

Particle Physics II – CP violation

(also known as “Physics of Anti-matter”)

Lecture 4

N. Tuning

Plan

- 1) Wed 12 Feb: Anti-matter + SM
- 2) Mon 17 Feb: CKM matrix + Unitarity Triangle
- 3) Wed 19 Feb: Mixing + Master eqs. + $B^0 \rightarrow J/\psi K_s$
- 4) Mon 9 Mar: CP violation in $B_{(s)}$ decays (I)
- 5) Wed 11 Mar: CP violation in $B_{(s)}$ and K decays (II)
- 6) Mon 16 Mar: Rare decays + Flavour Anomalies
- 7) Wed 18 Mar: Exam

➤ Final Mark:

- if (mark > 5.5) mark = max(exam, 0.85*exam + 0.15*homework)
- else mark = exam

➤ In parallel: Lectures on Flavour Physics by prof.dr. R. Fleischer

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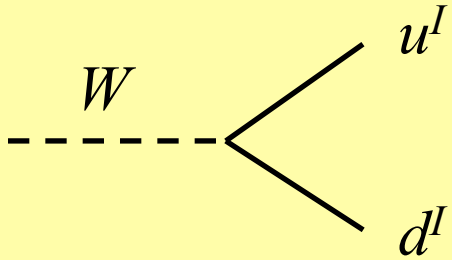
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Recap

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

$$-L_{Yuk} = Y_{ij}^d (\bar{u}_L^I, \bar{d}_L^I)_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^I + \dots$$

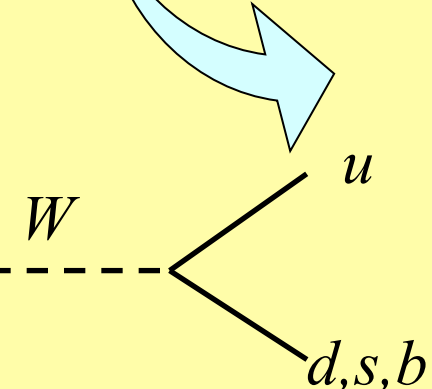
$$L_{Kinetic} = \frac{g}{\sqrt{2}} \bar{u}_{Li}^I \gamma^\mu W_\mu^- d_{Li}^I + \frac{g}{\sqrt{2}} \bar{d}_{Li}^I \gamma^\mu W_\mu^+ u_{Li}^I + \dots$$



Diagonalize Yukawa matrix Y_{ij}

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



$$-L_{Mass} = (\bar{d}, \bar{s}, \bar{b})_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + (\bar{u}, \bar{c}, \bar{t})_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \dots$$

$$L_{CKM} = \frac{g}{\sqrt{2}} \bar{u}_i \gamma^\mu W_\mu^- V_{ij} (1 - \gamma^5) d_j + \frac{g}{\sqrt{2}} \bar{d}_j \gamma^\mu W_\mu^+ V_{ij}^* (1 - \gamma^5) u_i + \dots$$

$$L_{SM} = L_{CKM} + L_{Higgs} + L_{Mass}$$

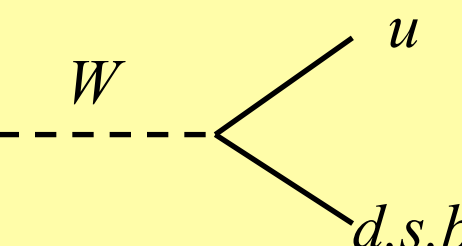
CKM-matrix: where are the phases?

- Possibility 1: simply 3 ‘rotations’, and put phase on smallest:

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

- Possibility 2: parameterize according to magnitude, in $O(\lambda)$:

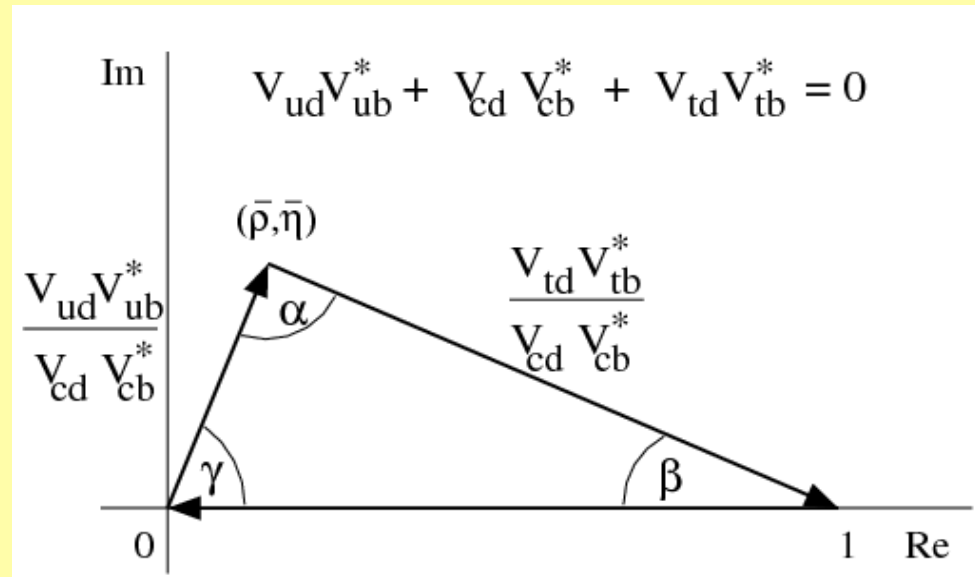


A Feynman diagram showing a dashed line representing a W boson on the left, which splits into two solid lines on the right. The upper solid line is labeled u and the lower solid line is labeled d, s, b .

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

This was theory, now comes experiment

- We already saw how the **moduli** $|V_{ij}|$ are determined
- Now we will work towards the measurement of the **imaginary** part
 - Parameter: η
 - Equivalent: angles α, β, γ .



- To measure this, we need the formalism of **neutral meson oscillations**...

Neutral Meson Oscillations (1)

- Start with Schrodinger equation:

$$i\frac{\partial\psi}{\partial t} = H\psi = (M - \frac{i}{2}\Gamma)\psi = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \psi$$

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

(2-component state in P^0 and \bar{P}^0 subspace)

- Find eigenvalue:

$$\begin{vmatrix} M - \frac{i}{2}\Gamma - \lambda & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma - \lambda \end{vmatrix} = 0$$

- Solve eigenstates:

$$\begin{aligned} |P_1\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \\ |P_2\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \end{aligned}$$

$$\psi_{\pm} = \begin{pmatrix} p \\ \pm q \end{pmatrix}$$

we find p and q by solving

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \lambda_{\pm} \begin{pmatrix} p \\ q \end{pmatrix} \longrightarrow \frac{q}{p} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}$$

- Eigenstates have diagonal Hamiltonian: **mass eigenstates!**

Neutral Meson Oscillations (2)

- Two mass eigenstates

$$|P_H\rangle = p|P^0\rangle + q|\bar{P}^0\rangle$$

$$|P_L\rangle = p|P^0\rangle - q|\bar{P}^0\rangle$$

- Time evolution:

$$|P_H(t)\rangle = e^{-im_H t - \frac{1}{2}\Gamma_H t} |P_H(0)\rangle$$

$$|P_L(t)\rangle = e^{-im_L t - \frac{1}{2}\Gamma_L t} |P_L(0)\rangle$$

$$|P^0\rangle = \frac{1}{2p} [|P_H\rangle + |P_L\rangle]$$

$$|\bar{P}^0\rangle = \frac{1}{2q} [|P_H\rangle - |P_L\rangle]$$

$$|P^0(t)\rangle = \frac{1}{2} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle$$

- Probability for $|P^0\rangle \rightarrow |\bar{P}^0\rangle$!
- Express in $M = m_H + m_L$ and $\Delta m = m_H - m_L \rightarrow \Delta m$ dependence

Meson Decays

- Formalism of meson oscillations:

$$|P^0(t)\rangle = \frac{1}{2} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle$$

$$|\langle \bar{P}^0(t) | P^0 \rangle|^2 = |g_-(t)|^2 \left(\frac{p}{q} \right)^2$$

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2} \left(\cosh \frac{1}{2} \Delta\Gamma t \pm \cos \Delta m t \right)$$

- Subsequent: decay

$$P^0 \rightarrow f$$

Notation: Define A_f and λ_f

$$\begin{aligned} A(f) &= \langle f|T|P^0\rangle & \bar{A}(f) &= \langle f|T|\bar{P}^0\rangle \\ A(\bar{f}) &= \langle \bar{f}|T|P^0\rangle & \bar{A}(\bar{f}) &= \langle \bar{f}|T|\bar{P}^0\rangle \end{aligned}$$

and define the complex parameter λ_f (not be confused with the Wolfenstein parameter λ !):

$$\lambda_f = \frac{q \bar{A}_f}{p A_f}, \quad \bar{\lambda}_f = \frac{1}{\lambda_f}, \quad \lambda_{\bar{f}} = \frac{q \bar{A}_{\bar{f}}}{p A_{\bar{f}}}, \quad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}} \quad (3.14)$$

The general expression for the time dependent decay rates, $\Gamma_{P^0 \rightarrow f}(t) = |\langle f|T|P^0(t)\rangle|^2$,

Some algebra for the decay $P^0 \rightarrow f$

$$|P^0(t)\rangle = \frac{1}{2} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle$$

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)] \right)$$

$$A(f) = \langle f|T|P^0\rangle$$

$$\bar{A}(f) = \langle f|T|\bar{P}^0\rangle$$

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$$

Interference

— $P^0 \rightarrow f$

— $P^0 \rightarrow \bar{P}^0 \rightarrow f$

Some algebra for the decay $P^0 \rightarrow f$

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)] \right) \\
 \Gamma_{P^0 \rightarrow \bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left| \frac{q}{p} \right|^2 \left(|g_-(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2 |g_+(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+(t) g_-^*(t)] \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 \left(|g_-(t)|^2 + |\lambda_f|^2 |g_+(t)|^2 + 2\Re[\lambda_f g_+(t) g_-^*(t)] \right) \\
 \Gamma_{\bar{P}^0 \rightarrow \bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left(|g_+(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2 |g_-(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+^*(t) g_-(t)] \right) \quad (3.15)
 \end{aligned}$$

$$\begin{aligned}
 |g_{\pm}(t)|^2 &= \frac{e^{-\Gamma t}}{2} \left(\cosh \frac{1}{2} \Delta\Gamma t \pm \cos \Delta m t \right) \\
 g_+^*(t) g_-(t) &= \frac{e^{-\Gamma t}}{2} \left(\sinh \frac{1}{2} \Delta\Gamma t + i \sin \Delta m t \right) \\
 g_+(t) g_-^*(t) &= \frac{e^{-\Gamma t}}{2} \left(\sinh \frac{1}{2} \Delta\Gamma t - i \sin \Delta m t \right) \quad (3.16)
 \end{aligned}$$

The 'master equations'

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 \frac{e^{-\Gamma t}}{2} \left((1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t + (1 - |\lambda_f|^2) \cos \Delta mt - 2\Im\lambda_f \sin \Delta mt \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 \frac{e^{-\Gamma t}}{2} \left((1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t - (1 - |\lambda_f|^2) \cos \Delta mt + 2\Im\lambda_f \sin \Delta mt \right)
 \end{aligned} \tag{3.17}$$

The diagram highlights the terms in the equations:

- (direct) Decay** (blue text and circles): $(1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t$ and $(1 - |\lambda_f|^2) \cos \Delta mt$
- Interference** (green text and circles): $2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t$ and $2\Im\lambda_f \sin \Delta mt$

The sinh- and sin-terms are associated to the interference between the decays with and without oscillation. Commonly, the master equations are expressed as:

The 'master equations'

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 \frac{e^{-\Gamma t}}{2} \left((1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t + (1 - |\lambda_f|^2) \cos \Delta mt - 2\Im\lambda_f \sin \Delta mt \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 \frac{e^{-\Gamma t}}{2} \left((1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t - (1 - |\lambda_f|^2) \cos \Delta mt + 2\Im\lambda_f \sin \Delta mt \right)
 \end{aligned} \tag{3.17}$$

