“Elementary Particles”
Lecture 4

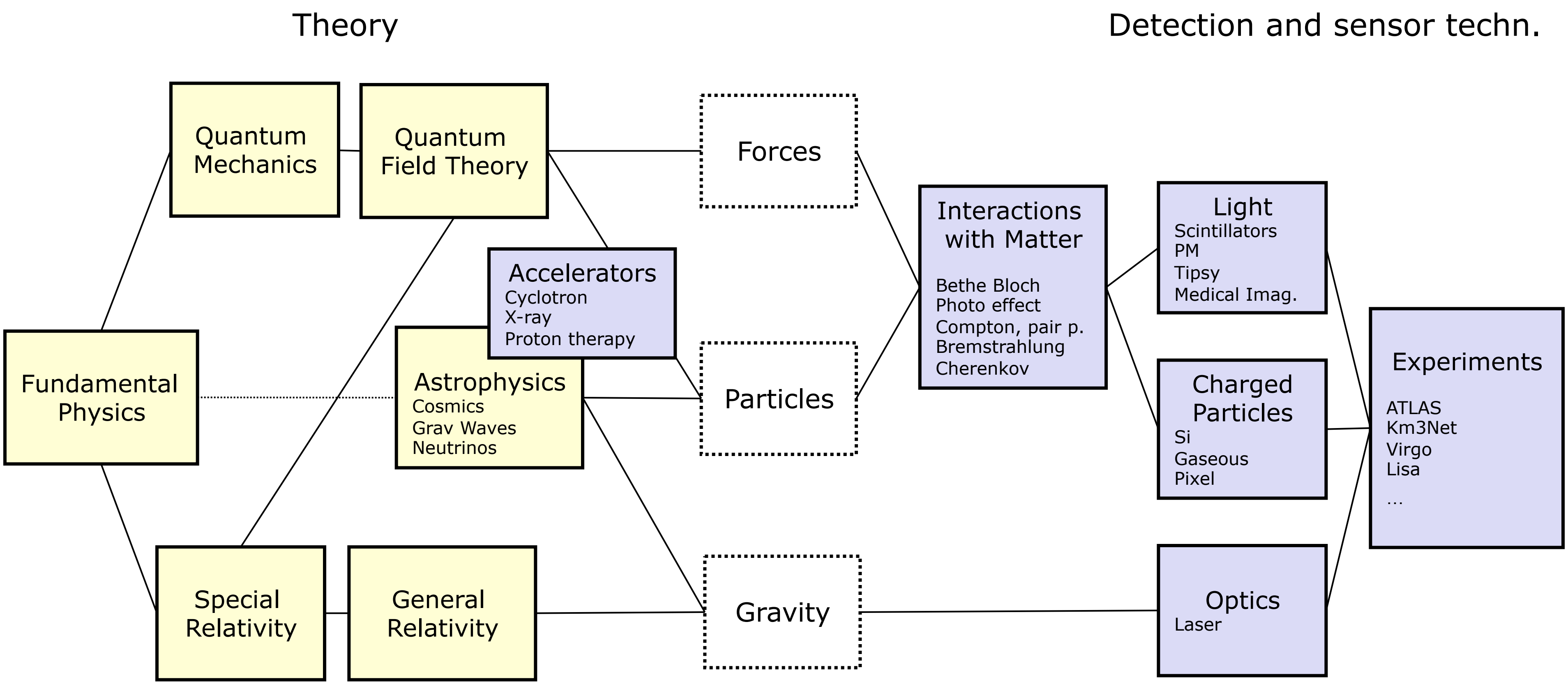
Niels Tuning

Harry van der Graaf

Thanks

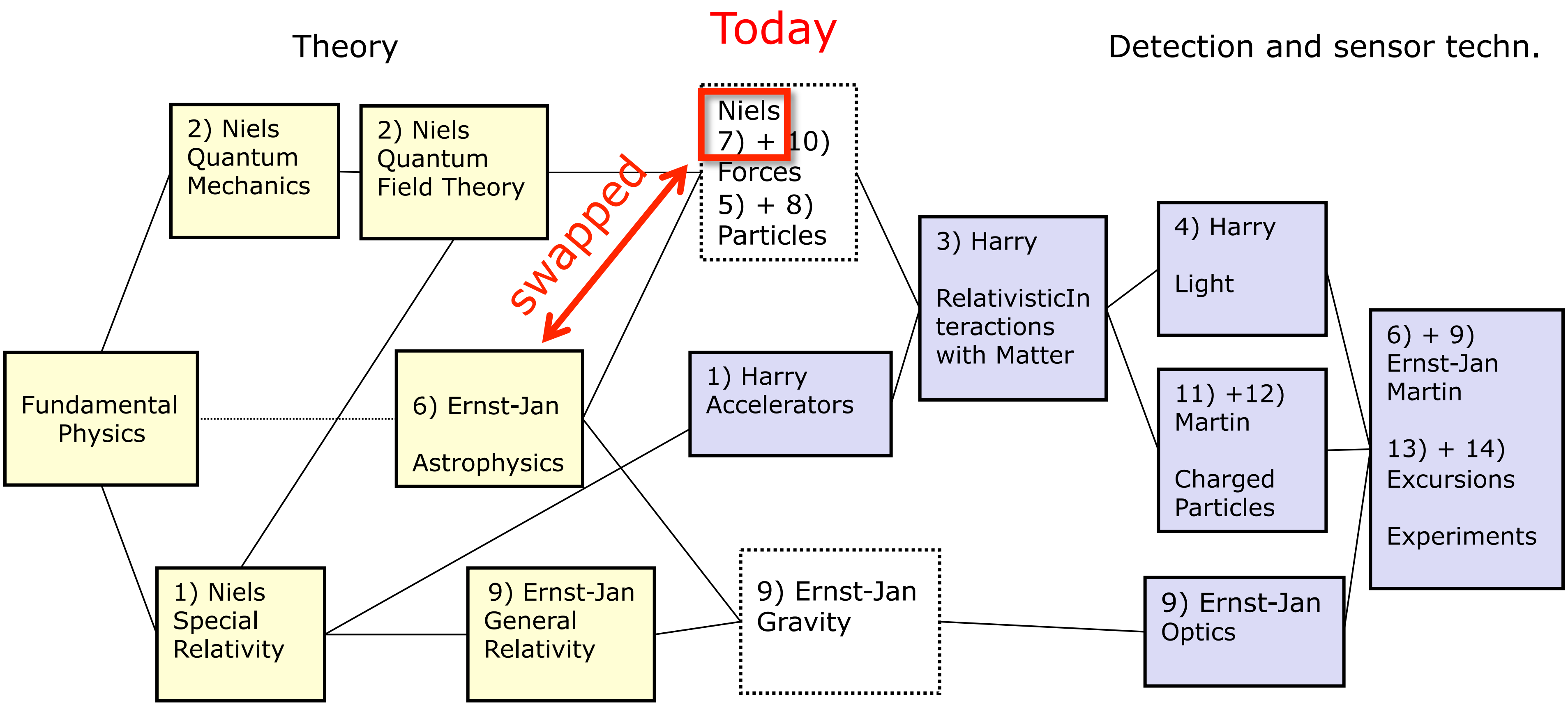
- Ik ben schatplichtig aan:
  - Dr. Ivo van Vulpen (UvA)
  - Prof. dr. ir. Bob van Eijk (UT)
  - Prof. dr. Marcel Merk (VU)

Plan



Theory
Detection and sensor techn.
Quantum Mechanics
Quantum Field Theory
Forces
Accelerators
Cyclotron
X-ray
Proton therapy
Fundamental Physics
Astrophysics
Cosmics
Grav Waves
Neutrinos
Particles
Interactions with Matter
Bethe Bloch
Photo effect
Compton, pair p.
Bremstrahlung
Cherenkov
Light
Scintillators
PM
Tipsy
Medical Imag.
Charged Particles
Si
Gaseous
Pixel
Experiments
ATLAS
Km3Net
Virgo
Lisa
...
Special Relativity
General Relativity
Gravity
Optics
Laser

Plan



Theory
Today
Detection and sensor techn.
2) Niels Quantum Mechanics
2) Niels Quantum Field Theory
Niels 7) + 10) Forces 5) + 8) Particles
swapped
3) Harry RelativisticInteractions with Matter
4) Harry Light
Fundamental Physics
6) Ernst-Jan Astrophysics
1) Harry Accelerators
11) +12) Martin Charged Particles
6) + 9) Ernst-Jan Martin 13) + 14) Excursions Experiments
1) Niels Special Relativity
9) Ernst-Jan General Relativity
9) Ernst-Jan Gravity
9) Ernst-Jan Optics

Schedule

1) 11 Feb: Accelerators (Harry vd Graaf) + Special relativity (Niels Tuning)
2) 18 Feb: Quantum Mechanics (Niels Tuning)
3) 25 Feb: Interactions with Matter (Harry vd Graaf)
4) 3 Mar: Light detection (Harry vd Graaf)
5) 10 Mar: Particles and cosmics (Niels Tuning)
6) 17 Mar: Forces (Niels Tuning)
7) 24 Mar: Astrophysics and Dark Matter (Ernst-Jan Buis)

break

8) 21 Apr: e^+e^- and ep scattering (Niels Tuning)
9) 28 Apr: Gravitational Waves (Ernst-Jan Buis)
10) 12 May: Higgs and big picture (Niels Tuning)
11) 19 May: Charged particle detection (Martin Franse)
12) 26 May: Applications: experiments and medical (Martin Franse)

13) 2 Jun: Nikhef excursie
14) 8 Jun: CERN excursie

Plan

	1) Intro: Standard Model & Relativity	***11 Feb***
1900-1940	2) Basis	***18 Feb***
	1) Atom model, strong and weak force	
	2) Scattering theory	
1945-1965	3) Hadrons	***10 Mar***
	1) Isospin, strangeness	
	2) Quark model, GIM	
1965-1975	4) Standard Model	***17 Mar***
	1) QED	
	2) Parity, neutrinos, weak inteaction	
	3) QCD	
1975-2000	5) e^+e^- and DIS	***21 Apr***
2000-2015	6) Higgs and CKM	***12 May***

Homework

1) Homework for this *and* previous lecture
2) Hand in before 21 April

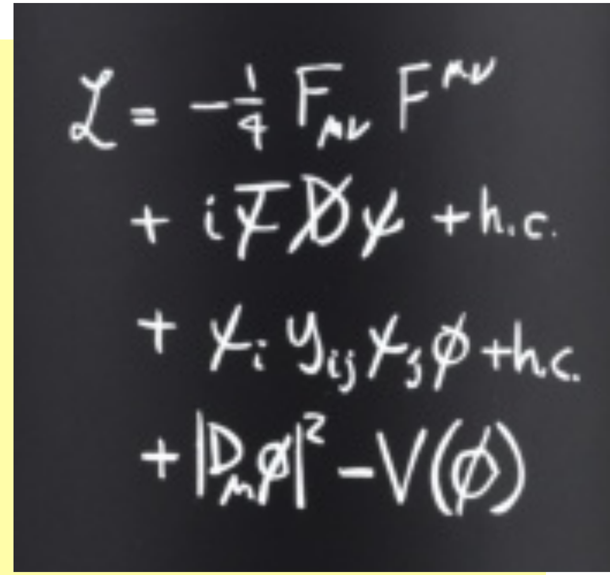
Outline for today: Interactions

1) Gauge invariance, and the Lagrangian
2) Electro-magnetic interaction
   - QED
3) Weak interaction
   - Parity violation
4) Strong interaction
   - QCD

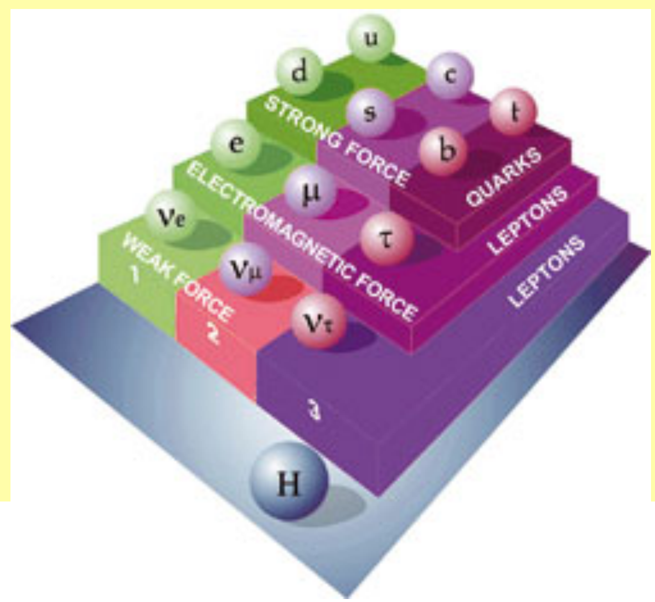
Summary

Lecture 1: Standard Model & Relativity

- Standard Model Lagrangian



- Standard Model Particles



Lecture 1: Standard Model & Relativity

- Theory of relativity
  - Lorentz transformations ("boost")
  - Calculate energy in collissions

$$x'^0 = \gamma\left(x^0 - \beta x^1\right) \qquad \beta \equiv \frac{v}{c}$$
$$x'^1 = \gamma\left(x^1 - \beta x^0\right) \qquad \textbf{met}$$
$$x'^2 = x^2 \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$
$$x'^3 = x^3$$

- 4-vector calculus

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \qquad \left(\mu = 0, 1, 2, 3\right)$$

$$p_\mu p^\mu = (E/c)^2 - |\vec{p}|^2 = (E^2 - c^2|\vec{p}|^2)/c^2 = (m_0 c^4)/c^2$$

- High energies needed to make (new) particles

$$s = \left(p_1 + p_2\right)^2 = 2m^2 + 2\left(E^2 + \vec{p}^2\right)$$
$$= 2m^2 + 2E^2 + 2\left(E^2 - m^2\right) = 4E^2$$

Lecture 2: Quantum Mechanics & Scattering

- Schrödinger equation
  - Time-dependence of wave function

$$E = \frac{\vec{p}^2}{2m}$$

$$i\frac{\partial}{\partial t}\psi = \frac{-1}{2m}\nabla^2\psi$$

- Klein-Gordon equation
  - Relativistic equation of motion of scalar particles

$$E^2 = \vec{p}^2 + m^2$$

$$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\,\phi$$

- Dirac equation
  - Relativistically correct, and linear
  - Equation of motion for spin-1/2 particles
  - Prediction of anti-matter

