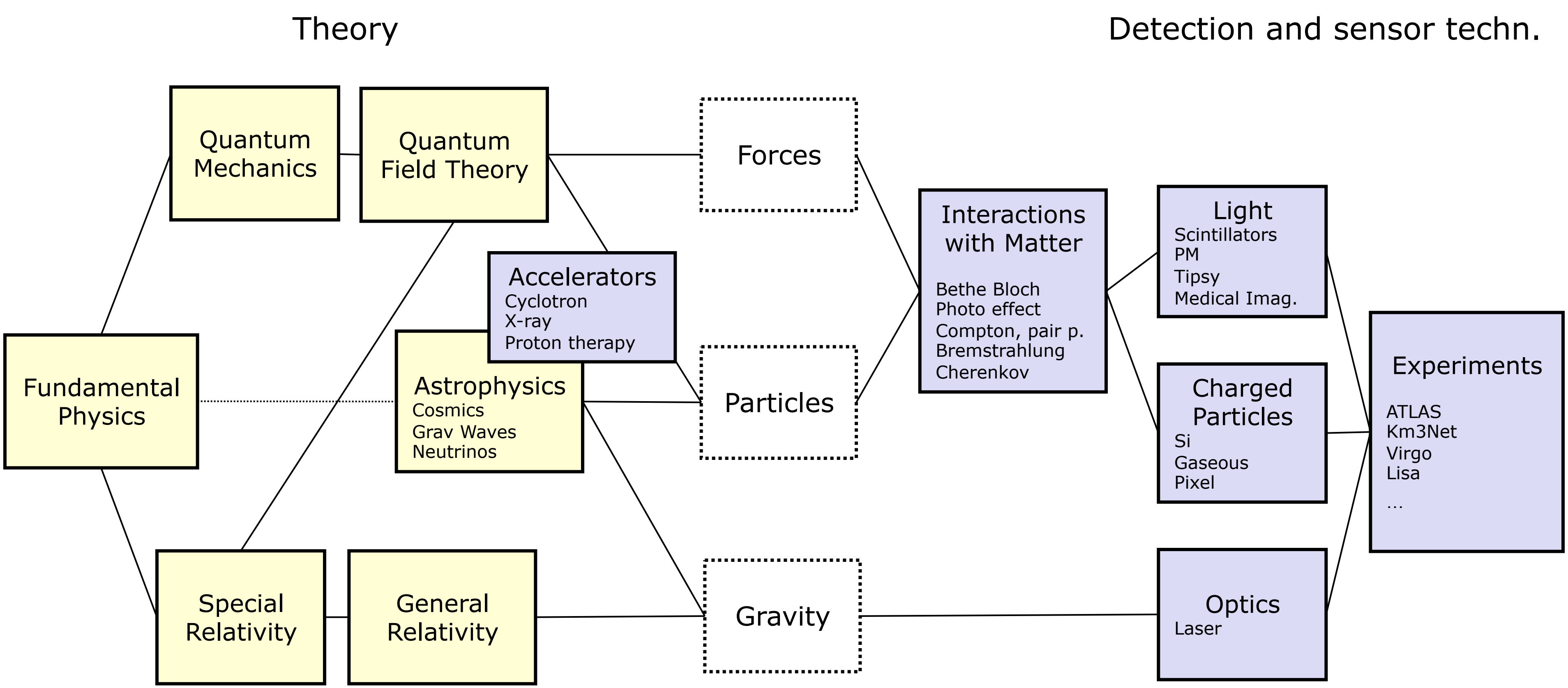
“Elementary Particles”
Lecture 3

Niels Tuning

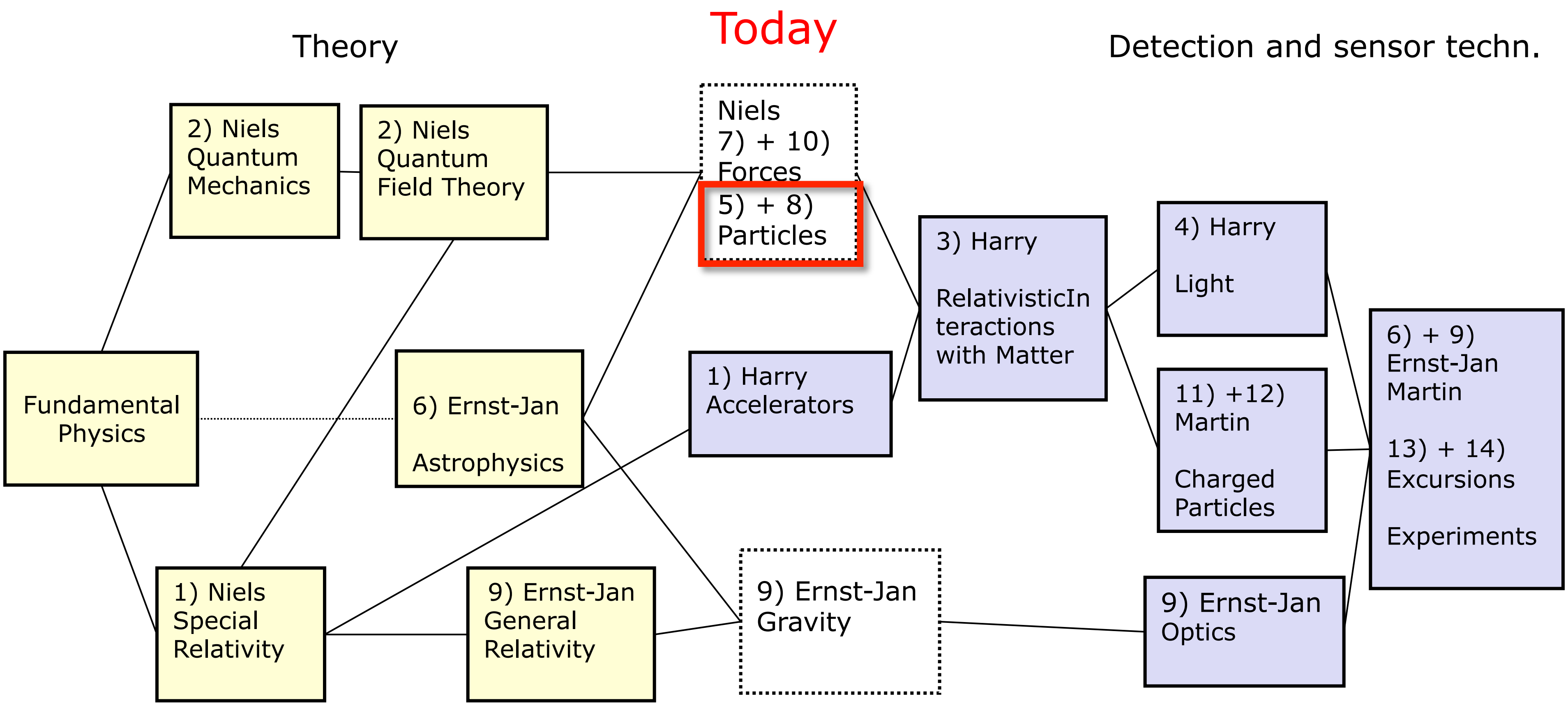
Harry van der Graaf

Plan



Theory
Detection and sensor techn.
Quantum Mechanics
Quantum Field Theory
Forces
Accelerators
Cyclotron
X-ray
Proton therapy
Fundamental Physics
Astrophysics
Cosmics
Grav Waves
Neutrinos
Particles
Interactions with Matter
Bethe Bloch
Photo effect
Compton, pair p.
Bremstrahlung
Cherenkov
Light
Scintillators
PM
Tipsy
Medical Imag.
Charged Particles
Si
Gaseous
Pixel
Experiments
ATLAS
Km3Net
Virgo
Lisa
...
Special Relativity
General Relativity
Gravity
Optics
Laser

Plan



Today
Theory
Detection and sensor techn.
2) Niels
Quantum
Mechanics
2) Niels
Quantum
Field Theory
Niels
7) + 10)
Forces
5) + 8)
Particles
3) Harry
RelativisticIn
teractions
with Matter
4) Harry
Light
Fundamental
Physics
6) Ernst-Jan
Astrophysics
1) Harry
Accelerators
11) +12)
Martin
Charged
Particles
6) + 9)
Ernst-Jan
Martin
13) + 14)
Excursions
Experiments
1) Niels
Special
Relativity
9) Ernst-Jan
General
Relativity
9) Ernst-Jan
Gravity
9) Ernst-Jan
Optics

Schedule

1) 11 Feb: Accelerators (Harry vd Graaf) + Special relativity (Niels Tuning)
2) 18 Feb: Quantum Mechanics (Niels Tuning)
3) 25 Feb: Interactions with Matter (Harry vd Graaf)
4) 3 Mar: Light detection (Harry vd Graaf)
5) 10 Mar: Particles and cosmics (Niels Tuning)
6) 17 Mar: Astrophysics and Dark Matter (Ernst-Jan Buis)
7) 24 Mar: Forces (Niels Tuning)

break

8) 21 Apr: e^+e^- and ep scattering (Niels Tuning)
9) 28 Apr: Gravitational Waves (Ernst-Jan Buis)
10) 12 May: Higgs and big picture (Niels Tuning)
11) 19 May: Charged particle detection (Martin Franse)
12) 26 May: Applications: experiments and medical (Martin Franse)

13) 2 Jun: Nikhef excursie
14) 8 Jun: CERN excursie

Plan

1) Intro: Standard Model & Relativity — *11 Feb*

1900-1940 2) Basis — *18 Feb*
   1) Atom model, strong and weak force
   2) Scattering theory

1945-1965 3) Hadrons — *10 Mar*
   1) Isospin, strangeness
   2) Quark model, GIM

1965-1975 4) Standard Model — *24 Mar*
   1) QED
   2) Parity, neutrinos, weak inteaction
   3) QCD

1975-2000 5) e^+e^- and DIS — *21 Apr*

2000-2015 6) Higgs and CKM — *12 May*

Thanks

- Ik ben schatplichtig aan:
  - Dr. Ivo van Vulpen (UvA)
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  - Prof. dr. M. Merk (VU)

Exercises Lecture 2: QM and Scattering

1 Celebrating Bohr

One of the "problems" that led to the birth of Quantum Mechanics was the fact that electrons do not spiral onto the nucleus. Let's briefly celebrate the 100^{th} aniversary of Bohr's atom model.

a) Consider the orbital momentum of the electron, $L = mvr$, and the classic situation of a stable orbit, $\alpha\frac{q_e q_p}{r^2} = \frac{mv^2}{r}$. Write the expression for L in terms of r (eliminating v).

b) Niels Bohr stated in his paper (Phil.Mag **26**, 1, 1913) that *"for a system consisting of a nucleus and an electron rotating round it, ... the angular momentum of the electron round the nucleus is equal to $h/2\pi$"*. What is then the radius of the orbit of the electron? With $E_{kin} = \frac{1}{2}mv^2$ and $E_{pot} = \frac{-\alpha q_e q_p}{r}$, what is the value for the total energy of the orbiting electron?

a) $v = \sqrt{(\alpha q_e q_p)/(mr)} \Rightarrow L = mvr = \sqrt{\alpha q_e q_p m r}$

b) $L = \sqrt{\alpha q_e q_p m r} = \hbar \Rightarrow r = \hbar^2/(\alpha q_e q_p m)$.

(I tried SI units: $\alpha qq = k_e qq = (9 \times 10^9 \mathrm{Nm}^2/\mathrm{C}^2) \times (1.6 \times 10^{-19} C)^2)$

$r = \hbar^2/(\alpha q_e q_p m) = (10^{-34})^2/(9 \times 10^9 \times (1.6 \times 10^{-19})^2 \times 10^{-30}) = 0.4 \times 10^{-10}\mathrm{m}$

$E_{tot} = E_{kin} + E_{pot} = \frac{\alpha q_e q_p}{2r} - \frac{\alpha q_e q_p}{r} = \frac{\alpha q_e q_p}{2r}$

$E_{tot} = -\frac{m(\alpha q_e q_p)^2}{2\hbar^2}$.

I tried natural units here:

$E_{tot} = -\alpha^2 m/2) = -(1/137)^2 \times 0.5$ MeV / 2 = -13.3 eV

Exercises Lecture 2: QM and Scattering

2 Yukawa's massive force carrier

Yukawa predicted a massive force carrier. Let's find out the predicted mass.

a) The strong force acts only at the scale of the nucleus. The nucleus has a size of $\sim 10^{-15}$m. To what time-scale does this correspond?

b) To what energy scale, i.e. mass scale, does this correspond?

a) $r \sim 10^{-15} m \Rightarrow t = r/c = 10^{-15}/(3 \times 10^{8}) = 3 \times 10^{-24}$ s.

b) $E \sim \hbar/t = 6.6 \times 10^{-22}\text{MeVs}/(3 \times 10^{-24} s) = 200$ MeV.

Exercises Lecture 2: QM and Scattering

3 Spinors

We saw that the requirement of a relativistically correct, but linear equation led to the Dirac equation, $(i\gamma^\mu\partial_\mu - m)\psi = 0$, with ψ being a four component spinor.

a) $H\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\psi$ gives $E^2 = p^2 + m^2$ if the matrices anticommute, $\{\alpha_i, \alpha_j\} = \alpha_i\alpha_j + \alpha_j\alpha_i = 0$. Usually we use the γ matrices, $\gamma = (\beta, \beta\vec{\alpha})$.
Show that indeed $\gamma_1\gamma_2 = \gamma_2\gamma_1$, using the Pauli-Dirac representation,

$$\beta = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}; \quad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}.$$

a) $\gamma_1\gamma_2 = -\gamma_2\gamma_1$:

$$\gamma_1 = \beta\alpha_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma_2 = \beta\alpha_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma_1\gamma_2 = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \qquad \gamma_2\gamma_1 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

Exercises

$$(\vec{\sigma}\cdot\vec{p})u_B = (E-m)u_A \quad (1)$$
$$(\vec{\sigma}\cdot\vec{p})u_A = (E+m)u_B, \quad (2)$$

where u_A and u_B are two-component objects. Let's inspect this two-fold degeneracy, and find the observable that distinguishes the two components.

b) Consider an electron with the momentum in the z-direction, $\vec{p} = (0,0,p)$. What do you find for $\vec{\sigma}\cdot\vec{p}$?

c) What is the eigenvalue of $\frac{1}{2}\vec{\sigma}\cdot\hat{p}$ for the eigenfunction

$$\chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

with $\hat{p} = \vec{p}/|\vec{p}|$ the vector in the direction of $\vec{p}$ with unit length. What does this value correspond to, you think?

d) Suppose $\hat{p}$ can point in any direction, what is then the meaning of $\frac{1}{2}\vec{\sigma}\cdot\hat{p}$? What are the possible eigenvalues?

b)

$$\vec{\sigma}\cdot\vec{p} = \sigma_3 p = \begin{pmatrix} p & 0 \\ 0 & -p \end{pmatrix}$$

c)

$$(\frac{1}{2}\vec{\sigma}\cdot\hat{p})\chi = \frac{1}{2}\sigma_3\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The eigenvalue -1/2 is the z-component of the spin.

d) $\frac{1}{2}\vec{\sigma}\cdot\hat{p}$ is the spin component in the direction of motion.
Possible eigenvalue: $\pm 1/2$.

Exercises Lecture 2: QM and Scattering

e) Let's consider the operator

$$\vec{\Sigma}\cdot\hat{p} \equiv \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix},$$

What are its eigenvalues for

$$u^{(1)} = \begin{pmatrix} u_A^{(1)} \\ u_B^{(1)} \end{pmatrix} \quad , \quad u^{(2)} = \begin{pmatrix} u_A^{(2)} \\ u_B^{(2)} \end{pmatrix}$$

where:

$$u_A^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad u_B^{(1)} = \vec{\sigma}\cdot\vec{p}/(E+m)\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad u_A^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad , \quad u_B^{(2)} = \vec{\sigma}\cdot\vec{p}/(E+m)\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(Hint: rotate your frame such that $\vec{p}$ points along the z-axis, such that you only need to worry about p_3.)

e) Positive and negative helicity:

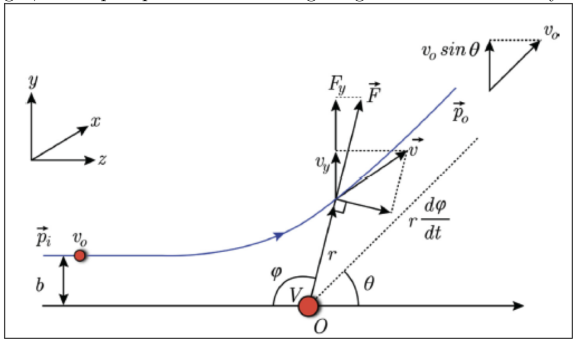
$$(\vec{\Sigma}\cdot\hat{p})u^{(1)} = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}\begin{pmatrix} u_A^{(1)} \\ u_B^{(1)} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ p_z/(E+m) \\ 0 \end{pmatrix} = +u^{(1)}$$

$$(\vec{\Sigma}\cdot\hat{p})u^{(2)} = -u^{(2)}$$

Exercises Lecture 2: QM and Scattering

4 Rutherford scattering

We calculate the distribution of scattering angles for charged particles on a charged target, like alpha particles scattering off gold nuclei as done by Ernest Rutherford in 1913.



a) The incoming particle arrives with an impact parameter b, and initial velocity v_0. The angular momentum of the initial state is $L = mbv_0$, whereas the angular momentum somewhere after the scatter can be given by $L = mrv_\perp = mr\,d\phi/dr$ Express r in terms of b.

Express r in terms of b.

$d\phi/dt\ r$

Before $L = mv_0 b$

After: $L = mr\dfrac{d\phi}{dt}r$

a)

$$L = mv_0 b = mr\frac{d\phi}{dt}r \rightarrow$$

$$r^2 = bv_0\frac{1}{d\phi/dt}$$

Exercises Lecture 2: QM and Scattering

b) The force perpendicular to the direction of the incoming particle is given by $F_y = m\, dv_y/dt$, and $F_y = F \sin\phi = (Z_1 Z_2 \alpha / r^2) \sin\phi$.
Give the expression for dv_y/dt, as a function of b (using the result from a).

c) We now multiply both sides with dt, and perform the integral from the start until the end, so the velocity on the left-hand side ranges from $v_y = 0$ to $v_y = v_0 \sin\theta$, and the angle on the right-hand side ranges from $\phi = 0$ to $\phi = \theta$.
Show that

$$\frac{\sin(\theta/2)}{\cos(\theta/2)} = \frac{Z_1 Z_2 \alpha}{m v_0^2} \frac{1}{b}$$

$$\tan\left(\tfrac{1}{2}x\right) = \frac{\sin x}{1 + \cos x}$$

b)

$$F_y = m\frac{dv_y}{dt} = F\sin\phi = \frac{Z_1 Z_2 \alpha}{r^2}\sin\phi = \frac{Z_1 Z_2 \alpha}{b v_0}\sin\phi\frac{d\phi}{dt}$$

c)

$$\int_0^{v_0 \sin\theta} dv_y = \frac{Z_1 Z_2 \alpha}{m b v_0} \int_{\cos\pi}^{\cos\theta} d\cos\theta$$

$$v_0 \sin\theta = \frac{Z_1 Z_2 \alpha}{m b v_0} (\cos\theta + 1)$$

$$\frac{\sin\theta}{\cos\theta - 1} = \frac{Z_1 Z_2 \alpha}{m b v_0^2}$$

Exercises Lecture 2: QM and Scattering

$$\cot\frac{\theta}{2} = \frac{mv_0^2}{Z_1 Z_2 \alpha} b$$

Show that

$$\frac{\sin(\theta/2)}{\cos(\theta/2)} = \frac{Z_1 Z_2 \alpha}{m v_0^2}\frac{1}{b}$$

d) For a given surface (ring) of possible incoming particles, $d\sigma = b\, db\, d\phi$, the particle is scattered in a certain solid angle $d\Omega = \sin\theta d\theta d\phi$. Show that the expression for the differential cross section is given by,

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\frac{db}{d\theta} = \left(\frac{Z_1 Z_2 \alpha}{m v_0^2}\right)^2 \frac{1}{4\sin^4\frac{\theta}{2}}$$

$$= \left| \left(\frac{Z_1 Z_2 \alpha}{m v_0^2}\right)^2 \frac{\cot\frac{\theta}{2}}{\sin\theta} \frac{d\cot\frac{\theta}{2}}{d\theta} \right|$$

$$= \left(\frac{Z_1 Z_2 \alpha}{m v_0^2}\right)^2 \frac{1}{4\sin^4\frac{\theta}{2}}$$

Rutherford

- 3d: incoming particle "sees" surface $d\sigma$, and scatters off solid angle $d\Omega$
  - Conservation of angular momentum:
  - Force:

a)

Before $L = mv_0 b$

After: $L = mr\dfrac{d\phi}{dt}r$

$$F(r) = \frac{Z_1 Z_2 \alpha}{r^2}$$

$$m\frac{dv_y}{dt} = F_y = F\sin\phi = \frac{Z_1 Z_2 \alpha}{r^2}\sin\phi$$

Replace r by b, using L conservation

b)

$$\frac{dv_y}{dt} = \frac{Z_1 Z_2 \alpha}{mv_0 b}\sin\phi\frac{d\phi}{dt}$$

c)

$$\int_0^{v_0\sin\theta} dv_y = \frac{Z_1 Z_2 \alpha}{mv_0 b}\int_{\cos\pi}^{\cos\theta} d\cos\phi$$

=(cosθ + 1)

$$\cot\frac{\theta}{2} = \frac{mv_0^2}{Z_1 Z_2 \alpha}b$$

Use expression $b = \ldots$

d)

$$\frac{d\sigma}{d\Omega} = \left|\frac{b}{\sin\theta}\frac{db}{d\theta}\right|$$

$$= \left|\left(\frac{Z_1 Z_2 \alpha}{mv_0^2}\right)^2 \frac{\cot\frac{\theta}{2}}{\sin\theta}\frac{d\cot\frac{\theta}{2}}{d\theta}\right|$$

$$= \left(\frac{Z_1 Z_2 \alpha}{mv_0^2}\right)^2 \frac{1}{4\sin^4\frac{\theta}{2}}$$

Exercises Lecture 2: QM and Scattering

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\frac{db}{d\theta} = \left(\frac{Z_1 Z_2 \alpha}{m v_0^2}\right)^2 \frac{1}{4\sin^4\frac{\theta}{2}}$$

e) Use the 4-vectors $p_i = (E, 0, 0, mv_0)$ and $p_o = (E, 0, mv_0\sin\theta, mv_0\cos\theta)$ for the incoming and outgoing particle, respectively, and express the differential cross section in terms of the 4-momentum transfer $q = p_o - p_i$, instead of θ.

e)

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 \alpha}{m v_0^2}\right)^2 \frac{1}{4\sin^4\frac{\theta}{2}} = (2mZ_1Z_2\alpha)^2\left(\frac{1}{4m^2v_0^2\sin^2\frac{\theta}{2}}\right)^2$$

$$-q^2 = -\begin{pmatrix} 0 \\ 0 \\ -mv_0\sin\theta \\ mv_0(1-\cos\theta) \end{pmatrix}^2 = (mv_0)^2(\sin^2\theta + (1-\cos\theta)^2) = 2(mv_0)^2(1-\cos\theta) = 4m^2v_0^2\sin^2\frac{\theta}{2}$$

b) $\pi R^2 = 60\text{mb} = 60 \times 10^{-3} \times 10^{-28}\text{m}^2 = 6 \times 10^{-30}\text{m}^2 \Rightarrow R \sim 10^{-15}$ m.
So, the total proton-proton cross section is similar to the surface of the proton.

Exercises Lecture 2: QM and Scattering

5 Cross section

Let's juggle a bit with cross sections and luminosities.

a) The total cross section for proton-proton scattering at the LHC is about $\sigma_{tot} = 60\,\text{mb}$. To what surface does this cross section correspond? (1 barn $= 10^{-28}\text{m}^2$.) What is the size of an object with similar surface?

b) The cross section for Higgs production at the LHC is approximately $\sigma_{pp\to H+X} = 30\,\text{pb}$. The "luminosity" is the number of particles produced for a given cross-section, and is an important characteristic of the performance of an accelerator. How many Higgs particles are then produced for a total luminosity of $\mathcal{L}_{tot} = 10\ \text{fb}^{-1}$?

c) The "instantaneous" luminosity at the LHC is about $\mathcal{L}_{inst} = 10^{34}\text{s}^{-1}\text{cm}^{-2}$. How many Higgs particles are thus produced per hour?

d) Compare the total proton-proton cross section with the cross section for Higgs production. In what fraction of the proton-proton collisions is a Higgs particle produced?

b) $\pi R^2 = 60\text{mb} = 60 \times 10^{-3} \times 10^{-28}\text{m}^2 = 6 \times 10^{-30}\text{m}^2 \Rightarrow R \sim 10^{-15}$ m.
So, the total proton-proton cross section is similar to the surface of the proton.

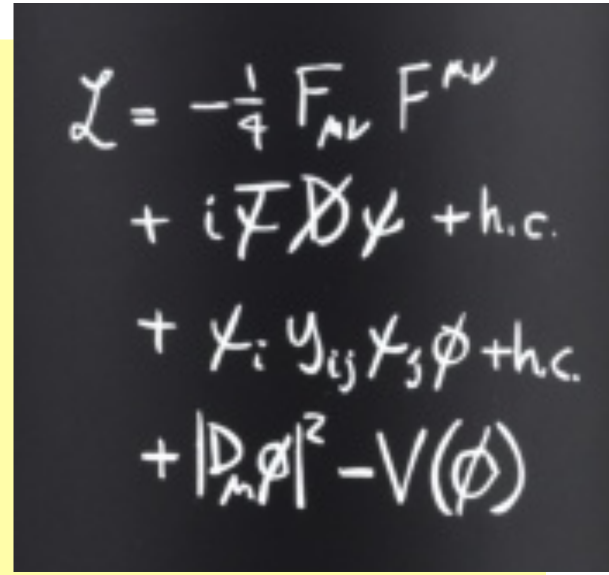
c) $N = \sigma \times \mathcal{L}_{tot} = 30 \times 10^{-12} \times 10 \times 10^{15} = 3 \times 10^5$.
However, we will see that not all Higgs particles are detected. Only a fraction of the Higgs particles decay to a final state that is easy to detect. Furthermore, out of the visible final states, a fraction passes the selection criteria.

d) $N = \sigma \times \mathcal{L}_{inst} = 30 \times 10^{-40}\text{m}^2 \times 10^{38}\, s^{-1}\text{m}^{-2} = 0.3\, s^{-1} \Rightarrow \sim 1000\ \text{h}^{-1}$.

