

# “Elementary Particles”

## *Lecture 1*

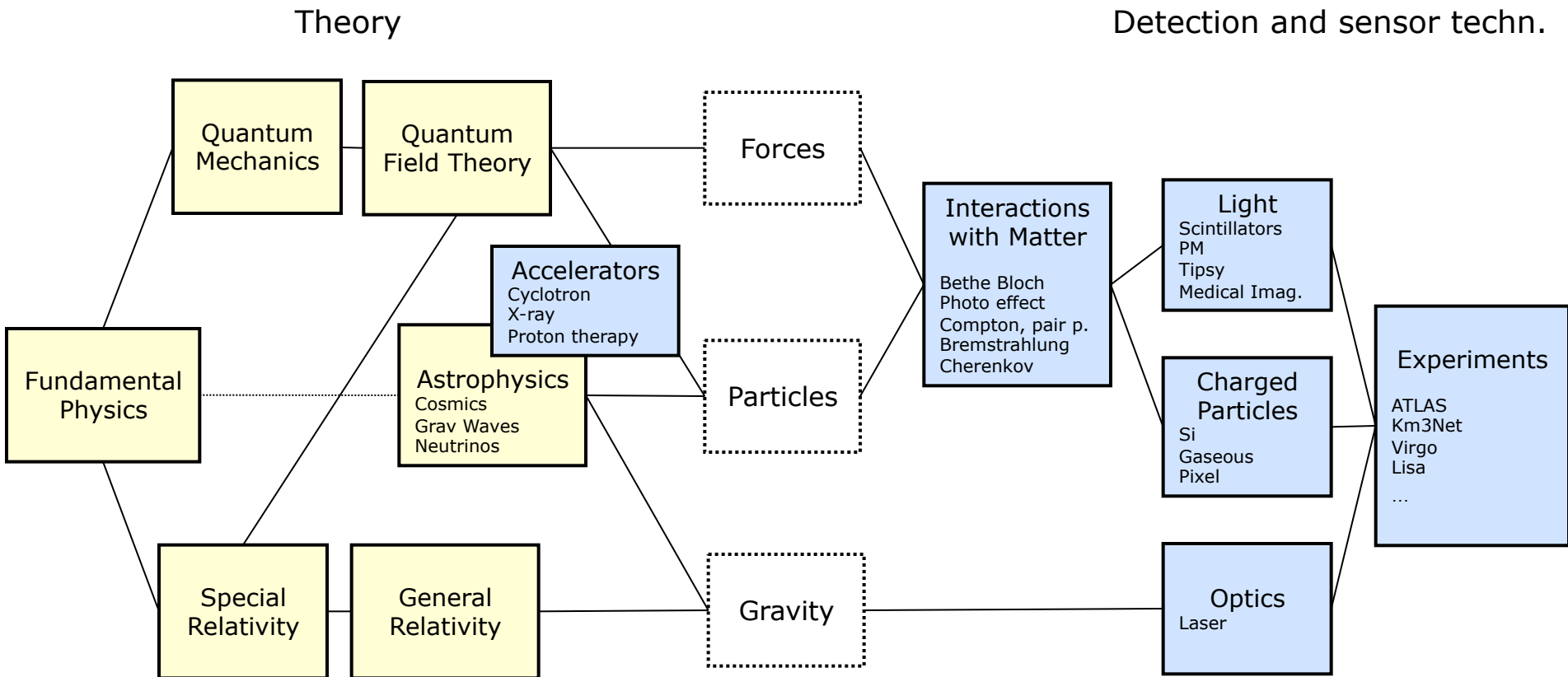
Niels Tuning

Harry van der Graaf

Martin Fransen

Ernst-Jan Buis

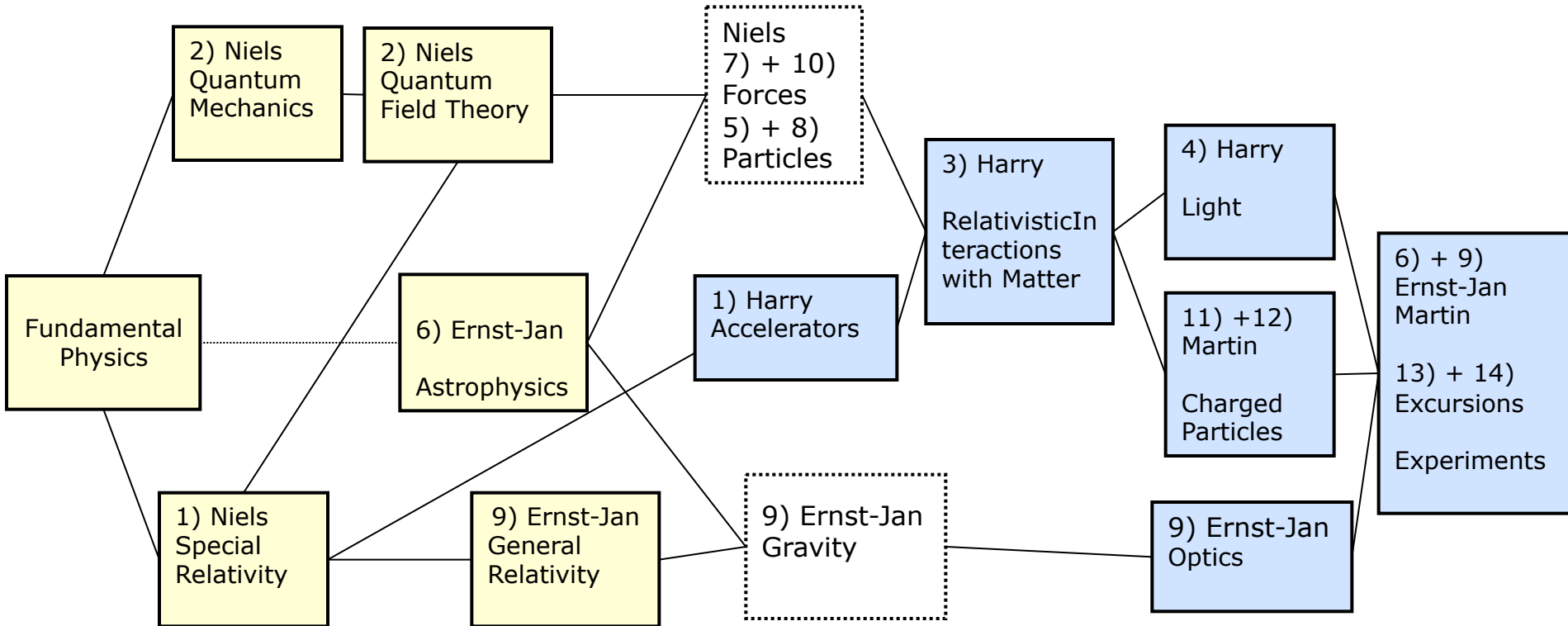
# Plan



# Plan

## Theory

## Detection and sensor techn.



# Schedule

## 1) 11 Feb: Accelerators (Harry) + Special relativity (Niels)

- Layout, structure
- Thomson Tube, vdGraaff, Cockroft Walton, cyclotron, synchrotron,
- **(Synchrotron radiation (ESRF), neutron sites (ESS), WakeField accelerators, proton beam therapy ?)**
- 4-vectors, Lorentz transformation, Special relativity

## 2) 18 Feb: Quantum Mechanics (Niels)

- QM basics, wave function, Schrodinger, Klein-Gordon, Dirac equation, Rutherford scattering

## 3) 25 Feb: Interactions with Matter (Harry)

- EM interactions, Bethe Bloch, Landau distributions, Ionisation in gas and Si
- Three photon interactions (Photo effect, Compton, Pair Production)
- Bremsstrahlung, Cherenkov radiation. Equivalence of Pair Production and Brehmstrahlung

## 4) 3 Mar: Light detection? (Harry)

- **Scintillators (including photon detectors, from ZISulfide to Topsy))**
- **Calorimeters?**

## 5) 10 Mar: Particles and cosmics (Niels)

- Cosmics, quark model, strangeness

## 6) 17 Mar: Astrophysics and Dark matter (Ernst-Jan)

- Cosmic rays (Showers (**protons/gammas/neutrinos/dark matter**); Signals (Cherenkov radiation, fluorescence,radio); Experiments (PA/IceCube/Anatares/KM3NeT/TA); Cherenkov gamma-ray telescope(Magic/Hess/CTA) )
- Low background experiments (PMTs; Shielding; Experiments (Kamiokande/Xenon/DAMA)
- Space based experiments (cosmic rays from space and spaceweather (AMS/ACE); Gamma/X-ray space based astrophysics, Optics/coded masks, Swift, Integral, XMM/Chandra, planetaire mission)

## 7) 24 Mar: Forces (Niels)

- Symmetries, Gauge invariance, QED, weak and strong interaction

# Schedule

## 8) 21 Apr: $e^+e^-$ and ep scattering (Niels)

- R (colors), running coupling, charm, gluon, tt, WZ, DIS

## 9) 28 Apr: Gravitational waves (Ernst-Jan)

- Interferometry (Michelson, Sagnac; lasers, optics)
- Ground based experiments (Virgo/LIGO/Karga/ET)
- Spaced based experiments (LISA)
- Multimessenger (Space+ground; triggers; Future, big questions)

## 10) 12 May: Higgs and big picture (Niels)

- Higgs mechanism and Standard Model completion

## 11) 19 May: Charged particle detection (Martin)

- **Gaseous detectors (from Geiger to GridPix)**
- **Semiconductor (Si) detectors; pixel detectors**

## 12) 26 May: Applications: experiments and medical (Martin)

- Pixels, ATLAS, 4D tracking
- medical imaging, CT, spectral X-ray, PET scan

## 13) 2 Jun: Nikhef excursie

- ATLAS? ALICE? Km3Net? Virgo? LHCb?

## 14) 8 Jun: CERN excursie

- CERN lecture (H. Ten Kate); ATLAS underground; Synchro-cyclotron; LHCb; AD antimatter ?

# Schedule

- 1) 11 Feb: Accelerators (Harry vd Graaf) + Special relativity (Niels Tuning)
- 2) 18 Feb: Quantum Mechanics (Niels Tuning)
- 3) 25 Feb: Interactions with Matter (Harry vd Graaf)
- 4) 3 Mar: Light detection (Harry vd Graaf)
- 5) 10 Mar: Particles and cosmics (Niels Tuning)
- 6) 17 Mar: Astrophysics and Dark Matter (Ernst-Jan Buis)
- 7) 24 Mar: Forces (Niels Tuning)
- break
- 8) 21 Apr:  $e^+e^-$  and ep scattering (Niels Tuning)
- 9) 28 Apr: Gravitational Waves (Ernst-Jan Buis)
- 10) 12 May: Higgs and big picture (Niels Tuning)
- 11) 19 May: Charged particle detection (Martin Franse)
- 12) 26 May: Applications: experiments and medical (Martin Franse)
  
- 13) 2 Jun: Nikhef excursie
- 14) 8 Jun: CERN excursie

# Plan

	1) Intro: Standard Model & Relativity	<b>11 Feb</b>
<b>1900-1940</b>	2) Basis	<b>18 Feb</b>
	1) Atom model, strong and weak force	
	2) Scattering theory	
<b>1945-1965</b>	3) Hadrons	<b>10 Mar</b>
	1) Isospin, strangeness	
	2) Quark model, GIM	
<b>1965-1975</b>	4) Standard Model	<b>24 Mar</b>
	1) QED	
	2) Parity, neutrinos, weak inteaction	
	3) QCD	
<b>1975-2000</b>	5) $e^+e^-$ and DIS	<b>21 Apr</b>
<b>2000-2015</b>	6) Higgs and CKM	<b>12 May</b>

# Books

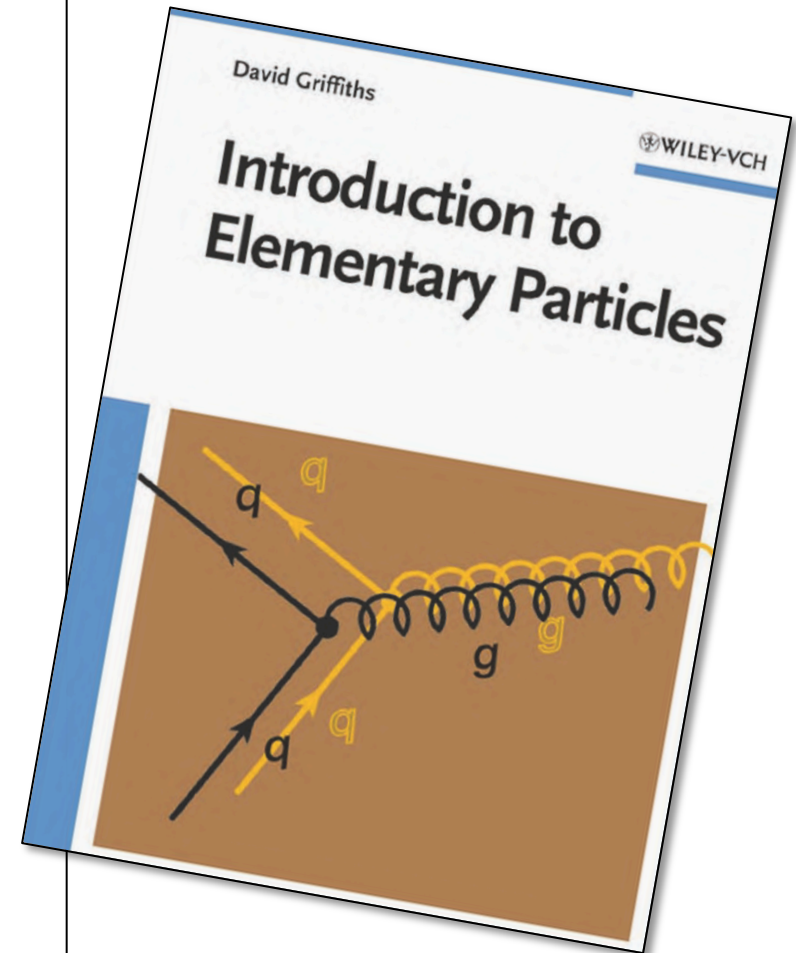
- M. Thomson                    “[Modern Particle Physics](#)”                   (2013, 49 EUR)
- D. Griffiths                   “[Introduction to Elementary Particles](#)”   (2008, 68 EUR)
- C. Tully                        “[Elementary Particle Physics in a Nutshell](#)” (2011, 65 EUR)
- F. Halzen & A.D.Martin   “[Quarks and Leptons](#)”                   (1984, 68 EUR)



## D. Griffiths

# "Introduction to Elementary Particles"

- Lecture 1:
  - ch.3 Relativistic kinematics
- Lecture 2:
  - ch.5.1 Schrodinger equation
  - ch.7.1 Dirac equation
  - ch.6.5 Scattering
- Lecture 3:
  - ch.1.7 Quarkmodel
  - ch.4 Symmetry/spin
- Lecture 4:
  - ch.7.4 QED
  - h.11.3 Gauge theories
- Lecture 5:
  - ch.8.2  $e+e^-$
  - ch.8.5  $e+p$
- Lecture 6:
  - ch.11.8 Higgs mechanism



# Outline of today

- Introduction
  - Start with the end... : Higgs!
  - The Standard Model
- How to calculate with high energies? A reminder.
  - Lorentz Transformation
  - Invariants
  - Colliding particles

Kairo is synoniem met seksueel geweld

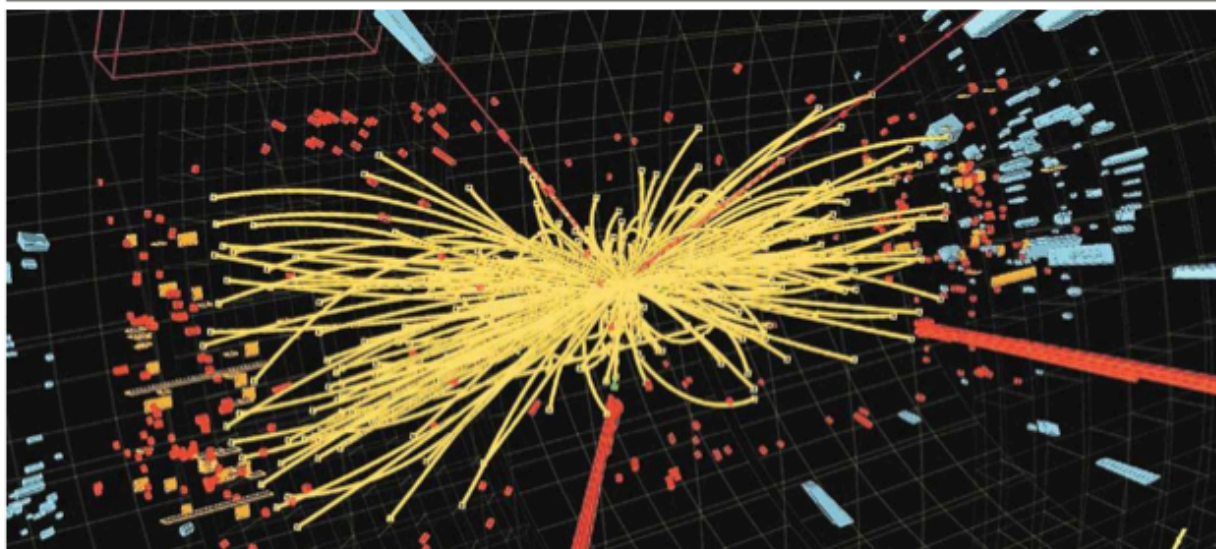
buitenland 10

Pininfarina gaf Ferrari een gezicht

het grote verhaal 12-13

Afstudeerfilms: lelijke kinderen, dolende zielen

film 18-19



Grafische weergave van de sporen van een proton-protonbotsing in een van de deeltjesdetectoren van CERN, het deeltjesversnellerinstituut bij Genève. Foto AFP / CERN

Historische stap in het onderzoek naar de bouwstenen waaruit heelal is opgebouwd

## Higgsdeeltje 'vrijwel zeker' ontdekt

**Door BRUNO VAN WAYENBURG**  
AMSTERDAM. Na twee uur spanningsrekkende praatjes komt CERN-directeur Rolf Heuer vanmorgen met de mededeling: „We hebben een ontdekking, de waarneming van een nieuw deeltje, met de eigenschappen van een Higgs-boson“.

