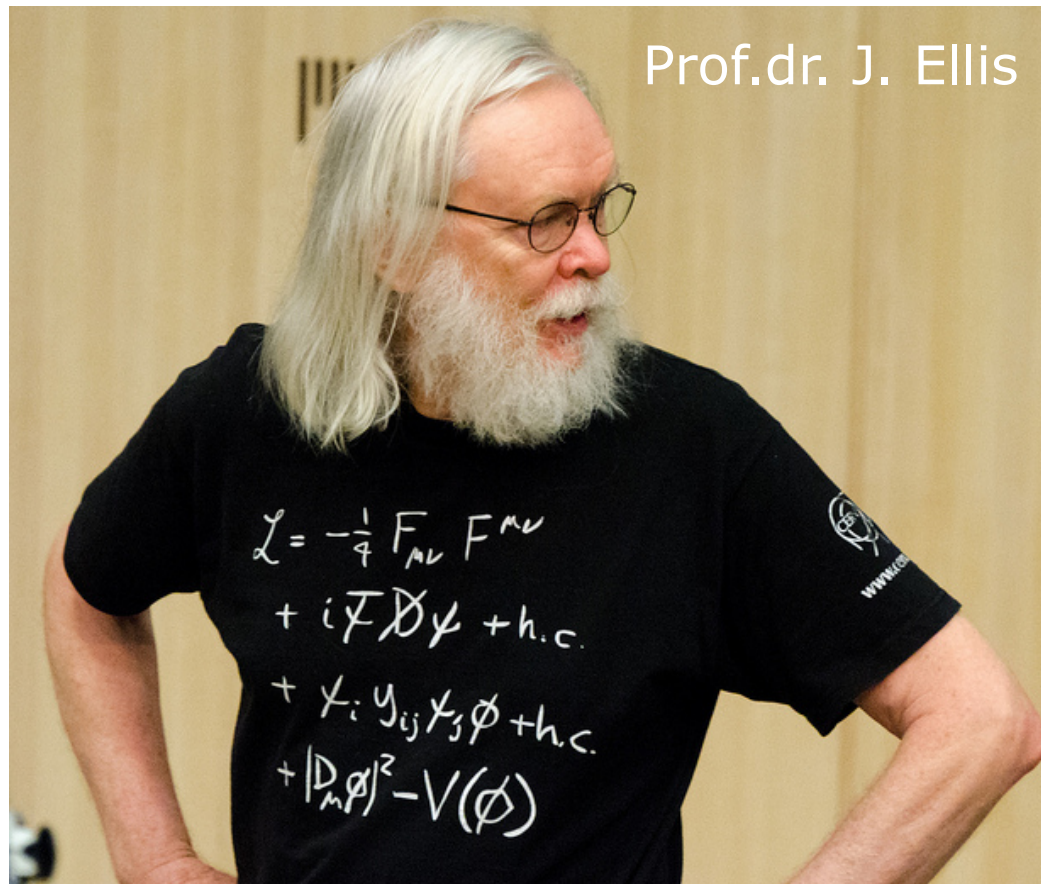
Fermion fields ψ  Gauge fields A_{μ}



Interactions through D^{μ}

Standard Model

$$\mathcal{L} = \bar{\psi} \left(i\gamma_{\mu} D^{\mu} - m \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$



Standard Model

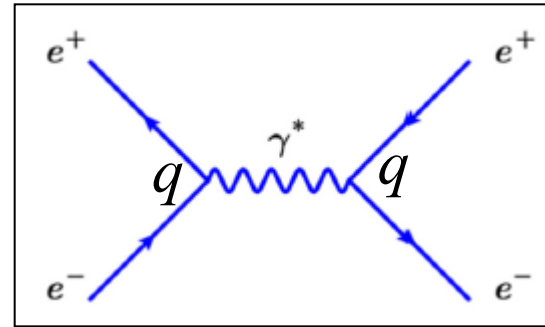
$$\mathcal{L} = \bar{\psi} \left(i\gamma_{\mu} D^{\mu} - m \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Todo-list:

- e^+e^- scattering
 - QED at work (LEP): R, neutrinos
- e^+p scattering
 - QCD at work (HERA): DIS, structure functions, scaling
- No masses for W, Z
 - (LHC/ATLAS) Higgs mechanism, Yukawa couplings
- Consequences of three families
 - (LHC/LHCb) CKM-mechanism, CP violation

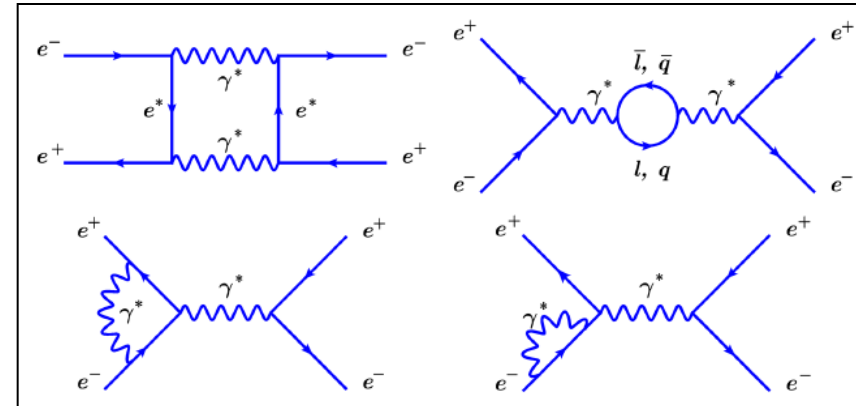
“Running” Coupling Constant (QED)

- Consider $e^+e^- \rightarrow e^+e^-$ scattering:



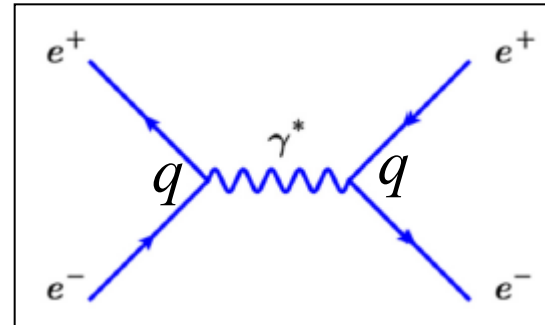
$$\alpha \sim q^2$$

- More possibilities!
 - “Higher order” diagrams
 - Each coupling has strength $1/137$
 - Perturbation series



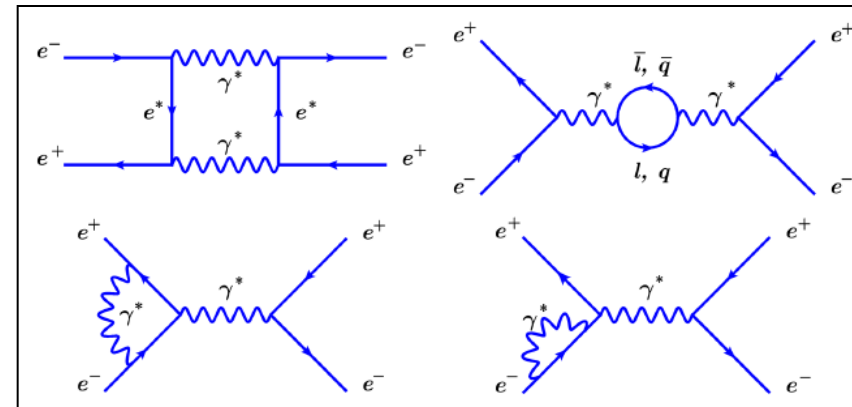
“Running” Coupling Constant (QED)

- Consider $e^+e^- \rightarrow e^+e^-$ scattering:

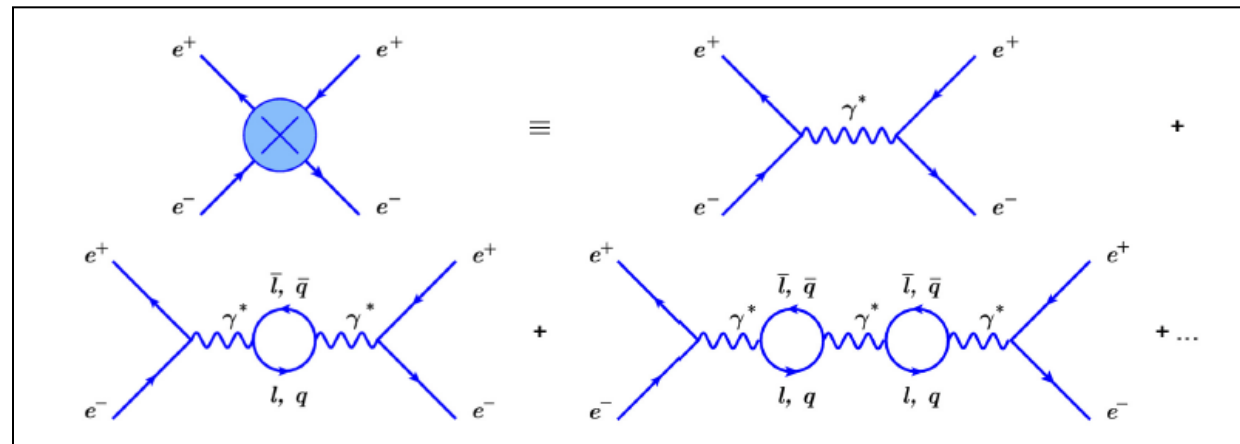


$$\alpha \sim q^2$$

- More possibilities!
 - “Higher order” diagrams
 - Each coupling has strength $1/137$
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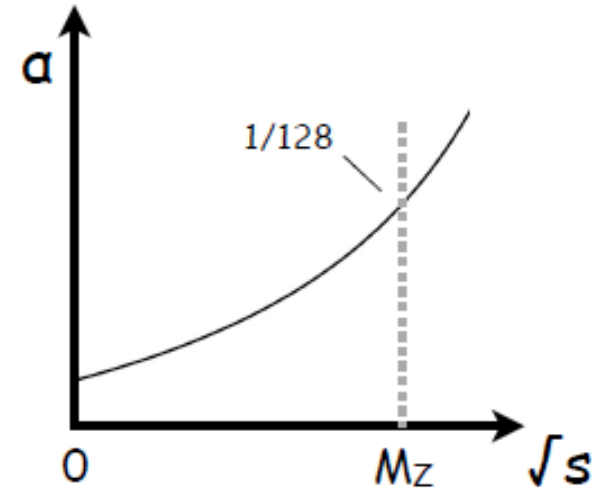
- Effectively:



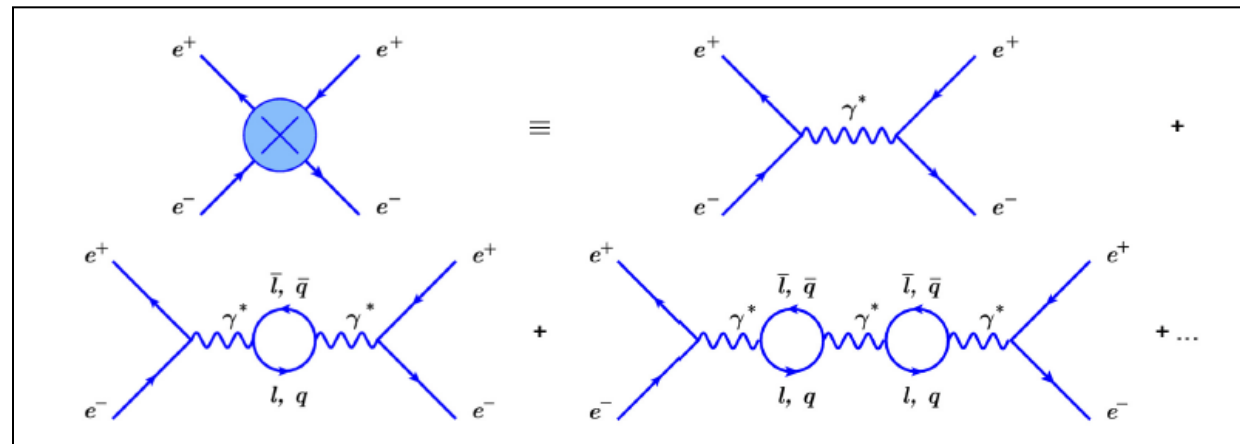
“Running” Coupling Constant (QED)

- Coupling depends on the scale!
 - $\alpha(0)=1/137$
 - $\alpha(M_Z^2)=1/128$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{Q^2}{\mu^2}\right)}$$

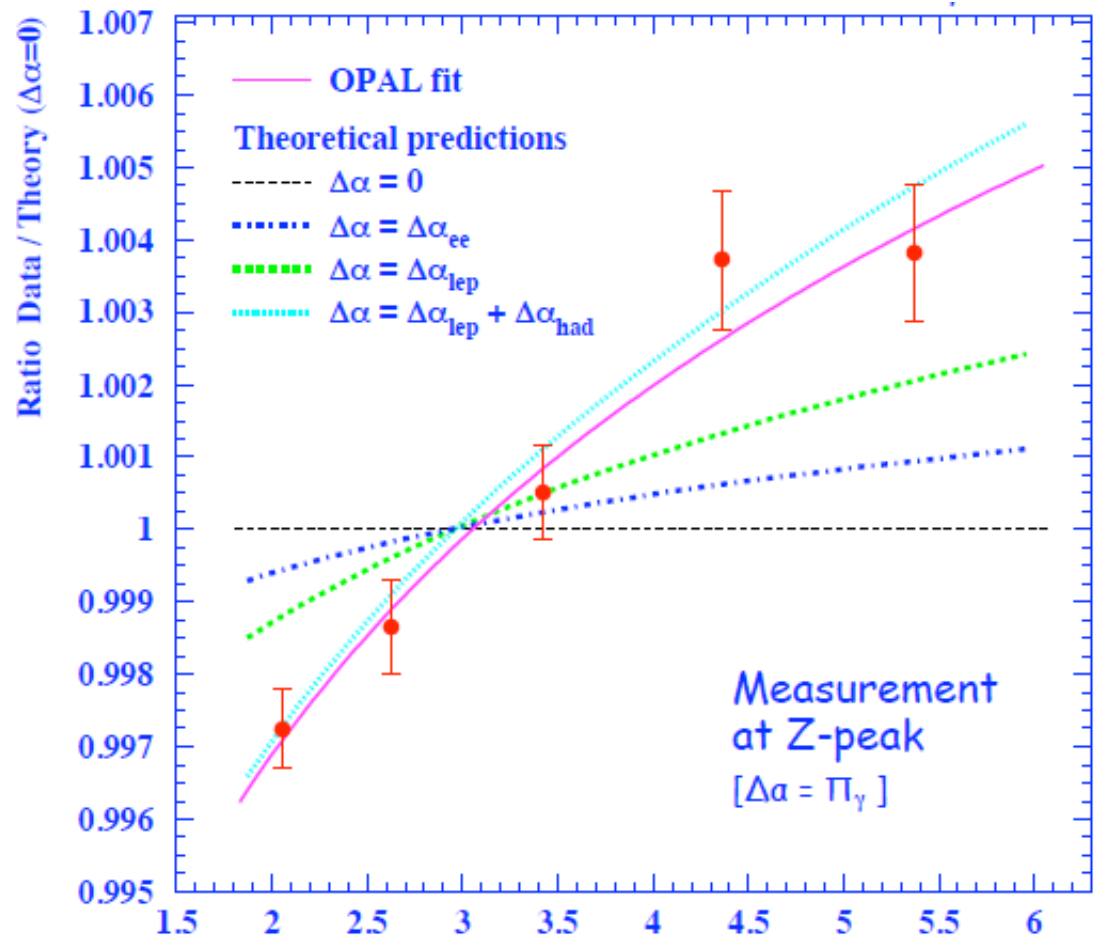
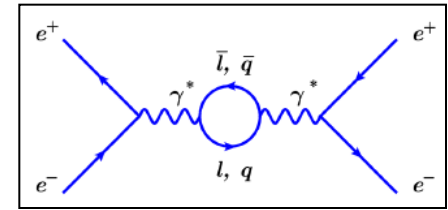


- Effectively:



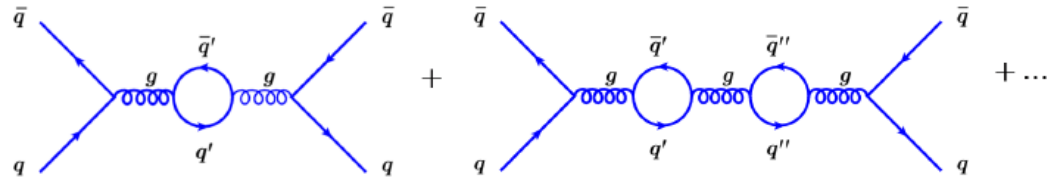
"Running" Coupling Constant

- Do you need all fermions in the loop??
- Yes:

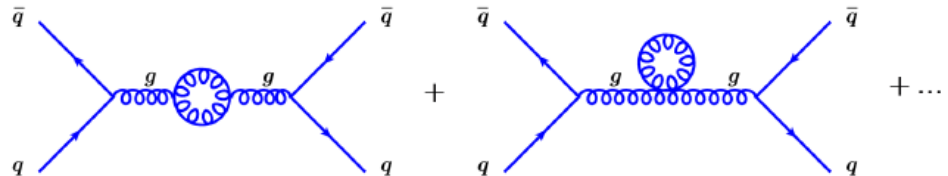


“Running” Coupling Constant

- Running in QCD



- Also gluon loops

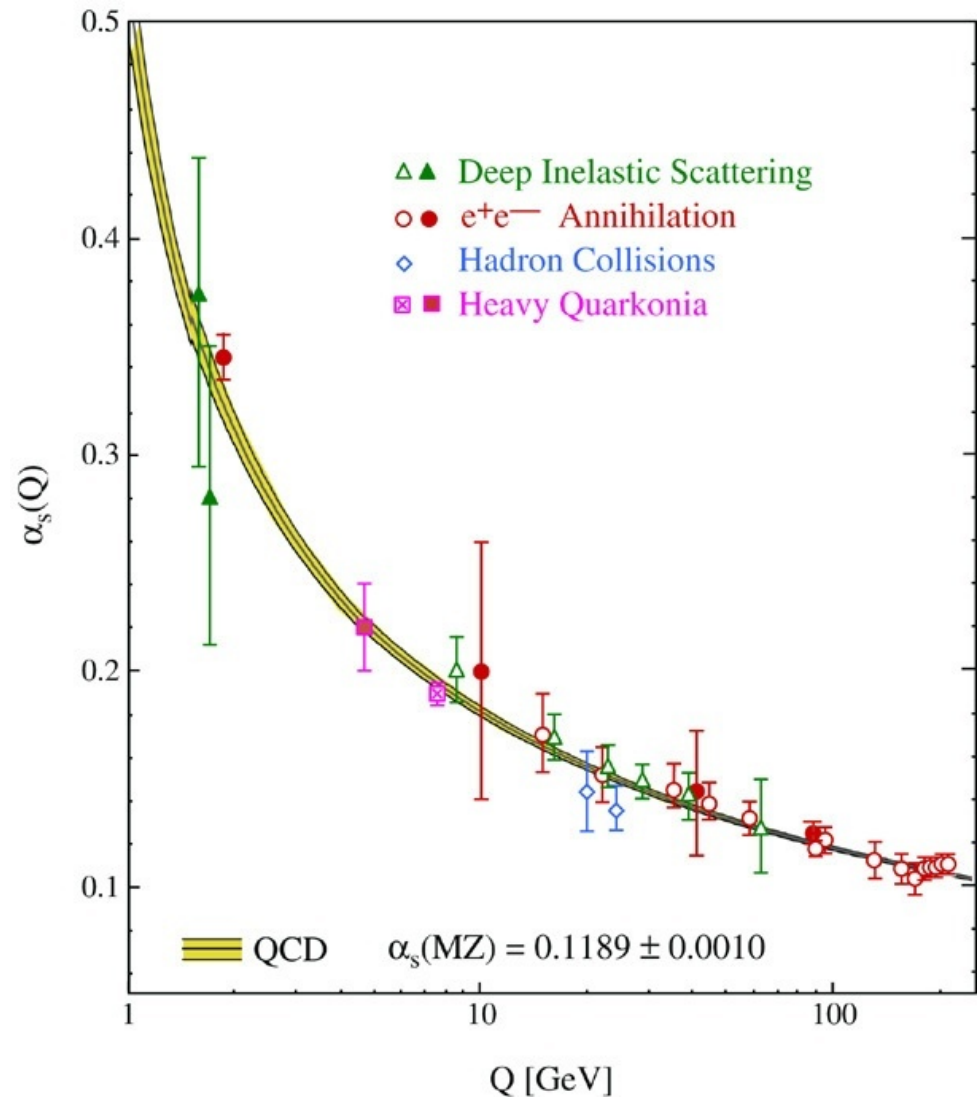


➤ It turns out, the gluon has opposite effect!

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi} (33 - 2n_f) \log(Q^2/\mu^2)}$$

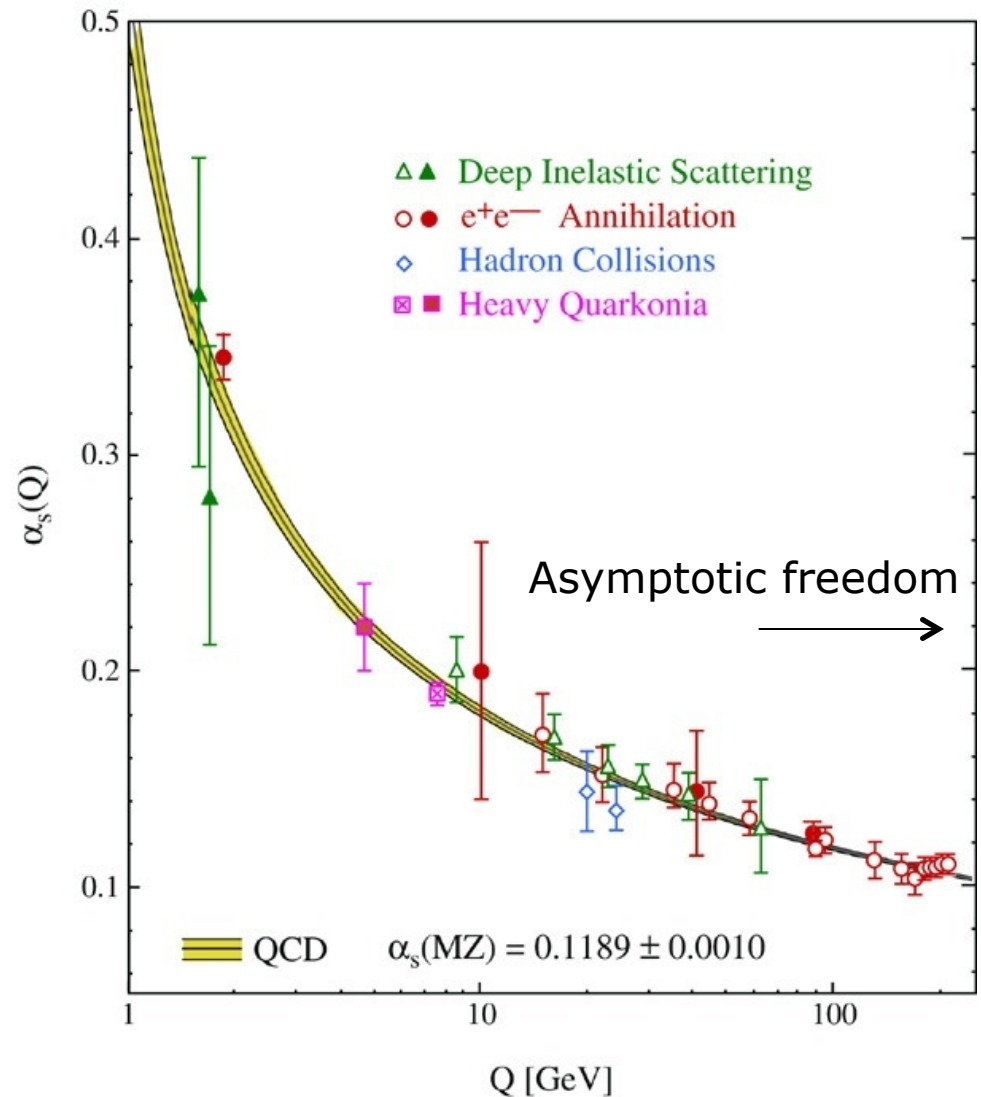
"Running" Coupling Constant

- Running in QCD
- NB: if $\alpha_s > 1$ perturbation theory breaks down...