('direct') Decay Interference

The sinh- and sin-terms are associated to the interference between the decays with and without oscillation. Commonly, the master equations are expressed as:

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left(\cosh \frac{1}{2} \Delta\Gamma t + D_f \sinh \frac{1}{2} \Delta\Gamma t + C_f \cos \Delta mt - S_f \sin \Delta mt \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left(\cosh \frac{1}{2} \Delta\Gamma t + D_f \sinh \frac{1}{2} \Delta\Gamma t - C_f \cos \Delta mt + S_f \sin \Delta mt \right)
 \end{aligned} \tag{3.18}$$

with

$$D_f = \frac{2\Re\lambda_f}{1 + |\lambda_f|^2} \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad S_f = \frac{2\Im\lambda_f}{1 + |\lambda_f|^2}. \tag{3.19}$$

Classification of CP Violating effects

1. CP violation in decay

$$\Gamma(P^0 \rightarrow f) \neq \Gamma(\bar{P}^0 \rightarrow \bar{f})$$

This is obviously satisfied (see Eq. (3.15)) when

$$\left| \frac{\bar{A}_f}{A_f} \right| \neq 1.$$

2. CP violation in mixing

$$\text{Prob}(P^0 \rightarrow \bar{P}^0) \neq \text{Prob}(\bar{P}^0 \rightarrow P^0)$$

$$\left| \frac{q}{p} \right| \neq 1.$$

3. CP violation in interference

$$\Gamma(P^0(\rightsquigarrow \bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

$$\Im \lambda_f = \Im \left(\frac{q \bar{A}_f}{p A_f} \right) \neq 0$$

Meson Decays

- Formalism of meson *oscillations*:

$$|P^0(t)\rangle = \frac{1}{2} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left(e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle$$

- Subsequent: *decay*

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)] \right)$$

— $P^0 \rightarrow f$

— $P^0 \rightarrow \bar{P}^0 \rightarrow f$

$$A(f) = \langle f|T|P^0\rangle$$

$$\bar{A}(f) = \langle f|T|\bar{P}^0\rangle$$

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$$

Interference

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \frac{e^{-\Gamma t}}{2} \left((1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t + (1 - |\lambda_f|^2) \cos \Delta m t - 2\Im\lambda_f \sin \Delta m t \right)$$

('direct') Decay
Interference

Classification of CP Violating effects

1. CP violation in decay

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This is obviously satisfied (see Eq. (3.15)) when

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$$\text{Prob}(P^0 \rightarrow \bar{P}^0) \neq \text{Prob}(\bar{P}^0 \rightarrow P^0)$$

$$\left| \frac{q}{p} \right| \neq 1.$$

3. CP violation in interference

$$\Gamma(P^0(\rightsquigarrow \bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

$$\Im \lambda_f = \Im \left(\frac{q \bar{A}_f}{p A_f} \right) \neq 0$$

Now: $\text{Im}(\lambda_f)$

1. CP violation in decay

$$\Gamma(P^0 \rightarrow f) \neq \Gamma(\bar{P}^0 \rightarrow \bar{f})$$

This is obviously satisfied (see Eq. (3.15)) when

$$\left| \frac{\bar{A}_f}{A_f} \right| \neq 1.$$

2. CP violation in mixing

$$\text{Prob}(P^0 \rightarrow \bar{P}^0) \neq \text{Prob}(\bar{P}^0 \rightarrow P^0)$$

$$\left| \frac{q}{p} \right| \neq 1.$$

3. CP violation in interference

$$\Gamma(P^0(\rightsquigarrow \bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

We will investigate λ_f for various final states f

$$\Im \lambda_f = \Im \left(\frac{q \bar{A}_f}{p A_f} \right) \neq 0$$

CP violation: type 3

$$\Im \lambda_f = \Im \left(\frac{q \bar{A}_f}{p A_f} \right) \neq 0$$

$$\Gamma(P^0(\rightsquigarrow \bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

$$A_{CP}(t) = \frac{\Gamma_{P^0(t) \rightarrow f} - \Gamma_{\bar{P}^0(t) \rightarrow f}}{\Gamma_{P^0(t) \rightarrow f} + \Gamma_{\bar{P}^0(t) \rightarrow f}}$$

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left(\cosh \frac{1}{2} \Delta \Gamma t + D_f \sinh \frac{1}{2} \Delta \Gamma t + C_f \cos \Delta m t - S_f \sin \Delta m t \right)$$

$$\Gamma_{\bar{P}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left(\cosh \frac{1}{2} \Delta \Gamma t + D_f \sinh \frac{1}{2} \Delta \Gamma t - C_f \cos \Delta m t + S_f \sin \Delta m t \right)$$

$$D_f = \frac{2\Re \lambda_f}{1 + |\lambda_f|^2} \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad S_f = \frac{2\Im \lambda_f}{1 + |\lambda_f|^2}$$

$$A_{CP}(t) = \frac{\Gamma_{P^0(t) \rightarrow f} - \Gamma_{\bar{P}^0(t) \rightarrow f}}{\Gamma_{P^0(t) \rightarrow f} + \Gamma_{\bar{P}^0(t) \rightarrow f}} = \frac{2C_f \cos \Delta m t - 2S_f \sin \Delta m t}{2 \cosh \frac{1}{2} \Delta \Gamma t + 2D_f \sinh \frac{1}{2} \Delta \Gamma t}$$

Classification of CP Violating effects - Nr. 3:

Consider $f=\bar{f}$:

$$A_{CP}(t) = \frac{\Gamma_{P^0(t)\rightarrow f} - \Gamma_{\bar{P}^0(t)\rightarrow f}}{\Gamma_{P^0(t)\rightarrow f} + \Gamma_{\bar{P}^0(t)\rightarrow f}} = \frac{2C_f \cos \Delta mt - 2S_f \sin \Delta mt}{2 \cosh \frac{1}{2}\Delta\Gamma t + 2D_f \sinh \frac{1}{2}\Delta\Gamma t}$$

If one amplitude dominates the decay, then $A_f = \bar{A}_f$

$$A_{CP}(t) = \frac{-\Im\lambda_f \sin \Delta mt}{\cosh \frac{1}{2}\Delta\Gamma t + \Re\lambda_f \sinh \frac{1}{2}\Delta\Gamma t}$$

3. CP violation in interference

$$\Gamma(P^0(\rightsquigarrow\bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

$$\Im\lambda_f = \Im\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

CP violation: a famous example

- The golden decay $B^0 \rightarrow J/\psi K_s$



Final state f : $J/\psi K_s$

- Interference between $B^0 \rightarrow f_{CP}$ and $B^0 \rightarrow \bar{B}^0 \rightarrow f_{CP}$
 - For example: $B^0 \rightarrow J/\psi K_s$ and $B^0 \rightarrow \bar{B}^0 \rightarrow J/\psi K_s$
 - Lets' s simplify ☺...

1) For B^0 we have:

$$\left| \frac{q}{p} \right| = 1$$

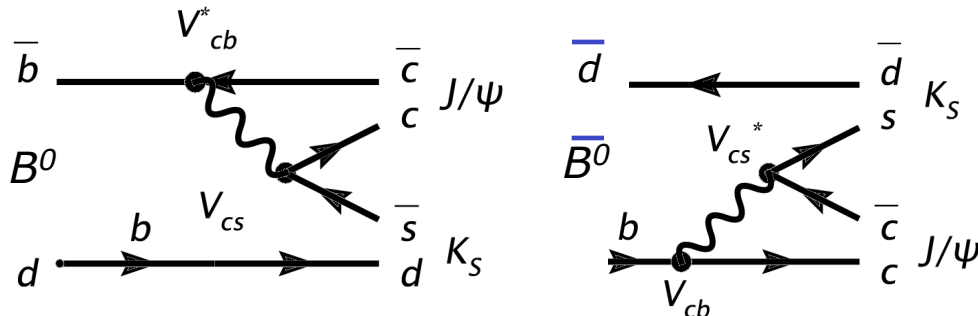
2) Since $f_{CP} = \bar{f}_{CP}$ we have:

$$\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f} = \frac{q}{p} \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} \equiv \lambda_{\bar{f}}$$

$|\lambda_f| = 1$

3) The amplitudes $|A(B^0 \rightarrow J/\psi K_s)|$ and $|A(\bar{B}^0 \rightarrow J/\psi K_s)|$ are equal:

$$|A_{f_{CP}}| = |\bar{A}_{f_{CP}}|$$



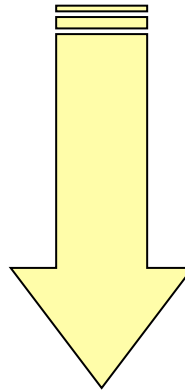
Relax: $B^0 \rightarrow J/\psi K_s$ simplifies...

$$D_f = \frac{2\Re\lambda_f}{1 + |\lambda_f|^2} \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad S_f = \frac{2\Im\lambda_f}{1 + |\lambda_f|^2}.$$

$$A_{CP}(t) = \frac{\Gamma_{P^0(t) \rightarrow f} - \Gamma_{\bar{P}^0(t) \rightarrow f}}{\Gamma_{P^0(t) \rightarrow f} + \Gamma_{\bar{P}^0(t) \rightarrow f}} = \frac{2C_f \cos \Delta m t - 2S_f \sin \Delta m t}{2 \cosh \frac{1}{2} \Delta \Gamma t + 2D_f \sinh \frac{1}{2} \Delta \Gamma t}$$

$$|\lambda_f|=1$$

$$\Delta\Gamma=0$$

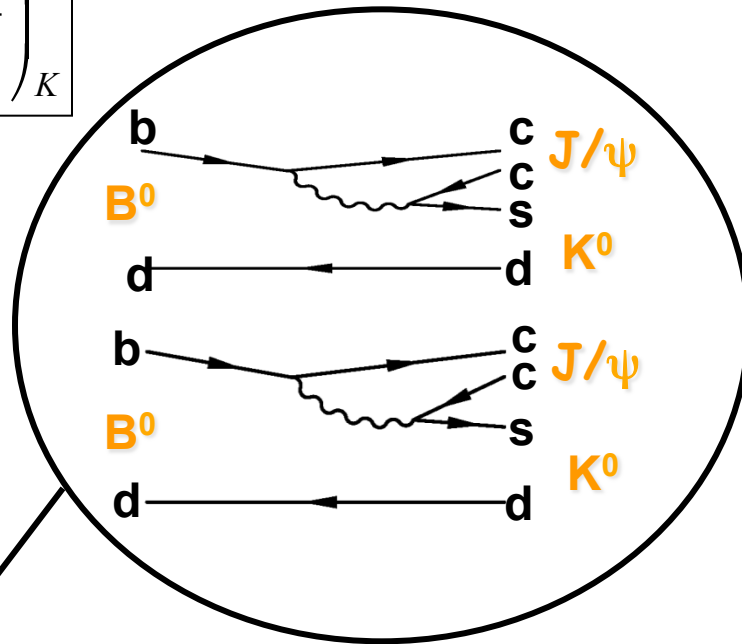
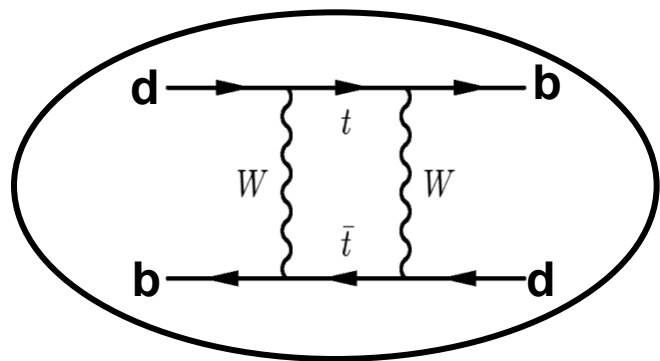


$$A_{CP}(t) = -\Im\lambda_f \sin(\Delta m t)$$

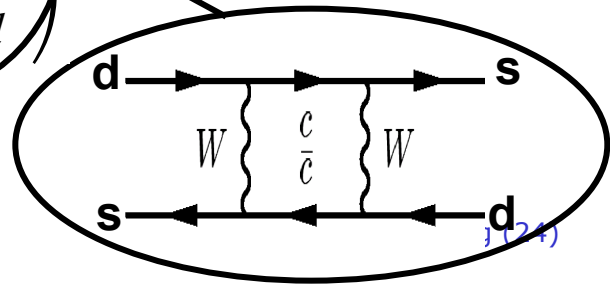
λ_f for $B^0 \rightarrow J/\psi K^0_s$

$$\lambda_f = \frac{q \bar{A}_f}{p A_f}$$

$$\lambda_{J/\psi K^0_s} = \left(\frac{q}{p} \right)_{B^0} \frac{\bar{A}_{J/\psi K^0_s}}{A_{J/\psi K^0_s}} = \left(\frac{q}{p} \right)_{B^0} \frac{\bar{A}_{J/\psi K^0}}{A_{J/\psi K^0}} \left(\frac{p}{q} \right)_K$$



$$\lambda_{J/\psi K^0_s} = - \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}$$