Het Higgsdeeltje dus, het enige deeltje in het Standaardmodel van de natuurkunde waarvan het bestaan wel voorspeld was maar nog niet aangetoond.

Het is het deeltje dat andere deeltjes hun massa geeft.

Het is groot nieuws: de mededeling leidt tot een ontlasting in de zaal bij het CERN, het deeltjesversnellerinstituut bij Genève. Het publiek van vooral natuurkundigen klappt en joelt.

Aanvankelijk aarzelend applausdissen ook de onderzoekers en pers die samengekomen zijn in Nikhef, het instituut voor deeltjesfysica in Amsterdam.

Het lijkt er nu toch echt op dat ze, zij het virtueel, aanwezig zijn bij een historische aankondiging. Uit presentaties van Joe Incandela van de CMS-deeltjesdetector en van Gianotti van de ATLAS-detector, blijkt dat er maar een hele kleine kans op toeval is: minder dan 1 op de 3,5 miljoen. „Er is absoluut een nieuw deeltje ontdekt, dat valt niet meer te ontkennen“, zegt ook Stan Bentvelsen, projectleider van de Nederlandse inbreng in het ATLAS-experiment, waarbij 45 onderzoekers betrokken zijn.

Om het Higgsdeeltje aan te tonen,

moet je nieuwe deeltjes maken. Dat is peperduur, maar eenvoudig. Je laat in een deeltjesversneller deeltjes met vrijwel de lichtsnelheid met elkaar botsen. Hoe harder de botsing, hoe meer energie er wordt omgezet in nieuwe deeltjes. Zoals het Higgs-boson, dat in de jaren zestig voorspeld werd door zes theoretisch natuurkundigen. Het werd naar een van hen genoemd, Peter Higgs.

De Higgs is nodig om te verklaren hoe het komt dat alles massa heeft: doordat deeltjes worden afgeremd door het zogeheten Higgs-veld. Zo alomtegenwoordig als het uitgesmeerde Higgs is, zo ongrijpbaar is het als deeltje.

Zo gauw het ontstaat uit de enorme energie die vrijkomt bij een botsing, zo snel valt het ook weer uit el-

kaar in verschillende elementaire deeltjes. Alleen die brokstukken zijn, meteen na een botsing, goed te zien in de detectoren.

Maar veel vaker ontstaat er bij een botsing geen Higgs, maar een mix van al bekende deeltjes. Onderscheid maken tussen 'Higgs- en niet-Higgs-botsingen' is een kwestie van netjes meten en turven en zware statistiek.

Daarover gaat het allemaal, in de presentaties van Gianotti en Incandela.

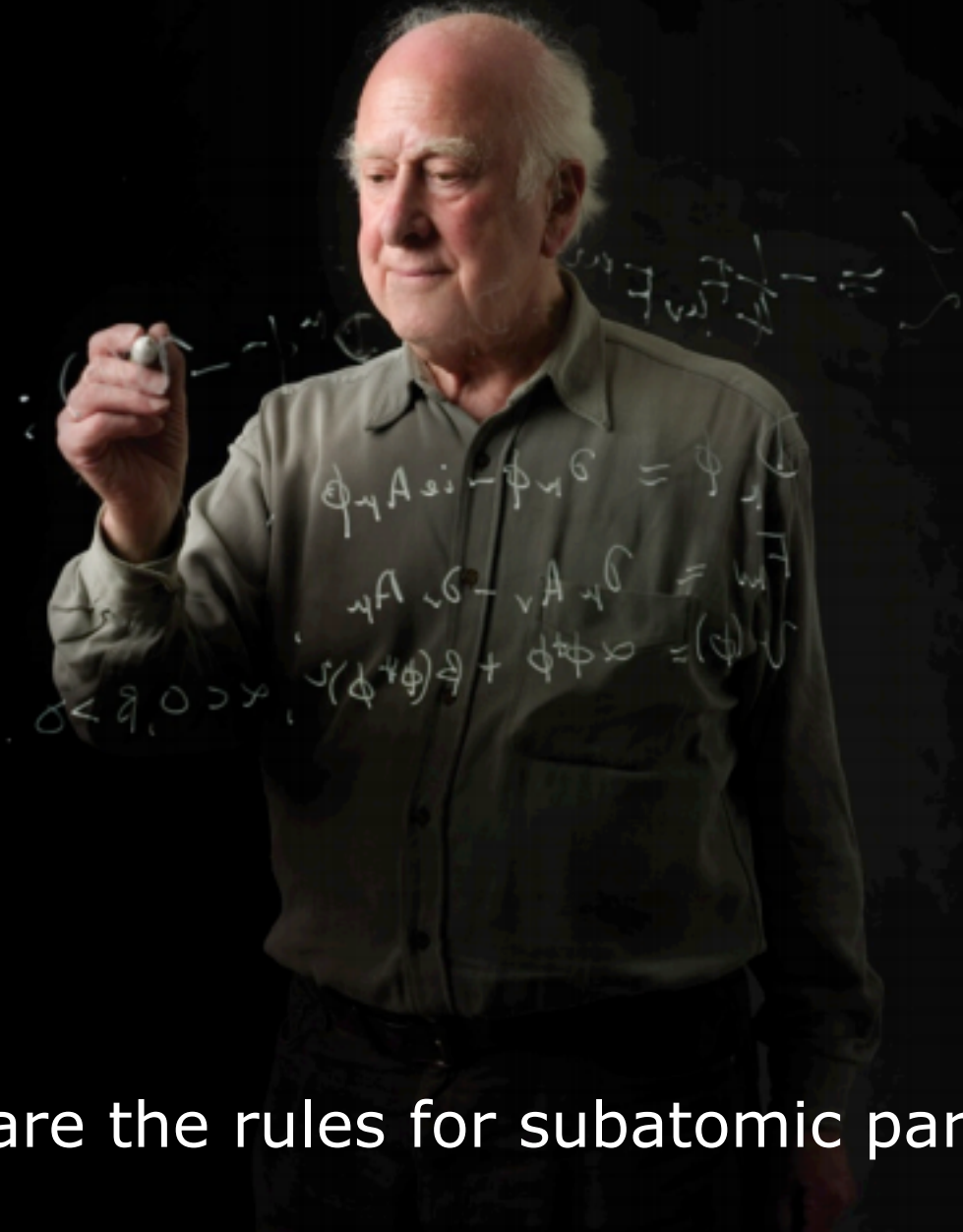
In het deeltjesjagen gaat het om de sigma-waarde: een statistische maat voor de kans dat de gevonden botsingen, ook al lijken ze op een nieuw deeltje, toch een toevallige uitschieter zijn. De afspraak is dat je een deeltje pas mag claimen bij een sigma van 5: de kans dat het om een toevallige

uitschieter zou gaan is dan 1 op de 3,5 miljoen. Incandela eindigt met een tergende 4,9, net geen 5. Maar Gianotti komt na eindeloze details uit op 5,0.

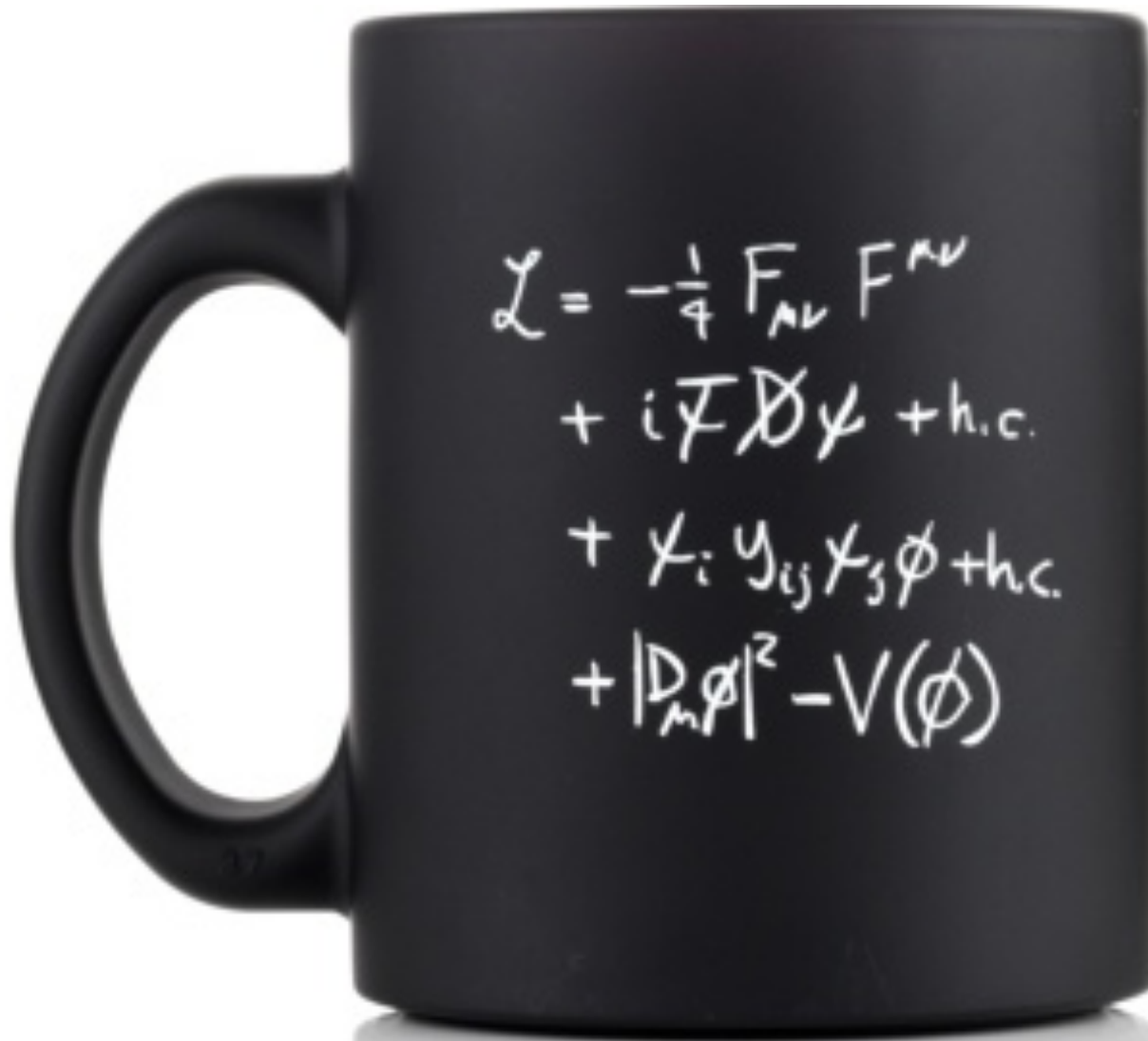
„Nu moeten we onderzoeken of het ook de Higgs is“, zegt Bentvelsen. Daar lijkt het wel op. „Al vervalt het wel iets vaker in twee fotonen dan je zou verwachten.“

Maar mocht het geen Higgs-boson zijn, of zelfs maar een licht afwijkend Higgs-boson, dan zou dat nog groter nieuws zijn. Want meer nog dan naar het Higgs-boson, snakken natuurkundigen naar 'nieuwe natuurkunde': metingen of deeltjes die eindelijk eens een keer niet overeenkomen met het Standaardmodel, is dus het devies. „Dit is pas het begin“, concludeert ook Gianotti.

- Why is the Higgs particle so special?
- The Standard Model



What are the rules for subatomic particles?



- Describes the behaviour of particles

## Photons $F_{\mu\nu}$

(Maxwell equations!  
E-field, B-field,  
electro-magnetic waves, ...)

## Particles $\psi$

("normal" matter, electrons,  
quarks, ...)

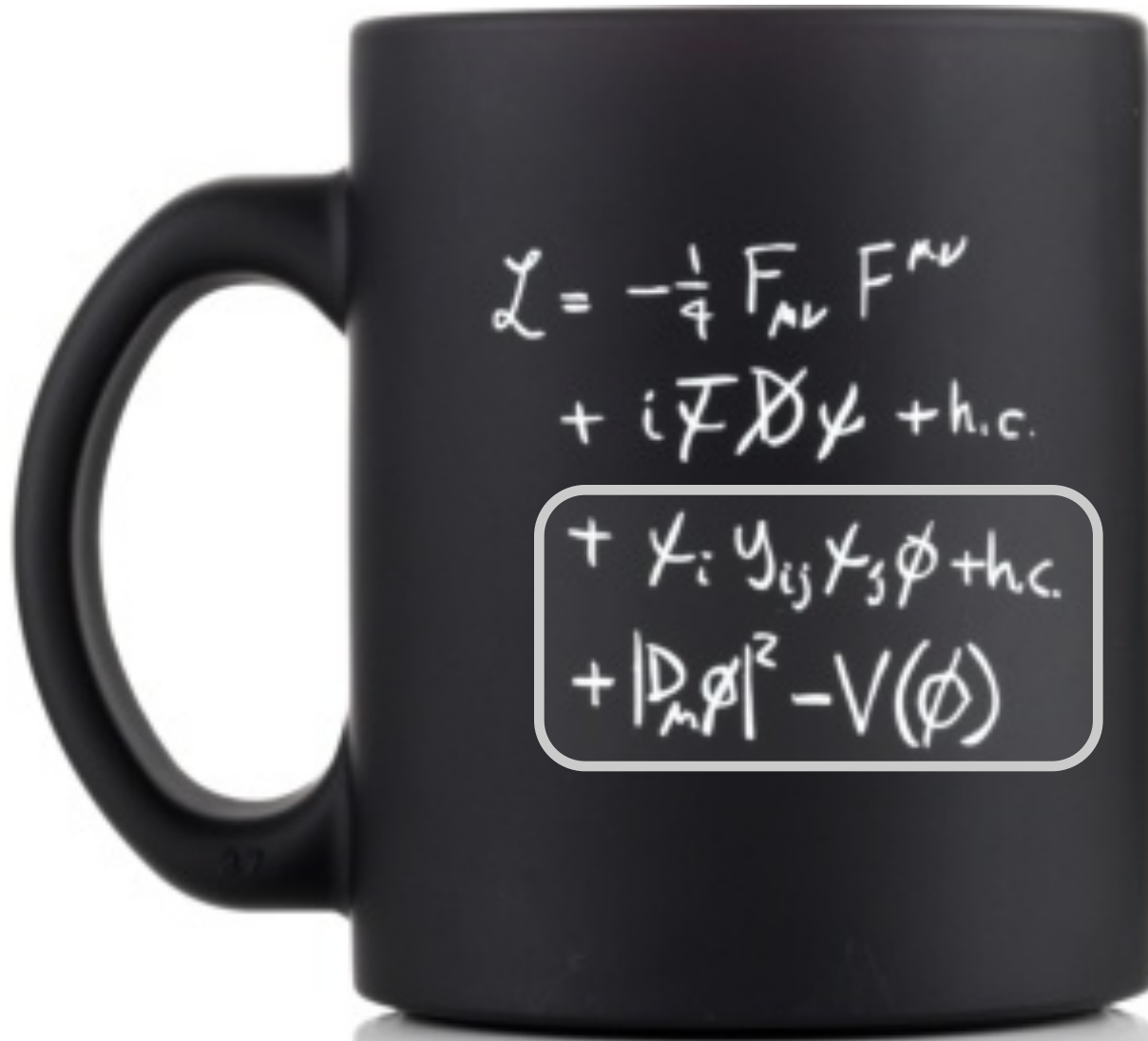
## Interactions $D$

(how the particles "feel"  
each other)

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + \bar{\psi} \not{D} \psi + \text{h.c.} \\ & + \bar{\chi}_i Y_{ij} \chi_j \phi + \text{h.c.} \\ & + |D_\mu \phi|^2 - V(\phi) \end{aligned}$$