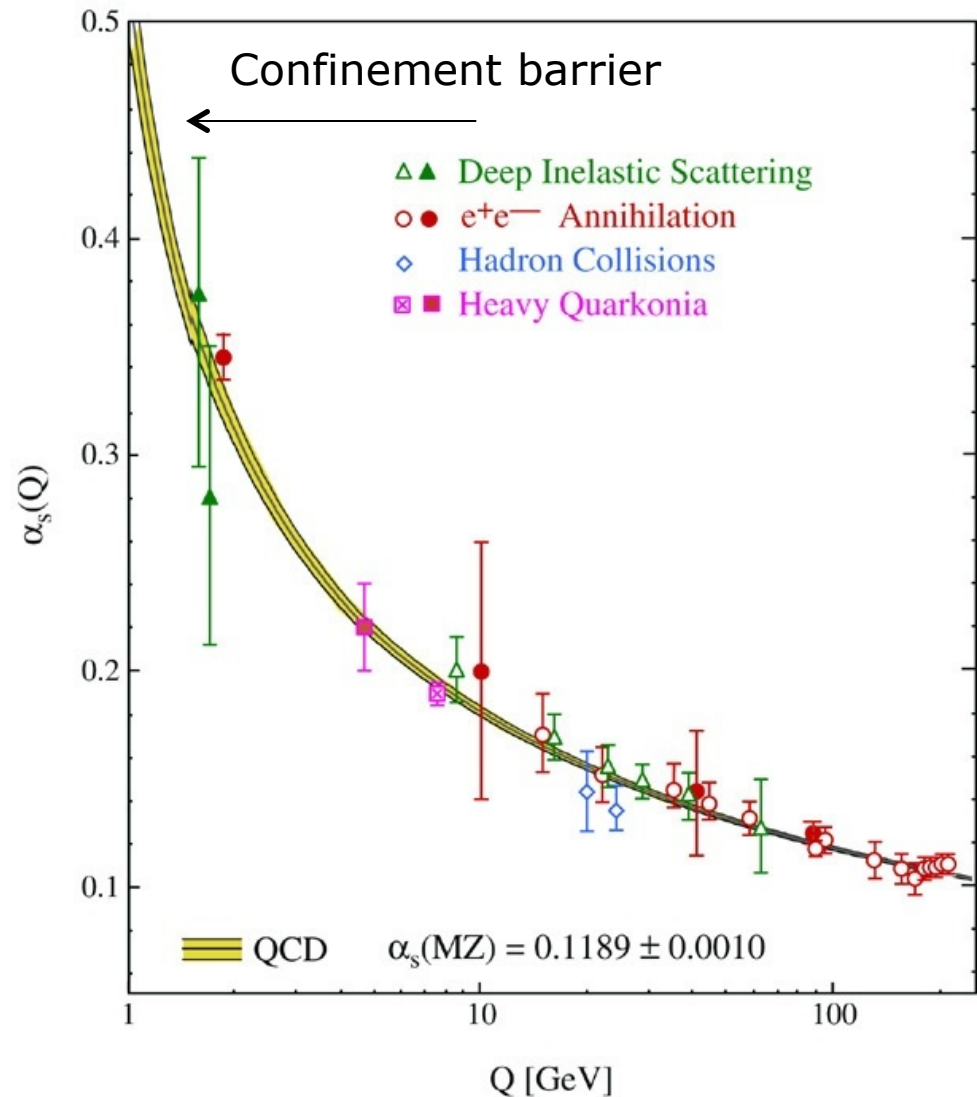
Asymptotic Freedom

- Running in QCD
- High energy:
coupling small

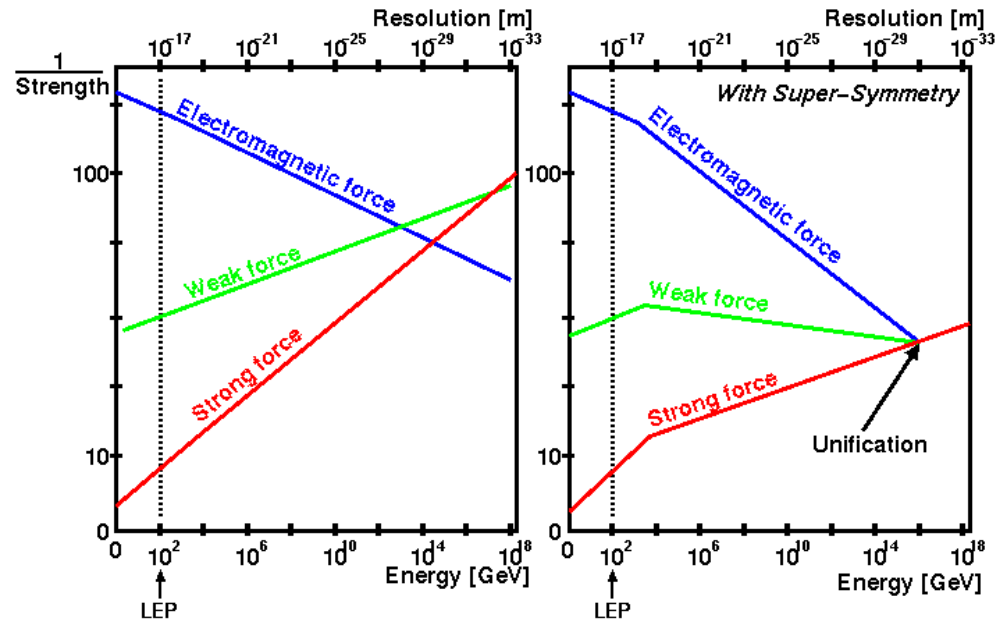


"Confinement"

- Running in QCD
- Low energy: coupling big



Unification?



- A reason to believe that there are more particles?

Feynman diagrams

Fermi's Golden Rule

Fermi's "golden rule" gives:

The transition probability to go from initial state i to final state f

$$T_{fi} = \frac{2\pi}{\hbar} |\langle f | H | i \rangle|^2 \rho(E_f)$$

1) Density of final states

- Integral of final states that can be reached
- "Phase space" Φ

2) Matrix element

- Scattering amplitude
- \mathcal{M}

$$d\sigma = \frac{1}{\text{Flux}} |\mathcal{M}|^2 d\Phi$$

3) Flux factor

- "Number of incident particles per unit area"

(Explicit Example:)

- Process: $A+B \rightarrow C+D$

$$d\sigma = \frac{(2\pi)^4 \delta^4(p_A + p_B - p_C - p_D)}{4\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2}} \cdot |\mathcal{M}|^2 \cdot \frac{d^3 p_C}{(2\pi)^3 2E_C} \frac{d^3 p_D}{(2\pi)^3 2E_D}$$

- Decay: $A \rightarrow C+D$

$$d\Gamma = \frac{(2\pi)^4 \delta^4(p_A - p_C - p_D)}{2E_A} \cdot |\mathcal{M}|^2 \cdot \frac{d^3 p_C}{(2\pi)^3 2E_C} \frac{d^3 p_D}{(2\pi)^3 2E_D}$$

$$d\sigma = \frac{1}{\text{Flux}} |\mathcal{M}|^2 d\Phi$$

2) Density of final states

- “How many quantum states can be put in volume V ?”
- “Phase space” Φ

3) Flux factor

- “Number of incident particles per unit area”

Feynman Rules

- How to calculate \mathcal{M} ?

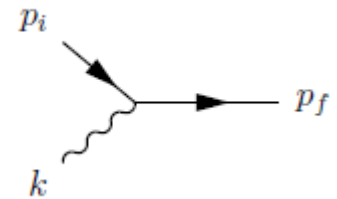
- Example QM:

$$\mathcal{M} \propto \int \phi_f^*(x) V(x) \phi_i(x) dx$$

$$\mathcal{M} \propto \int (e^{-ip_f x})^* e^{-ikx} e^{-ip_i x} dx$$

$$= \int e^{-i(p_i + k - p_f)x} dx$$

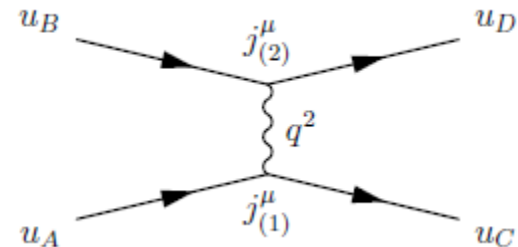
$$= (2\pi)^4 \delta(E_i + \omega - E_f) \delta^3(\vec{p}_i + \vec{k} - \vec{p}_f)$$



- Example spin-1/2 scattering:

$$T_{fi} = -i(2\pi)^4 \delta^4(p_D + p_C - p_B - p_A) \cdot \mathcal{M}$$

$$-i\mathcal{M} = \underbrace{ie(\bar{u}_C \gamma^\mu u_A)}_{\text{vertex}} \cdot \underbrace{\frac{-ig_{\mu\nu}}{q^2}}_{\text{propagator}} \cdot \underbrace{ie(\bar{u}_D \gamma^\nu u_B)}_{\text{vertex}}$$

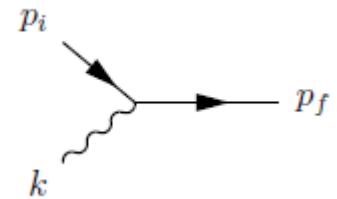


Feynman Rules

- How to calculate \mathcal{M} ?

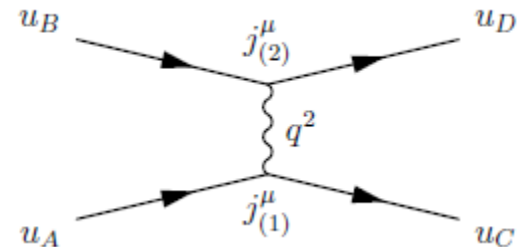
- Example QM:

$$\begin{aligned} \mathcal{M} &\propto \int \phi_f^*(x) V(x) \phi_i(x) dx \\ \mathcal{M} &\propto \int (e^{-ip_f x})^* e^{-ikx} e^{-ip_i x} dx \\ &= \int e^{-i(p_i+k-p_f)x} dx \\ &= (2\pi)^4 \delta(E_i + \omega - E_f) \delta^3(\vec{p}_i + \vec{k} - \vec{p}_f) \end{aligned}$$




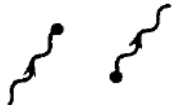







- Example spin-1/2 scattering:

$$\begin{aligned} T_{fi} &= -i(2\pi)^4 \delta^4(p_D + p_C - p_B - p_A) \cdot \mathcal{M} \\ -i\mathcal{M} &= \underbrace{ie(\bar{u}_C \gamma^\mu u_A)}_{\text{vertex}} \cdot \underbrace{\frac{-ig_{\mu\nu}}{q^2}}_{\text{propagator}} \cdot \underbrace{ie(\bar{u}_D \gamma^\nu u_B)}_{\text{vertex}} \end{aligned}$$

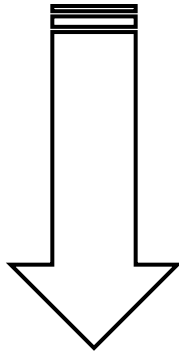


➤ Remember: \mathcal{M} is “just” a complex number

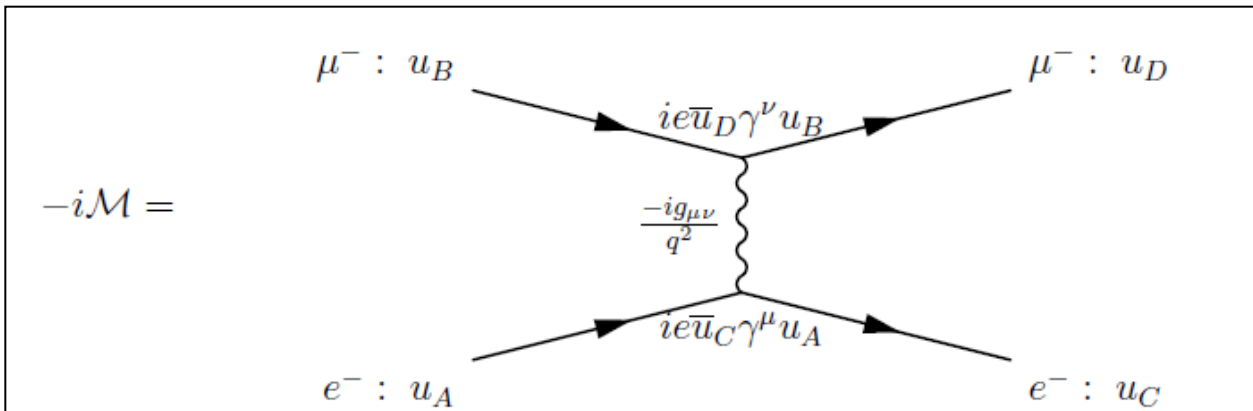
		Multiplicative Factor
● External Lines	Spin 0 boson (or antiboson)	 1
	Spin $\frac{1}{2}$ fermion (in, out)	 u, \bar{u}
	antifermion (in, out)	 \bar{v}, v
	Spin 1 photon (in, out)	 $\epsilon_\mu, \epsilon_\mu^*$
● Internal Lines—Propagators (need $+i\epsilon$ prescription)	Spin 0 boson	 $\frac{i}{p^2 - m^2}$
	Spin $\frac{1}{2}$ fermion	 $\frac{i(\not{p} + m)}{p^2 - m^2}$
	Massive spin 1 boson	 $\frac{-i(g_{\mu\nu} - p_\mu p_\nu / M^2)}{p^2 - M^2}$
	Massless spin 1 photon (Feynman gauge)	 $\frac{-ig_{\mu\nu}}{p^2}$
	● Vertex Factors	Photon—spin 0 (charge $-e$)
Photon—spin $\frac{1}{2}$ (charge $-e$)		 $ie\gamma^\mu$

Feynman rules: Example

- Process: $e^- \mu^- \rightarrow \mu^- e^-$



Spin $\frac{1}{2}$ fermion (in, out)		u, \bar{u}
antifermion (in, out)		\bar{v}, v
Massless spin 1 photon (Feynman gauge)		$\frac{-ig_{\mu\nu}}{p^2}$
Photon—spin $\frac{1}{2}$ (charge $-e$)		$ie(p + p')^\mu$ $ie\gamma^\mu$

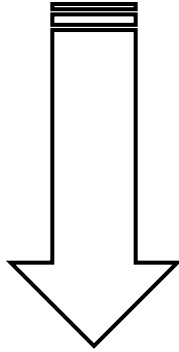


$$-i\mathcal{M} = -e^2 \bar{u}_C \gamma^\mu u_A \frac{-i}{q^2} \bar{u}_D \gamma_\mu u_B$$

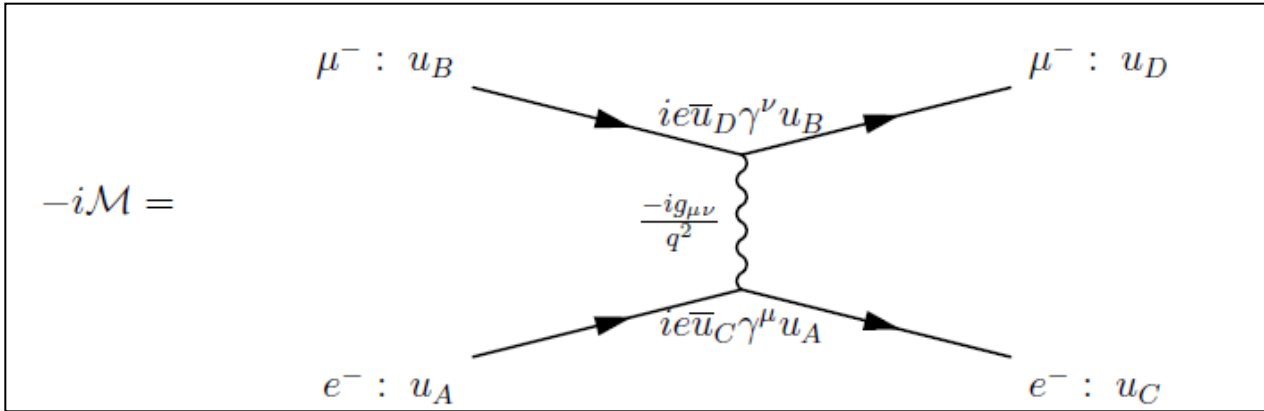
$$|\mathcal{M}|^2 = e^4 \left[(\bar{u}_C \gamma^\mu u_A) \frac{1}{q^2} (\bar{u}_D \gamma_\mu u_B) \right] \left[(\bar{u}_C \gamma^\nu u_A) \frac{1}{q^2} (\bar{u}_D \gamma_\nu u_B) \right]^*$$

Feynman rules: Example

- Process: $e^- \mu^- \rightarrow \mu^- e^-$



Spin $\frac{1}{2}$ fermion (in, out)		u, \bar{u}
antifermion (in, out)		\bar{v}, v
Massless spin 1 photon (Feynman gauge)		$\frac{-ig_{\mu\nu}}{p^2}$
Photon—spin $\frac{1}{2}$ (charge $-e$)		$ie(p + p')^\mu$ $ie\gamma^\mu$



$$-i\mathcal{M} = -e^2 \bar{u}_C \gamma^\mu u_A \frac{-i}{q^2} \bar{u}_D \gamma_\mu u_B$$

$$|\mathcal{M}|^2 = e^4 \left[(\bar{u}_C \gamma^\mu u_A) \frac{1}{q^2} (\bar{u}_D \gamma_\mu u_B) \right] \left[(\bar{u}_C \gamma^\nu u_A) \frac{1}{q^2} (\bar{u}_D \gamma_\nu u_B) \right]^*$$

Remember the 4-component spinors in Dirac-space:

$$\underbrace{\left[(\bar{u} \) \left(\gamma^\mu \right) \left(u \right) \right]}_{\text{a number}}$$

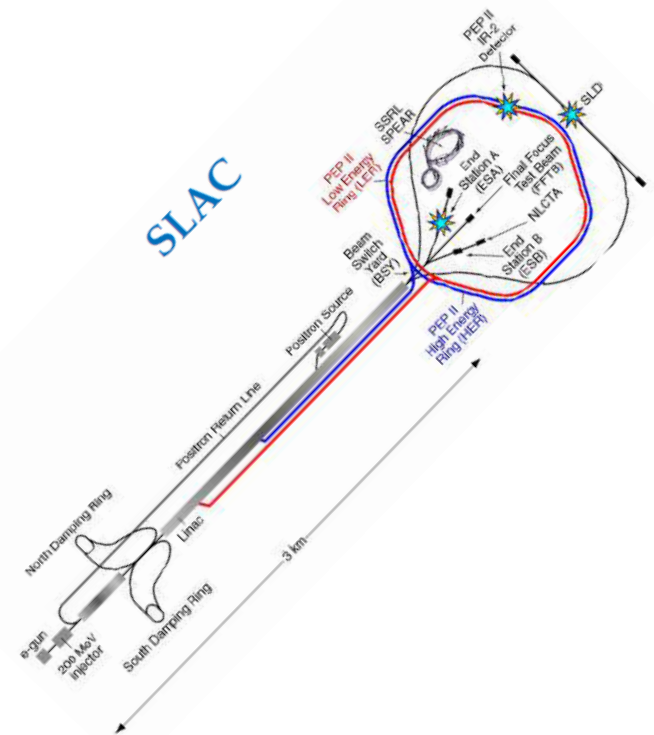
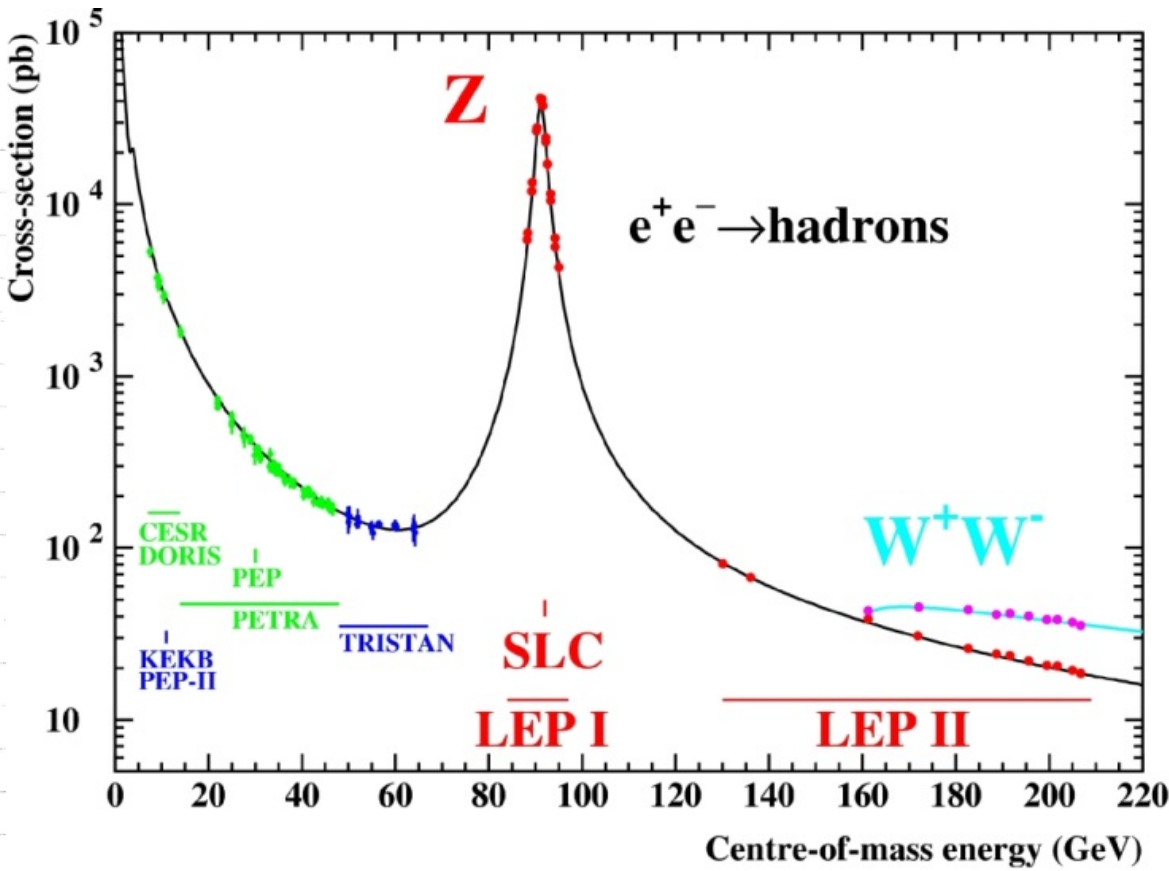
e^+e^- Scattering

Shopping list

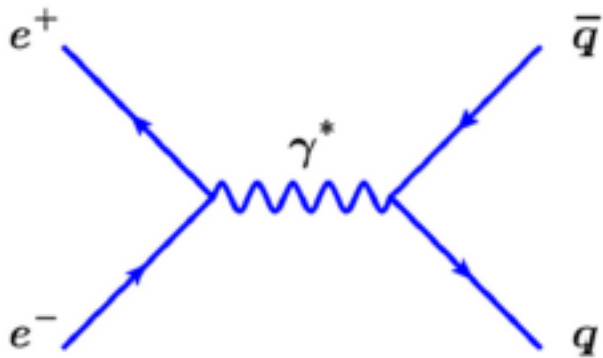
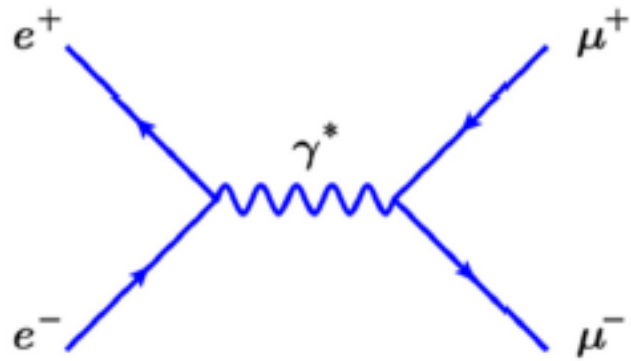
- $e^+e^- \rightarrow \mu^+\mu^-$
- $e^+e^- \rightarrow cc$
 - Confirmation of “color”
 - Discovery of charm
- $e^+e^- \rightarrow qq g$
 - Discovery of the gluon
- $e^+e^- \rightarrow tt$
 - Hunt for the top
- $e^+e^- \rightarrow Z$
 - 3 neutrino's
- $e^+e^- \rightarrow WW$

e^+e^- colliders

Accelerator	Lab	\sqrt{s} (GeV)	Year
SPEAR	SLAC	2 - 8	1972 - 1990
DORIS I, II	DESY	10	1974 - 1993
CESR(-c)	Cornell	3.5 - 12	1979 - 2008
PEP	SLAC	20 - 29	1980 - 1990
PETRA	DESY	12 - 47	1978 - 1988
TRISTAN	KEK	50 - 60	1987 - 1995
SLC	SLAC	90	1988 - 1998
LEP I, II	CERN	90 - 208	1989 - 2000



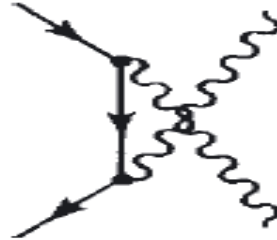
Examples of e^+e^- processes



time



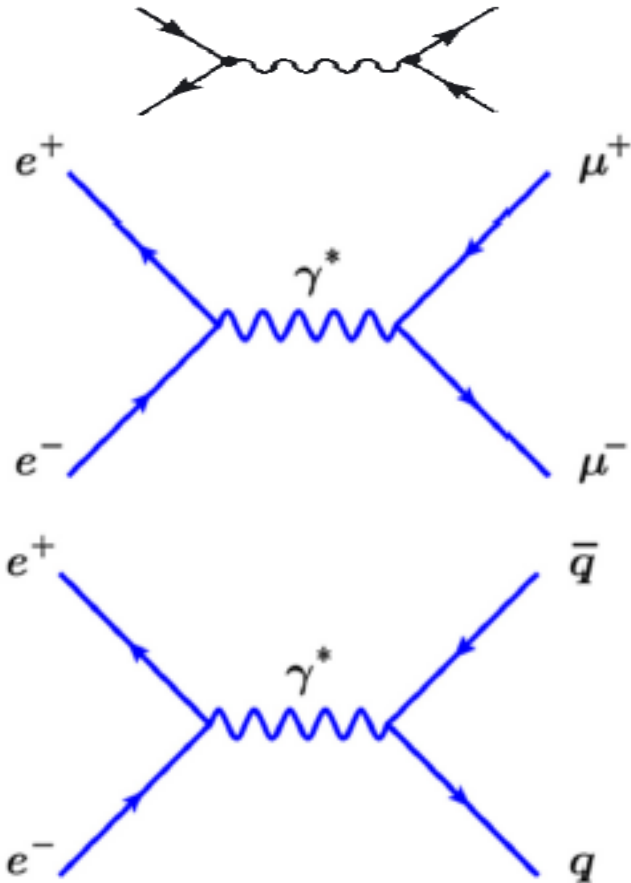
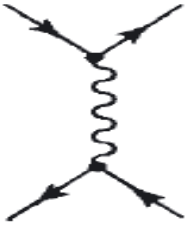
Pair annihilation ($e^- + e^+ \rightarrow \gamma + \gamma$)



Electron-positron scattering ($e^- + e^+ \rightarrow e^- + e^+$)
(Bhabha scattering)



Examples of e^+e^- processes



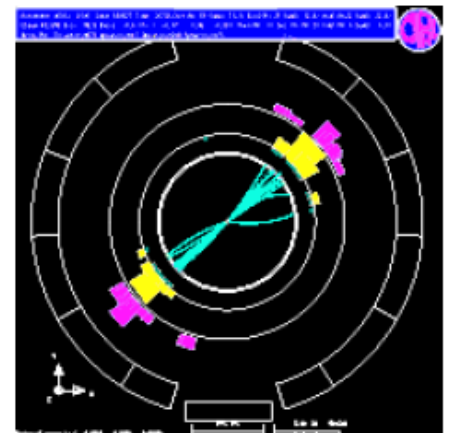
e^-e^+



$\mu^- \mu^+$



$q \bar{q}$



Scattering

- Rutherford scattering
(scattering off static point charge)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\frac{1}{2}\theta)}$$

- Mott scattering
(now with high energy, taking into account recoil of target and magnetic moment: spin $\frac{1}{2}$)