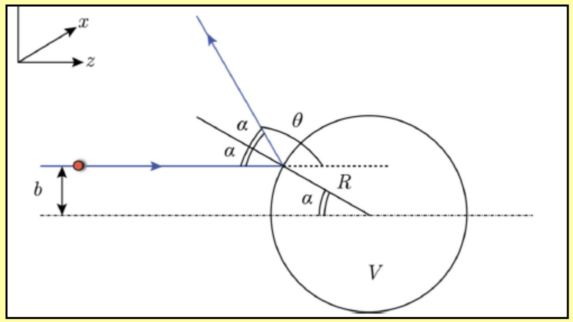
$$(i\gamma^\mu\partial_\mu - m)\;\psi \;\;=\;\; 0$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

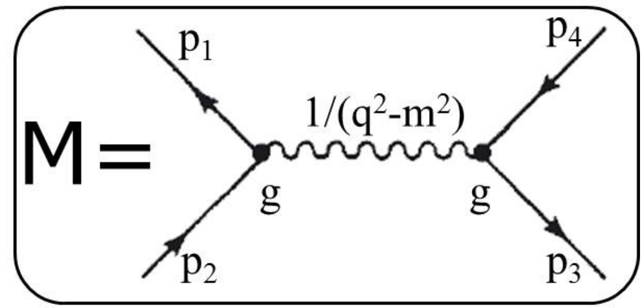
Lecture 2: Quantum Mechanics & Scattering

- Scattering Theory
  - (Relative) probability for certain process to happen
  - Cross section

$$\frac{d\sigma}{d\Omega} = D(\theta, \varphi)$$



Classic



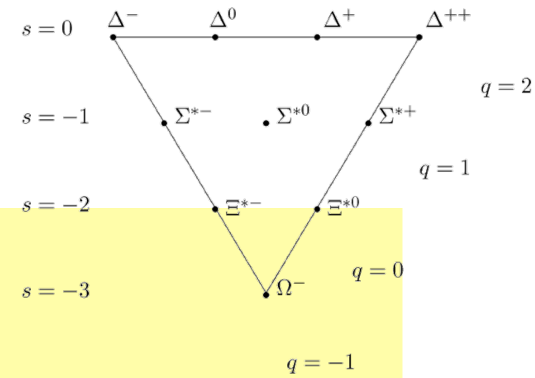
Scattering amplitude in Quantum Field Theory

- Fermi's Golden Rule

$$\text{transition rate} = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \times (\text{phase space})$$

  - Decay: "decay width" Γ $\quad a \rightarrow b + c$
  - Scattering: "cross section" σ $\quad a + b \rightarrow c + d$

Lecture 3: Quarkmodel & Isospin



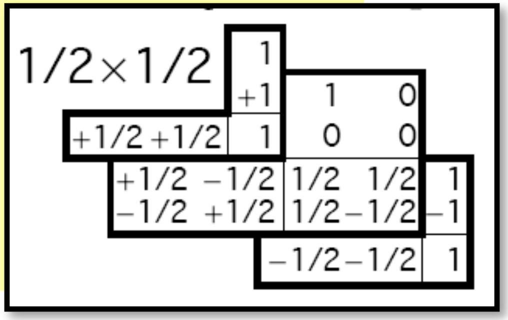
- "Partice Zoo" not elegant

- Hadrons consist of quarks

	d	u	s
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0
S – strangeness	0	0	-1

➢ Observed symmetries
  - Same mass of hadrons: isospin
  - Slow decay of K, Λ: strangeness
  - Fermi-Dirac statistics Δ^{++},. Ω: color

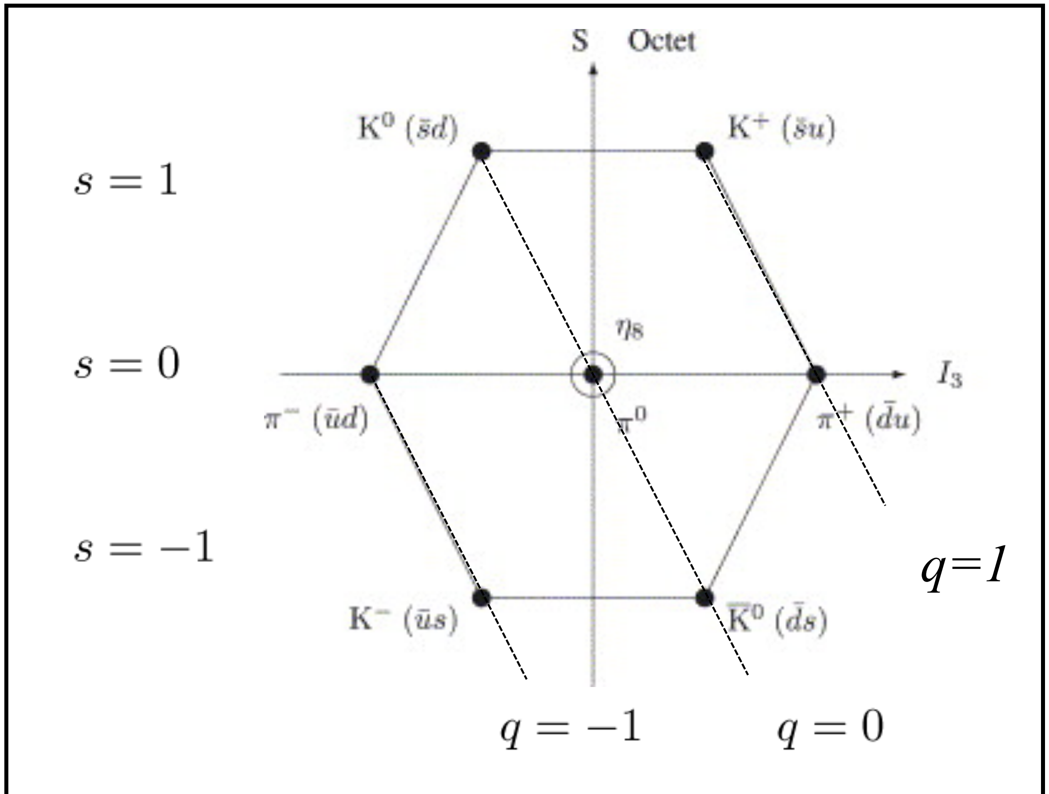
- Combining/decaying particles with (iso)spin
  - Clebsch-Gordan coefficients

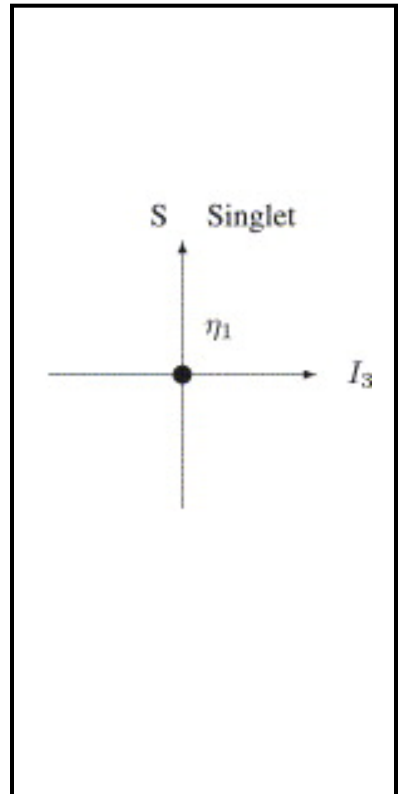


Group theory

$$3 \otimes \bar{3} = 8 \oplus 1$$

- Mesons:
  - 2 quarks, with 3 possible flavours: u, d, s
  - 3^2 =9 possibilities = 8 + 1

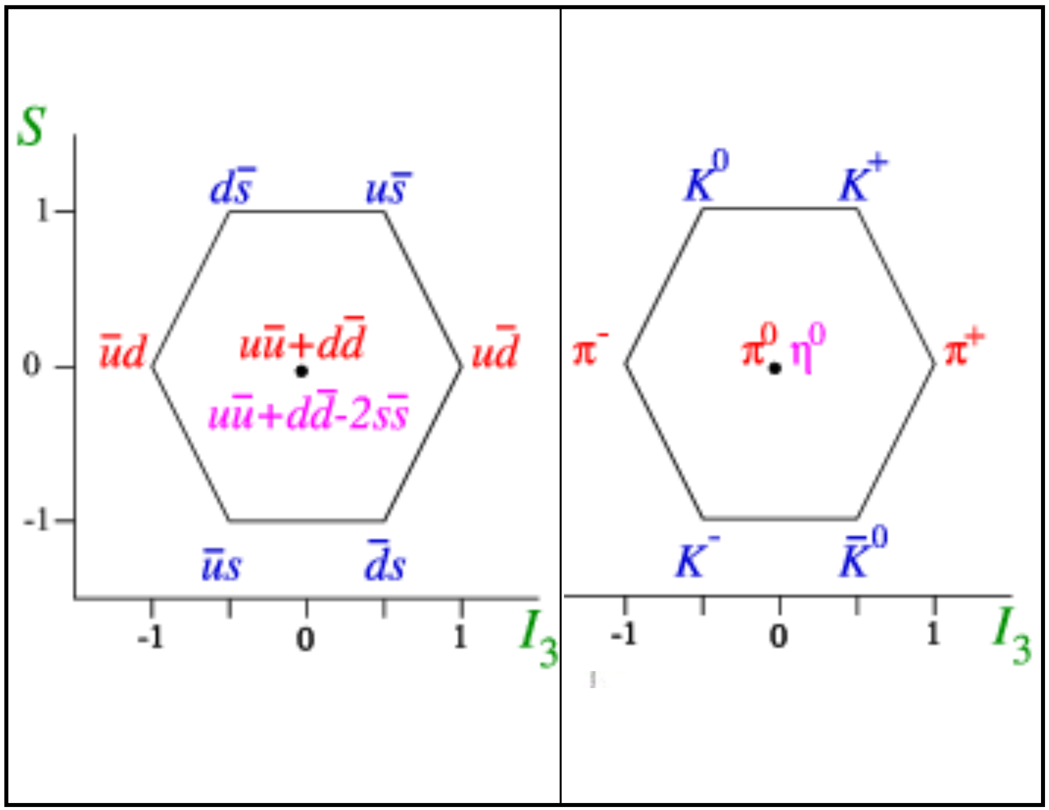


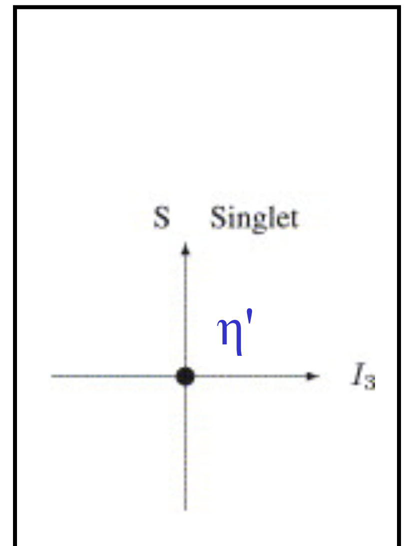


Group theory

$$3 \otimes \bar{3} = 8 \oplus 1$$

- Mesons:
  - 2 quarks, with 3 possible flavours: u, d, s
  - 3^2 =9 possibilities = 8 + 1





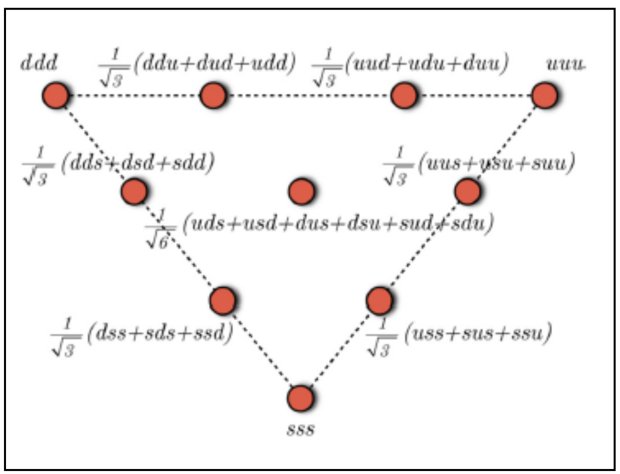
$$f' = \psi_8 \cos\theta - \psi_1 \sin\theta$$
$$f = \psi_8 \sin\theta + \psi_1 \cos\theta$$
$$\psi_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$
$$\psi_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

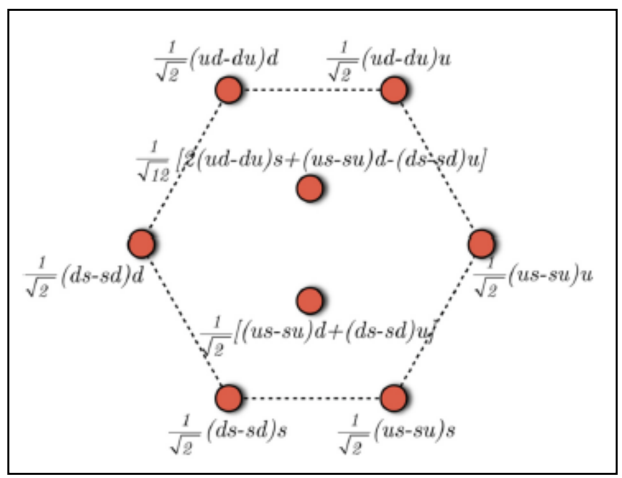
Group theory

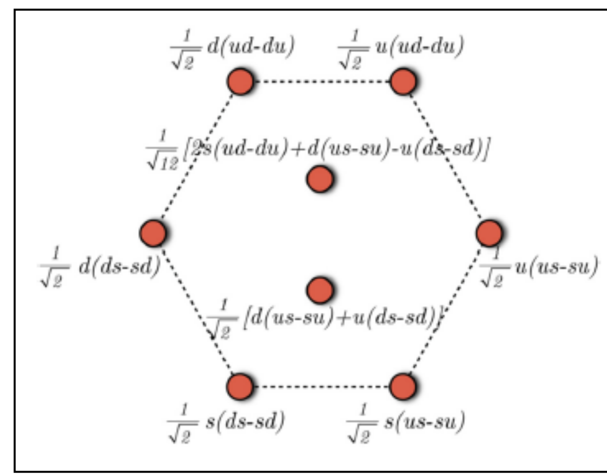
- Baryons:
  - 3 quarks, with 3 possible flavours: u, d, s
  - 3^3 =27 possibilities = 10 + 8 + 8 + 1

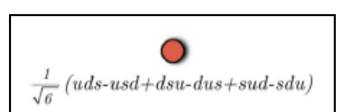
$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

ψ_{sym} | $\psi_{anti-sym}(1 \leftrightarrow 2)$ | $\psi_{anti-sym}(2 \leftrightarrow 3)$