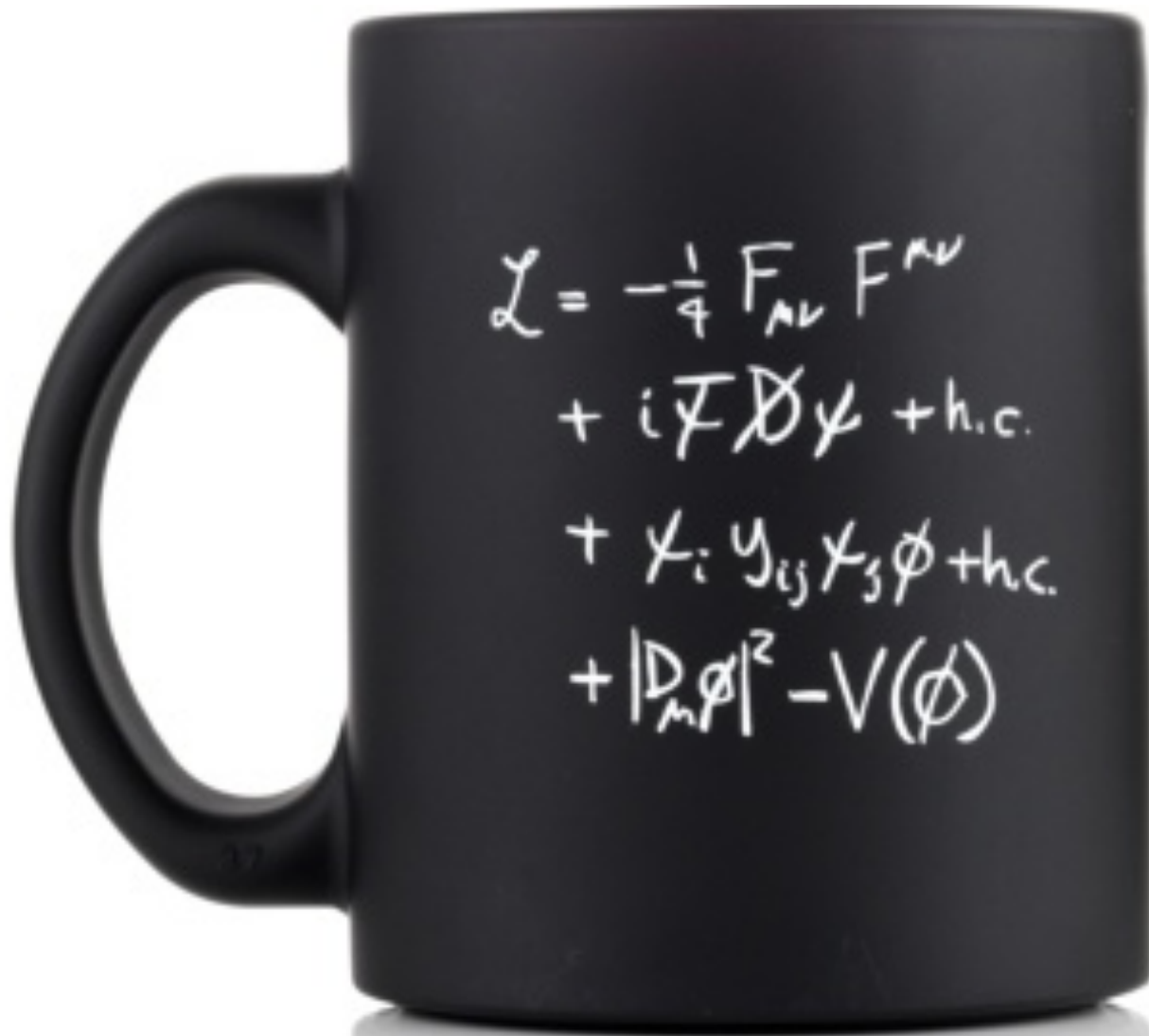
$\psi\psi\phi$  **Mass**  
(for "normal" particles)

$\phi$  **Higgs**



➤ Half of the mug is about Higgs!





For sale in the CERN shop...

# Higgs and Mass?

- Mass is “exchange rate” between force and acceleration  
But... what **is** it ?

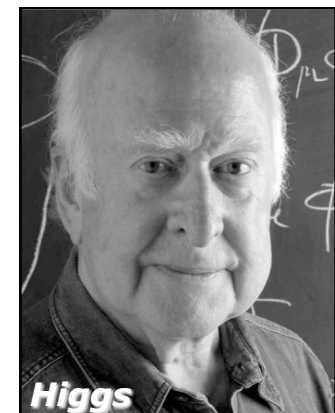
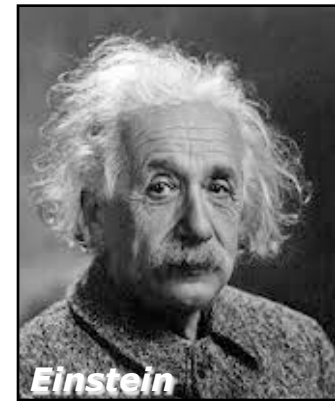
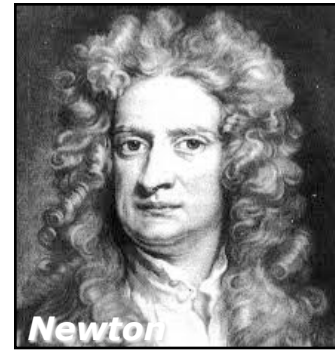
$$F = m \times a$$

- Mass is energy  
But... where does it **come from** ?

$$E = m \times c^2$$

- Mass is friction with Higgs field!

$$m: \psi\psi\phi$$



*“Wij zwemmen in een oceaan van Higgs deeltjes,  
... alsof we vissen zijn en nu hebben vastgesteld dat  
er water om ons heen is.”*

Prof. Robbert Dijkgraaf



# Outline of today

- Introduction
  - Start with the end... : Higgs!
  - The Standard Model
- How to calculate with high energies? A reminder.
  - Lorentz Transformation
  - Invariants
  - Colliding particles

# The Standard Model

These lectures deal with the

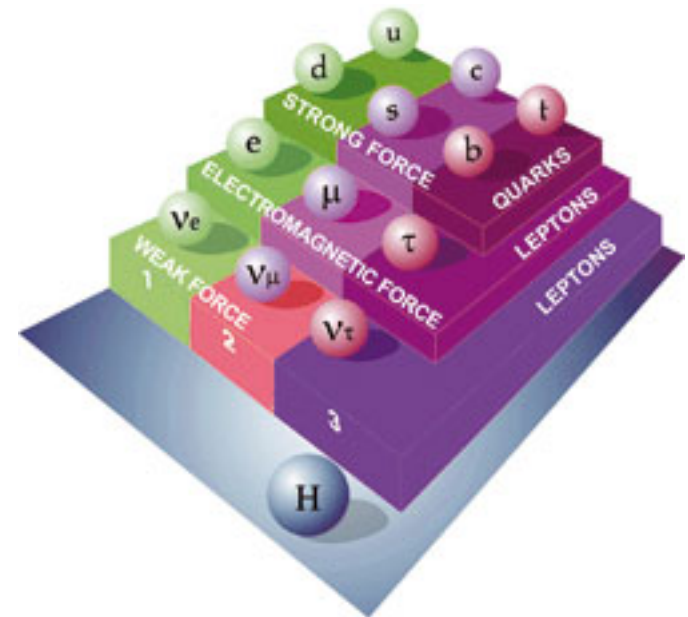
- Formalism
- Concepts

on

- Particles
- Interactions

jointly known as the Standard Model

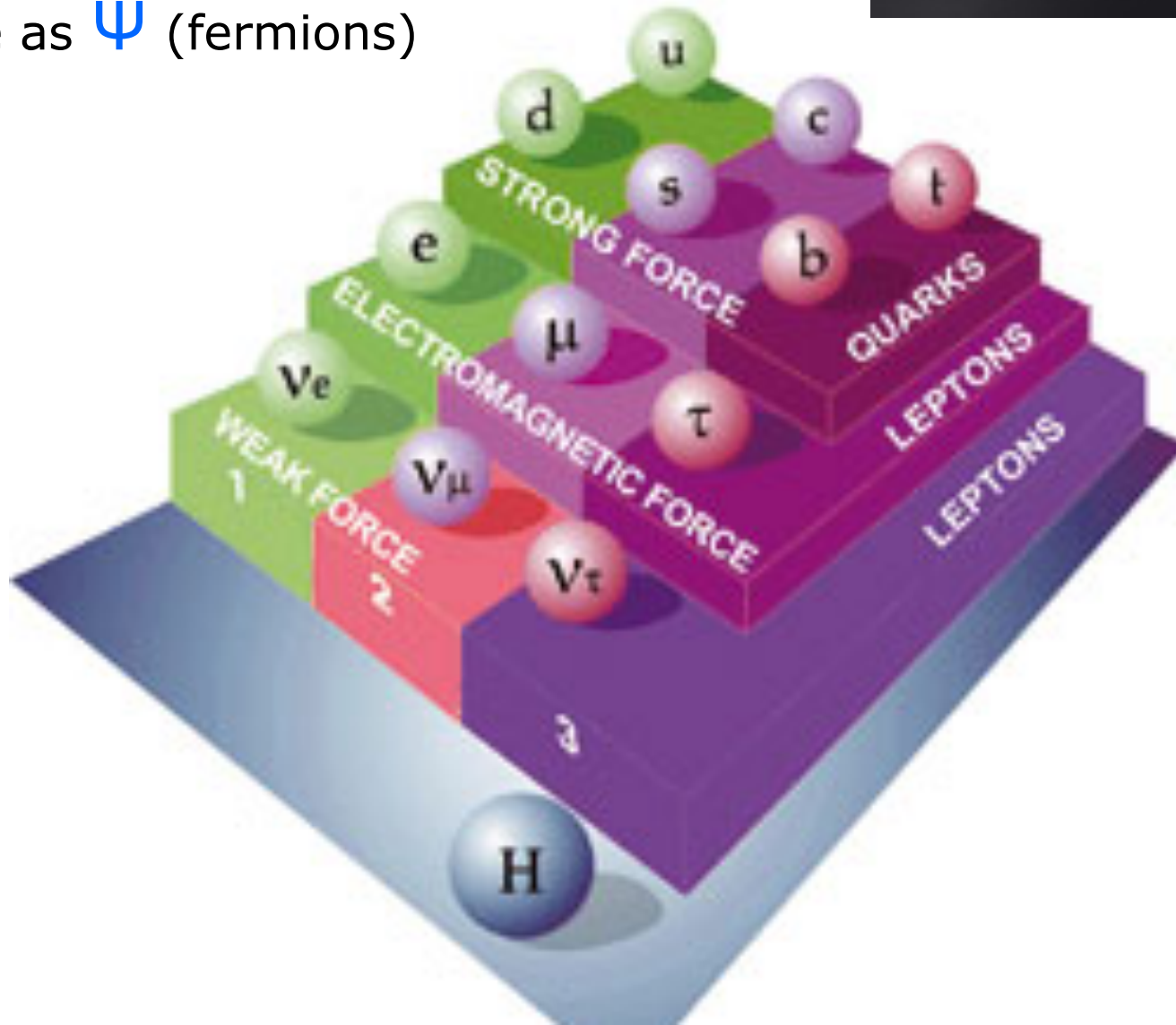
$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i\bar{\psi} \not{D} \psi + \text{h.c.} \\ & + \bar{\psi}_i Y_{ij} \psi_j \phi + \text{h.c.} \\ & + |D_\mu \phi|^2 - V(\phi) \end{aligned}$$



# The Standard Model

All "matter" particles are described here as  $\Psi$  (fermions)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}\not{D}\Psi + \text{h.c.}$$

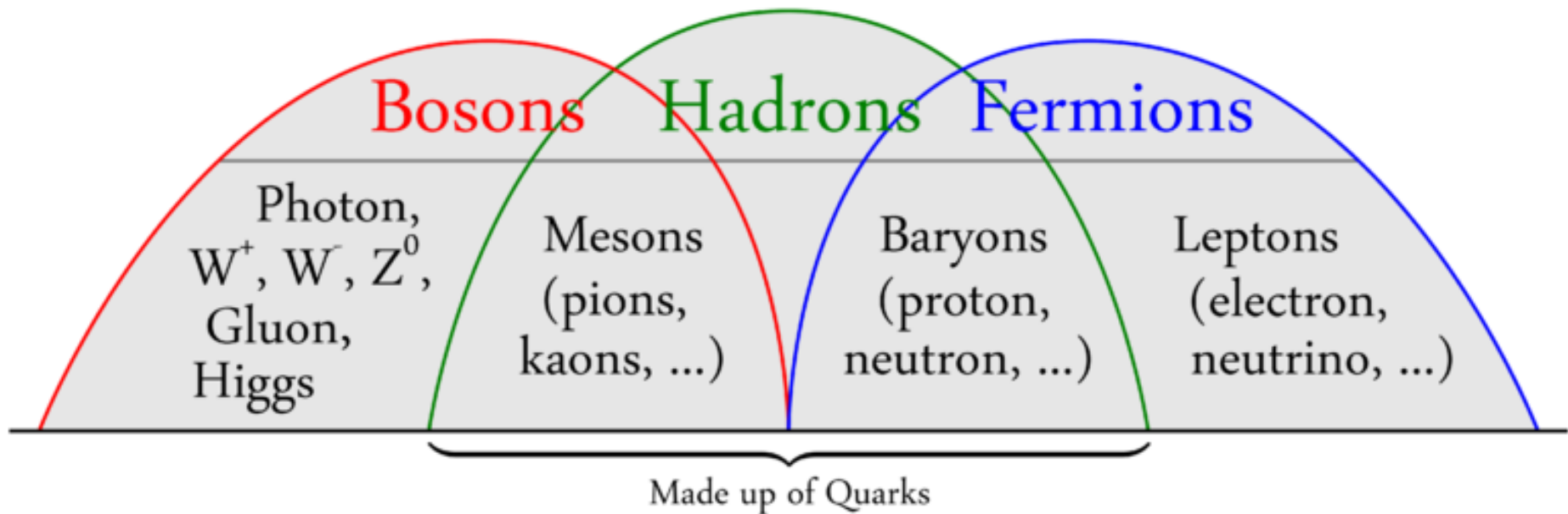
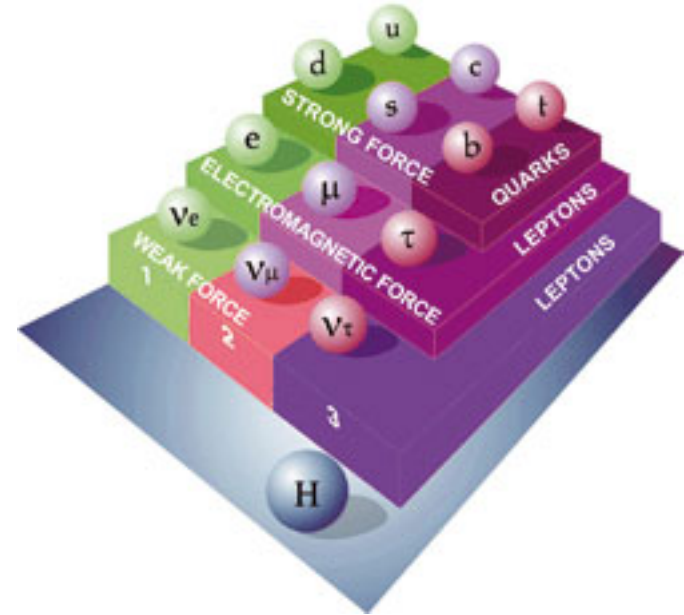


# The Standard Model

			Quarks
			Leptons
			Anti-Quarks
			Anti-Leptons
			Bosons

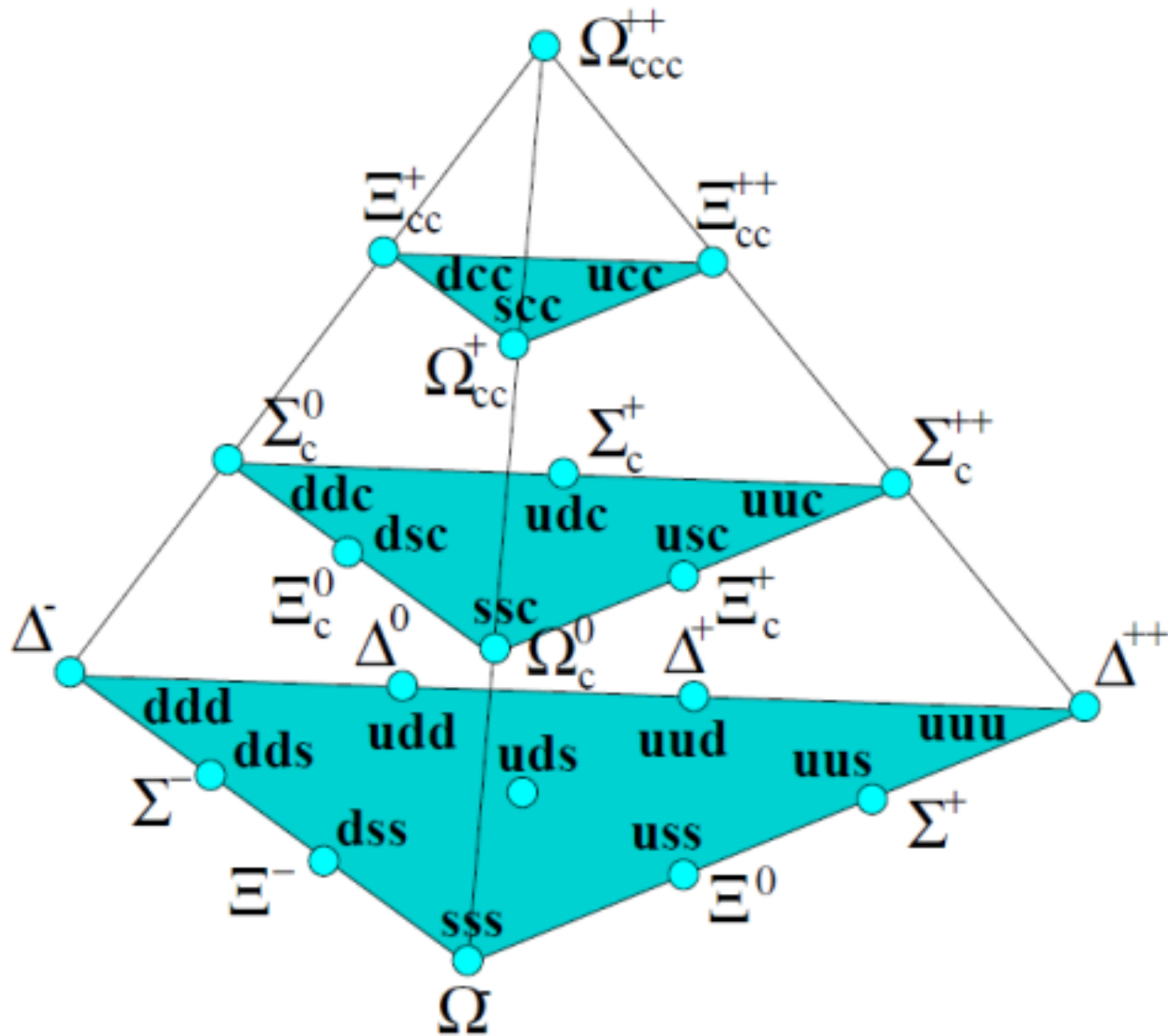
# Particles

- Quarks and leptons...:





# Particles...



# Particles

Three generations:

	I	II	III	<u>Charge</u>
quarks	u (1947)	c (1976)	t (1995)	+2/3 e
	d (1947)	s (1978)	b (1978)	-1/3 e
leptons	e (1895)	$\mu$ (1936)	$\tau$ (1973)	-1 e
	$\nu_e$ (1956)	$\nu_\mu$ (1963)	$\nu_\tau$ (2000)	0 e

Particles

and

Anti-particles

Three generations

quarks

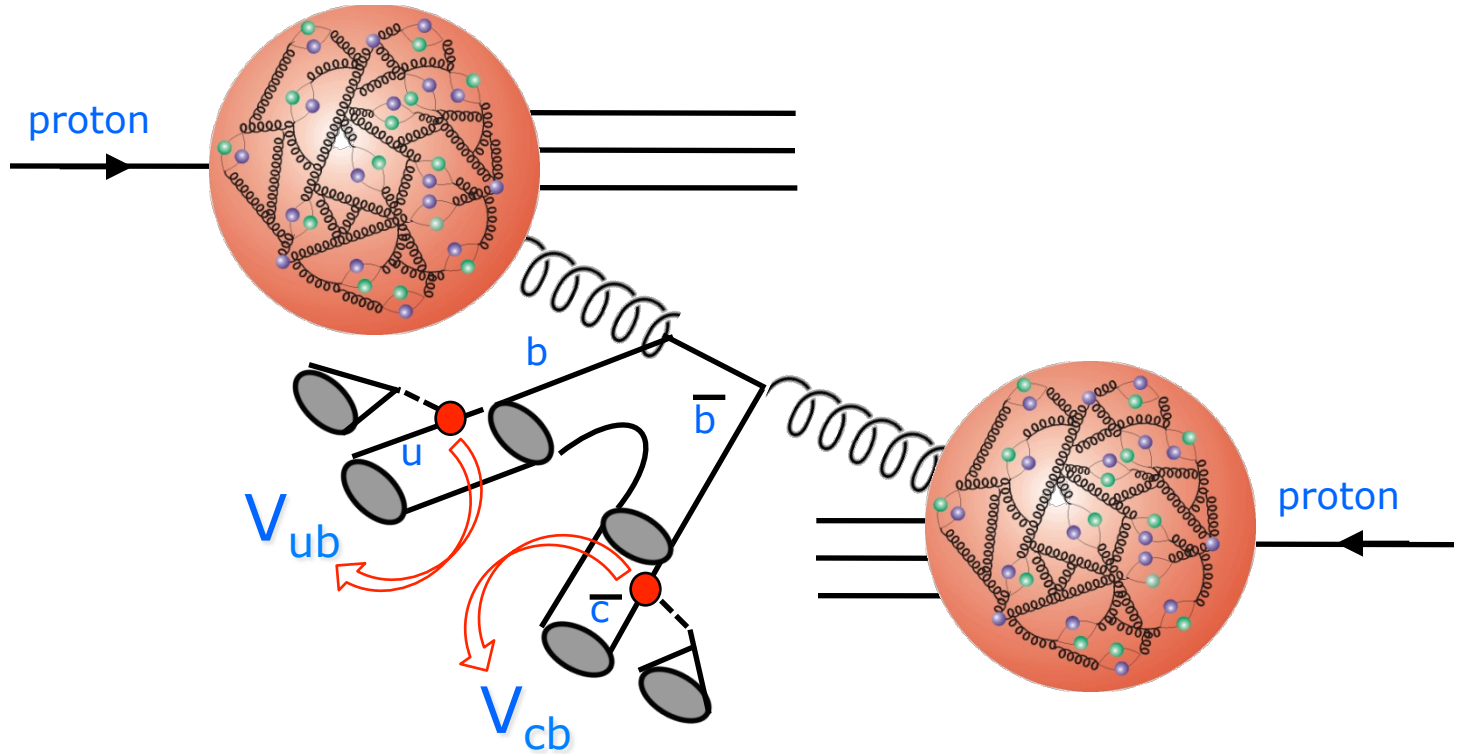
leptons

	I	II	III	<u>Charge</u>	<u>Charge</u>	I	II	III
	u	c (1976)	t (1995)	+2/3 e	-2/3 e	$\bar{u}$	$\bar{c}$	$\bar{t}$
	d	s (1947)	b (1978)	-1/3 e	+1/3 e	$\bar{d}$	$\bar{s}$	$\bar{b}$
	e (1895)	$\mu$ (1936)	$\tau$ (1973)	-1 e	+1 e	$\bar{e}$	$\bar{\mu}$	$\bar{\tau}$
	$\nu_e$ (1956)	$\nu_\mu$ (1963)	$\nu_\tau$ (2000)	0 e	0 e	$\bar{\nu}_e$	$\bar{\nu}_\mu$	$\bar{\nu}_\tau$

***Where did the anti-matter go?***



# Personal Intermezzo



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\not{D}\psi + \text{h.c.} + \bar{\psi}_i y_{ij} \psi_j \phi + \text{h.c.} + |D_\mu \phi|^2 - V(\phi)$$

$Y_{ij} \rightarrow V_{cb}, V_{ub}$

Difference between matter and anti-matter



SUISSE  
FRANCE

CMS

LHCb

CERN Prévessin

ATLAS

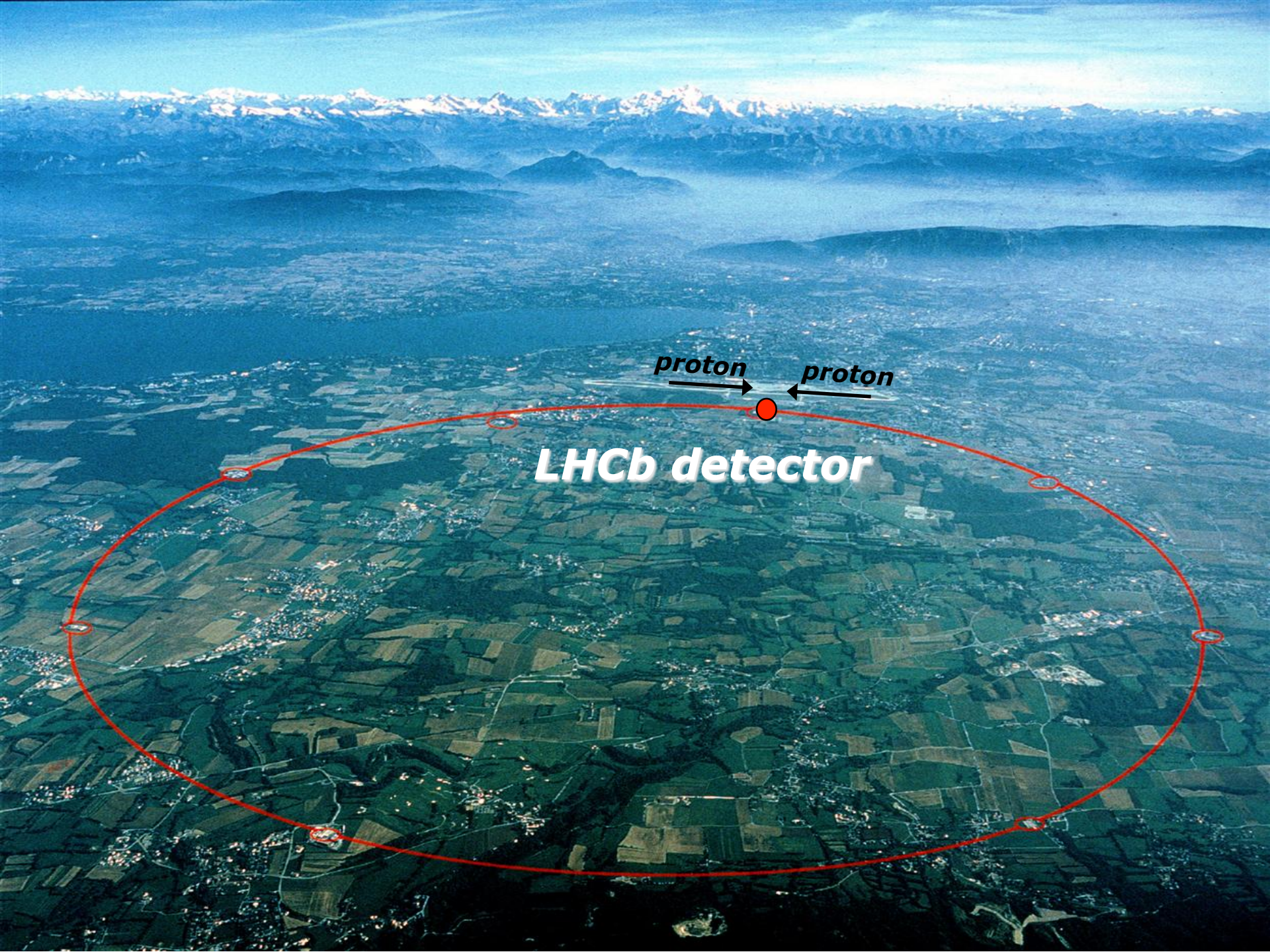
CERN Meyrin

SPS 7 km

ALICE

LHC 27 km

PS 6.30 km

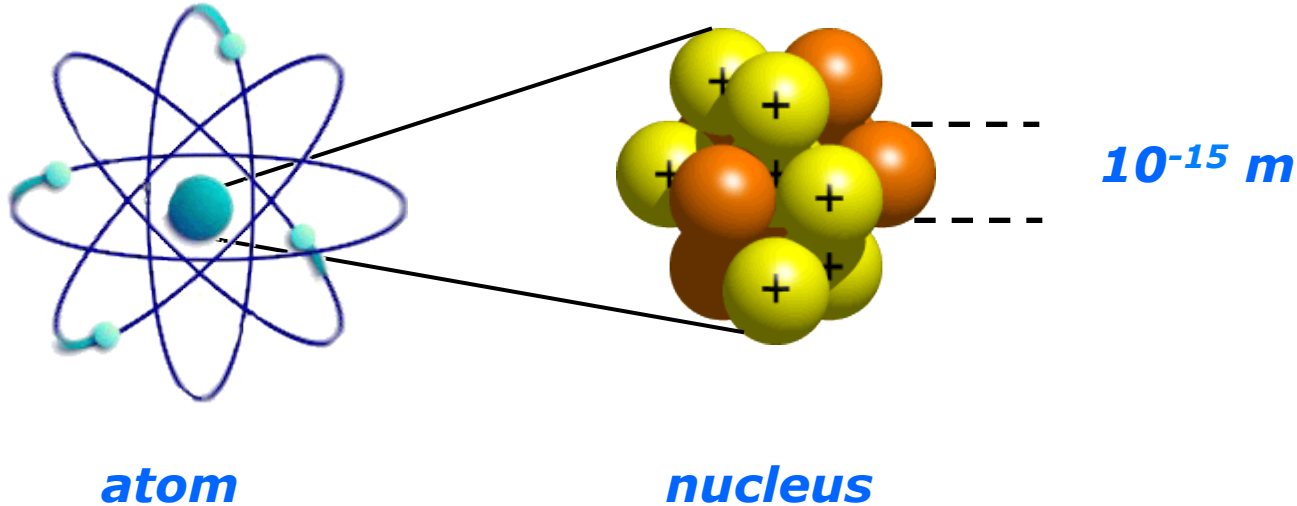


*proton*

*proton*

**LHCb detector**

# What energy is needed?



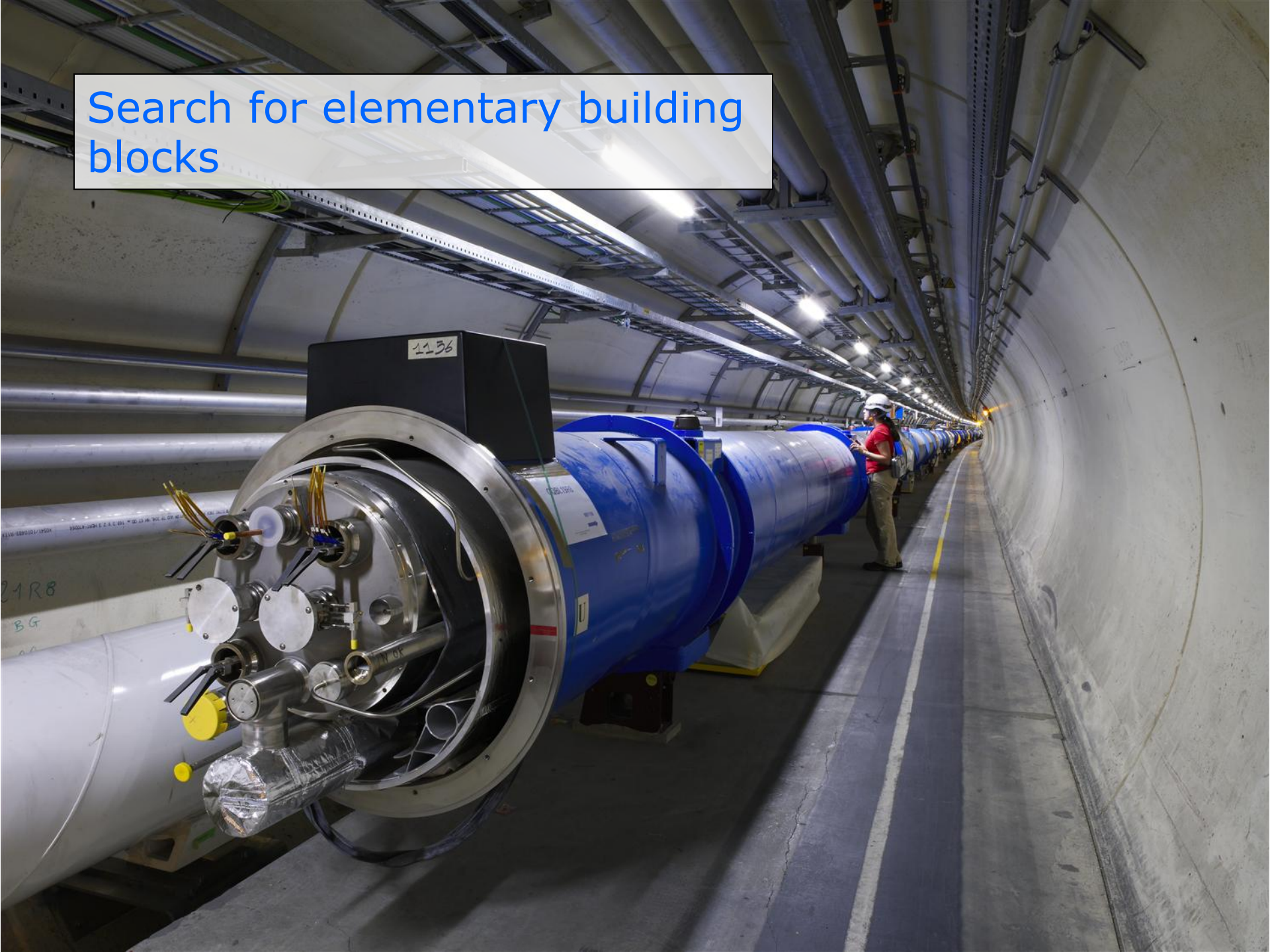
Object	Grootte	Energie
Atoom	$10^{-10} \text{ m}$	10keV
Kern	$10^{-14} \text{ m}$	10MeV
Nucleon	$10^{-15} \text{ m}$	0.1GeV
Quark	$<10^{-18} \text{ m}$	$>1 \text{ GeV}$

How to make energies around 100.000.000 eV or more ?

