Particle Physics II – CP violation

(also known as "Physics of Anti-matter")

Lecture 3

N. Tuning

Plan

1) Wed 12 Feb: Anti-matter + SM
2) Mon 17 Feb: CKM matrix + Unitarity Triangle
3) Wed 19 Feb: Mixing + Master eqs. + $B^0 \to J/\psi K_s$
4) Mon 20 Feb: CP violation in $B_{(s)}$ decays (I)
5) Wed 9 Mar: CP violation in $B_{(s)}$ and K decays (II)
6) Mon 16 Mar: Rare decays + Flavour Anomalies
7) Wed 18 Mar: Exam

- Final Mark:
  - if (mark > 5.5) mark = max(exam, 0.85*exam + 0.15*homework)
  - else mark = exam
- In parallel: Lectures on Flavour Physics by prof.dr. R. Fleischer

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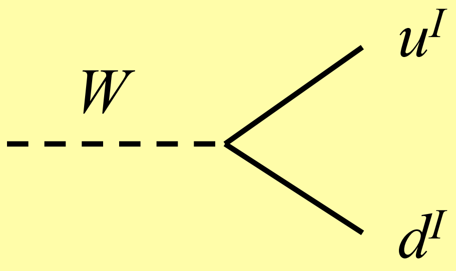
Recap

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$$-\mathcal{L}_{Yuk} = Y^{d}_{ij}\,(\overline{u^I_L}, \overline{d^I_L})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d^I_{Rj} + \ldots$$

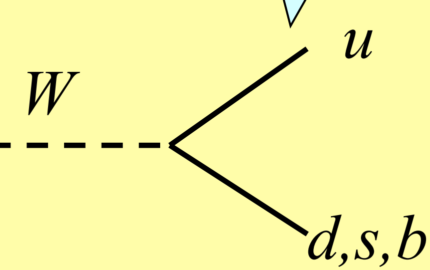
$$\mathcal{L}_{Kinetic} = \frac{g}{\sqrt{2}}\overline{u^I_{Li}}\gamma^\mu W^-_\mu d^I_{Li} + \frac{g}{\sqrt{2}}\overline{d^I_{Li}}\gamma^\mu W^+_\mu u^I_{Li} + \ldots$$



Diagonalize Yukawa matrix Y_{ij}

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



$$-\mathcal{L}_{Mass} = (\bar{d}, \bar{s}, \bar{b})_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + (\bar{u}, \bar{c}, \bar{t})_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \ldots$$

$$\mathcal{L}_{CKM} = \frac{g}{\sqrt{2}}\overline{u_i}\gamma^\mu W^-_\mu V_{ij}\left(1-\gamma^5\right)d_j + \frac{g}{\sqrt{2}}\overline{d_j}\gamma^\mu W^+_\mu V^*_{ij}\left(1-\gamma^5\right)u_i + \ldots$$

$$\mathcal{L}_{SM} = \mathcal{L}_{CKM} + \mathcal{L}_{Higgs} + \mathcal{L}_{Mass}$$

Charged Currents

The charged current term reads:

$$L_{CC} = \frac{g}{\sqrt{2}}\overline{u^I_{Li}}\gamma^\mu W^-_\mu d^I_{Li} \quad + \quad \frac{g}{\sqrt{2}}\overline{d^I_{Li}}\gamma^\mu W^+_\mu u^I_{Li} \quad = \quad J^{\mu-}_{CC}\, W^-_\mu + J^{\mu+}_{CC}\, W^+_\mu$$

$$= \frac{g}{\sqrt{2}}\overline{u_i}\left(\frac{1-\gamma^5}{2}\right)\gamma^\mu W^-_\mu V_{ij}\left(\frac{1-\gamma^5}{2}\right)d_j \quad + \quad \frac{g}{\sqrt{2}}\overline{d_j}\left(\frac{1-\gamma^5}{2}\right)\gamma^\mu W^+_\mu V^\dagger_{ji}\left(\frac{1-\gamma^5}{2}\right)u_i$$

$$= \frac{g}{\sqrt{2}}\overline{u_i}\gamma^\mu W^-_\mu V_{ij}\left(1-\gamma^5\right)d_j \quad + \quad \frac{g}{\sqrt{2}}\overline{d_j}\gamma^\mu W^+_\mu V^*_{ij}\left(1-\gamma^5\right)u_i$$

Under the CP operator this gives:

(Together with (x,t) -> (-x,t))

$$L_{CC} \xrightarrow{CP} \frac{g}{\sqrt{2}}\overline{d_j}\gamma^\mu W^+_\mu V_{ij}\left(1-\gamma^5\right)u_i \quad + \quad \frac{g}{\sqrt{2}}\overline{u_i}\gamma^\mu W^i_\mu V^*_{ij}\left(1-\gamma^5\right)d_j$$

A comparison shows that CP is conserved only if $V_{ij} = V_{ij}^*$

In general the charged current term is CP violating

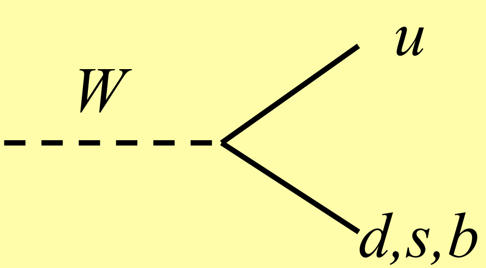
CKM-matrix: where are the phases?

- Possibility 1: simply 3 'rotations', and put phase on smallest:

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

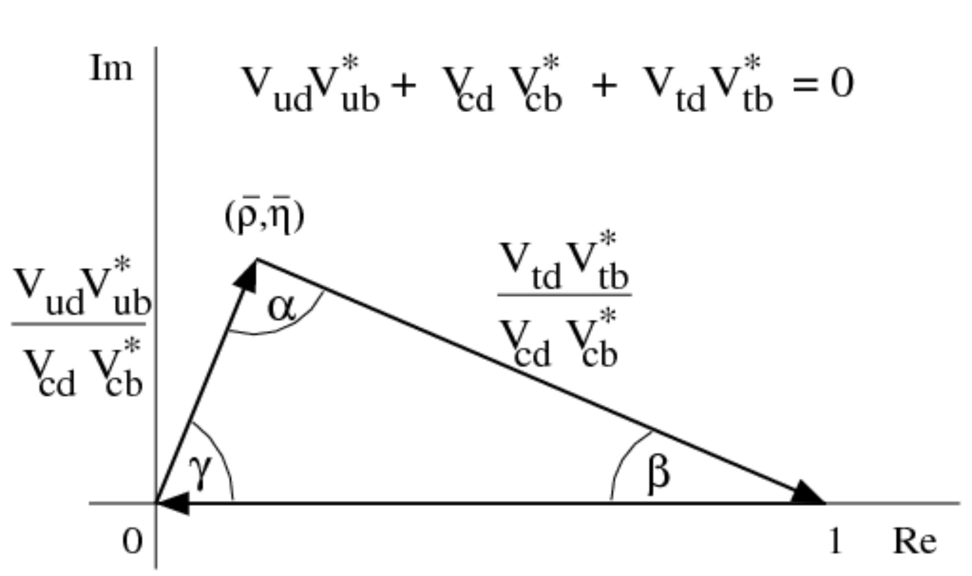
- Possibility 2: parameterize according to magnitude, in O(λ):