Group theory

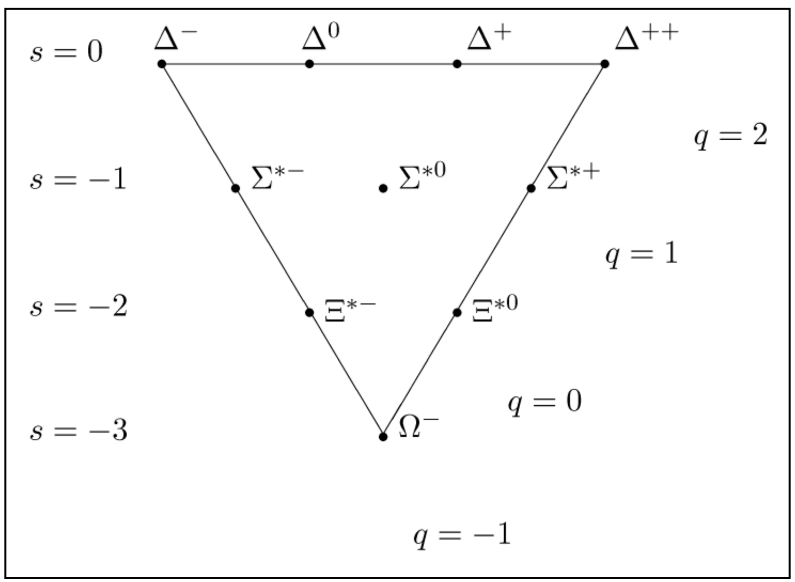
$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

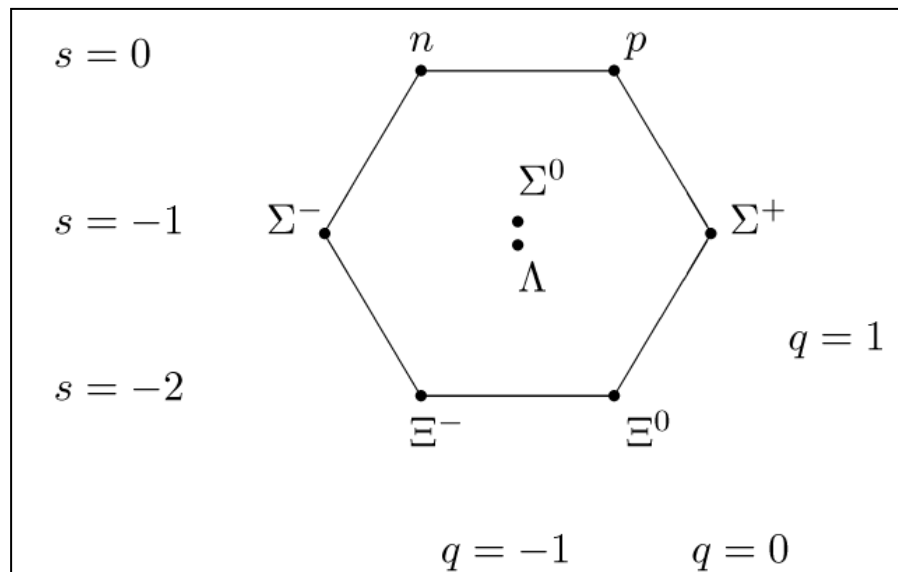
- Baryons:
  - 3 quarks, with 3 possible flavours: u, d, s
  - 3^3 =27 possibilities = 10 + 8 + 8 + 1

$$\psi_{sym}$$

$$\psi_{anti-sym}(1 \leftrightarrow 2)$$

$$\psi_{anti-sym}(2 \leftrightarrow 3)$$





What did we learn about quarks

Quarks:

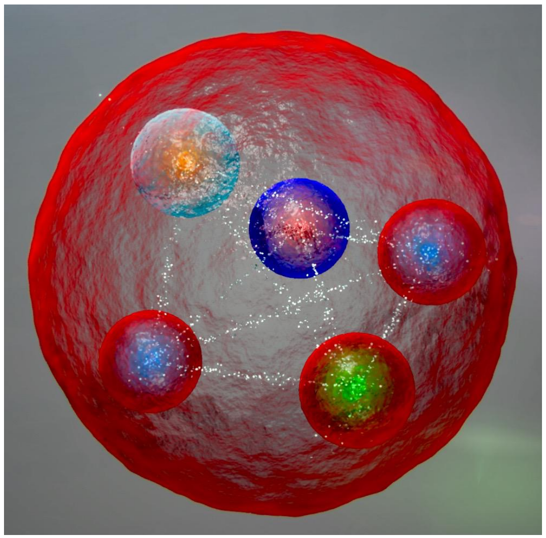
- Associate production, but long lifetime: strangeness
- Many (degenerate) particles: isospin
- Pauli exclusion principle: color

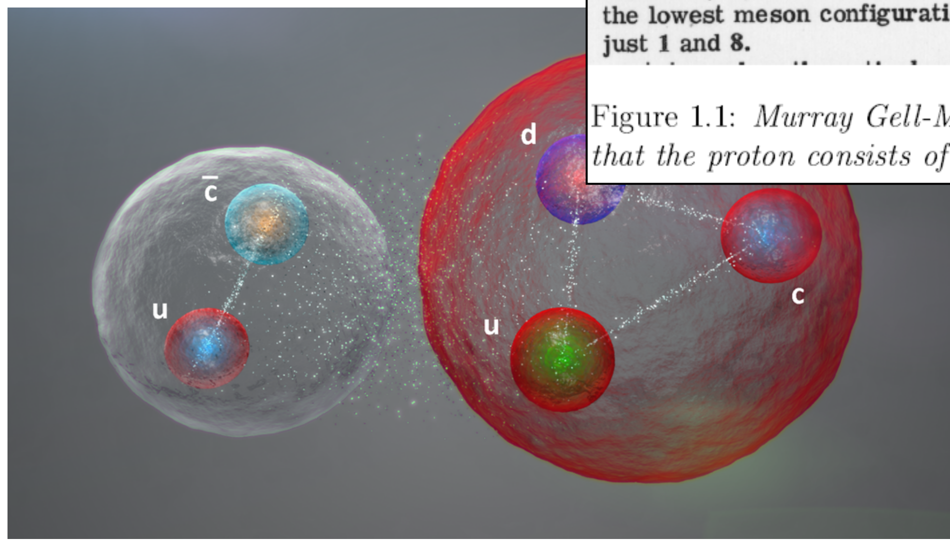
	d	u	s
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0
S – strangeness	0	0	-1

- How they combine into hadrons: multiplets
- How to add (iso)spin: Clebsch-Gordan

[LHCb, Phys. Rev. Lett. 115 (20

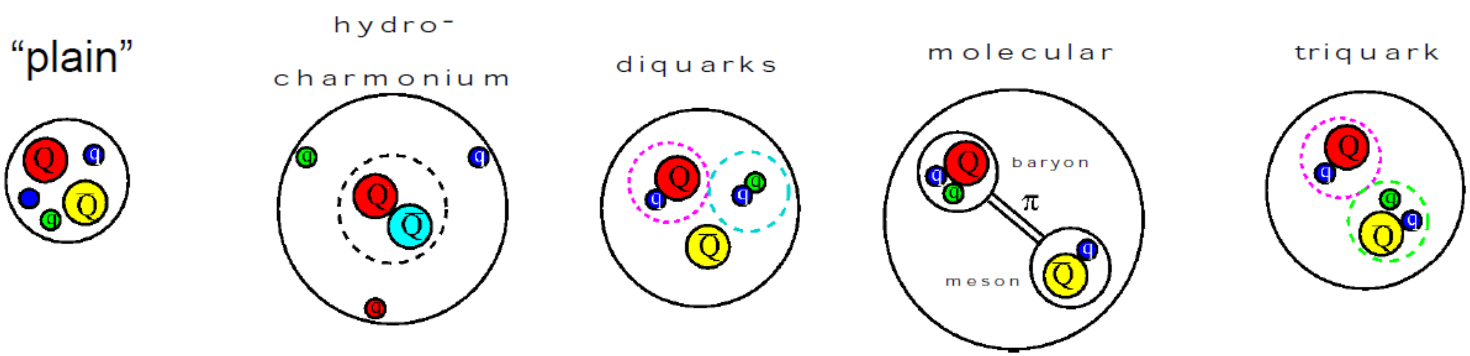
What is a Pentaquark?





A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

Figure 1.1: *Murray Gell-Mann suggested in 1964 that the proton consists of three "quarks"* [6] *[1].*

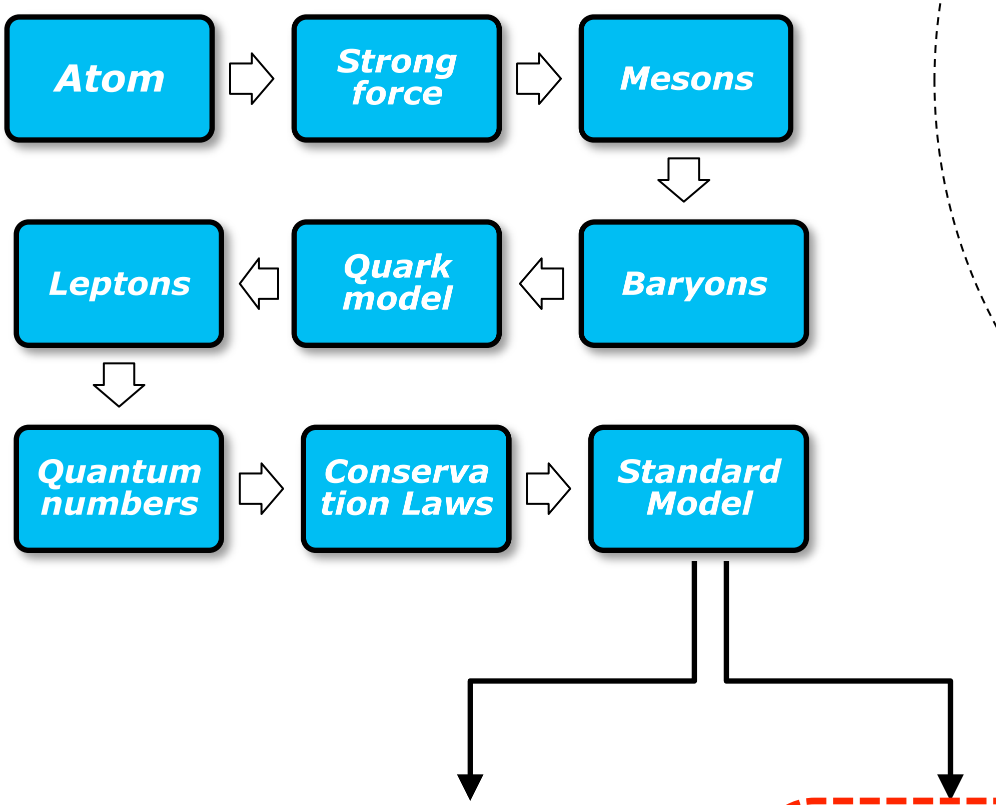


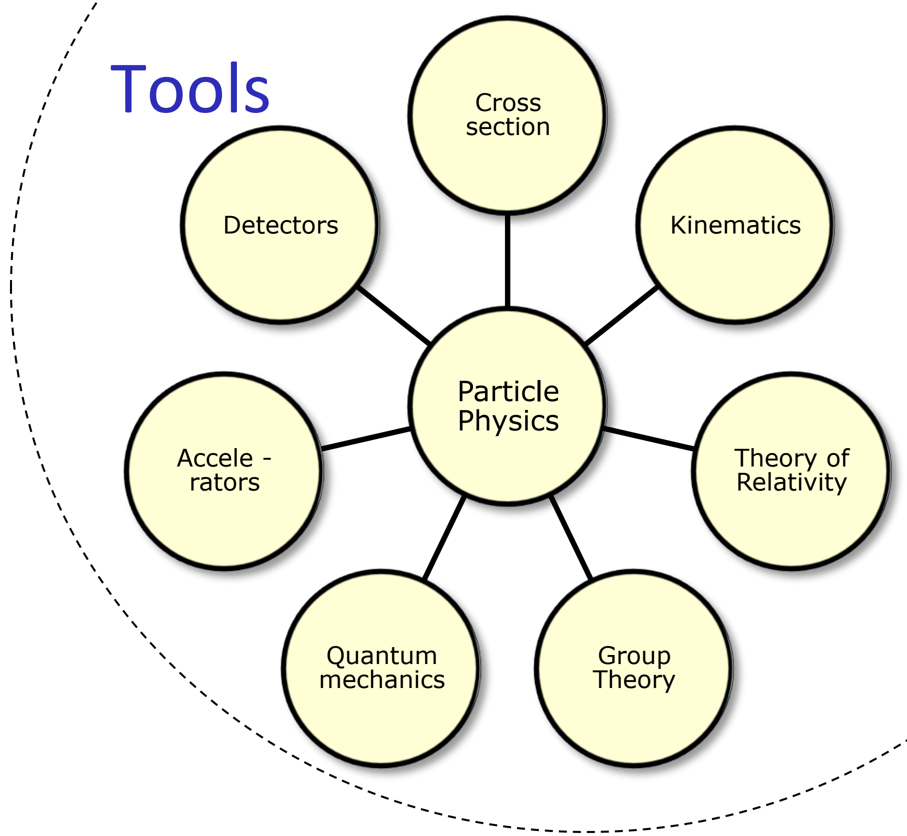
>300 papers citing the result, with many possible interpretations.