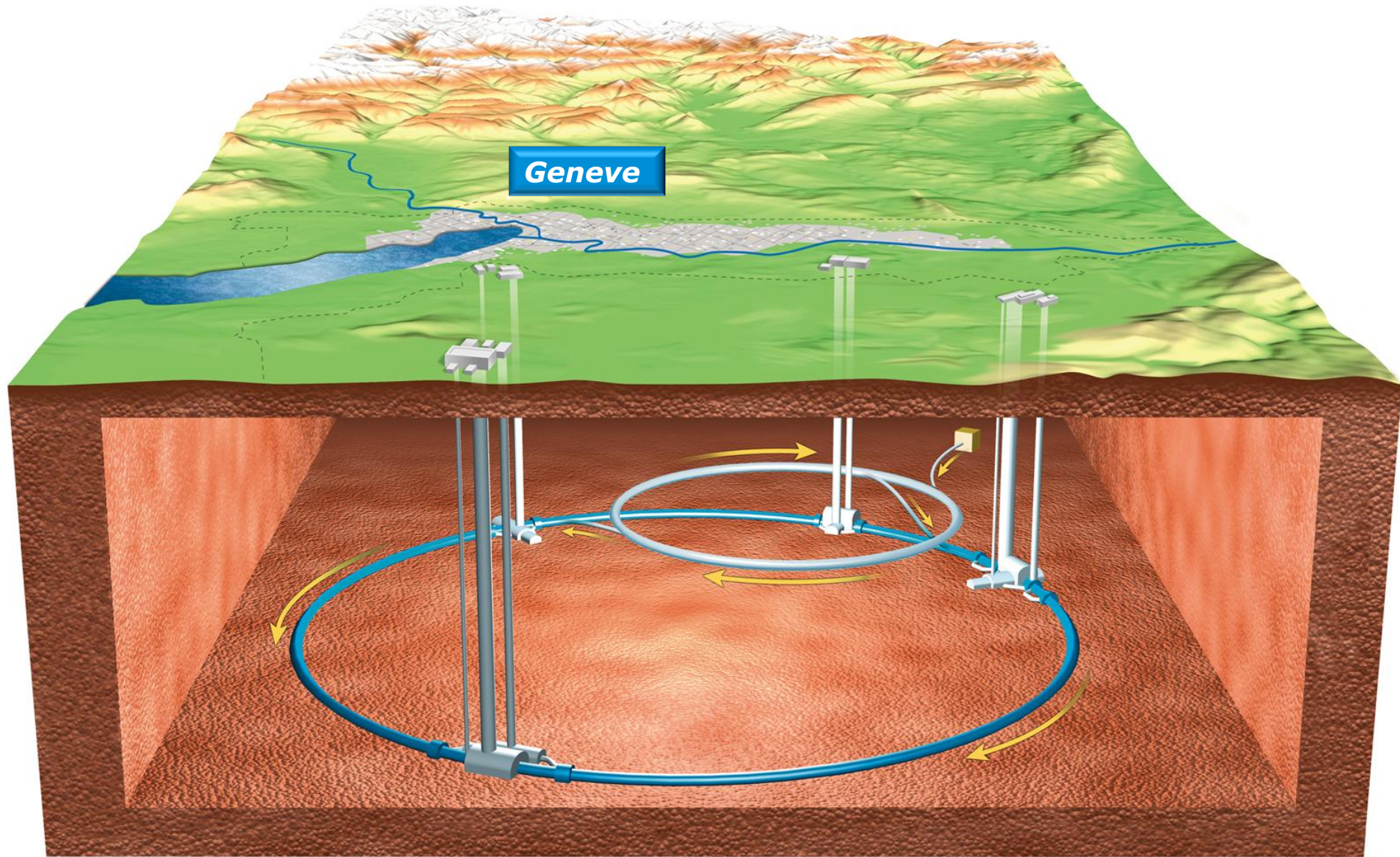
Energy of 1  $e^-$  that passes a potential difference of 1 V: 1 eV  
Energy of mass of 1 proton:  $m = E/c^2$ : 1 GeV



Search for elementary building blocks



# LHC accelerator





LHC

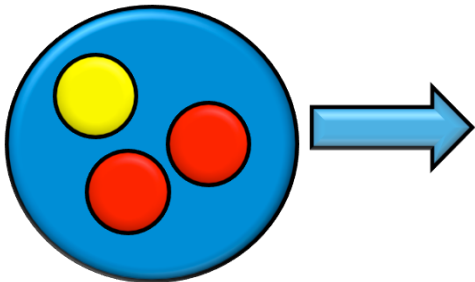
Energy limited by field of 1232 dipole magnets:  $B = 8.4 \text{ T}$



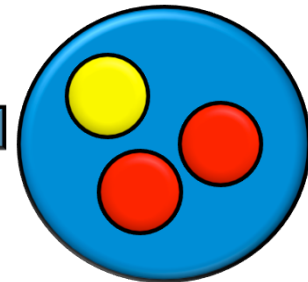
## *Klassiek botsen*

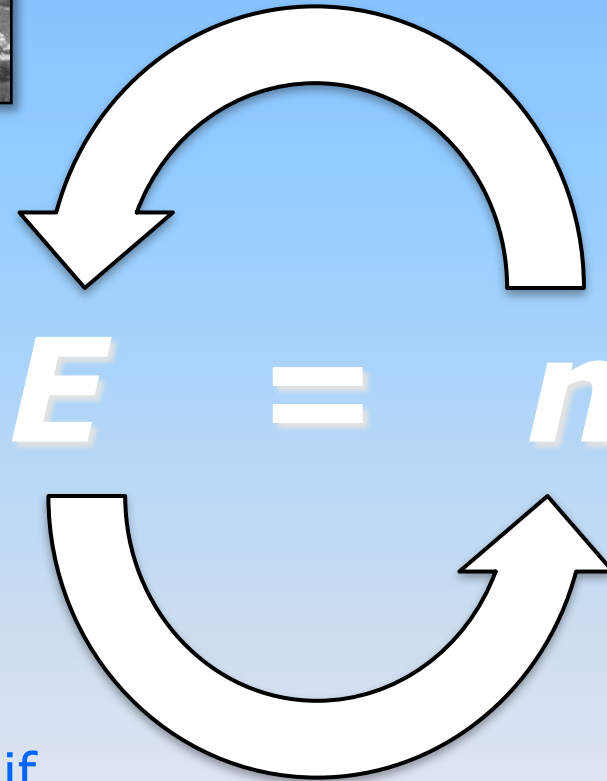
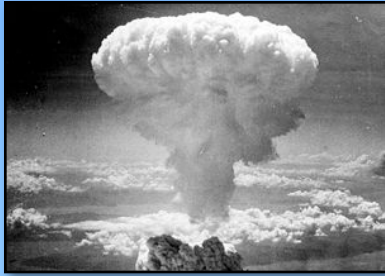
## *Quantummechanisch botsen*

*proton*



*proton*





$$E = mc^2$$

Create new particles if  
energy is large enough  
(and if they exist...)



# Outline of today

- Introduction

- Start with the end... : Higgs!
- The Standard Model

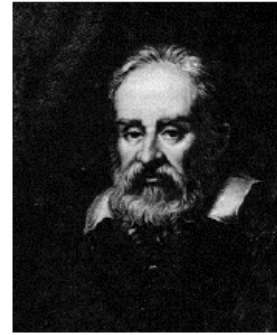


## How to calculate with high energies? A reminder.

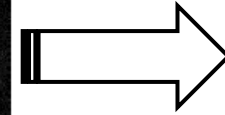
- Lorentz Transformation
- Invariants
- Colliding particles

# Summary special relativity

- Lorentz transformation
- Length contraction & Time dilatation
- Adding velocities
  
- Relativistic energies
- Relativistic kinematics
- Collision
- Decay



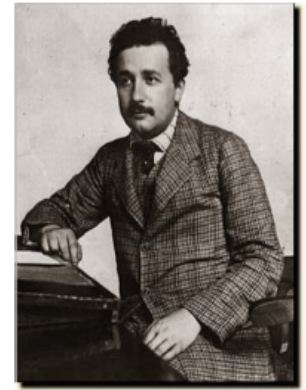
Galileo Galilei



Hendrik Antoon Lorentz

# Lorentz transformation

- 1) Speed of light constant
- 2) Every (inertial) coordinate system equivalent



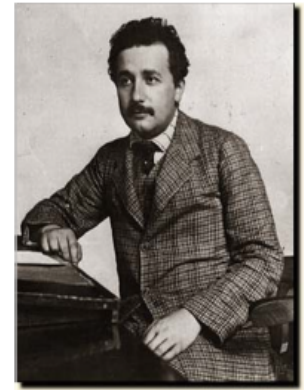
Albert Einstein

$x=ct$  becomes  $x' = ct'$



# Lorentz transformation

- 1) Speed of light constant
- 2) Every (inertial) coordinate system equivalent
- Find transformation rules:



Albert Einstein

Galilei:

Lorentz:

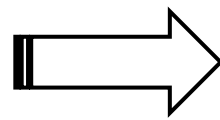
$x=ct$  becomes  $x' = ct'$  :

Newton :  $x' = x - vt$     wordt :  $x' = \gamma(x - vt)$      $x' = ct' = \gamma(ct - vt)$

Newton :  $x = x' + vt'$     wordt :  $x = \gamma(x' + vt')$      $x = ct = \gamma(ct' + vt')$

➤ Find  $\gamma$  :

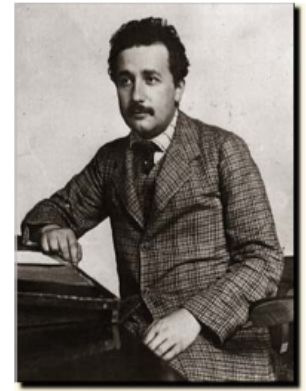
$ct' = \gamma(ct - vt)$   
 $ct = \gamma(ct' + vt')$



$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

# Lorentz transformation

- 1) Speed of light constant
  - 2) Every (inertial) coordinate system equivalent
- Find transformation rules:



Albert Einstein

Galilei:

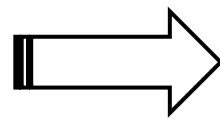
Lorentz:

$x=ct$  becomes  $x' = ct'$  :

$$\begin{array}{l}
 \text{Newton : } x' = x - vt \quad \text{wordt : } x' = \gamma(x - vt) \\
 \text{Newton : } x = x' + vt' \quad \text{wordt : } x = \gamma(x' + vt')
 \end{array}
 \rightarrow
 \begin{array}{l}
 x' = \boxed{ct' = \gamma(ct - vt)} \\
 x = ct = \gamma(ct' + vt')
 \end{array}$$

➤ Find  $\gamma$  :

$$\begin{array}{l}
 ct' = \gamma(ct - vt) \\
 ct = \gamma(ct' + vt')
 \end{array}$$



$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \gamma(x - vt)$$

$$x = \gamma(x' + vt')$$

# Consequences: Lorentz contraction

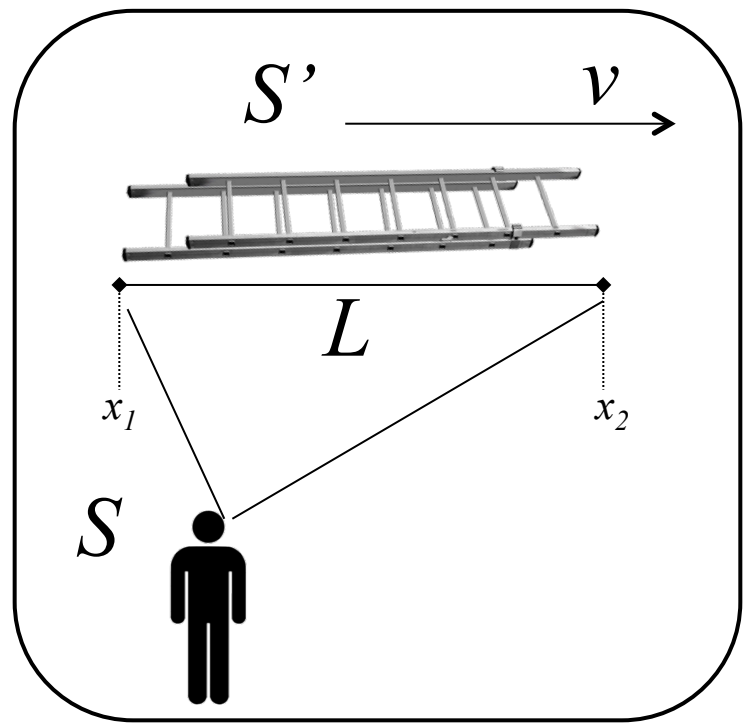
- Stick with length  $L_0$  in system  $S'$  :
  - moving relative to system  $S$  with speed  $v$
  - Observer in  $S$  sees length  $L$
  - At same time  $t$  in fixed frame:  $t_1 = t_2$

$$L_0 = x'_2 - x'_1$$

$$t = 0 : x'_2 = \gamma x_2; \quad x'_1 = \gamma x_1$$

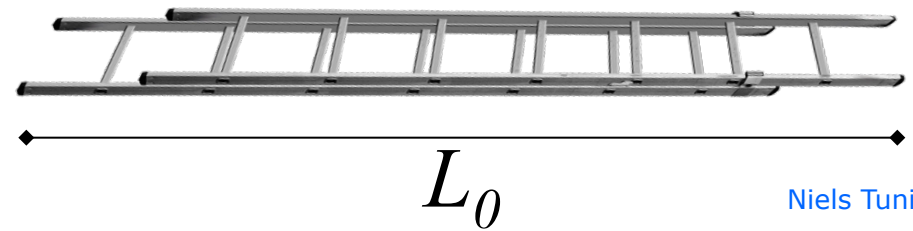
(Length  $L_0$  as seen in moving frame  $S'$ , is at rest)

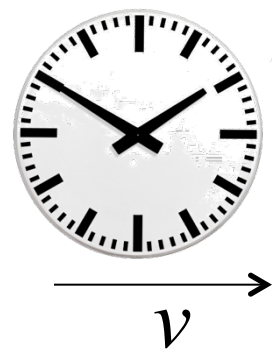
➤ Length  $L$  is factor  $1/\gamma$  smaller in rest frame  $S$ :



$$L = x_2 - x_1 = \frac{x'_2 - x'_1}{\gamma} = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

(Length  $L$  as seen in frame  $S$ , is difference between coordinates  $x_2$  and  $x_1$  in frame  $S$ .)





# Consequences: Time dilatation

- Clock is moving in frame  $S'$  with relative speed  $v$
- Suppose clock is emitting light pulses
  - Time interval between pulses in frame  $S'$  :
  - Light pulses are emitted from same point  $x'$  in **moving** frame:

$$\Delta t' = t_2' - t_1'$$

$$x_1' = x_2'$$

- What sees the observer at rest in frame  $S$ ?