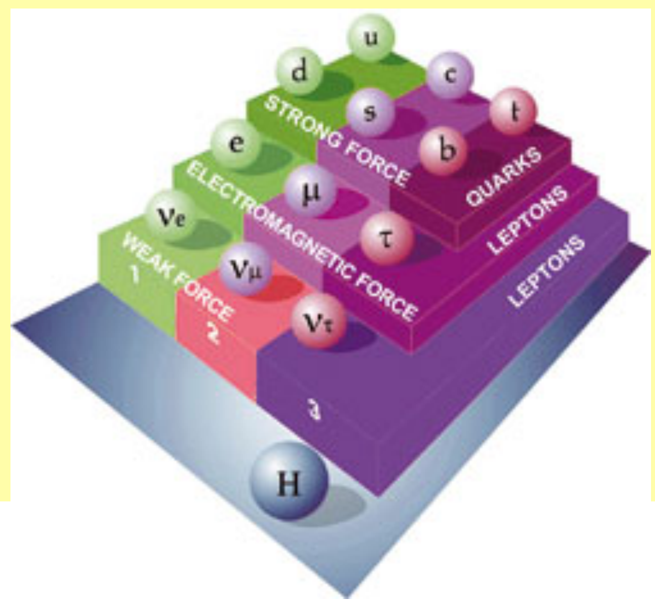
e) This is a very small number, compared to the fact that about 20 proton-proton collisions occur every 25 ns: 1 out of 3×10^9 pp collisions produces a Higgs particle.

Lecture 1: Standard Model & Relativity

- Standard Model Lagrangian



- Standard Model Particles



Lecture 1: Accelerators & Relativity

- Theory of relativity
  - Lorentz transformations ("boost")
  - Calculate energy in collissions

$$x'^0 = \gamma\left(x^0 - \beta x^1\right)$$
$$x'^1 = \gamma\left(x^1 - \beta x^0\right)$$
$$x'^2 = x^2$$
$$x'^3 = x^3$$

met

$$\beta \equiv \frac{v}{c}$$
$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

- 4-vector calculus

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \quad (\mu = 0, 1, 2, 3)$$

$$p_\mu p^\mu = (E/c)^2 - |\vec{p}|^2 = (E^2 - c^2|\vec{p}|^2)/c^2 = (m_0 c^4)/c^2$$

- High energies needed to make (new) particles

$$s = \left(p_1 + p_2\right)^2 = 2m^2 + 2\left(E^2 + \vec{p}^2\right)$$
$$= 2m^2 + 2E^2 + 2\left(E^2 - m^2\right) = 4E^2$$

Lecture 2: Quantum Mechanics & Scattering

- Schrödinger equation
  - Time-dependence of wave function

$$E = \frac{\vec{p}^2}{2m}$$

$$i\frac{\partial}{\partial t}\psi = \frac{-1}{2m}\nabla^2\psi$$

- Klein-Gordon equation
  - Relativistic equation of motion of scalar particles

$$E^2 = \vec{p}^2 + m^2$$

$$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\,\phi$$

- Dirac equation
  - Relativistically correct, and linear
  - Equation of motion for spin-1/2 particles
  - Prediction of anti-matter

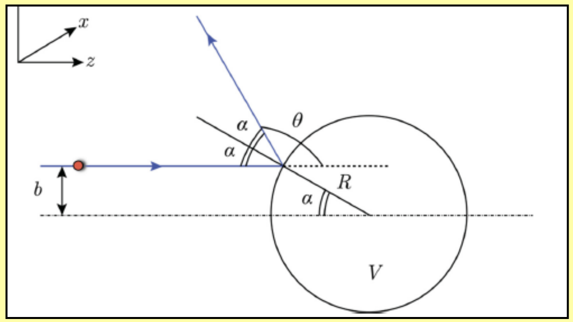
$$(i\gamma^\mu\partial_\mu - m)\,\psi = 0$$

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix}$$

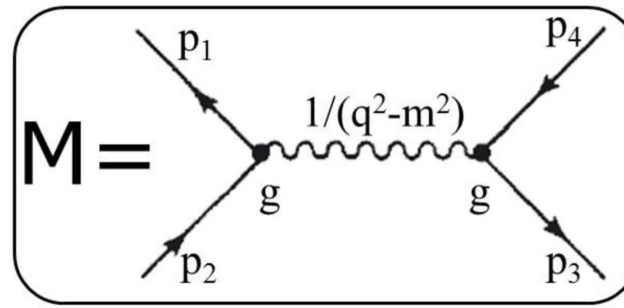
Lecture 2: Quantum Mechanics & Scattering

- Scattering Theory
  - (Relative) probability for certain process to happen
  - Cross section

$$\frac{d\sigma}{d\Omega} = D(\theta, \varphi)$$



Classic



Scattering amplitude in Quantum Field Theory

- Fermi's Golden Rule

$$\text{transition rate} = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \times (\text{phase space})$$

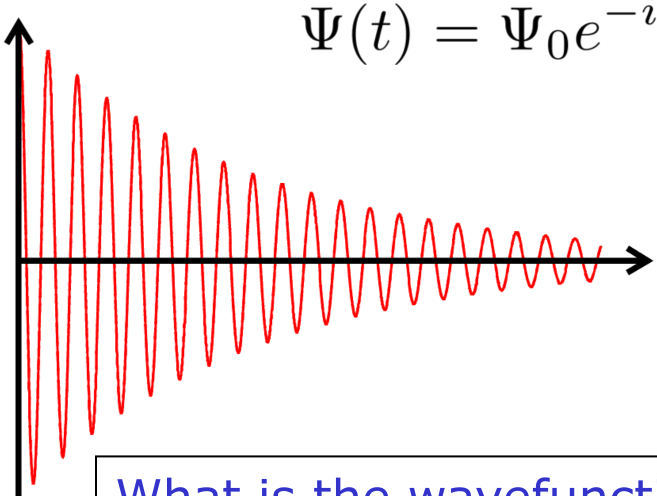
  - Decay: "decay width" Γ $\quad a \rightarrow b + c$
  - Scattering: "cross section" σ $\quad a + b \rightarrow c + d$

Resonances

Quantum mechanical description of decay

State with energy E_0 $(\hbar\omega)$ and lifetime τ

To allow for decay, we need to change the time-dependence:



$$\Psi(t) = \Psi_0 e^{-iE_0 t} \quad \Longrightarrow \quad \Psi(t) = \Psi_0 e^{-iE_0 t} e^{-\frac{t}{2\tau}}$$

$$\Psi^* \Psi = \Psi_0^* \Psi_0 e^{-\frac{t}{\tau}}$$

What is the wavefunction in terms of energy (instead of time) ?

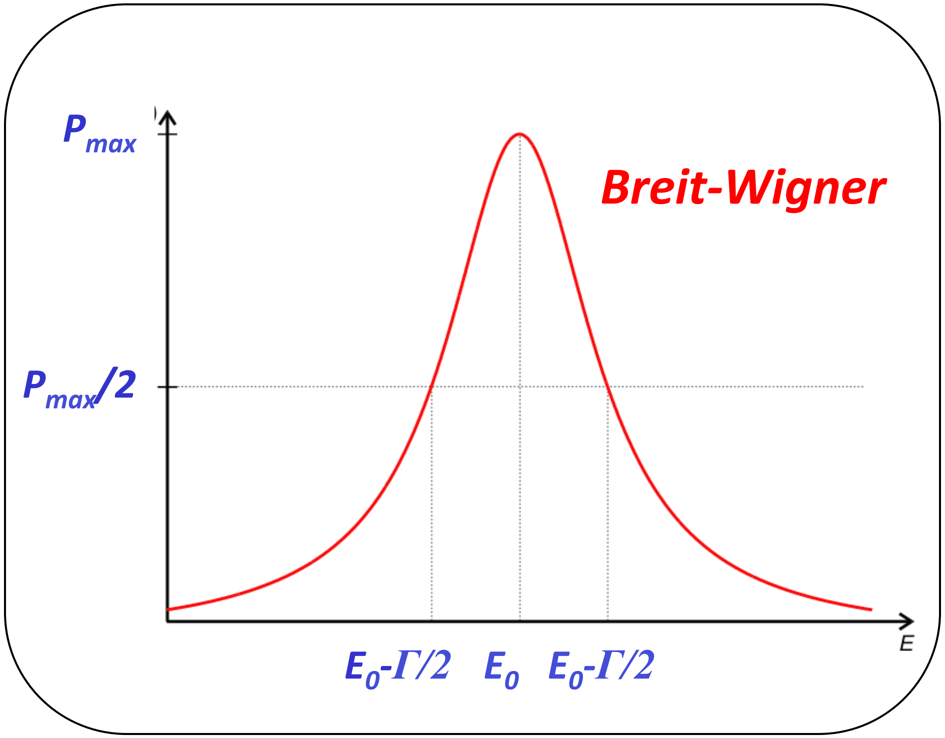
- Infinite sum of flat waves, each with own energy
- Fourier transformation:

$$f(\omega) = f(E) = \int_0^\infty \Psi_0 e^{-t(iE_0 + \frac{1}{2\tau})} e^{iEt} dt = \Psi_0 \frac{1}{i\left((E_0 - E) - \frac{i}{2}\Gamma\right)}$$

Resonance

Probability to find particle with energy E:

$$f(E)^* f(E) = \Psi_0^* \Psi_0 \frac{1}{(E_0 - E)^2 + \frac{1}{4}\Gamma^2}$$



Resonance-structure contains information on:

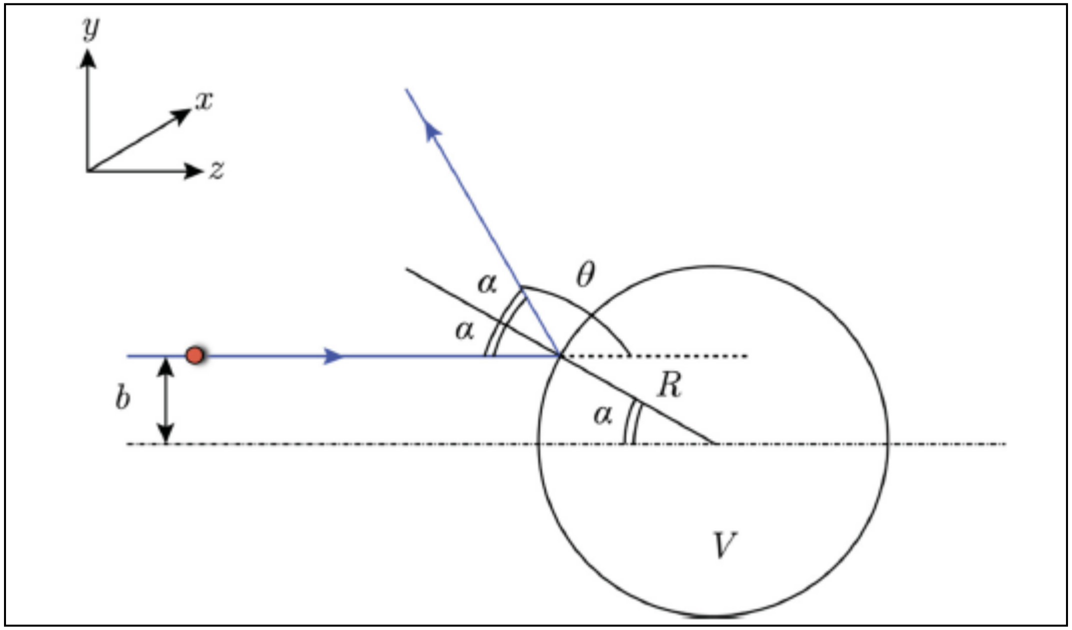
- Mass
- Lifetime
- Decay possibilities

Rutherford

- 3d: incoming particle "sees" surface dσ, and scatters off solid angle dΩ
- Calculate:

$$\frac{d\sigma}{d\Omega} = D(\theta, \varphi)$$

$$d\sigma = |b\, db\, d\varphi|$$
$$d\Omega = |\sin\theta\, d\theta\, d\varphi|$$



$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 \alpha}{m v_0^2}\right)^2 \frac{1}{4\sin^4\frac{\theta}{2}}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2mZ_1Z_2\alpha}{q^2}\right)^2$$

Scattering Theory

Let's try some potentials

$$\Phi(\vec{r}) = \phi_a(\vec{r}) - \frac{m}{2\pi\hbar^2}\int \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} V(\vec{r}')\phi_a(\vec{r}')\, d^3r'$$

$$f^{[1]}(\vec{k_a},\vec{k_b}) = \frac{m}{2\pi\hbar^2}\int e^{i(\vec{k_a}-\vec{k_b})\cdot\vec{r}'} V(\vec{r}')\, d^3r'$$

- **Yukawa:**
  (Pion exchange)

$$V(r) = \frac{Z_1 Z_2 e^2}{r} e^{-ar}$$

$$\frac{d\sigma}{d\Omega} = |f|^2 = \frac{m^2}{(2\pi\hbar^2)^2}\left[\frac{4\pi Z_1 Z_2 e^2}{q^2+a^2}\right]^2$$

- **Coulomb:**
  (Elastic scattering)

$$V(r) = \frac{Z_1 Z_2 e^2}{r}$$

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{(2\pi\hbar^2)^2}\left[\frac{4\pi Z_1 Z_2 e^2}{q^2}\right]^2 = \left[\frac{Z_1 Z_2 e^2}{2mv^2 \sin^2\frac{\theta}{2}}\right]^2$$

- **Centrifugal Barier:**
  (Resonances)

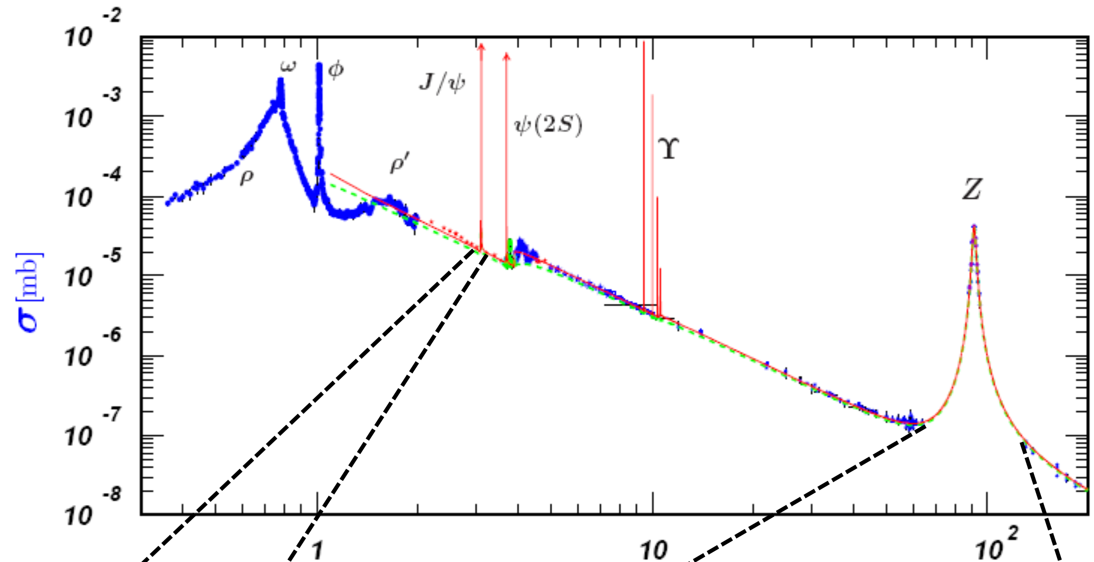
$$V_{eff} = V(r) + \frac{\hbar^2 l(l+1)}{2mr^2}$$

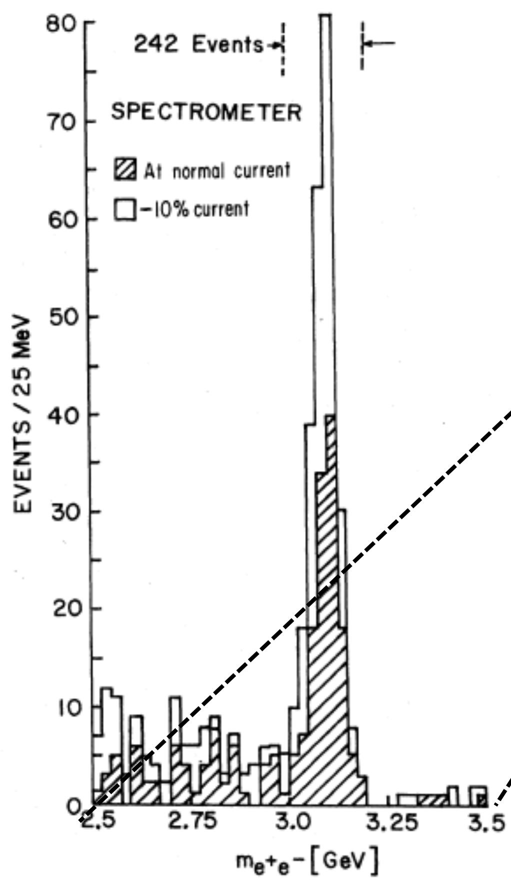
$$\begin{aligned}\frac{d\sigma}{d\Omega} &= |f^{res}(\theta)|^2 \\ &= \frac{(2l+1)^2}{k^2}\frac{\frac{\Gamma^2}{4}}{(E_r-E)^2+\frac{\Gamma^2}{4}}|P_l(\cos\theta)|^2\end{aligned}$$

Well-known resonances

$$e^+e^- \rightarrow R \rightarrow e^+e^-$$

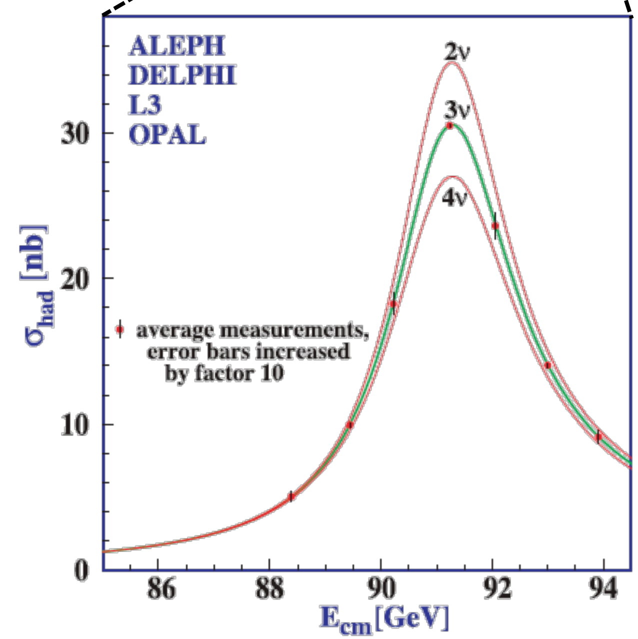
e$^+$e$^-$ cross-section





J/ψ

Z-boson

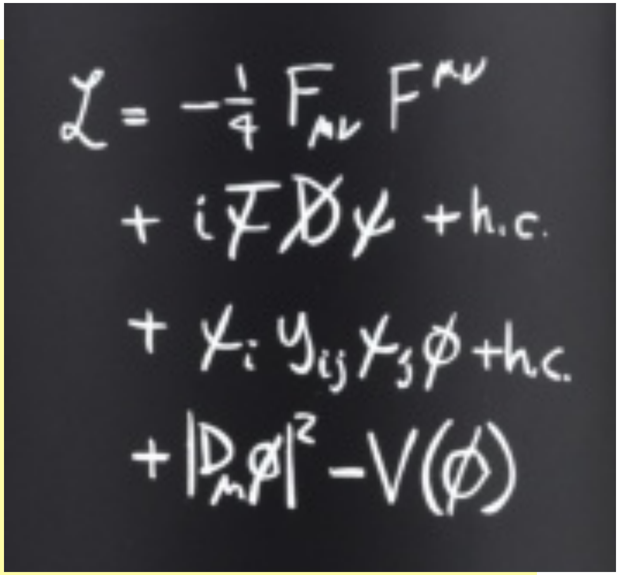


Outline for today

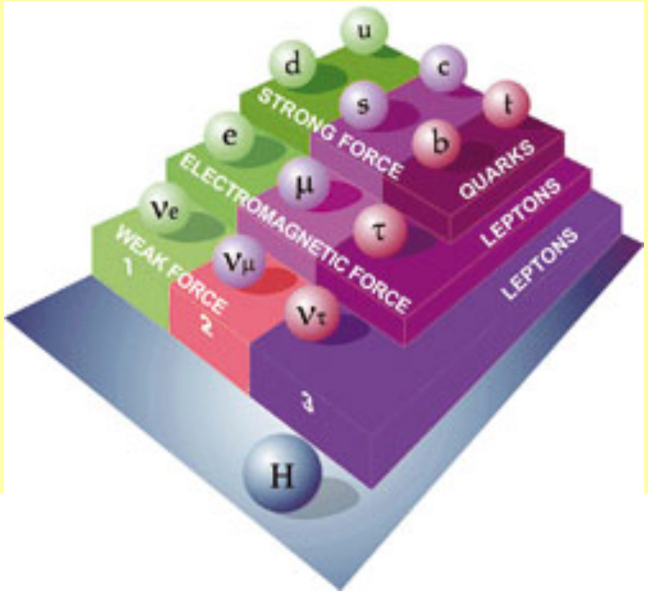
- Resonances
- Quarkmodel
  - Strangeness
  - Color
- Symmetries
  - Isospin
  - Adding spin
  - Clebsch Gordan coefficients

Lecture 1: Standard Model & Relativity

- Standard Model Lagrangian

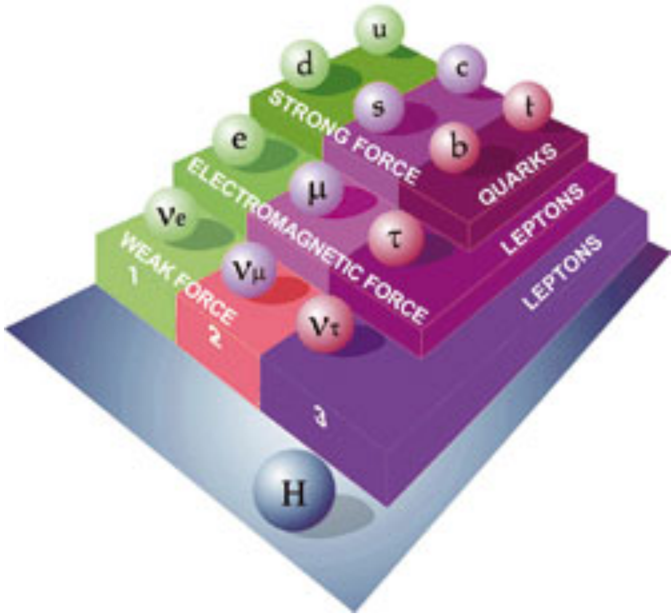


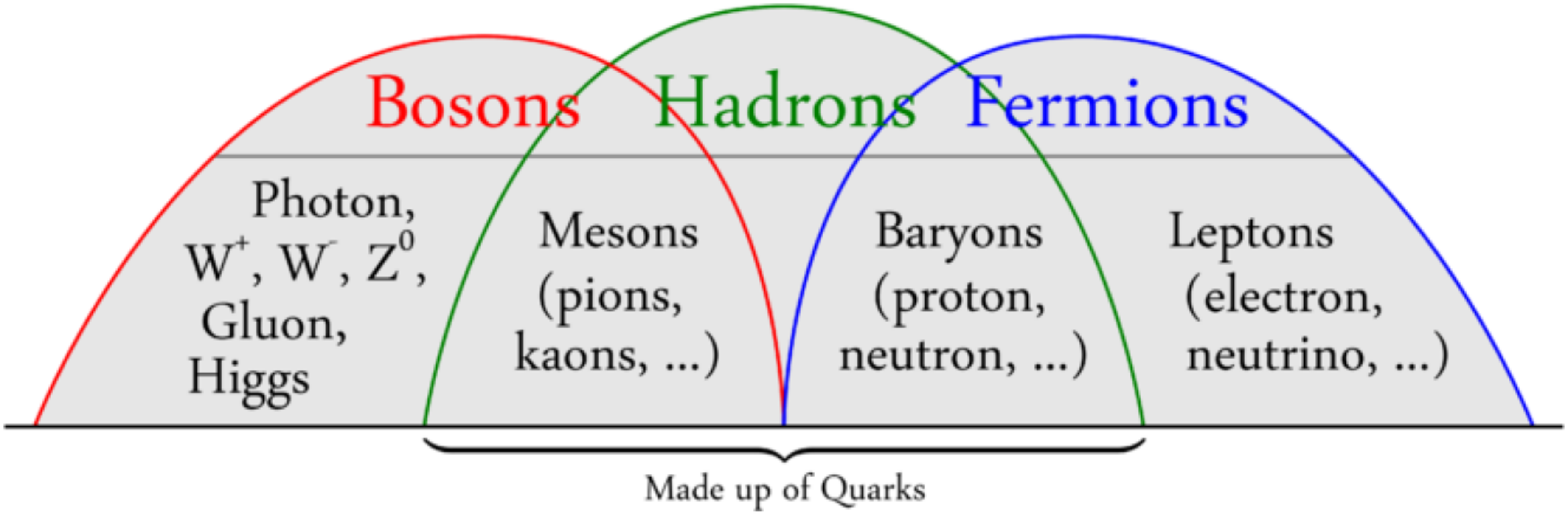
- Standard Model Particles



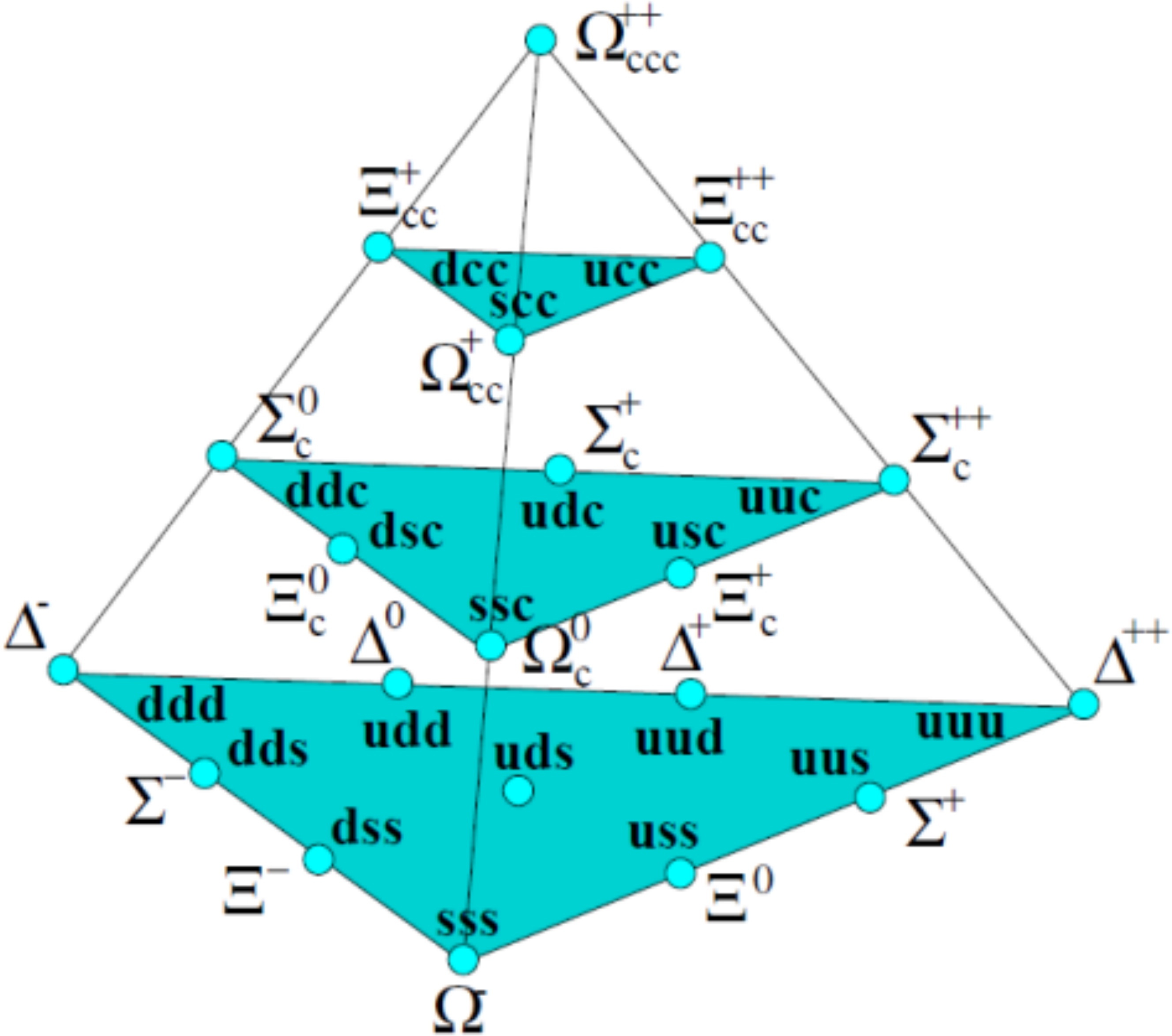
Particles

- Quarks and leptons…:



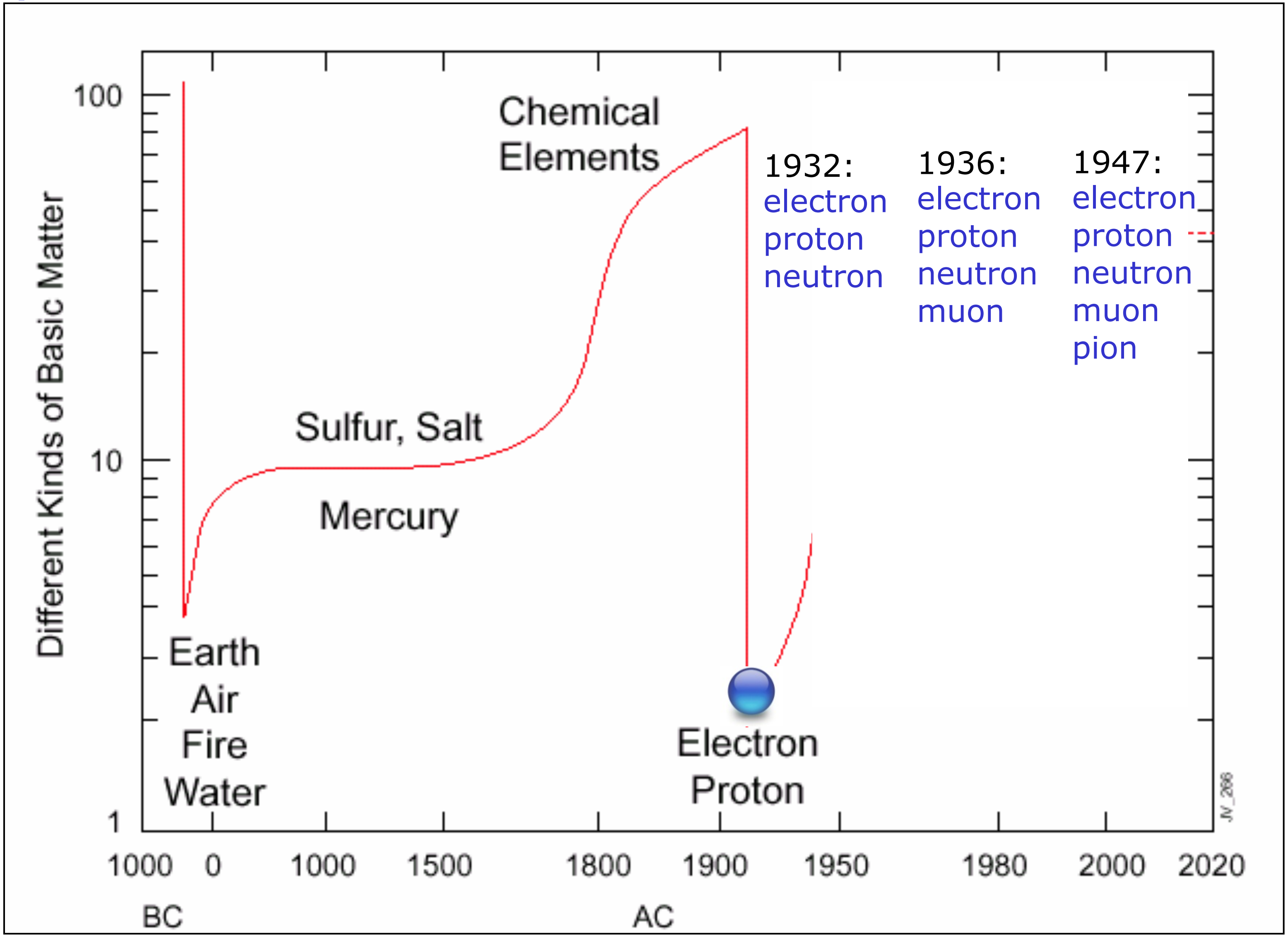


Particles…



Ω_{ccc}^{++}
Ξ_{cc}^{+}
Ξ_{cc}^{++}
dcc
ucc
scc
Ω_{cc}^{+}
Σ_c^0
Σ_c^+
Σ_c^{++}
ddc
udc
uuc
dsc
usc
Ξ_c^0
Ξ_c^+
ssc
Ω_c^0
Δ^-
Δ^0
Δ^+
Δ^{++}
ddd
udd
uud
uuu
dds
uds
uus
Σ^-
Σ^+
dss
uss
Ξ^-
Ξ^0
sss
Ω^-

The number of ‘elementary’ particles

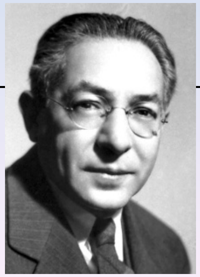


1947

- 1932: the ***positron*** had been observed to confirm Dirac's theory,
- 1947: and the ***pion*** had been identified as Yukawa's strong force carrier,

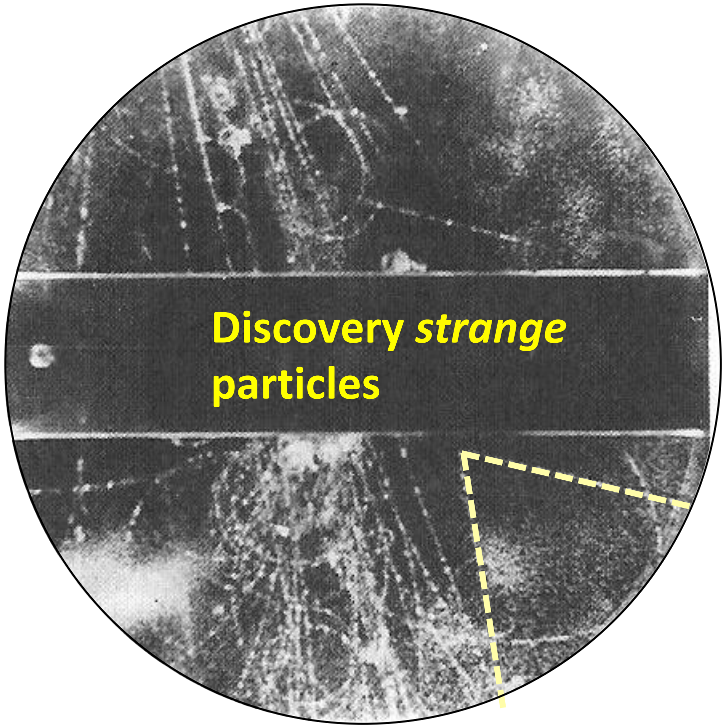
➢ So, things seemed under control!?

- Ok, the ***muon*** was a bit of a mystery…
  - Rabi: "Who ordered *that*?"



Quark model

Discovery *strange* particles



Discovery strange particles

- Why were these particles called *strange*?
- ➢ Large production cross section (10^{-27} cm^2)
- ➢ Long lifetime (corresponding to process with cross section 10^{-40} cm^2)

?

Discovery strange particles

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  - ➢ Large production cross section (10^{-27} cm^2)
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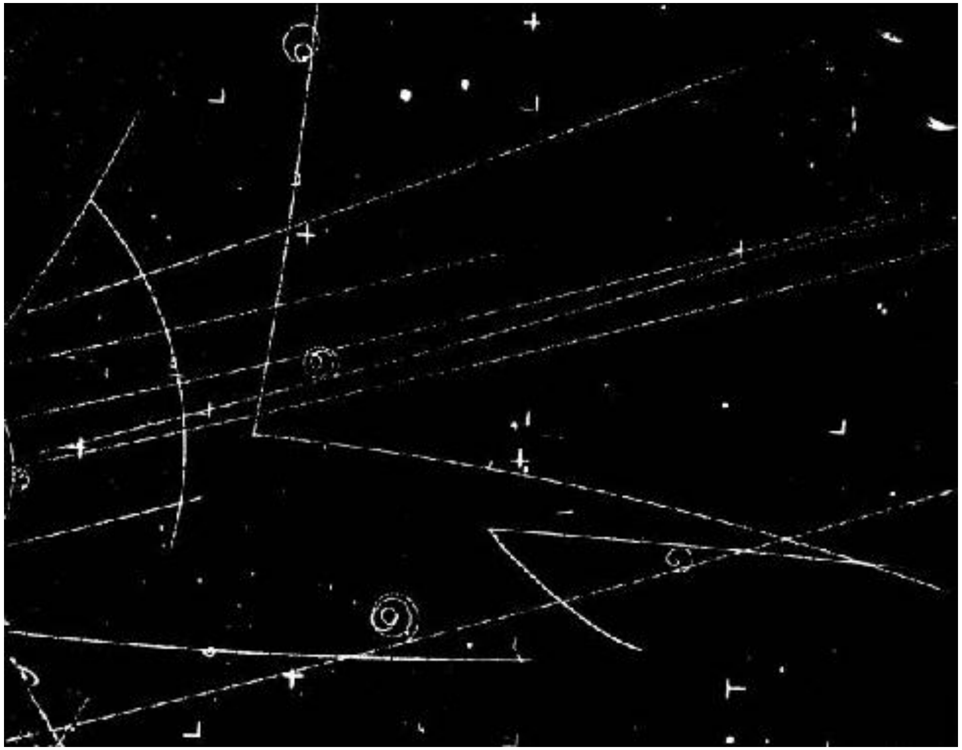
- *Associated* production!



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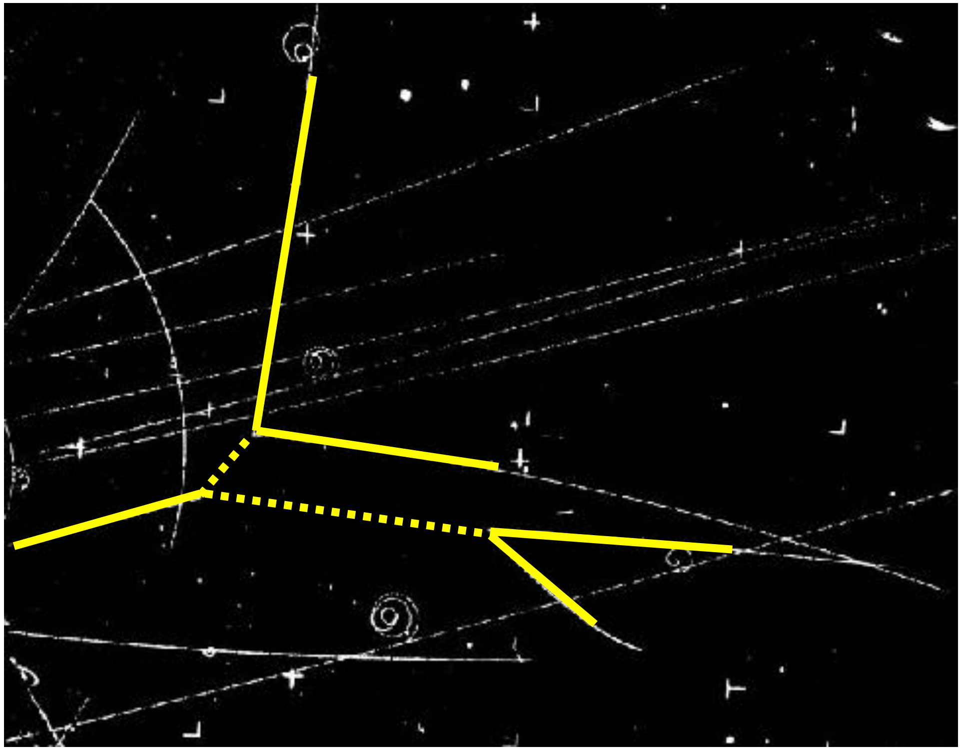

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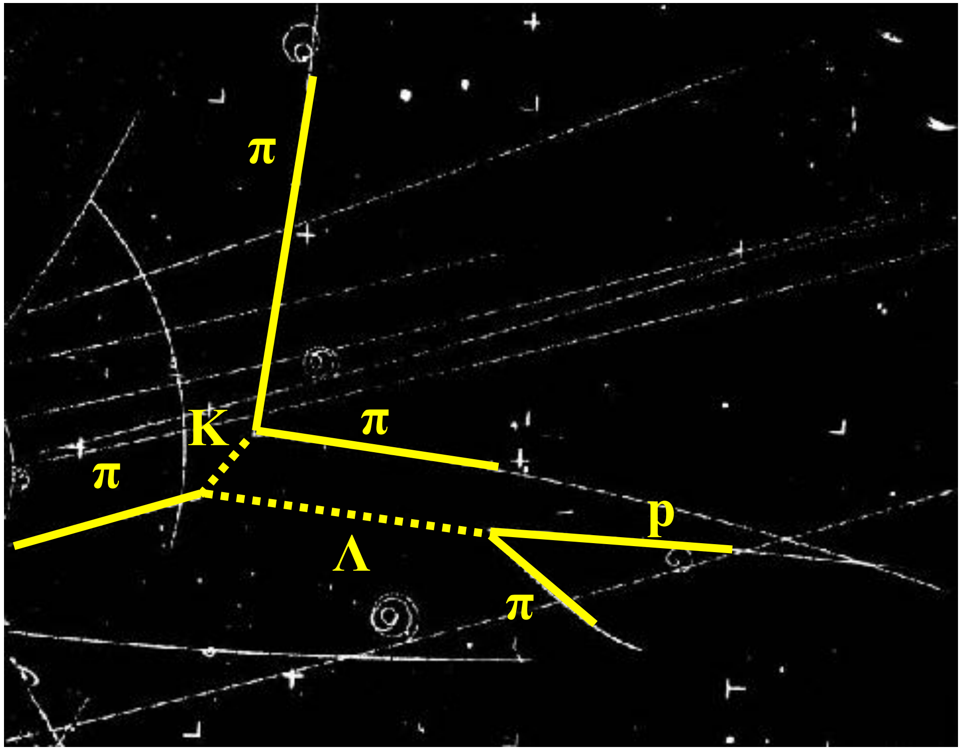
New quantum number:

- ➢ Strangeness, S
- ➢ Conserved in the strong interaction, ΔS=0
  - Particles with S=+1 and S=-1 simultaneously produced
- ➢ Not conserved in individual decay, ΔS=1



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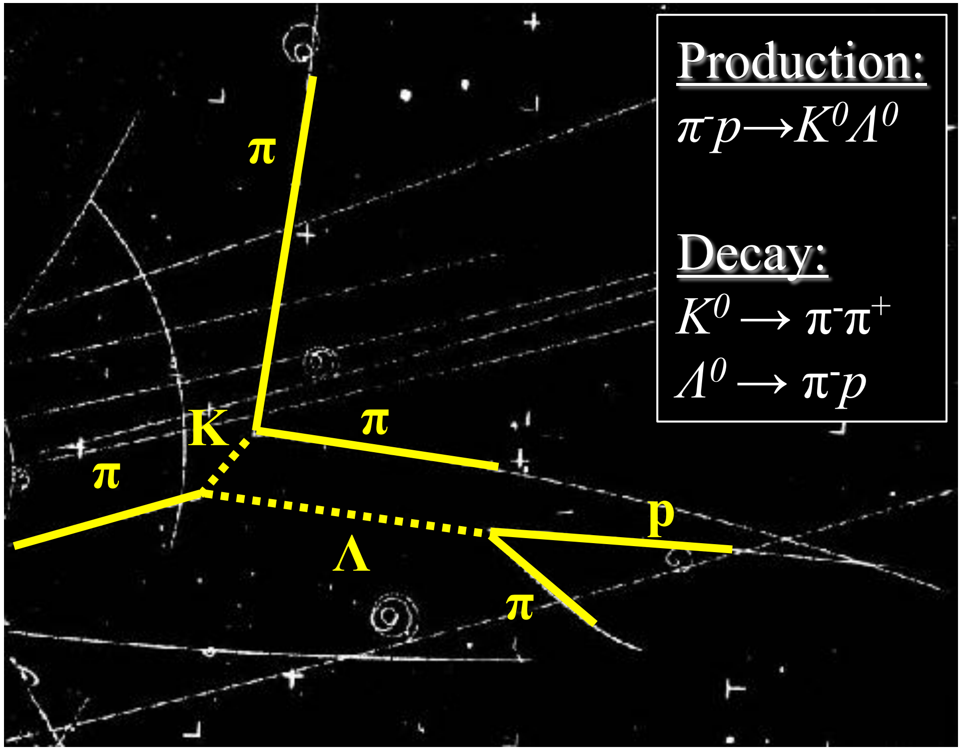
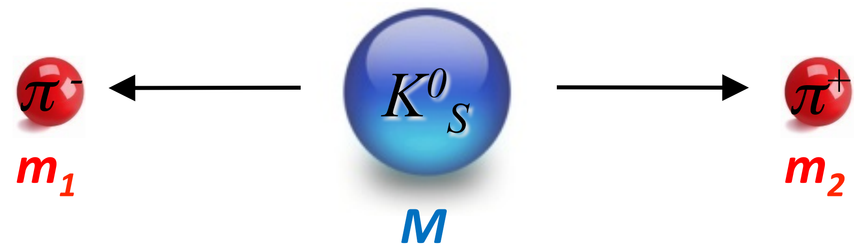


Intermezzo: conserved quantities

- What is conserved in interactions?
  - Decays & Scattering

- ➢ Energy, momentum
- ➢ Electric charge
- ➢ Total angular momentum (not just spin)
- Strangeness?
- Baryon number
- **Lepton flavour**
- Colour?
- Parity?
- CP ?
- …

Kinematics



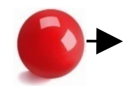
Specific (m_1=m_2=m):

before

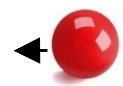
$$(E, \vec{p}) = (M, 0, 0, 0)$$

$$p_\mu p^\mu = M^2$$

after



$$(E, \vec{p}) = (\sqrt{p^2 + m^2}, 0, 0, p)$$



$$(E, \vec{p}) = (\sqrt{p^2 + m^2}, 0, 0, -p)$$

$$p_\mu p^\mu = 4(p^2 + m^2)$$

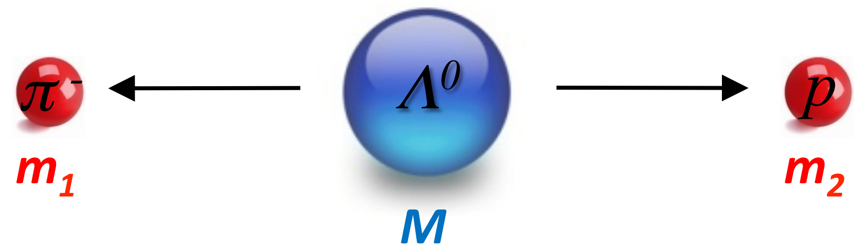
What is the energy of final-state particles?

$$M^2 = 4(p^2 + m^2) \longrightarrow p = \frac{1}{2} M \sqrt{1 - \frac{4m^2}{M^2}}$$

$$E^2 = p^2 + m^2$$

$$E = \frac{1}{2} M$$

Kinematics



Specific: ***(m_1=m_2=m)***

$$E_1 = \frac{1}{2}M \qquad p_1 = \frac{1}{2}M\sqrt{1-\frac{4m^2}{M^2}}$$

What if masses of final-state particles differ, $m_1 \neq m_2$?

General:

$$\boxed{E_{1,2} = \frac{M^2 \pm \Delta\left(m^2\right)}{2M}} \qquad \boxed{p_{1,2} = ?}$$

Strange particles

Mesons

Particle	Mass	S (Strangeness)
K^0	497.7	+1
K^+	493.6	+1
K^-	493.6	-1
$\bar{K}^0$	497.7	-1

Baryons

Particle	Mass	S (Strangeness)
Σ^+	1189.4	-1
Σ^0	1192.6	-1
Σ^-	1197.4	-1
Λ^0	1115.6	-1
Ξ^0	1314.9	-2
Ξ^-	1321.3	-2

What is different...?

Corresponding anti-baryons have positive Strangeness

50's – 60's

- Many particles discovered → 'particle zoo'

- Will Lamb:

"The finder of a new particle used to be awarded the Nobel Prize, but such a discovery now ought to be punished with a $10,000 fine."

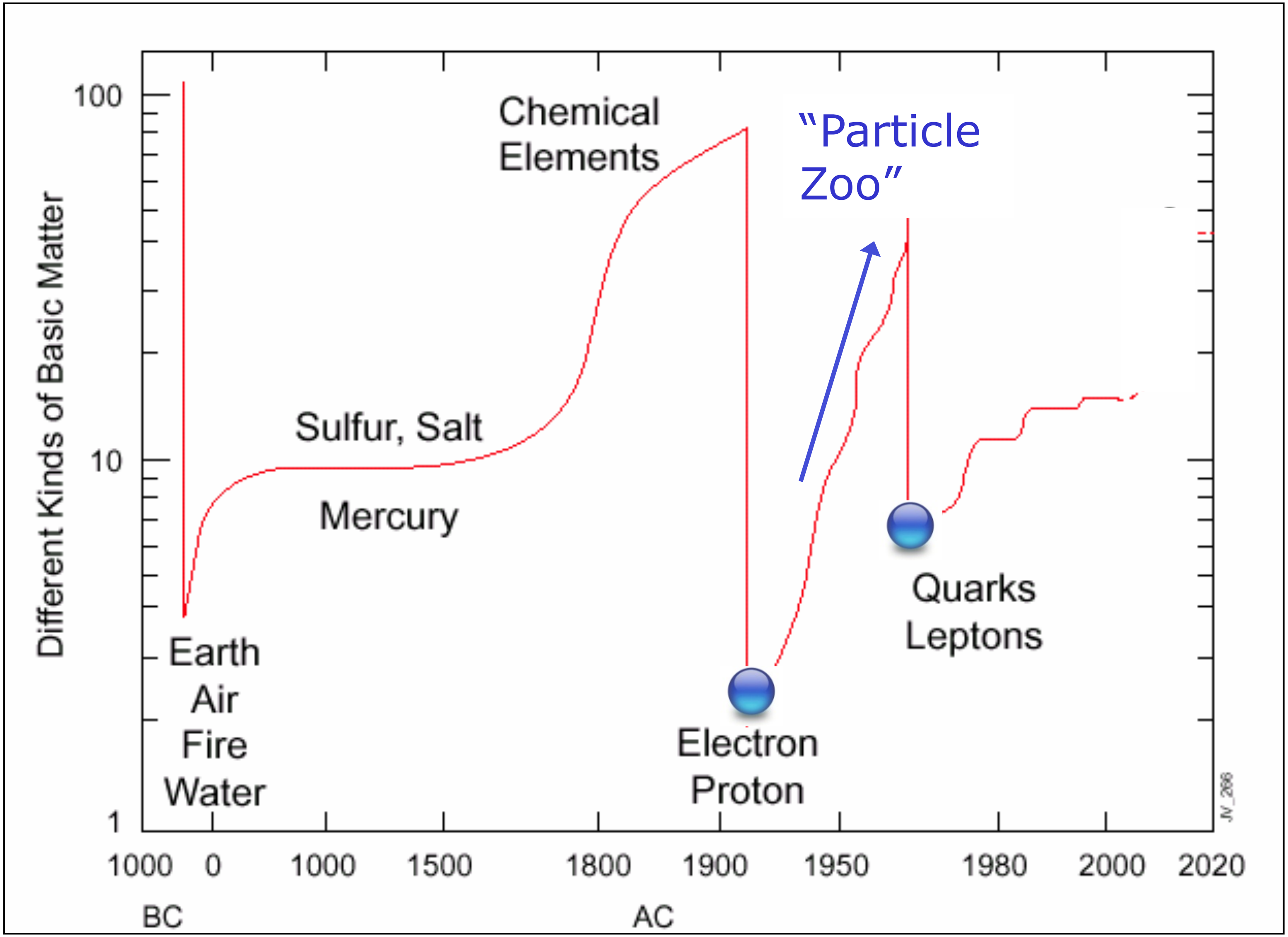
- Enrico Fermi:

"If I could remember the names of all these particles, I'd be a botanist."

- Wolfgang Pauli:

"Had I foreseen that, I would have gone into botany."

The number of 'elementary' particles



Strange particles

The 8 lightest strange baryons: baryon octet

Particle	Mass	S
n	**938.3**	**0**
p	**939.6**	**0**
Σ^+	**1189.4**	**-1**
Σ^0	**1192.6**	**-1**
Σ^-	**1197.4**	**-1**
Λ^0	**1115.6**	**-1**
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Ξ^-	**1321.3**	**-2**

Breakthrough in 1961 (Murray Gell-Mann): **"The eight-fold way"** (Nobel prize 1969)

Also works for: Eight lightest mesons - meson octet
Other baryons - baryon decuplet

Strange particles

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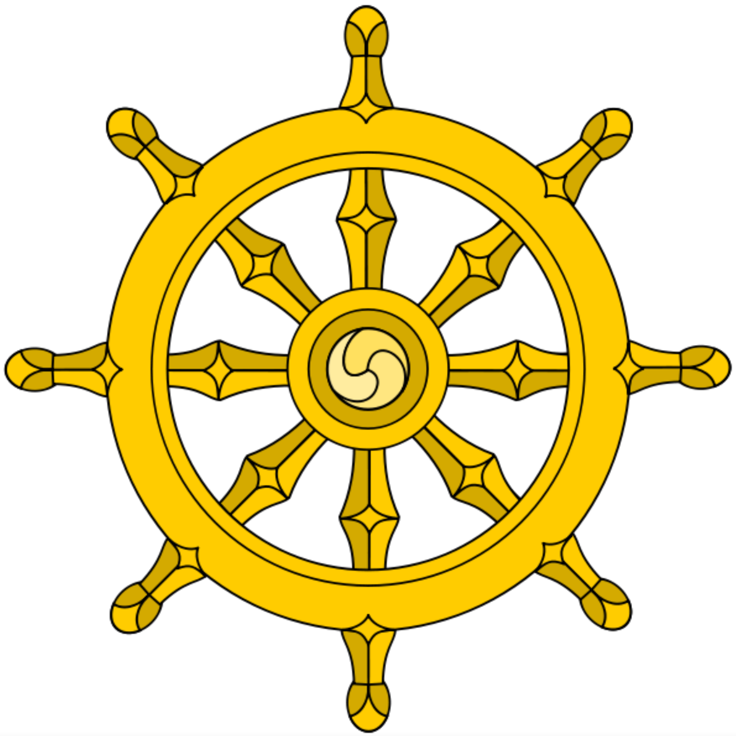
The Noble Eightfold Path is one of the principal teachings of the Buddha, who described it as the way leading to the cessation of suffering and the achievement of self-awakening.