Plan

	1) Intro: Standard Model & Relativity	***11 Feb***
1900-1940	2) Basis	***18 Feb***
	1) Atom model, strong and weak force	
	2) Scattering theory	
1945-1965	3) Hadrons	***10 Mar***
	1) Isospin, strangeness	
	2) Quark model, GIM	
1965-1975	4) Standard Model	***17 Mar***
	1) QED	
	2) Parity, neutrinos, weak inteaction	
	3) QCD	
1975-2000	5) e^+e^- and DIS	***21 Apr***
2000-2015	6) Higgs and CKM	***12 May***

Model elementary particles





Particles

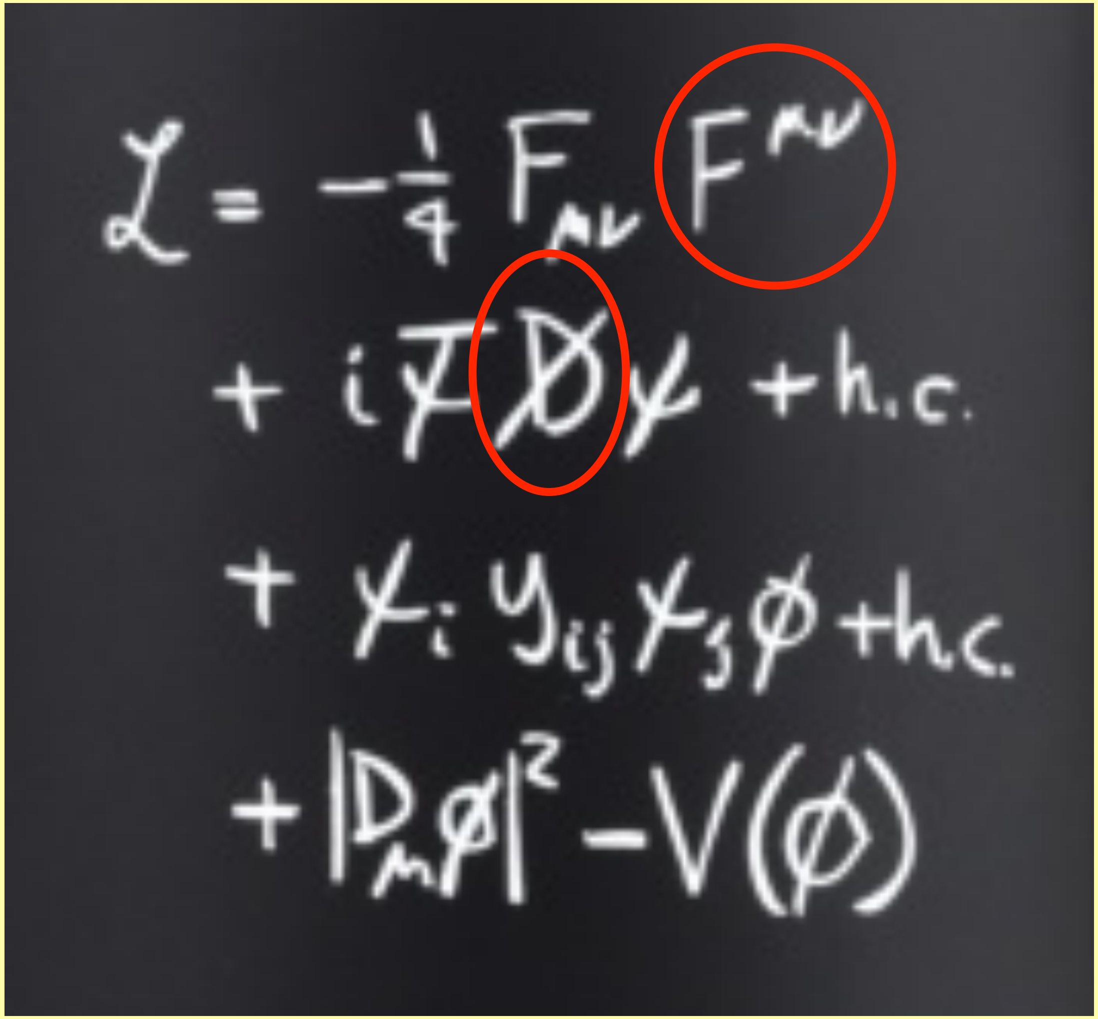
quarks/leptons

Forces

Electromagnetic
Weak
Strong

Quantum-field theory & Local gauge invariance

Lecture 4: Forces

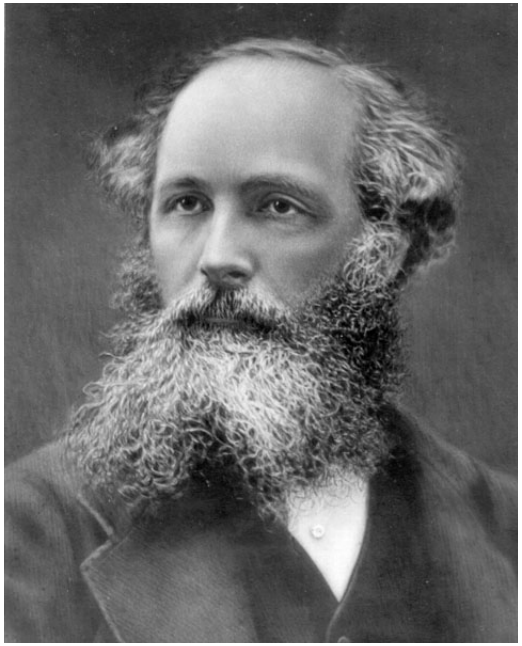


Electro-Magnetism

Electro-magnetism

- Towards a particle interacting with photon
  - Quantum Electro Dynamics, QED

➢ Start with electric and magnetic fields



J.C. Maxwell

Start: Classical electro-magnetism

Maxwell equations

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} + \vec{j}$$

- We wish to work relativistically
- Can we formulate this in Lorentz covariant form?

➢ Introduce a *mathematical tool*: $A^{\mu} = (V, \vec{A})$

Scalar potential also called φ:

$$(\phi, \vec{A})$$

Start: Classical electro-magnetism

Maxwell equations

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} + \vec{j}$$

- We wish to work relativistically
- Can we formulate this in Lorentz covariant form?

- Introduce a *mathematical tool*: $A^\mu = (V, \vec{A})$
- Note: $\vec{E}, \vec{B}$ are physical, A^μ is not!

Scalar potential also called φ:

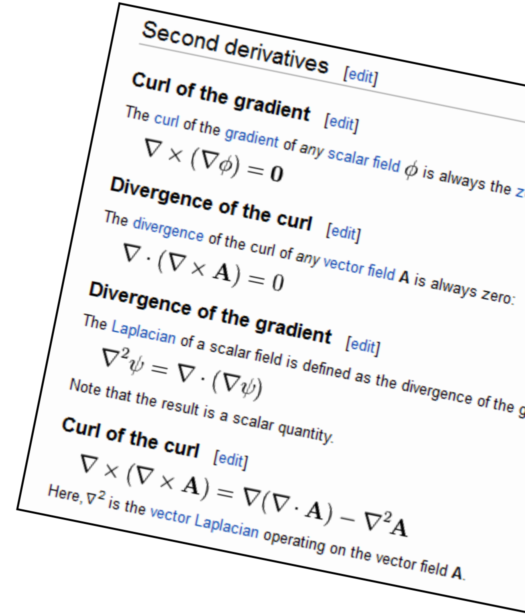
$$(\phi, \vec{A})$$

➤ Choose:

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\varphi$$

Start: Classical electro-magnetism


Second derivatives [edit]
Curl of the gradient [edit]
The curl of the gradient of any scalar field ϕ is always the z
$\nabla \times (\nabla\phi) = \mathbf{0}$
Divergence of the curl [edit]
The divergence of the curl of any vector field **A** is always zero:
$\nabla \cdot (\nabla \times \mathbf{A}) = 0$
Divergence of the gradient [edit]
The Laplacian of a scalar field is defined as the divergence of the g
$\nabla^2\psi = \nabla \cdot (\nabla\psi)$
Note that the result is a scalar quantity.
Curl of the curl [edit]
$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$
Here, ∇^2 is the vector Laplacian operating on the vector field **A**.

Maxwell equations

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} + \vec{j}$$

- We wish to work relativistically
- Can we formulate this in Lorentz covariant form?

- Introduce a *mathematical tool*: $A^\mu = (V, \vec{A})$

Scalar potential also called φ: $(\phi, \vec{A})$

- Note: $\vec{E}, \vec{B}$ are physical, A^μ is not!

➢ Choose:

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\varphi$$

Then automatically:

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Rewrite Maxwell

Maxwell equations

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} + \vec{j}$$

- Maxwell eqs. can then be written quite *economically*…:

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\varphi$$

$$\Rightarrow \quad \partial_\mu\partial^\mu A^\nu - \partial^\nu\partial_\mu A^\mu = j^\nu$$

$$j^\nu = (\rho, \vec{j})$$

Rewrite Maxwell

Maxwell equations

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} + \vec{j}$$

- Maxwell eqs. can then be written quite *economically*…:

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\varphi$$

$$\Rightarrow \quad \partial_\mu\partial^\mu A^\nu - \partial^\nu\partial_\mu A^\mu = j^\nu \qquad j^\nu = (\rho, \vec{j})$$

or even : $\partial_\mu F^{\mu\nu} = j^\nu$

$$F^{\mu\nu} = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x) = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

➢ Electromagnetic tensor
Unification of electromagnetism

Gauge Invariance

Gauge invariance: Classical

Maxwell equations

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} + \vec{j}$$

Physical Fields:

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\phi$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\Longleftrightarrow$$

Potentials:

A = Vector potential

ϕ = Scalar potential

Invariant under:

$$\phi \rightarrow \phi' = \phi - \frac{\partial\Lambda}{\partial t}$$

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \vec{\nabla}\Lambda$$

$$\Lambda = \Lambda(\vec{r}, t)$$

Gauge transformations

Maxwell invariant → gauge symmetry

$\vec{\nabla}\times\left(\vec{\nabla}V\right) = 0$ (rotation of gradient is 0)

$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right) = 0$ (divergence of a rotation is 0)

Gauge invariance: Classical

Maxwell equations

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} + \vec{j}$$

Physical Fields:

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\phi$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

Potentials:

A = Vector potential

ϕ = Scalar potential

$$\Lambda = \Lambda(\vec{r}, t)$$

Invariant under: $A^\mu \rightarrow A^\mu = A^\mu + \partial^\mu \Lambda$

Gauge transformations

Maxwell invariant → gauge symmetry

Not unique! Can choose extra constraints:

Coulomb-gauge: $\vec{\nabla} \cdot \vec{A} = 0$

Lorenz-gauge: $\vec{\nabla} \cdot \vec{A} - \frac{\partial \varphi}{\partial t} = 0$

$\partial^\mu A_\mu = 0$

Advantage: Lorentz-invariant

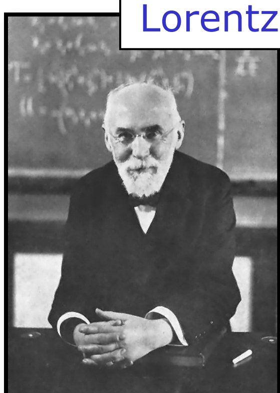
Not unique! Can choose extra constraints:

Coulomb-gauge: $\vec{\nabla}\cdot\vec{A}=0$

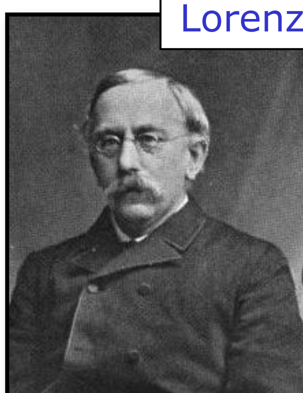
(Ludvig) Lorenz-gauge: $\vec{\nabla}\cdot\vec{A}-\dfrac{\partial\varphi}{\partial t}=0$

$\partial^\mu A_\mu = 0$

(Hendrik)
Advantage: Lorentz-invariant



Lorentz



Lorenz

Total confusion:
Lorentz-Lorenz formula

The most general form of the Lorentz–Lorenz equation is

$$\frac{n^2-1}{n^2+2}=\frac{4\pi}{3}N\alpha$$

Classically: $p \rightarrow p - qA$

Try: $L = \frac{1}{2}mv^2 - qV + q\vec{v}\cdot\vec{A}$

Then indeed: $F = q\left(E + \vec{v}\times\vec{B}\right)$

$\Rightarrow p_x = \frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial v_x} = mv_x + qA_x$

Gauge invariance : QM

Charged particle moving in electro-magnetic field:

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi$$

Original

$$H = \frac{\mathbf{p}^2}{2m} + q\phi$$

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + q\phi\right]\Psi$$

Schrödinger eq.

A,φ

$$H = \frac{1}{2m}(\mathbf{p} - q\mathbf{A})^2 + q\phi$$

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[\frac{1}{2m}(-i\hbar\nabla - q\mathbf{A})^2 + q\phi\right]\Psi$$

Pauli eq. (spin-1/2 in EM-field)

Theory gauge invariant: (same physics with A, φ as with A′, φ’) ?

Rewrite and $\hbar = c = 1$

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[\frac{1}{2m}(-i\hbar\nabla - q\mathbf{A})^2 + q\phi\right]\Psi$$

$$i\left(\frac{\partial}{\partial t} + iq\phi\right)\Psi = \frac{1}{2m}(\nabla - iq\mathbf{A})^2\Psi$$

Wave equation (A,φ)

$$A \to A' = A + \nabla\Lambda$$

$$\phi \to \phi' = \phi - \frac{\partial\Lambda}{\partial t}$$

Theory gauge invariant: (same physics with A, φ as with A′, φ’) ??

Rewrite and $\hbar = c = 1$

$$i\hbar\frac{\partial \Psi}{\partial t} = \left[\frac{1}{2m}(-i\hbar\nabla - q\mathbf{A})^2 + q\phi\right]\Psi$$

$$i\left(\frac{\partial}{\partial t} + iq\phi\right)\Psi = \frac{1}{2m}(\nabla - iq\mathbf{A})^2\Psi$$

Wave equation (A,φ)

$$A \to A' = A + \nabla\Lambda$$

$$\phi \to \phi' = \phi - \frac{\partial \Lambda}{\partial t}$$

Theory gauge invariant: (same physics with A, φ as with A’, φ’) ??

$$i\left(\frac{\partial}{\partial t} + iq\phi' + iq\frac{\partial \Lambda}{\partial t}\right)\Psi = -\frac{1}{2m}(\nabla - iq\mathbf{A}' - iq\nabla\Lambda)^2\Psi$$

Wave equation (A’,φ’)

Yes! If $\Psi \to \Psi' = e^{iq\Lambda}\Psi$

$$i\left(\frac{\partial}{\partial t} + iq\phi'\right)\Psi' = -\frac{1}{2m}(\nabla - iq\mathbf{A}')^2\Psi'$$

Local gauge symmetry

Schrödinger equation (time-independent):

$$H\Psi(\vec{r}) = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r})\right)\Psi(\vec{r}) = E\Psi(\vec{r})$$

Global phase: $\Psi(\vec{r}) \rightarrow \Psi'(\vec{r}) = e^{i\alpha}\Psi(\vec{r})$ Ψ' stays solution of Schrödinger eq!

Local gauge symmetry

Schrödinger equation (time-independent):

$$H\Psi(\vec{r}) = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r})\right)\Psi(\vec{r}) = E\Psi(\vec{r})$$

Global phase: $\Psi(\vec{r}) \rightarrow \Psi'(\vec{r}) = e^{i\alpha}\Psi(\vec{r})$ Ψ' stays solution of Schrödinger eq!

Local phase: $\Psi(\vec{r}) \rightarrow \Psi'(\vec{r}) = e^{i\alpha(\vec{r})}\Psi(\vec{r})$ Ψ' solution of Schrödinger eq?