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

This was theory, now comes experiment

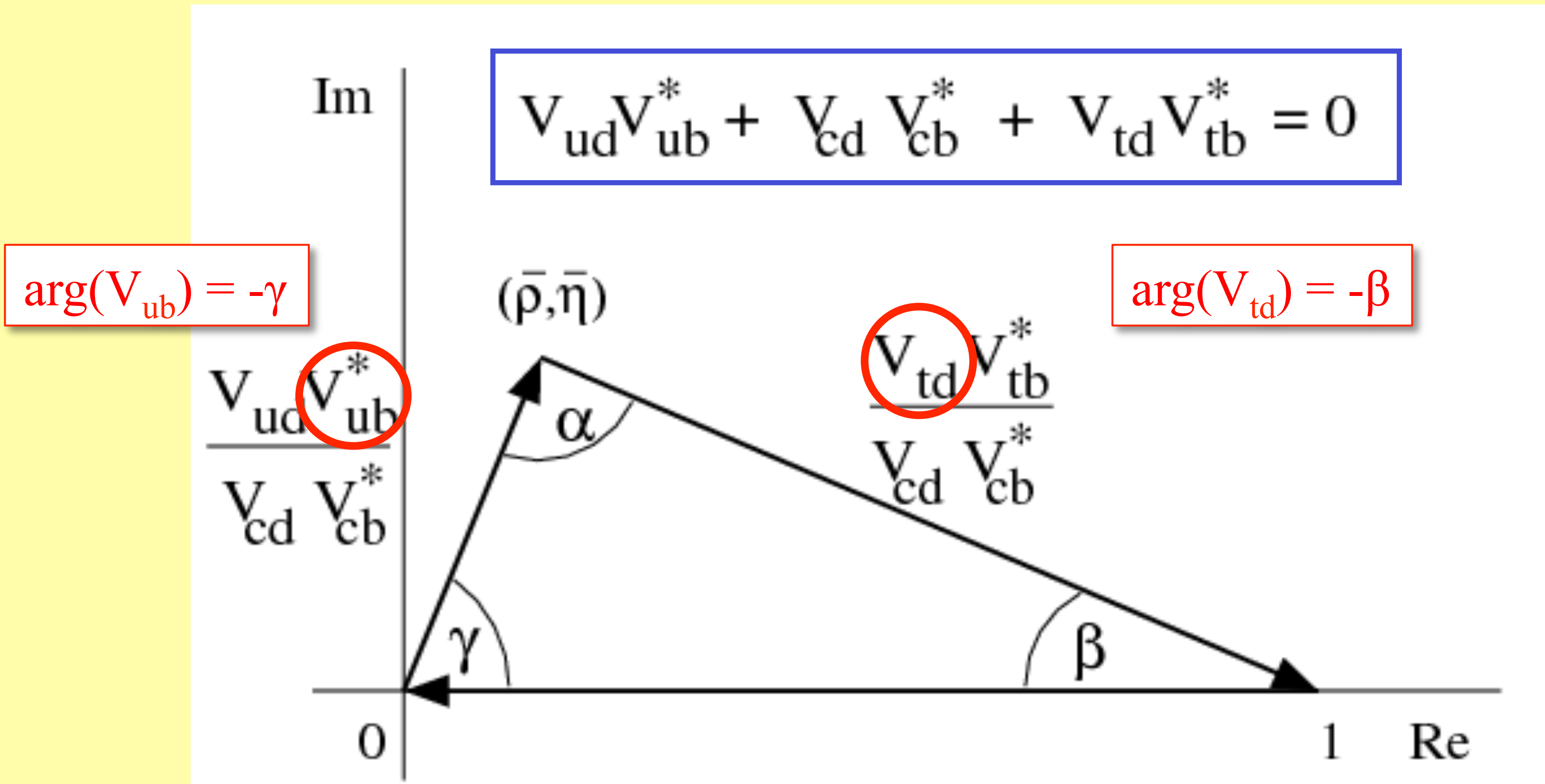
- We already saw how the moduli $|V_{ij}|$ are determined
- Now we will work towards the measurement of the imaginary part
  - Parameter: η
  - Equivalent: angles α, β, γ .



- To measure this, we need the formalism of neutral meson oscillations…

"The" Unitarity triangle

- We can visualize the CKM-constraints in (ρ,η) plane

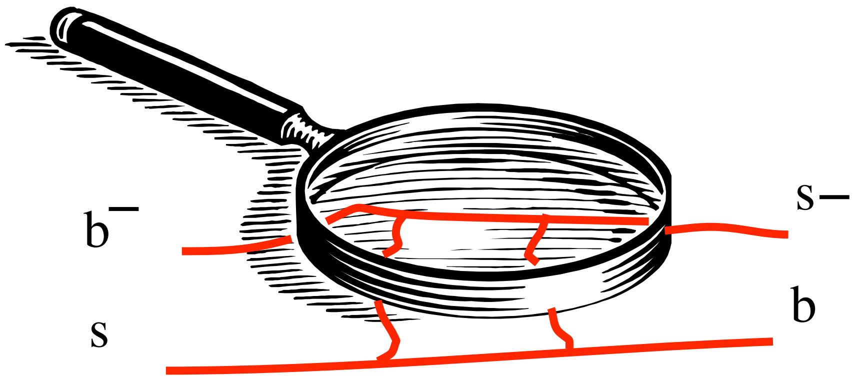


$$V_{CKM,\text{Wolfenstein}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$

Neutral Meson Oscillations

Why?

- Loop diagram: sensitive to new particles
- Provides a second amplitude
  - interference effects in B-decays



Dynamics of Neutral B (or K) mesons…

Time evolution of B^0 and $\overline{B}^0$ can be described by an *effective* Hamiltonian:

$$i\frac{\partial}{\partial t}\Psi = H\Psi \qquad \Psi(t) = a(t)\left|B^0\right\rangle + b(t)\left|\overline{B}^0\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}}$$

No mixing, no decay…

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

No mixing, but with decays…
(i.e.: H is not Hermitian!)

➔With decays included, probability of observing *either* B^0 *or* B^0 must go down as time goes by:

$$\frac{d}{dt}\left(\left|a(t)\right|^2 + \left|b(t)\right|^2\right) = -\begin{pmatrix} a(t)^* & b(t)^* \end{pmatrix}\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \qquad \Rightarrow \Gamma > 0$$

Describing Mixing…

Time evolution of B^0 and $\overline{B}^0$ can be described by an *effective* Hamiltonian:

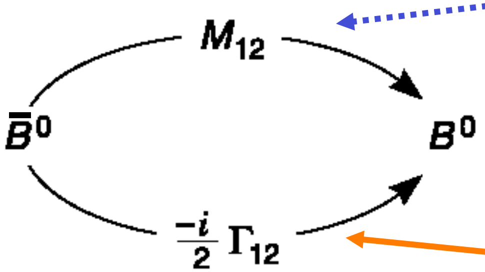
$$i\frac{\partial}{\partial t}\Psi = H\Psi \qquad \Psi(t) = a(t)\left|B^0\right\rangle + b(t)\left|\overline{B}^0\right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

Where to put the mixing term?

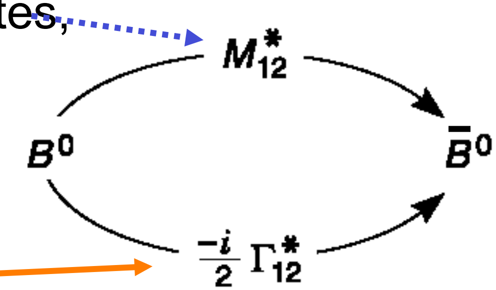
$$H = \underbrace{\begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2}\underbrace{\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

Now with mixing – but what is the difference between M_{12} and Γ_{12}?