λ_f for $B^0 \rightarrow J/\psi K_S^0$

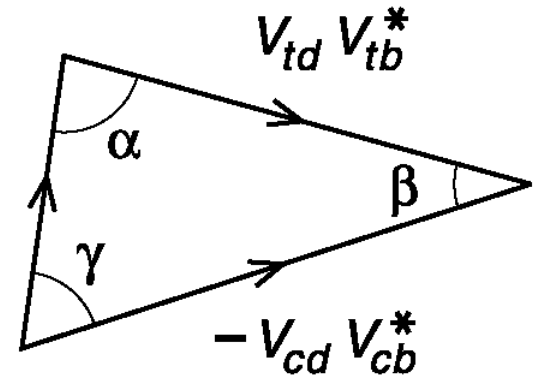
$$\lambda_{J/\psi K_S} = - \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left(\frac{V_{cb} \cancel{V_{cs}^*}}{V_{cb}^* \cancel{V_{cs}}} \right) \left(\frac{\cancel{V_{cs}} V_{cd}^*}{\cancel{V_{cs}^*} V_{cd}} \right)$$

$$= -e^{-2i\beta}$$

Time-dependent CP asymmetry

$$A_{CP}(t) = -\sin 2\beta \sin(\Delta mt)$$

- Theoretically clean way to measure β
- Clean experimental signature
- Branching fraction: $O(10^{-4})$
 - “Large” compared to other CP modes!



Remember: C and P eigenvalue

- C and P are good symmetries (not involving the weak interaction)
 - Can associate a conserved value with them (Noether Theorem)
- Each hadron has a conserved P and C quantum number
 - What are the values of the quantum numbers
 - Evaluate the eigenvalue of the P and C operators on each hadron
 $\mathbf{P}|\psi\rangle = p|\psi\rangle$
- What values of C and P are possible for hadrons?
 - **Symmetry operation squared gives unity** so eigenvalue squared must be 1
 - Possible C and P values are +1 and -1.
- Meaning of P quantum number
 - If $P=1$ then $P|\psi\rangle = +1|\psi\rangle$ (wave function symmetric in space)
 - if $P=-1$ then $P|\psi\rangle = -1|\psi\rangle$ (wave function anti-symmetric in space)

Remember: P eigenvalues for hadrons

- The π^+ meson
 - Quark and anti-quark composite: intrinsic $P = (1)*(-1) = -1$
 - Orbital ground state \rightarrow no extra term
 - **$P(\pi^+) = -1$**

Meaning: $P|\pi^+\rangle = -1|\pi^+\rangle$

- The neutron
 - Three quark composite: intrinsic $P = (1)*(1)*(1) = 1$
 - Orbital ground state \rightarrow no extra term
 - **$P(n) = +1$**

- The $K_1(1270)$
 - Quark anti-quark composite: intrinsic $P = (1)*(-1) = -1$
 - Orbital excitation with $L=1 \rightarrow$ extra term $(-1)^1$
 - **$P(K_1) = +1$**

Remember: C eigenvalues for hadrons

- Only particles that are their own anti-particles are C eigenstates because $C|x\rangle \equiv |\bar{x}\rangle = c|x\rangle$
 - E.g. $\pi^0, \eta, \eta', \rho^0, \phi, \omega, \psi$ and photon
- C eigenvalues of quark-anti-quark pairs is determined by L and S angular momenta: $C = (-1)^{L+S}$
 - Rule applies to all above mesons
- C eigenvalue of photon is -1
 - Since photon is carrier of EM force, which obviously changes sign under C conjugation
- Example of C conservation:
 - Process $\pi^0 \rightarrow \gamma\gamma$ $C=+1(\pi^0 \text{ has spin } 0) \rightarrow (-1)*(-1)$
 - Process $\pi^0 \rightarrow \gamma\gamma\gamma$ does not occur (and would violate C conservation)

Experimental proof of C-invariance:
 $BR(\pi^0 \rightarrow \gamma\gamma\gamma) < 3.1 \cdot 10^{-5}$

Intermezzo: CP eigenvalue

- Remember:

- $P^2 = 1$ ($x \rightarrow -x \rightarrow x$)

- $C^2 = 1$ ($\psi \rightarrow \bar{\psi} \rightarrow \psi$)

- $\rightarrow CP^2 = 1$

- $CP | f \rangle = \pm | f \rangle$

- Knowing this we can evaluate the effect of CP on the K^0

$$CP |\bar{K}^0\rangle = -1 |K^0\rangle$$

$$CP |K^0\rangle = -1 |\bar{K}^0\rangle$$

- K^0 is not CP eigenstate, but flavour eigenstate (sd) !

- Mass eigenstates:

$$|K_S\rangle = p |K^0\rangle + q |\bar{K}^0\rangle$$

$$|K_L\rangle = p |K^0\rangle - q |\bar{K}^0\rangle$$

$$(S(K)=0 \rightarrow L(\pi\pi)=0)$$

$$|K_S\rangle (CP=+1) \rightarrow \pi \pi \quad (CP = (-1)(-1)(-1)^{l=0} = +1)$$

$$|K_L\rangle (CP=-1) \rightarrow \pi \pi \pi \quad (CP = (-1)(-1)(-1)(-1)^{l=0} = -1)$$

CP eigenvalue of final state $J/\psi K^0_S$

- CP $|J/\psi\rangle = +1 |J/\psi\rangle$
- CP $|K^0_S\rangle = +1 |K^0_S\rangle$
- CP $|J/\psi K^0_S\rangle = (-1)^l |J/\psi K^0_S\rangle$

$$\lambda_{J/\psi K_S} = - \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left(\frac{V_{cb} \cancel{V_{cs}^*}}{V_{cb}^* \cancel{V_{cs}}} \right) \left(\frac{\cancel{V_{cs}} V_{cd}^*}{\cancel{V_{cs}^*} V_{cd}} \right)$$

$$= -e^{-2i\beta}$$

Relative minus-sign between state and CP-conjugated state:

$$A_{CP}(t) = \frac{\Gamma_{\bar{B} \rightarrow f}(t) - \Gamma_{B \rightarrow f}(t)}{\Gamma_{\bar{B} \rightarrow f}(t) + \Gamma_{B \rightarrow f}(t)} = \text{Im}(\lambda_f) \sin \Delta m t$$

$$A_{CP}(t) = -\sin 2\beta \sin(\Delta m t)$$

CP eigenvalue of final state $J/\psi K^0_S$

- CP $|J/\psi\rangle = +1 |J/\psi\rangle$
- CP $|K^0_S\rangle = +1 |K^0_S\rangle$
- CP $|J/\psi K^0_S\rangle = (-1)^l |J/\psi K^0_S\rangle$

$$\left. \begin{array}{l} S(J/\psi)=1, L(J/\psi)=0 : \\ C \text{ eigenvalue } (-1)^{L+S} = -1 \\ P \text{ eigenvalue } (-1)^L = -1 \end{array} \right\} \text{CP}|J/\psi\rangle = +|J/\psi\rangle$$

$$\lambda_{J/\psi K_S} = - \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left(\frac{V_{cb} \cancel{V_{cs}^*}}{V_{cb}^* \cancel{V_{cs}}} \right) \left(\frac{\cancel{V_{cs}} V_{cd}^*}{\cancel{V_{cs}^*} V_{cd}} \right)$$

$$= -e^{-2i\beta}$$

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$$A_{CP}(t) = -\sin 2\beta \sin(\Delta m t)$$

CP eigenvalue of final state $J/\psi K^0_S$

- CP $|J/\psi\rangle = +1 |J/\psi\rangle$
- CP $|K^0_S\rangle = +1 |K^0_S\rangle$
- CP $|J/\psi K^0_S\rangle = (-1)^1 |J/\psi K^0_S\rangle$

(S(B)=0 \rightarrow L(J/ ψ K $_S^0$)=1 !)

$$\lambda_{J/\psi K_S} = - \begin{pmatrix} V_{tb}^* V_{td} \\ V_{tb} V_{td}^* \end{pmatrix} \begin{pmatrix} V_{cb} \cancel{V_{cs}^*} \\ V_{cb}^* \cancel{V_{cs}} \end{pmatrix} \begin{pmatrix} \cancel{V_{cs}} V_{cd}^* \\ \cancel{V_{cs}^*} V_{cd} \end{pmatrix}$$

$$= -e^{-2i\beta}$$

Relative minus-sign between state and CP-conjugated state:

$$A_{CP}(t) = \frac{\Gamma_{\bar{B} \rightarrow f}(t) - \Gamma_{B \rightarrow f}(t)}{\Gamma_{\bar{B} \rightarrow f}(t) + \Gamma_{B \rightarrow f}(t)} = \text{Im}(\lambda_f) \sin \Delta m t$$

$$A_{CP}(t) = -\sin 2\beta \sin(\Delta m t)$$

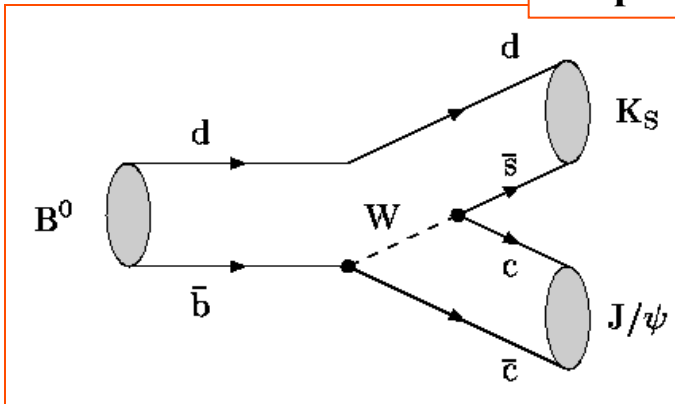
Time dependent CP violation

- If final state is CP eigenstate then 2 amplitudes (w/o mixing):

$$B^0 \rightarrow f \quad \text{and} \quad B^0 \rightarrow \bar{B}^0 \rightarrow f$$

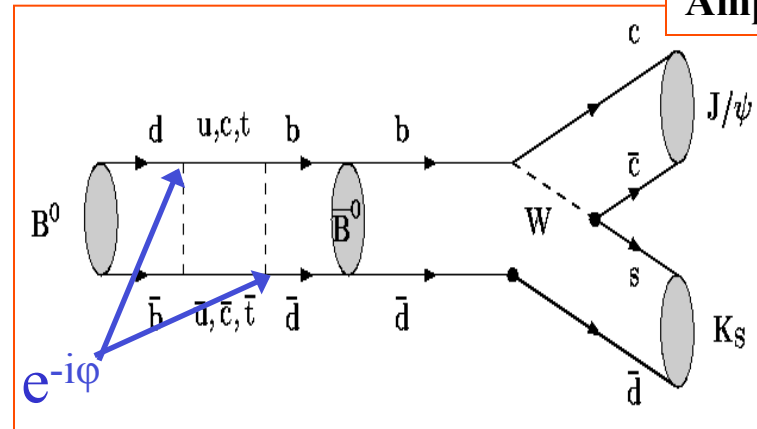
- $B^0 - \bar{B}^0$ oscillation is periodic in time \rightarrow CP violation time dependent