Local gauge symmetry

Schrödinger equation (time-independent):

$$H\Psi(\vec{r}) = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r})\right)\Psi(\vec{r}) = E\Psi(\vec{r})$$

Global phase: $\Psi(\vec{r}) \to \Psi'(\vec{r}) = e^{i\alpha}\Psi(\vec{r})$ Ψ' stays solution of Schrodinger eq!

Local phase: $\Psi(\vec{r}) \to \Psi'(\vec{r}) = e^{i\alpha(\vec{r})}\Psi(\vec{r})$ Ψ' solution of Schrodinger eq?

No:

$$\vec{\nabla}\left[e^{i\alpha(\vec{r})}\Psi(\vec{r})\right] = e^{i\alpha(\vec{r})}\left[i\vec{\nabla}\alpha(\vec{r})\Psi(\vec{r}) + \nabla\Psi(\vec{r})\right] \neq e^{i\alpha(\vec{r})}\vec{\nabla}\Psi(\vec{r})$$

- (How) can you keep the Schrödinger equation invariant ?

Before going to Quantum Field Theory,
lets remind ourselves of the
Lagrange formalism

Lagrangian

Euler-Lagrange

$$H = T + V$$

$$L = T - V$$

- Euler-Lagrange equations:

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$$

$$\Updownarrow$$

- Least-action, or Hamilton's principle:

$$\partial S = \partial \int_{t_1}^{t_2} dt L(q, \dot{q}) = 0$$

Lagrangian and Equation of motion: Example

$$L = T - V$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

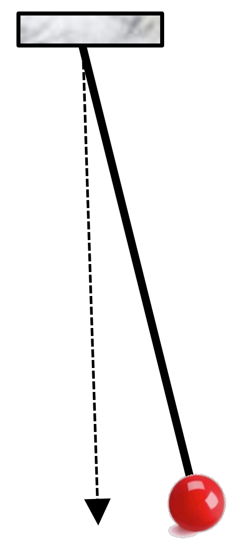
q: generalized coordinates

→ Equation of motion

$$T = \frac{1}{2}m\left(l\dot{\theta}\right)^2$$

$$V = -mgl\cos(\theta)$$

$$\ddot{\theta} - (\frac{g}{l})\sin(\theta) = 0$$



pendulum

Lagrangian in Field Theory

- Replace Lagrangian by a *Lagrangian density*

  in terms of field $\varphi(x)$:

$$\mathcal{L}\left(\phi\left(x\right),\partial_\mu\phi\left(x\right)\right) \quad \text{where} \quad L \equiv \int d^3x\,\mathcal{L}\left(\phi,\partial_\mu\phi\right)$$

- Least-action principle:

$$\delta\int_{t_1}^{t_2} d^4x\,\mathcal{L}\left(\phi,\partial_\mu\phi\right) = 0$$

- Euler-Lagrange equations:

$$\frac{\partial\mathcal{L}}{\partial\phi\left(x\right)} = \partial_\mu\frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\phi\left(x\right)\right)}$$

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$$ (classic)

Lagrangian ➔ Equation of motion

- spin-0 particles (Klein-Gordon)

$$\mathcal{L} = \mathcal{L}_{KG}^{free} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 \quad \Rightarrow \quad \left(\partial_\mu \partial^\mu + m^2\right)\phi(x) = 0$$

Klein-Gordon equation

- spin-1/2 fermions (Dirac)

$$\mathcal{L} = \mathcal{L}_{Dirac}^{free} = i\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi \quad \Rightarrow \quad (i\gamma^\mu \partial_\mu - m)\,\psi(x) = 0$$

Dirac equation

- Photons

$$\mathcal{L} = \mathcal{L}_{EM} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) - j^\mu A_\mu = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu$$

$$\Rightarrow \quad \partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = j^\nu$$

Maxwell equations

QED

Local gauge symmetry

Schrödinger equation (time-independent):

$$H\Psi(\vec{r}) = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r})\right)\Psi(\vec{r}) = E\Psi(\vec{r})$$

Global phase: $\Psi(\vec{r}) \to \Psi'(\vec{r}) = e^{i\alpha}\Psi(\vec{r})$ Ψ' stays solution of Schrödinger eq!

Local phase: $\Psi(\vec{r}) \to \Psi'(\vec{r}) = e^{i\alpha(\vec{r})}\Psi(\vec{r})$ Ψ' solution of Schrödinger eq?

No:

$$\vec{\nabla}\left[e^{i\alpha(\vec{r})}\Psi(\vec{r})\right] = e^{i\alpha(\vec{r})}\left[i\vec{\nabla}\alpha(\vec{r})\Psi(\vec{r}) + \nabla\Psi(\vec{r})\right] \neq e^{i\alpha(\vec{r})}\vec{\nabla}\Psi(\vec{r})$$

- (How) can you keep the Schrödinger equation invariant ?

Phase Invariance

- Quantum Mechanics:
  - Expectation value:
  - Observation is invariant under transformation:

$$\langle O \rangle = \int \psi^* O \psi$$

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

- But what happens to the Lagrangian density, $\mathcal{L}\left(\phi\left(x\right), \partial_\mu \phi\left(x\right)\right)$?
  - Depends also on derivative..., so:

$$\partial_\mu \psi(x) \rightarrow \partial_\mu \psi'(x) = e^{i\alpha(x)} \; \left(\partial_\mu \psi(x) + i\partial_\mu \alpha(x) \psi(x)\right)$$

Phase Invariance

- Quantum Mechanics:
  - Expectation value:

$$\langle O \rangle = \int \psi^* O \psi$$

  - Observation is invariant under transformation:

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

- But what happens to the Lagrangian density, $\mathcal{L}\left(\phi\left(x\right), \partial_\mu \phi\left(x\right)\right)$?
  - Depends also on derivative..., so:

$$\partial_\mu \psi(x) \rightarrow \partial_\mu \psi'(x) = e^{i\alpha(x)} \left(\partial_\mu \psi(x) + i\partial_\mu \alpha(x)\psi(x)\right)$$

➢ Replace by "*gauge-covariant derivative*":

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + iqA_\mu$$

Phase Invariance

- Quantum Mechanics:
  - Expectation value:
  - Observation is invariant under transformation:

$$\langle O \rangle = \int \psi^* O \psi$$

$$\psi(x) \to \psi'(x) = e^{i\alpha(x)}\psi(x)$$

- But what happens to the Lagrangian density, $\mathcal{L}\left(\phi\left(x\right), \partial_\mu \phi\left(x\right)\right)$?
  - Depends also on derivative..., so:

$$\partial_\mu \psi(x) \to \partial_\mu \psi'(x) = e^{i\alpha(x)} \left(\partial_\mu \psi(x) + i\partial_\mu \alpha(x)\psi(x)\right)$$

➢ Replace by "*gauge-covariant derivative*":

$$\partial_\mu \to D_\mu \equiv \partial_\mu + iqA_\mu$$

- And with $A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{q}\partial_\mu \alpha(x)$:

$$\begin{aligned} D_\mu \psi(x) \to D_\mu \psi'(x) &= e^{i\alpha(x)} \left(\partial_\mu \psi(x) + i\partial_\mu \alpha(x)\psi(x) + iqA_\mu(x)\psi(x) - iq\frac{1}{q}\partial_\mu \alpha(x)\psi(x)\right) \\ &= e^{i\alpha(x)} D_\mu \psi(x) \end{aligned}$$

Gauge Invariance

We started globally:

1) Arbitrary gauge $A^{\mu} \to A'^{\mu} = A^{\mu} + \partial^{\mu}\Lambda$
2) Keep Eqs valid

➢ Need $\psi \to \psi' = \psi e^{i\alpha}$

Then we went local:

1) Assume symmetry $\psi \to \psi' = \psi e^{i\alpha(x)}$
2) Keep Eqs valid

➢ Covariant derivative $\partial^{\mu} \to D^{\mu} \equiv \partial^{\mu} + iqA^{\mu}$

Gauge Invariance

We started globally:

1) Arbitrary gauge $A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu \Lambda$
2) Keep Eqs valid

➢ Need $\psi \rightarrow \psi' = \psi e^{i\alpha}$

Then we went local:

1) Assume symmetry $\psi \rightarrow \psi' = \psi e^{i\alpha(x)}$
2) Keep Eqs valid

➢ Covariant derivative $\partial^\mu \rightarrow D^\mu \equiv \partial^\mu + iqA^\mu$

Why would you want to do that?

Quantum Electro Dynamics - QED

- Let's replace the derivative in the Dirac equation by the gauge-covariant derivative:

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + iqA_\mu(x)$$

$$\begin{aligned} \mathcal{L} &= \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi \\ &= \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi - qA_\mu \bar{\psi}\gamma^\mu \psi \\ &= \mathcal{L}_{free} - \mathcal{L}_{int} \end{aligned}$$

➢ Gauge invariance leads to:

gauge fields, and their interactions!

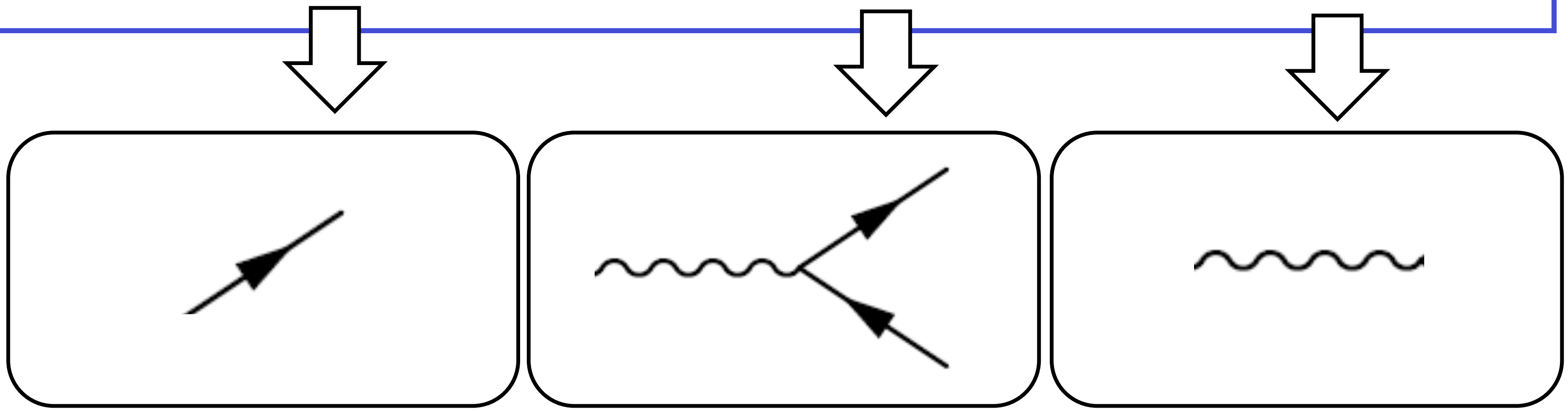
Quantum Electro Dynamics - QED

- For completeness, let's also add the piece that describes the free photons:

$$\mathcal{L}_{QED} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi - qA_{\mu}\bar{\psi}\gamma^{\mu}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

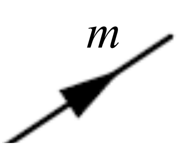
Quantum Electro Dynamics - QED

$$\mathcal{L}_{QED} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi - qA_{\mu}\bar{\psi}\gamma^{\mu}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

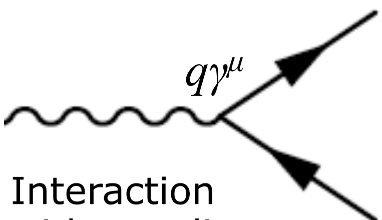


Quantum Electro Dynamics - QED

$$\mathcal{L}_{QED} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi - qA_{\mu}\bar{\psi}\gamma^{\mu}\psi \; - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



Fermion ψ with mass m



Interaction
with coupling q

Photon field A^{μ}

Local gauge symmetry

Basis of forces in
Standard Model

Fundamental
symmetry

and

Forces arise
beautifully!

Gauge field(+ interactions)