M_{12} describes $B^0 \leftrightarrow B^0$ via *off-shell* states, e.g. the weak box diagram

Γ_{12} describes $B^0 \leftrightarrow f \leftrightarrow B^0$ via *on-shell* states, eg. $f=\pi^+\pi^-$



Solving the Schrödinger Equation

$$i\frac{\partial}{\partial t}\psi(t) = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\psi(t)$$

Eigenvalues:

– Mass and lifetime of physical states: mass eigenstates

$$\begin{vmatrix} M - \frac{i}{2}\Gamma - \lambda & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma - \lambda \end{vmatrix} = 0$$

$$\text{notation } F = \sqrt{(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*)}$$

$$\begin{aligned} m_1 + \frac{i}{2}\Gamma_1 &= M - \Re F - \frac{i}{2}\Gamma - \Im F \\ m_2 + \frac{i}{2}\Gamma_2 &= M + \Re F - \frac{i}{2}\Gamma + \Im F \end{aligned}$$

$$\Delta m = 2\Re\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$$

$$\Delta\Gamma = 4\Im\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$$

Solving the Schrödinger Equation

$$i\frac{\partial}{\partial t}\psi(t) = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\psi(t)$$

Eigenvectors:

– mass eigenstates

$$\begin{aligned} |P_1\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \\ |P_2\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \end{aligned}$$

find p and q by solving

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \lambda_\pm \begin{pmatrix} p \\ q \end{pmatrix}$$

$$|B_H\rangle = p|B\rangle + q|\overline{B}\rangle$$

$$|B_L\rangle = p|B\rangle - q|\overline{B}\rangle$$

$$q/p = \sqrt{\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)\Big/\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)}$$

Time evolution

- With diagonal Hamiltonian, usual time evolution is obtained:

$$\begin{aligned} |P_H(t)\rangle &= e^{-im_H t - \frac{1}{2}\Gamma_H t}|P_H(0)\rangle \\ |P_L(t)\rangle &= e^{-im_L t - \frac{1}{2}\Gamma_L t}|P_L(0)\rangle \end{aligned}$$

$$\begin{aligned} |P^0\rangle &= \frac{1}{2p}\left[|P_H\rangle + |P_L\rangle\right] \\ |\bar{P}^0\rangle &= \frac{1}{2q}\left[|P_H\rangle - |P_L\rangle\right] \end{aligned}$$

$$\begin{aligned} |P_H\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \\ |P_L\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \end{aligned}$$

$$\begin{aligned} |P^0(t)\rangle &= \frac{1}{2p}\left\{e^{-im_H t - \frac{1}{2}\Gamma_H t}|P_H(0)\rangle + e^{-im_L t - \frac{1}{2}\Gamma_L t}|P_L(0)\rangle\right\} \\ &= \frac{1}{2p}\left\{e^{-im_H t - \frac{1}{2}\Gamma_H t}(p|P^0\rangle + q|\bar{P}^0\rangle) + e^{-im_L t - \frac{1}{2}\Gamma_L t}(p|P^0\rangle - q|\bar{P}^0\rangle)\right\} \\ &= \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)|P^0\rangle + \frac{q}{2p}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)|\bar{P}^0\rangle \\ &= g_+(t)|P^0\rangle + \left(\frac{q}{p}\right)g_-(t)|\bar{P}^0\rangle \qquad (3.6) \end{aligned}$$

B Oscillation Amplitudes

For an initially produced B^0 or a $\overline{B}^0$ it then follows: (using:

$$\left|B^0\right\rangle = \frac{1}{2p}\left(\left|B_H\right\rangle + \left|B_L\right\rangle\right)$$

$$\left|\overline{B}^0\right\rangle = \frac{1}{2q}\left(\left|B_H\right\rangle - \left|B_L\right\rangle\right)$$

$$\left|\psi(t)\right\rangle :$$

$$\left|B^0(t)\right\rangle = g_+(t)\left|B^0\right\rangle + \frac{q}{p} g_-(t)\left|\overline{B^0}\right\rangle$$

$$\left|\overline{B^0}(t)\right\rangle = g_+(t)\left|\overline{B^0}\right\rangle + \frac{p}{q} g_-(t)\left|B^0\right\rangle$$

with

$$g_\pm(t) = \frac{e^{-i\omega_+ t} \pm e^{-i\omega_- t}}{2}$$

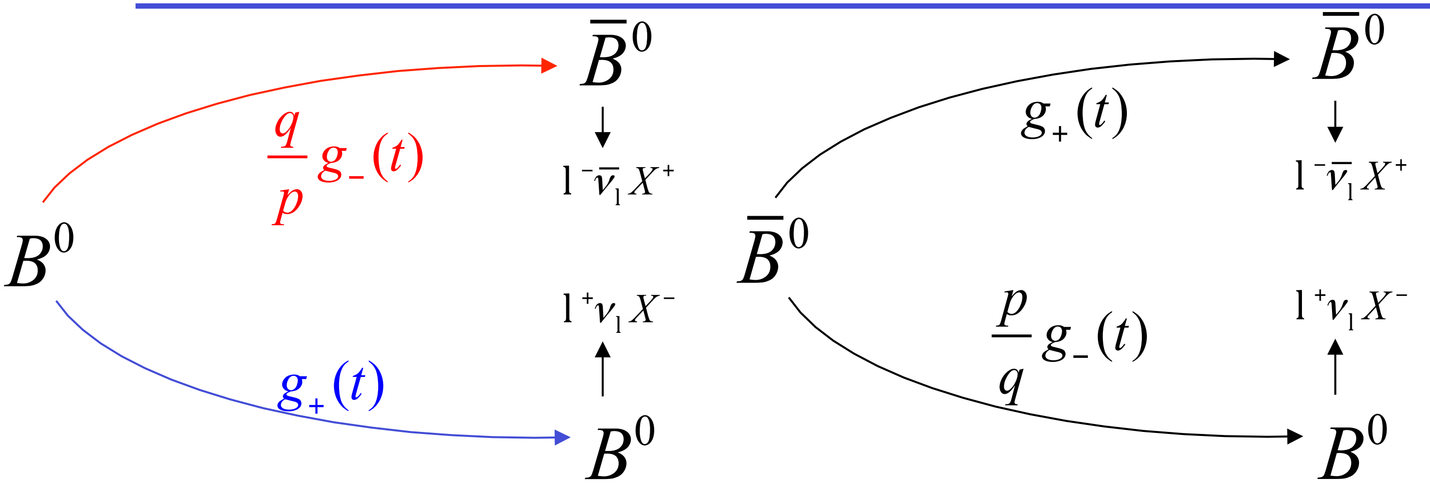
For B^0, expect:
$\Delta\Gamma \sim 0$,
$|q/p|=1$

$$g_\pm(t) \sim e^{-imt} e^{-\Gamma t/2}\left[\frac{e^{-\frac{1}{2}i\Delta m t} \pm e^{+\frac{1}{2}i\Delta m t}}{2}\right]$$