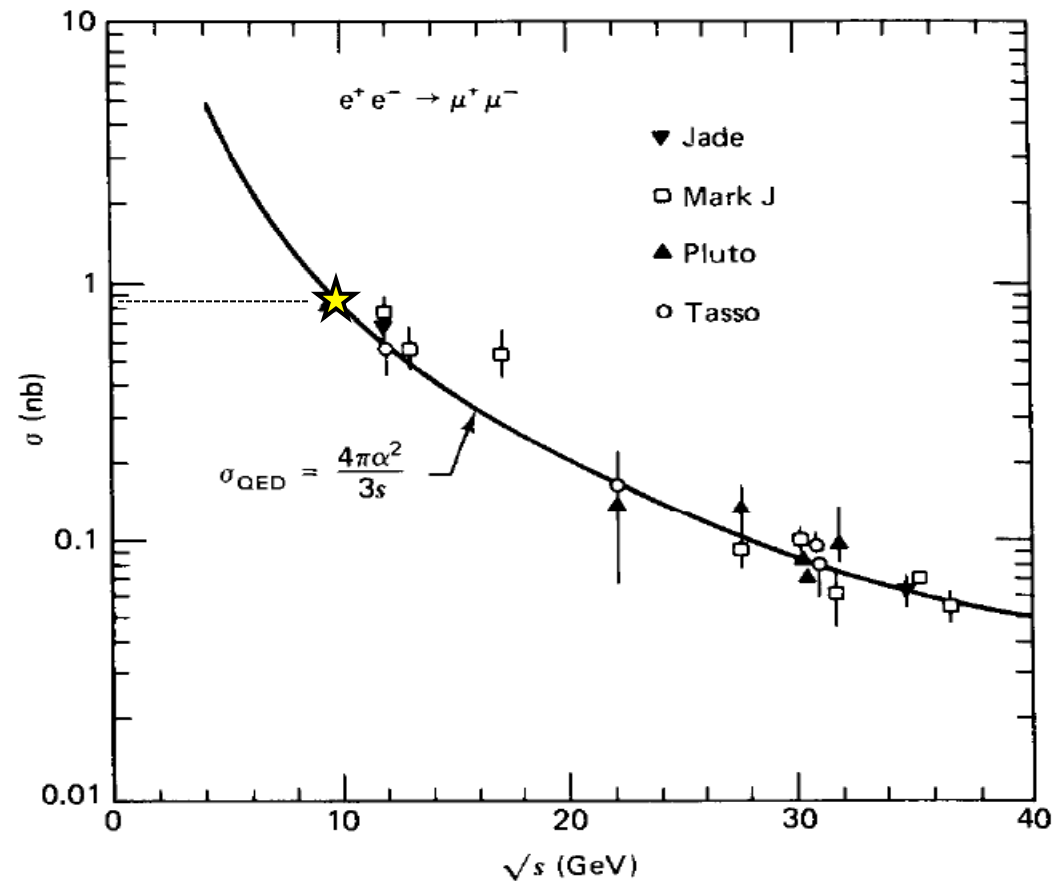
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\frac{1}{2}\theta)}{4E^2 \sin^4(\frac{1}{2}\theta)}$$

- spin- $\frac{1}{2}$ spin- $\frac{1}{2}$ scattering
(average over incoming spin, sum over outgoing spin)

$$\left. \frac{d\sigma}{d\Omega} \right|_{cm} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta)$$

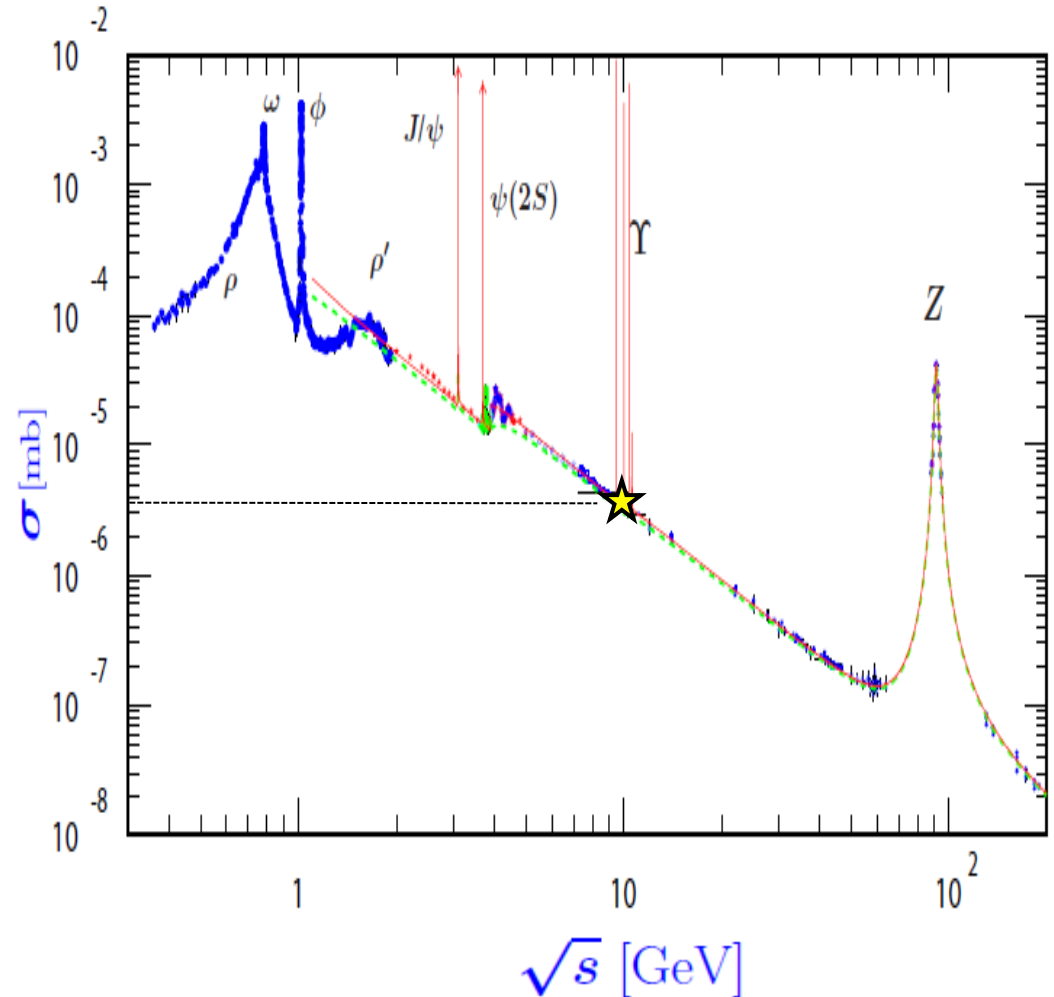
$e^+e^- \rightarrow \mu^+\mu^-$

- Point cross section
- At $\sqrt{s}=10$ GeV:
 $\sigma(e^+e^- \rightarrow \mu^+\mu^-) \sim 0.9$ nb

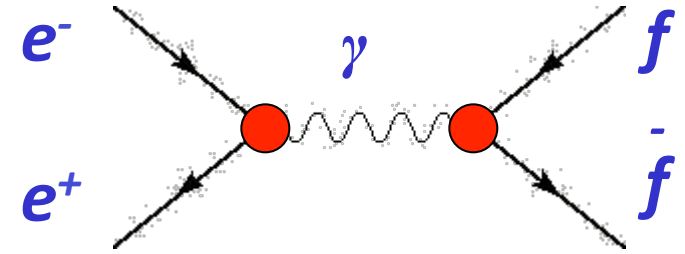


$e^+e^- \rightarrow \text{hadrons}$

- Point cross section
- At $\sqrt{s}=10$ GeV:
 $\sigma(e^+e^- \rightarrow q^+q^-) \sim 3.6$ nb



$e^+e^- \rightarrow \text{hadrons}$



- Compare cross section of $\sigma(e^+e^- \rightarrow q^+q^-)$ to $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$
 - Photon couples to fermion charge! $\sigma(e^+e^- \rightarrow f\bar{f}) \propto Q_f Q_{\bar{f}}$
 - Color of quarks?

- Inspect Ratio R:

$$R = \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$$R = N_c (q_u^2 + q_d^2 + q_s^2 + \dots)$$

Prediction R: $m_{e^+e^-} < 2m_c$

muons $\left\{ \begin{array}{l} R(e^+e^- \rightarrow \mu^+\mu^-) \propto Q_{\mu^+}Q_{\mu^-} \propto (1)^2 \end{array} \right.$

quarks $\left\{ \begin{array}{l} R(e^+e^- \rightarrow u\bar{u}) \propto Q_uQ_{\bar{u}} \propto \left(\frac{2}{3}\right)^2 \\ R(e^+e^- \rightarrow d\bar{d}) \propto Q_dQ_{\bar{d}} \propto \left(\frac{1}{3}\right)^2 \\ R(e^+e^- \rightarrow s\bar{s}) \propto Q_sQ_{\bar{s}} \propto \left(\frac{1}{3}\right)^2 \end{array} \right.$

$$R = \frac{\alpha(e^+e^- \rightarrow q\bar{q})}{\alpha(e^+e^- \rightarrow \mu^+\mu^-)} = \frac{(2/3)^2 + (1/3)^2 + (1/3)^2}{1^2} = \frac{2}{3}$$

Without color: $R = 2/3$

$$R = \frac{\alpha(e^+e^- \rightarrow q\bar{q})}{\alpha(e^+e^- \rightarrow \mu^+\mu^-)} = \frac{3 \cdot (2/3)^2 + 3 \cdot (1/3)^2 + 3 \cdot (1/3)^2}{1^2} = 2$$

With 3 colors: $R = 2$

Prediction R: $m_{e^+e^-} > 2m_c$

muons $\left\{ \begin{array}{l} R(e^+e^- \rightarrow \mu^+\mu^-) \propto Q_{\mu^+}Q_{\mu^-} \propto (1)^2 \end{array} \right.$

quarks $\left\{ \begin{array}{l} R(e^+e^- \rightarrow u\bar{u}) \propto Q_uQ_{\bar{u}} \propto \left(\frac{2}{3}\right)^2 \\ R(e^+e^- \rightarrow d\bar{d}) \propto Q_dQ_{\bar{d}} \propto \left(\frac{1}{3}\right)^2 \\ R(e^+e^- \rightarrow s\bar{s}) \propto Q_sQ_{\bar{s}} \propto \left(\frac{1}{3}\right)^2 \\ R(e^+e^- \rightarrow c\bar{c}) \propto Q_cQ_{\bar{c}} \propto \left(\frac{2}{3}\right)^2 \end{array} \right.$

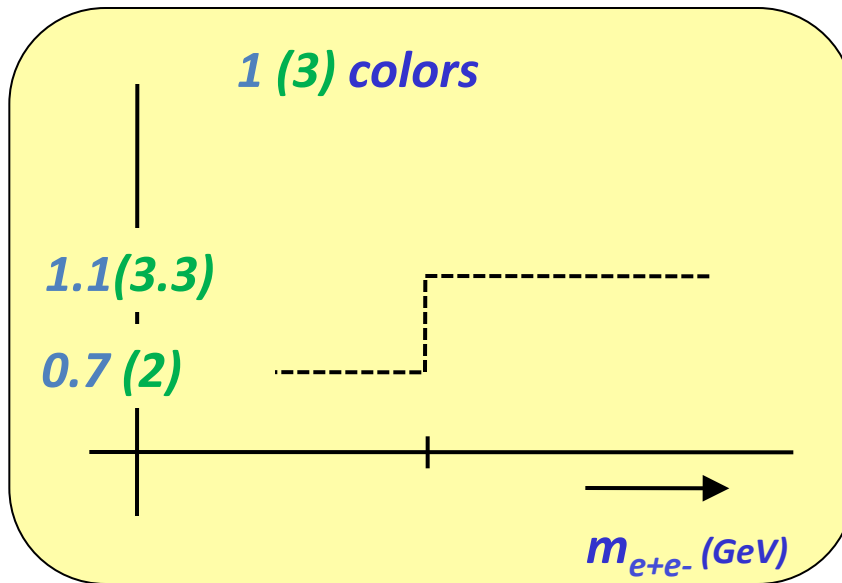
$$R = \frac{\alpha(e^+e^- \rightarrow q\bar{q})}{\alpha(e^+e^- \rightarrow \mu^+\mu^-)} = \frac{(2/3)^2 + (1/3)^2 + (1/3)^2 + (2/3)^2}{1^2} = \frac{10}{9}$$

Without color: R = 10/9

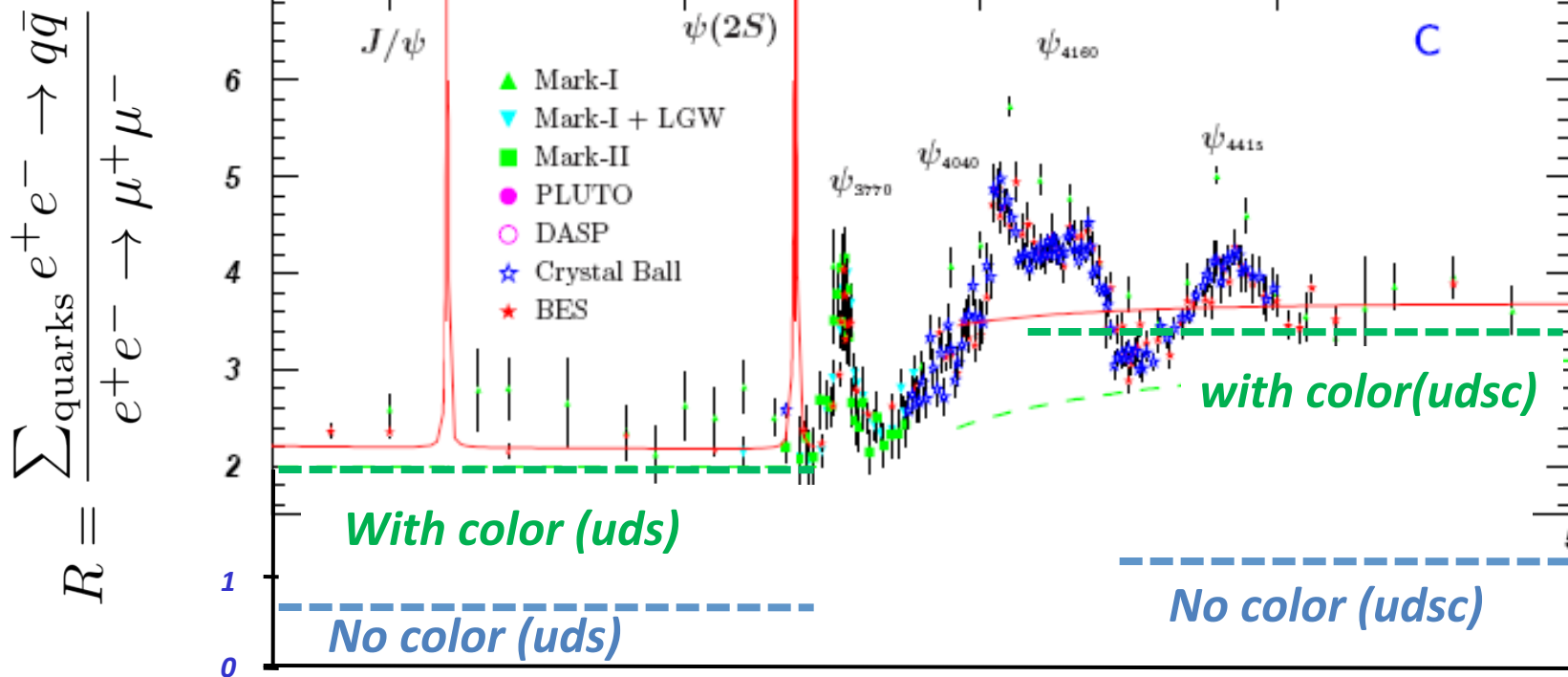
$$R = \frac{\alpha(e^+e^- \rightarrow q\bar{q})}{\alpha(e^+e^- \rightarrow \mu^+\mu^-)} = \frac{3 \cdot (2/3)^2 + 3 \cdot (1/3)^2 + 3 \cdot (1/3)^2 + 3 \cdot (2/3)^2}{1^2} = \frac{30}{9}$$

With 3 colors: R = 30/9

prediction



data



$e^+e^- \rightarrow \text{hadrons}$

- So, quarks have color!
- How about **spin**?

$$\left. \frac{d\sigma}{d\Omega} \right|_{cm} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta)$$

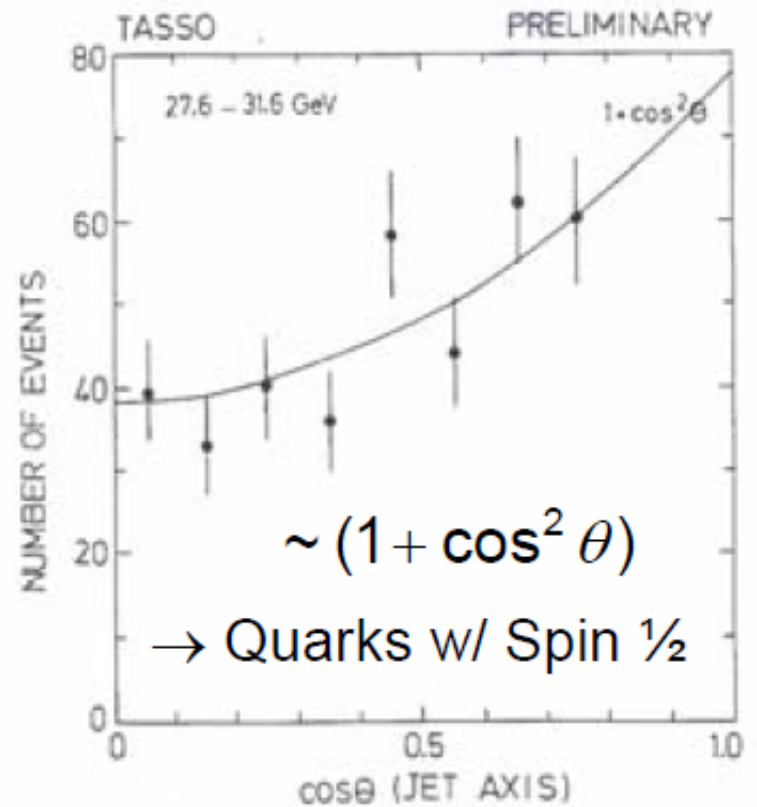
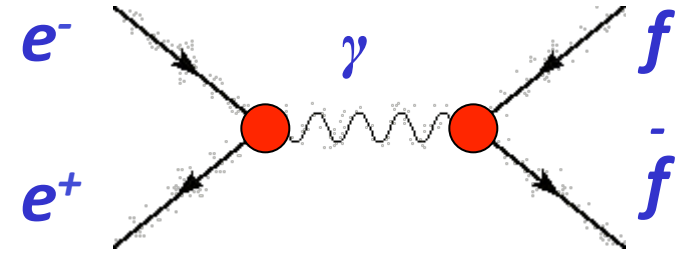
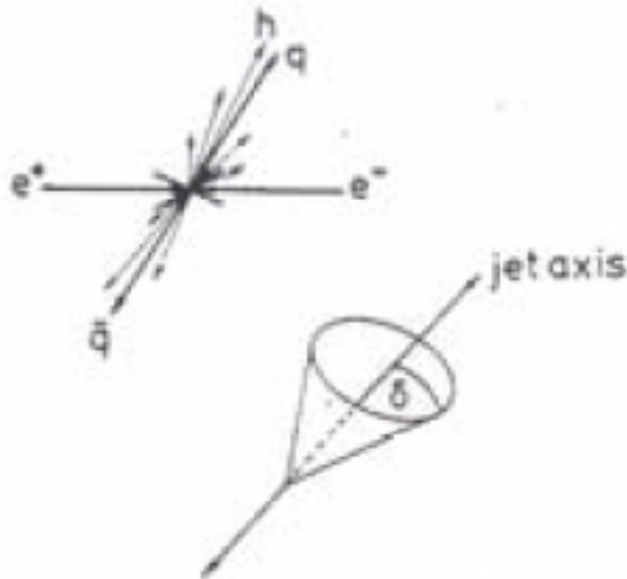


Fig.7 Angular distribution of the jet axis with respect to the beam.

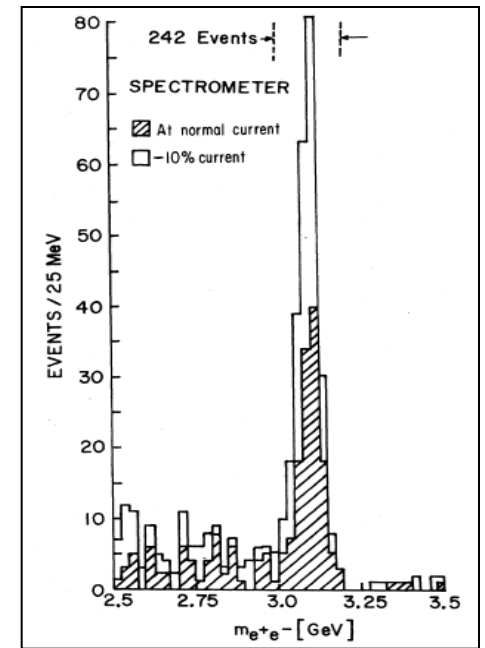
$$e^+e^- \rightarrow c\bar{c}$$

Discovery of charm (J/ψ)

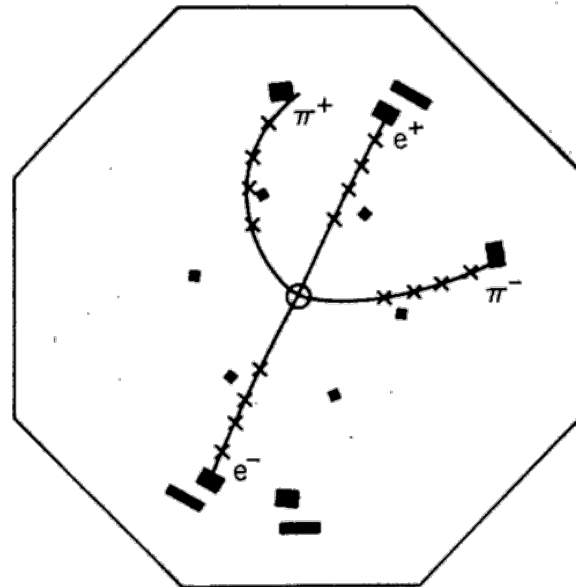
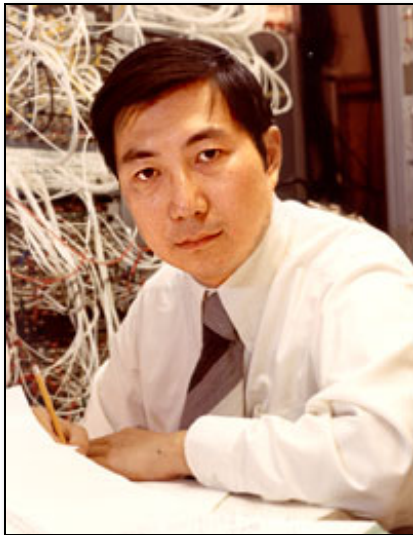
November 1976: 'November revolution'

New meson discovered (J/ψ) with mass 3.16 GeV

Quark model: J/ψ consists of pair of c-quarks

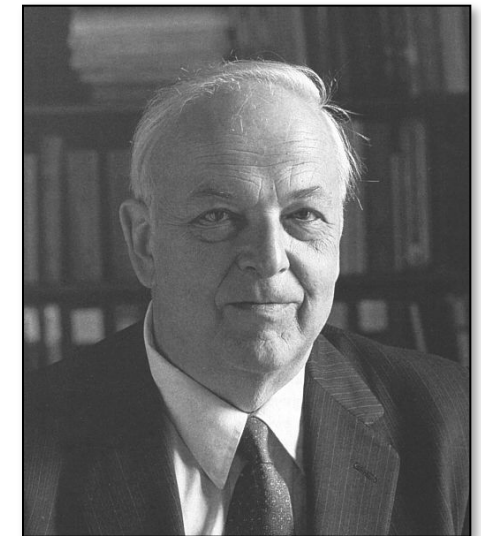


Sam 'J' Ting.



$$p + \text{Be} \rightarrow J(\rightarrow e^+e^-) + X$$

Burt 'psi' Richter



$$e^+e^- \rightarrow \Psi \rightarrow e^+e^-$$

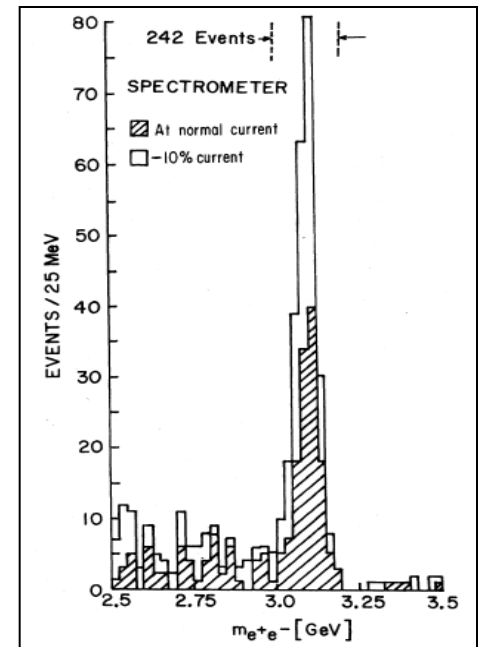
$$e^+e^- \rightarrow c\bar{c}$$

Discovery of charm (J/ψ)

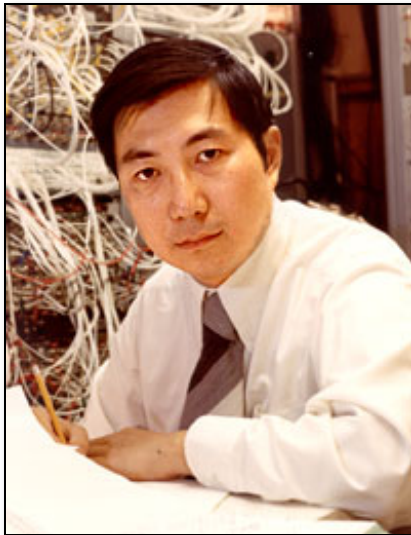
November 1976: 'November revolution'

New meson discovered (J/ψ) with mass 3.16 GeV

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Sam 'J' Ting.

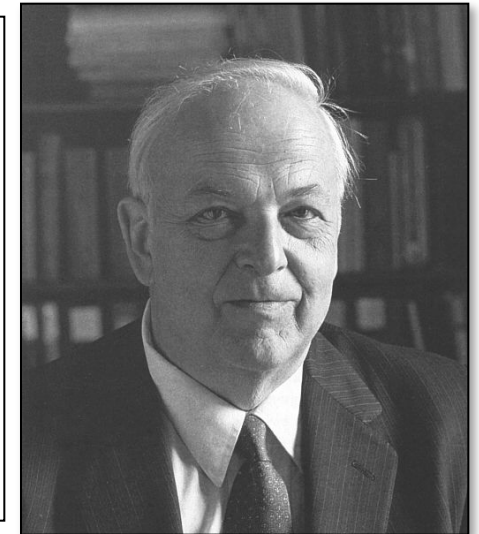


Because of the nearly simultaneous discovery, the J/ψ is the only elementary particle to have a **two-letter name**.