Local gauge symmetry

Pragmatic:
We have forces

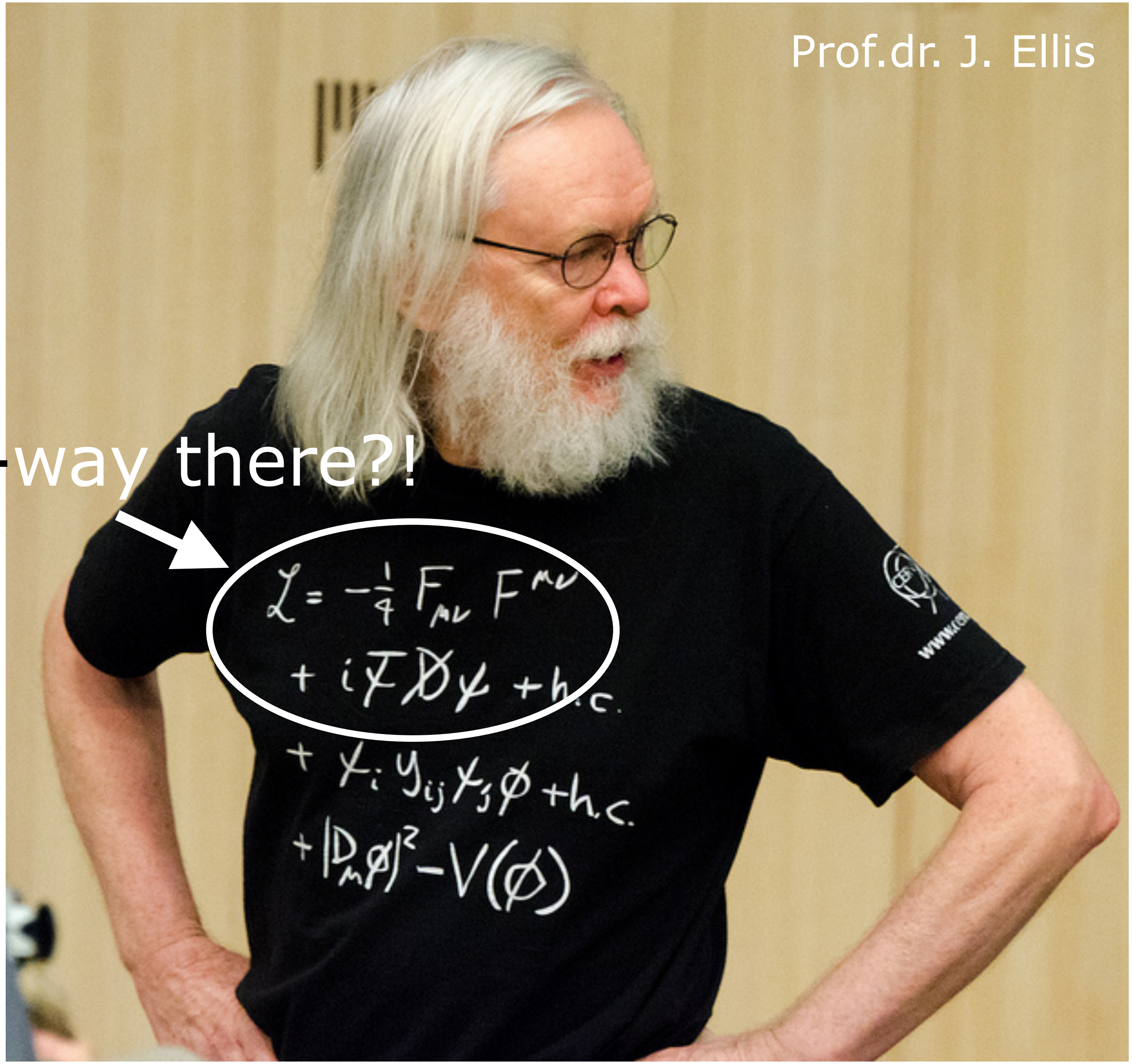
and

Can be described
mathematically

Basis of forces in
Standard Model

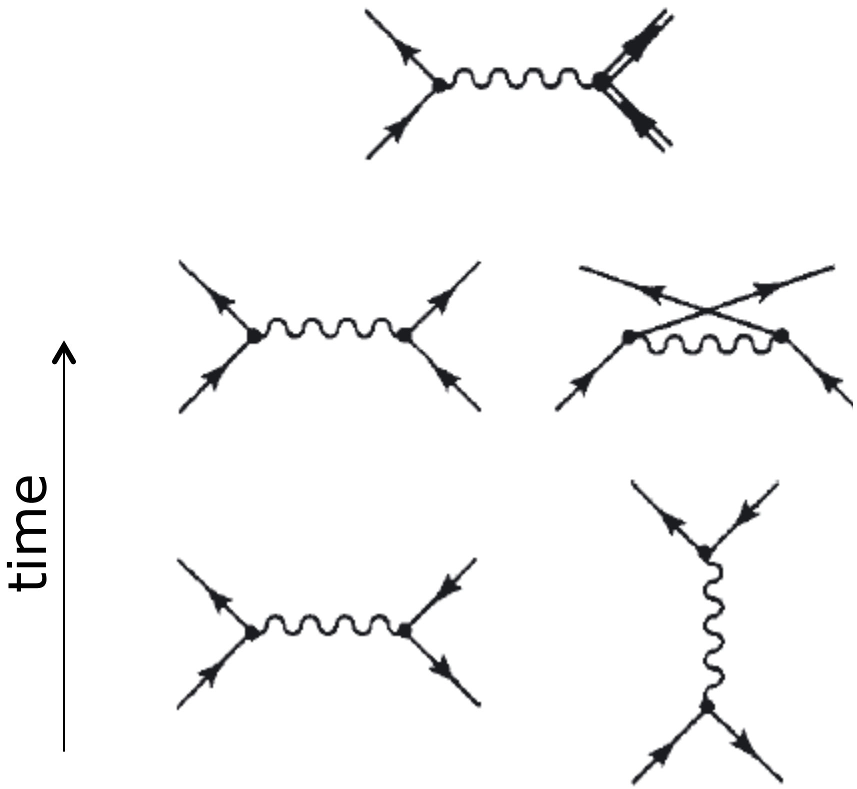
Gauge field(+ interactions)

Half-way there?!



“Famous” QED processes (I)

Elastic



Electron-muon scattering $(e + \mu \to e + \mu)$

(Mott scattering $(M \gg m)$ ⇒ Rutherford scattering $(v \ll c)$)

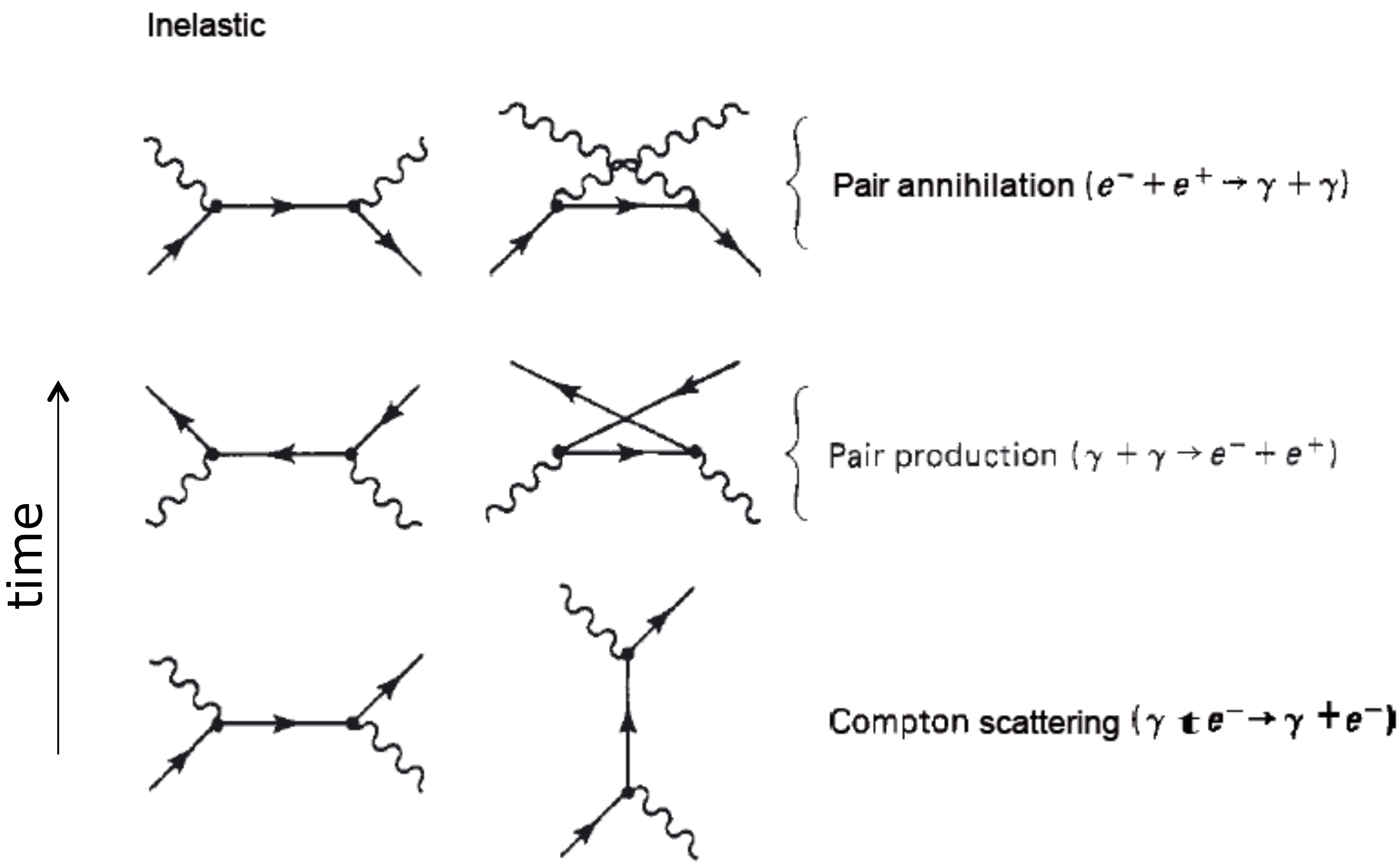
Electron-electron scattering $(e^- + e^- \to e^- + e^-)$

(Møller scattering)

Electron-positron scattering $(e^- + e^+ \to e^- + e^+)$

(Bhabha scattering)

“Famous” QED processes (II)



Weak Interaction

More gauge transformations

- We had:

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

- How about?

$$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$

More gauge transformations

- We had:

$$\psi(x) \to \psi'(x) = e^{i\alpha(x)}\psi(x)$$

- How about?

$$\psi \to \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$

- Why? Historical origin:

  proton and neutron exhibit isospin symmetry

$$\mathcal{L} = \bar{p}\,\left(i\gamma^\mu\partial_\mu - m\right)\,p + \bar{n}\,\left(i\gamma^\mu\partial_\mu - m\right)\,n$$

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}$$

$$\mathcal{L} = \bar{\psi}\,\left(i\gamma^\mu\, I\,\partial_\mu - I\,m\right)\,\psi$$

More gauge transformations

- We had:

$$\psi(x) \to \psi'(x) = e^{i\alpha(x)}\psi(x)$$

- How about?

$$\psi \to \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$

- Introduce new covariant derivative:

$$I\partial_\mu \to D_\mu = I\partial_\mu + igB_\mu$$

- B^μ is now 2x2 matrix:

$$B_\mu = \frac{1}{2}\vec{\tau}\cdot\vec{b}_\mu = \frac{1}{2}t^a b^a_\mu = \frac{1}{2}\begin{pmatrix} b_3 & b_1 - ib_2 \\ b_1 + ib_2 & -b_3 \end{pmatrix}$$

➢ with three gauge fields:

$$\vec{b}_\mu = (b_1, b_2, b_3)$$

Btw, $\vec{\tau}$ are the Pauli matrices from the SU(2) group

Electro-weak theory

$$\psi \to \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$

$$\partial_\mu \to D_\mu = \partial_\mu + igB_\mu \text{ with } B_\mu = \tfrac{1}{2}\vec{\tau}\cdot\vec{b}_\mu$$

$$\begin{aligned}\mathcal{L}_{SU(2)} &= \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}\vec{F}_{\mu\nu}\vec{F}^{\mu\nu} \\ &= \mathcal{L}^{free}_{SU(2)} - \frac{g}{2}\vec{b}_\mu\cdot\bar{\psi}\gamma^\mu\vec{\tau}\psi - \frac{1}{4}\vec{F}_{\mu\nu}\vec{F}^{\mu\nu}\end{aligned}$$

- The three b, are associated to the W^+,W^- and Z^0

$$W^\pm_\mu \equiv \frac{b^1_\mu \mp i b^2_\mu}{\sqrt{2}}$$

Electroweak theory

$$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$

- We measured that left and right are different!

- Instead of “strong” isospin, switch to “weak” isospin:

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix} \quad \Rightarrow \quad \psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \text{and} \quad \psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

- Formalism the same

$$L_{kinetic}(\psi_L) = i\psi_L\gamma_\mu D^\mu \psi_L = i\overline{\psi}_L\gamma_\mu\left(\partial^\mu + ig\frac{1}{2}\vec{\tau}\cdot\vec{b}^\mu + iqA^\mu\right)\psi_L$$

(What is difference between chirality and helicity…?)

Electroweak theory

$$\psi \to \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$

- We measured that left and right are different!

- Instead of "strong" isospin, switch to "weak" isospin:

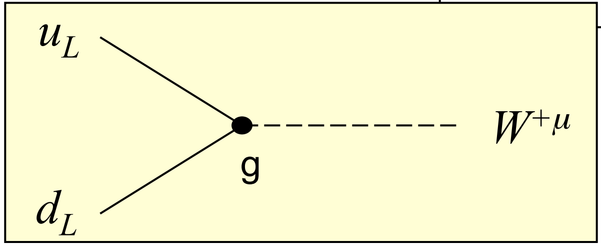
$$\psi = \begin{pmatrix} p \\ n \end{pmatrix} \quad \Rightarrow \quad \psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \text{and} \quad \psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

- Formalism the same

$$L_{kinetic}(\psi_L) = i\psi_L\gamma_\mu D^\mu\psi_L = i\overline{\psi}_L\gamma_\mu\left(\partial^\mu + ig\frac{1}{2}\vec{\tau}\cdot\vec{b}^\mu + iqA^\mu\right)\psi_L$$

$$L^{weak}_{kinetic}(u,d)_L = i(u,d)_L\gamma_\mu\left(\partial^\mu + ig\frac{1}{2}\left(b_1^\mu\tau_1 + b_2^\mu\tau_2 + b_3^\mu\tau_3\right)\right)\begin{pmatrix} u \\ d \end{pmatrix}_L$$

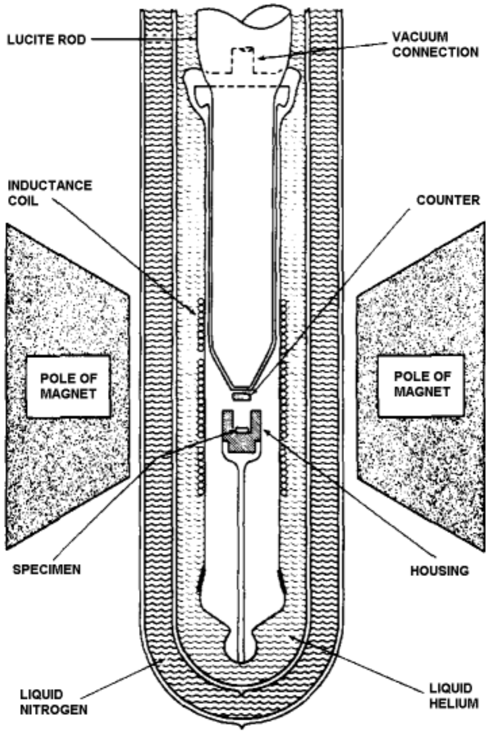
$$= i\bar{u}_L\gamma_\mu\partial^\mu u_L + i\bar{d}_L\gamma_\mu\partial^\mu d_L - \frac{g}{\sqrt{2}}\bar{u}_L\gamma_\mu W^{-\mu}d_L - \frac{g}{\sqrt{2}}\bar{d}_L\gamma_\mu W^{+\mu}u_L - \ldots$$

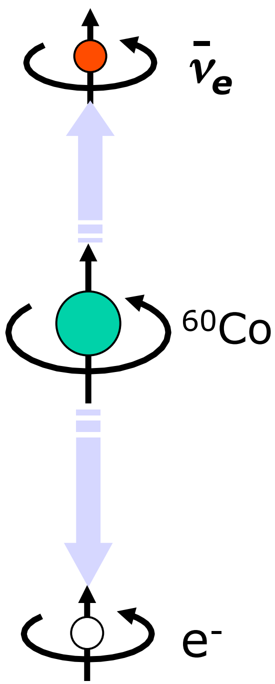


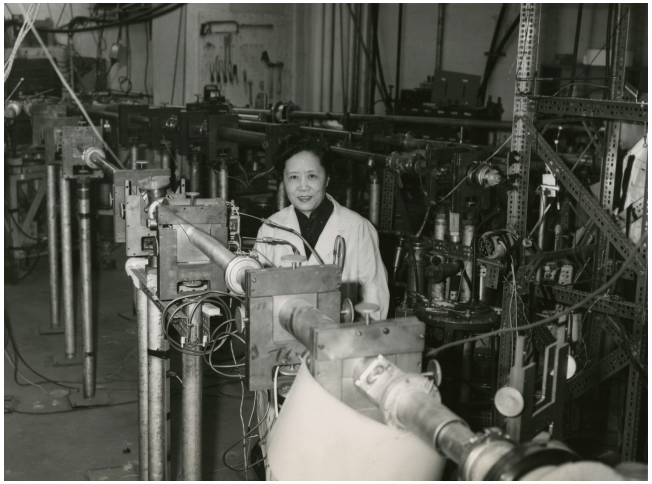
$$W^\pm_\mu \equiv \frac{b^1_\mu \mp i b^2_\mu}{\sqrt{2}}$$