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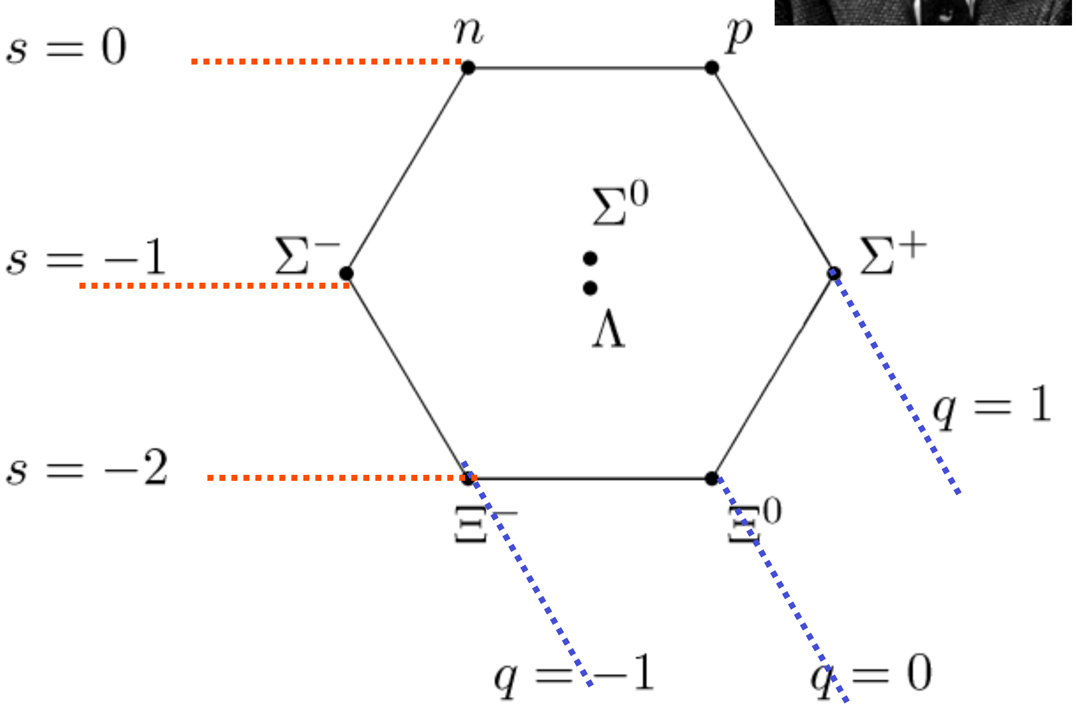

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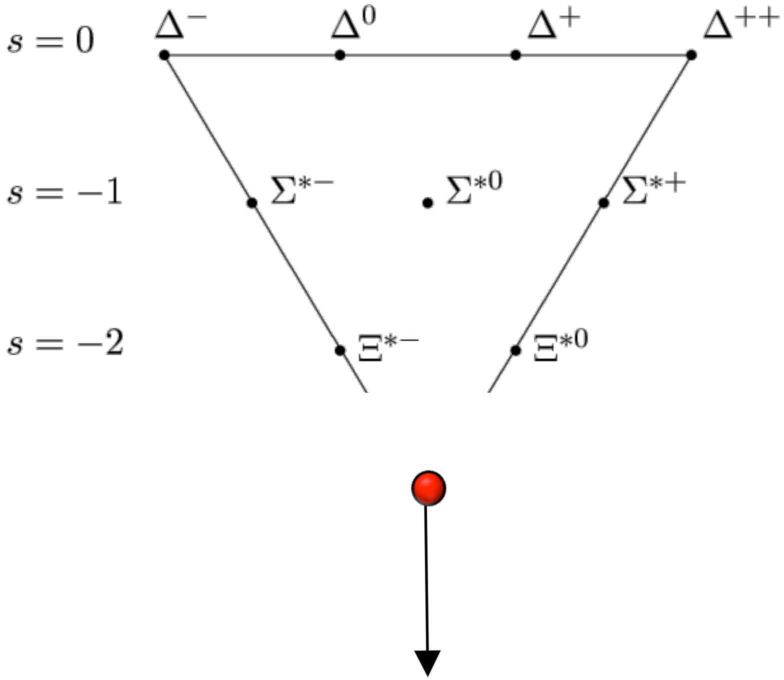
Discovery of Ω⁻

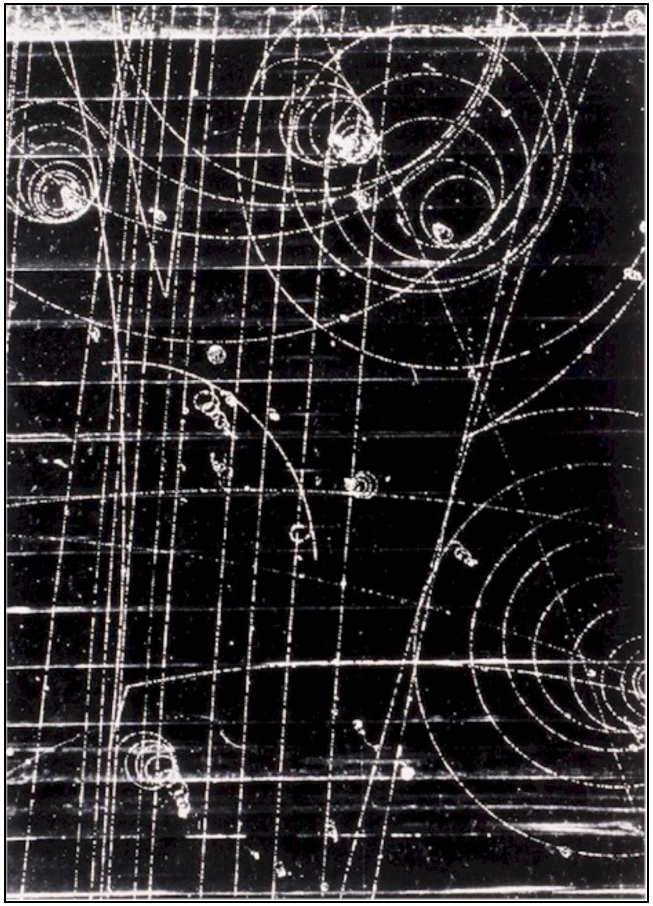
Not all multiplets complete…



1232 MeV

1385 MeV

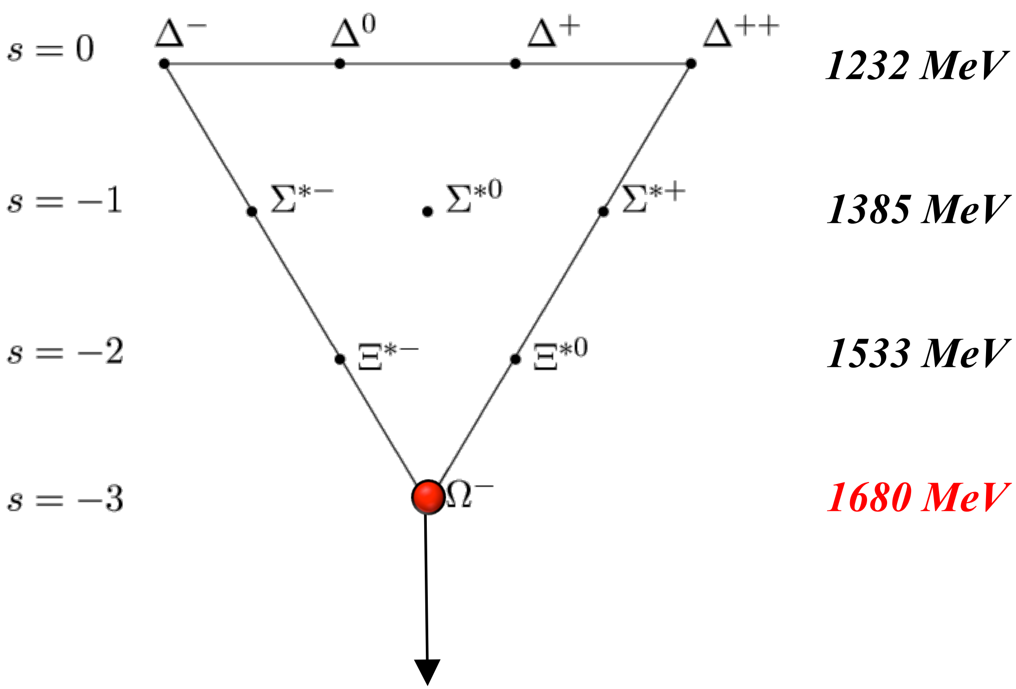
1533 MeV

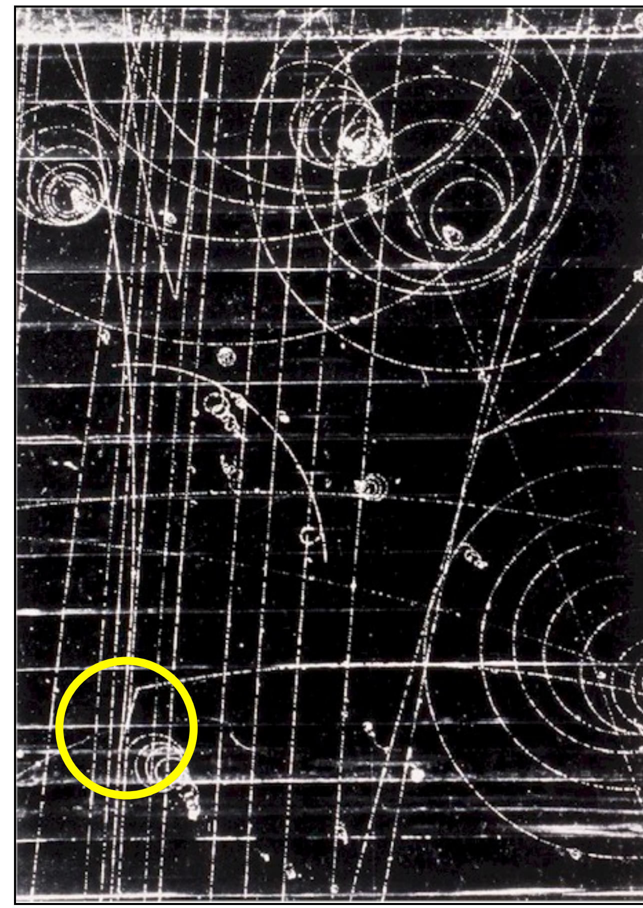


Gell-Mann and Zweig predicted the Ω⁻
… and its properties

Discovery of Ω^-

Not all multiplets complete…





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… and its properties

Discovery of Ω⁻

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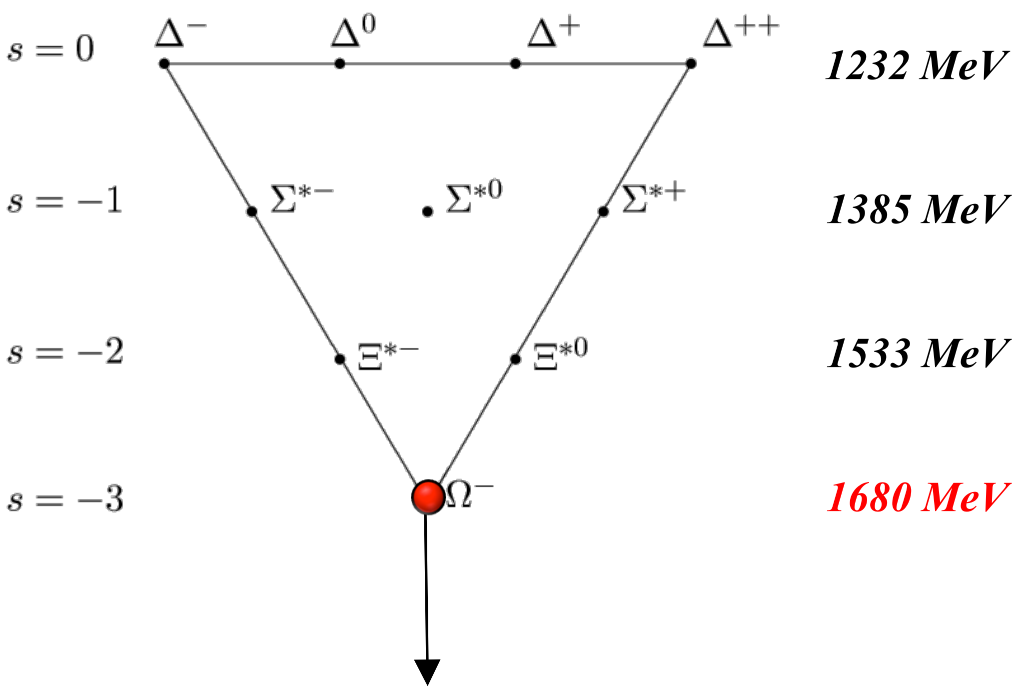
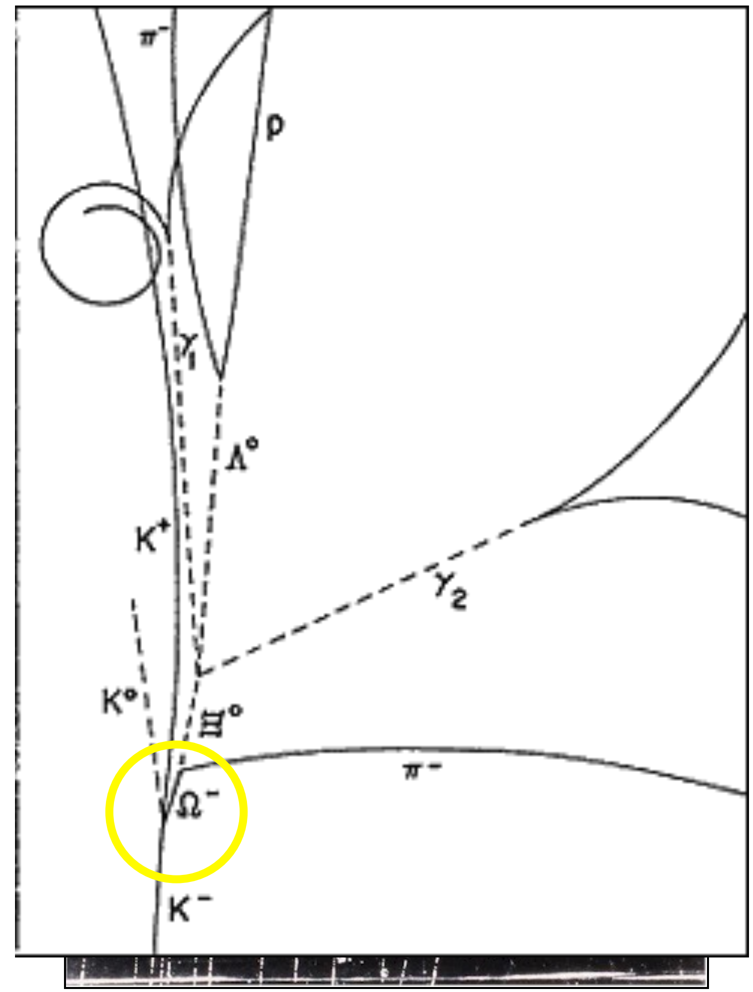


Gell-Mann and Zweig predicted the Ω⁻
… and its properties

Discovered in 1964:

$K^- + p \rightarrow \Omega^- + K^+ + K^0$



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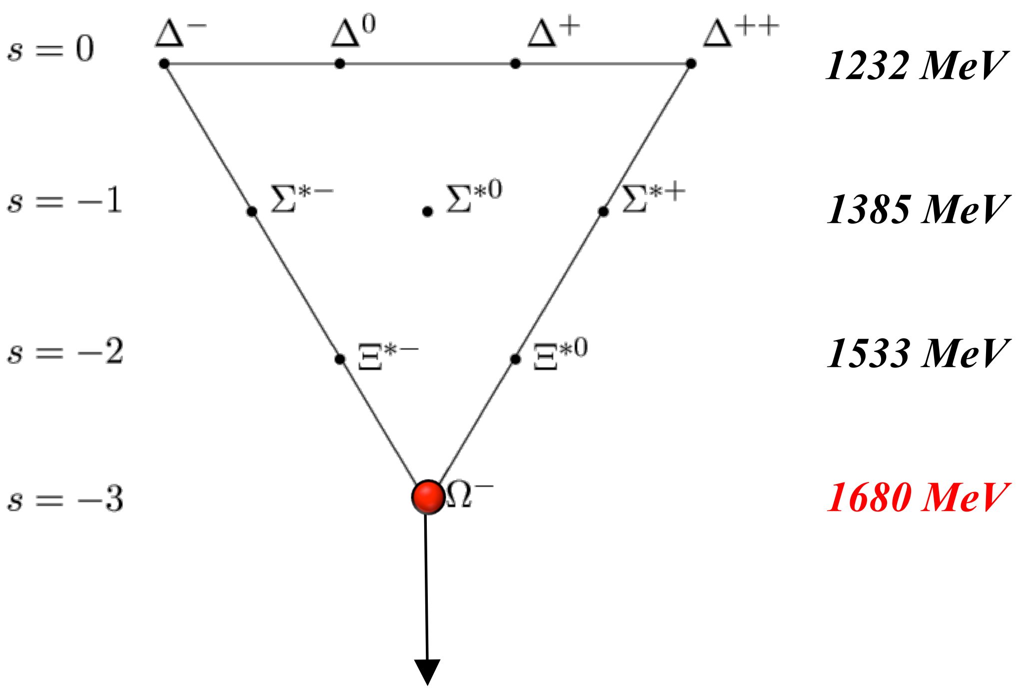
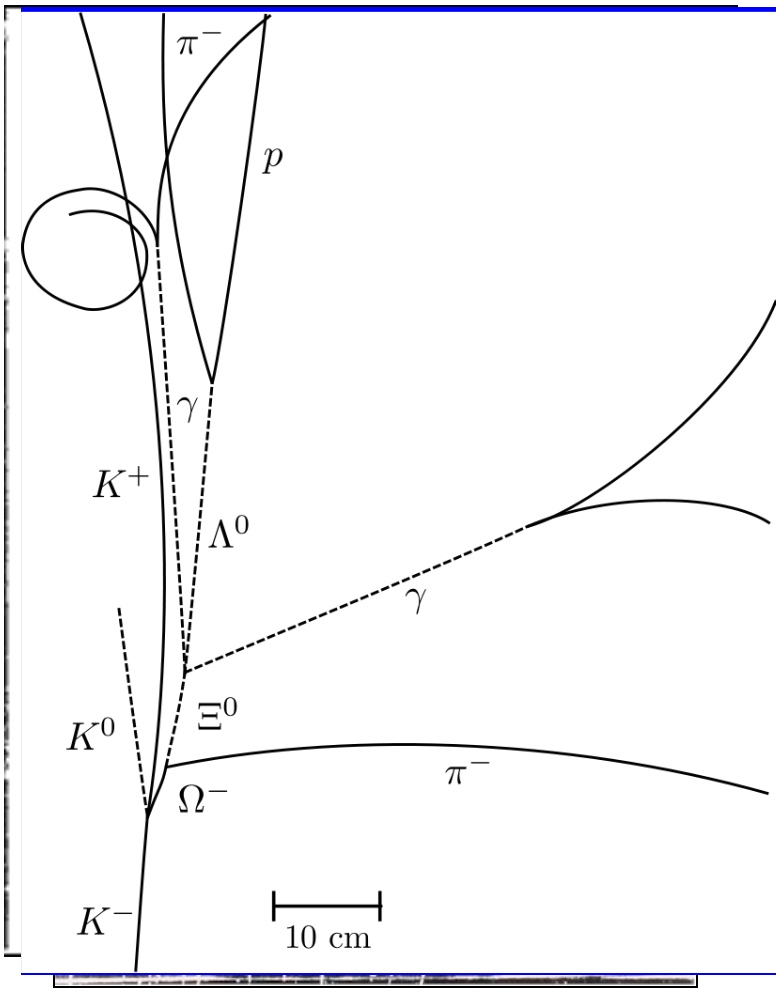




Gell-Mann and Zweig predicted the Ω⁻
… and its properties

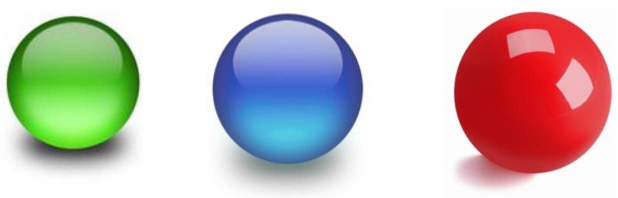
Quark model

Gell-Mann en Zweig (1964):

"All multiplet patterns can be explained if you assume hadrons are composite particles built from more elementary constituents: quarks"

- First quark model:
  - 3 types: up, down en strange (and anti-quarks)
  - Baryons: 3 quarks
  - Mesons: 2 quarks

26 → 3+3



up ***down*** ***strange***

$q = +\frac{2}{3}$ (up)

$q = -\frac{1}{3}$ (down, strange)

mesonen

$\pi^+ = u\bar{d}$ $K^+ = u\bar{s}$

$\pi^- = \bar{u}d$ $K^0 = d\bar{s}$

baryonen

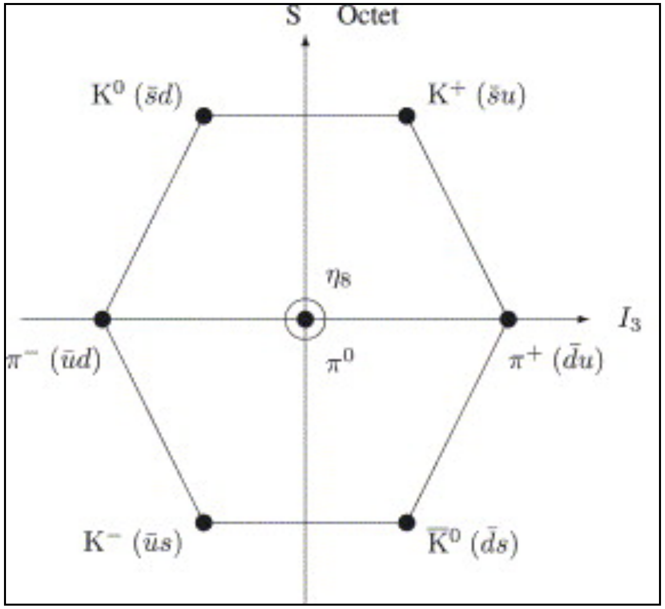
$p = uud$ $n = udd$

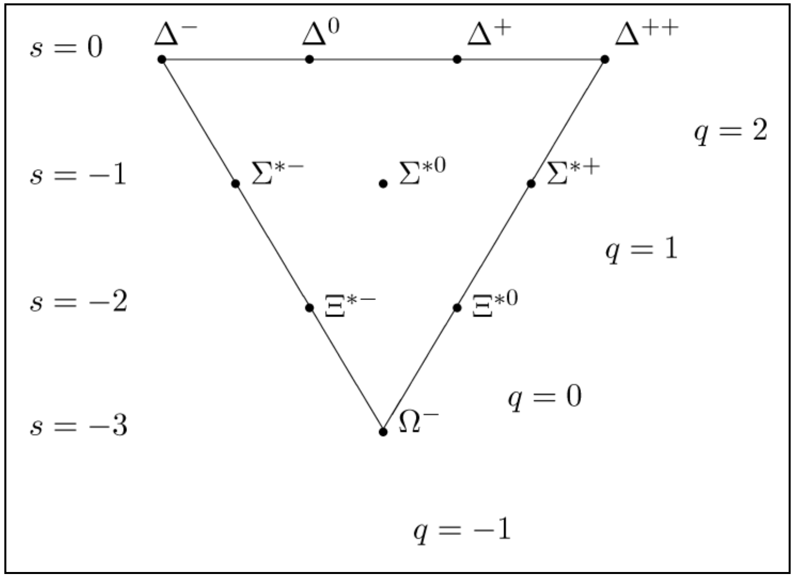
$\Sigma^+ = uus$ $\Lambda^0 = uds$

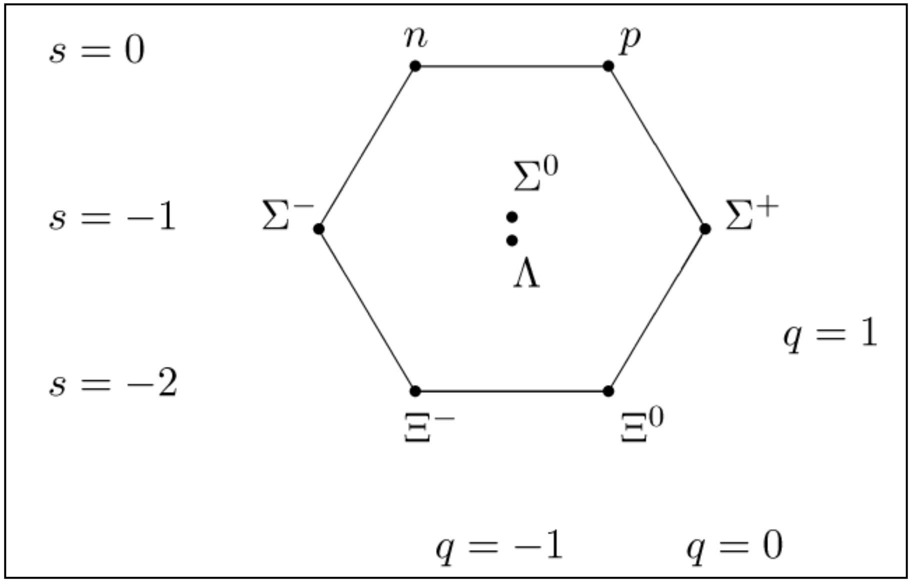
$\Xi^0 = uss$

Quark model

- Mesons:
  - Octet
- Baryons:
  - Octet
  - Decuplet



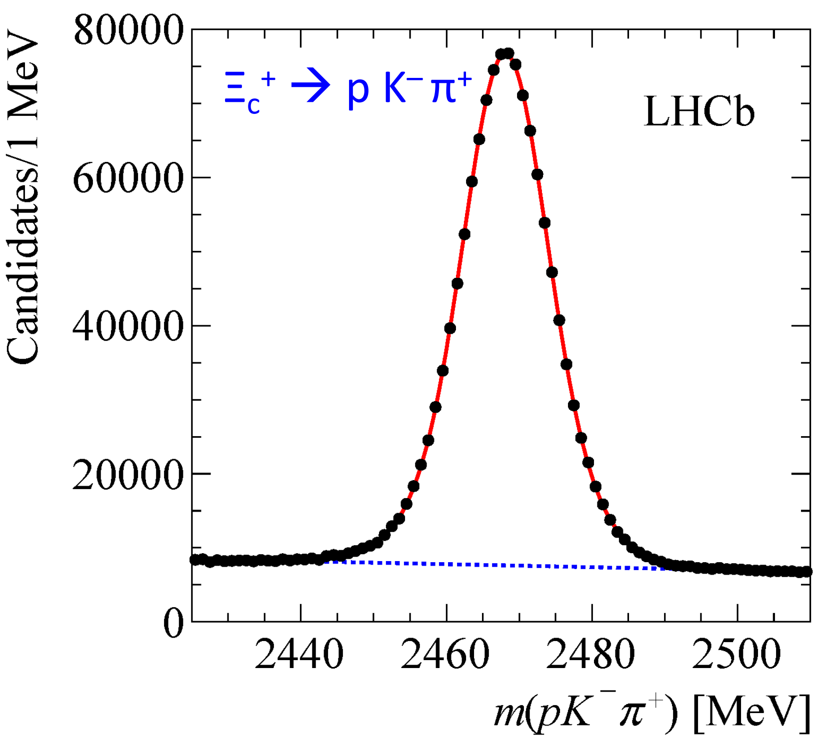




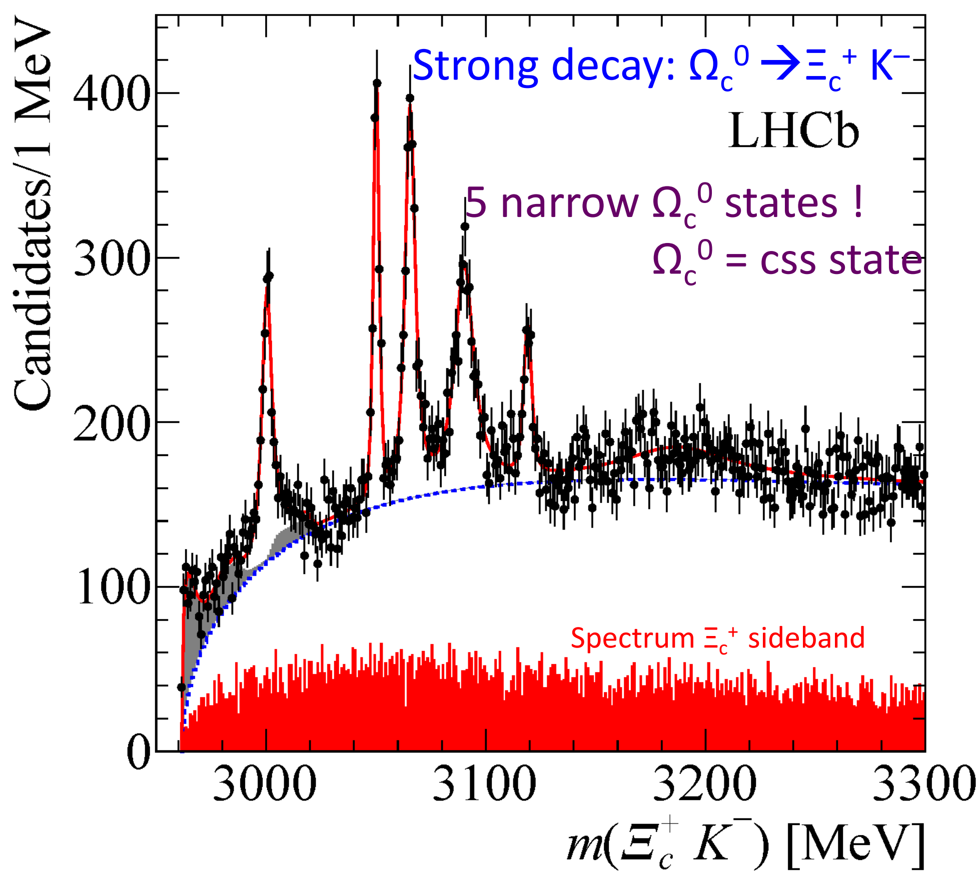
New last year: Ω_c^0 (css)

- Just discovered 5 excited (ccs) states
- Still active research!