- First pulse:  $t_1 = \gamma(t_1' + vx_1'/c^2)$

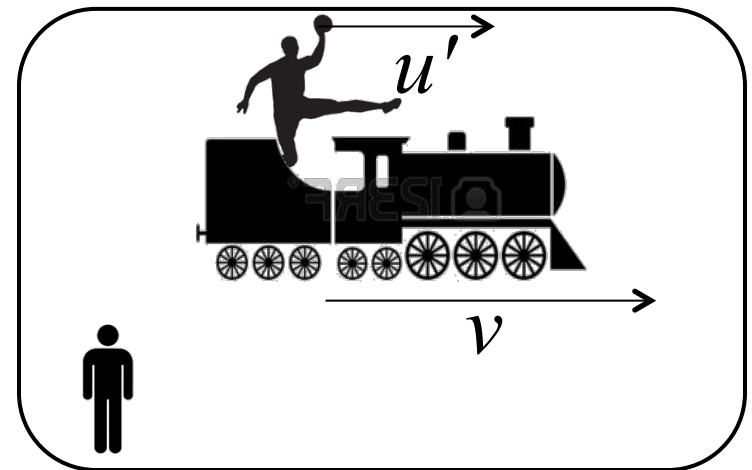
- Second pulse:  $t_2 = \gamma(t_2' + vx_2'/c^2)$

- Hence: 
$$\Delta t = t_2 - t_1 = \gamma(t_1' - t_2' + \cancel{v/c^2 (x_1' - x_2')}^{=0}) = \gamma \Delta t'$$

$$\Delta t = \gamma \Delta t'$$

➤ Clock period is seen factor  $\gamma$  longer for observer at rest

# Adding velocities



- Time and space transformation:

$$\begin{array}{l}
 x = \gamma(x' + vt') \\
 t = \gamma\left(t' + \frac{vx'}{c^2}\right)
 \end{array}
 \quad \longrightarrow \quad
 \begin{array}{l}
 dx = \gamma(dx' + vdt') \\
 dt = \gamma\left(dt' + \frac{vdx'}{c^2}\right)
 \end{array}$$

- Hence observer in frame S sees velocity  $u_x$ :

$$u_x = \frac{dx}{dt} = \frac{\gamma(dx' + vdt')}{\gamma\left(dt' + \frac{vdx'}{c^2}\right)} \stackrel{/dt'}{=} \frac{(u'_{x'} + v)}{\left(1 + u'_{x'} \frac{v}{c^2}\right)}$$

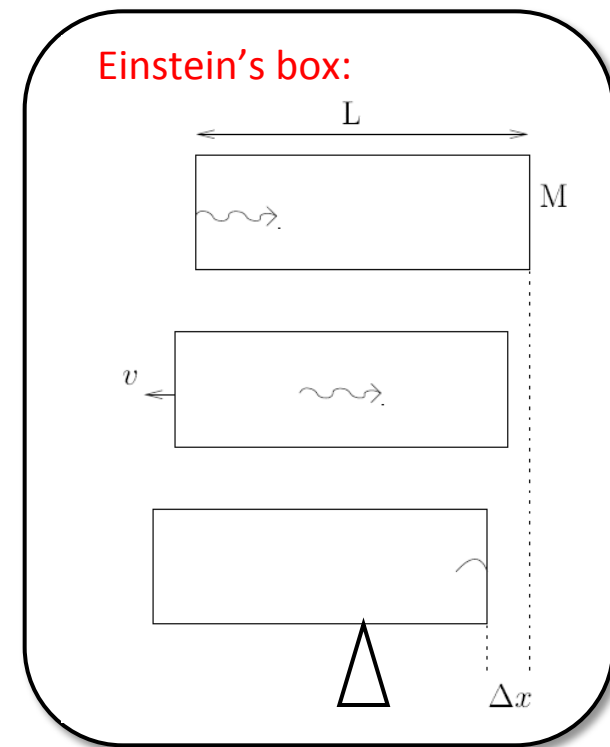
(Galilei:  $u = u' + v$ )

- Ex: If train goes fast ( $v=c$ ), then velocity  $u_x$  seen by observer:

$$u_x = (u' + c)/(1 + u'/c) = c$$

# $E=mc^2$

- 1) Photon is emitted from box
- 2) Momentum conservation: box moves
- 3) Photon is absorbed by box: box stops



NB: Centre-of-mass of entire system remains at rest

- Photon must carry a "mass equivalent to the energy of the photon,  $m$ "

- Box: mass  $M$  over length  $\Delta x$ :  $M\Delta x$
- Photon: mass  $m$  over length  $L$ :  $mL$
- System at rest:  $(M\Delta x + mL) = 0$

$$\Delta x = v\Delta t$$

$$v = \frac{p_{box}}{M} = -\frac{p_{photon}}{M} = -\frac{E_{photon}}{Mc}, \Delta t = \frac{L}{c}$$

$$\Delta x = -\frac{E_{photon}L}{Mc^2}$$



$$(M\Delta x + mL) = 0 \Rightarrow$$

$$L\left(-\frac{E}{c^2} + m\right) = 0 \Rightarrow$$

$$E = mc^2$$

# Relativistic energies

- Momentum:

- In rest:  $p = m_0 v$  (or at *low speeds*, to satisfy Newtonian dynamics)

- Moving mass:  $p = \gamma m_0 v$  (Relativistic momentum must be conserved in all frames)

- Einstein: equivalence between energy and mass

- In rest:  $E = m_0 c^2$

- Moving mass:  $E = \gamma m_0 c^2$

$$E = pc^2/v \rightarrow v/c = pc/E$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - p^2 c^2 / E^2}}$$

➤ E:

$$E_{tot} = \frac{m_0 c^2}{\sqrt{1 - \frac{(\bar{p}c)^2}{E_{tot}^2}}}$$

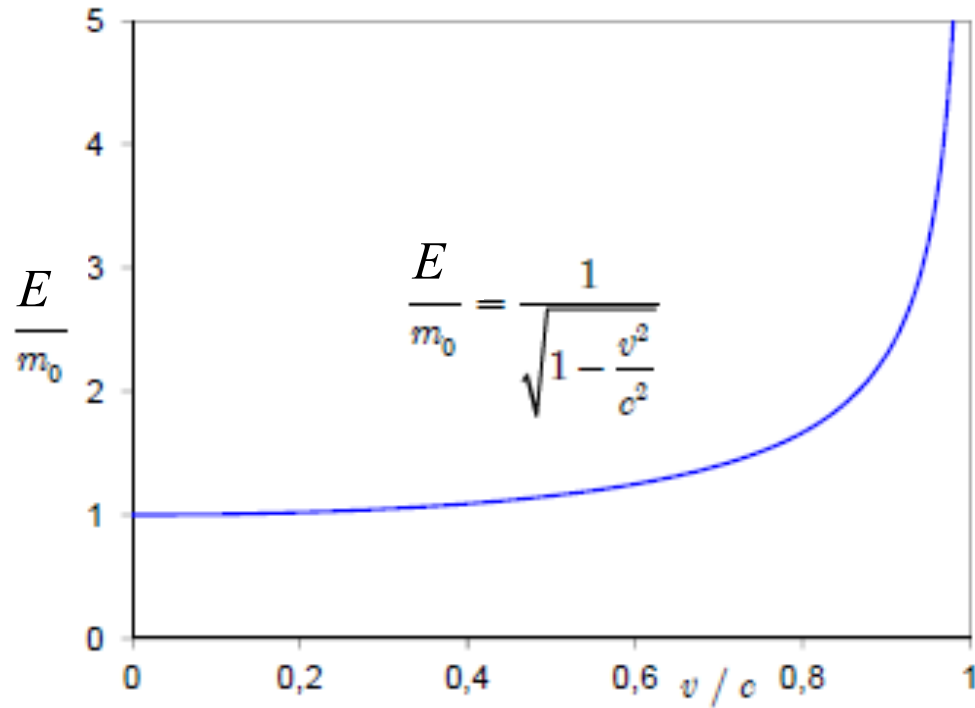
$$E_{tot}^2 - \bar{p}^2 c^2 = m_0^2 c^4$$

$$E = \gamma m_0 c^2$$

- Btw, a Taylor expansion gives classical kinetic energy:

$$E = mc^2 \left( 1 + \frac{1}{2} \frac{v^2}{c^2} + \frac{3}{8} \frac{v^4}{c^4} + \dots \right) = mc^2 + \frac{1}{2} mv^2 + \frac{3}{8} m \frac{v^4}{c^2} + \dots$$

# Relativistic energies



$$E = \gamma m_0 c^2$$



# 4-vectors

- Write  $(t, x)$  as 4-vector  $x^\mu$ :

$$x^0 \equiv ct, \quad x^1 \equiv x, \quad x^2 \equiv y, \quad x^3 \equiv z$$

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \quad (\mu = 0, 1, 2, 3)$$

- Nicely symmetric form of Lorentz transformation:

$$\begin{aligned} x'^0 &= \gamma(x^0 - \beta x^1) \\ x'^1 &= \gamma(x^1 - \beta x^0) \\ x'^2 &= x^2 \\ x'^3 &= x^3 \end{aligned} \quad \text{met} \quad \begin{aligned} \beta &\equiv \frac{v}{c} \\ \gamma &\equiv \frac{1}{\sqrt{1 - \beta^2}} \end{aligned}$$

$$x'^\mu = \sum_{\nu=0}^3 \Lambda^\mu_\nu x^\nu \quad (\mu = 0, 1, 2, 3)$$

“Boost” in x-direction:

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Invariants (“fixed length”)

- Write  $(t, x)$  as 4-vector  $x^\mu$ :

$$x^0 \equiv ct, \quad x^1 \equiv x, \quad x^2 \equiv y, \quad x^3 \equiv z$$

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \quad (\mu = 0, 1, 2, 3)$$

- Covariant and contravariant 4-vector related through metric  $g$ :

$$x_\mu = g_{\mu\nu} x^\nu$$

$$x_0 = x^0, \quad x_1 = -x^1, \quad x_2 = -x^2, \quad x_3 = -x^3$$

$$g_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Any pair of 4-vectors is invariant as:

$$a^\mu b_\mu = a_\mu b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 \text{ **invariant**}$$

– (similar to the *length* of a vector in Euclidean space)

# Spacetime

- Special relativity
  - Flat ("Minkowski") spacetime
  
- General relativity
  - Curved spacetime

$$g_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix}$$

# Spacetime

- General relativity

- Curved spacetime

- Line element (invariant)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

- Christoffel symbols:

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left( \frac{\partial g_{\rho\mu}}{\partial x^\nu} + \frac{\partial g_{\rho\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho} \right)$$

- Riemann curvature tensor:

$$R^\rho_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma}$$

- Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$T_{\mu\nu}$ : Energy-momentum tensor

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix}$$

# Intermezzo: Use of 4-vectors

- 4-vectors
  - Use for relativistic kinematics in particle collisions
  - Use for quantum-field description of matter fields:

The famous Dirac equation:

$$(i\gamma^\mu \partial_\mu - m) \psi = 0$$

with :  $\gamma^\mu = (\beta, \beta\vec{\alpha}) \equiv$  Dirac  $\gamma$ -matrices

Remember!

- $\mu$  : [Lorentz index](#)
- 4x4  $\gamma$  matrix: [Dirac index](#)

Less compact notation:

$$\left[ \left( \begin{array}{cc} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{array} \right) \frac{i\partial}{\partial t} - \left( \begin{array}{cc} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{array} \right) \frac{i\partial}{\partial x} - \left( \begin{array}{cc} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{array} \right) \frac{i\partial}{\partial y} - \left( \begin{array}{cc} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{array} \right) \frac{i\partial}{\partial z} - \left( \begin{array}{cc} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{array} \right) m \right] \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

# Energy-momentum 4-vector

- Example of invariant: **rest mass** (“*invariant mass*”)

$$p^\mu = \left( \frac{E}{c}, p_x, p_y, p_z \right)$$



$$p_\mu p^\mu = \frac{E^2}{c^2} - \mathbf{p}^2 = m^2 c^2$$

$$E_{tot}^2 - \vec{p}^2 c^2 = m_0^2 c^4$$

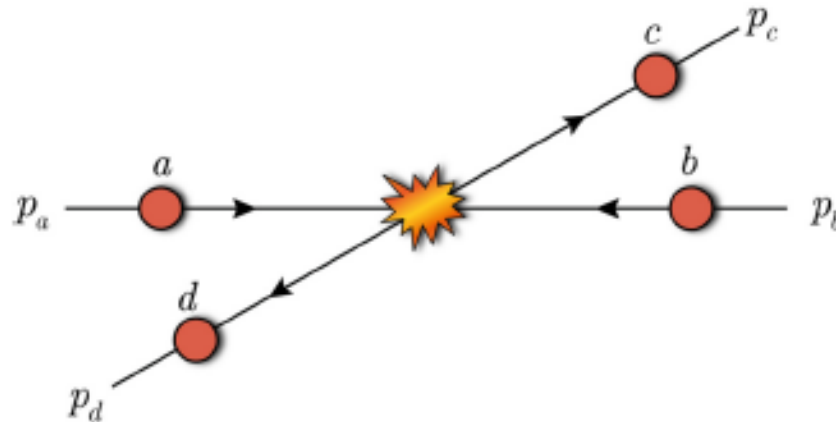
- Lorentz transformation on energy-momentum 4-vector:

$$x'^\mu = \sum_{\nu=0}^3 \Lambda_{\nu}^{\mu} x^\nu \quad (\mu = 0, 1, 2, 3)$$

$$\begin{pmatrix} \frac{E'}{c} \\ p_{x'} \\ p_{y'} \\ p_{z'} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{E}{c} \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

## Calculate with 4-vectors: colliding particles

- Elastic collision of two particles  $a$  and  $b$ :  $a + b \rightarrow c + d$
- Take  $c=1$  (“*natural units*”)
- **Invariant** mass of initial state:  $(p_a + p_b)^2 = m_{ab}^2$



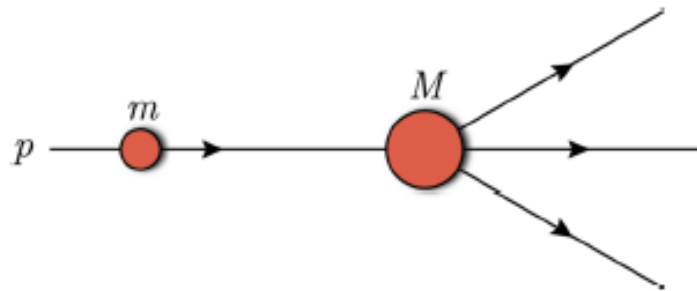
- **Invariant** mass of initial state = invariant mass of final state:  
= “center-of-mass energy” ,  $\sqrt{s}$ :

$$\begin{aligned} s &= (p_a + p_b)^2 = (p_c + p_d)^2 \\ &= m_c^2 + m_d^2 + 2(E_c E_d - \vec{p}_c \cdot \vec{p}_d) \geq (m_c + m_d)^2 \end{aligned}$$

# “Fixed target” vs “colliding beams”

- Calculate center-of-mass energy for beam of 450 GeV protons:

## 1) Fixed target:



$$s = (p + P)^2 = m^2 + M^2 + 2EM$$

## 2) Colliding beams:



$$\begin{aligned} s &= (p_1 + p_2)^2 = 2m^2 + 2(E^2 + \vec{p}^2) \\ &= 2m^2 + 2E^2 + 2(E^2 - m^2) = 4E^2 \end{aligned}$$

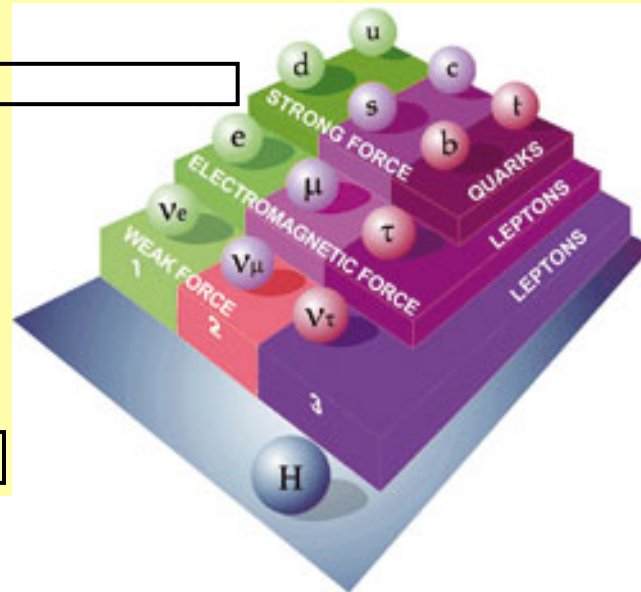
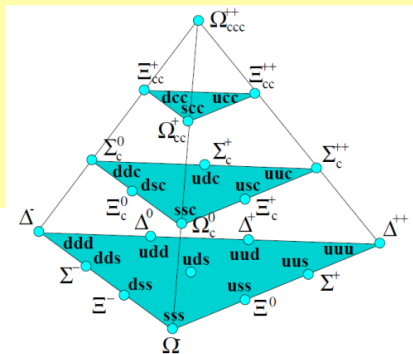
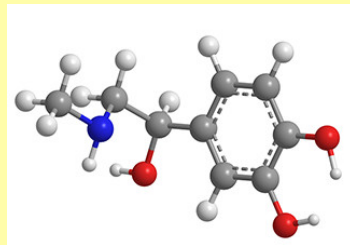


# Summary: Standard Model

- Standard Model Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\not{D}\psi + h.c. + \bar{\psi}_i Y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

- Standard Model Particles



# Summary: Relativity

- Theory of relativity
  - Lorentz transformations ("boost")
  - Calculate energy in collisions

$$\begin{aligned}x^{10} &= \gamma(x^0 - \beta x^1) \\x^{11} &= \gamma(x^1 - \beta x^0) \\x^{12} &= x^2 \\x^{13} &= x^3\end{aligned} \quad \text{met} \quad \begin{aligned}\beta &\equiv \frac{v}{c} \\ \gamma &\equiv \frac{1}{\sqrt{1 - \beta^2}}\end{aligned}$$

- 4-vector calculus

$$p_\mu p^\mu = (E/c)^2 - |\vec{p}|^2 = (E^2 - c^2|\vec{p}|^2)/c^2 = (m_0 c^4)/c^2$$

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \quad (\mu = 0, 1, 2, 3)$$

- High energies needed to make (new) particles



$$\begin{aligned}s &= (p_1 + p_2)^2 = 2m^2 + 2(E^2 + \vec{p}^2) \\ &= 2m^2 + 2E^2 + 2(E^2 - m^2) = 4E^2\end{aligned}$$

## Next: QM

- Introduce “matter particles”
  - spinor  $\psi$  from Dirac equation
- Introduce “force particles”

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i\bar{\psi}\not{D}\psi + \text{h.c.} \\ & + \chi_i y_{ij} \chi_j \phi + \text{h.c.} \\ & + |D_\mu\phi|^2 - V(\phi)\end{aligned}$$

- Introduce basic concepts of scattering processes



# Plan

	1) Intro: Standard Model & Relativity	<i>12 Feb</i>
<i>1900-1940</i>	2) Basis	<i>19 Feb</i>
	1) Atom model, strong and weak force	
	2) Scattering theory	
<i>1945-1965</i>	3) Hadrons	<i>12 Mar</i>
	1) Isospin, strangeness	
	2) Quark model, GIM	
<i>1965-1975</i>	4) Standard Model	<i>19 Mar</i>
	1) QED	
	2) Parity, neutrinos, weak interaction	
	3) QCD	
<i>1975-2000</i>	5) $e^+e^-$ and DIS	<i>7 May</i>
<i>2000-2015</i>	6) Higgs and CKM	<i>21 May</i>