Parity violation

- 1956:
  - C.-S. Wu discovered that neutrino's are always left-handed
  - The process involved: ${}^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + e^- + \bar{\nu}_e$
  - ${}^{60}_{27}Co$ is spin-5 and ${}^{60}_{28}Ni$ is spin-4, both e- and ν_e are spin-½



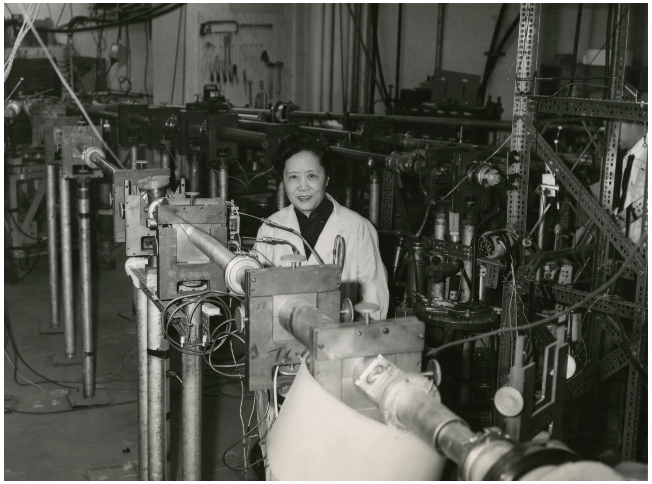


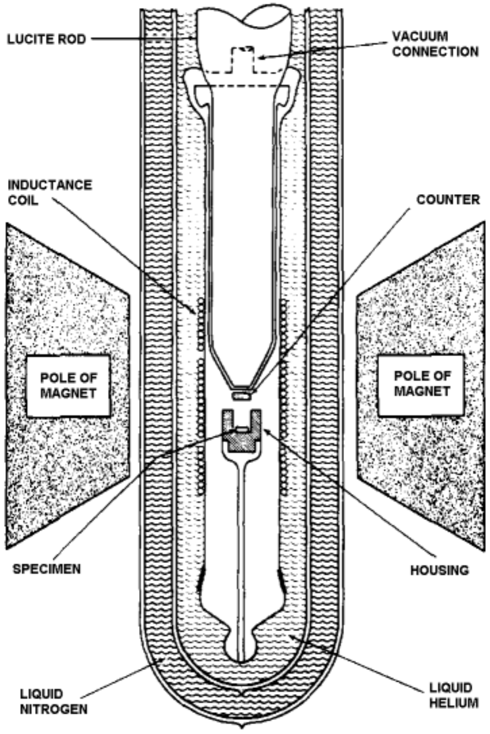


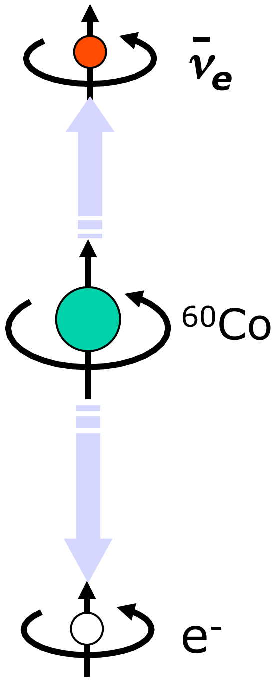
- All neutrino's are left-handed
- All antineutrinos are right-handed

Parity violation

- W. Pauli: *"That's total nonsense!" "Then it must be repeated!" "I refuse to believe that God is a weak left-hander."*
- 1956:
  - C.-S. Wu discovered that neutrino's are always left-handed
  - The process involved: ${}^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + e^- + \bar{\nu}_e$
  - ${}^{60}_{27}Co$ is spin-5 and ${}^{60}_{28}Ni$ is spin-4, both e- and ν_e are spin-½



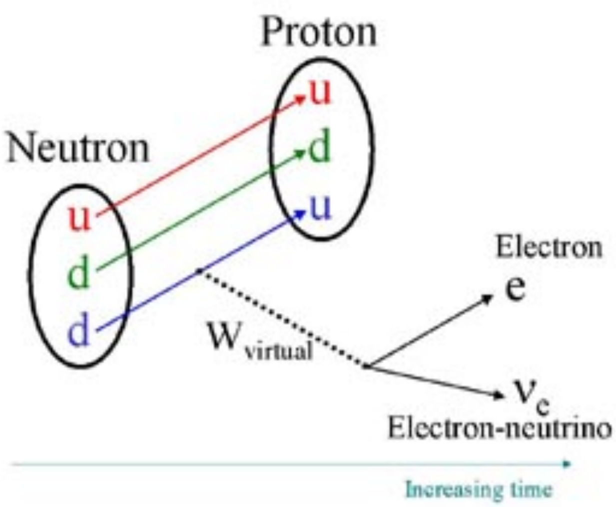


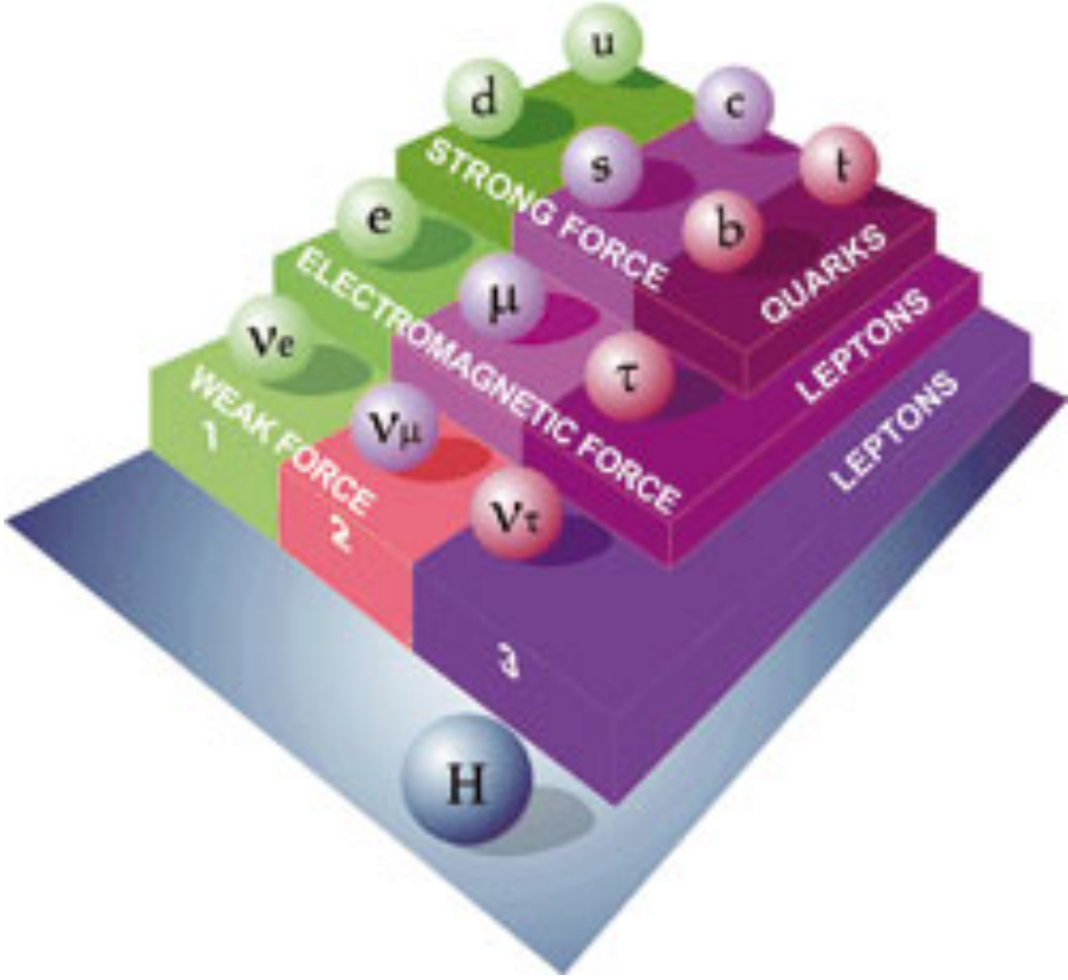


- ➢ All neutrino's are left-handed
- ➢ All antineutrinos are right-handed

Fermions and interactions

- Neutrino's only feel the weak force
  - Parity violation is most obvious for neutrino's





Electroweak theory

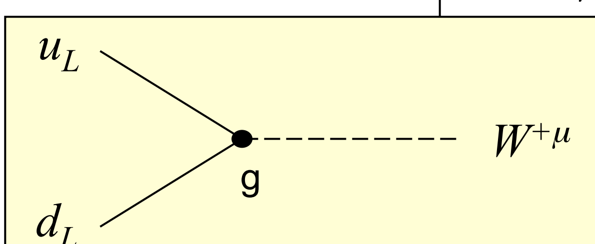
- We measured that left and right are different!
- Weak interaction only couples to left-handed particles (or right-handed anti-particles)

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \text{and} \quad \psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$L_{kinetic}(\psi_L) = i\psi_L \gamma_\mu D^\mu \psi_L = i\overline{\psi}_L \gamma_\mu \left(\partial^\mu + ig\frac{1}{2}\vec{\tau}\cdot\vec{b}^\mu + iqA^\mu \right)\psi_L$$

$$L^{weak}_{kinetic}(u,d)_L = i(u,d)_L \gamma_\mu \left(\partial^\mu + ig\frac{1}{2}\left(b_1^\mu \tau_1 + b_2^\mu \tau_2 + b_3^\mu \tau_3\right)\right)\begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$= i\overline{u}_L \gamma_\mu \partial^\mu u_L + i\overline{d}_L \gamma_\mu \partial^\mu d_L - \frac{g}{\sqrt{2}}\overline{u}_L \gamma_\mu W^{-\mu} d_L - \frac{g}{\sqrt{2}}\overline{d}_L \gamma_\mu W^{+\mu} u_L - \ldots$$



$$W_\mu^\pm \equiv \frac{b_\mu^1 \mp i b_\mu^2}{\sqrt{2}}$$

QCD

Symmetries

- Charge $\boxed{\psi}$

- Isospin $\boxed{\psi = \begin{pmatrix} u \\ d \end{pmatrix}_L}$

- Color $\boxed{\psi = \begin{pmatrix} r \\ g \\ b \end{pmatrix}}$

More gauge transformations

- We had: $$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$ U(1) (QED)
- Then: $$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$ SU(2) (Weak)
- How about: $$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8}\frac{i}{2}\theta_a(x)\lambda_a\right)\psi$$ SU(3) (QCD)

(Why 8…? Group theory: 3x3=8+1 …)

SU(2) → SU(3)

- We had: $$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$ U(1) (QED)
- Then: $$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$ SU(2) (Weak)
- How about: $$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8}\frac{i}{2}\theta_a(x)\lambda_a\right)\psi$$ SU(3) (QCD)

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

➢ **Gell-Mann matrices:**
SU(3)-equivalent of Pauli-matrices

Symmetries

- Charge ψ $$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$ U(1) (QED)

- Isospin $\psi = \begin{pmatrix} u \\ d \end{pmatrix}_L$ $$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$ SU(2) (Weak)

- Color $\psi = \begin{pmatrix} r \\ g \\ b \end{pmatrix}$ $$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8}\frac{i}{2}\theta_a(x)\lambda_a\right)\psi$$ SU(3) (QCD)

(Why 8…? Group theory: 3x3=8+1 …)

$$\begin{array}{ll} (r\bar{b}+b\bar{r})/\sqrt{2} & -i(r\bar{b}-b\bar{r})/\sqrt{2} \\ (r\bar{g}+g\bar{r})/\sqrt{2} & -i(r\bar{g}-g\bar{r})/\sqrt{2} \\ (b\bar{g}+g\bar{b})/\sqrt{2} & -i(b\bar{g}-g\bar{b})/\sqrt{2} \\ (r\bar{r}-b\bar{b})/\sqrt{2} & (r\bar{r}+g\bar{g}-2b\bar{b})/\sqrt{6}. \end{array}$$

More gauge transformations

- We had: $$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$ U(1) (QED)
- Then: $$\psi \rightarrow \psi' = \exp\left(i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\right)\psi$$ SU(2) (Weak)
- How about: $$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8}\frac{i}{2}\theta_a(x)\lambda_a\right)\psi$$ SU(3) (QCD)

a=1,8: 8 gluons

Another covariant derivative: $$D_\mu = \partial_\mu - i\,g\,A_\mu(x)$$

with 8 A_μ fields: $$A_\mu(x) = \sum_{a=1}^{8} t_a\,A_\mu^a(x) \quad , \qquad t_a = \frac{\lambda_a}{2}$$