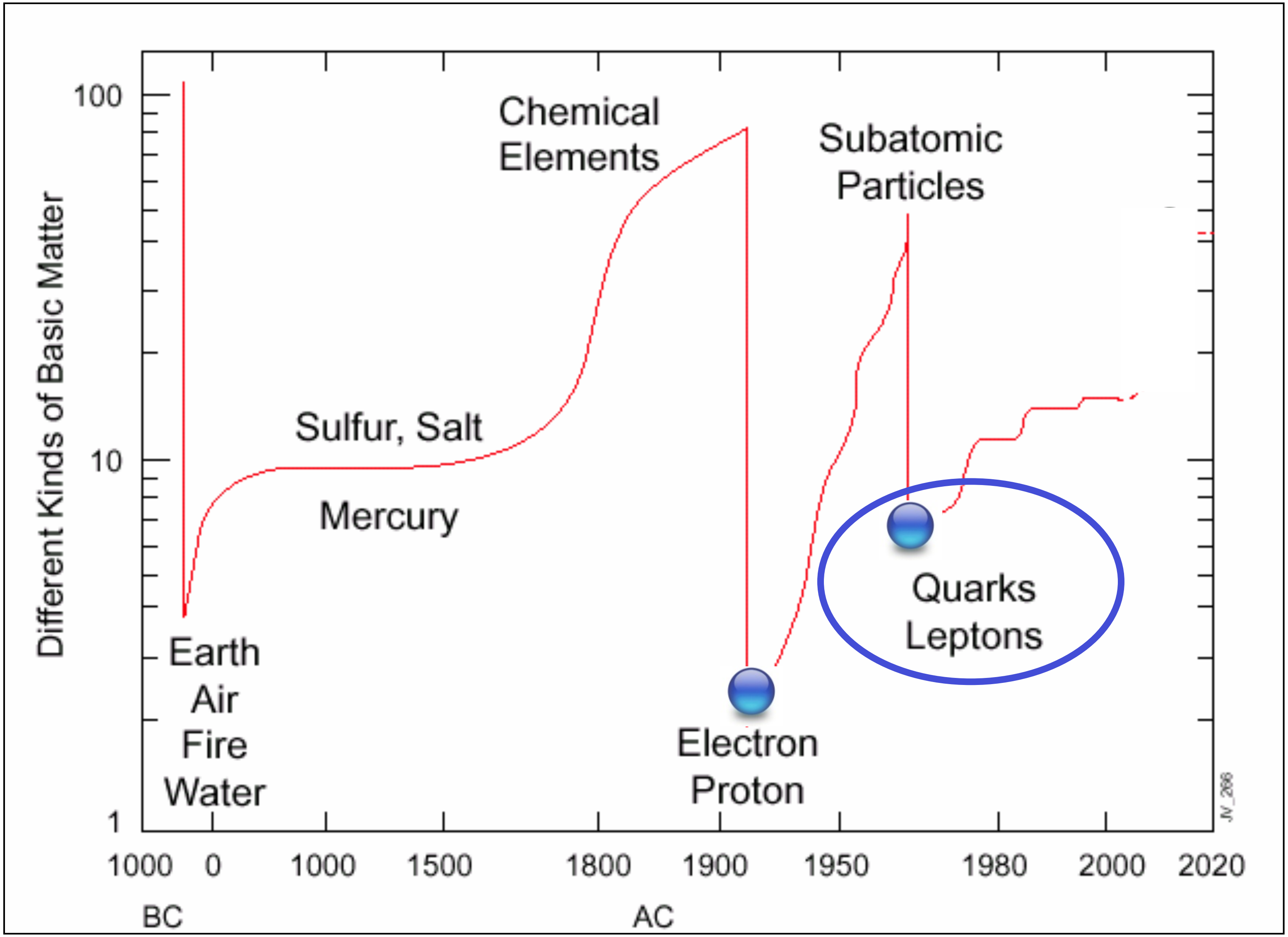
1. Reconstruct Ξ_c^+ = csu state



2. Combine Ξ_c^+ with K^- :



The number of ‘elementary’ particles

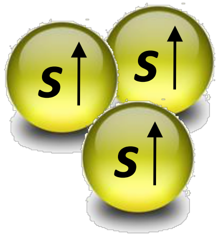


“Problems”

1) Are quarks ‘real’ or a mathematical tric?
2) How can a baryon exist, like Δ^{++} with ($u\uparrow u\uparrow u\uparrow$), given the Pauli exclusion principle?

"Problem" of quark model

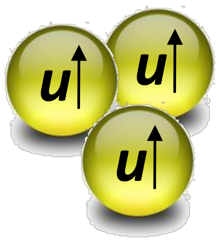
Ω^-



Intrinsic spin: $|\frac{3}{2},+\frac{3}{2}\rangle = \uparrow\uparrow\uparrow$ = symmetric

quarks: $|sss\rangle$ = symmetric

Δ^{++}



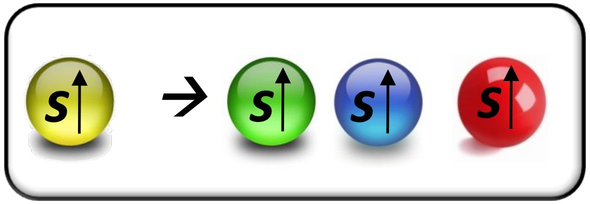
Intrinsic spin: $|\frac{3}{2},+\frac{3}{2}\rangle = \uparrow\uparrow\uparrow$ = symmetric

quarks: $|uuu\rangle$ = symmetric

J=3/2, ie. fermion, ie. obey Fermi-Dirac statistics: anti-symmetric wavefunction

New quantum number: color!

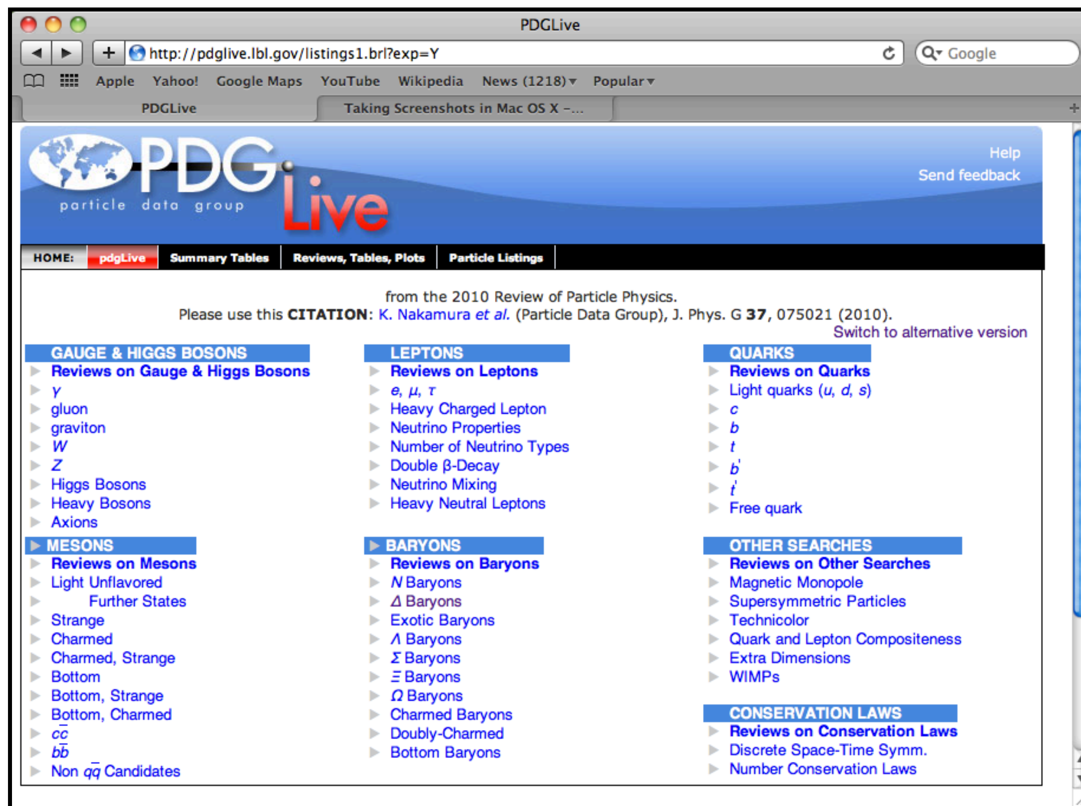
- ***3 values: red, green, blue***
- ***Only quarks, not the leptons***



The Particle Zoo

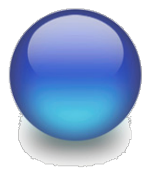
	mass
Force carier: γ	$<1\text{x }10^{-18}$ eV
Leptons: $e^-, \mu^-, \tau^-, \upsilon_e, \upsilon_\mu, \upsilon_\tau$	~0 – 1.8 GeV
Mesons: $\pi^+, \pi^0, \pi^-, K^+, K^-, K^0, \rho^+, \rho^0, \rho^-$	0.1-1 GeV
Baryons: $p, n, \Lambda, \Sigma^+, \Sigma^-, \Sigma^0, \Delta^{++}, \Delta^+, \Delta^0, \Delta^-, \Omega, \ldots$	1-few GeV

http://pdg.lbl.gov/



Protons and neutrons

Proton and neutron *identical* under strong interaction



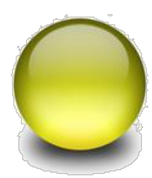
proton

$m_p = 938.272$ ***MeV***

neutron

$m_n = 939.565$ ***MeV***

?

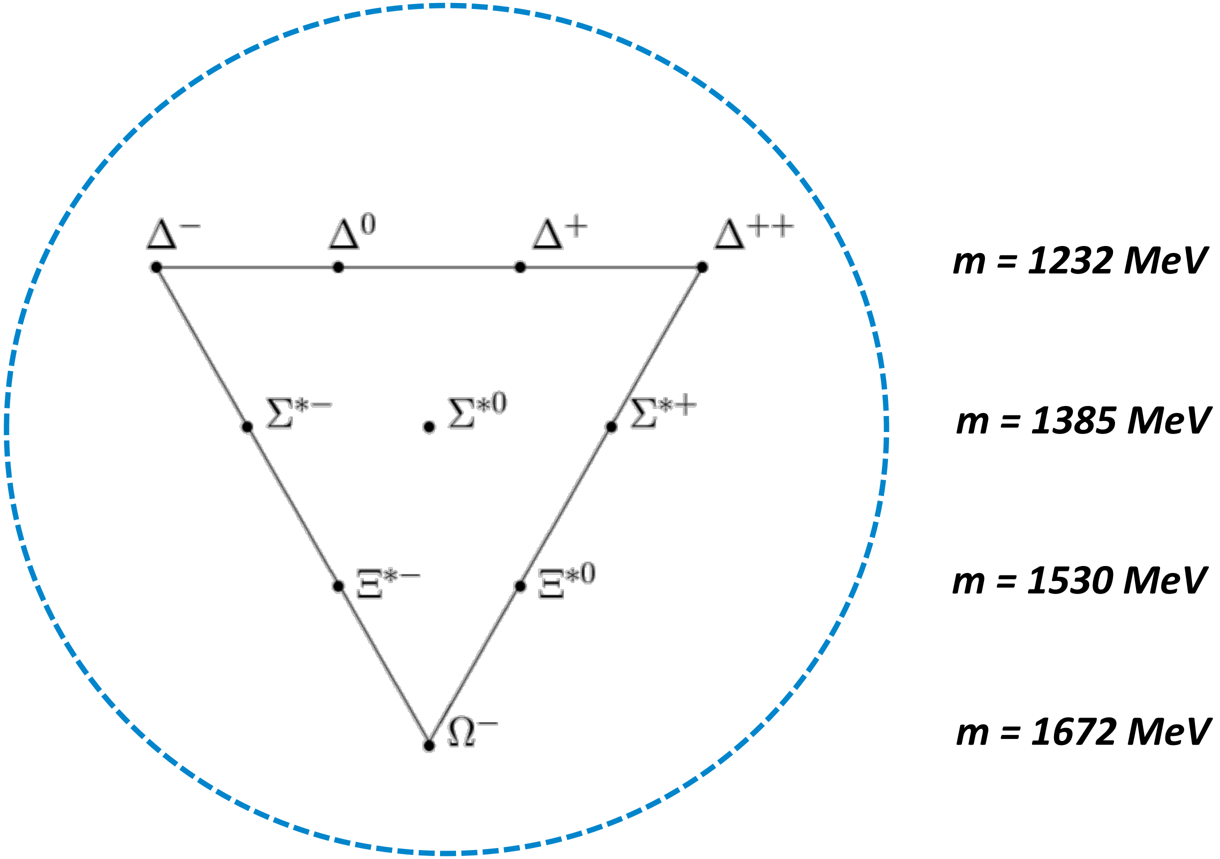


Nucleon

+ internal degree of freedom
to distinguish the two

Multiplets

Pattern (mass degeneracy) suggest internal degree of freedom



Baryon decuplet

Eightfold way

- Introduction of quarks
- Introduction of quantum numbers
  - Strangeness
  - Isospin

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

Figure 1.1: *Murray Gell-Mann suggested in 1964 that the proton consists of three "quarks"* [6] *[1].*

M. Gell-Mann, *A schematic model of baryons and mesons.* Phys. Lett. **8**, 214 (1964).

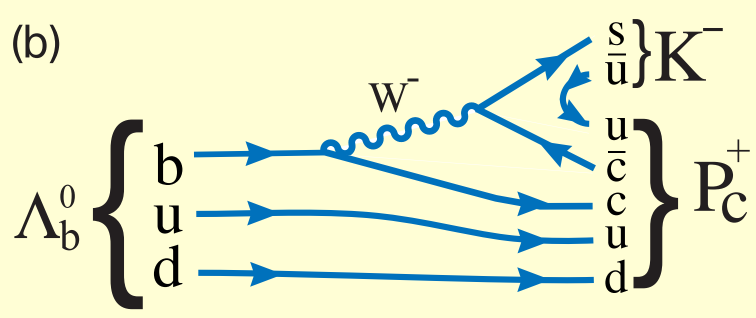
	d	u	s
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0
S – strangeness	0	0	-1

Tetra- and pentaquarks ??

- Tetraquark discovered in 2003
  - X(3872)
  - Also *charged* cc and bb states…

$B^+ \to X(3872)K^+$ decays, where $X(3872) \to \pi^+\pi^- J/\psi$ and $J/\psi \to \mu^+\mu^-$.

- Pentaquark discovered in 2016
  - $P_c^+(4450)$

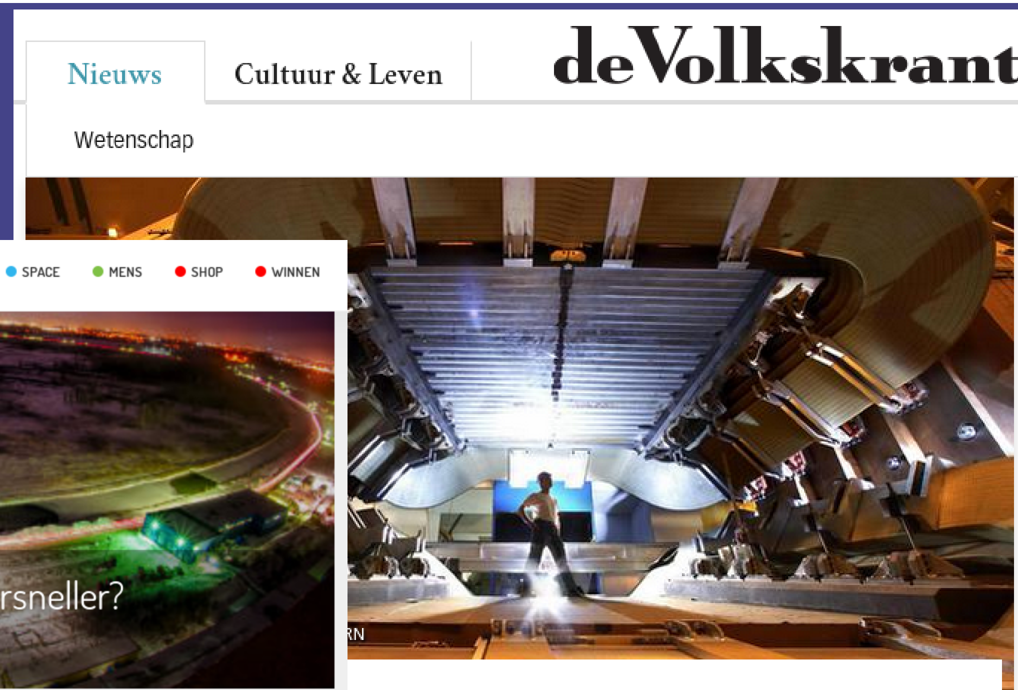


$\Lambda_b^0 \to P_c^+ K^-$, $P_c^+ \to \psi p$, $\psi \to \mu^+\mu^-$ decay sequence

Timeline

- Active research…:

In the News



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Wetenschap

CERN ontdekt familie vreemde zware deeltjes

KIJK

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Mogelijk kan het nieuwe deeltje worden bevestigd door LHCb, een van de vier grote experimenten rond de deeltjesversneller LHC bij Genève. "We gaan er ook zeker naar zoeken", zegt natuurkundige Patrick Koppenburg, een van de tetra- en pentaquark-experts van het LHCb-team. "Maar onder andere doordat wij naar een andere energie kijken, is er geen garantie dat wij het deeltje kunnen zien." Maar áls LHCb het deeltje ziet, zal ook gelijk kunnen worden bepaald wat precies de quark-samenstelling is, zegt Koppenburg.

Nieuw deeltje ontdekt dankzij afgedankte versneller?

JEAN-PAUL KEULEN - 29 FEBRUARI 2016

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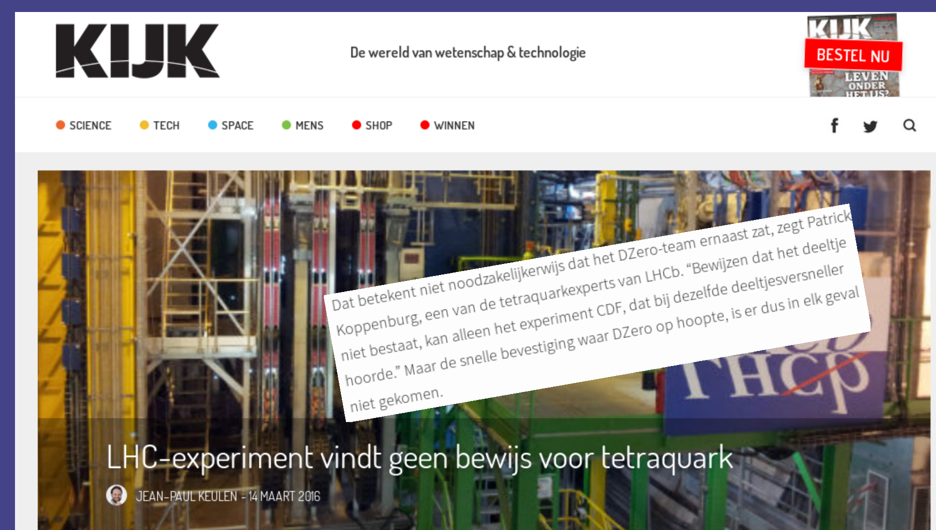
LHC ontdekt een nieuwe klasse van deeltjes: de pentaquark

"Het is een hele geschiedenis met de pentaquarks, wat de reden is waarom we zeer voorzichtig zijn geweest voor we deze studie openbaar hebben gemaakt", zei Patrick Koppenburg, een van de coördinatoren van het LHCb aan de BBC. "Het lijkt of het woord 'pentaquark' op een of andere manier vervloekt is, aangezien er veel ontdekkingen zijn geweest die dan te niet gedaan werden door nieuwe resultaten, die aantoonden dat de vorige in werkelijkheid fluctuaties waren en geen echte aanwijzingen."

Een pentaquark, met 5 sterk verbonden quarks (illustratie: LHCb/Daniel Dominguez).



KIJK

De wereld van wetenschap & technologie

BESTEL NU

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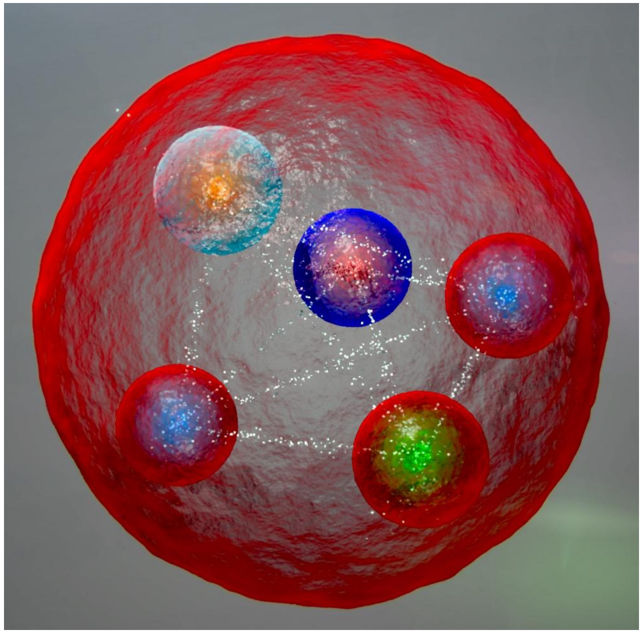
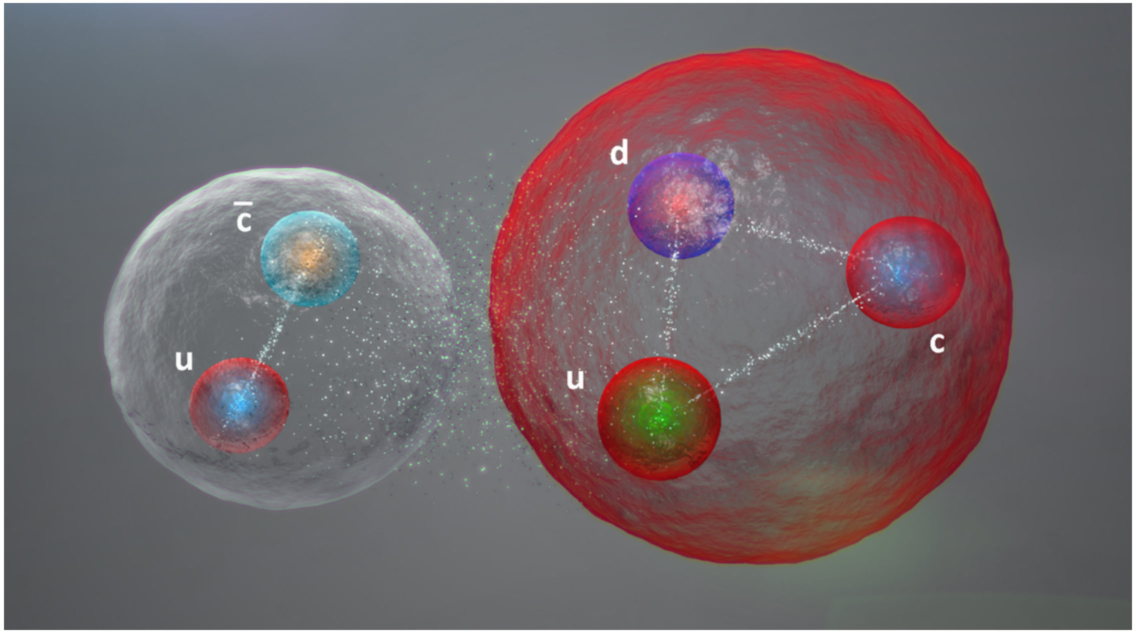
Dat betekent niet noodzakelijkerwijs dat het DZero-team ernaast zat, zegt Patrick Koppenburg, een van de tetraquarkexperts van LHCb. "Bewijzen dat het deeltje niet bestaat, kan alleen het experiment CDF, dat bij dezelfde deeltjesversneller hoorde." Maar de snelle bevestiging waar DZero op hoopte, is er dus in elk geval niet gekomen.