$$g_+(t) = e^{-imt} e^{-\Gamma t/2} \cos\frac{\Delta m t}{2}$$

$$g_-(t) = e^{-imt} e^{-\Gamma t/2}\, i \sin\frac{\Delta m t}{2}$$

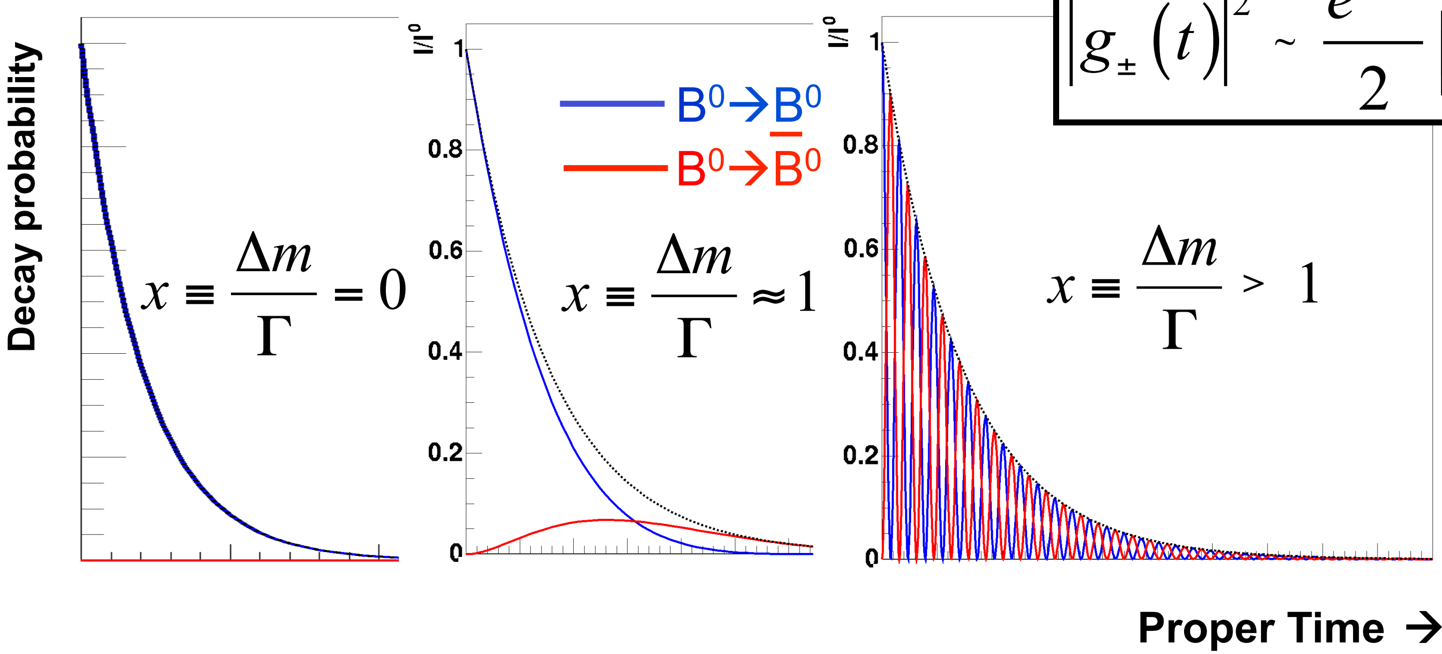
Measuring B Oscillations



For B^0, expect:
$\Delta\Gamma \sim 0$,
$|q/p|=1$

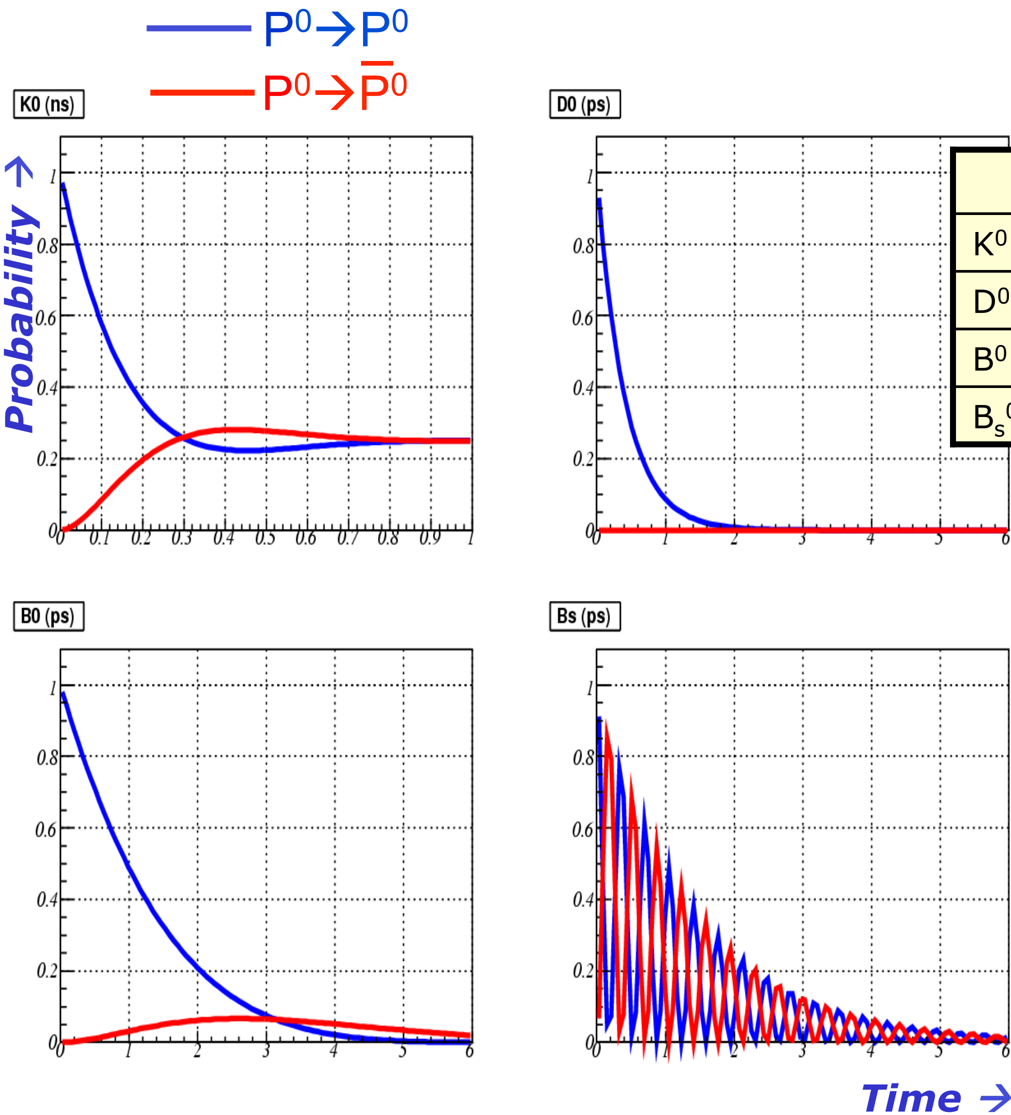
$$\left|g_\pm(t)\right|^2 \sim \frac{e^{-\Gamma t}}{2}\left[1 \pm \cos\left(\Delta m \cdot t\right)\right]$$

Examples:



Compare the mesons:

Probability to measure P or $\overline{P}$, when we start with 100% P



	<τ>	Δm	x=Δm/Γ	y=ΔΓ/2Γ
K^0	2.6 10^{-8} s	5.29 ns^{-1}	$\Delta m/\Gamma_S$=0.49	~1
D^0	0.41 10^{-12} s	0.001 fs^{-1}	~0	0.01
B^0	1.53 10^{-12} s	0.507 ps^{-1}	0.78	~0
B_s^0	1.47 10^{-12} s	17.8 ps^{-1}	12.1	~0.05

By the way,
$\hbar$=6.58 10^{-22} MeVs

x=Δm/Γ: avg nr of oscillations before decay

Summary (1)

- Start with Schrodinger equation:

$$i\frac{\partial\psi}{\partial t} = H\psi = (M - \frac{i}{2}\Gamma)\psi = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\psi$$

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

(2-component state in P^0 and $\overline{P^0}$ subspace)

- Find eigenvalue:

$$\begin{vmatrix} M - \frac{i}{2}\Gamma - \lambda & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma - \lambda \end{vmatrix} = 0$$

- Solve eigenstates:

$$\psi_\pm = \begin{pmatrix} p \\ \pm q \end{pmatrix}$$

$$\begin{aligned} |P_1\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \\ |P_2\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \end{aligned}$$

we find p and q by solving

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \lambda_\pm \begin{pmatrix} p \\ q \end{pmatrix} \longrightarrow \frac{q}{p} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}$$

- Eigenstates have diagonal Hamiltonian: mass eigenstates!

Summary (2)

- Two mass eigenstates

$$
\begin{aligned}
|P_H\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \\
|P_L\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle
\end{aligned}
$$

- Time evolution:

$$
\begin{aligned}
|P_H(t)\rangle &= e^{-im_H t - \frac{1}{2}\Gamma_H t}|P_H(0)\rangle \\
|P_L(t)\rangle &= e^{-im_L t - \frac{1}{2}\Gamma_L t}|P_L(0)\rangle
\end{aligned}
$$

$$
\begin{aligned}
|P^0\rangle &= \frac{1}{2p}\left[|P_H\rangle + |P_L\rangle\right] \\
|\bar{P}^0\rangle &= \frac{1}{2q}\left[|P_H\rangle - |P_L\rangle\right]
\end{aligned}
$$

$$
\left|P^0(t)\right\rangle = \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)|P^0\rangle + \frac{q}{2p}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)|\bar{P}^0\rangle
$$

- Probability for $|P^0\rangle \rightarrow |\overline{P^0}\rangle$!
- Express in $M=m_H+m_L$ and $\Delta m=m_H-m_L$ → Δm dependence

Summary

- p, q: $\left|B_H\right\rangle = p\left|B^0\right\rangle + q\left|\overline{B}^0\right\rangle$