Amplitude 1



+

Amplitude 2



$$A_{CP}(t) = \frac{\Gamma(B^0 \rightarrow J/\psi K_S) - \Gamma(\bar{B}^0 \rightarrow J/\psi K_S)}{\Gamma(B^0 \rightarrow J/\psi K_S) + \Gamma(\bar{B}^0 \rightarrow J/\psi K_S)} = -\sin 2\beta \sin \Delta m t$$

$$(i = e^{i\pi/2} \rightarrow \delta = 90^\circ)$$

$$(\varphi = 2\beta)$$

Time dependent CP violation

- If final state is CP eigenstate then 2 amplitudes (w/o mixing):

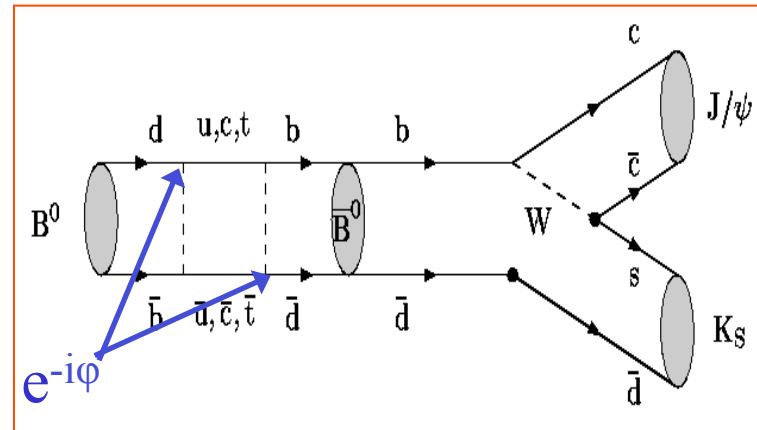
$$B^0 \rightarrow f \quad \text{and} \quad B^0 \rightarrow \bar{B}^0 \rightarrow f$$

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)] \right)$$

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2} \begin{pmatrix} 1 & \pm \cos \Delta m t \end{pmatrix}$$

$$g_+^*(t) g_-(t) = \frac{e^{-\Gamma t}}{2} \begin{pmatrix} + i \sin \Delta m t \end{pmatrix}$$

$$g_+(t) g_-^*(t) = \frac{e^{-\Gamma t}}{2} \begin{pmatrix} - i \sin \Delta m t \end{pmatrix}$$



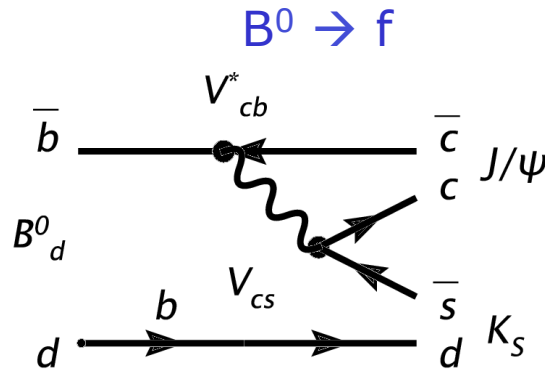
$$\Gamma(B^0 \rightarrow J/\psi K_S) = \left| A e^{-imt - \Gamma t} \left(\cos \frac{\Delta m t}{2} + e^{i\phi} \sin \frac{\Delta m t}{2} \right) \right|^2$$

$$A_{CP}(t) = \frac{\Gamma(B^0 \rightarrow J/\psi K_S) - \Gamma(\bar{B}^0 \rightarrow J/\psi K_S)}{\Gamma(B^0 \rightarrow J/\psi K_S) + \Gamma(\bar{B}^0 \rightarrow J/\psi K_S)} = -\sin 2\beta \sin \Delta m t$$

$$\begin{aligned} (i = e^{i\pi/2} \rightarrow \delta = 90^\circ) \\ (\phi = 2\beta) \end{aligned}$$

Sum of 2 amplitudes: sensitivity to phase

- What do we know about the relative phases of the diagrams?

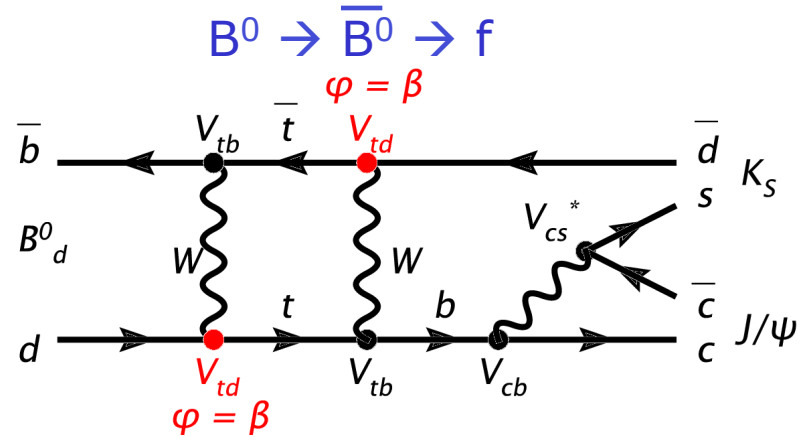


$\phi(\text{strong}) = \phi$

$\phi(\text{weak}) = 0$

Decays are identical

K^0 mixing exactly cancels V_{cs}



$\phi(\text{strong}) = \phi$

$\phi(\text{weak}) = 2\beta$

$\phi(\text{mixing}) = \pi/2$

$|\psi(t)\rangle:$

$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle$

$|\bar{B}^0(t)\rangle = g_+(t)|\bar{B}^0\rangle + \frac{p}{q}g_-(t)|B^0\rangle$

$g_+(t) = e^{-imt} e^{-\Gamma t/2} \cos \frac{\Delta mt}{2}$

$g_-(t) = e^{-imt} e^{-\Gamma t/2} i \sin \frac{\Delta mt}{2}$

$$\Psi_0(x,t) = e^{-imt-t/2\tau} \left(|B^0\rangle \cos(\Delta mt/2) + i \left(\frac{q}{p} \right) |\bar{B}^0\rangle \sin(\Delta mt/2) \right)$$

There is a phase difference of i between the B^0 and \bar{B}^0

Sum of 2 amplitudes: sensitivity to phase

- Now also look at CP-conjugate process
- Investigate situation at time t , such that $|A_1| = |A_2|$:

$\Gamma(B \rightarrow f) =$

$\Gamma(\bar{B} \rightarrow f) =$

$N(B^0 \rightarrow f) \propto |A|^2 \propto (1 - \cos\phi)^2 + \sin^2\phi$
 $= 1 - 2\cos\phi + \cos^2\phi + \sin^2\phi$
 $= 2 - 2\cos(\pi/2 - 2\beta)$
 $\propto 1 - \sin(2\beta)$

$A_{CP} = \frac{N_{\bar{B}^0 \rightarrow f} - N_{B^0 \rightarrow f}}{N_{B^0 \rightarrow f} + N_{\bar{B}^0 \rightarrow f}} = \sin(2\beta)$

$N(\bar{B}^0 \rightarrow f) \propto (1 + \cos\phi)^2 + \sin^2\phi$
 $= 2 + 2\cos(\pi/2 - 2\beta)$
 $\propto 1 + \sin(2\beta)$

- Directly observable result (essentially just from counting) measure CKM phase β directly!

$$A_{CP}(t = \pi / 2\Delta m) = \frac{N_{\bar{B}^0 \rightarrow f} - N_{B^0 \rightarrow f}}{N_{B^0 \rightarrow f} + N_{\bar{B}^0 \rightarrow f}} = \sin(2\beta)$$

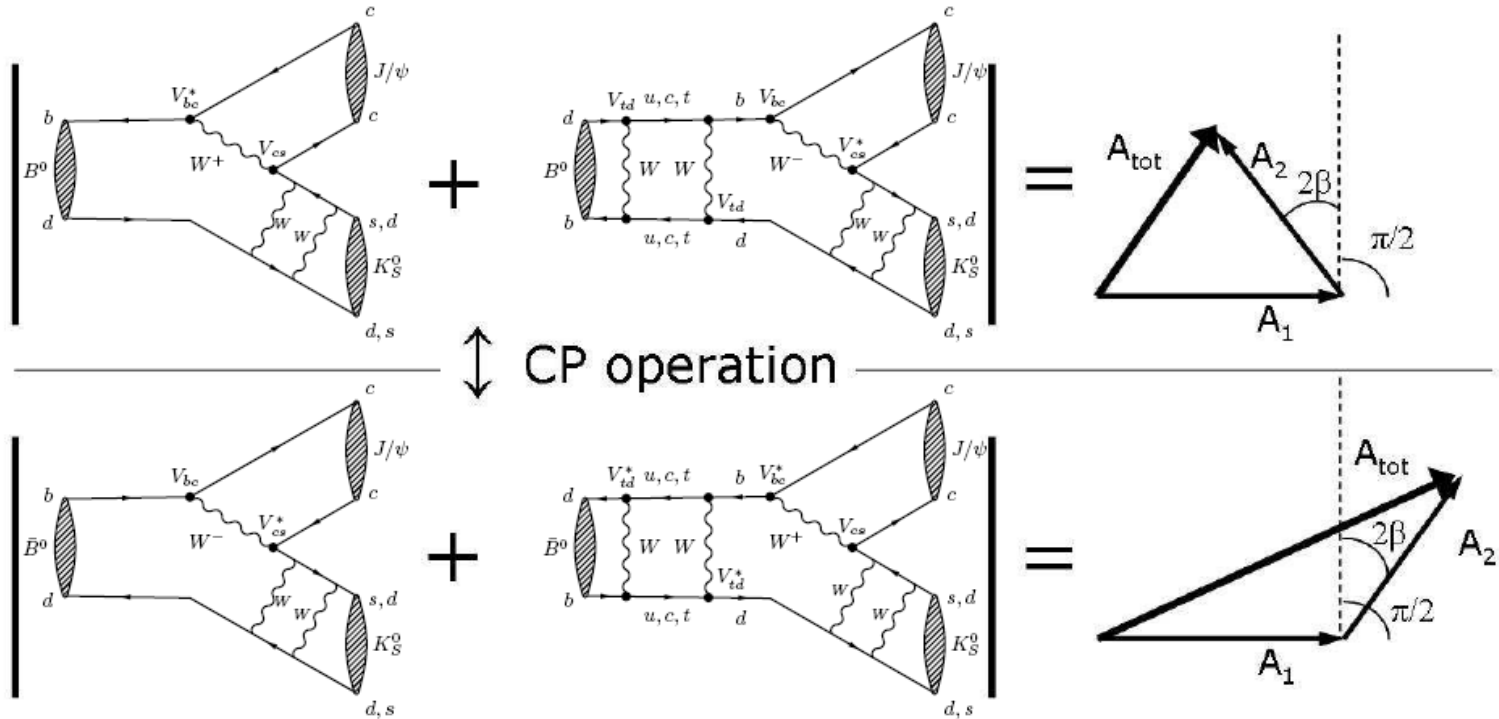
Remember!

Necessary ingredients for CP violation:

- 1) Two (interfering) amplitudes
- 2) Phase difference between amplitudes
 - one CP conserving phase (‘strong’ phase)
 - one CP violating phase (‘weak’ phase)

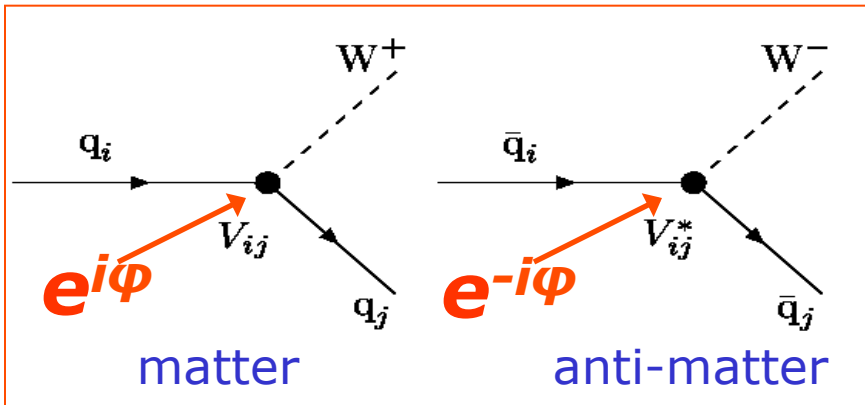
2 amplitudes
2 phases

Remember!