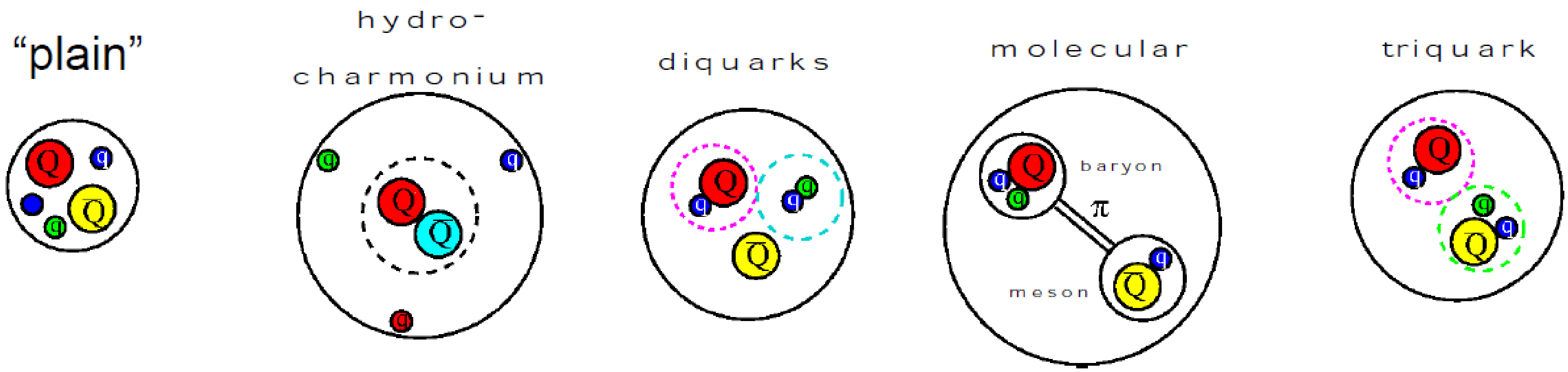
LHC-experiment vindt geen bewijs voor tetraquark

JEAN-PAUL KEULEN - 14 MAART 2016

WHAT IS A PENTAQUARK?







>300 papers citing the result, with many possible interpretations.

Symmetries

Conserved quantities

Time dependence of observable U:

Hamilton formalism:

$$i\hbar \frac{\mathrm{d}}{\mathrm{dt}}\Psi = H\Psi$$

$$\begin{aligned}
\frac{d}{dt}\langle U\rangle &= \frac{d}{dt}\langle\Psi|U|\Psi\rangle \\
&= \langle\frac{\partial\Psi}{\partial t}|U|\Psi\rangle + \langle\Psi|\frac{\partial U}{\partial t}|\Psi\rangle + \langle\Psi|U|\frac{\partial\Psi}{\partial t}\rangle \\
&= -\frac{1}{i\hbar}\langle H\Psi|U|\Psi\rangle + \langle\Psi|\frac{\partial U}{\partial t}|\Psi\rangle + \frac{1}{i\hbar}\langle\Psi|U|H\Psi\rangle \\
&= \frac{1}{i\hbar}\langle[U,H]\rangle + \langle\Psi|\frac{\partial U}{\partial t}|\Psi\rangle
\end{aligned}$$

If *U* commutes with *H*, *[U,H]=0*
(and if *U* does not depend on time, dU/dt=0)

Then U is conserved: d/dt<U> = 0

U conserved → U generates a symmetry of the system

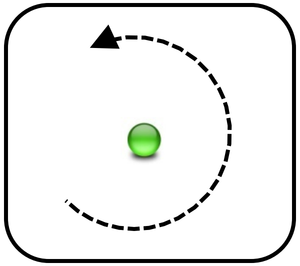
Other symmetries:

Transformation	**Conserved quantity**
Translation (space)	Momentum
Translation (time)	Energy
Rotation (space)	Orbital momentum
Rotation (iso-spin)	Iso-spin

Quantum mechanics: orbital momentum

$$L = \vec{r} \times \vec{p} = -i\hbar(\vec{r} \times \vec{\nabla})$$

$$\begin{aligned} L_x &= -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right) = yp_z - zp_y \\ L_y &= -i\hbar\left(z\frac{\partial}{\partial y} - x\frac{\partial}{\partial z}\right) = zp_x - xp_z \\ L_z &= -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) = xp_y - yp_x \end{aligned}$$



$$[L_x, L_y] = i\hbar L_z$$

L_x and L_y cannot be known simultaneously
Sequence matters!

$$[x, p_x] = i\hbar$$

$$[L^2, L_i] = 0$$

L^2 and L_i (i=x,y,z) can be known simultaneously
Can both be used to label states

$$[L^2, H] = [L_z, H] = 0$$

Provided V = V(r), ie not θ dependent
L^2 and L_z label eigenstates

Quantum mechanics: orbital momentum

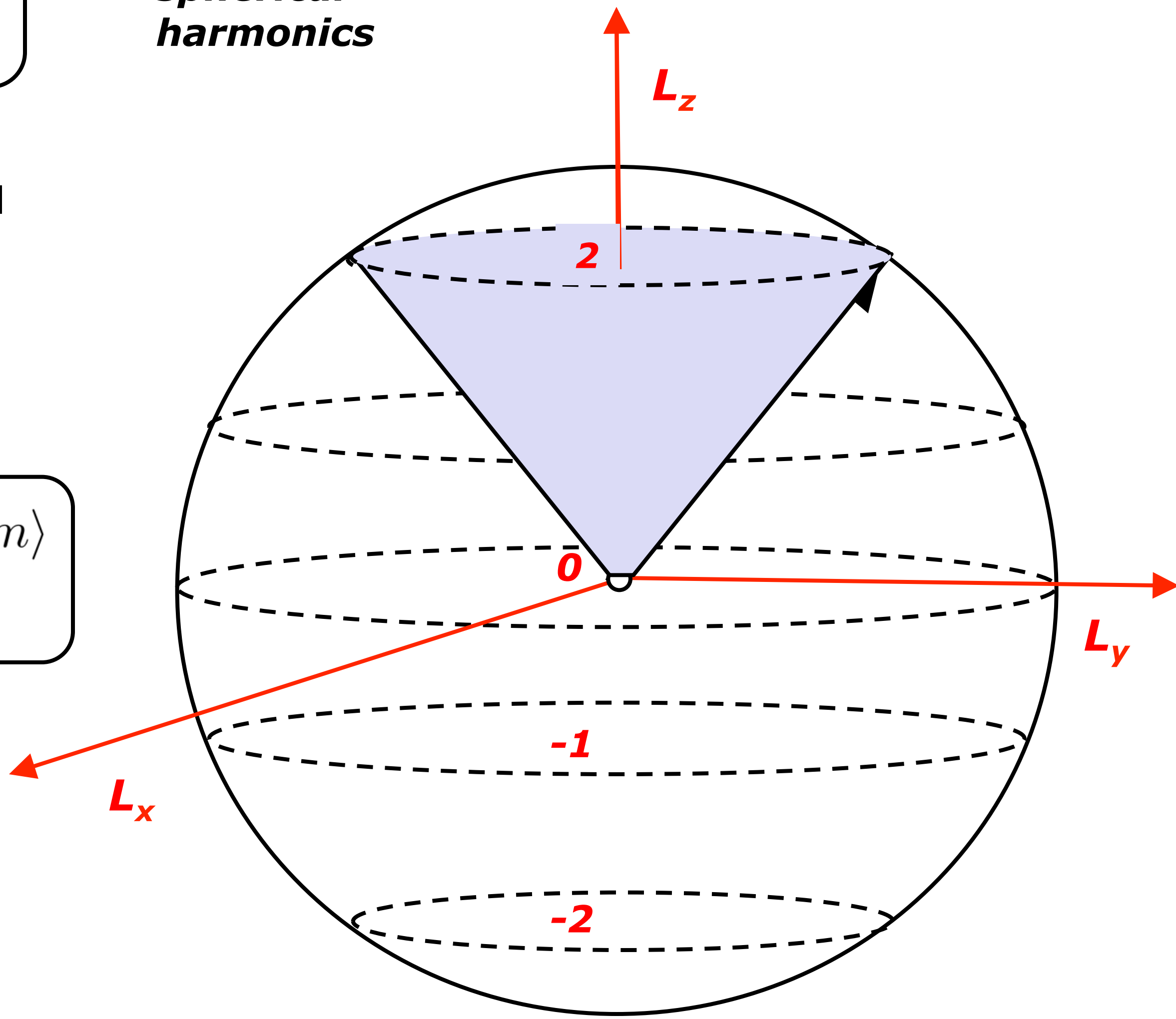
$$\begin{aligned} L^2 f_l^m &= \hbar^2 l(l+1) f_l^m \\ L_z f_l^m &= \hbar m f_l^m \end{aligned}$$

$f_l^m = Y_l^m$ spherical harmonics

m = -l, -l+1, …, 0, … , l-1, l

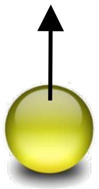
Different notation:

$$\begin{aligned} L^2 |l,m\rangle &= \hbar^2 l(l+1) |l,m\rangle \\ L_z |l,m\rangle &= \hbar m |l,m\rangle \end{aligned}$$



Quantum mechanics: (intrinsic) spin

Spin is characterized by: - total spin S
- spin projection S_z



Rotations: ***SO(3) group***
Internal symmetry: ***SU(2) group***
} ***similar***

$[S_x, S_y] = i\hbar S_z$ $[S^2, S_i] = 0$

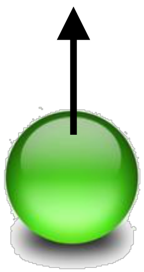
Spin is quantized,
just as orbital momentum

$$S = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$$
$$S_z = -S, -S+1, \ldots, S-1, S$$

Eigenfunctions $|s, m_s>$:

$$S^2 |s, m_s\rangle = \hbar^2 s(s+1) |s, m_s\rangle$$
$$S_z |s, m_s\rangle = \hbar m_s |s, m_s\rangle$$

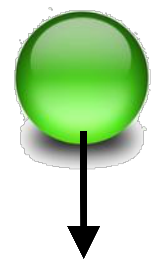
spin-½ particles



$$\begin{pmatrix}1\\0\end{pmatrix}$$

spin-up

$$|\uparrow\rangle = |\frac{1}{2}, +\frac{1}{2}\rangle$$



$$\begin{pmatrix}0\\1\end{pmatrix}$$

spin-down

$$|\downarrow\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle$$

spin-½ particles

Complex numbers

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \alpha\begin{pmatrix}1\\ 0\end{pmatrix} + \beta\begin{pmatrix}0\\ 1\end{pmatrix}$$

general

$|\alpha|^2$ prob for $S_z = +\frac{1}{2}\hbar$

$|\beta|^2$ prob for $S_z = -\frac{1}{2}\hbar$

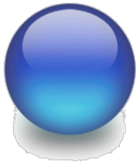
$$S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad S_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \qquad S_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Pauli matrices: any complex 2x2 matrix can be written as: $A = a\sigma_1 + b\sigma_2 + c\sigma_3$

Isospin

Protons and neutrons

Proton and neutron ***identical*** under strong interaction



proton ***neutron***

m_p = 938.272 MeV ***m_n = 939.565 MeV***

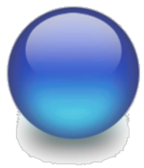
?

Nucleon

\+ internal degree of freedom
to distinguish the two

Protons and neutrons: Isospin

Proton and neutron *identical* under strong interaction



proton

m_p ***= 938.272 MeV***

neutron

m_n ***= 939.565 MeV***

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

Nucleon

+ internal degree of freedom

Introduce new quantum number: isospin

Proton and neutron ('nucleons'): I en I_3

$$p = |\frac{1}{2}, +\frac{1}{2}\rangle \text{ en } n = |\frac{1}{2}, -\frac{1}{2}\rangle$$

Isospin 'up' ***Isospin 'down'***

Possible states for given value of the Isospin

$$I = 0, \frac{1}{2}, 1, \frac{3}{2}, ...$$
$$I_z = -I, -I+1, ..., I-1, I$$

$$\mathbf{I_z = \frac{1}{2}}$$

$$\mathbf{I_z = 1}$$

$$\mathbf{I_z = \frac{3}{2}}$$

Possible states for given value of the Isospin

$$
\begin{aligned}
I &= 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots \\
I_z &= -I, -I+1, \ldots, I-1, I
\end{aligned}
$$

$$\mathbf{I_z = \frac{1}{2}} \begin{cases} I_z = +1/2 \\ I_z = -1/2 \end{cases}$$

$$\mathbf{I_z = 1} \begin{cases} I_z = +1 \\ I_z = 0 \\ I_z = -1 \end{cases}$$

$$\mathbf{I_z = \frac{3}{2}} \begin{cases} I_z = +3/2 \\ I_z = +1/2 \\ I_z = -1/2 \\ I_z = -3/2 \end{cases}$$

Possible states for given value of the Isospin

$$I = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$$
$$I_z = -I, -I+1, \ldots, I-1, I$$

$\mathbf{I_z = \frac{1}{2}}$
- $I_z = +1/2$
- $I_z = -1/2$

$\mathbf{I_z = 1}$
- $I_z = +1$
- $I_z = 0$
- $I_z = -1$

$\mathbf{I_z = \frac{3}{2}}$
- $I_z = +3/2$
- $I_z = +1/2$
- $I_z = -1/2$
- $I_z = -3/2$

proton $|\frac{1}{2}, +\frac{1}{2}\rangle$

neutron $|\frac{1}{2}, -\frac{1}{2}\rangle$

π^+ $|1, +1\rangle$

π^0 $|1, 0\rangle$

π^- $|1, -1\rangle$

Δ^{++} $|\frac{3}{2}, +\frac{3}{2}\rangle$

Δ^{+} $|\frac{3}{2}, +\frac{1}{2}\rangle$

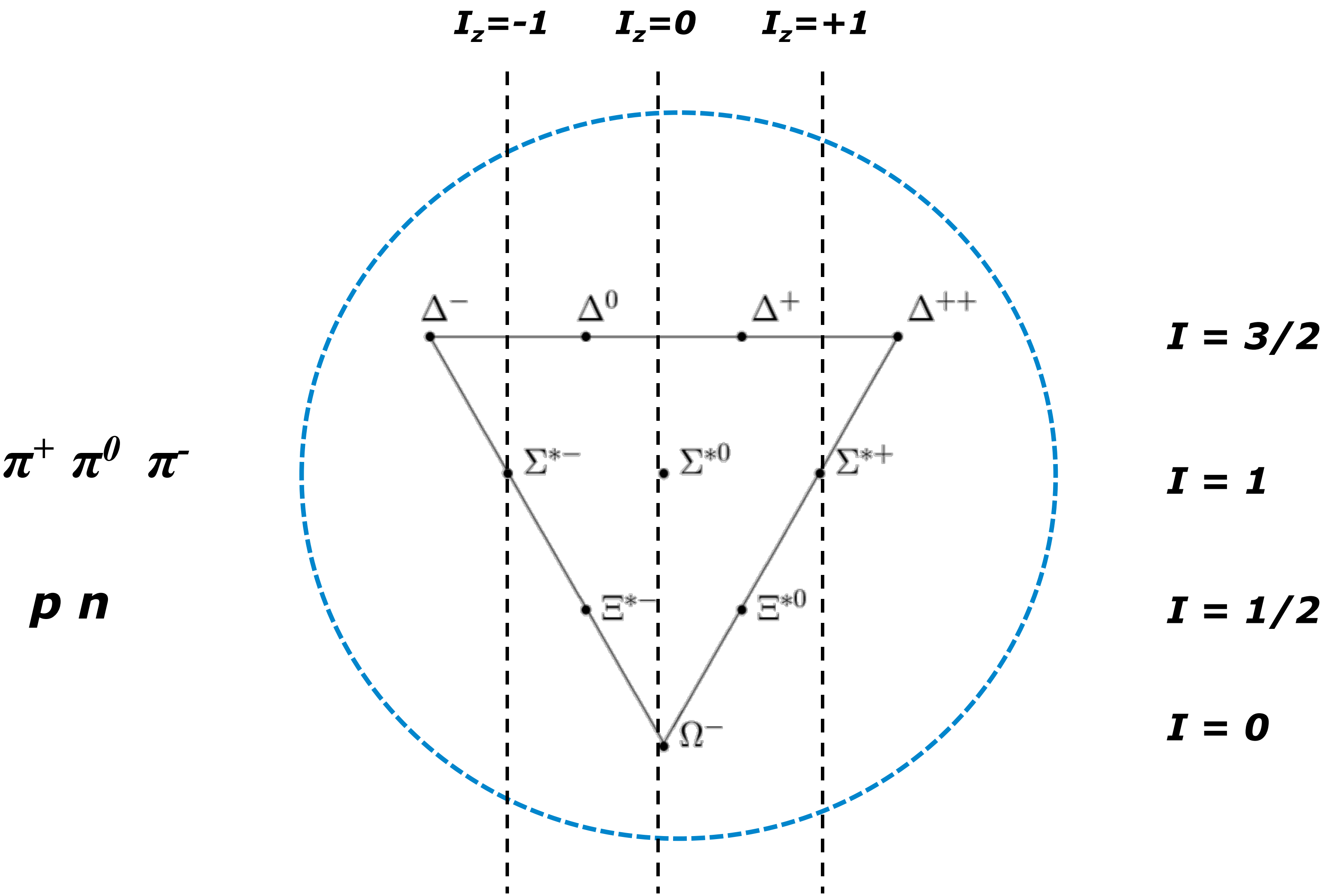
Δ^{0} $|\frac{3}{2}, -\frac{1}{2}\rangle$

Δ^{-} $|\frac{3}{2}, -\frac{3}{2}\rangle$

m_p ~ 939 MeV

m_π ~ 140 MeV

m_Δ ~ 1232 MeV



Baryon decuplet

Adding spin

Quantum mechanica: adding spin

$|s_1,m_1> + |s_2,m_2> \rightarrow |s,m>$

1) Conditions:

$m = m_1 + m_2$
$S = |s_1 - s_2|, |s_1 - s_2| + 1, .., s_1 + s_2 - 1, s_1 + s_2$

- S_z add up
- S can vary between difference and sum

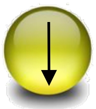
2) Notation:

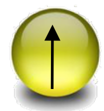
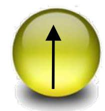
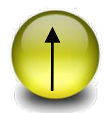
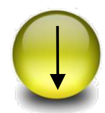
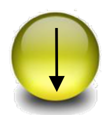
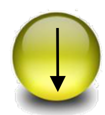
$$|s,m\rangle = \sum_{m_1+m_2=m} C^{s_1,s_2,s}_{m_1,m_2,m} |s_1,m_1\rangle |s_2,m_2\rangle$$

C: Clebsch-Gordan coefficient

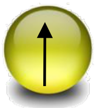
Adding spin of two spin-½ particles

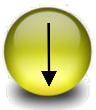
 $\alpha = |\frac{1}{2}, +\frac{1}{2}\rangle$

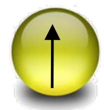
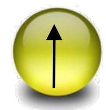
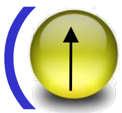
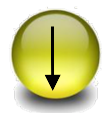
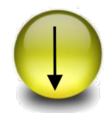
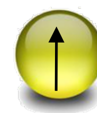
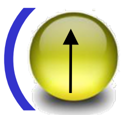
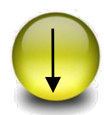
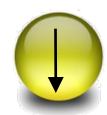
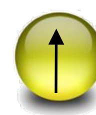
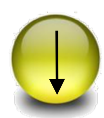
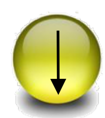
 $\beta = |\frac{1}{2}, -\frac{1}{2}\rangle$

			S_z	S
(1)			*+1*	*1*
(2)			*0*	*?*
(3)			*-1*	*1*

Adding spin of two spin-½ particles

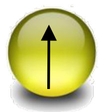
 $\alpha = |\frac{1}{2}, +\frac{1}{2}\rangle$

 $\beta = |\frac{1}{2}, -\frac{1}{2}\rangle$

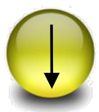
		S_z	S
(1)	 	+1	1
(2a)	$\sqrt{\frac{1}{2}}$ (  +  )	0	1
(2b)	$\sqrt{\frac{1}{2}}$ (  -  )	0	0
(3)	 	-1	1

Adding spin of two spin-½ particles

$$2 \otimes 2 = 3 \oplus 1$$



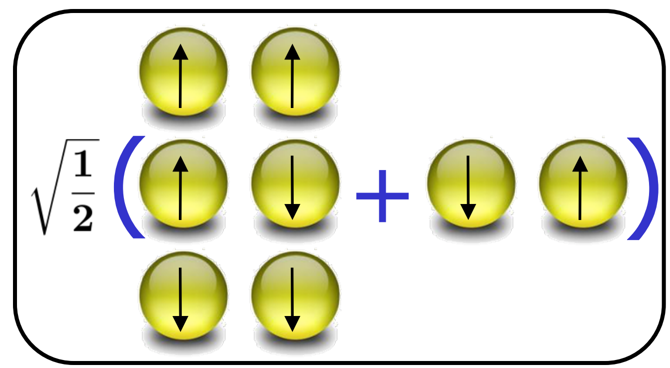
$$\alpha = |\tfrac{1}{2}, +\tfrac{1}{2}\rangle$$



$$\beta = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

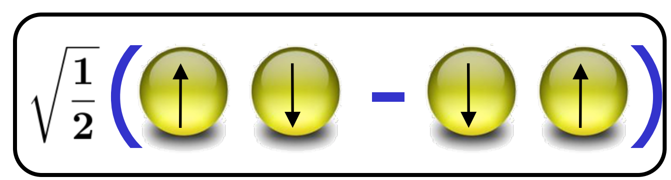
S=1 *Triplet (symmetric)*

$$\left\{ \begin{array}{ll} |1, +1\rangle & = \\ |1, \;\; 0\rangle & = \\ |1, -1\rangle & = \end{array} \right.$$



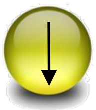
S=0 *Singlet (anti-symmetric)*

$$\left\{ \; |0, \;\; 0\rangle \; = \right.$$



Adding spin of two spin-½ particles

 $\alpha = |\frac{1}{2}, +\frac{1}{2}\rangle$

 $\beta = |\frac{1}{2}, -\frac{1}{2}\rangle$

$$\boxed{2 \otimes 2 = 3 \oplus 1}$$

Triplet (symmetric)

$$\left\{ \begin{array}{lcl} |1, +1\rangle & = & \alpha(1)\alpha(2) \\ |1, \ \ 0\rangle & = & \sqrt{\frac{1}{2}}[\alpha(1)\beta(2) + \beta(1)\alpha(2)] \\ |1, -1\rangle & = & \beta(1)\beta(2) \end{array} \right.$$

Singlet (anti-symmetric)

$$\left\{ \ |0, \ \ 0\rangle = \sqrt{\frac{1}{2}}[\alpha(1)\beta(2) - \beta(1)\alpha(2)] \right.$$

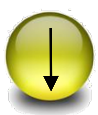
Quantum mechanics: adding spin

$$|s,m\rangle = \sum_{m_1+m_2=m} C^{s_1,s_2,s}_{m_1,m_2,m} |s_1,m_1\rangle |s_2,m_2\rangle$$

→ ***Clebsch-Gordan coefficient***

***Specific:* adding spin of two spin-1/2 particles:**

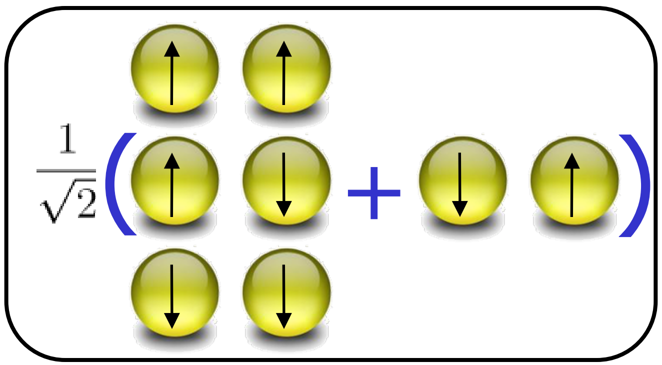
$\alpha = |\frac{1}{2}, +\frac{1}{2}\rangle$



$\beta = |\frac{1}{2}, -\frac{1}{2}\rangle$

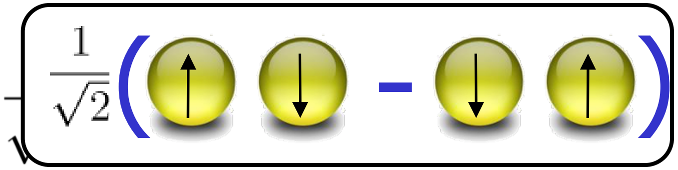
Triplet (symmetric)

$$\begin{cases} |1,+1\rangle = \\ |1,\ \ 0\rangle = \\ |1,-1\rangle = \end{cases}$$