  $\left|B_L\right\rangle = p\left|B^0\right\rangle - q\left|\overline{B}^0\right\rangle$

- Δm, ΔΓ: $\Delta m = 2\Re\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$

  $\Delta\Gamma = 4\Im\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$

q,p,M_{ij},Γ_{ij} related through:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}$$

- x,y: mixing often quoted in *scaled* parameters: $x = \frac{\Delta m}{\Gamma}$ $\quad y = \frac{\Delta\Gamma}{2\Gamma}$

$$\cos(\Delta m t) = \cos\left(\frac{\Delta m}{\Gamma}\frac{t}{\tau}\right) = \cos\left(x\frac{t}{\tau}\right)$$

Time dependence (if ΔΓ~0, like for B^0):

$$\left|B^0(t)\right\rangle = g_+(t)\left|B^0\right\rangle + \frac{q}{p}g_-(t)\left|\overline{B^0}\right\rangle$$

$$\left|\overline{B^0}(t)\right\rangle = g_+(t)\left|\overline{B^0}\right\rangle + \frac{p}{q}g_-(t)\left|B^0\right\rangle$$

with

$$g_+(t) = e^{-imt}e^{-\Gamma t/2}\cos\frac{\Delta m t}{2}$$

$$g_-(t) = e^{-imt}e^{-\Gamma t/2}\,i\sin\frac{\Delta m t}{2}$$

Personal impression:

- People think it is a complicated part of the Standard Model (me too:-). Why?

1) Non-intuitive concepts?

  - *Imaginary phase* in transition amplitude, T ~ $e^{i\varphi}$
  - *Different bases* to express quark states, d' =0.97 d + 0.22 s + 0.003 b
  - *Oscillations* (mixing) of mesons: $|K^0\rangle \leftrightarrow |\overline{K^0}\rangle$

2) Complicated calculations?

$$\Gamma\left(B^0 \to f\right) \propto \left|A_f\right|^2 \left[\left|g_+(t)\right|^2 + \left|\lambda\right|^2 \left|g_-(t)\right|^2 + 2\Re\left(\lambda g_+^*(t) g_-(t)\right)\right]$$

$$\Gamma\left(\overline{B}^0 \to f\right) \propto \left|\overline{A}_f\right|^2 \left[\left|g_+(t)\right|^2 + \frac{1}{\left|\lambda\right|^2} \left|g_-(t)\right|^2 + \frac{2}{\left|\lambda\right|^2}\Re\left(\lambda^* g_+^*(t) g_-(t)\right)\right]$$

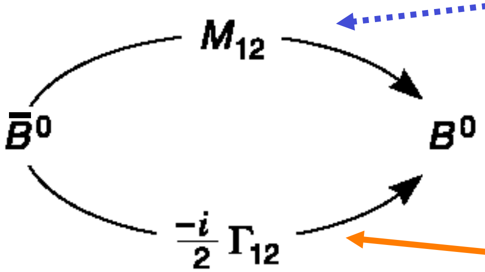
3) Many decay modes? *"Beetopaipaigamma…"*

  - PDG reports 347 decay modes of the B^0-meson:
    - Γ_1 $l^+ \nu_l$ *anything* $(10.33 \pm 0.28) \times 10^{-2}$
    - Γ_{347} $\nu \nu \gamma$ $<4.7 \times 10^{-5}$ *CL=90%*
  - And for one decay there are often more than one decay *amplitudes*…

Describing Mixing

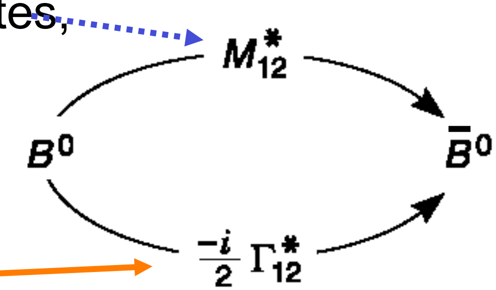
Time evolution of B^0 and ? B^0 can be described by an *effective* Hamiltonian:

$$i\frac{\partial\psi}{\partial t} = H\psi = (M - \frac{i}{2}\Gamma)\psi = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\psi$$



M_{12} describes $B^0 \leftrightarrow B^0$ via *off-shell* states, e.g. the weak box diagram

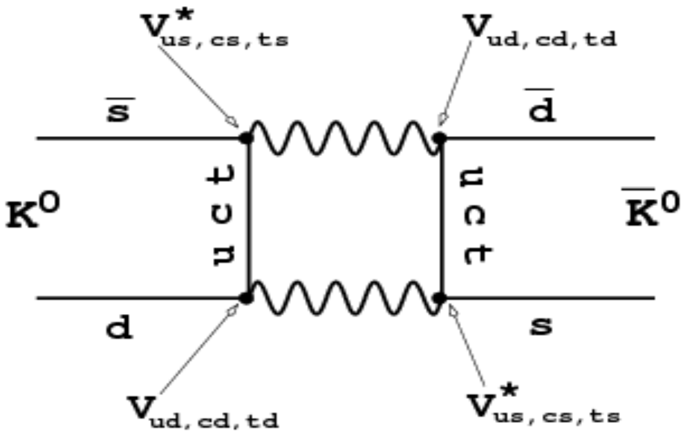
Γ_{12} describes $B^0 \leftrightarrow f \leftrightarrow B^0$ via *on-shell* states, eg. $f=\pi^+\pi^-$

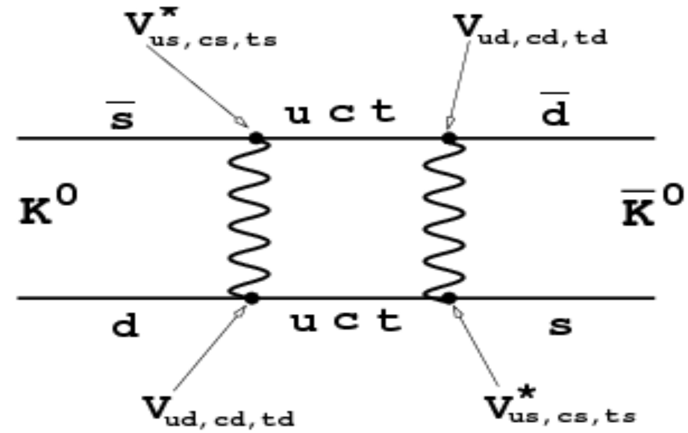


Box diagram and Δm

Inami and Lim,
Prog.Theor.Phys.65:297,1981

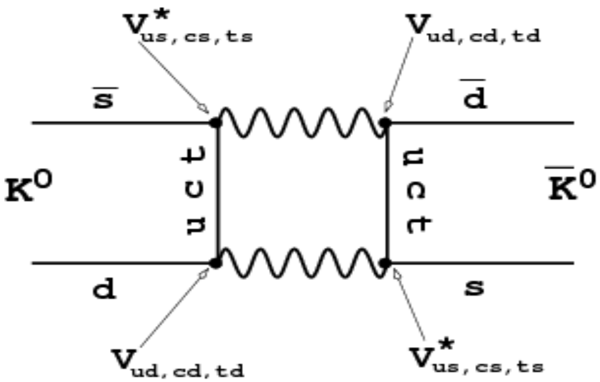
$$\Delta m = m_{P_H^0} - m_{P_L^0} = \left\langle P_H^0 \right| H \left| P_H^0 \right\rangle - \left\langle P_L^0 \right| H \left| P_L^0 \right\rangle$$