2 amplitudes
2 phases

What do we measure?



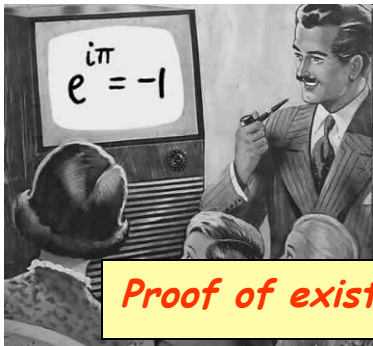
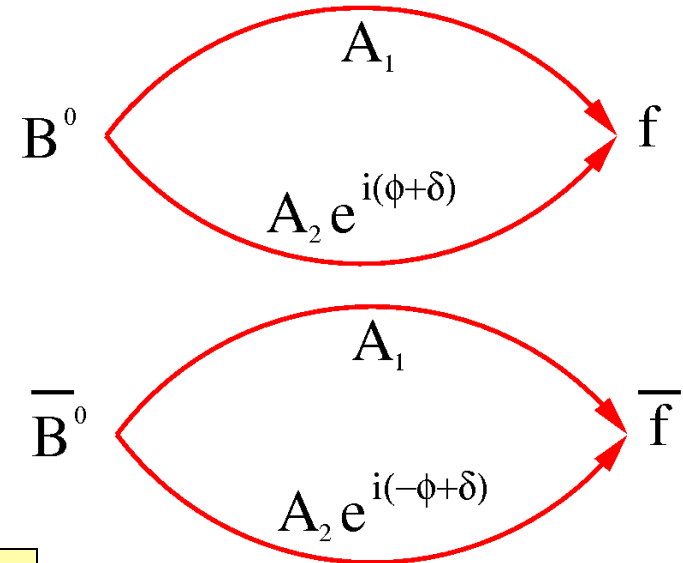
We measure **quark couplings**

There is a **complex phase** in couplings!

Visible when there are **2 amplitudes**:

$$\Gamma(B \rightarrow f) = |A_1 + A_2 e^{i(\phi + \delta)}|^2$$

$$\Gamma(\bar{B} \rightarrow \bar{f}) = |A_1 + A_2 e^{i(-\phi + \delta)}|^2$$

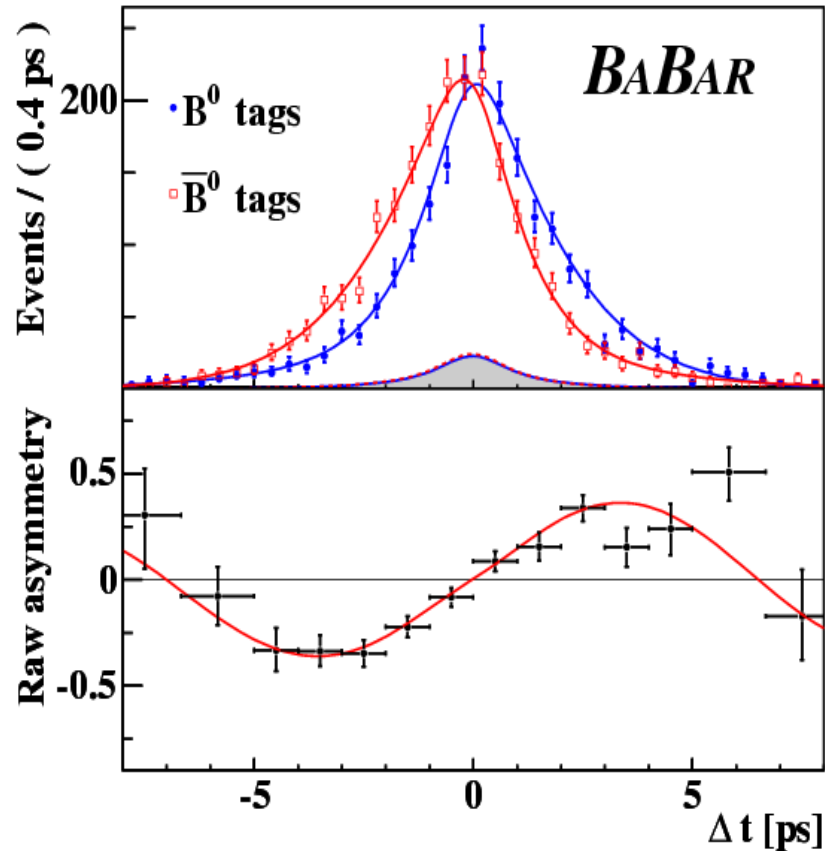


Proof of existence of complex numbers...

B-system - Time-dependent CP asymmetry

$$B^0 \rightarrow J/\psi K_S$$

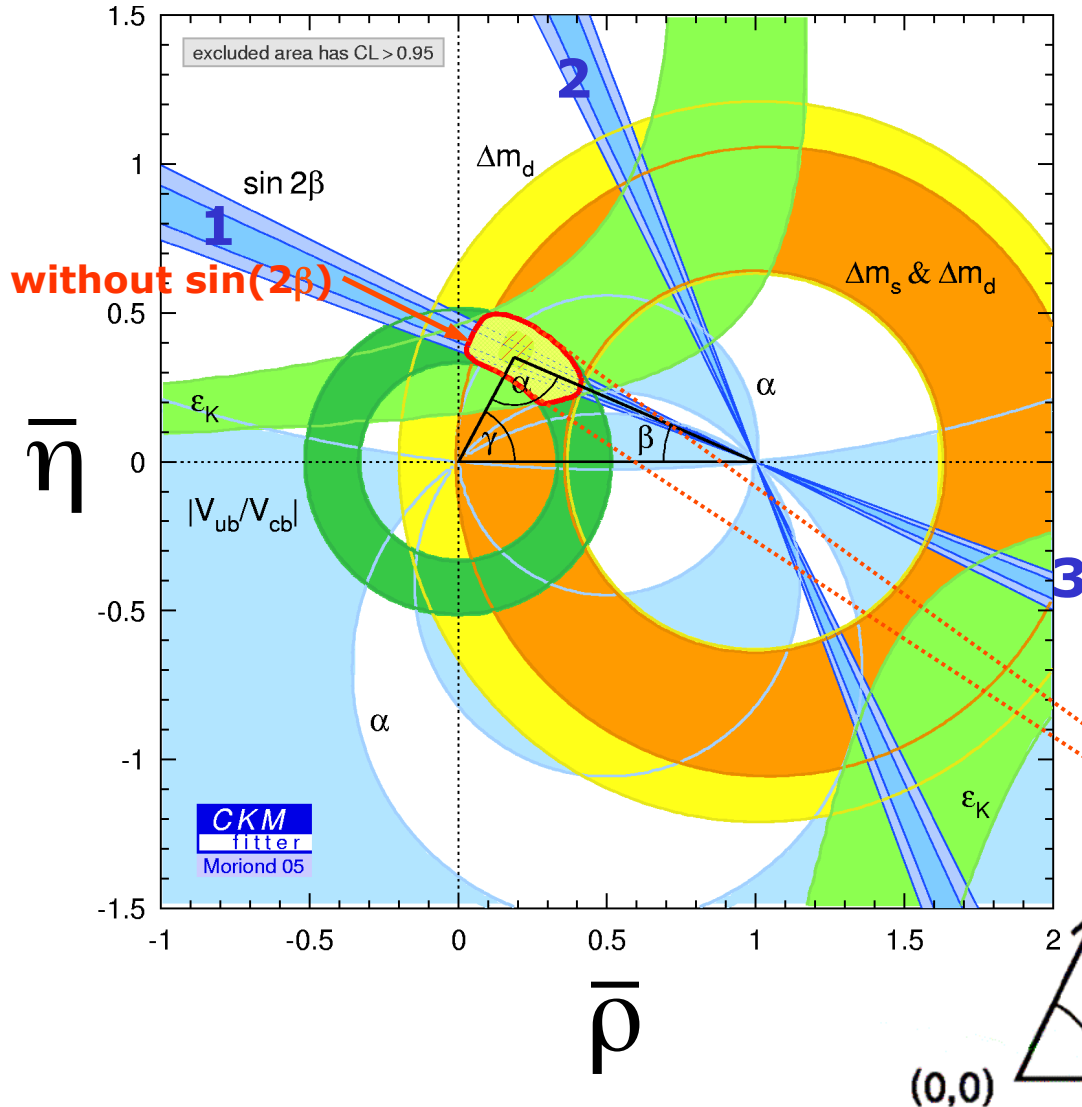
$$A_{CP}(t) = \frac{N_{\bar{B}^0 \rightarrow f} - N_{B^0 \rightarrow f}}{N_{\bar{B}^0 \rightarrow f} + N_{B^0 \rightarrow f}} = \sin(2\beta) \sin(\Delta m t)$$



BaBar (2002)

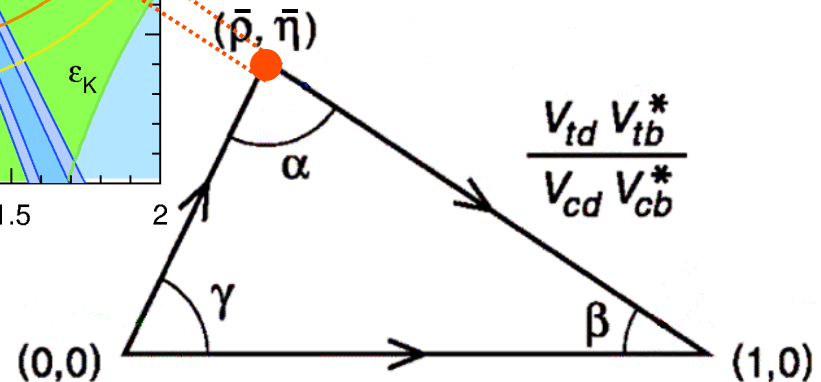
Consistency with other measurements in (ρ, η) plane

4-fold ambiguity because we measure $\sin(2\beta)$, not β

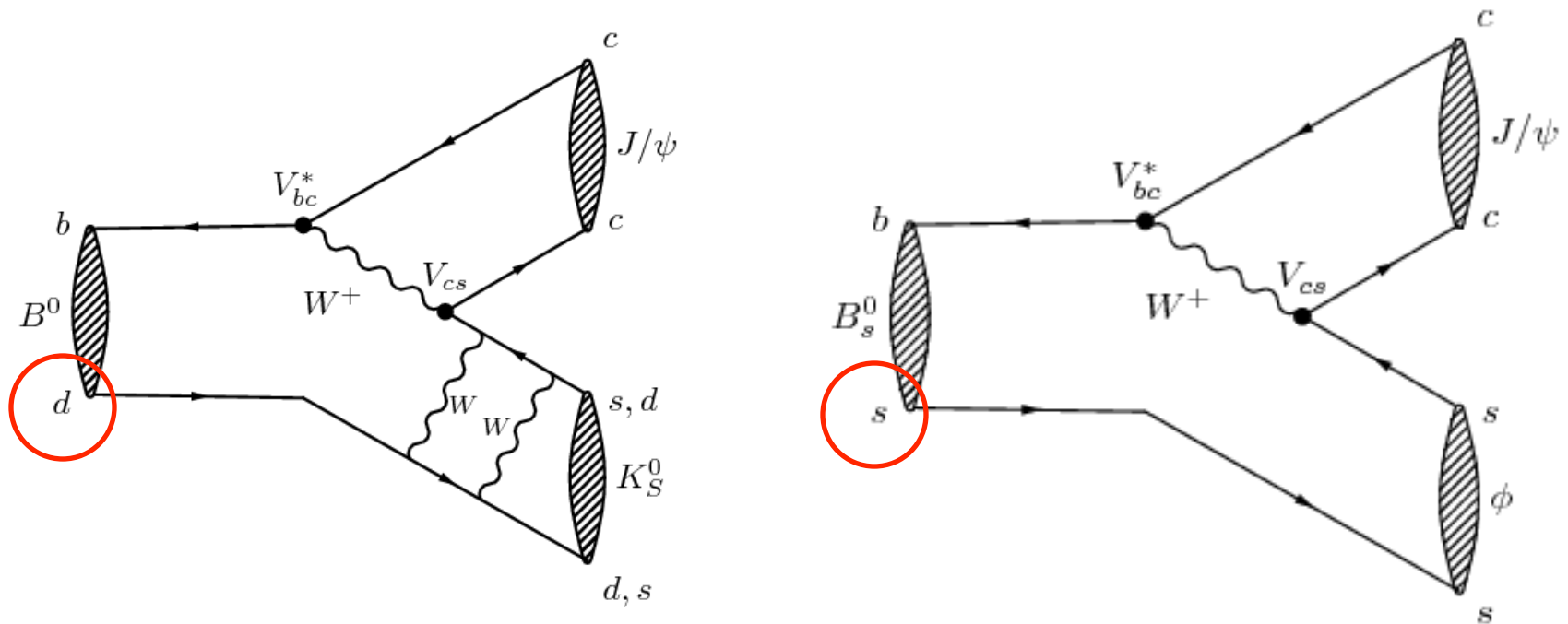


Prices measurement of $\sin(2\beta)$ agrees perfectly with other measurements and CKM model assumptions

The CKM model of CP violation experimentally confirmed with high precision!