# Backup slides: on accelerators

*How do you create enough energy?*

***Accelerators***

# From bubble chamber to LHC

## Discoveries made with the help of Accelerators:

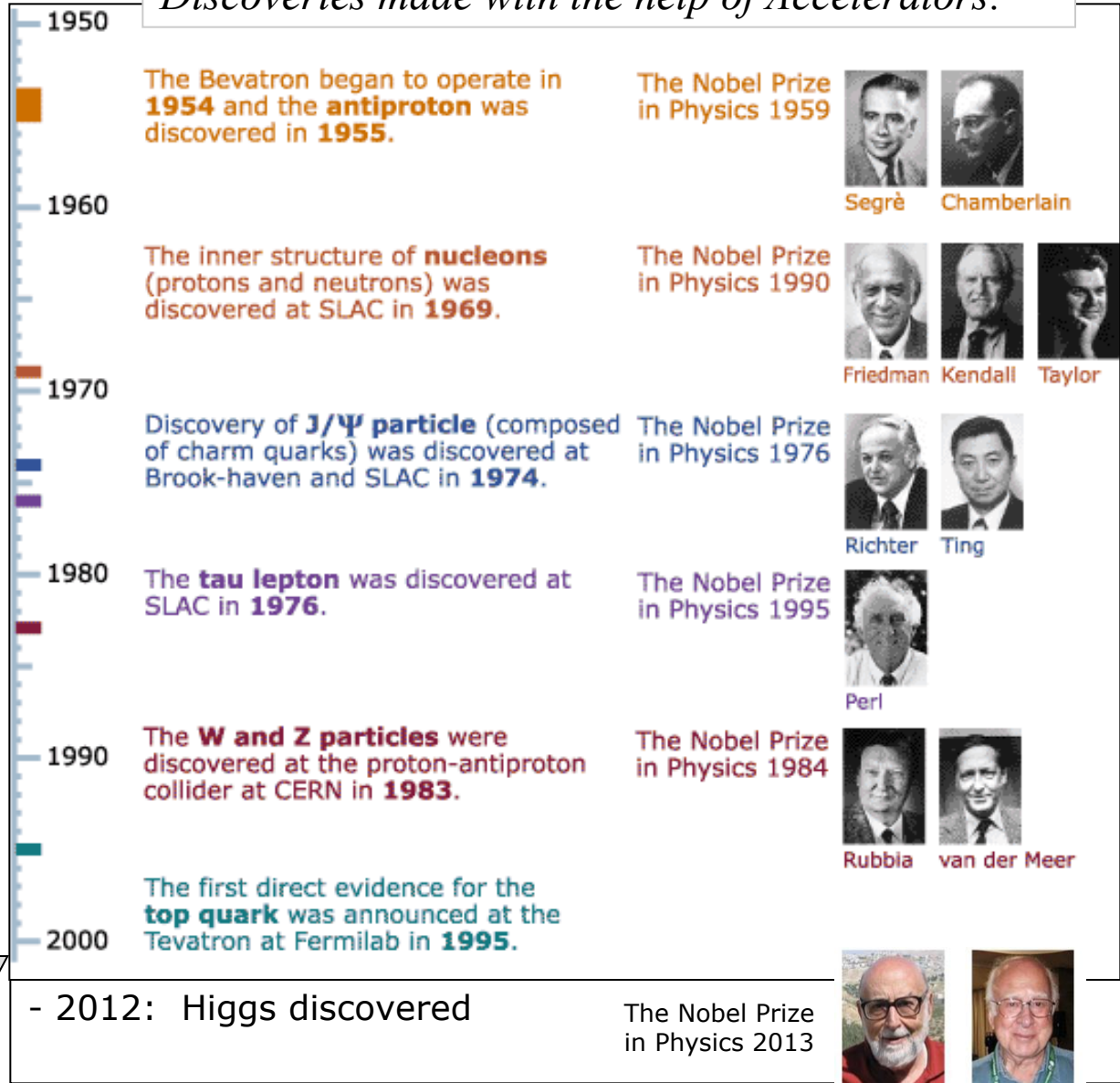
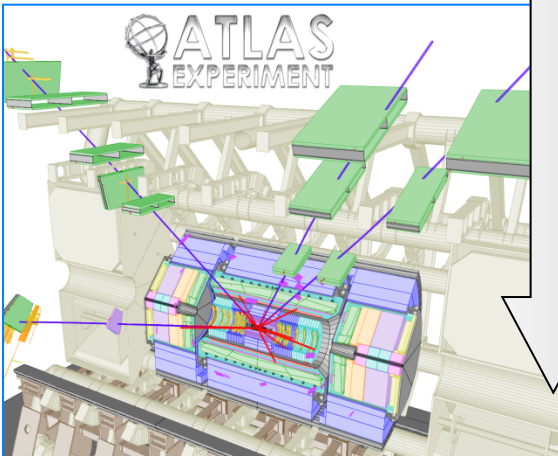
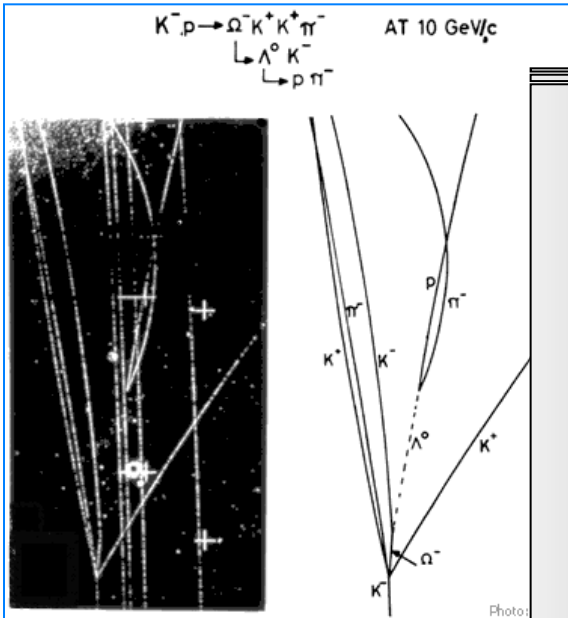


Photo: Pnicolet via Wikimedia Commons  
François Englert

Photo: G-M Greuel via Wikimedia Commons  
Peter W. Higgs

# Cockcroft-Walton



Bart Hommels

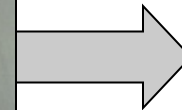


Cockcroft

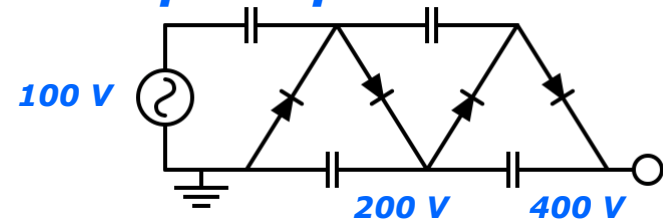


Walton

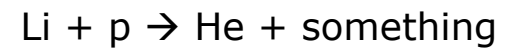
Cavendish lab Cambridge



## Operation principle



1932: 800 kV 0.8 MeV:  
energy threshold to split atoms



1951: Nobelprize



# Van de Graaff

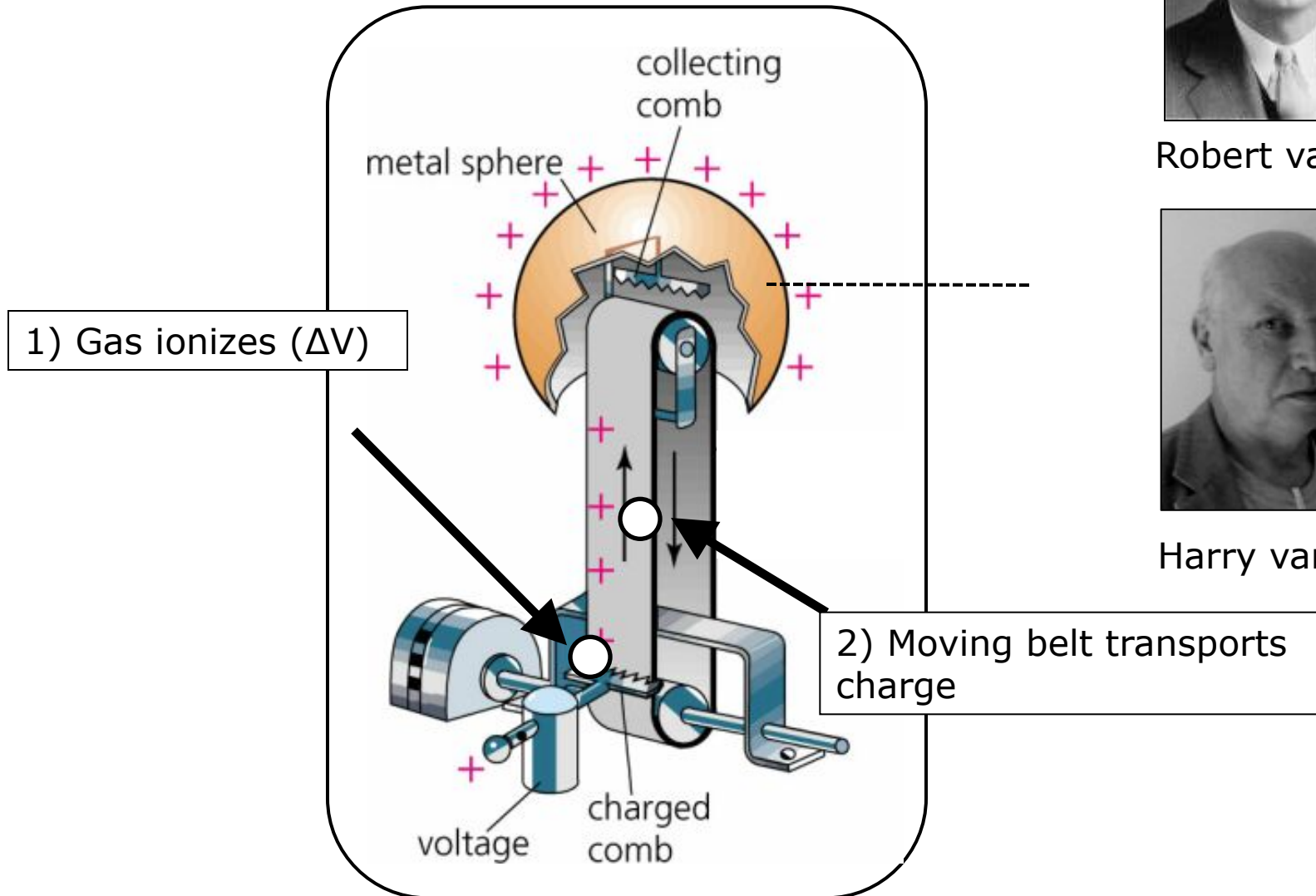
High voltage electro static generator



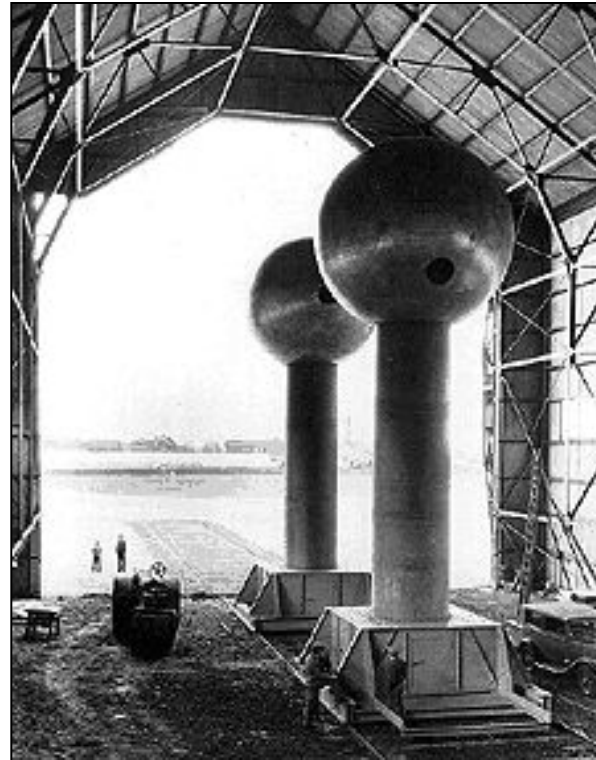
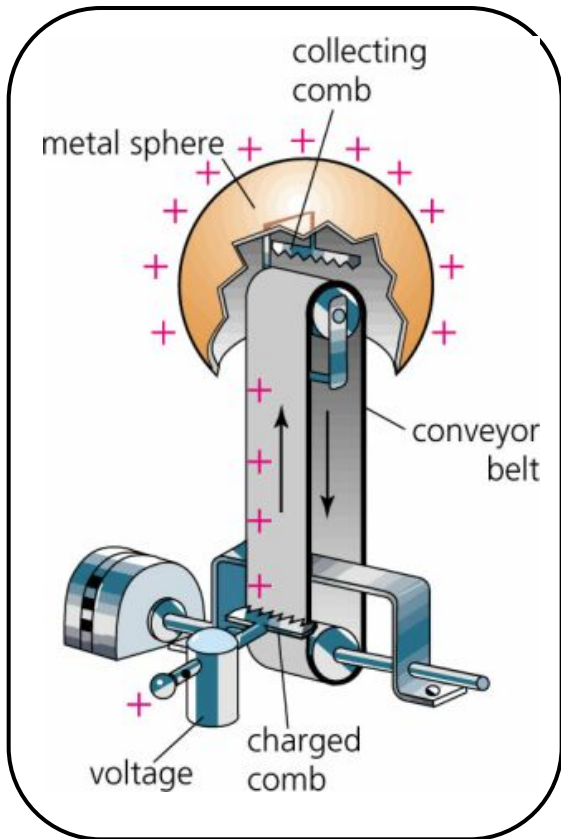
Robert van de Graaff



Harry van der Graaf



# Van de Graaff

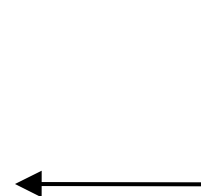


Robert van de Graaff



Harry van der Graaf

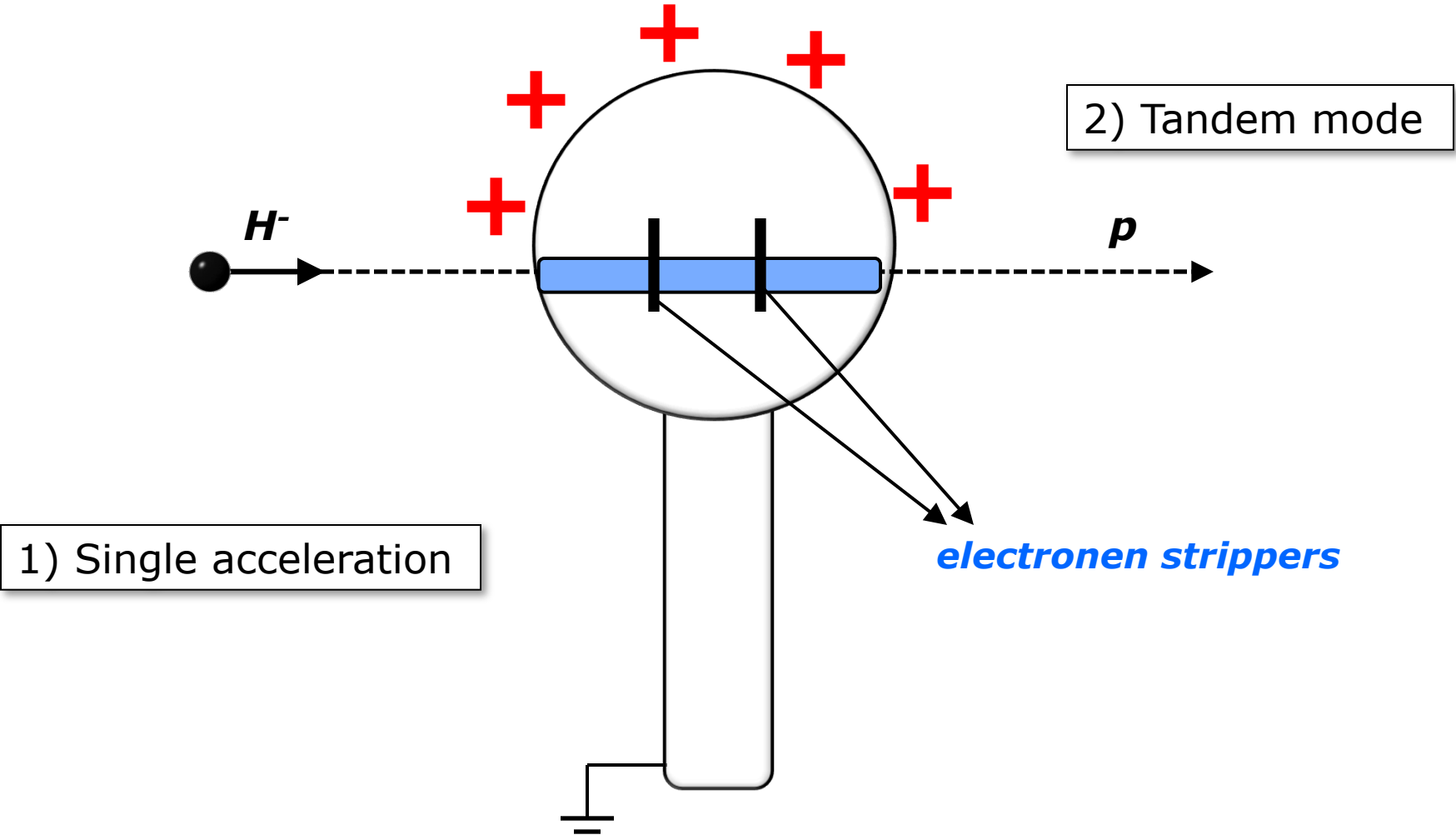
1929: 80,000 volt  
1931: 1,000,000 volt  
1933: 7,000,000 volt