Singlet (anti-symmetric)

$$\left\{ |0,\ \ 0\rangle = \right.$$

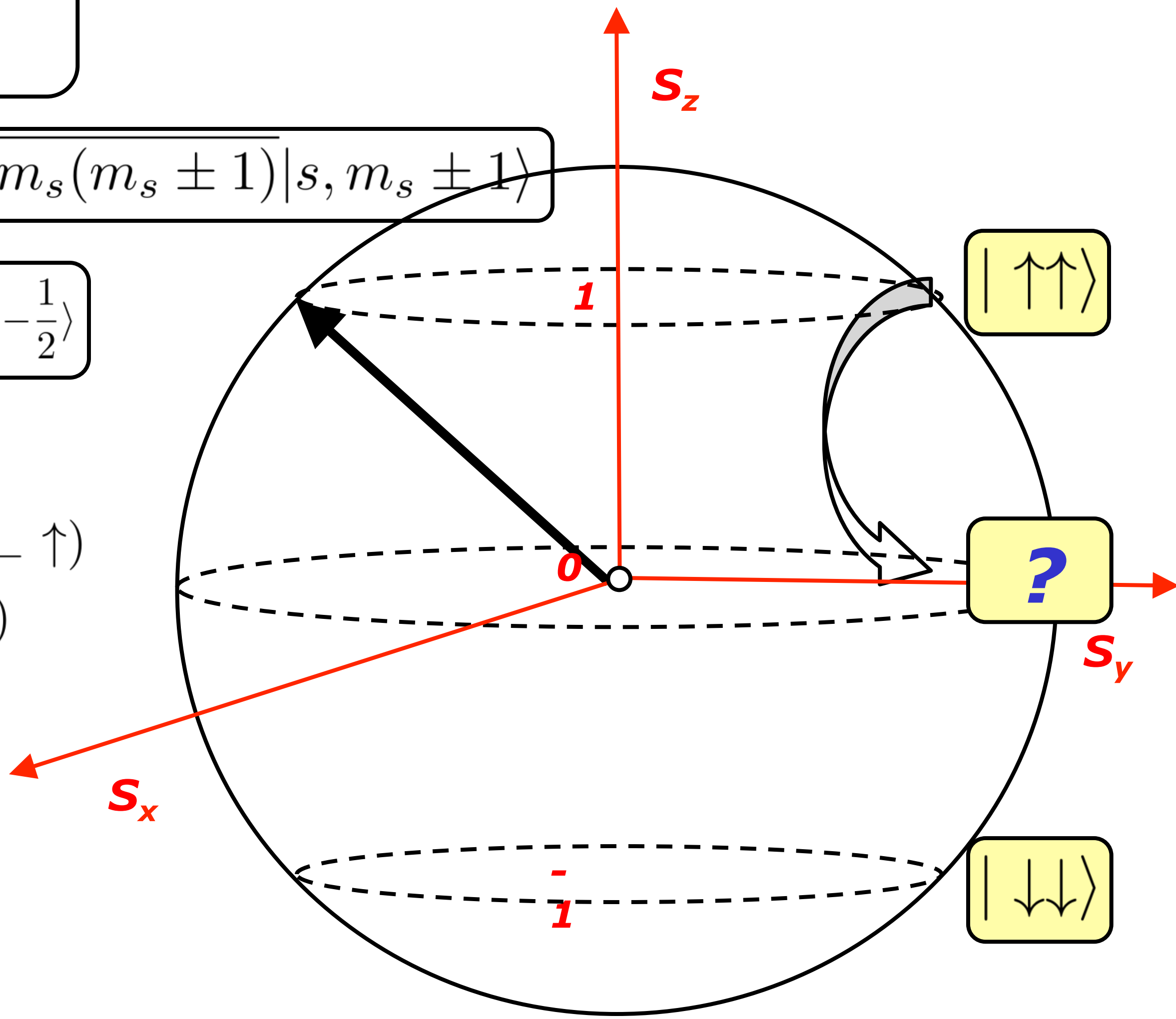


Why is $|1,0\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow + \downarrow\uparrow\rangle$ and not $|1,0\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow - \downarrow\uparrow\rangle$?

$$\begin{aligned} S^2|s,m_s\rangle &= \hbar^2 s(s+1)|s,m_s\rangle \\ S_z|s,m_s\rangle &= \hbar m_s|s,m_s\rangle \end{aligned}$$

$$S_\pm|s,m_s\rangle = \hbar\sqrt{s(s+1)-m_s(m_s\pm 1)}|s,m_s\pm 1\rangle$$

$$S_-|\frac{1}{2},+\frac{1}{2}\rangle = \hbar|\frac{1}{2},-\frac{1}{2}\rangle$$



$$\begin{aligned} S_-|\uparrow\uparrow\rangle &= (S_-\uparrow)\uparrow + \uparrow(S_-\uparrow) \\ &= (\hbar\downarrow)\uparrow + \uparrow(\hbar\downarrow) \\ &= \hbar(\downarrow\uparrow + \uparrow\downarrow) \end{aligned}$$

Griffiths Par 4.4.3

Clebsch-Gordan coefficients

Coefficients can be used "both ways":

1) add

$$|\frac{3}{2},\frac{1}{2}\rangle|1,0\rangle = \sqrt{\frac{3}{5}}|\frac{5}{2},\frac{1}{2}\rangle + \sqrt{\frac{1}{15}}|\frac{3}{2},\frac{1}{2}\rangle - \sqrt{\frac{1}{3}}|\frac{1}{2},\frac{1}{2}\rangle$$

$|s_1,m_1\rangle + |s_2,m_2\rangle \rightarrow |s,m\rangle$

2) decay

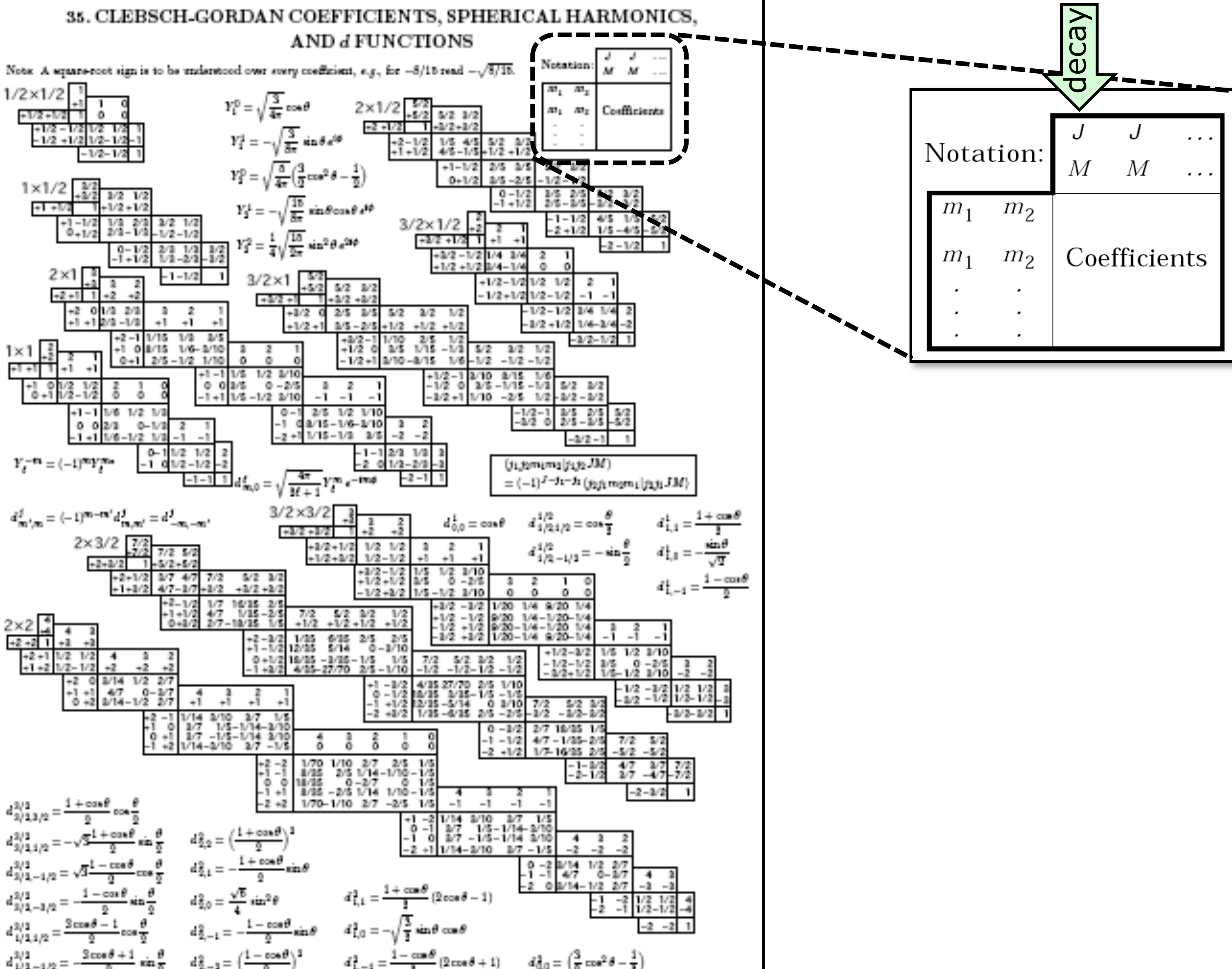
$$|3,0\rangle = \sqrt{\frac{1}{5}}|2,1\rangle|1,-1\rangle + \sqrt{\frac{3}{5}}|2,0\rangle|1,0\rangle - \sqrt{\frac{1}{5}}|2,-1\rangle|1,1\rangle$$

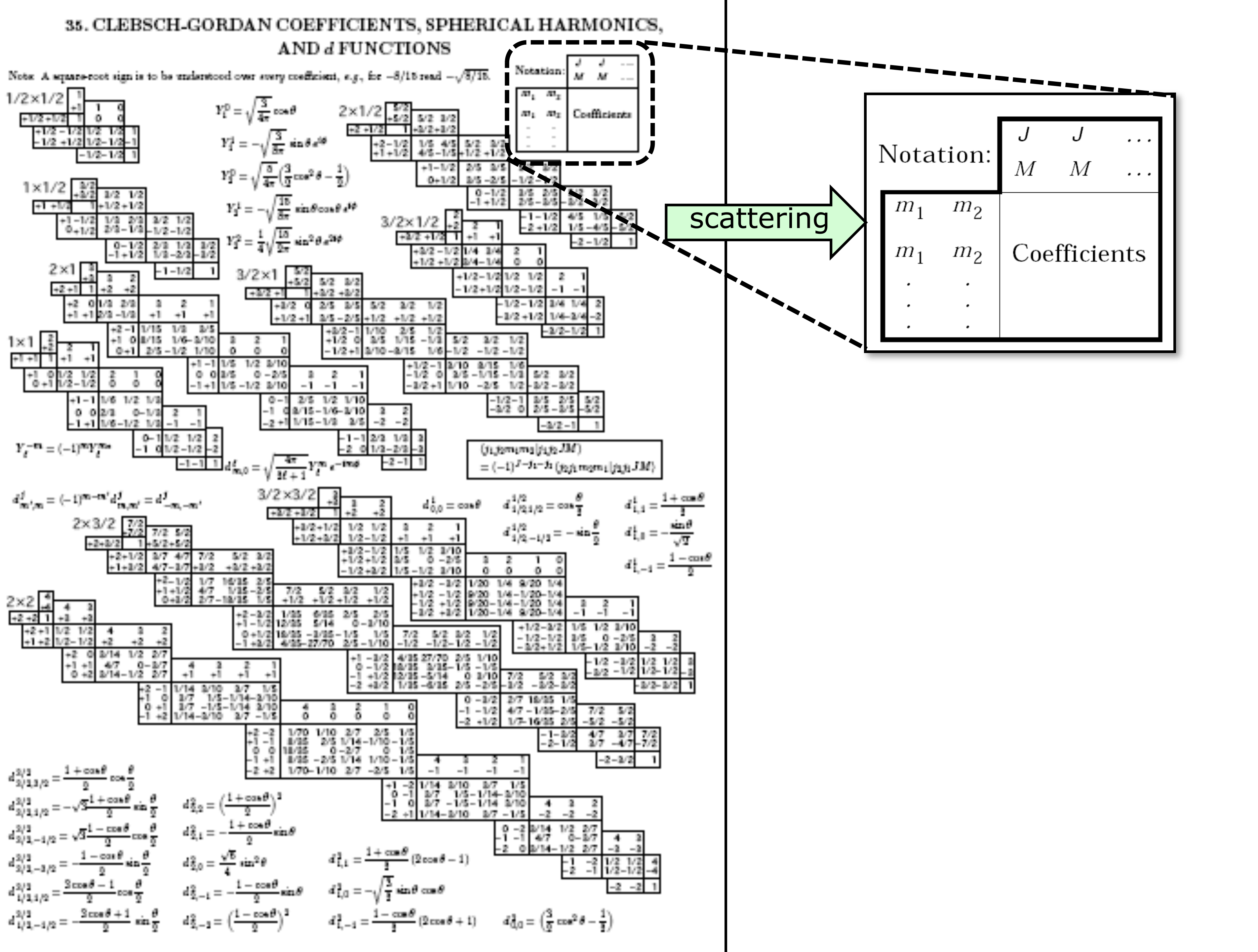
$|s,m\rangle \rightarrow |s_1,m_1\rangle + |s_2,m_2\rangle$

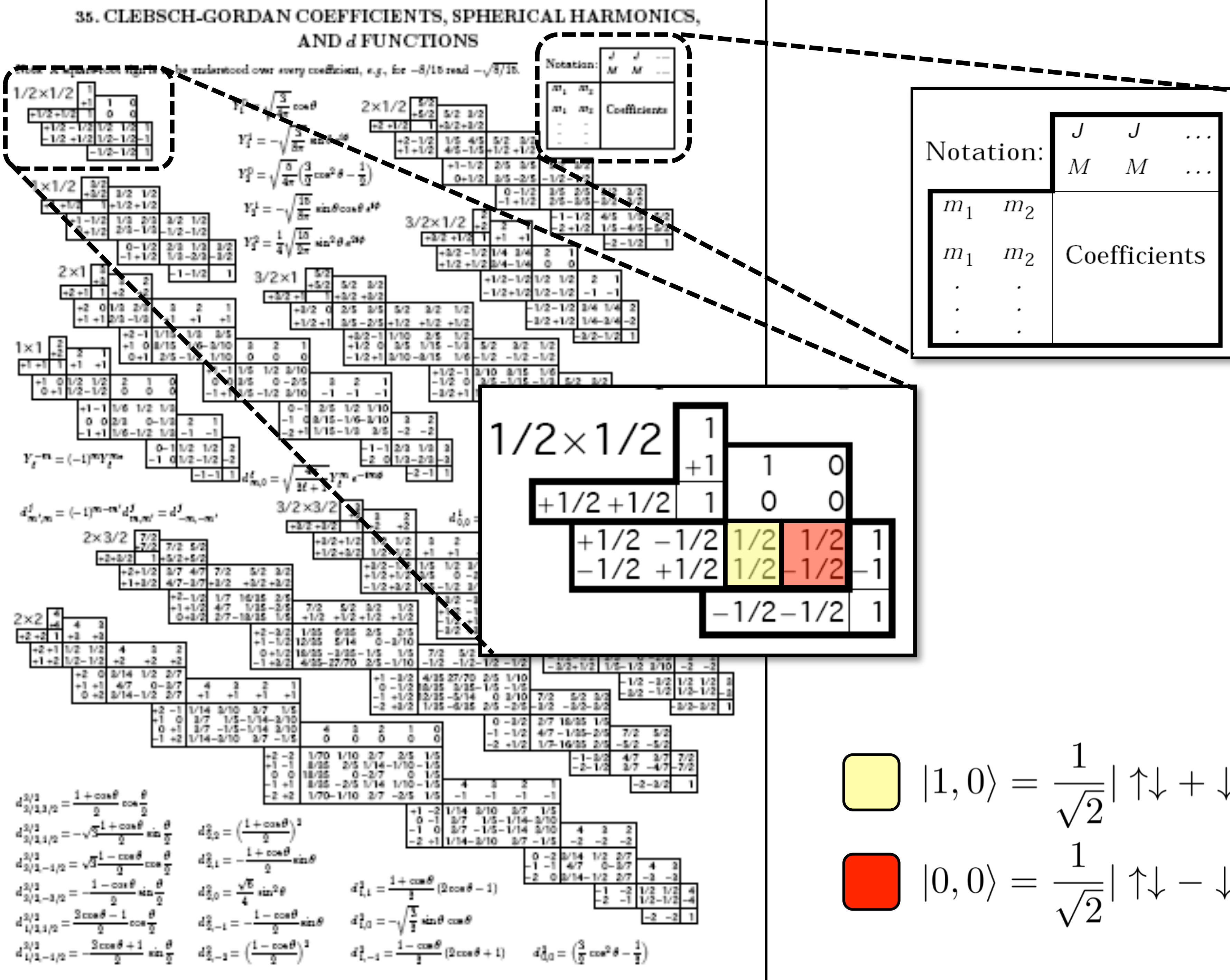
Clebsch-Gordan coefficients

A) Find out yourself (doable, but bit messy…)

$$
\begin{aligned}
C^{m_1,m_2,m}_{s_1,s_2,s} &= \delta_{m,m_1+m_2}\sqrt{\frac{(2j+1)(j+j_1-j_2)!(j-j_1+j_2)!(j_1+j_2+j)}{(j_1+j_2+j+1)!}} \\
&\times \sqrt{(j+m)!(j-m)!(j_1-m_1)!(j_1+m_1)!(j_2-m_2)!(j_2+m_2)!} \\
&\times \sum_k \frac{-1^k}{k!(j_1+j_2-j-k)!(j_1-m_1-k)!(j_2+m_2-k)!(j-j_2+m_1-k)!(j-j_1-m_2+k)!}
\end{aligned}
$$

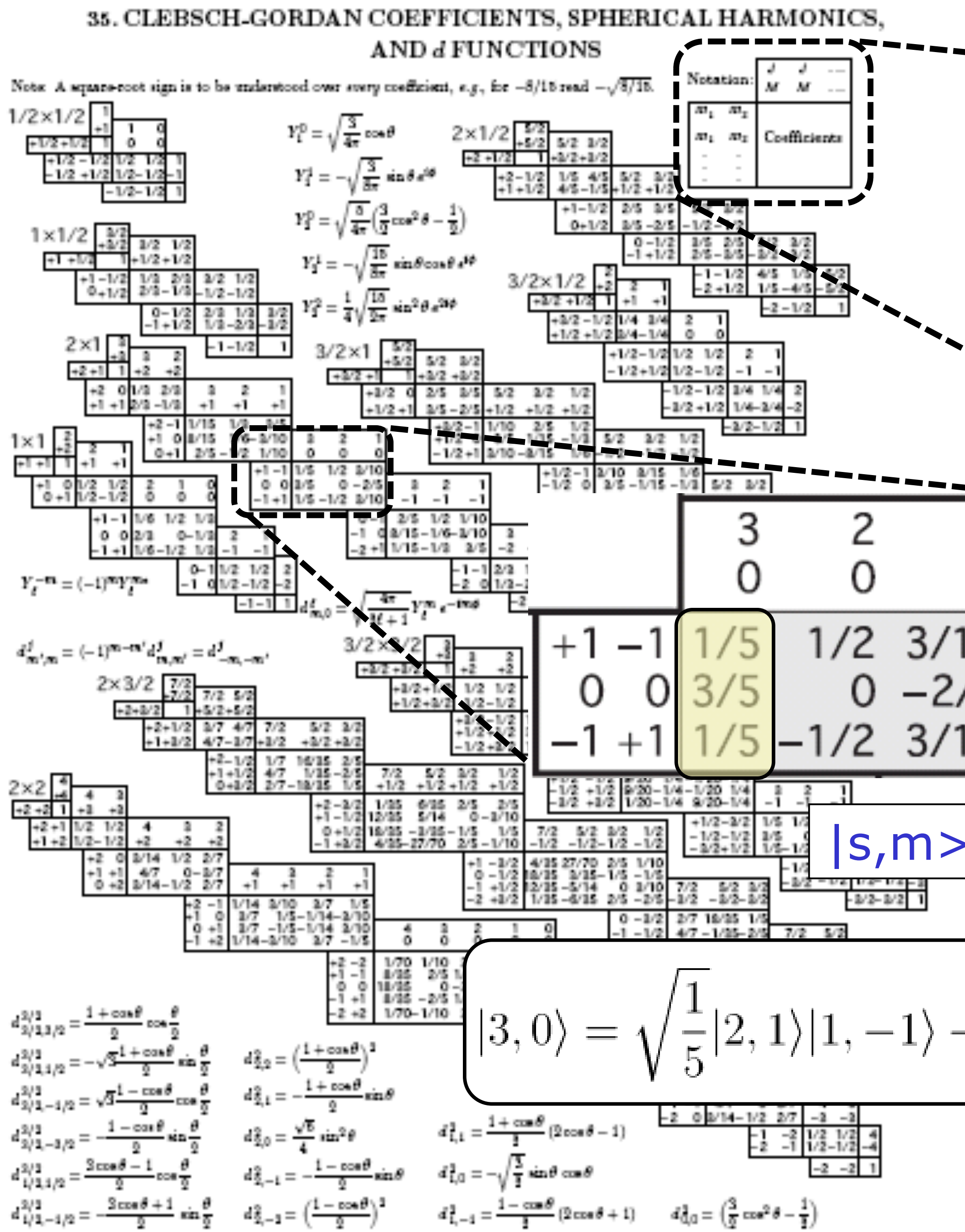

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS
Note: A square-root sign is to be understood over every coefficient, e.g., for −8/15 read −√8/15.
Notation:
Coefficients
decay
Notation:
J J . . .
M M . . .
m1 m2
m1 m2
Coefficients


35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS
Notation:
J J ...
M M ...
m1 m2
m1 m2
Coefficients
scattering



$$|1,0\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow + \downarrow\uparrow\rangle$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow - \downarrow\uparrow\rangle$$



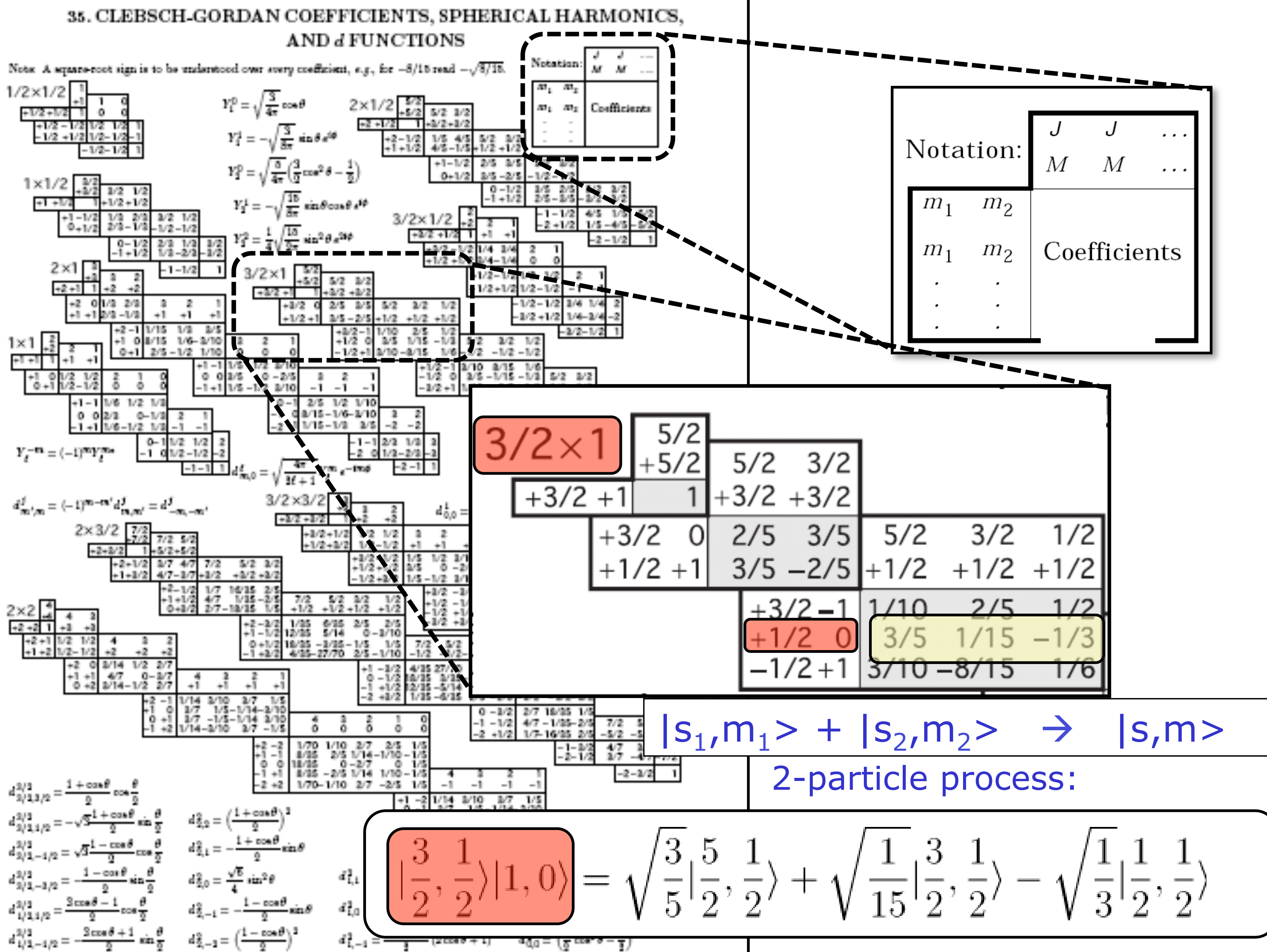
Notation:	J	J	...
	M	M	...
m_1 m_2	Coefficients		
m_1 m_2			
. .			

		3	2	1
		0	0	0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

|s,m> → |s$_1$,m$_1$> + |s$_2$,m$_2$>

Decay:

$$|3,0\rangle = \sqrt{\frac{1}{5}}|2,1\rangle|1,-1\rangle + \sqrt{\frac{3}{5}}|2,0\rangle|1,0\rangle - \sqrt{\frac{1}{5}}|2,-1\rangle|1,1\rangle$$



$|s_1,m_1\rangle + |s_2,m_2\rangle \;\rightarrow\; |s,m\rangle$

2-particle process:

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle|1,0\rangle = \sqrt{\frac{3}{5}}\left|\frac{5}{2},\frac{1}{2}\right\rangle + \sqrt{\frac{1}{15}}\left|\frac{3}{2},\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2},\frac{1}{2}\right\rangle$$

Example: πp scattering

1) $\pi^+ p \rightarrow \pi^+ p$

- $I_z = 3/2$
- ➔ Pure I = 3/2 !

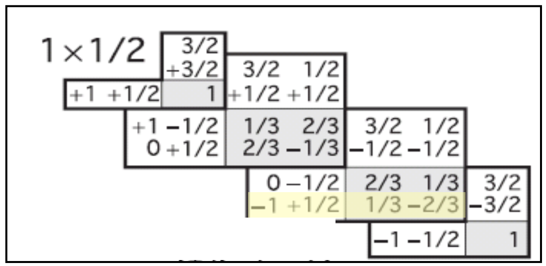
$$I_3^{\pi^+ p} = I_3^{\pi +} + I_3^{p} = \frac{3}{2}$$

$$|I^{\pi +} - I^p| \leq I^{\pi^+ p} \leq I^{\pi +} + I^p$$

$$\frac{1}{2} \leq I^{\pi^+ p} \leq \frac{3}{2}$$

2) $\pi^- p \rightarrow \pi^- p$

- $I_z = 3/2$
- ➔ Mixed I !



$$|1,-1\rangle \, |\tfrac{1}{2},\tfrac{1}{2}\rangle = \sqrt{\frac{1}{3}} \, |\tfrac{3}{2},-\tfrac{1}{2}\rangle - \sqrt{\frac{2}{3}} \, |\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

What is relative cross section to make the I=3/2 resonance?