- Richter named it "SP", after the SPEAR accelerator used at SLAC; however, none of his coworkers liked that name. After consulting with Greek-born Leo Resvanis to see which Greek letters were still available, and rejecting "iota" because its name implies insignificance, Richter chose "**psi**" – a name which, as Gerson Goldhaber pointed out, contains the original name "SP", but in reverse order.^[2] Coincidentally, later spark chamber pictures often resembled the psi shape.

- Ting assigned the name "J" to it, which is one letter removed from "K", the name of the already-known strange meson; possibly by coincidence, "**J**" strongly resembles the Chinese character for Ting's name (T). J is also the first letter of Ting's oldest daughter's name, Jeanne.

Burt 'ψ' Richter



$$p + \text{Be} \rightarrow J(\rightarrow e^+e^-) + X$$

$$e^+e^- \rightarrow \Psi \rightarrow e^+e^-$$

Intermezzo: "indirect discoveries"

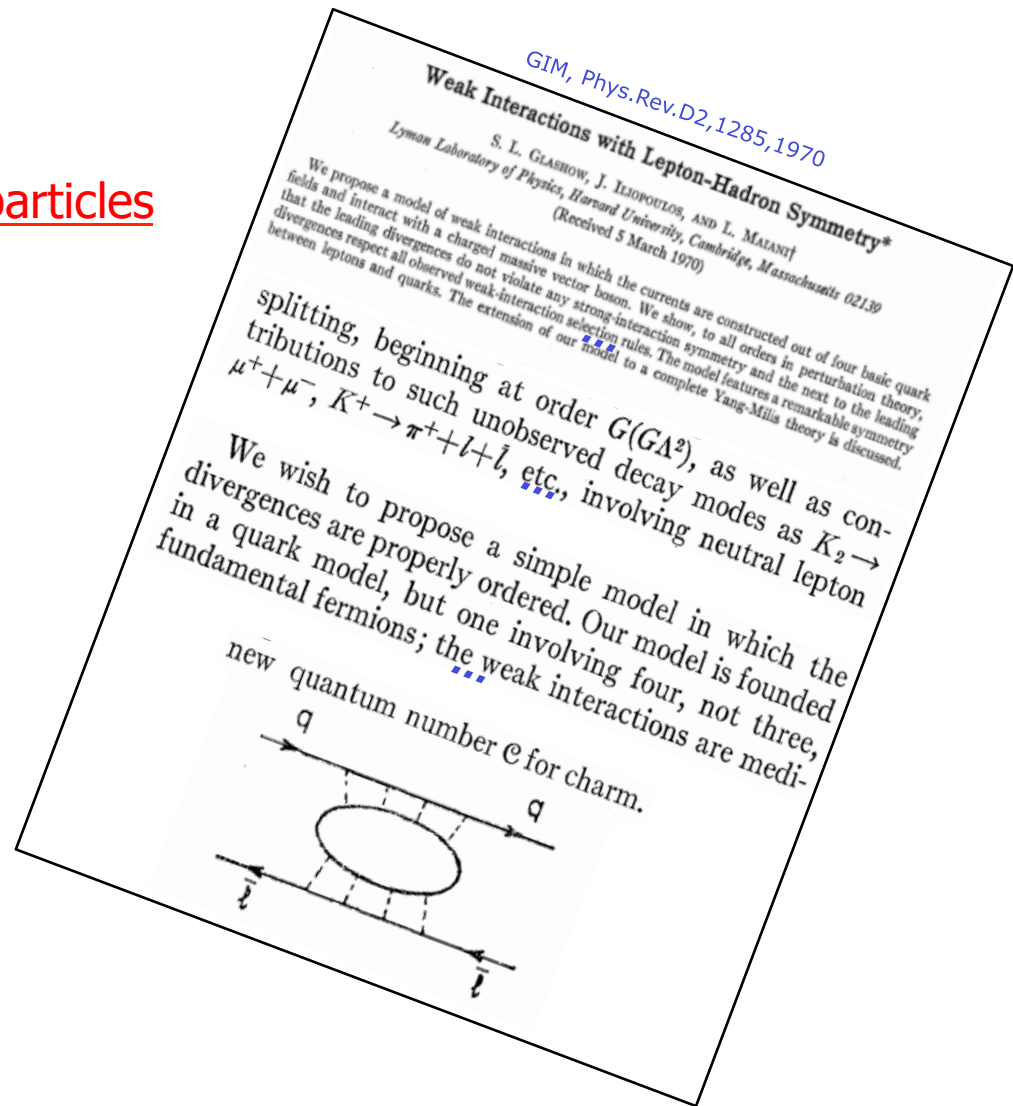
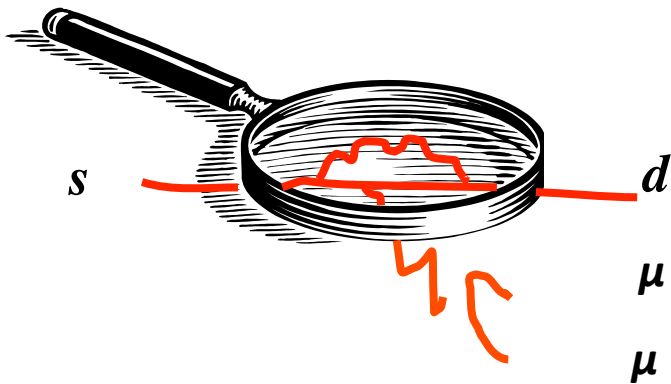
1970:

- Prediction of charm quark
- Observation $K^0 \rightarrow \mu\mu$

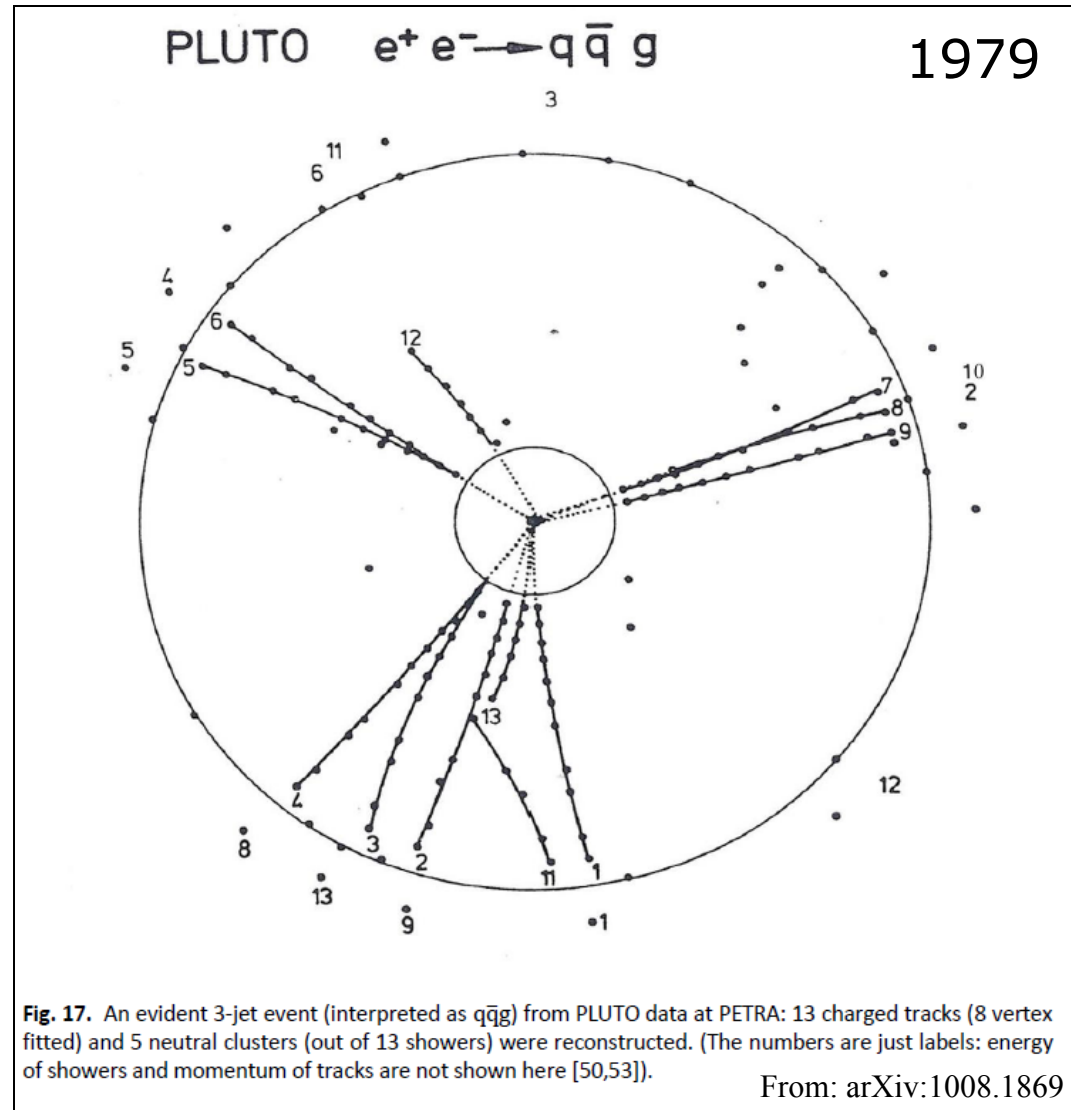
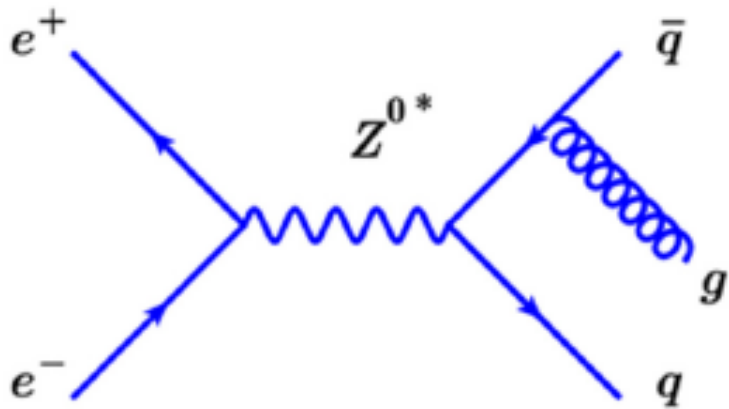
Lesson:

Loop diagrams are sensitive to heavy particles

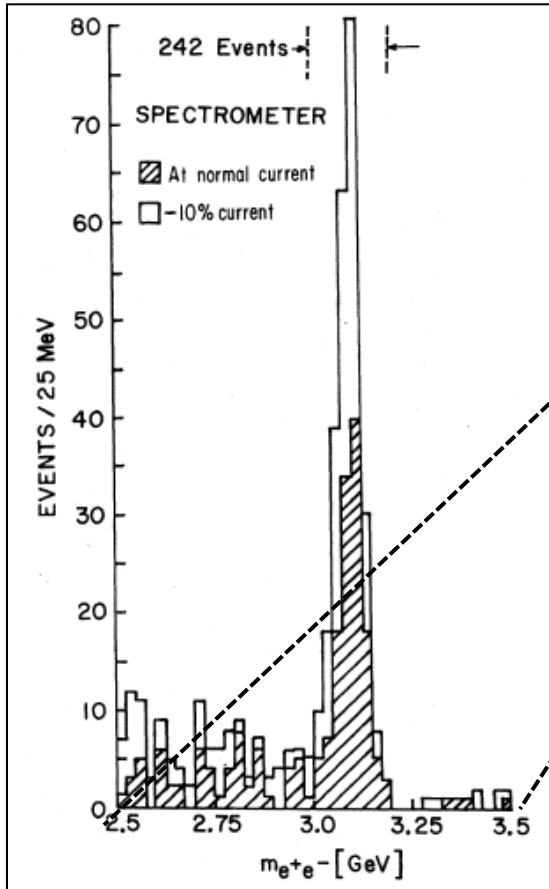
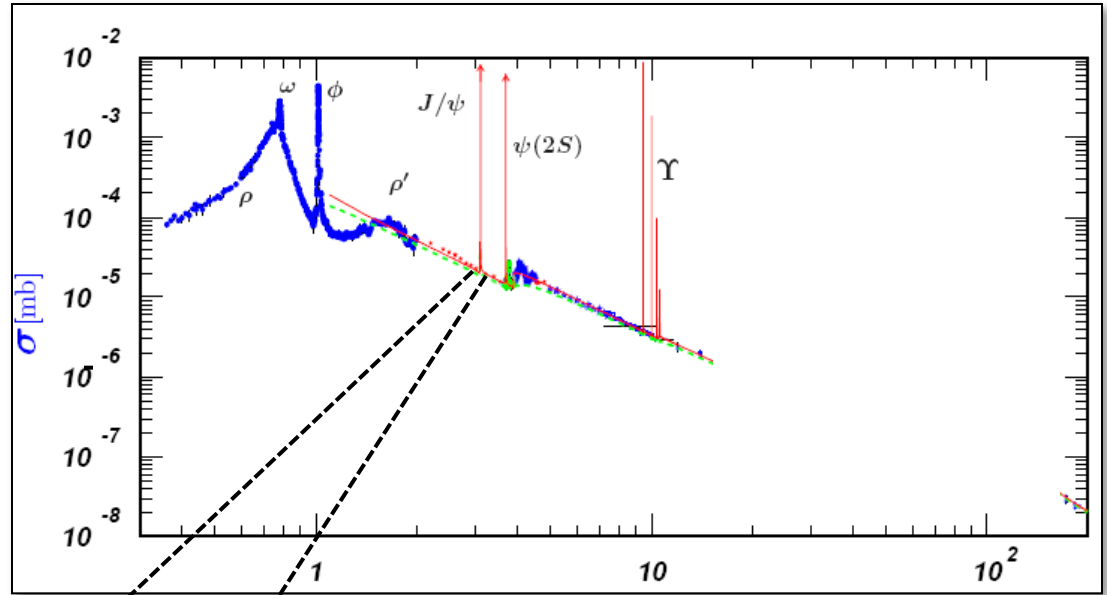
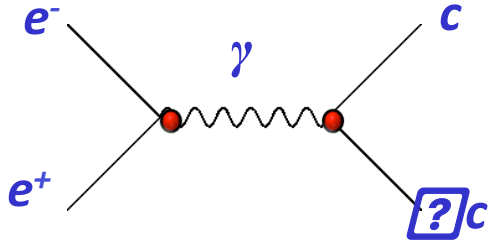
"Loop" diagram:



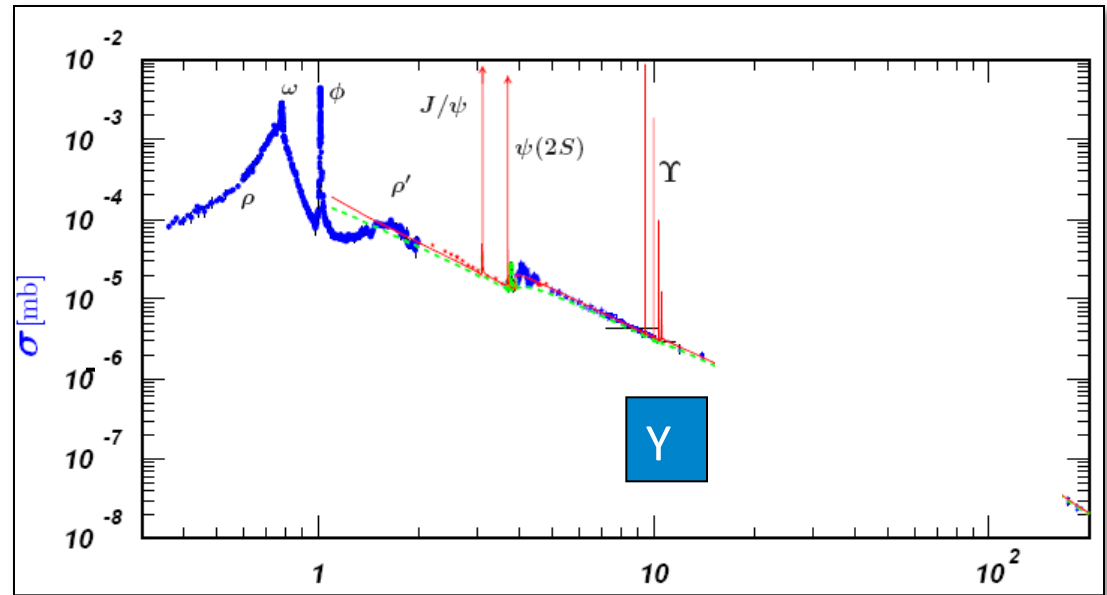
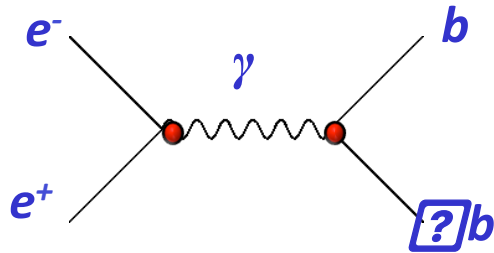
$$e^+e^- \rightarrow qqg$$



$$e^+e^- \rightarrow cc$$



$e^+e^- \rightarrow bb$?

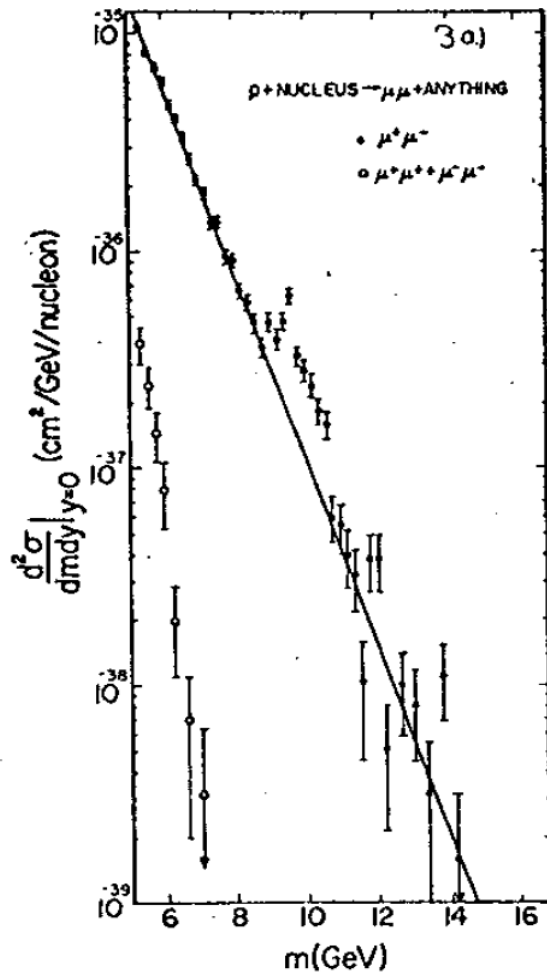


- In modern times: yes!
 - Babar experiment at SLAC (USA)
 - Belle experiment at KEK (Japan)
 - $e^+e^- \rightarrow Y(4S) \rightarrow B^0 \bar{B}^0$
 - $M(Y(4S)) = 10579.4$ MeV
 - $2 \times M(B^0) = 10559.2$ MeV (coincidence??)
- “B-factories”

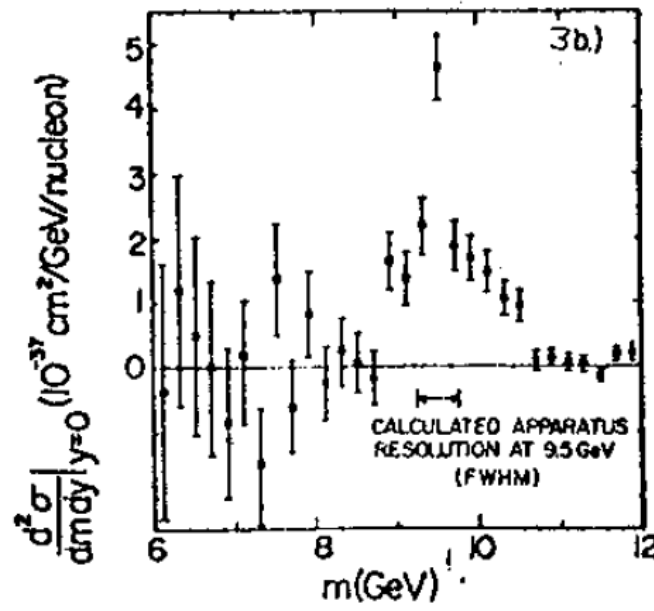
Intermezzo: bottom discovery

- Fermilab (Chicago), 400 GeV proton – fixed target
 - $p + (\text{Cu, Pt}) \rightarrow \mu^+ + \mu^- + \text{anything}$
 - $m(\text{bb}) \sim 9.5 \text{ GeV}$, so $m(\text{b}) \sim 5 \text{ GeV}$

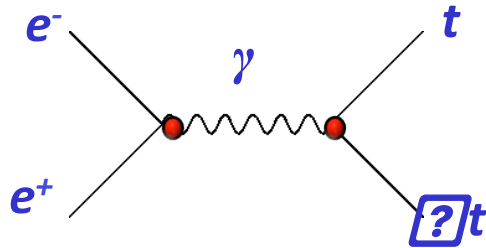
$$p + (\text{Cu, Pt}) \rightarrow \mu^+ + \mu^- + \text{anything}$$



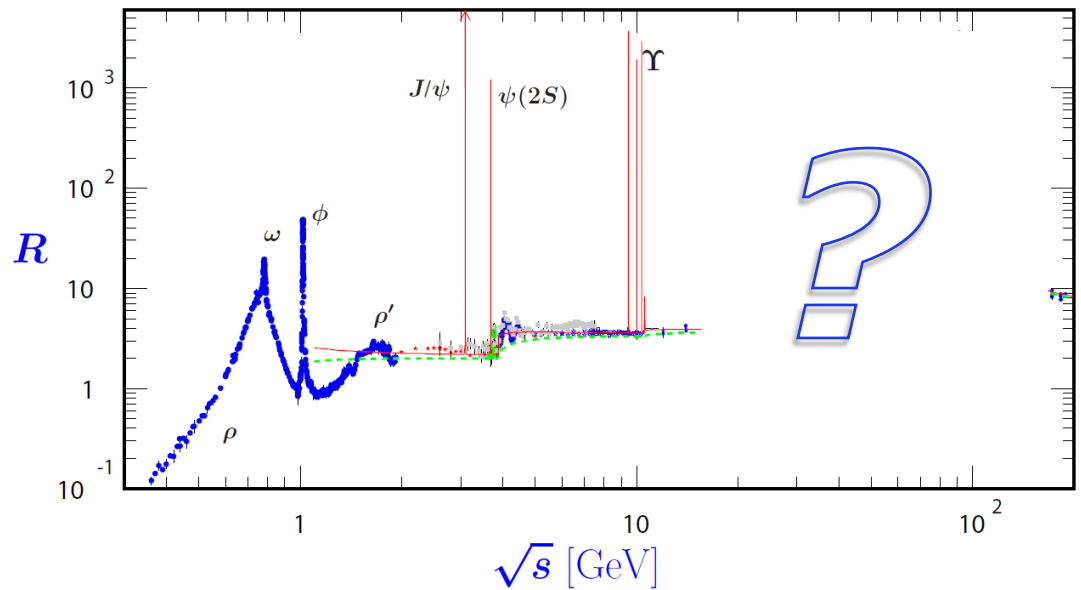
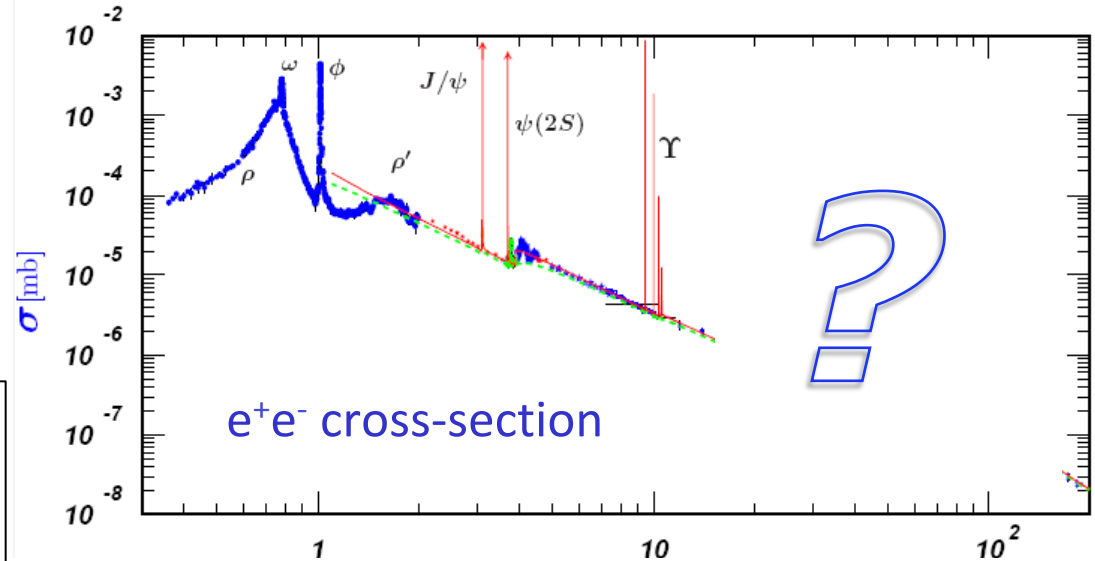
“background subtracted”:



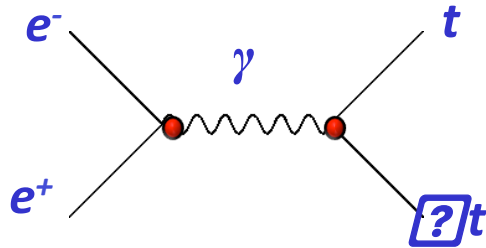
$$e^+e^- \rightarrow tt$$



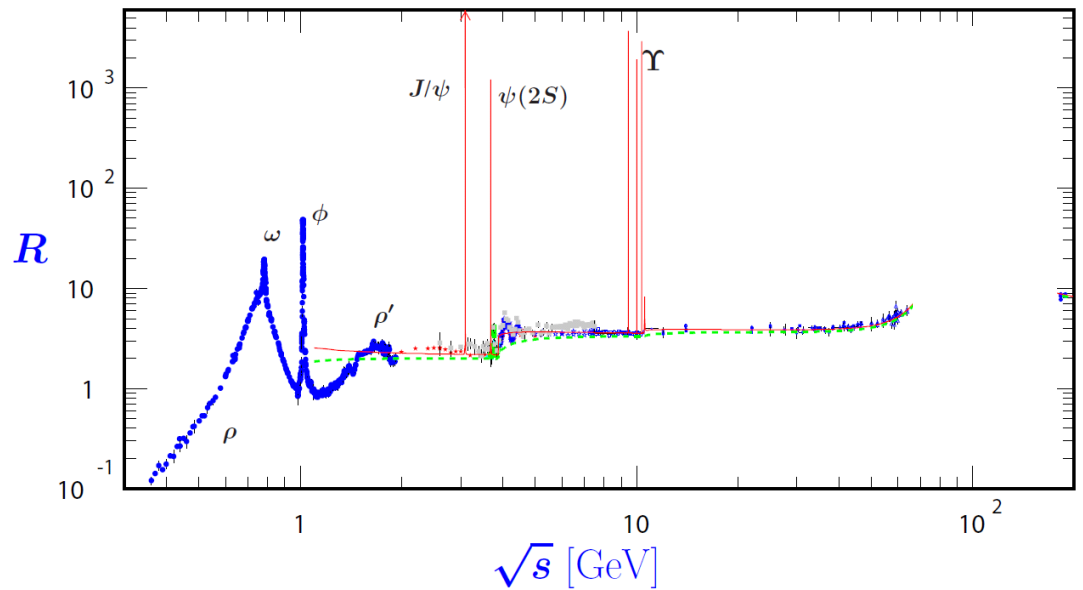
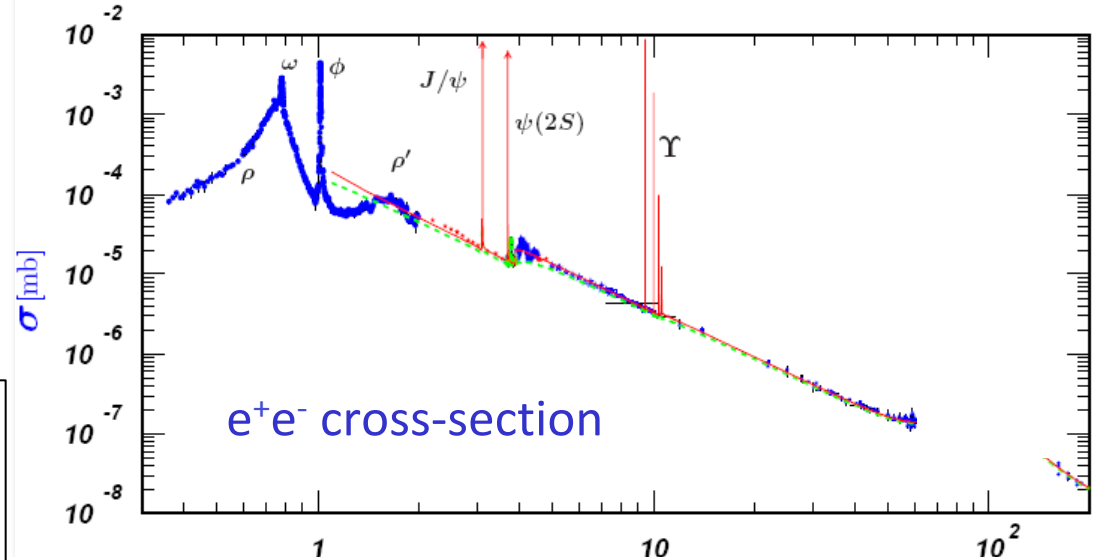
- 1977: bottom discovered
- How heavy could the **top** be??
 - 20 GeV ?
 - 25 GeV ??
 - 30 GeV ???
- Built Tristram accelerator, with Topaz detector (Japan)
 - Reached 25.5 GeV in 1986
- At minimum of cross section...



$$e^+e^- \rightarrow tt$$



- 1977: bottom discovered
- How heavy could the **top** be??
 - 20 GeV ?
 - 25 GeV ??
 - 30 GeV ???
- Built Tristan accelerator, with Topaz detector (Japan)
 - Reached 25.5 GeV in 1986
- At minimum of cross section...



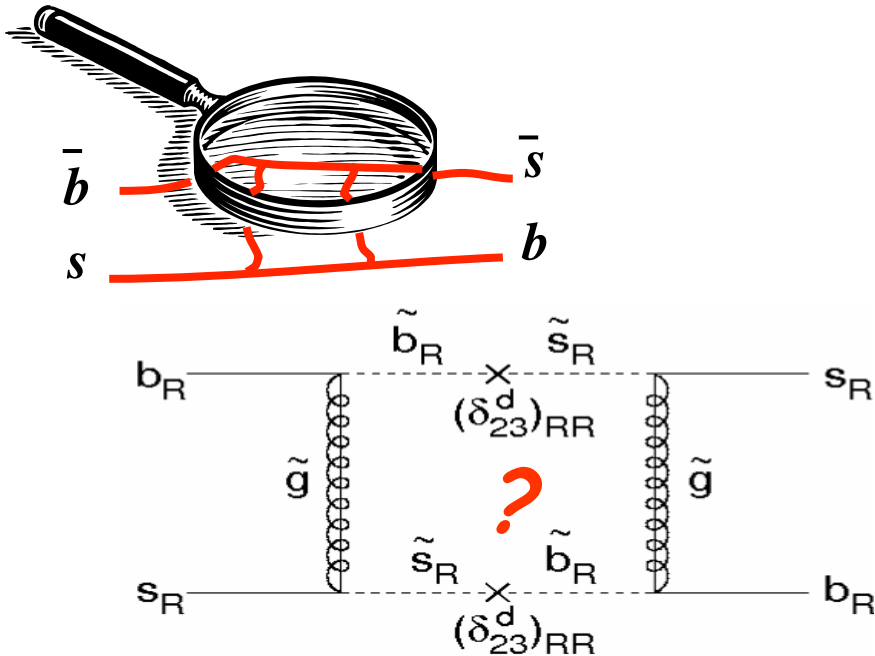
Intermezzo: "indirect discoveries"

ARGUS almost discovered the top quark:

➤ $m_{\text{top}} > 50 \text{ GeV}$

Lesson:

Loop diagrams are sensitive to heavy particles



ARGUS Coll, Phys.Lett.B192:245,1987

DESY 87-029
April 1987

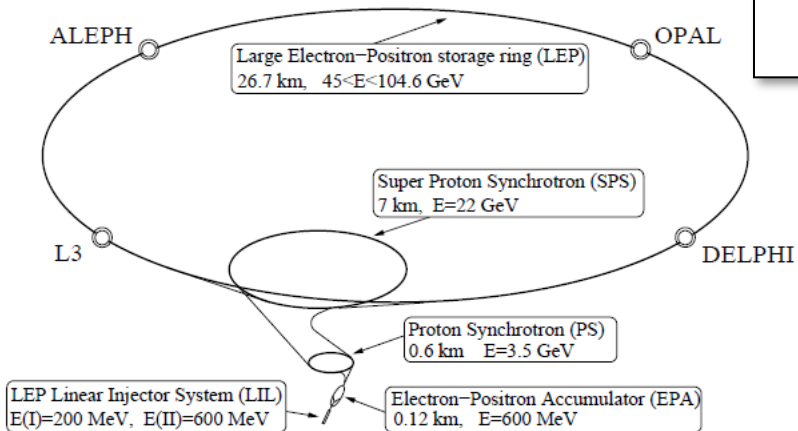
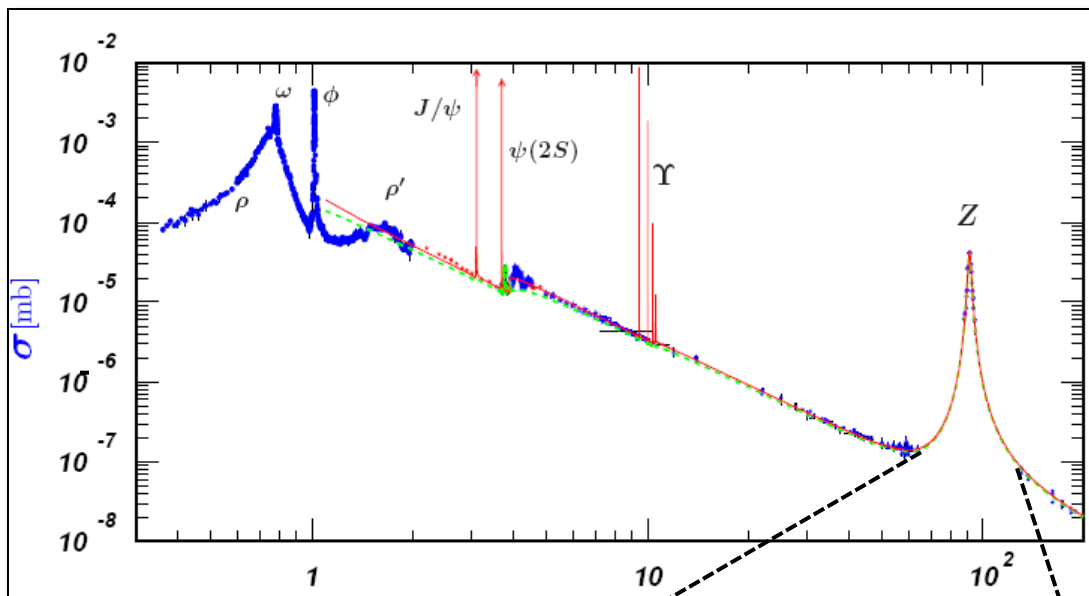
OBSERVATION OF $B^0 - \bar{B}^0$ MIXING

The ARGUS Collaboration

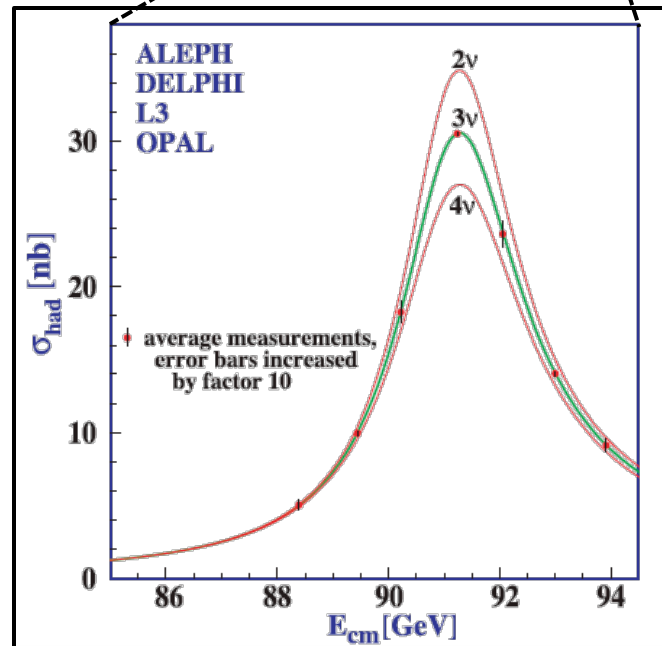
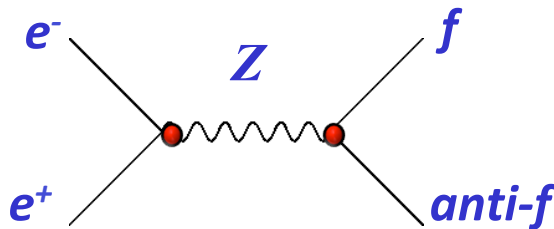
In summary, the combined evidence of the investigation of B^0 meson pairs, lepton pairs and B^0 meson-lepton events on the $\Upsilon(4S)$ leads to the conclusion that $B^0 - \bar{B}^0$ mixing has been observed and is substantial.

Parameters	Comments
$r > 0.09 \text{ } 90\%CL$	This experiment This experiment B meson (\approx pion) decay constant b-quark mass B meson lifetime Kobayashi-Maskawa matrix element QCD correction factor [17] t quark mass
$x > 0.44$	
$B^{\frac{1}{2}} f_B \approx f_\pi < 160 \text{ MeV}$	
$m_b < 5 \text{ GeV}/c^2$	
$\tau_b < 1.4 \cdot 10^{-12} \text{ s}$	
$ V_{td} < 0.018$	
$ V_{cb} < 0.80$	
$m_t > 50 \text{ GeV}/c^2$	

$$e^+e^- \rightarrow Z$$



Z-boson



$e^+e^- \rightarrow Z$

Remember:

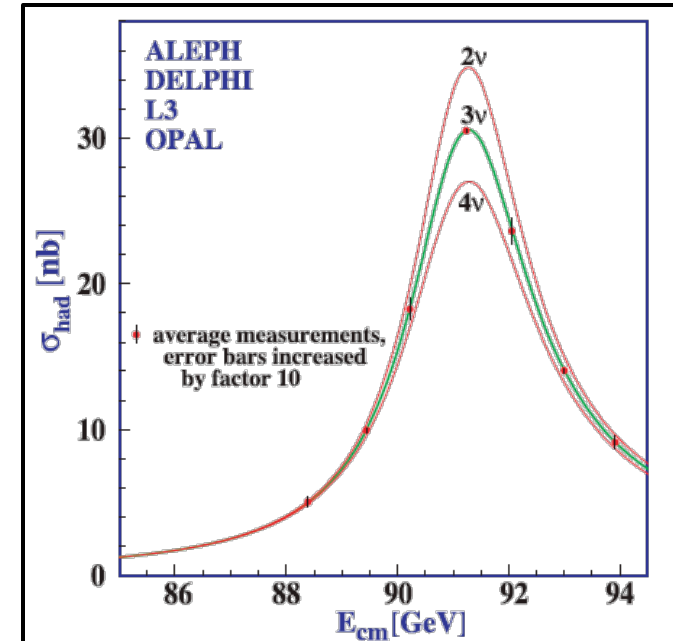
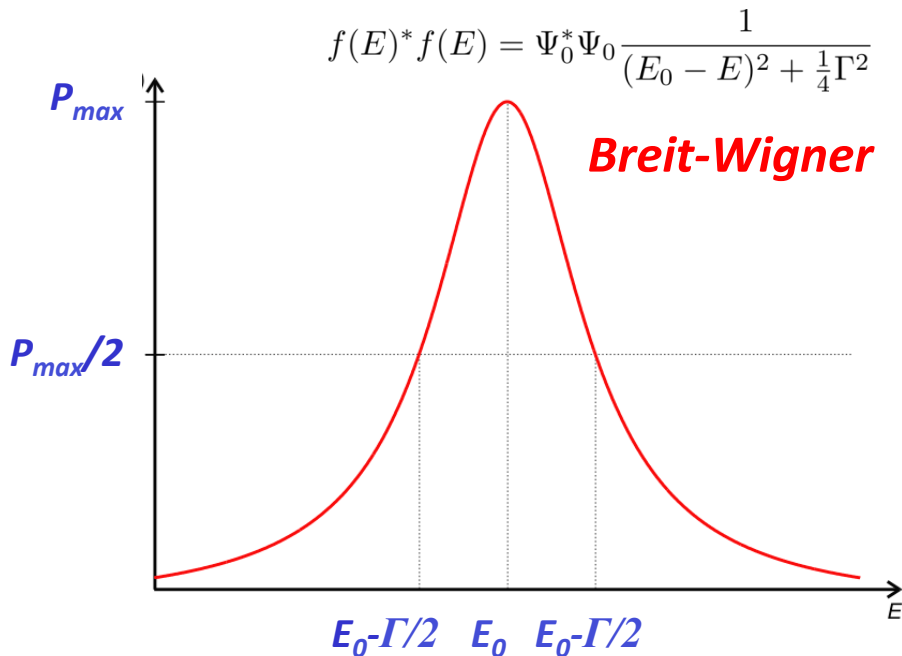
- $A \rightarrow B+C$: Total decay rate

- $\tau = \hbar / \Gamma$

- $A+B \rightarrow R \rightarrow C+D$: Resonance:

$$d\Gamma = \frac{(2\pi)^4 \delta^4(p_A - p_C - p_D)}{2E_A} \cdot |\mathcal{M}|^2 \cdot \frac{d^3p_C}{(2\pi)^3 2E_C} \frac{d^3p_D}{(2\pi)^3 2E_D}$$

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= |f^{res}(\theta)|^2 \\ &= \frac{(2l+1)^2}{k^2} \frac{\frac{\Gamma^2}{4}}{(E_r - E)^2 + \frac{\Gamma^2}{4}} |P_l(\cos\theta)|^2 \end{aligned}$$



$e^+e^- \rightarrow Z$

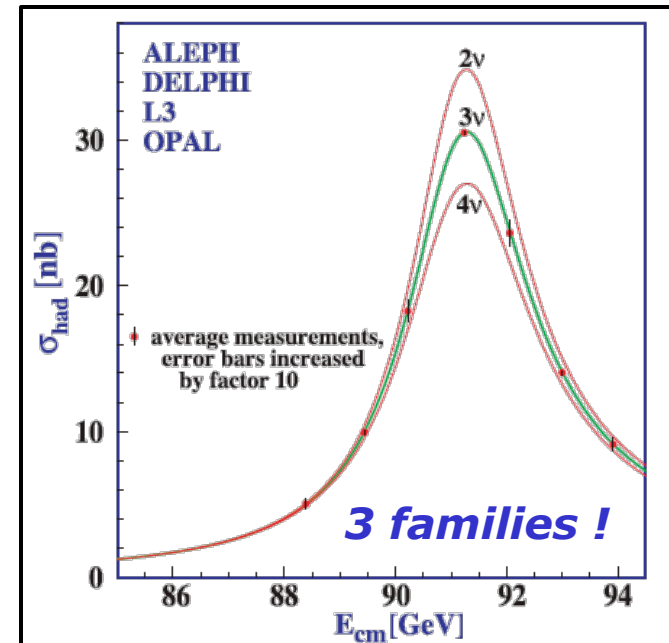
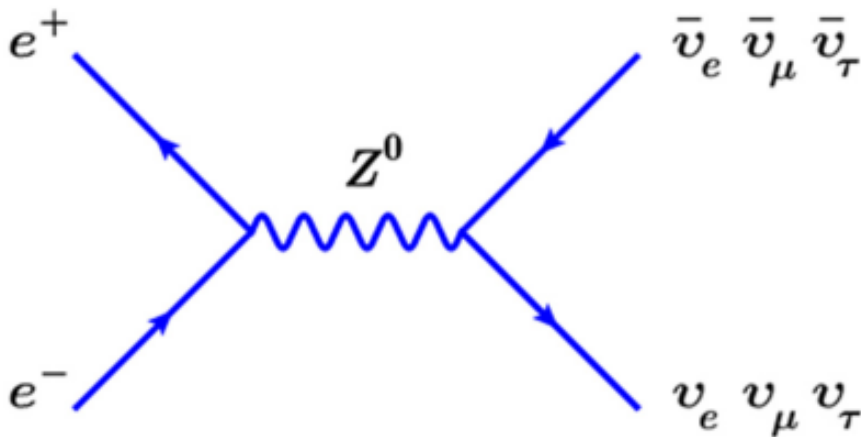
- Measure total width, with energy scan: “Z-lineshape”

□ Total width is sum of “partial widths”

$$\Gamma_{Z,total} = \Gamma_{ee} + \Gamma_{\mu\mu} + \dots$$

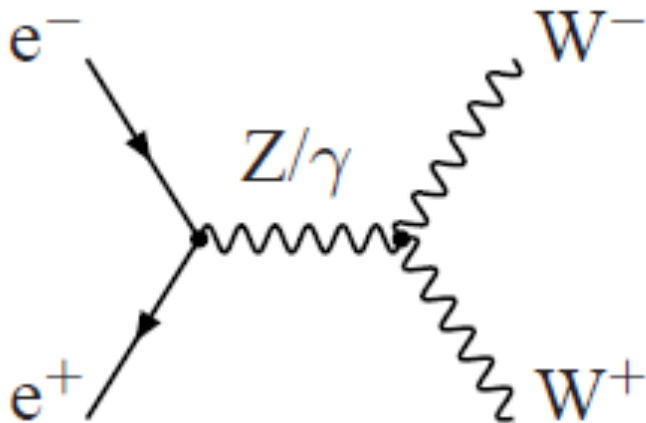
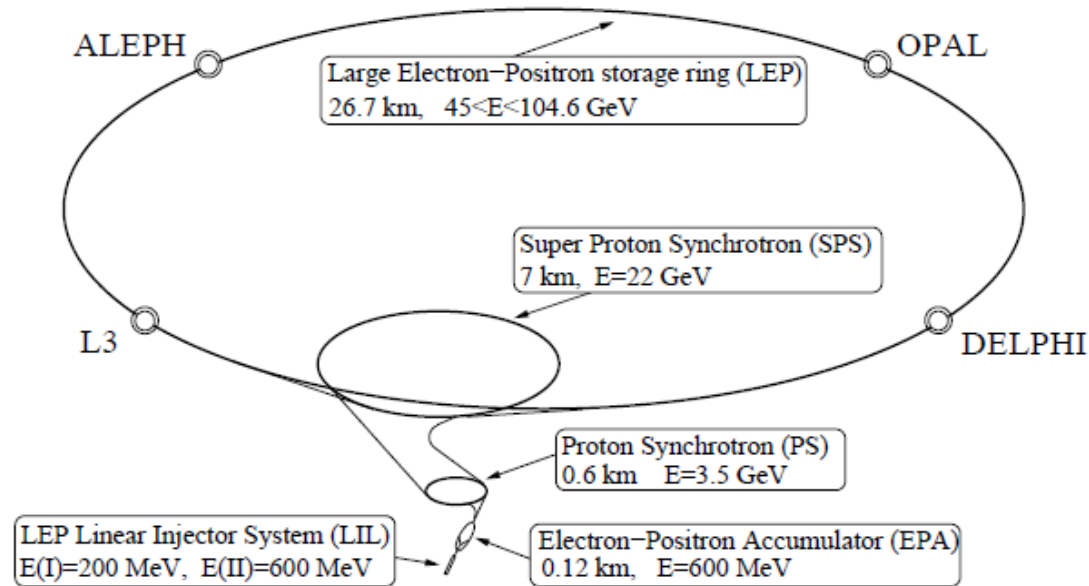
- Measure branching ratios of $Z \rightarrow [?]qq$ and $Z \rightarrow l^+l^-$

➤ “Invisible width” due to neutrinos



$e^+e^- \rightarrow WW$

- Upgrade of LEP:
90 \rightarrow 208 GeV!
- Enough energy to create WW-pairs
□ and ZH-pairs?!

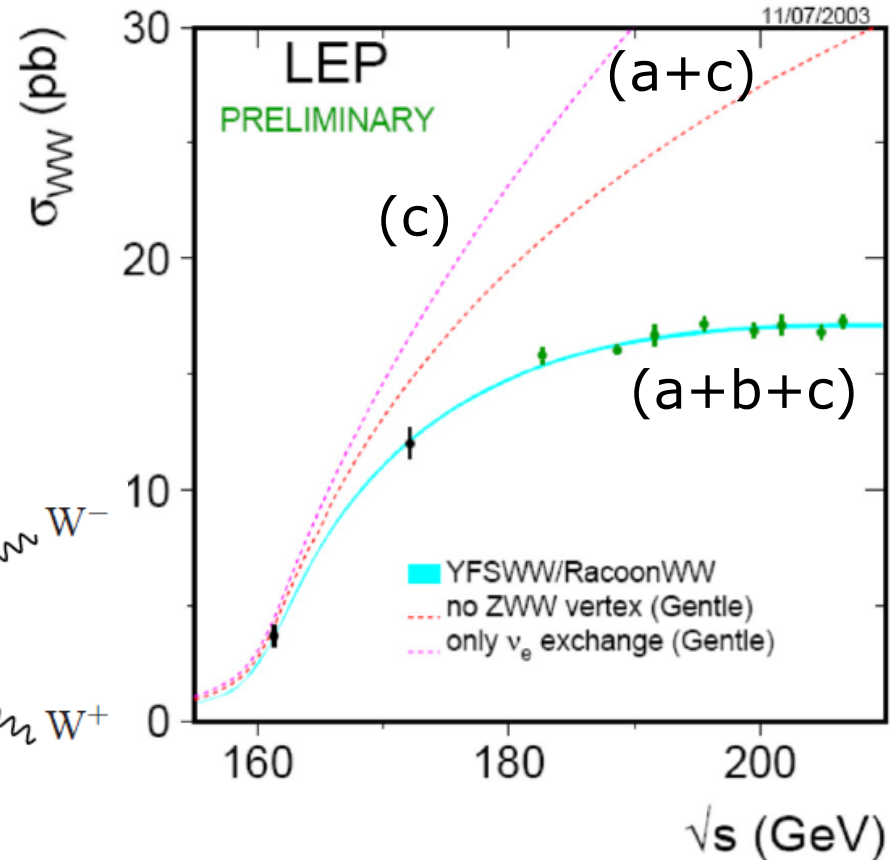
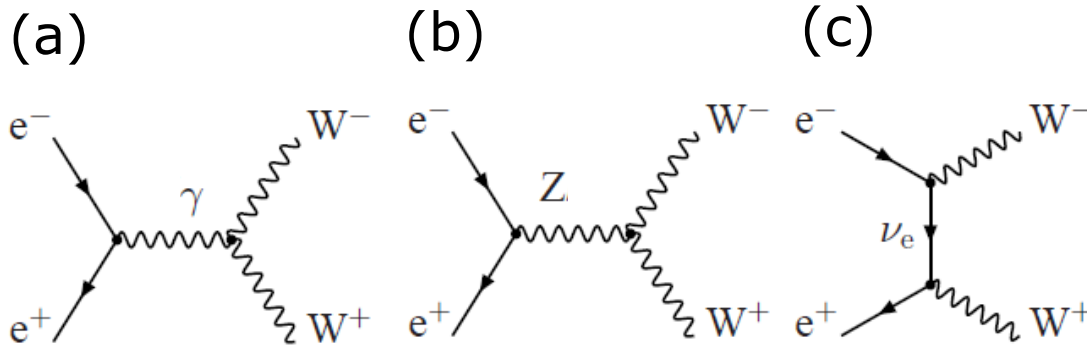
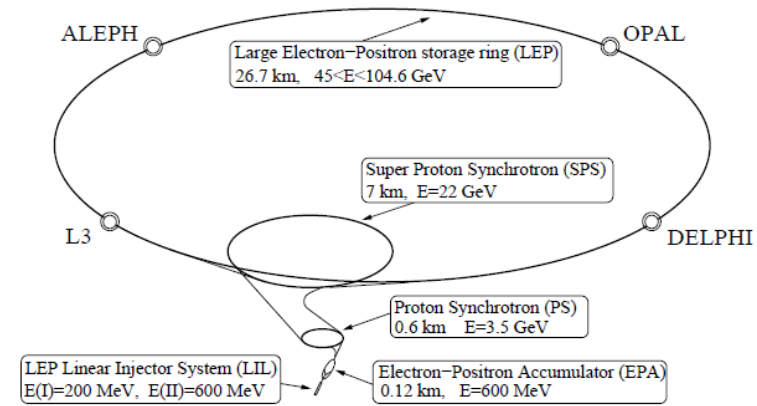


year	1996		1997	1998	1999			
\sqrt{s} (GeV)	161	172	183	189	192	196	200	202
\mathcal{L} (pb^{-1})	10.0	10.0	54.7	158	25.9	76.9	84.3	41.1