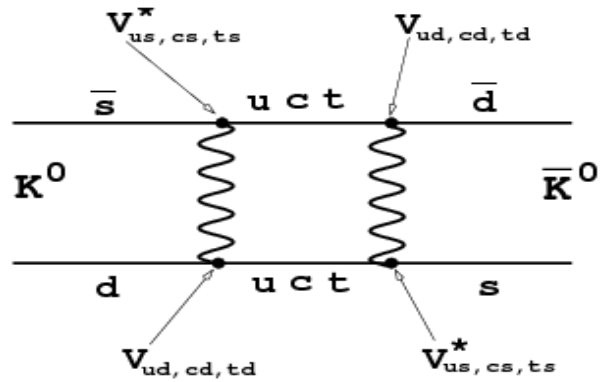




$$\mathcal{M}_{uu} = i\left(\frac{-ig_w}{2\sqrt{2}}\right)^4 (V_{us}^* V_{ud} V_{us}^* V_{ud}) \int \frac{d^4k}{(2\pi)^4}\left(\frac{-ig^{\lambda\sigma} - k^\lambda k^\sigma/m_W^2}{k^2 - m_W^2}\right)\left(\frac{-ig^{\alpha\rho} - k^\alpha k^\rho/m_W^2}{k^2 - m_W^2}\right) \left[\bar{u}_s\gamma_\lambda(1-\gamma^5)\frac{\not{k} + m_u}{k^2 - m_u^2}\gamma_\rho(1-\gamma^5)u_d\right]\left[\bar{v}_s\gamma_\alpha(1-\gamma^5)\frac{\not{k} + m_u}{k^2 - m_u^2}\gamma_\sigma(1-\gamma^5)v_d\right]$$

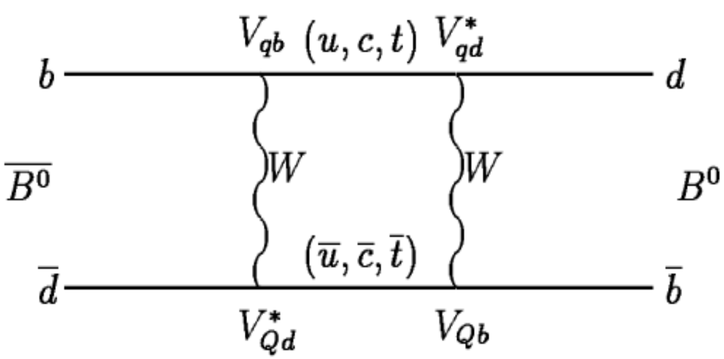
Box diagram and Δm

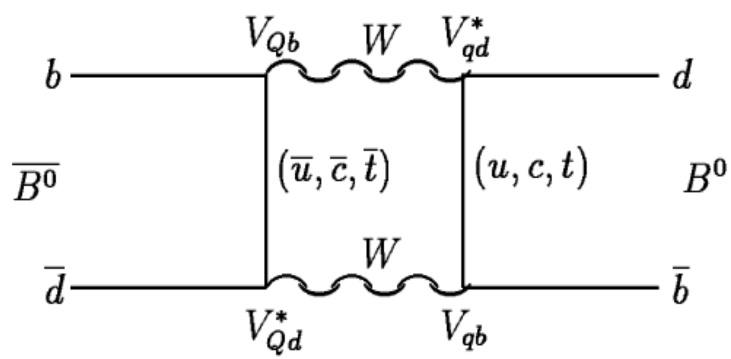




$$\Delta m_K = \frac{G_F^2 m_W^2}{6\pi^2}\eta_{QCD} B_K f_K^2 m_K \left[S_0(m_c^2/m_W^2)|V_{cd}V_{cs}|^2 \right]$$

$$\Delta m_B = \frac{G_F^2 m_W^2}{6\pi^2}\eta_{QCD} B_B f_B^2 m_B \left[S_0(m_t^2/m_W^2)|V_{td}V_{tb}|^2 \right]$$





Box diagram and Δm: Inami-Lim

- K-mixing

Table 2: Factors entering the matrix element, which is proportional to the product of the Inami-Lim function and the CKM term.

Internal quarks	I-L factor	B^0 CKM	B_s^0 CKM	K^0 CKM
c,c	3.5×10^{-4}	$A^2\lambda^6$ (7.4×10^{-5})	$A^2\lambda^4$ (1.4×10^{-3})	λ^2 (2.7×10^{-2})
c,t	3.0×10^{-3}	$A^2\lambda^6\|1-\rho-i\eta\|$ (7.3×10^{-5})	$A^2\lambda^4$ (1.5×10^{-3})	$A^2\lambda^6\|1-\rho-i\eta\|$ (8.8×10^{-6})
t,t	2.5	$A^2\lambda^6\|1-\rho-i\eta\|^2$ (7.2×10^{-5})	$A^2\lambda^4$ (1.5×10^{-3})	$A^4\lambda^{10}\|1-\rho-i\eta\|^2$ (1.1×10^{-7})

C.Gay, B Mixing, hep-ex/0103016

Box diagram and Δm: Inami-Lim

- B^0-mixing

Table 2: Factors entering the matrix element, which is proportional to the product of the Inami-Lim function and the CKM term.

Internal quarks	I-L factor	B^0 CKM	B^0_s CKM	K^0 CKM
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Box diagram and Δm: Inami-Lim

- B_s^0-mixing

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Next: measurements of oscillations

1. B^0 mixing:
   - 1987: Argus, first
   - 2001: Babar/Belle, precise
2. B_s^0 mixing:
   - 2006: CDF: first
   - 2010: D0: anomalous ??

B^0 mixing

B^0 mixing

- What is the probability to observe a $B^0/\overline{B}^0$ at time t, when it was produced as a B^0 at t=0?
  - Calculate observable probility $\Psi^*\Psi(t)$

$$prob(B^0(t)\,|\,B^0) \propto \frac{e^{-t/\tau}}{2}\left(1+\cos(\Delta m t)\right)$$

$$prob(\overline{B}^0(t)\,|\,B^0) \propto \frac{e^{-t/\tau}}{2}\left(1-\cos(\Delta m t)\right)$$

- A simple B^0 decay experiment.
  - Given a source B^0 mesons produced in a flavor eigenstate $|B^0\rangle$
  - You measure the decay time of each meson that decays into a flavor eigenstate (either B^0 or $\overline{B}^0$) you will find that

$$\frac{N_{B^0\to B^0}(t)-N_{B^0\to\overline{B}^0}(t)}{N_{B^0\to B^0}(t)+N_{B^0\to\overline{B}^0}(t)}=\cos(\Delta m\cdot t)$$

B^0 mixing: 1987 Argus

B^0 oscillations:

- First evidence of heavy top
- → m_{top}>50 GeV
- Needed to break GIM cancellations

NB: loops can reveal heavy particles!

Phys.Lett.B192:245,1987

DESY 87-029
April 1987

OBSERVATION OF B^0 - $\overline{B}^0$ MIXING

The ARGUS Collaboration

In summary, the combined evidence of the investigation of B^0 meson pairs, lepton pairs and B^0 meson-lepton events on the $\Upsilon(4S)$ leads to the conclusion that B^0-$\overline{B}^0$ mixing has been observed and is substantial.