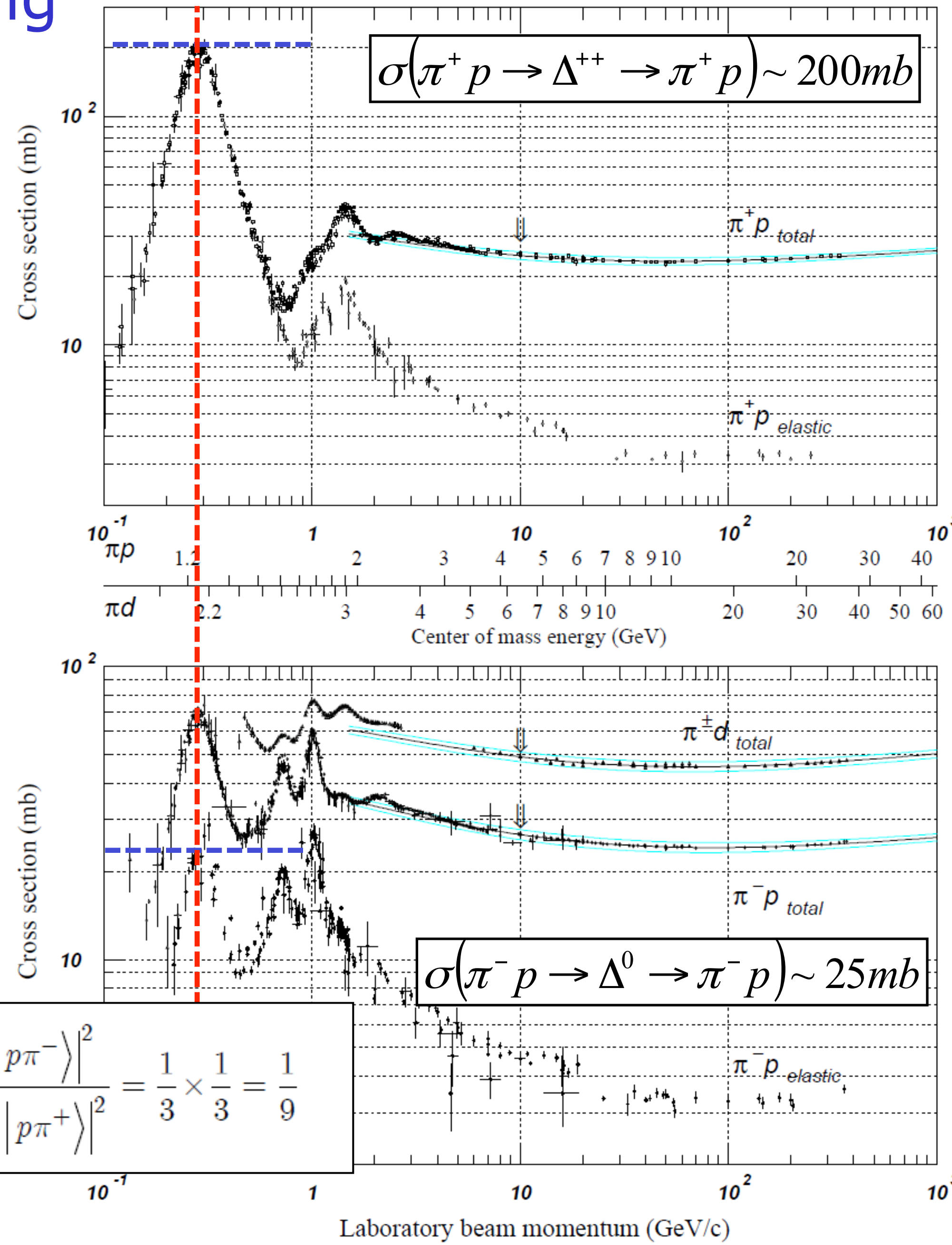
$$\frac{\sigma\left(p\pi^- \rightarrow \Delta^0\right)}{\sigma\left(p\pi^+ \rightarrow \Delta^{++}\right)} = \frac{\left|\left\langle p\pi^- \middle| \Delta^0 \right\rangle\right|^2}{\left|\left\langle p\pi^+ \middle| \Delta^{++} \right\rangle\right|^2} = \frac{\left|\sqrt{1/3}\right|^2}{1} = \frac{1}{3}$$

Example: πp scattering

Compare Δ resonance in elastic scattering:

1) $\pi^+ p \rightarrow \pi^+ p$

2) $\pi^- p \rightarrow \pi^- p$

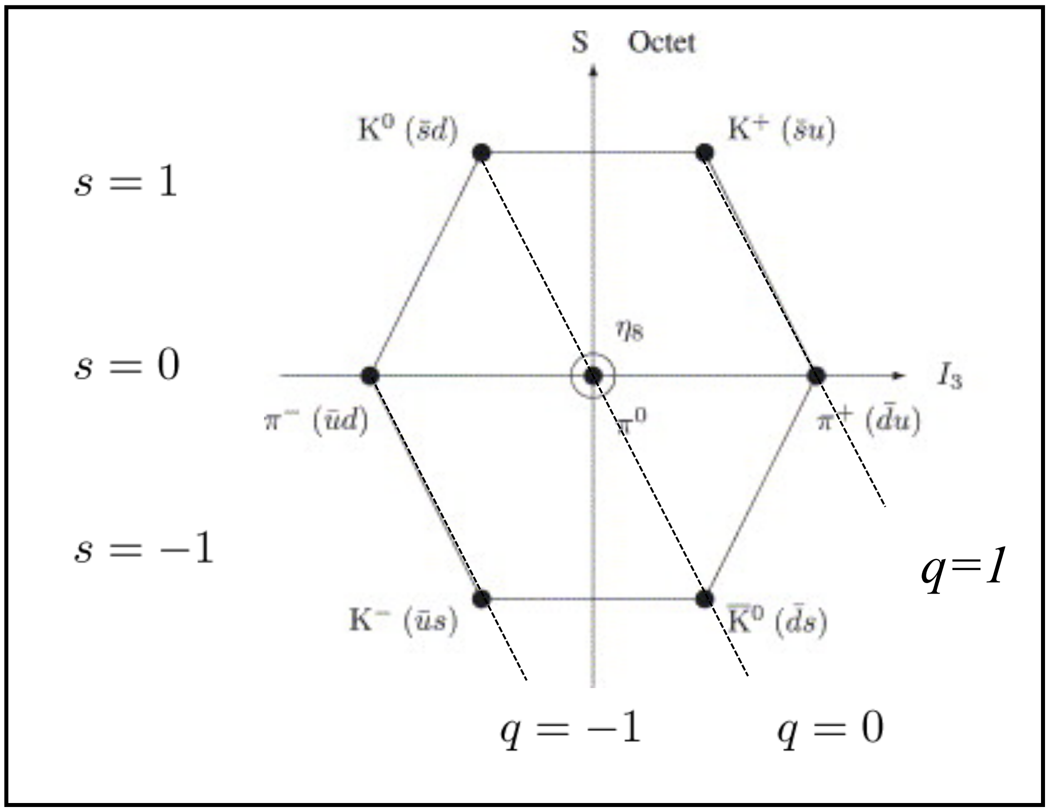


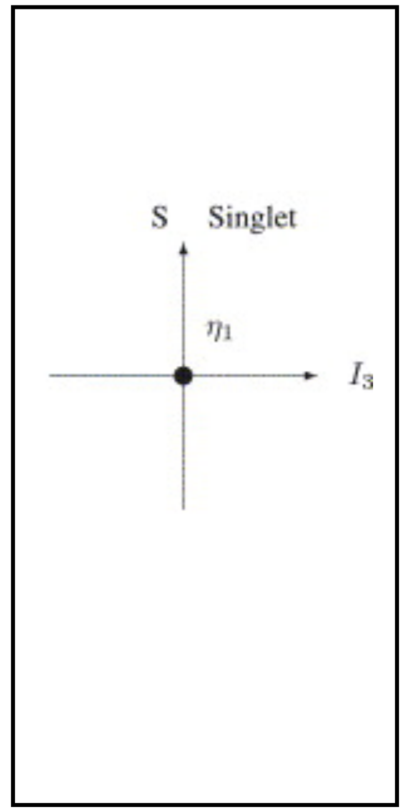
$$\frac{\sigma\left(p\pi^- \rightarrow \Delta^0 \rightarrow p\pi^-\right)}{\sigma\left(p\pi^+ \rightarrow \Delta^{++} \rightarrow p\pi^+\right)} = \frac{\left|\left\langle p\pi^- \middle| \Delta^0 \right\rangle\right|^2}{\left|\left\langle p\pi^+ \middle| \Delta^{++} \right\rangle\right|^2} \times \frac{\left|\left\langle \Delta^0 \middle| p\pi^- \right\rangle\right|^2}{\left|\left\langle \Delta^{++} \middle| p\pi^+ \right\rangle\right|^2} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

Group theory

$$3 \otimes \bar{3} = 8 \oplus 1$$

- Mesons:
  - 2 quarks, with 3 possible flavours: u, d, s
  - 3^2 =9 possibilities = 8 + 1





Group theory

$$3 \otimes \bar{3} = 8 \oplus 1$$

- Mesons:
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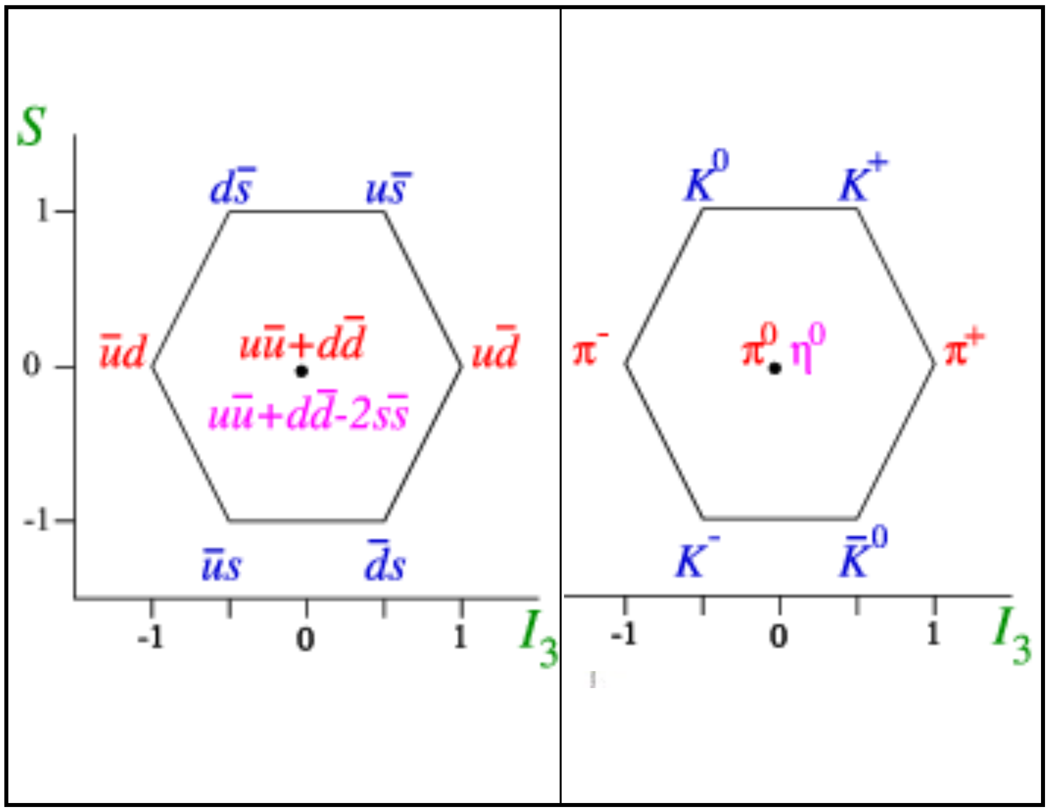
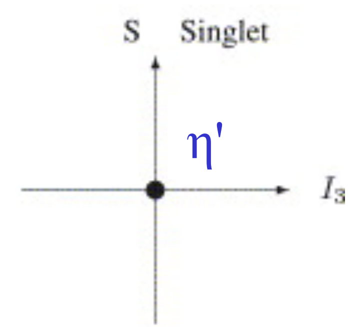
  - $3^2 = 9$ possibilities = 8 + 1





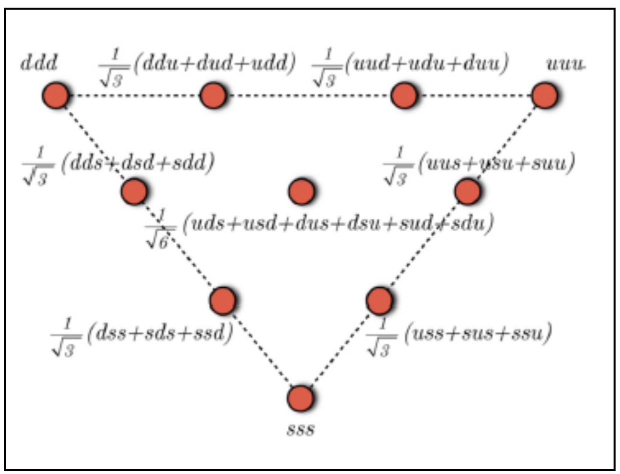
$$f' = \psi_8 \cos\theta - \psi_1 \sin\theta$$
$$f = \psi_8 \sin\theta + \psi_1 \cos\theta$$
$$\psi_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$
$$\psi_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

Group theory

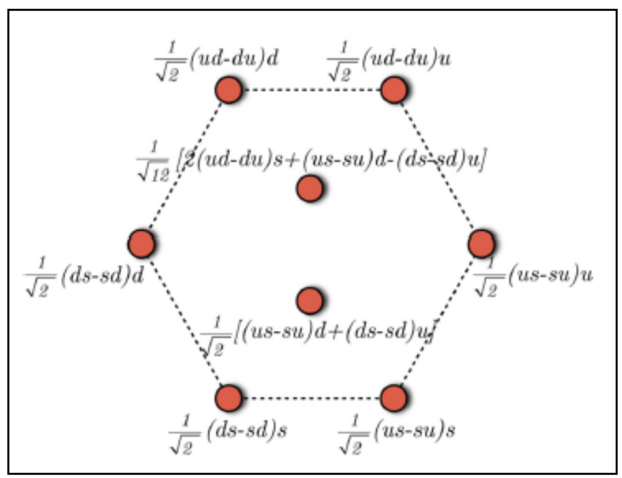
$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

- Baryons:
  - 3 quarks, with 3 possible flavours: u, d, s
  - 3^3 =27 possibilities = 10 + 8 + 8 + 1

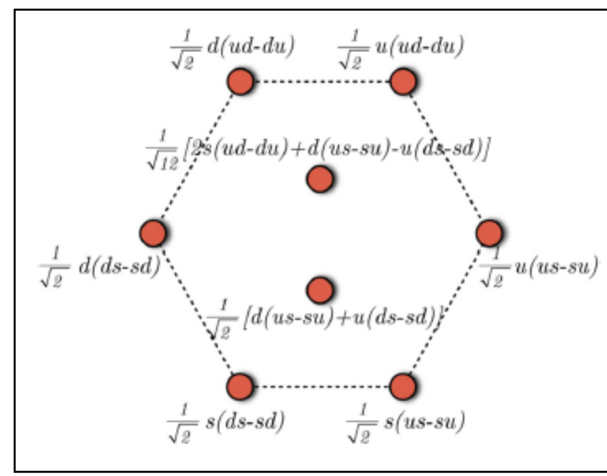
ψ_{sym}

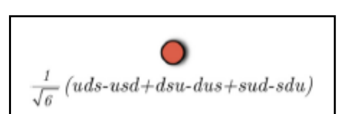


$\psi_{anti-sym}(1 \leftrightarrow 2)$



$\psi_{anti-sym}(2 \leftrightarrow 3)$



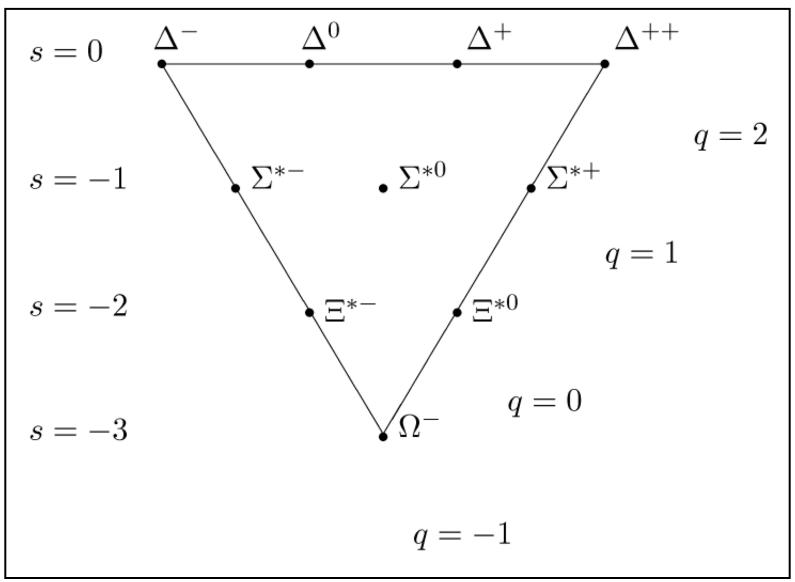


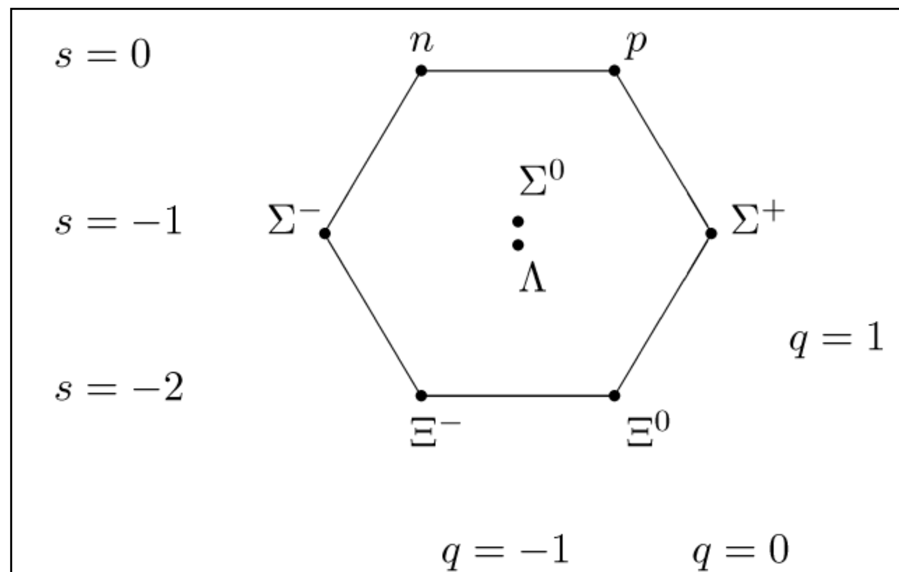
Group theory

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

- Baryons:
  - 3 quarks, with 3 possible flavours: u, d, s
  - 3^3 =27 possibilities = 10 + 8 + 8 + 1

$$\psi_{sym} \qquad \psi_{anti-sym}(1 \leftrightarrow 2) \qquad \psi_{anti-sym}(2 \leftrightarrow 3)$$





What did we learn about quarks

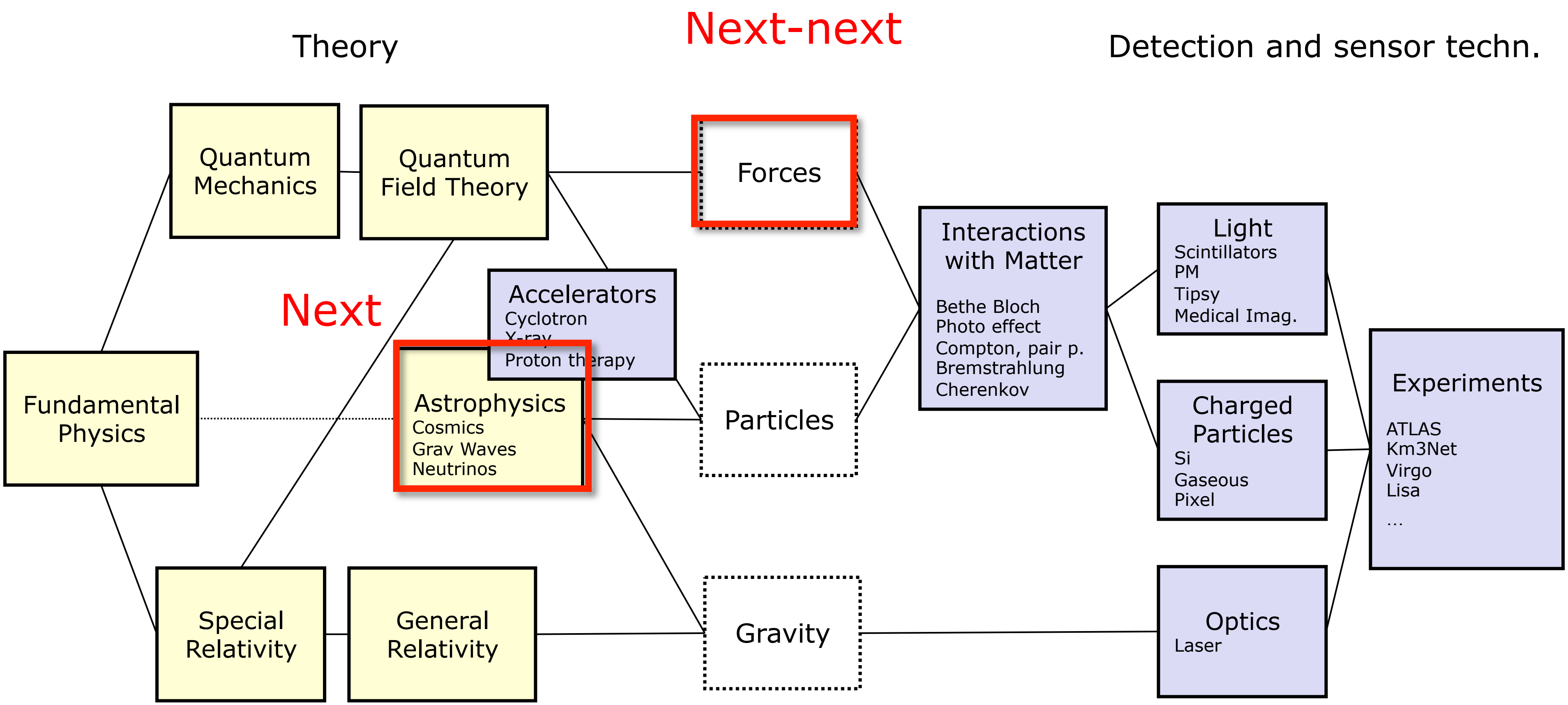
Quarks:

- Associate production, but long lifetime: strangeness
- Many (degenerate) particles: isospin
- Pauli exclusion principle: color

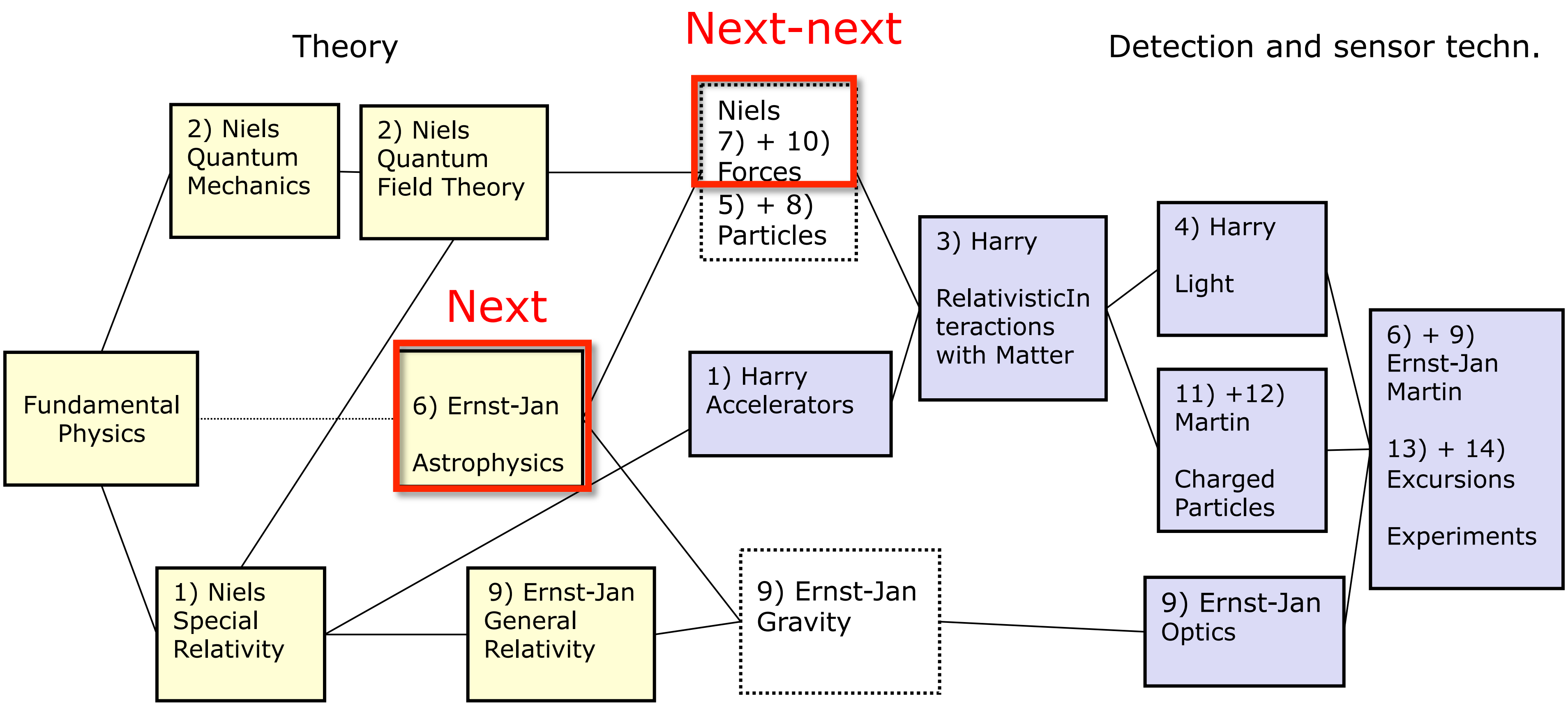
	d	u	s
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0
S – strangeness	0	0	-1

- How they combine into hadrons: multiplets
- How to add (iso)spin: Clebsch-Gordan

Plan


Theory
Next-next
Detection and sensor techn.
Quantum Mechanics
Quantum Field Theory
Forces
Interactions with Matter
Bethe Bloch
Photo effect
Compton, pair p.
Bremstrahlung
Cherenkov
Light
Scintillators
PM
Tipsy
Medical Imag.
Accelerators
Cyclotron
X-ray
Proton therapy
Next
Fundamental Physics
Astrophysics
Cosmics
Grav Waves
Neutrinos
Particles
Charged Particles
Si
Gaseous
Pixel
Experiments
ATLAS
Km3Net
Virgo
Lisa
...
Special Relativity
General Relativity
Gravity
Optics
Laser

Plan


Theory
Next-next
Detection and sensor techn.
2) Niels Quantum Mechanics
2) Niels Quantum Field Theory
Niels 7) + 10) Forces
5) + 8) Particles
3) Harry RelativisticInteractions with Matter
4) Harry Light
Next
Fundamental Physics
6) Ernst-Jan Astrophysics
1) Harry Accelerators
11) +12) Martin Charged Particles
6) + 9) Ernst-Jan Martin 13) + 14) Excursions Experiments
1) Niels Special Relativity
9) Ernst-Jan General Relativity
9) Ernst-Jan Gravity
9) Ernst-Jan Optics

Plan

1) Intro: Standard Model & Relativity — *11 Feb*

1900-1940 2) Basis — *18 Feb*
   1) Atom model, strong and weak force
   2) Scattering theory

1945-1965 3) Hadrons — *10 Mar*
   1) Isospin, strangeness
   2) Quark model, GIM

1965-1975 4) Standard Model — *24 Mar*
   1) QED
   2) Parity, neutrinos, weak inteaction
   3) QCD

1975-2000 5) e^+e^- and DIS — *21 Apr*

2000-2015 6) Higgs and CKM — *12 May*