**Nowadays: Oak Ridge  
Vivitron**

**25 MeV  
35 MeV**

# Van de Graaff

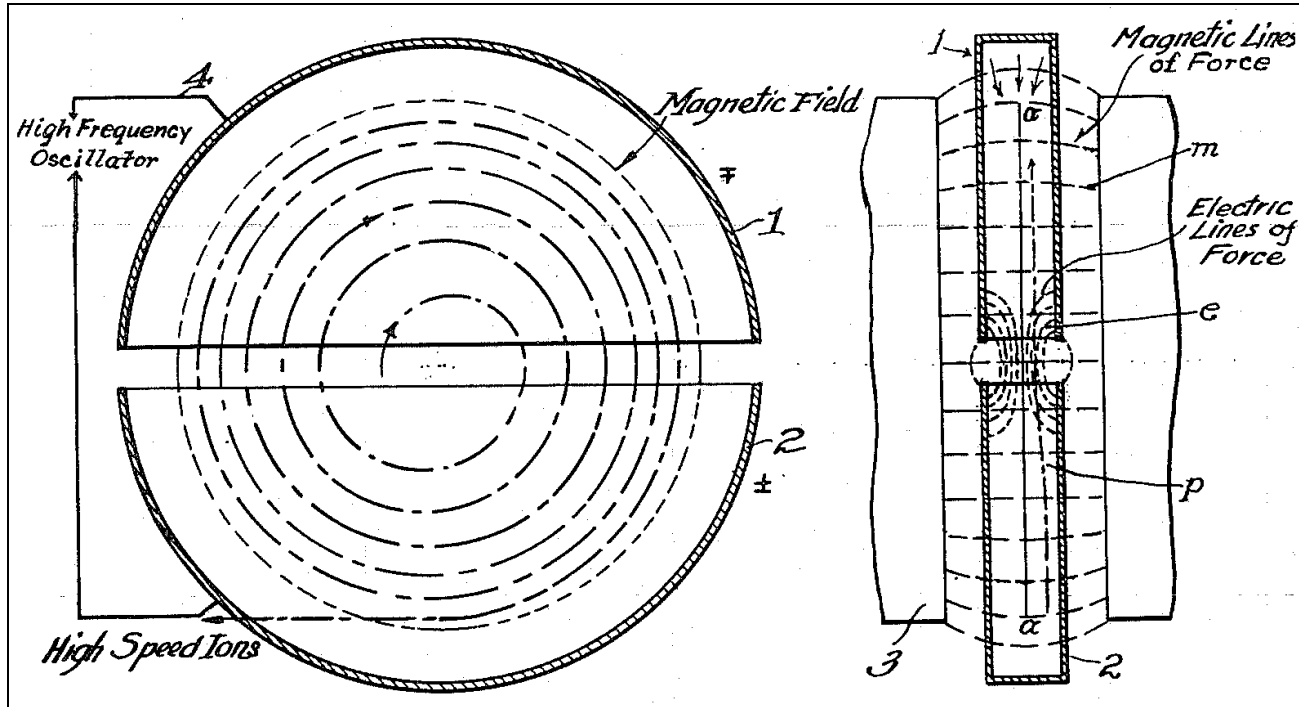


# Cyclotron

Ernest "atom smasher" Lawrence

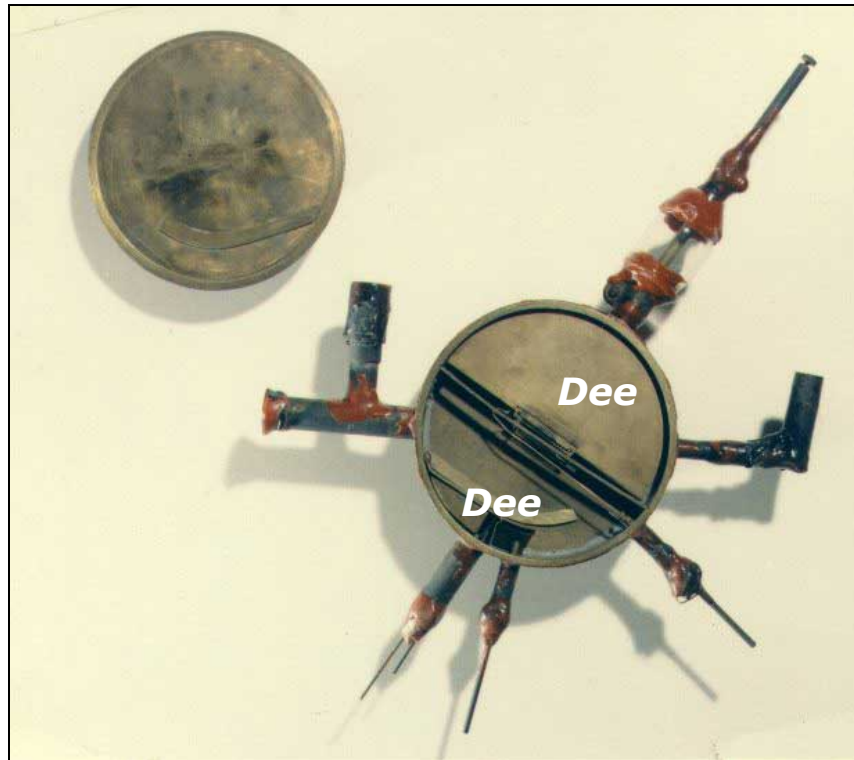


Nobelprijs 1939



First cyclotron 1930

# Cyclotrons in real life



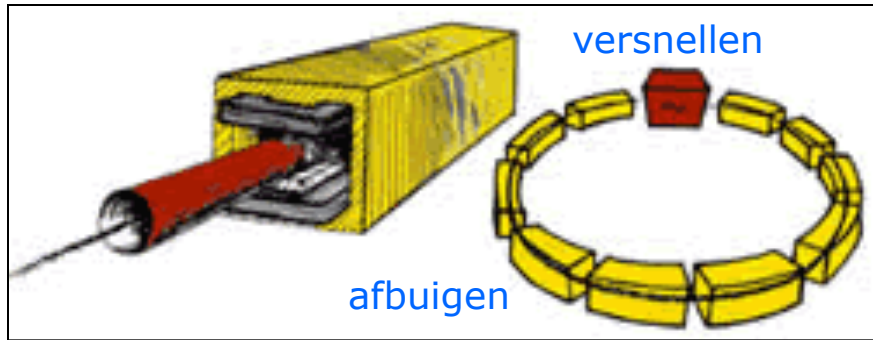
1931:  $r = 12 \text{ cm}$   
→ 1 MeV protons



1974:  $B = 0.46 \text{ [T]}$ ,  $r = 9 \text{ [m]}$   
→ 520 MeV protons

# Synchrotron

In a synchrotron, particles move in fixed orbit



Accelerate: higher E  $\rightarrow$  higher p  
r constant: also higher B



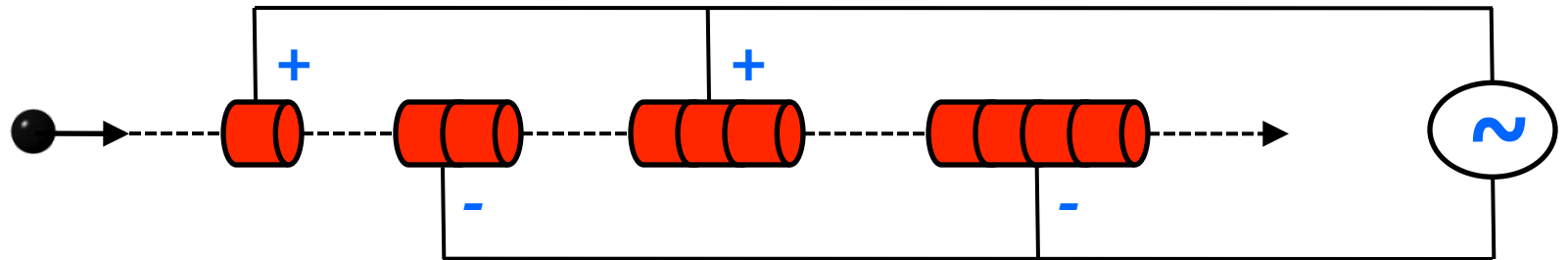
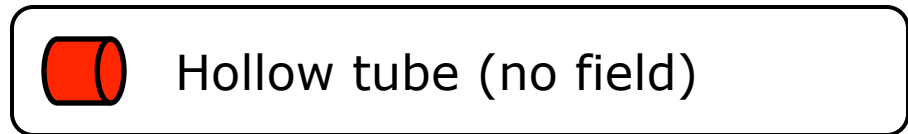
M. Oliphant

$$r = \frac{p}{qB}$$

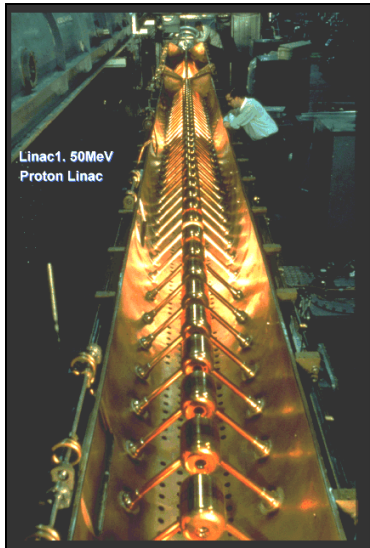
Known synchrotrons:

- |                       |          |
|-----------------------|----------|
| - Bevatron            |          |
| - Tevatron (Fermilab) | collider |
| - LEP (CERN)          | collider |
| - LHC (CERN)          | collider |

# Linac (principle)



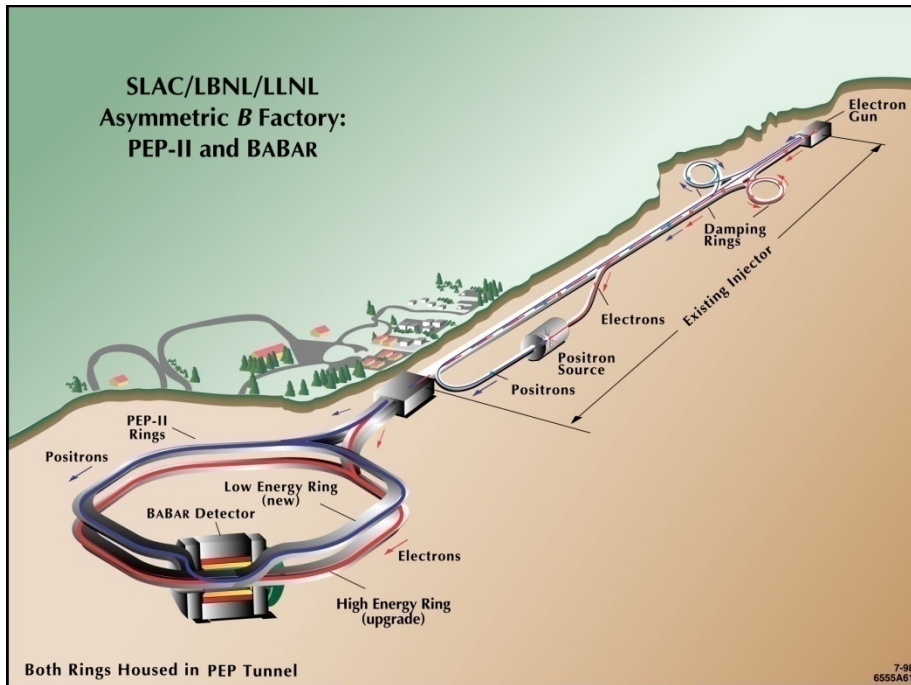
Equal frequency, larger velocity  
→ (space between) tubes increasingly larger



Linac typically first step in acceleration chain  
Typical: ~50m, ~100 MeV

# Linac's & traveling wave guide

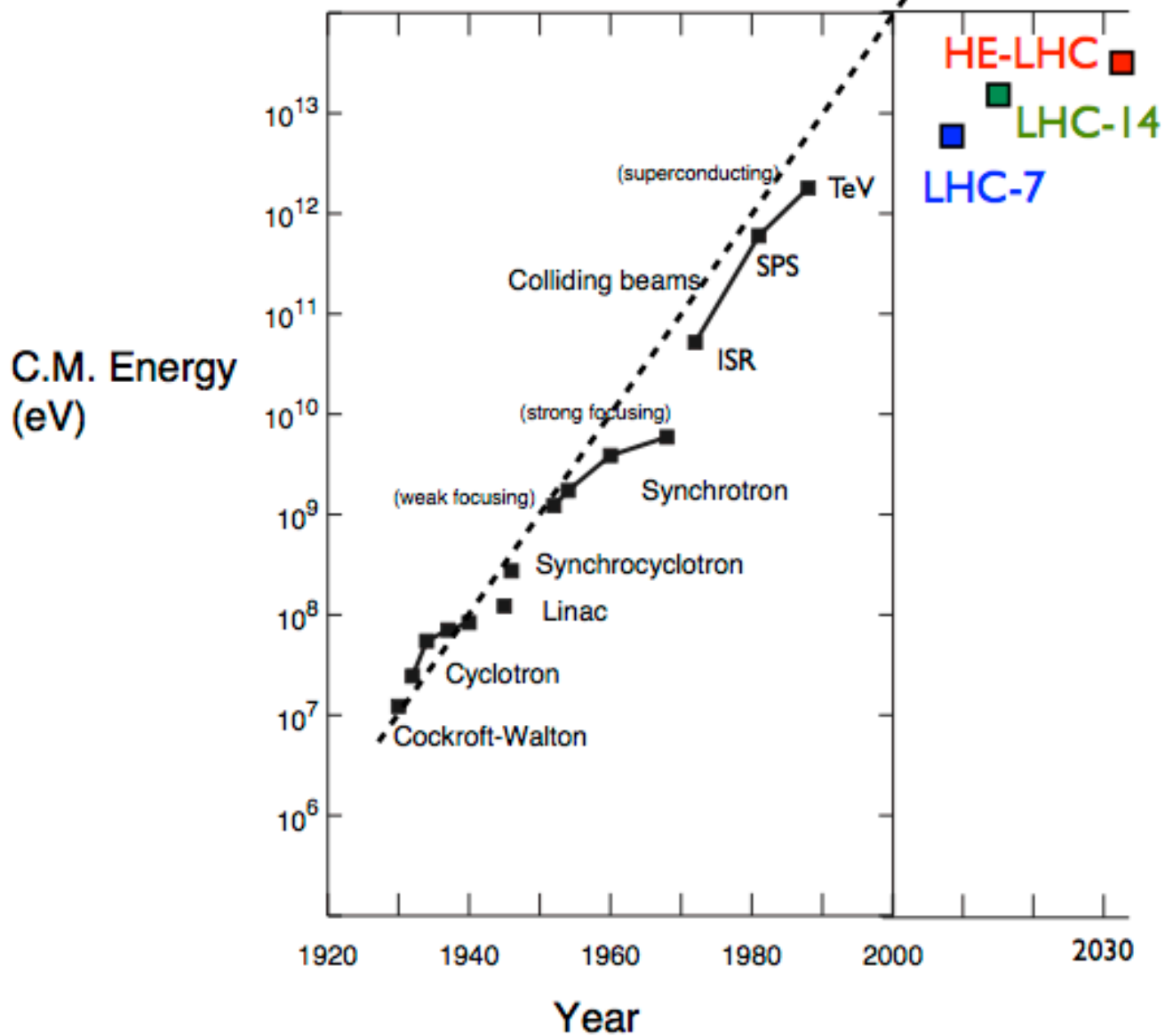
## Big Linac's



SLAC: Stanford Linear Accelerator Center (San Francisco)  
3.2 km long  $\rightarrow$  50 GeV electrons



# Livingston line



# Future Circular Collider (FCC) ???

- 80-100 km tunnel infrastructure in Geneva area
- design driven by pp-collider requirements
- with possibility of  $e^+e^-$  (TLEP) and p-e (VLHeC)
- *CERN-hosted study performed in international collaboration*

**16 T  $\Rightarrow$  100 TeV in 100 km**  
**20 T  $\Rightarrow$  100 TeV in 80 km**

## LEGEND

- LHC tunnel
- ..... HE\_LHC 80km option
- potential shaft location