\sqrt{s} (GeV)	\mathcal{L} (pb^{-1})		
	< 205.5 (205)	> 205.5 (207)	all energies
C-period	75.6	87.8	163.4
S-period	6.3	54.3	60.7
year 2000	82.0	142.2	224.2

$$e^+e^- \rightarrow WW$$

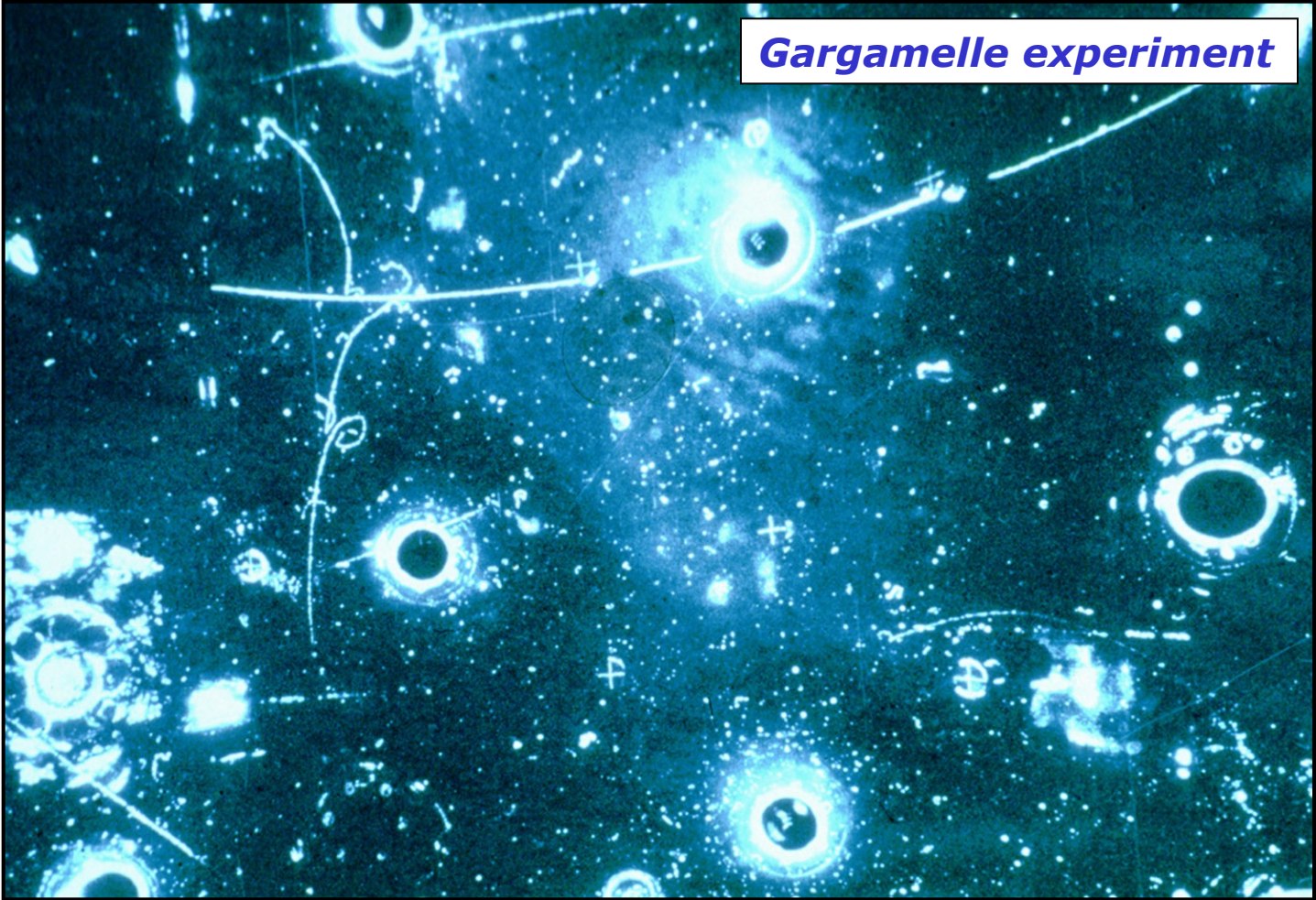
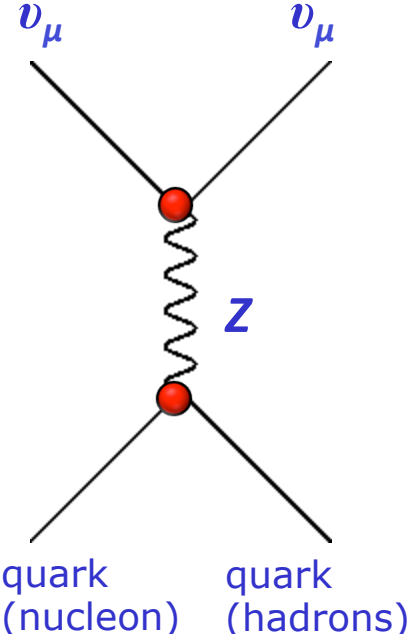
- Triumph for Standard Model:



***Intermezzo:
Discovery Z and W***

Discovery neutral current

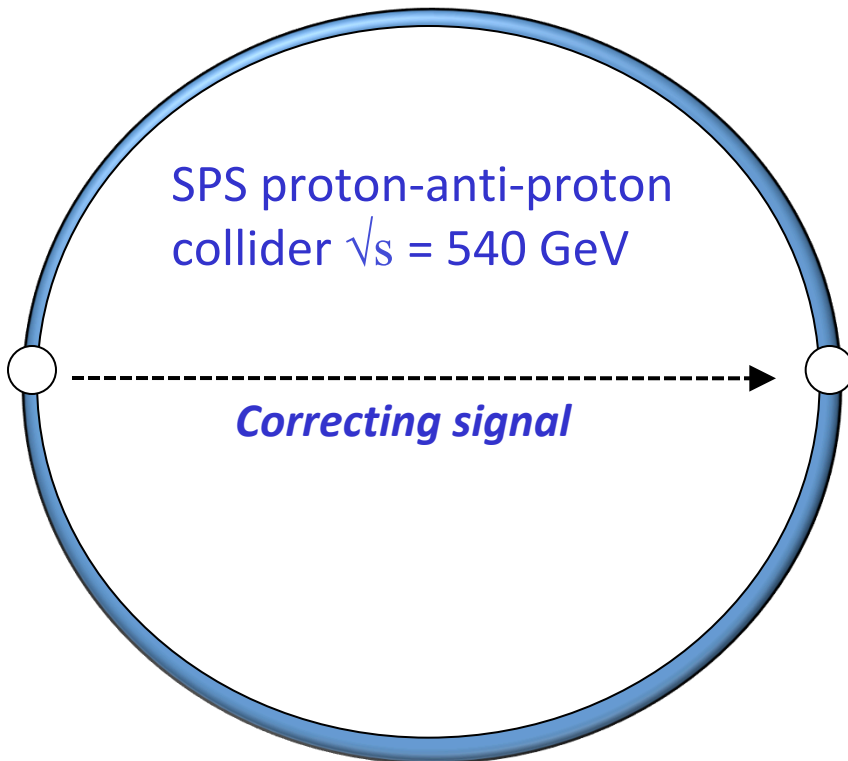
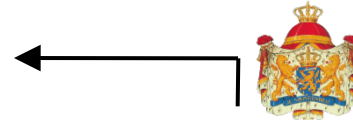
1973



Neutrinos on target (12,000 liter freon (CF_3Br))

Direct observation weak-force carrier

stochastic cooling



Simon van der Meer



Carlo Rubbia



- Spokesman UA1 experiment
- In 1989 Director General CERN
- Initiator of Energy Amplifier

3 June 1983

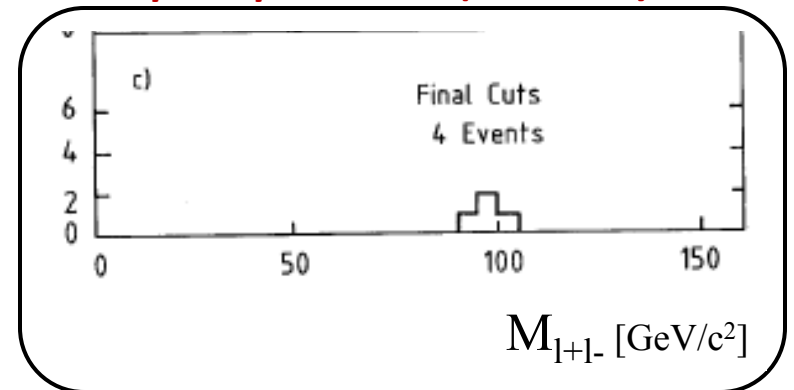
Discovery Z-boson



Nederlands tintje

“Experimental observation of lepton pairs of invariant mass around 95 GeV/c² at the CERN SPS collider”

Lepton pair mass (4 events)



EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/83-73
3 June 1983

EXPERIMENTAL OBSERVATION OF LEPTON PAIRS OF INVARIANT MASS
AROUND 95 GeV/c² AT THE CERN SPS COLLIDER

UAI Collaboration, CERN, Geneva, Switzerland

Aachen¹-Annecy (LAPP)²-Birmingham³-CERN⁴-Helsinki⁵-Queen Mary College, London⁶-
Paris (Coll. de France)⁷-Riverside⁸-Rome⁹-Rutherford Appleton Lab.¹⁰-
Saclay (CEA)¹¹-Vienna¹² Collaboration

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M. Kinnunen⁴, W. Kozanecki⁶, D. Kryn^{4,7}, P. Lacava³, J.-P. Laugier¹¹, J.-P. Lees²,
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T. Markiewicz⁴, G. Maurin⁴, T. Nctahon⁸, J.-P. Mendiburu⁷, M.-W. Minord²,
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E. Radernacher¹, J. Ransdell⁶, H. Reithler¹, J.-P. Revol⁴, J. Rich¹¹,
M. Rijssenbaek⁴, C. Roberts¹⁰, J. Bohl⁴, P. Bossi⁴, G. Rubbia⁴, B. Sadoulet⁴,
G. Sajot⁷, G. Salvi⁶, G. Salvini³, J. Sass¹¹, J. Soudraix¹¹, A. Savoy-Navarro¹¹,
D. Schinzel⁴, W. Scott¹⁰, T.P. Shah¹⁰, M. Spiro¹¹, J. Strauss¹², J. Straats³,
K. Sumorok⁴, F. Szponco¹², D. Smith⁴, G. Tso⁴, G. Thompson⁶, J. Timmer⁴,
E. Tchesonog¹, J. Tuominen⁵, B. Van Eijk⁴, J.-P. Vialle⁴, J. Vrana⁷,
V. Vuillemin⁴, H.D. Wahl¹², P. Watkins³, J. Wilson³, C. Wulz¹², C.Y. Xie⁴,
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(Submitted to Physics Letters B)

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“Experimental observation of isolated large transverse energy electrons with associated missing energy at $\sqrt{s}=540$ GeV”

Discovery W-boson

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EXPERIMENTAL OBSERVATION OF ISOLATED LARGE TRANSVERSE ENERGY ELECTRONS WITH ASSOCIATED MISSING ENERGY AT $\sqrt{s} = 540$ GeV

UA1 Collaboration, CERN, Geneva, Switzerland

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Aachen^k—Annecy (LAPP)^b—Birmingham^c—CERN^d—Helsinki^e—Queen Mary College, London^f—Paris (Coll. de France)^g—Riverside^h—Romeⁱ—Rutherford Appleton Lab.¹—Saclay (CEN)^k—Vienna¹ Collaboration

Received 23 January 1983

We report the results of two searches made on data recorded at the CERN SPS Proton—Antiproton Collider: one for isolated large- E_T electrons, the other for large- E_T neutrinos using the technique of missing transverse energy. Both searches converge to the same events, which have the signature of a two-body decay of a particle of mass ~ 80 GeV/ c^2 . The topology as well as the number of events fits well the hypothesis that they are produced by the process $\bar{p} + p \rightarrow W^{\pm} + X$, with $W^{\pm} \rightarrow e^{\pm} + \nu$; where W^{\pm} is the Intermediate Vector Boson postulated by the unified theory of weak and electromagnetic interactions.

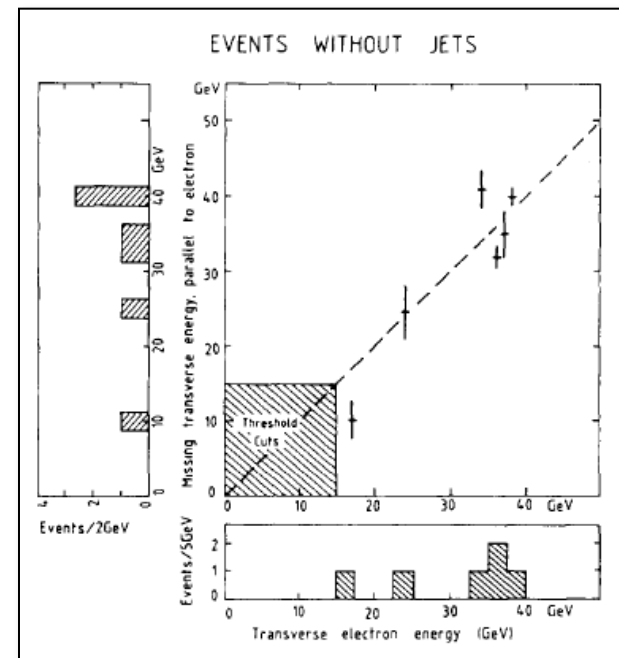
¹ University of Wisconsin, Madison, WI, USA.

² NIKHEF, Amsterdam, The Netherlands.

Prediction: $M_W = 82 \pm 2.4$ GeV

Observation: $M_W = 81 \pm 5.0$ GeV

2012: $M_W = 80.399 \pm 0.023$ GeV



Deep Inelastic Scattering

Lepton – proton scattering

or:

Hitting something big, using something small

Shopping list

- Quarkmodel: do quarks exist??
- Substructure
- Bjorken-x, sum rules
- Scaling
- 'Parton density functions' (pdf) and 'structure functions'
- Scaling violations

Scattering

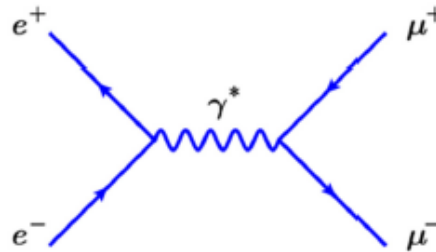
- Rutherford scattering

(scattering off static point charge)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\frac{1}{2}\theta)}$$

- $e^+e^- \rightarrow \mu^+\mu^-$ scattering

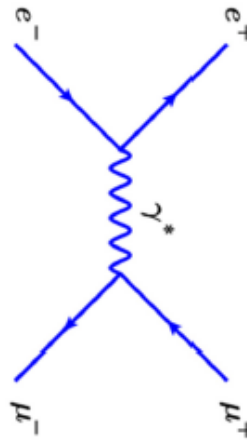
(s-channel)



$$\left. \frac{d\sigma}{d\Omega} \right|_{cm} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta)$$

- $e^-\mu^+ \rightarrow e^-\mu^+$ scattering

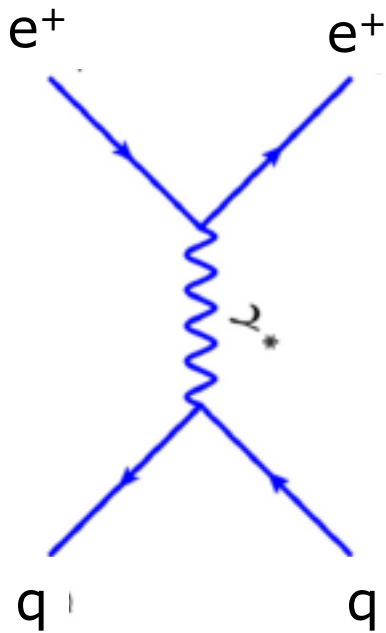
(t-channel)



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} e^2 \frac{4 + (1 + \cos \vartheta)^2}{(1 - \cos \vartheta)^2}$$

$e^+q \rightarrow e^+q$

- Point cross section



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} e_q^2 \frac{4 + (1 + \cos\vartheta)^2}{(1 - \cos\vartheta)^2}$$

$$Q^2 = 2E_e^2(1 - \cos\theta)$$

$$y = \sin^2 \frac{\theta}{2}$$

$$\frac{d\sigma^{eq \rightarrow eq}}{dQ^2} = \frac{2\pi\alpha^2}{Q^4} e_q^2 \left[2(1 - y) + y^2 \right]$$

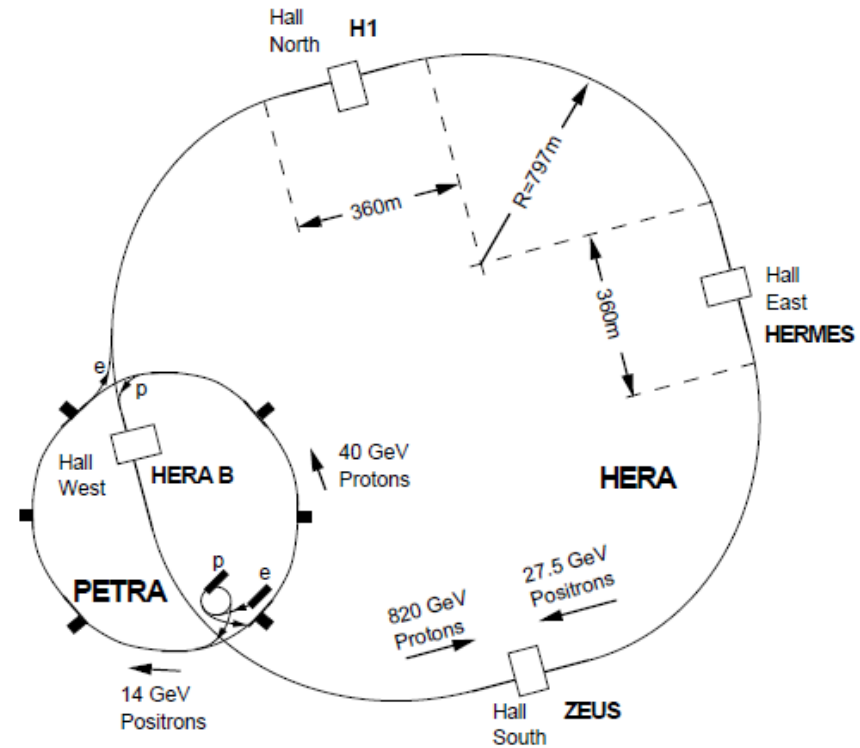
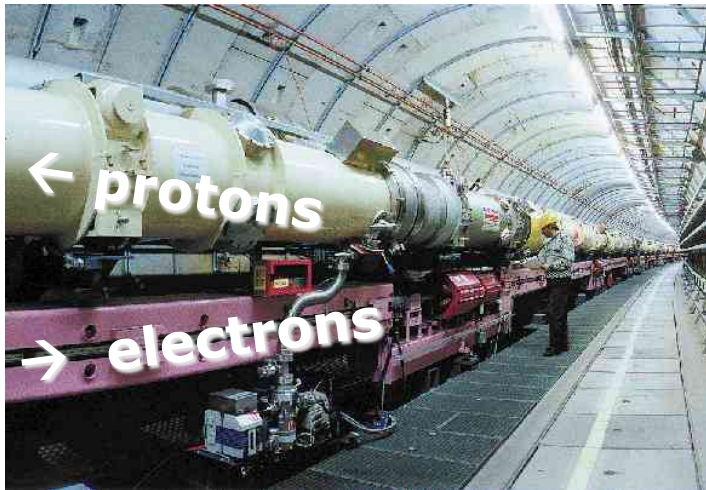
DIS experiments

- Easiest: fixed target
 - ep scattering
 - μp scattering
 - νp scattering

Experiment	Accel	Lab	lepton	E_{lep}	E_{had}	Year
SLAC-MIT		SLAC	e	20	fixed	1967-1973
Gargamelle		CERN	ν		fixed	
E80 -	SLC	SLAC			fixed	
CHORUS	SPS	CERN	ν	10-200	fixed	1998
CCFR	Tevatron	Fermilab	ν	30-360	fixed	
NMC	SPS	CERN	μ	90-280	fixed	1986-1989
EMC/SMC	SPS	CERN	μ	100-190	fixed	1984-1994
BCDMS	SPS	CERN	μ	100-280	fixed	
ZEUS, H1	HERA	DESY	e	27.5	920	1992-2007

NB: Table not complete

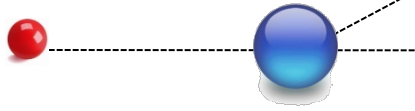
- 1990's: ep collider



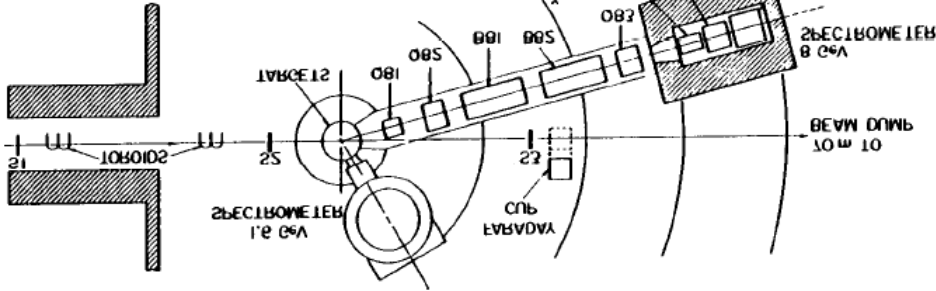
electron beam

proton target

detector

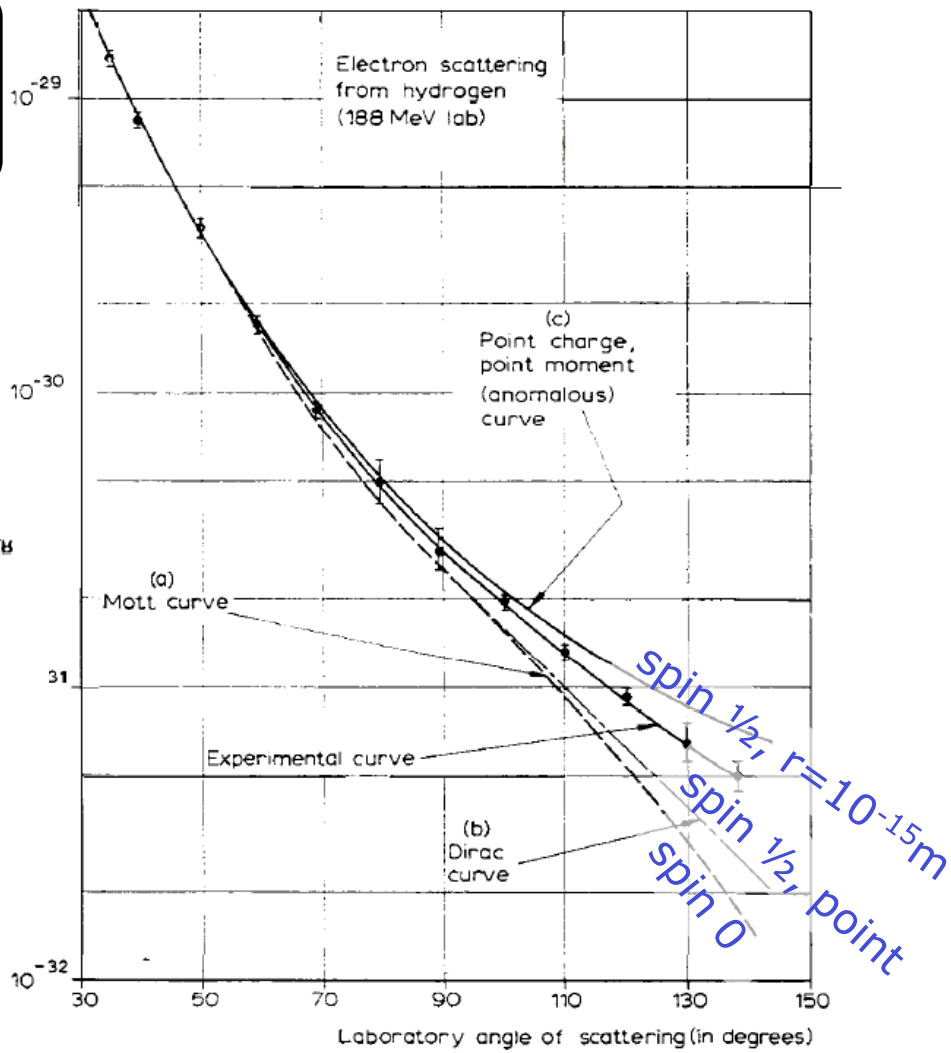


PLAN VIEW



$$\frac{d\sigma}{d\Omega}$$

Cross section



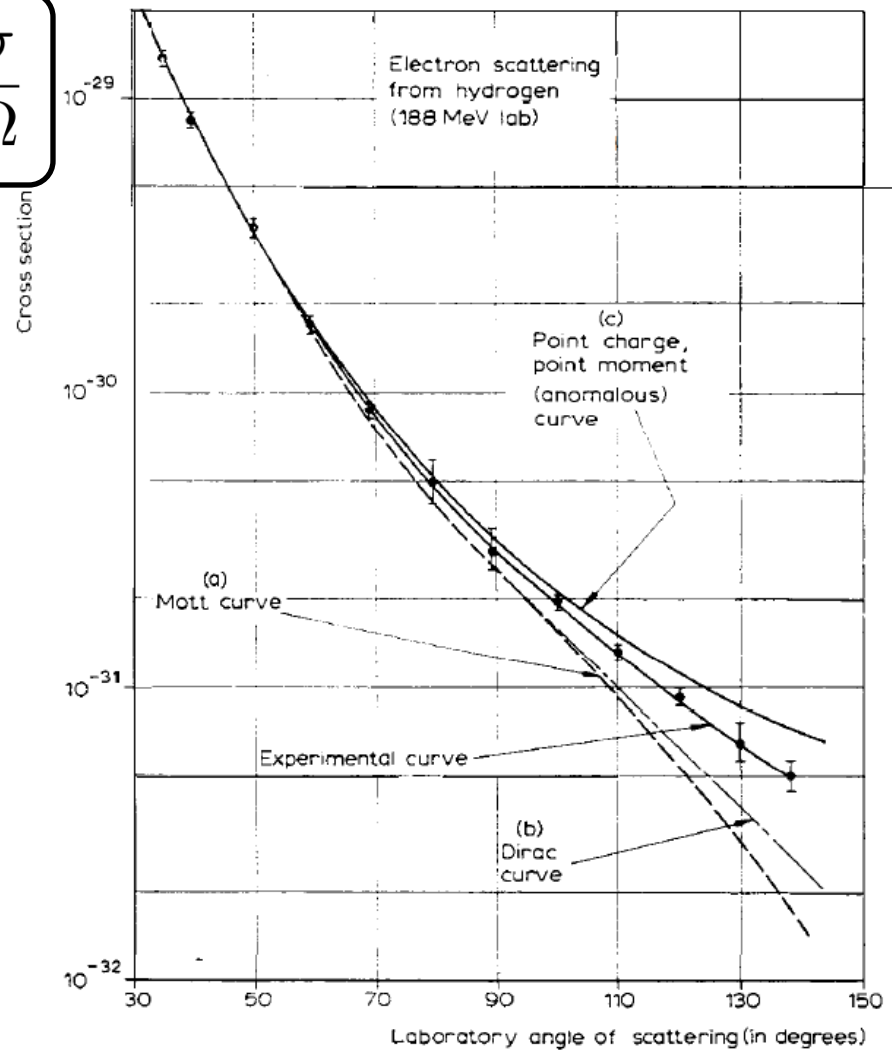
Robert Hofstadter

Fig. 9. Electron scattering from the proton at an incident energy of 188 MeV. Curve (a) shows the theoretical Mott curve for a spinless point proton. Curve (b) shows the theoretical curve for a point proton with a Dirac magnetic moment alone. Curve (c) shows the theoretical behavior of a point proton having the anomalous Pauli contribution in addition to the Dirac value of the magnetic moment. The deviation of the experimental curve from the Curve (c) represents the effect of form factors for the proton and indicates structure within the proton. The best fit in this figure indicates an rms radius close to $0.7 \cdot 10^{-13}$ cm.