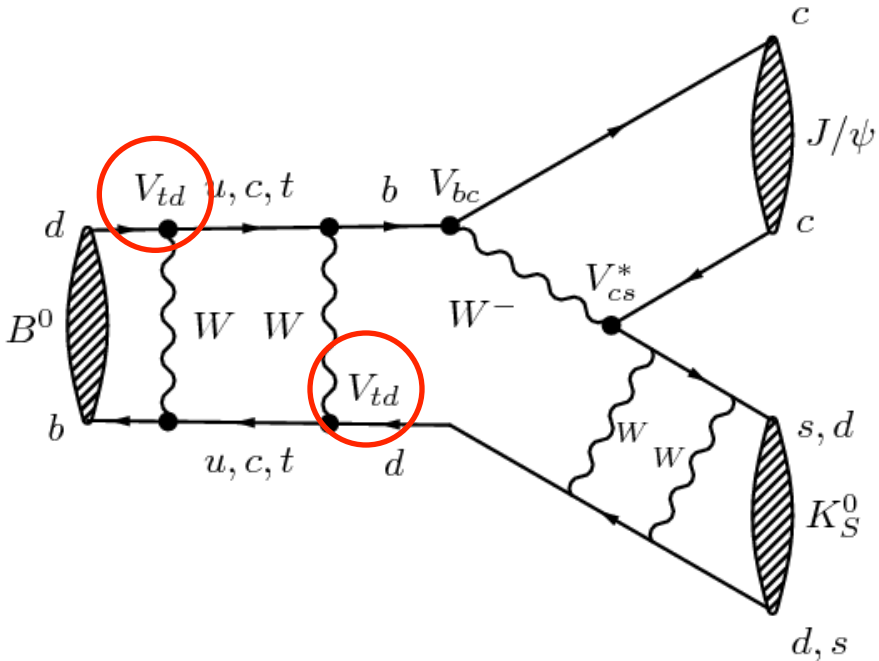


β_s : $B_s^0 \rightarrow J/\psi\phi$: B_s^0 analogue of $B^0 \rightarrow J/\psi K_S^0$

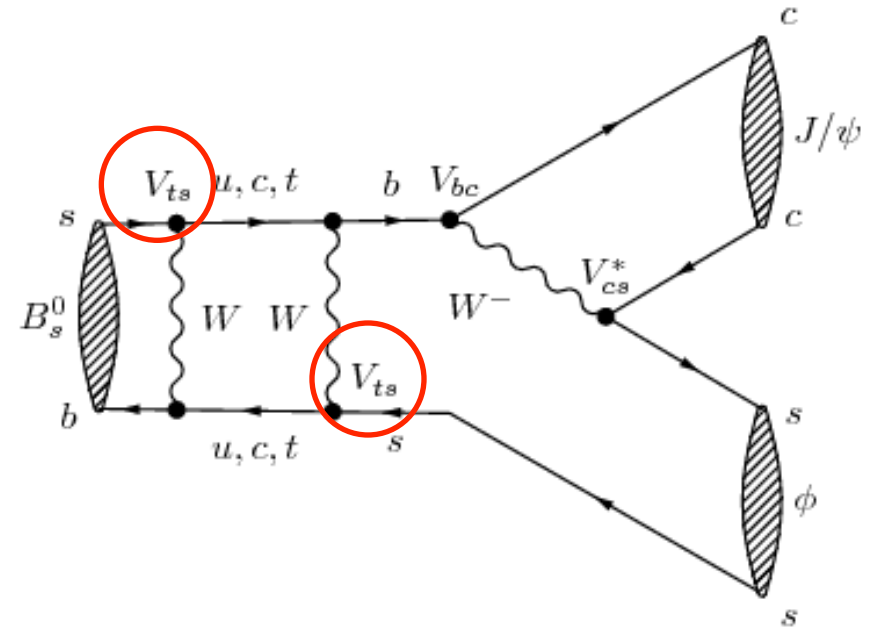


- Replace spectator quark $d \rightarrow s$

β_s : $B_s^0 \rightarrow J/\psi\phi$: B_s^0 analogue of $B^0 \rightarrow J/\psi K_S^0$



$$\beta \equiv \arg \left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right]$$

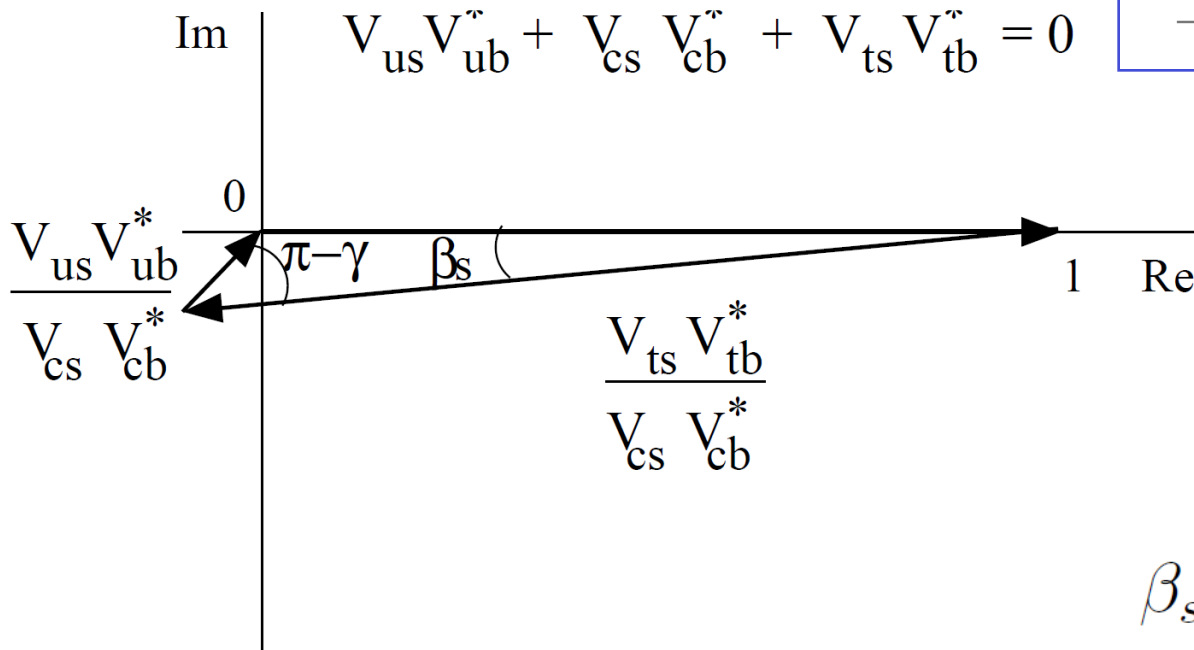
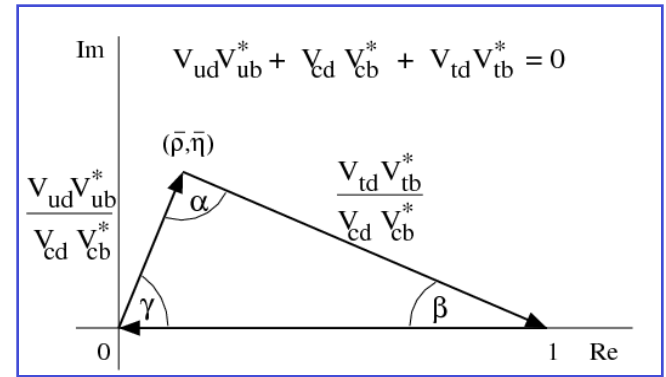


$$\beta_s \equiv \arg \left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right]$$

$$V_{CKM, \text{Wolfenstein}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

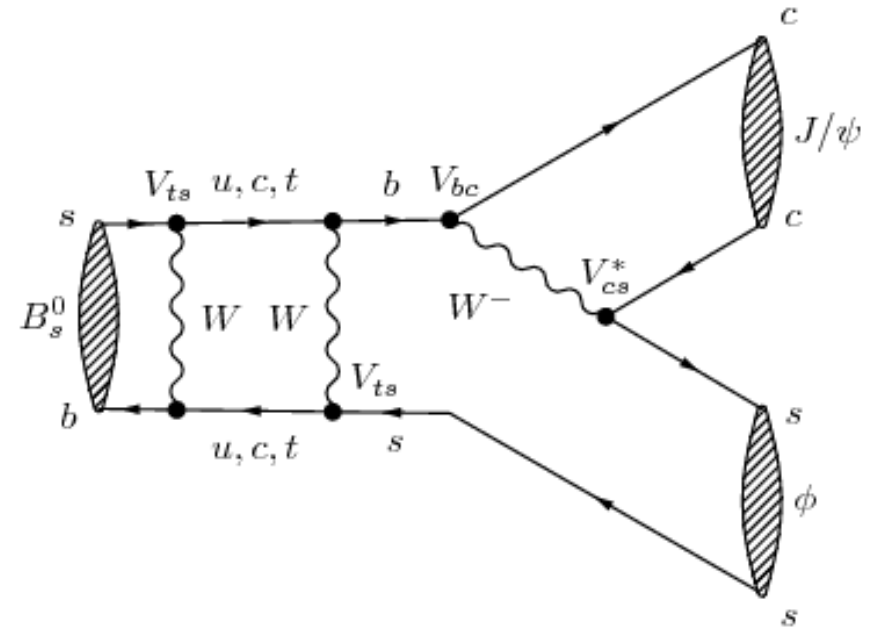
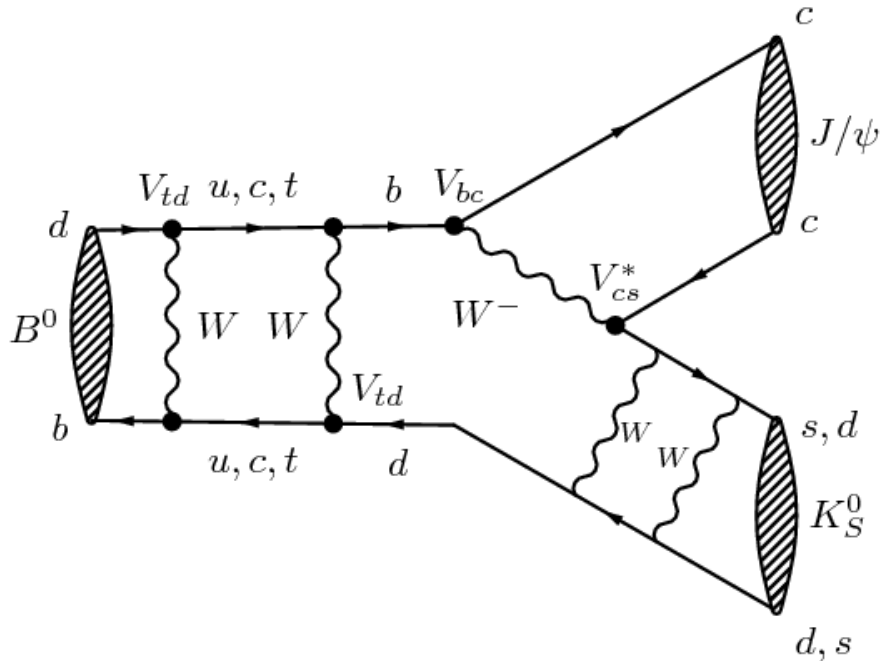
Remember: The “B_s-triangle”: β_s