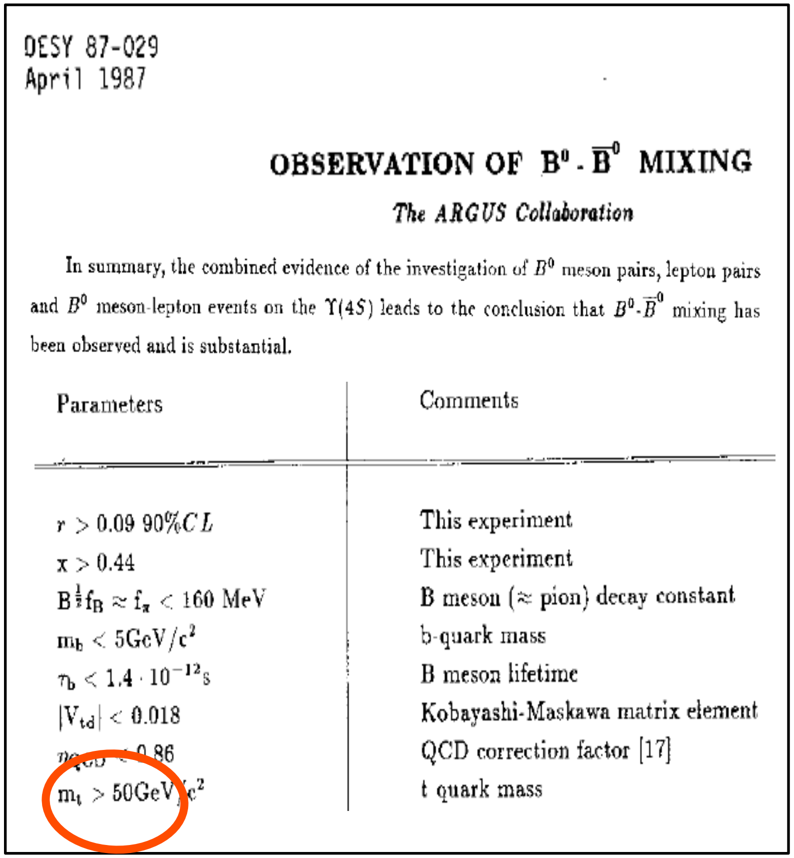
Parameters	Comments
$r > 0.09$ 90%CL	This experiment
$x > 0.44$	This experiment
$B^{\frac{1}{2}} f_B \approx f_\pi < 160$ MeV	B meson ($\approx$ pion) decay constant
$m_b < 5\mathrm{GeV}/c^2$	b-quark mass
$\tau_b < 1.4 \cdot 10^{-12}$s	B meson lifetime
$\lvert V_{td}\rvert < 0.018$	Kobayashi-Maskawa matrix element
$\eta_{QCD} < 0.86$	QCD correction factor [17]
$m_t > 50\mathrm{GeV}/c^2$	t quark mass

B^0 mixing: t quark

GIM: c quark

B^0 mixing pointed to the top quark:

ARGUS Coll, Phys.Lett.B192:245,1987



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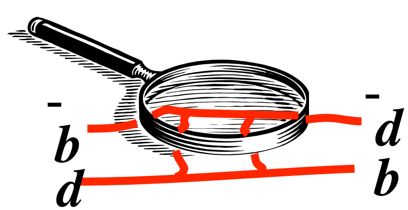
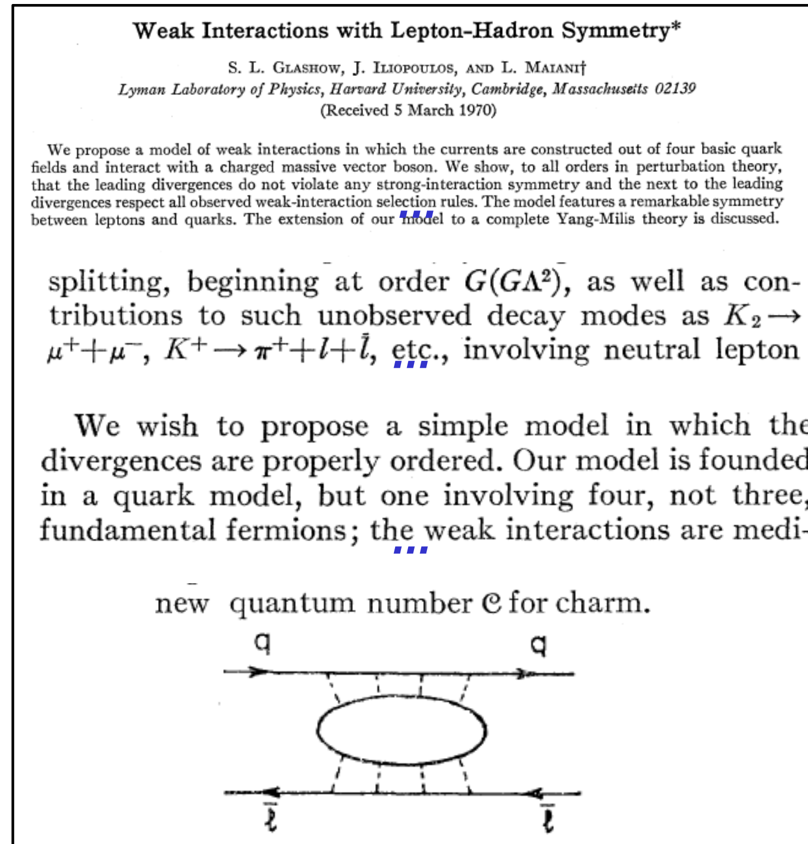


$K^0 \to \mu\mu$ pointed to the charm quark:

GIM, Phys.Rev.D2,1285,1970



Weak Interactions with Lepton-Hadron Symmetry*

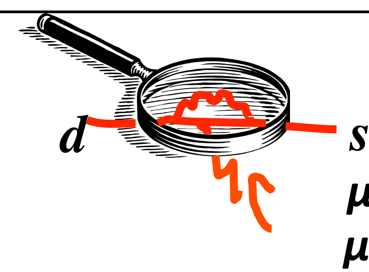
S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our ... model to a complete Yang-Milis theory is discussed.

splitting, beginning at order $G(G\Lambda^2)$, as well as contributions to such unobserved decay modes as $K_2 \to \mu^+ + \mu^-$, $K^+ \to \pi^+ + l + \bar{l}$, etc., involving neutral lepton ...

We wish to propose a simple model in which the divergences are properly ordered. Our model is founded in a quark model, but one involving four, not three, fundamental fermions; the weak interactions are medi- ...

new quantum number $\mathcal{C}$ for charm.

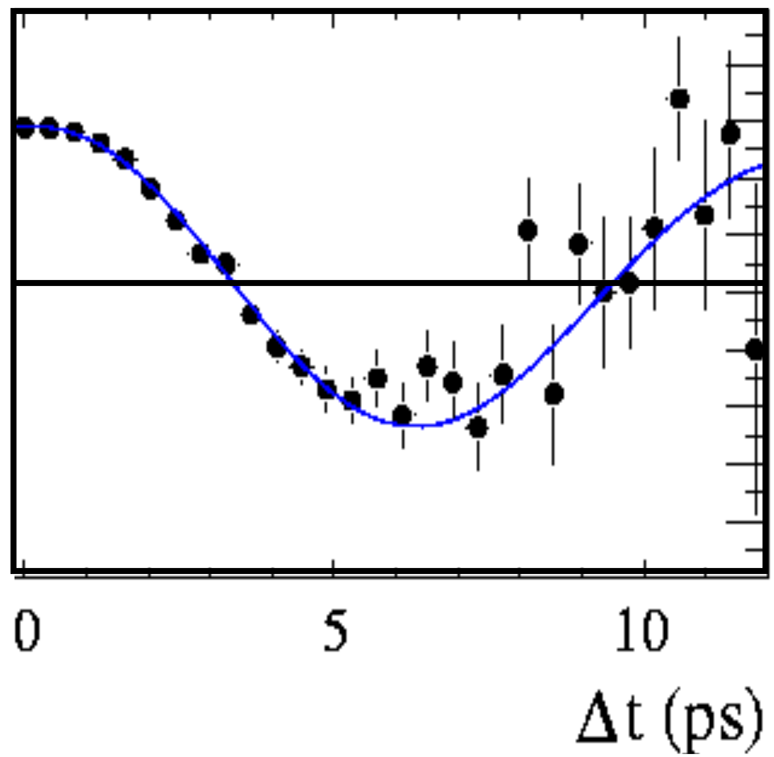


B^0 mixing: 2001 B-factories

- You can really see this because (amazingly) B^0 mixing has same time scale as decay
  - τ=1.54 ps
  - Δm=0.5 ps^{-1}
  - 50/50 point at $\pi\Delta m \approx \tau$
  - Maximal oscillation at $2\pi\Delta m \approx 2\tau$

- Actual measurement of $B^0/\overline{B}^0$ oscillation
  - Also precision measurement of Δm!

$$\frac{N_{B^0\to B^0}(t) - N_{B^0\to \overline{B}^0}(t)}{N_{B^0\to B^0}(t) + N_{B^0\to \overline{B}^0}(t)} = \cos(\Delta m \cdot t)$$