Sub-structure

- Remember Rutherford
 - Back-scatter of α from nucleus
- Now:
 - Back-scatter of e from quarks

$$\frac{d\sigma}{d\Omega}$$



Scaling



J.D. Bjorken “scaling hypothesis” (1967):

- If scattering is caused by pointlike constituents structure functions must be independent of Q^2
- *Would you expect a Q^2 dependence?*

R. Feynmans “parton model” (1969):



- Proton consists of ‘constituents’
- *“Physicists were reluctant to identify these objects with quarks at the time, instead calling them “partons” – a term coined by Richard Feynman.”*

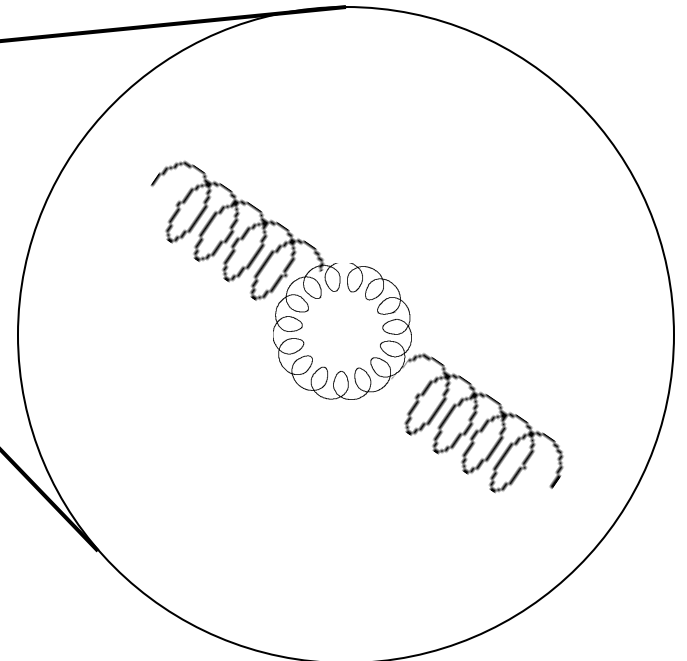
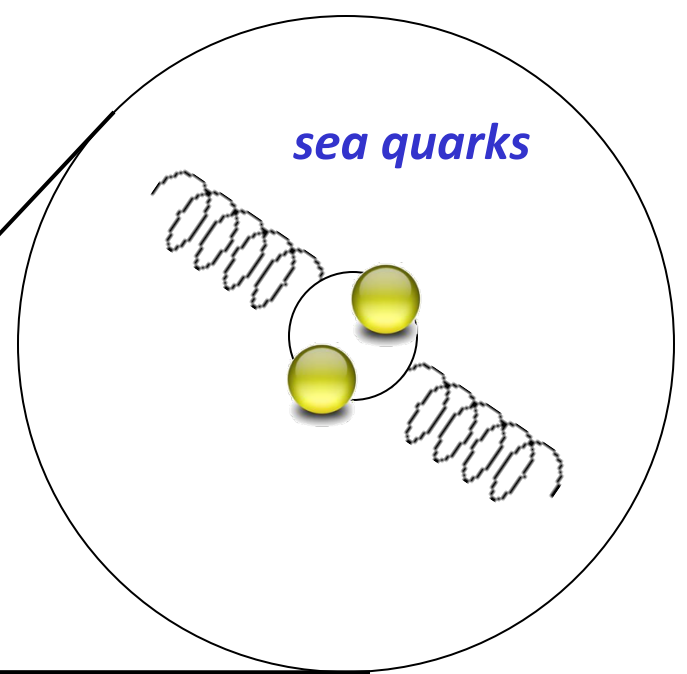
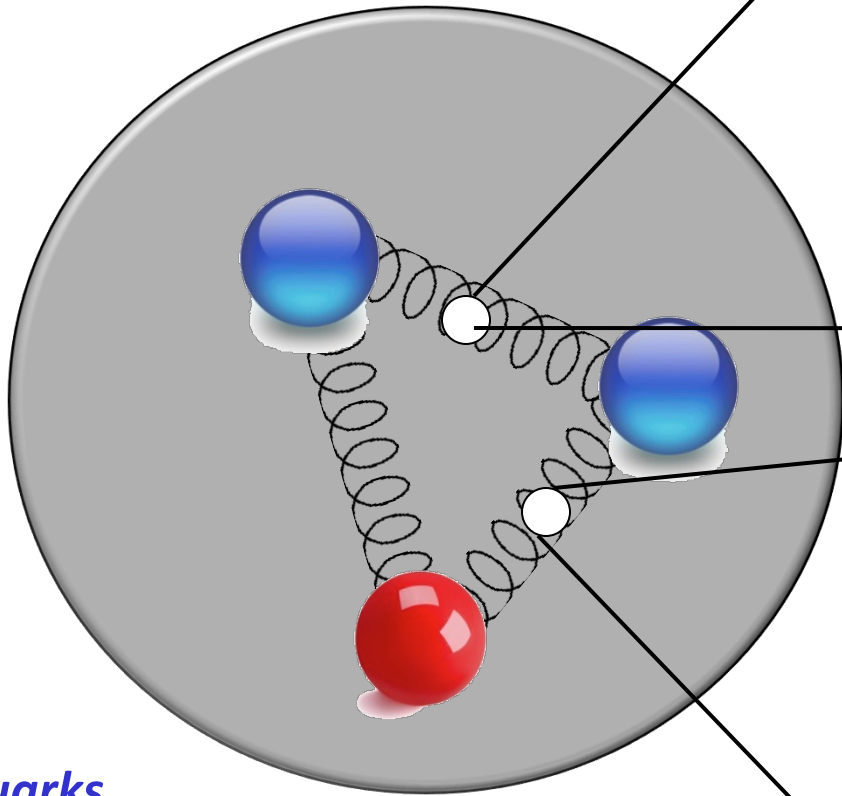
A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as “quarks” q and the members of the anti-triplet as anti-quarks \bar{q} . Baryons can now be constructed from quarks by using the combinations (qqq) , $(qqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration (qqq) gives just the representations 1, 8, and 10 that have been observed, while the lowest meson configuration $(q\bar{q})$ similarly gives just 1 and 8.

Figure 1.1: Murray Gell-Mann suggested in 1964 that the proton consists of three “quarks” ⁶ [1].

QCD: deep in the proton

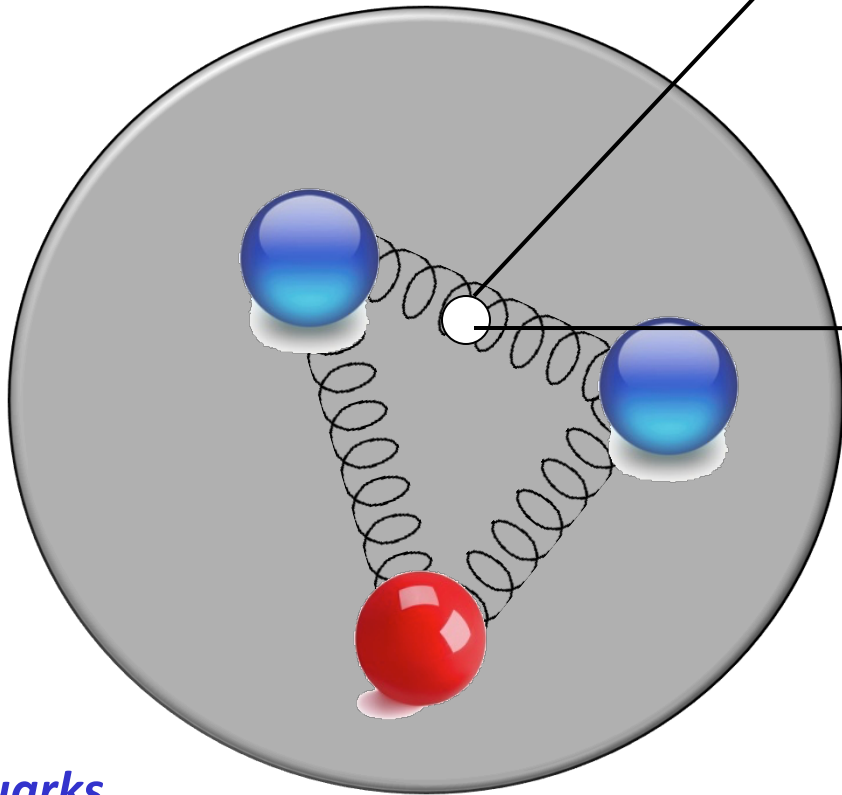
Proton

valence quarks

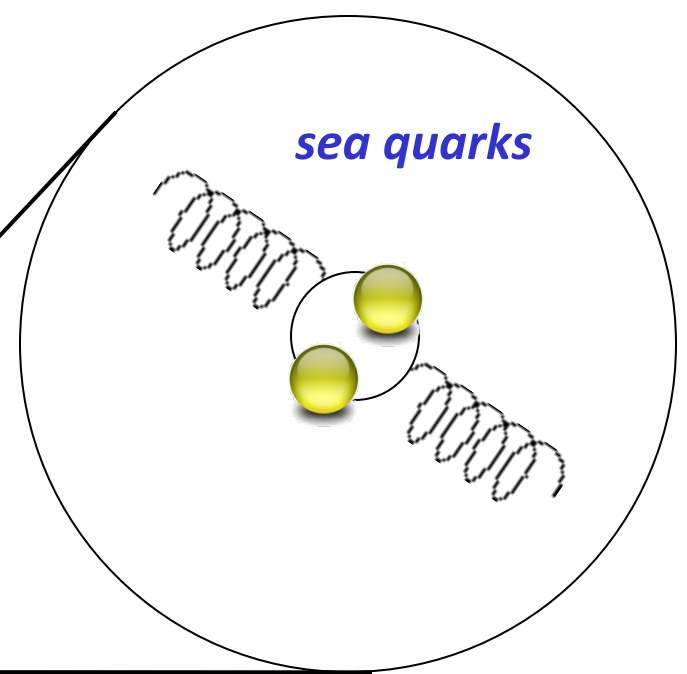


QCD: deep in the proton

Proton



valence quarks



sea quarks

Two important variables:

- Q^2 : 4-mom. transfer, scale
- x : fractional momentum of quark

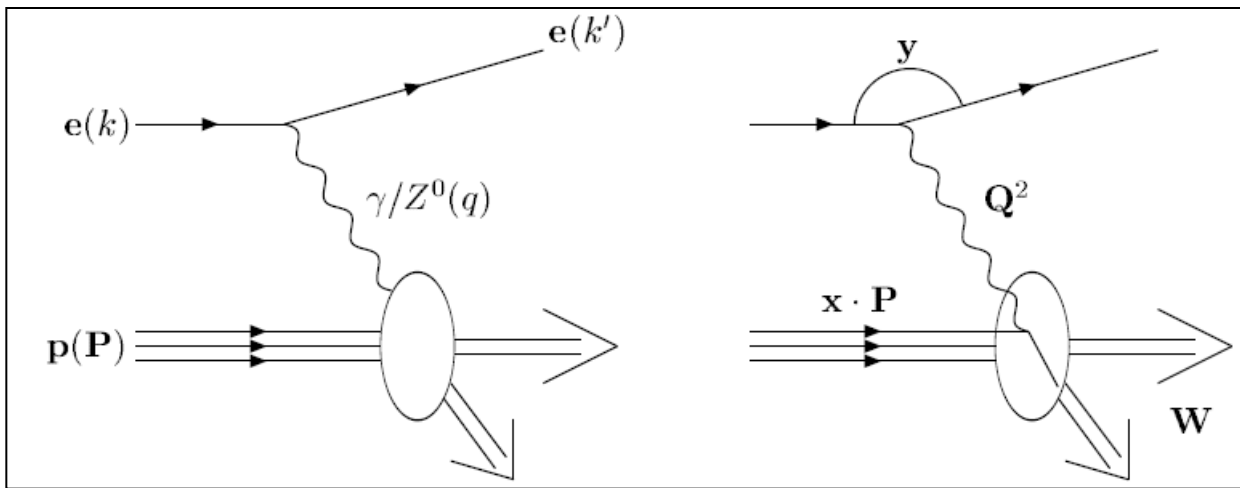
Deep Inelastic Scattering

- eq scattering:

$$\frac{d\sigma^{eq \rightarrow eq}}{dQ^2} = \frac{2\pi\alpha^2}{Q^4} e_q^2 \left[2(1-y) + y^2 \right]$$

- ep scattering:

$$\frac{d^2\sigma^{ep \rightarrow eX}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} (1 + (1-y)^2) F_2(x)$$



Form factor

$$Q^2 \equiv -q^2 = (k - k')^2 \quad : \text{Virtuality of the photon}$$

$$x \equiv \frac{-q^2}{2P \cdot q} \quad : \text{4-Momentum fraction carried by the struck quark}$$

$$y \equiv \frac{P \cdot q}{P \cdot k} \quad : \text{Inelasticity}$$

$$W^2 \equiv (P + q)^2 \quad : \text{Square of the invariant mass of the hadronic final state}$$

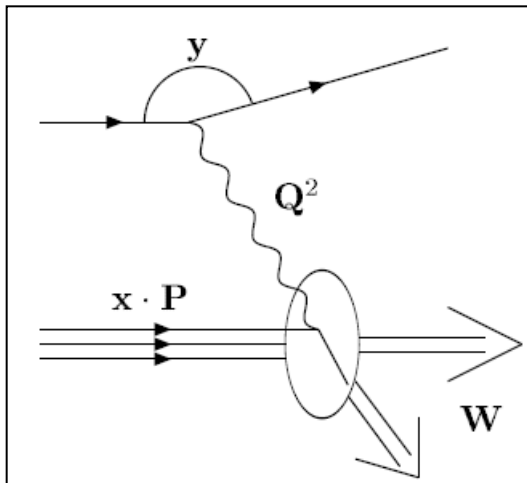
(From: PhD thesis N. Tuning)

Deep Inelastic Scattering

- ep scattering:

$$\frac{d^2\sigma^{ep \rightarrow eX}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} (1 + (1-y)^2) F_2(x)$$

- $F_2(x)$: proton structure function
- $q(x)$: parton density function



$$F_2(x) = \sum_q e_q^2 (xq(x) + x\bar{q}(x))$$

Parton Densities

- ep scattering:

$$\frac{d^2\sigma^{ep\rightarrow eX}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} (1 + (1-y)^2) F_2(x)$$

- $F_2(x)$: proton structure function
- $q(x)$: parton density function

$$F_2(x) = \sum_q e_q^2 (xq(x) + x\bar{q}(x))$$

- But... the proton had 3 quarks?!
- Sum rules:

$$\begin{aligned} \int_0^1 (u(x) - \bar{u}(x)) dx &= 2; \\ \int_0^1 (d(x) - \bar{d}(x)) dx &= 1; \\ \int_0^1 (s(x) - \bar{s}(x)) dx &= 0, \end{aligned}$$

Proton: x

- What is 'momentum fraction' distribution of quarks??
- Quarks:
 - "Valence"
 - "Sea"

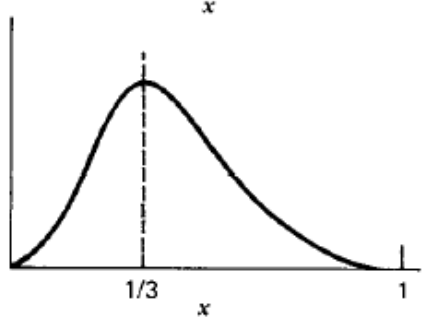
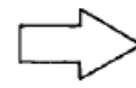
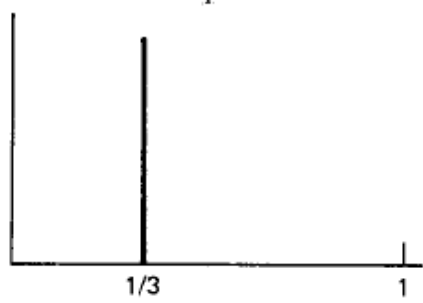
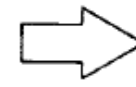
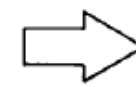
If the Proton is

A quark

Three valence quarks

Three bound valence quarks

then $F_2^{ep}(x)$ is

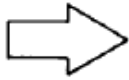


Proton: x

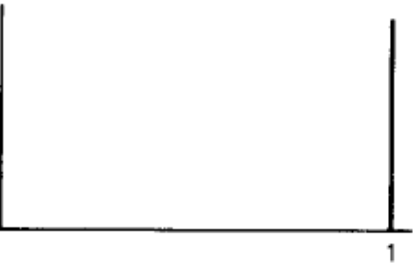
- What is 'momentum fraction' distribution of quarks??
- Quarks:
 - "Valence"
 - "Sea"
- Dynamic, QCD !

If the Proton is

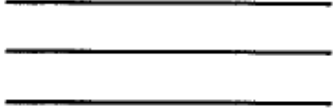
A quark



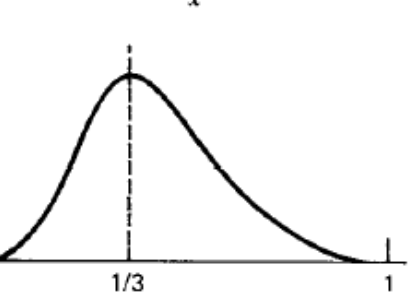
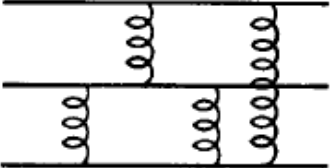
then $F_2^{ep}(x)$ is



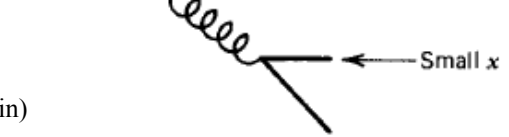
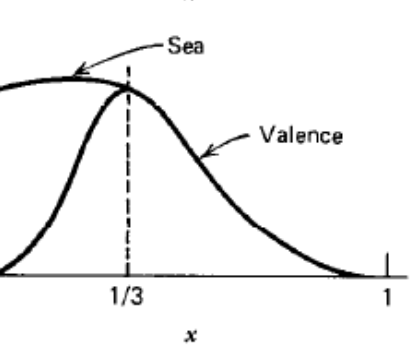
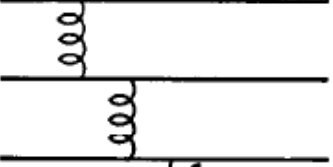
Three valence quarks



Three bound valence quarks



Three bound valence quarks + some slow debris, e.g., $g \rightarrow q\bar{q}$

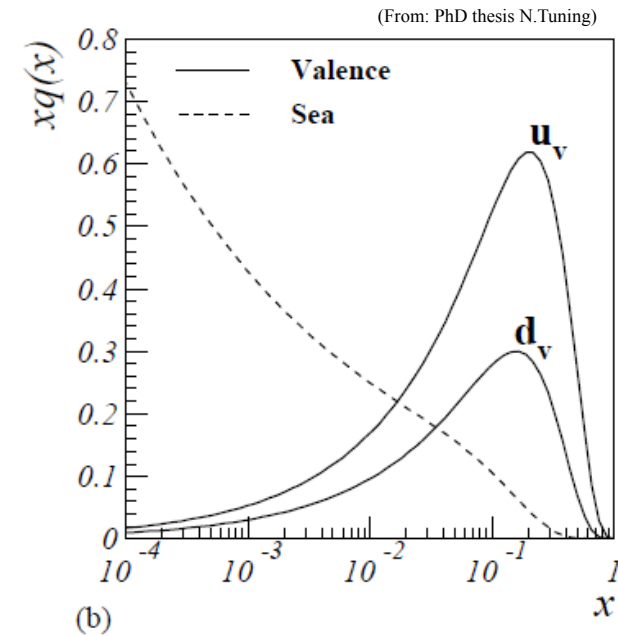
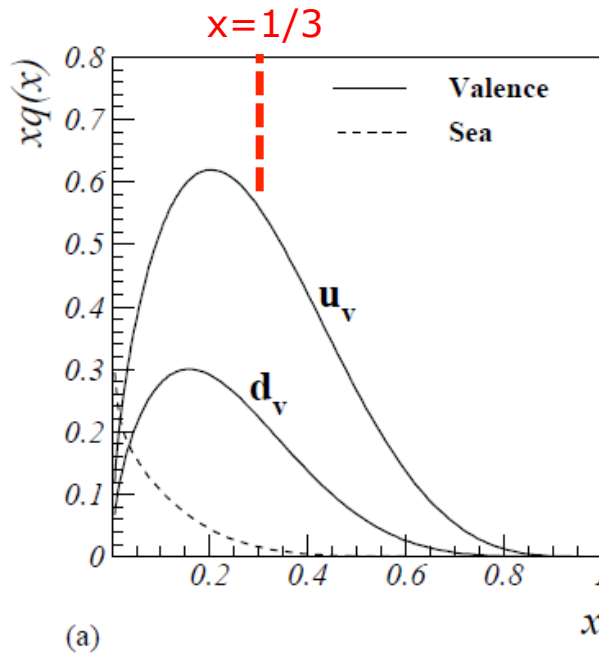


(From: Halzen & Martin)

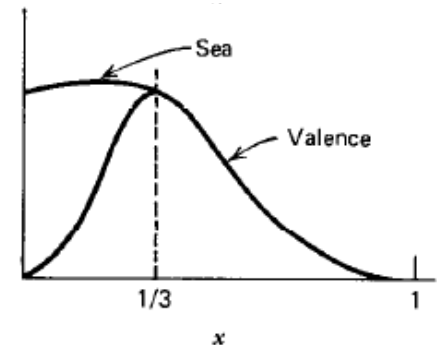
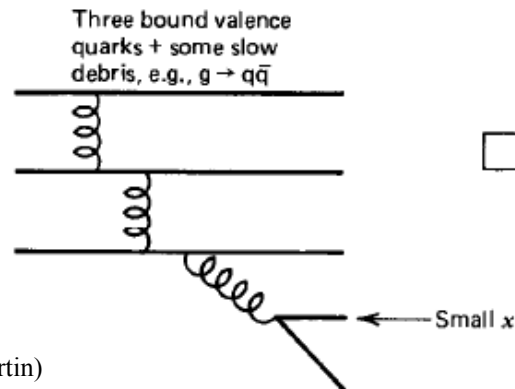
Proton: x

- What is ‘momentum fraction’ distribution of quarks??
- Quarks:
 - “Valence”
 - “Sea”

➤ Dynamic, QCD !