- Replace *d* by *s*:



$$\beta_s \equiv \arg \left[-\frac{V_{ts} V_{tb}^*}{V_{cs} V_{cb}^*} \right]$$

β_S : $B_s^0 \rightarrow J/\psi\phi$: B_s^0 analogue of $B^0 \rightarrow J/\psi K_S^0$



Differences:

	B^0	B_s^0
CKM	V_{td}	V_{ts}
$\Delta\Gamma$	~ 0	~ 0.1
Final state (spin)	K^0 : $s=0$	ϕ : $s=1$
Final state (K)	K^0 mixing	-

$$\beta_s: B_s^0 \rightarrow J/\psi\phi$$

$$A_{CP}(t) = \frac{\Gamma_{B_s^0(t) \rightarrow J/\psi\phi} - \Gamma_{\bar{B}_s^0(t) \rightarrow J/\psi\phi}}{\Gamma_{B_s^0(t) \rightarrow J/\psi\phi} + \Gamma_{\bar{B}_s^0(t) \rightarrow J/\psi\phi}} = \frac{\Im\lambda_{J/\psi\phi} \sin \Delta m t}{\cosh \frac{1}{2} \Delta\Gamma t + \Re\lambda_{J/\psi\phi} \sinh \frac{1}{2} \Delta\Gamma t}$$

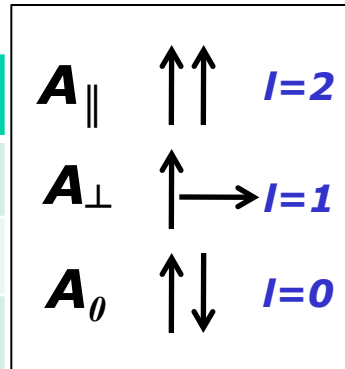
$$\lambda_{J/\psi\phi} = \left(\frac{q}{p}\right)_{B_s^0} \left(\eta_{J/\psi\phi} \frac{\bar{A}_{J/\psi\phi}}{A_{J/\psi\phi}}\right) = (-1)^l \left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right) \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right)$$

$$\Im\lambda_{J/\psi\phi} = (-1)^l \sin(-2\beta_s)$$

$$CP|J/\psi\phi\rangle_l = (-1)^l |J/\psi\phi\rangle_l$$

*V_{ts} large, oscillations fast,
need good vertex detector*
3 amplitudes

	B⁰	B_s⁰
CKM	V _{td}	V _{ts}
$\Delta\Gamma$	~ 0	~ 0.1
Final state (spin)	K ⁰ : s=0	ϕ : s=1
Final state (K)	K ⁰ mixing	-



“Recent” excitement (5 March 2008)

FIRST EVIDENCE OF NEW PHYSICS IN $b \leftrightarrow s$ TRANSITIONS

(*UTfit* Collaboration)

M. Bona,¹ M. Ciuchini,² E. Franco,³ V. Lubicz,^{2,4} G. Martinelli,^{3,5} F. Parodi,⁶ M. Pierini,¹
P. Roudeau,⁷ C. Schiavi,⁶ L. Silvestrini,³ V. Sordini,⁷ A. Stocchi,⁷ and V. Vagnoni⁸

We combine all the available experimental information on B_s mixing, including the very recent tagged analyses of $B_s \rightarrow J/\Psi\phi$ by the CDF and DØ collaborations. We find that the phase of the B_s mixing amplitude deviates more than 3σ from the Standard Model prediction. While no single measurement has a 3σ significance yet, all the constraints show a remarkable agreement with the combined result. **This is a first evidence of physics beyond the Standard Model.** This result disfavors New Physics models with Minimal Flavour Violation with the same significance.

In the Standard Model (SM), all flavour and CP violating phenomena in weak decays are described in terms of quark masses and the four independent parameters in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. In particular, there is only one source of CP violation, which is connected to the area of the Unitarity Triangle (UT). A peculiar prediction of the SM, due to the hierarchy among CKM matrix elements, is that CP violation in B_s mixing should be tiny. This property is also valid in models of Minimal Flavour Violation (MFV) [2], where flavour and CP violation are still governed by the CKM matrix. Therefore, the experimental observation of sizable CP violation in B_s mixing is a clear (and clean) signal of New Physics (NP) and a violation of the MFV paradigm. In the past decade, B factories have collected an impressive amount of data on B_d flavour- and CP-violating processes. The CKM paradigm has passed unscathed all the tests performed at the B factories down to an accuracy just below 10% [3, 4]. This has been often considered as an indication pointing to the MFV hypothesis, which has received considerable attention in recent years. The only possible hint of non-MFV NP is found in the penguin-dominated $b \rightarrow s$ non-leptonic decays. Indeed, in the SM, the $S_{q\bar{q}s}$ coefficient of the time-dependent CP asymmetry in these channels is equal to the $S_{c\bar{c}s}$ measured with $b \rightarrow c\bar{c}s$ decays, up to hadronic uncertainties related to subleading terms in the decay amplitudes. Present data show a systematic, although not statistically significant, downward shift of $S_{q\bar{q}s}$ with respect to $S_{c\bar{c}s}$ [5], while hadronic models predict a shift in the opposite direction in many cases [6, 7].

From the theoretical point of view, the hierarchical structure of quark masses and mixing angles of the SM calls for an explanation in terms of flavour symmetries or of other dynamical mechanisms, such as, for example, fermion localization in models with extra dimensions. All

such explanations depart from the MFV paradigm, and generically cause deviations from the SM in flavour violating processes. Models with localized fermions [8], and more generally models of Next-to-Minimal Flavour Violation [9], tend to produce too large effects in ε_K [10, 11]. On the contrary, flavour models based on nonabelian flavour symmetries, such as $U(2)$ or $SU(3)$, typically suppress NP contributions to $s \leftrightarrow d$ and possibly also to $b \leftrightarrow d$ transitions, but easily produce large NP contributions to $b \leftrightarrow s$ processes. This is due to the large flavour symmetry breaking caused by the top quark Yukawa coupling. Thus, if (nonabelian) flavour symmetry models are relevant for the solution of the SM flavour problem, one expects on general grounds NP contributions to $b \leftrightarrow s$ transitions. On the other hand, in the context of Grand Unified Theories (GUTs), there is a connection between leptonic and hadronic flavour violation. In particular, in a broad class of GUTs, the large mixing angle observed in neutrino oscillations corresponds to large NP contributions to $b \leftrightarrow s$ transitions [12].

In this Letter, we show that present data give evidence of a B_s mixing phase much larger than expected in the SM, with a significance of more than 3σ . This result is obtained by combining all available experimental information with the method used by our collaboration for UT analyses and described in Ref. [13].

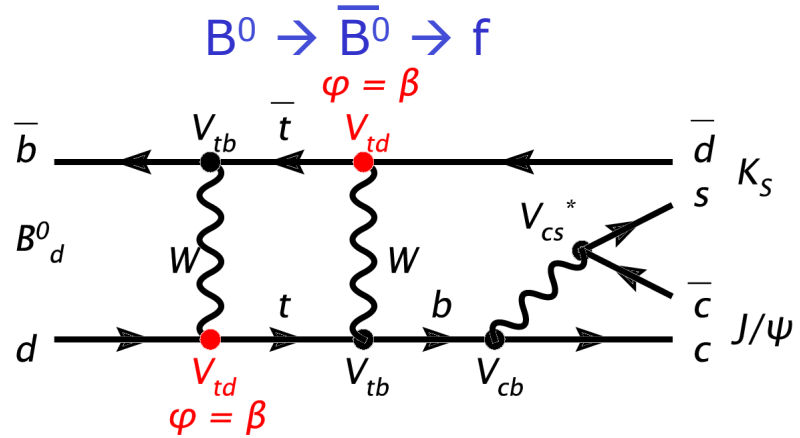
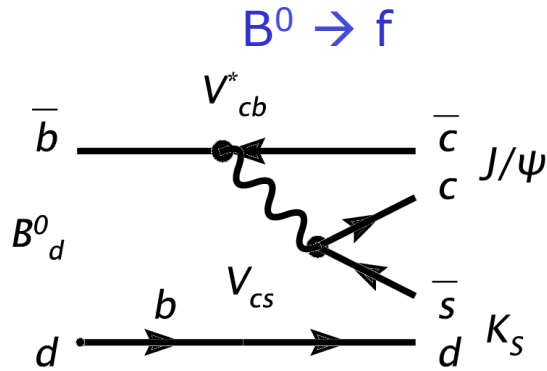
We perform a model-independent analysis of NP contributions to B_s mixing using the following parametrization [14]:

$$\begin{aligned} C_{B_s} e^{2i\phi_{B_s}} &= \frac{A_s^{\text{SM}} e^{-2i\beta_s} + A_s^{\text{NP}} e^{2i(\phi_s^{\text{NP}} - \beta_s)}}{A_s^{\text{SM}} e^{-2i\beta_s}} = \\ &= \frac{\langle B_s | H_{\text{eff}}^{\text{full}} | \bar{B}_s \rangle}{\langle B_s | H_{\text{eff}}^{\text{SM}} | \bar{B}_s \rangle}, \end{aligned} \quad (1)$$

where $H_{\text{eff}}^{\text{full}}$ is the effective Hamiltonian generated

$B_s \rightarrow J/\psi\Phi$: B_s equivalent of $B \rightarrow J/\psi K_s$!

- The mixing phase (V_{td}): $\varphi_d=2\beta$

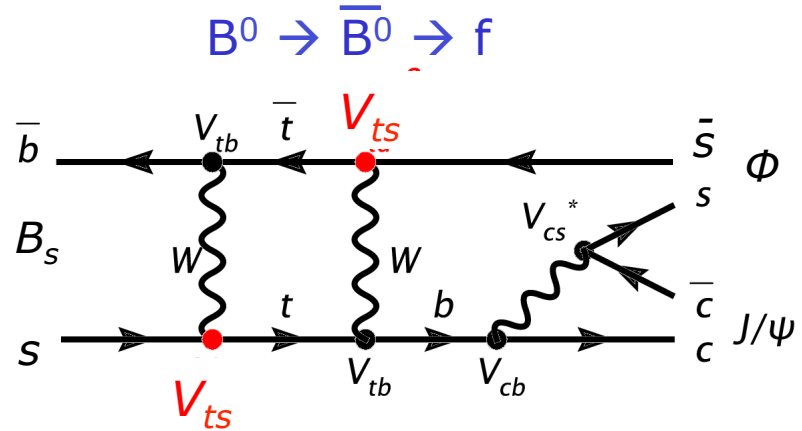
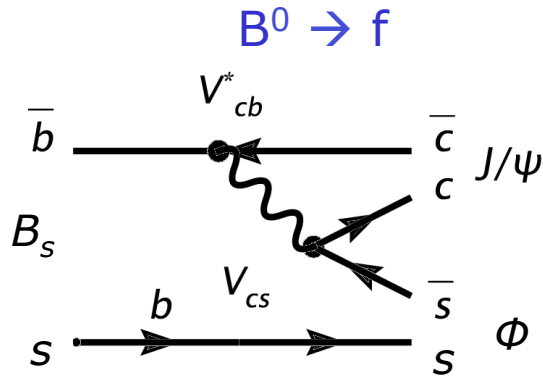


Wolfenstein parametrization to $O(\lambda^5)$:

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

$B_s \rightarrow J/\psi\Phi$: B_s equivalent of $B \rightarrow J/\psi K_s$!