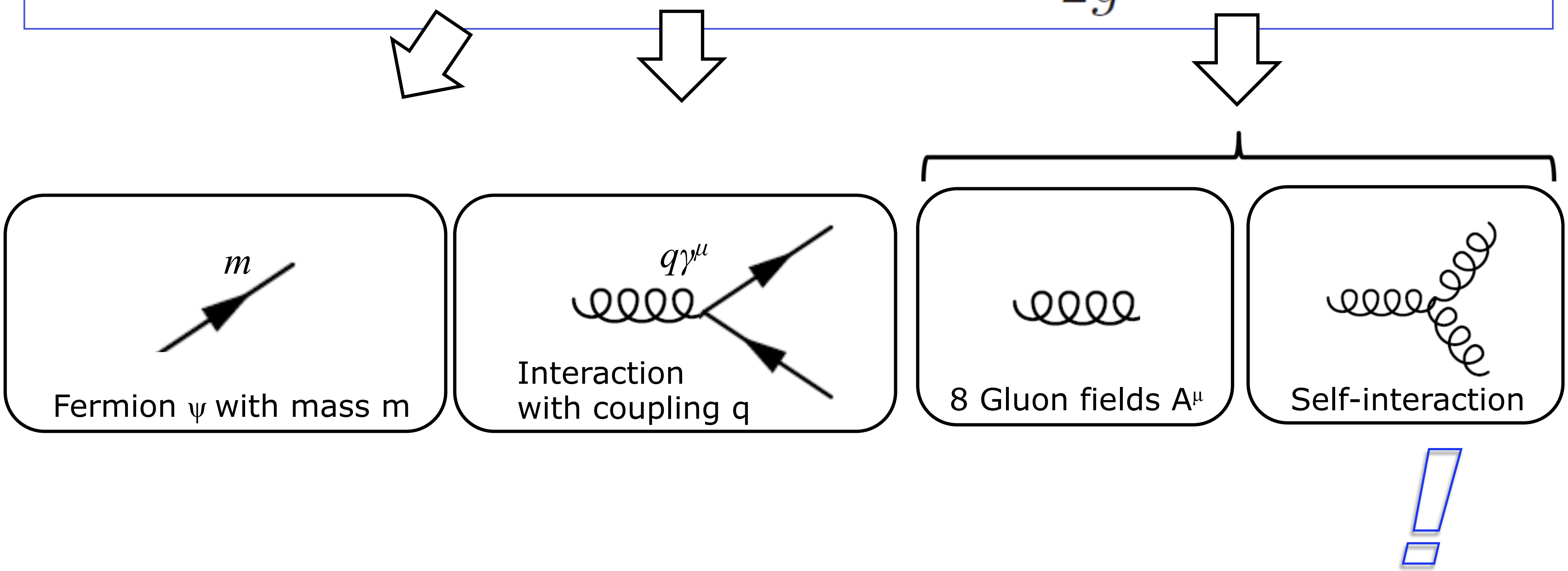
which transform as:

$$A_a^\mu(x) \rightarrow \tilde{A}_a^\mu(x) = A_a^\mu(x) - \frac{1}{g}\partial^\mu\theta_a(x) + f_{abc}\theta_b(x)A_c^\mu(x)$$

Gluonic field tensor: $$G_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g\,f_{abc}\,A_\mu^b(x)\,A_\nu^c(x)$$

QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}\left(i\gamma_\mu(\partial^\mu - iA^\mu) - m\right)\psi - \frac{1}{2g^2}\,\mathrm{tr}\{G_{\mu\nu}\,G^{\mu\nu}\}$$



QED and QCD

QED

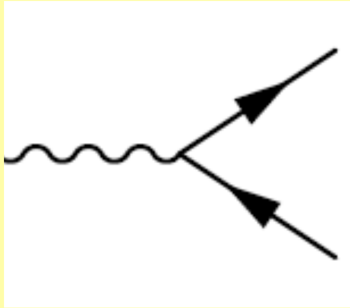
- Local U(1) gauge transformation

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

- Introduce 1 A_μ gauge field
- "Abelian" theory,

$$F^{\mu\nu} = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x)$$

- No self-interacting photon
  - Photons do not have (electric) charge
- Different "running"



QCD

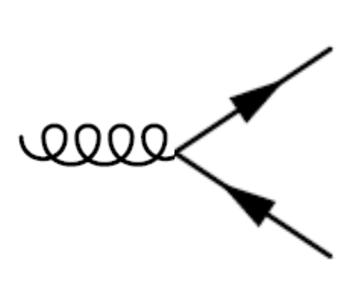
- Local SU(3) gauge transformation

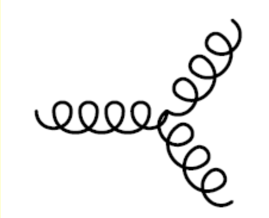
$$\psi \rightarrow \psi' = \exp\left(\sum_{a=1,8}\frac{i}{2}\theta_a(x)\lambda_a\right)\psi$$

- Introduce 8 A_μ^a gauge fields
- Non-"Abelian" theory,

$$G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g\, f_{abc}\, A^b_\mu(x)\, A^c_\nu(x)$$

- Self-interacting gluons
  - Gluons have (color) charge
- Different "running"





Which symmetries do we impose ?

$$U(1)_Y \otimes SU(2)_L \otimes SU(3)_C$$

Rotations in hypercharge space

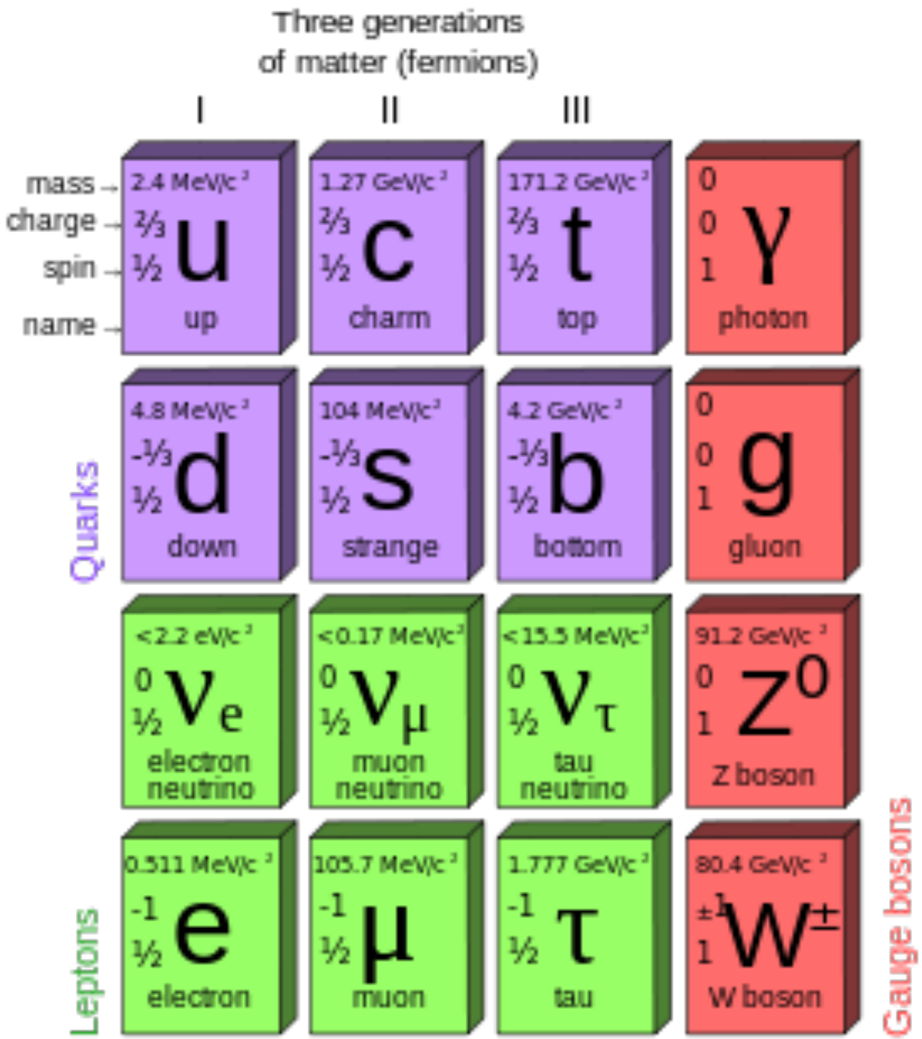
Rotations in weak isospin space

Rotations in color space

For example $SU(2)_L$:
2x2 complex matrices (det=1) → 3 basis-rotations → 3 vector fields

QED:	$U(1)_Y$	→ 1 degree of freedom:	γ	All spin-1
Weak Force:	$SU(2)_L$	→ 3 degrees of freedom:	W^+, W^- en Z^0	
Strong Force:	$SU(3)_C$	→ 8 degrees of freedom:	8 gluons	

Standard Model now (almost) complete!

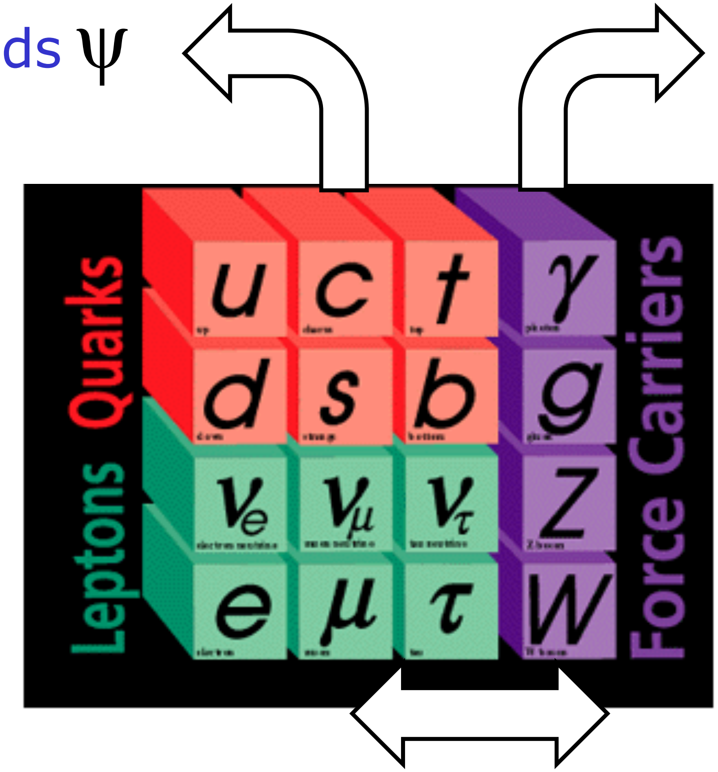


Standard Model

$$\mathcal{L} = \bar{\psi}\left(i\gamma_{\mu}D^{\mu} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Fermion fields ψ

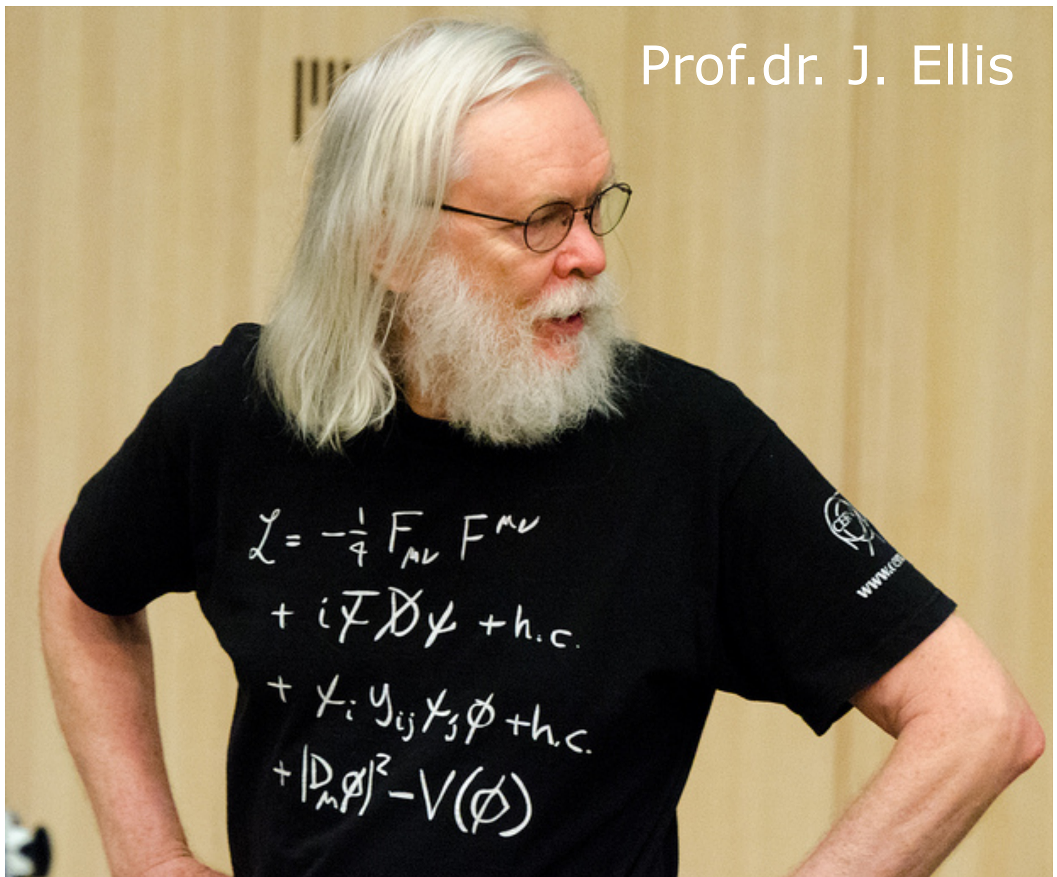
Gauge fields A_{μ}



Interactions through D^{μ}

Standard Model

$$\mathcal{L} = \overline{\psi}\left(i\gamma_{\mu}D^{\mu} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



Standard Model

$$\mathcal{L} = \overline{\psi}\left(i\gamma_\mu D^\mu - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Todo-list:

- e^+e^- scattering
  - QED at work (LEP): R, neutrinos
- e^+p scattering
  - QCD at work (HERA): DIS, structure functions, scaling
- No masses for W, Z
  - (LHC/ATLAS) Higgs mechanism, Yukawa couplings
- Consequences of three families
  - (LHC/LHCb) CKM-mechanism, CP violation

Schedule

1) 11 Feb: Accelerators (Harry vd Graaf) + Special relativity (Niels Tuning)
2) 18 Feb: Quantum Mechanics (Niels Tuning)
3) 25 Feb: Interactions with Matter (Harry vd Graaf)
4) 3 Mar: Light detection (Harry vd Graaf)
5) 10 Mar: Particles and cosmics (Niels Tuning)
6) 17 Mar: Forces (Niels Tuning)

You are here

7) 24 Mar: Astrophysics and Dark Matter (Ernst-Jan Buis)

break

8) 21 Apr: e^+e^- and ep scattering (Niels Tuning)
9) 28 Apr: Gravitational Waves (Ernst-Jan Buis)
10) 12 May: Higgs and big picture (Niels Tuning)
11) 19 May: Charged particle detection (Martin Franse)
12) 26 May: Applications: experiments and medical (Martin Franse)

13) 2 Jun: Nikhef excursie
14) 8 Jun: CERN excursie

Plan

	1) Intro: Standard Model & Relativity	***11 Feb***
1900-1940	2) Basis	***18 Feb***
	1) Atom model, strong and weak force	
	2) Scattering theory	
1945-1965	3) Hadrons	***10 Mar***
	1) Isospin, strangeness	
	2) Quark model, GIM	
1965-1975	4) Standard Model	***17 Mar***
	1) QED	
	2) Parity, neutrinos, weak inteaction	
	3) QCD	
1975-2000	5) e^+e^- and DIS	***21 Apr***
2000-2015	6) Higgs and CKM	***12 May***