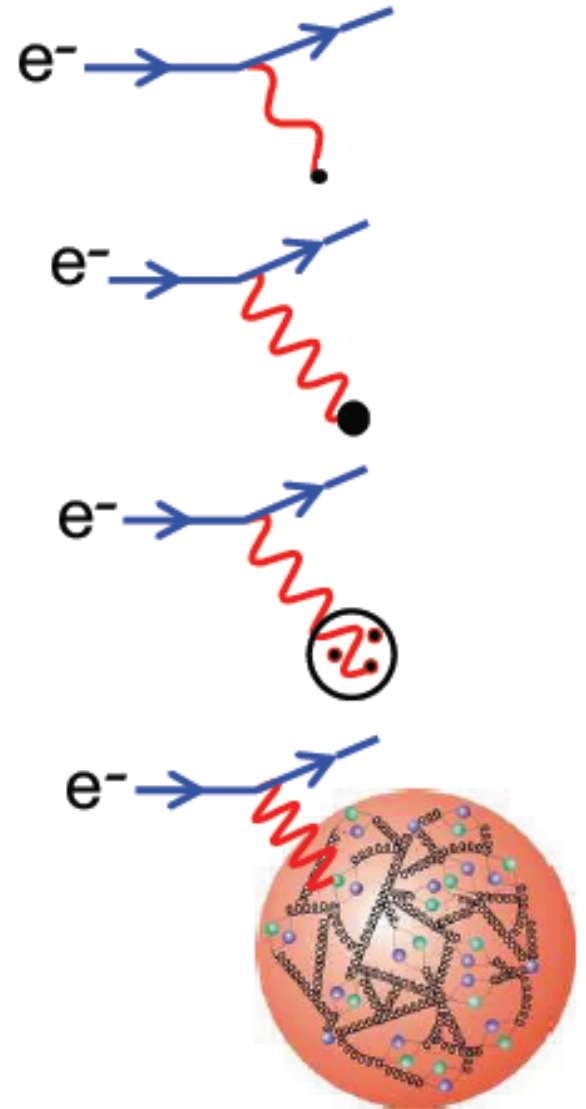
(From: PhD thesis N.Tuning)



(From: Halzen & Martin)

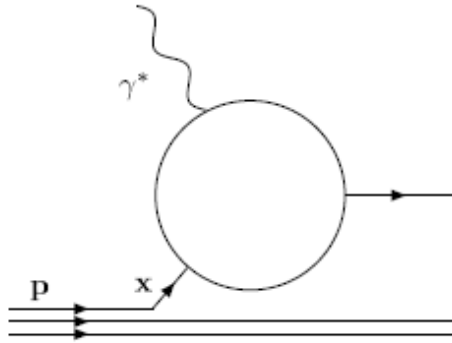
Proton: Q^2

- The “deeper” one looks into the proton, the more quarks and gluons

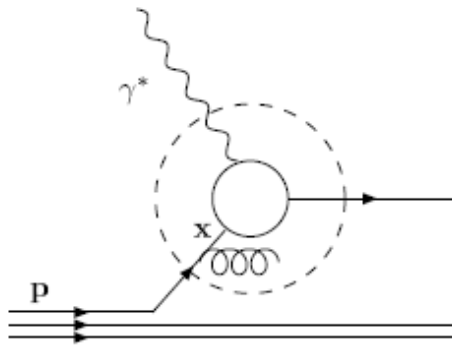


Proton: x, Q^2

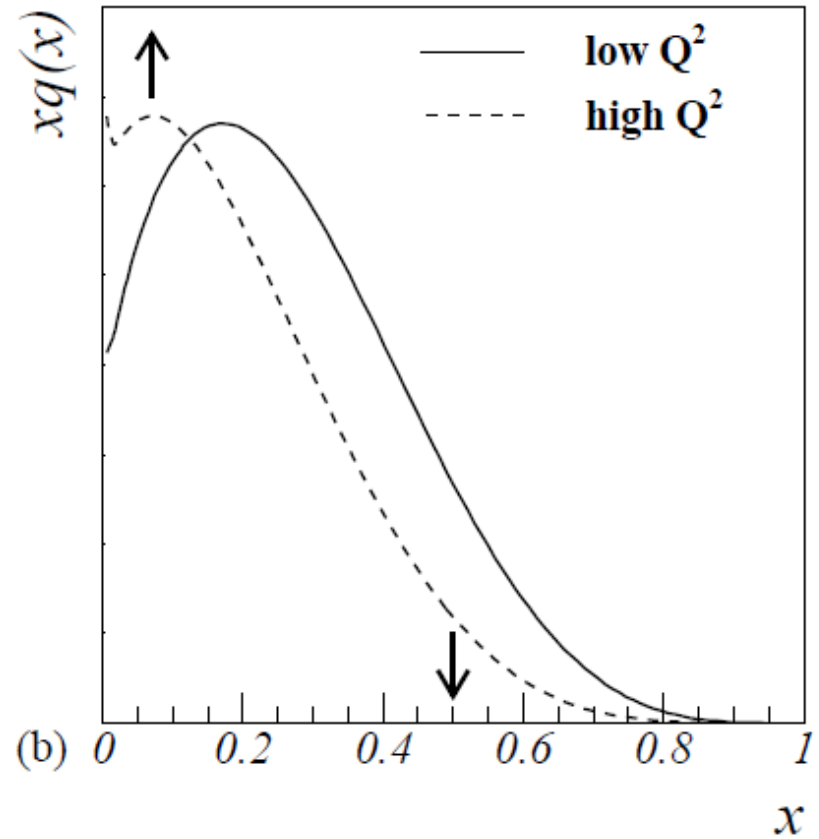
low Q^2 :



high Q^2 :



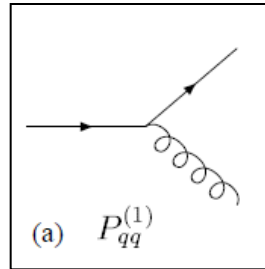
(a)



(b)

Proton: x, Q^2

- The “deeper” one looks into the proton, the more quarks and gluons
- “QCD evolution”
- Describes quark-gluon splitting



$$\frac{d\sigma^{\gamma^* q \rightarrow qq}}{dp_T^2} = \frac{4\pi\alpha^2}{s} e_q^2 \frac{1}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z)$$

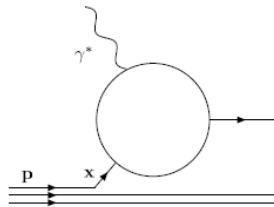
$$\sigma^{\gamma^* q \rightarrow qq} = \frac{4\pi\alpha^2}{s} e_q^2 \frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2}$$

➤ DGLAP evolution eqs:

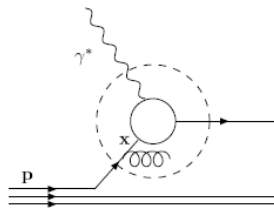
$$\frac{dq(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left(q(y, Q^2) P_{qq}\left(\frac{x}{y}\right) + g(y, Q^2) P_{qg}\left(\frac{x}{y}\right) \right)$$

$$\frac{dg(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left(\sum_q q(y, Q^2) P_{gq}\left(\frac{x}{y}\right) + g(y, Q^2) P_{gg}\left(\frac{x}{y}\right) \right)$$

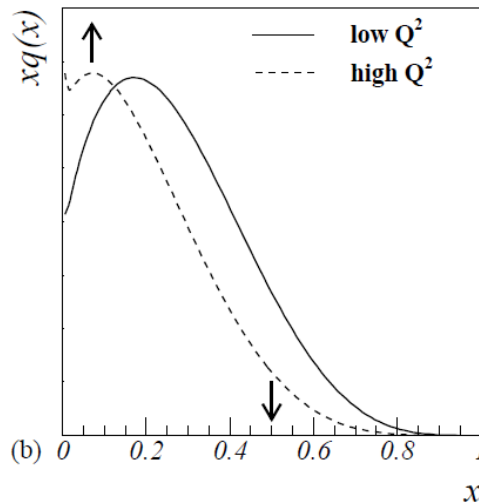
low Q^2 :



high Q^2 :



(a)



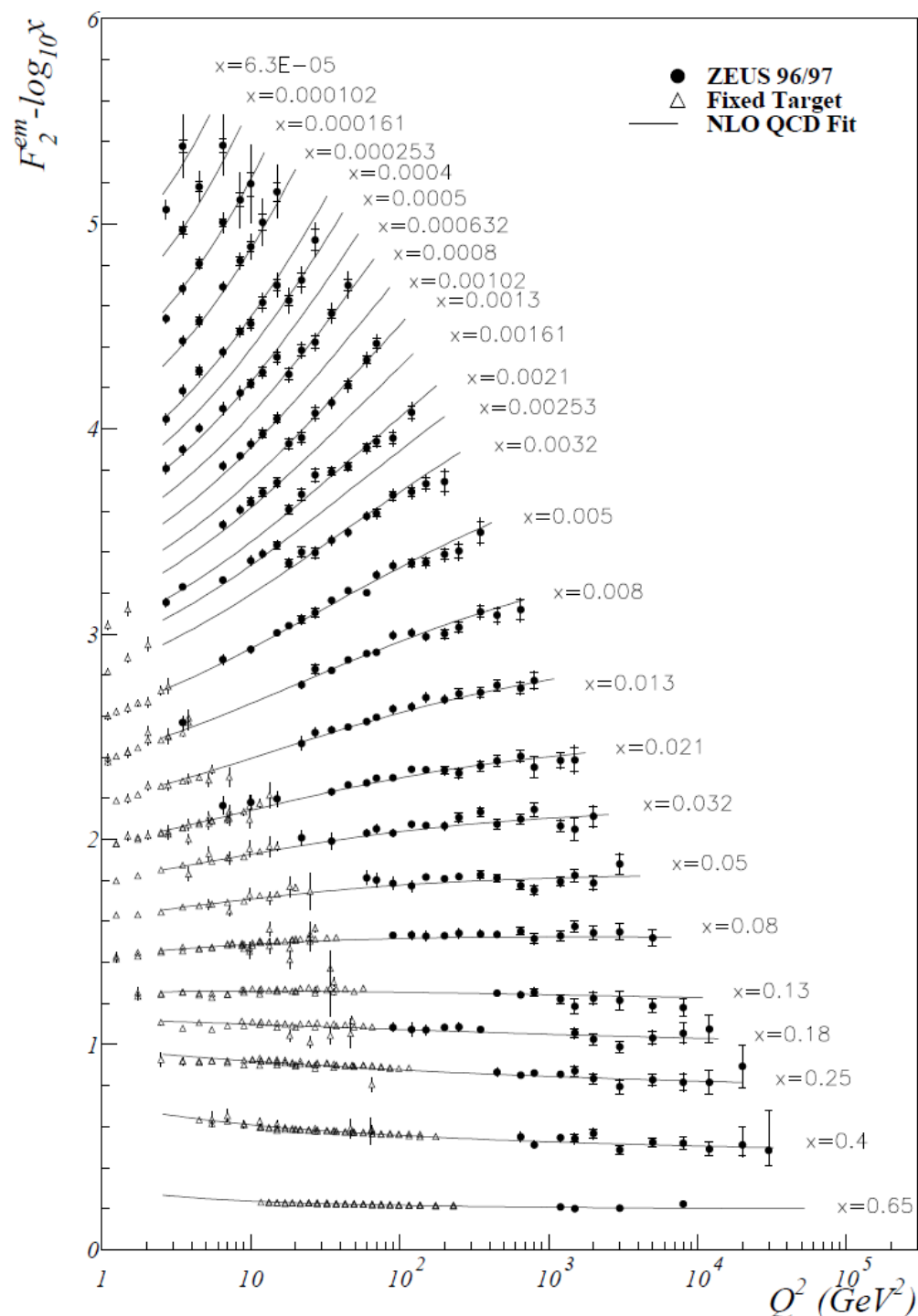
Scaling violations

J.D. Bjorken “scaling hypothesis” (1967):

- If scattering is caused by pointlike constituents structure functions must be independent of Q^2
 - *Would you expect a Q^2 dependence?*
-
- Yes, due to QCD, ie. quark/gluon splitting !
 - Matured in mid '70s
 - The proton is “dynamic” !
 - Measurement of $F_2(x, Q^2)$ very accurate test of QCD

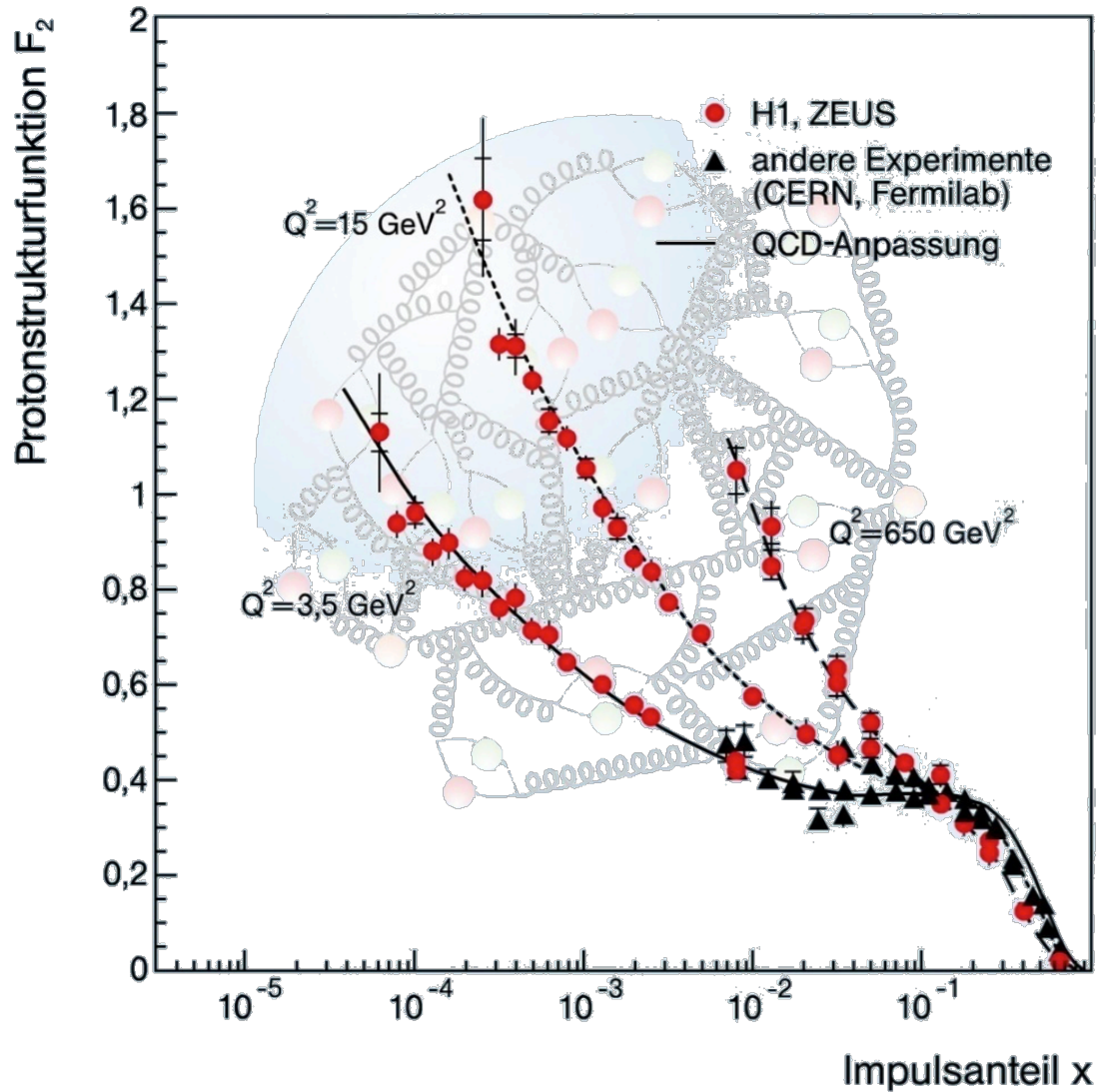
Scaling violations

- Measurement of $F_2(x, Q^2)$
very accurate test of QCD



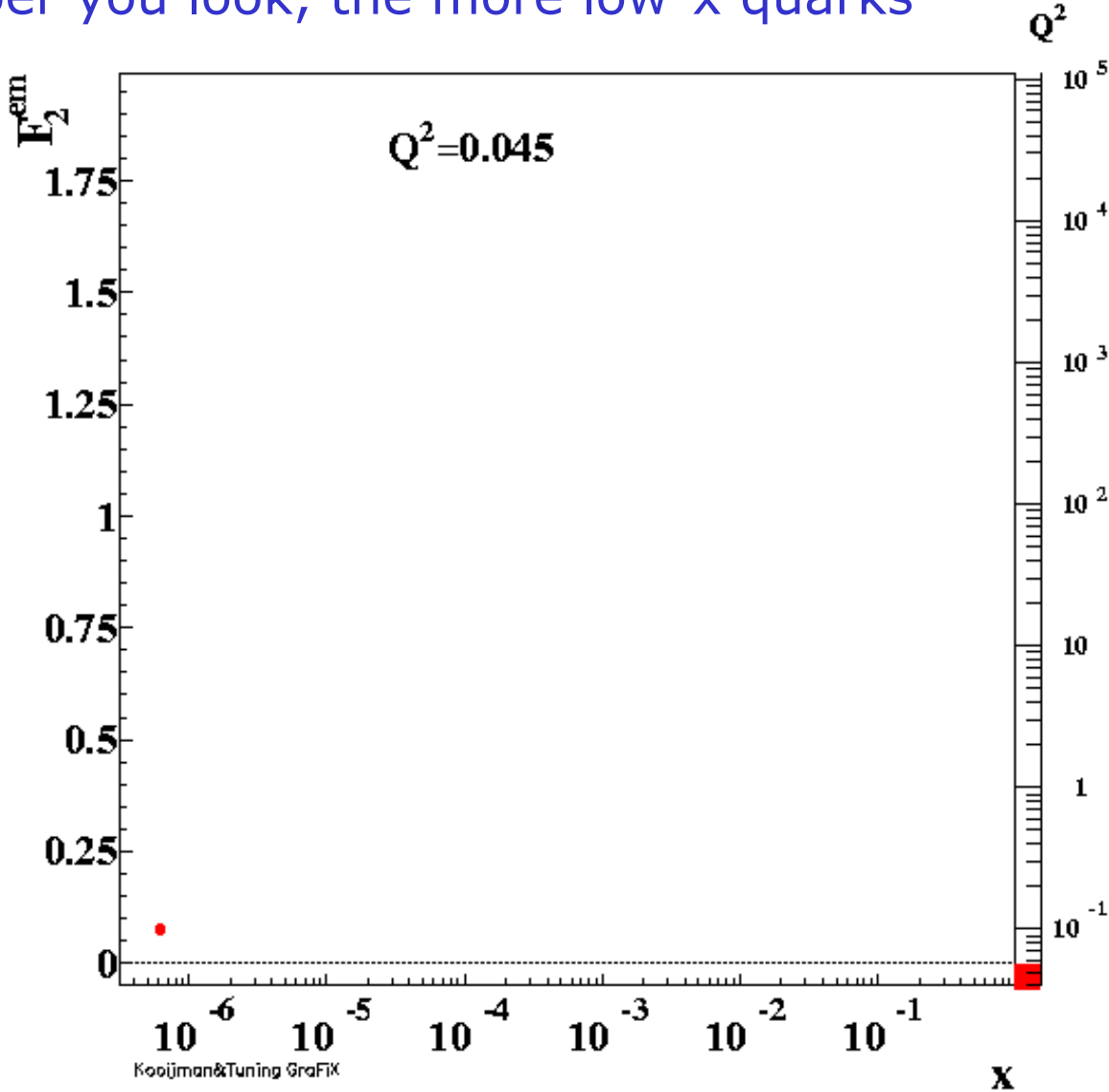
Proton Structure

- The deeper you look, the more low-x quarks

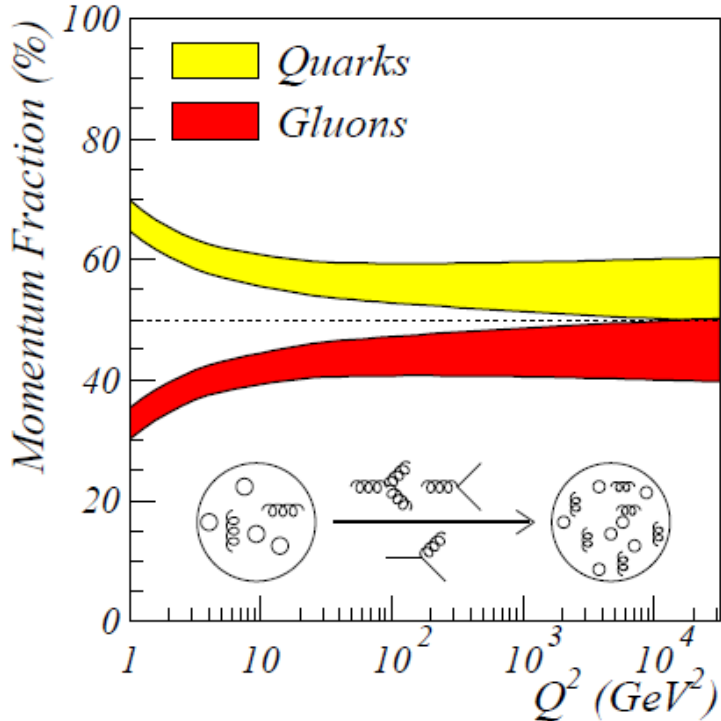
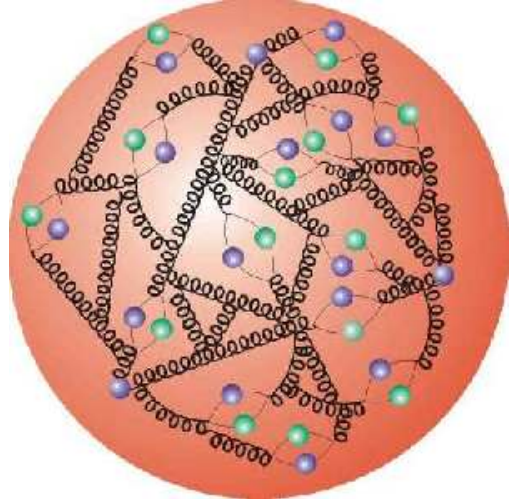
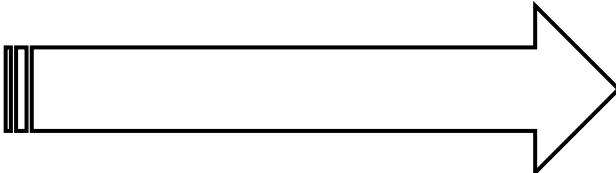
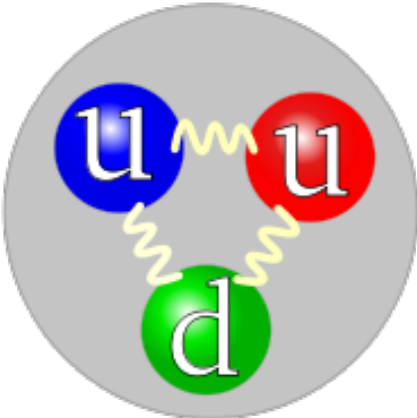


Proton Structure

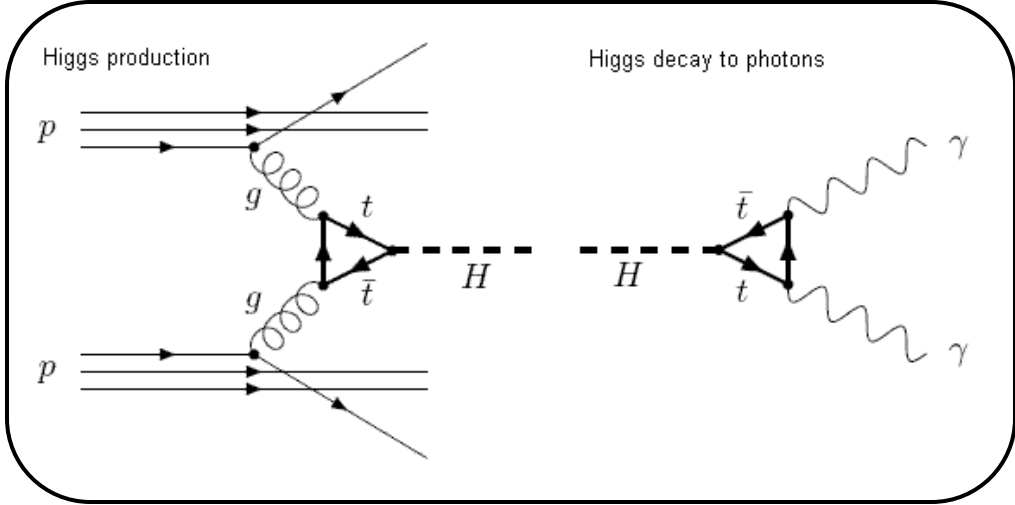
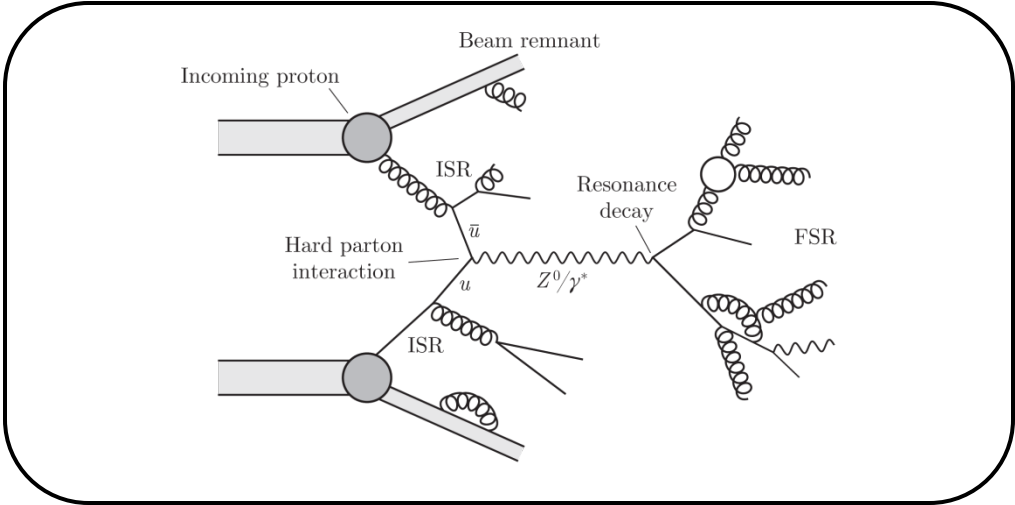
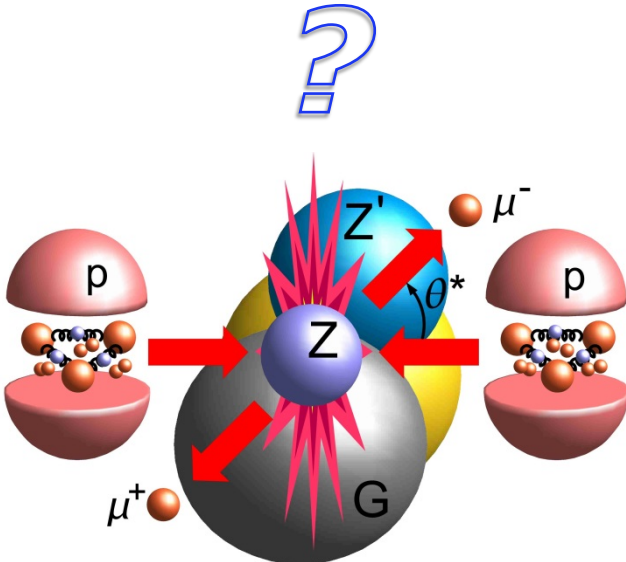
➤ The deeper you look, the more low-x quarks



Proton Structure



Proton Structure: knowledge needed for predictions



Plan

	1) Intro: Standard Model & Relativity	20 Feb
1900-1940	2) Basis	27 Feb
	1) Atom model, strong and weak force	
	2) Scattering theory	
1945-1965	3) Hadrons	13 Mar
	1) Isospin, strangeness	
	2) Quark model, GIM	
1965-1975	4) Standard Model	27 Mar
	1) QED	
	2) Parity, neutrinos, weak inteaction	
	3) QCD	
1975-2000	5) e^+e^- and DIS	15 May
2000-2015	6) Higgs and CKM	22 May