- The mixing phase (V_{ts}): $\varphi_s = -2\beta_s$

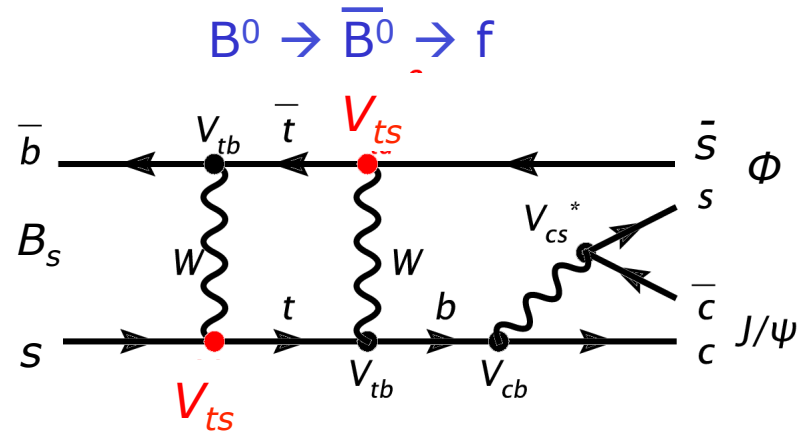
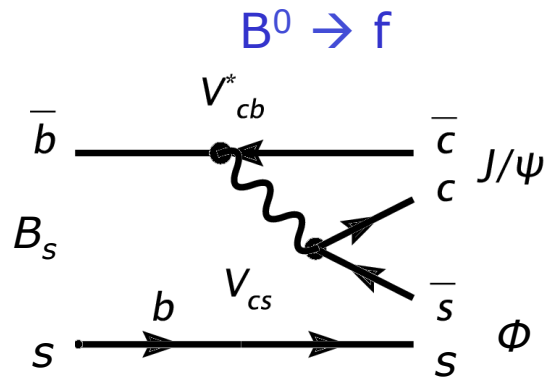


Wolfenstein parametrization to $O(\lambda^5)$:

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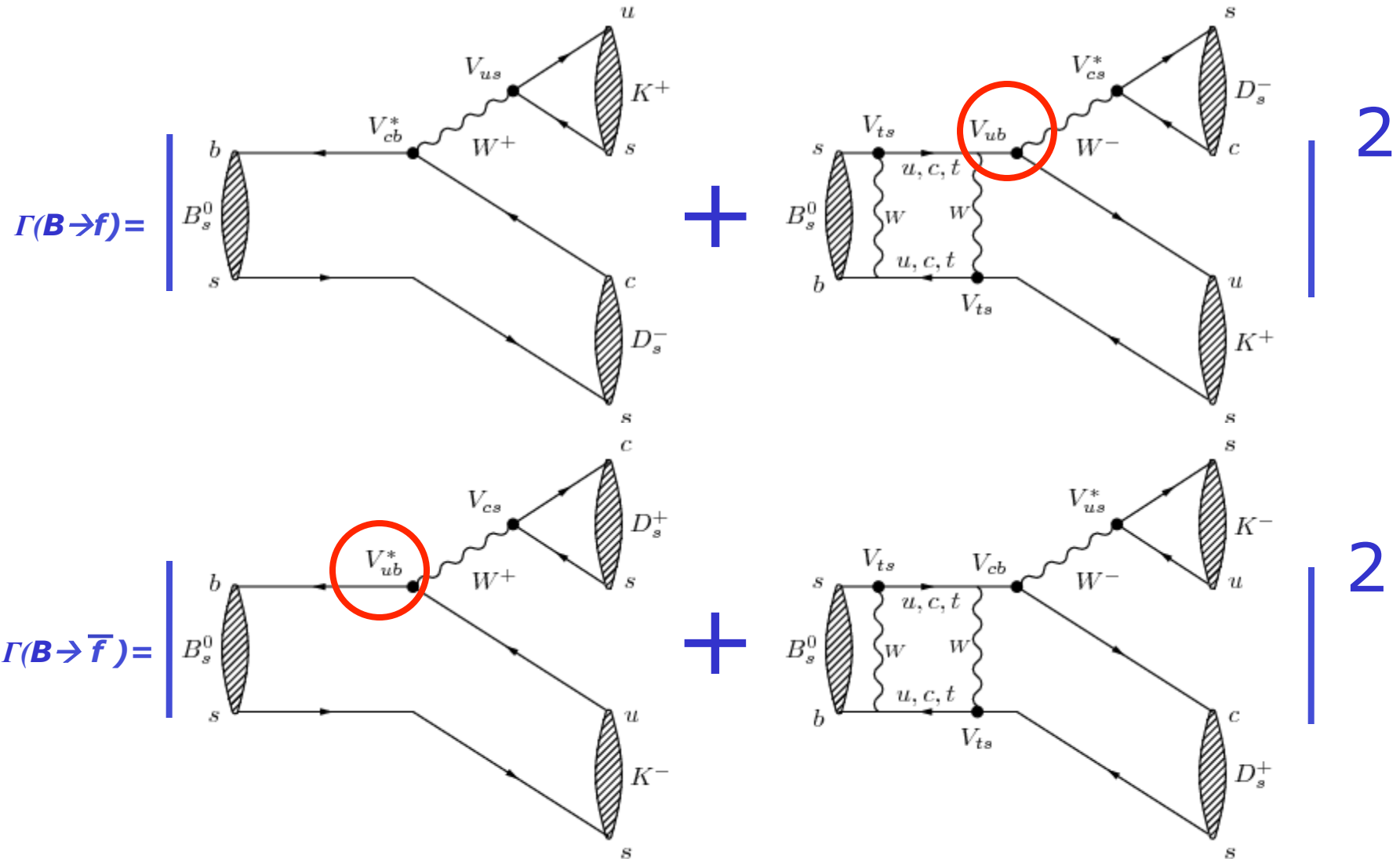
$B_s \rightarrow J/\psi\Phi$: B_s equivalent of $B \rightarrow J/\psi K_s$!

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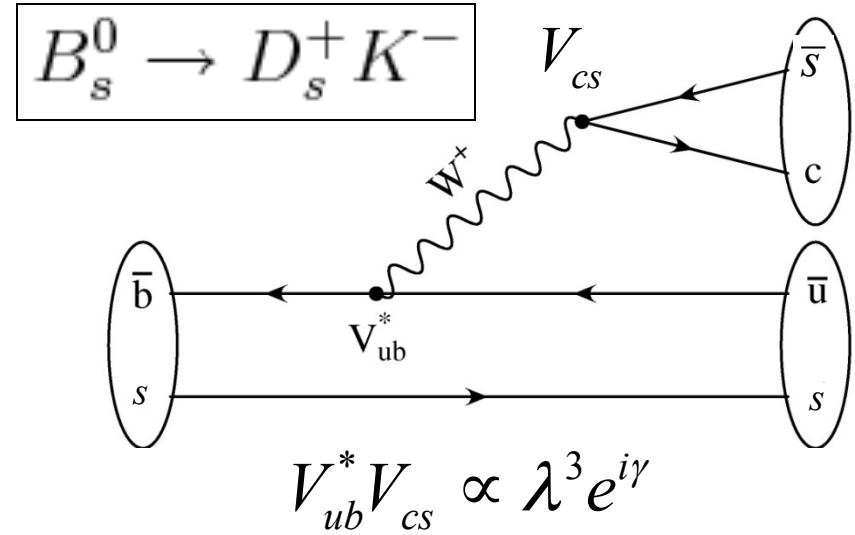
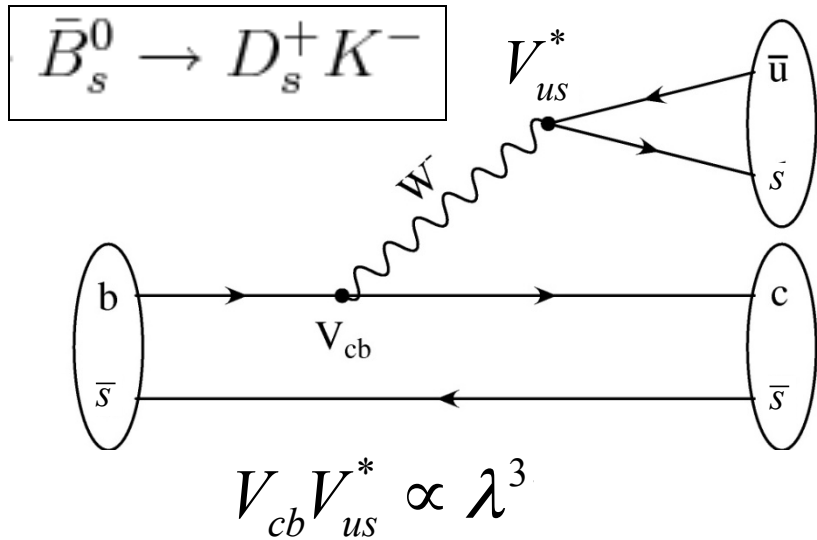
$$V_{CKM, \text{Wolfenstein}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

Measure γ : $B_s^0 \rightarrow D_s^\pm K^{-/+}$: both λ_f and $\lambda_{\bar{f}}$



NB: In addition $\bar{B}_s \rightarrow D_s^\pm K^{-/+}$: both $\bar{\lambda}_f$ and $\bar{\lambda}_{\bar{f}}$

Measure γ : $B_s \rightarrow D_s^\pm K^{-/+}$ --- first one f : $D_s^+ K^-$



- This time $|A_f| \neq |\bar{A}_f|$, so $|\lambda| \neq 1$!

$$\left(\frac{\bar{A}_{D_s^- K^+}}{A_{D_s^- K^+}} \right) = \left(\frac{V_{ub} V_{cs}^*}{V_{cb}^* V_{us}} \right) \left(\frac{A_2}{A_1} \right) \quad \lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$$

- In fact, not only magnitude, but also phase difference:

$$\frac{A_{D_s^- K^+}}{\bar{A}_{D_s^- K^+}} = \frac{|A_{D_s^- K^+}|}{|\bar{A}_{D_s^- K^+}|} e^{i(\delta_s - \gamma)}$$

Measure γ : $B_s \rightarrow D_s^\pm K^{-/+}$

- $B_s^0 \rightarrow D_s^- K^+$ has phase difference $(\delta - \gamma)$:

$$\frac{A_{D_s^- K^+}}{\bar{A}_{D_s^- K^+}} = \frac{|A_{D_s^- K^+}|}{|\bar{A}_{D_s^- K^+}|} e^{i(\delta_s - \gamma)}$$

- Need $B_s^0 \rightarrow D_s^+ K^-$ to disentangle δ and γ :

$$\lambda_{D_s^- K^+} = \left(\frac{q}{p}\right)_{B_s^0} \left(\frac{\bar{A}_{D_s^- K^+}}{A_{D_s^- K^+}}\right) = \left|\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right| \left|\frac{V_{ub} V_{cs}^*}{V_{cb}^* V_{us}}\right| \left|\frac{A_2}{A_1}\right| e^{i(-2\beta_s - \gamma + \delta_s)}$$

$$\lambda_{D_s^+ K^-} = \left(\frac{q}{p}\right)_{B_s^0} \left(\frac{\bar{A}_{D_s^+ K^-}}{A_{D_s^+ K^-}}\right) = \left|\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right| \left|\frac{V_{us}^* V_{cb}}{V_{cs} V_{ub}^*}\right| \left|\frac{A_1}{A_2}\right| e^{i(-2\beta_s - \gamma - \delta_s)}$$

Next

1. CP violation in decay

$$\Gamma(P^0 \rightarrow f) \neq \Gamma(\bar{P}^0 \rightarrow \bar{f})$$

This is obviously satisfied (see Eq. (3.15)) when

$$\left| \frac{\bar{A}_f}{A_f} \right| \neq 1.$$

2. CP violation in mixing

$$\text{Prob}(P^0 \rightarrow \bar{P}^0) \neq \text{Prob}(\bar{P}^0 \rightarrow P^0)$$

$$\left| \frac{q}{p} \right| \neq 1.$$

3. CP violation in interference

$$\Gamma(P^0(\rightsquigarrow \bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

$$\Im \lambda_f = \Im \left(\frac{q}{p} \frac{\bar{A}_f}{A_f} \right) \neq 0$$

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