B_s^0 mixing

B_s^0 mixing: 2006

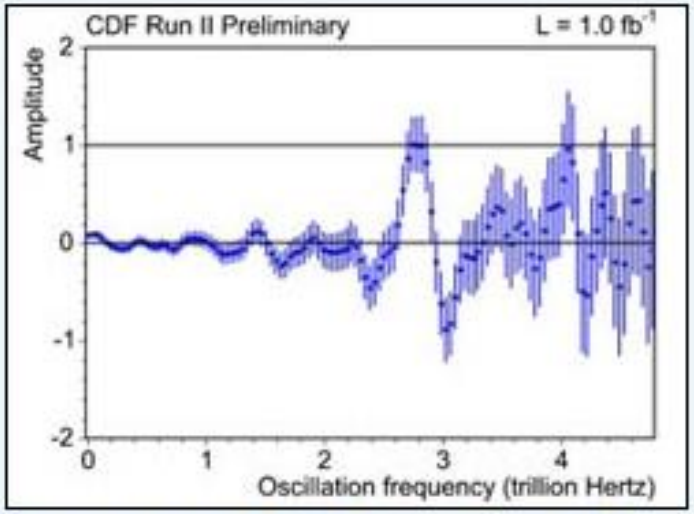
■ Physics Astronomy

DOE/Fermi National Accelerator Laboratory 12.04.2006

Fermilab's CDF scientists present a precision measurement of a subtle dance between matter and antimatter

Scientists of the CDF collaboration at the Department of Energy's Fermi National Accelerator Laboratory announced today (April 11, 2006) the precision measurement of extremely rapid transitions between matter and antimatter. As amazing as it may seem, it has been known for 50 years that very special species of subatomic particles can make spontaneous transitions between matter and antimatter. In this exciting new result, CDF physicists measured the rate of the matter-antimatter transitions for the Bs (pronounced "B sub s") meson, which consists of the heavy bottom quark bound by the strong nuclear interaction to a strange anti-quark, a staggering rate that challenges the imagination - 3 trillion times per second.

Dr. Raymond Orbach, Director of the DOE Office of Science, congratulated the CDF collaboration on "this important and fascinating new result" from the experiment.

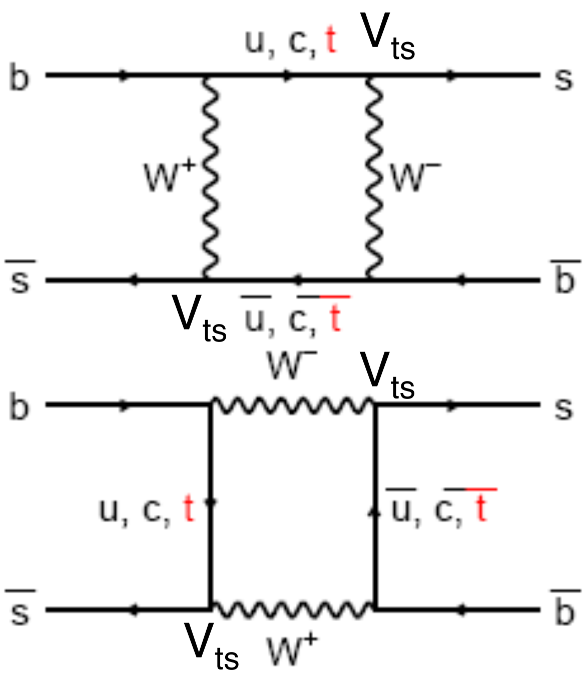


The figure shows the CDF measurement of the Bs oscillation frequency at 2.8 trillion times per second. The analysis is designed such that possible oscillation frequencies have an amplitude consistent with 1.0 while those not present in the data will have an amplitude consistent with zero. Image courtesy CDF collaboration.

B_s^0 mixing (Δm_s): SM Prediction

CKM Matrix — Wolfenstein parameterization

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$



Ratio of frequencies for B^0 and B_s

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{Bs}}{m_{Bd}} \frac{f_{Bs}^2 B_{Bs}}{f_{Bd}^2 B_{Bd}} \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{m_{Bs}}{m_{Bd}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$

$V_{ts} \sim \lambda^2$

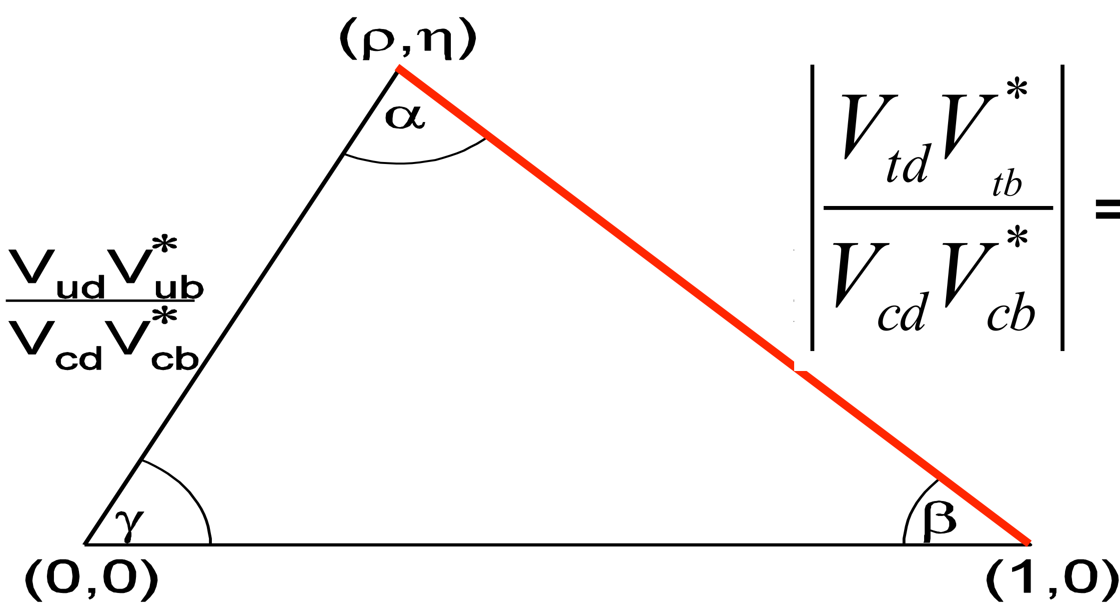
$V_{td} \sim \lambda^3$ → $\Delta m_s \sim (1/\lambda^2)\Delta m_d \sim 25\ \Delta m_d$

$\xi = 1.239 \pm 0.046$ from lattice QCD

B_s^0 mixing (Δm_s): Unitarity Triangle

CKM Matrix Unitarity Condition

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



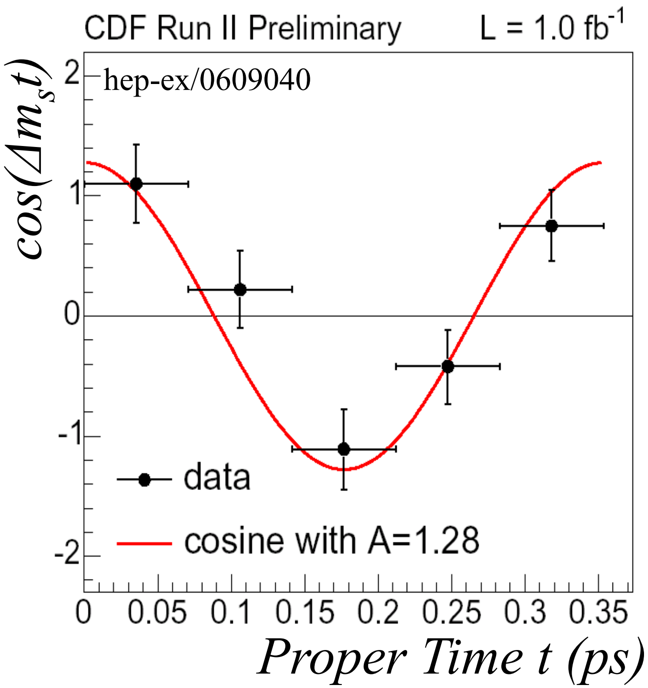
$$\left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right| = \frac{|V_{td}|}{|V_{ts}|} \times \frac{1}{|V_{cd}|}$$

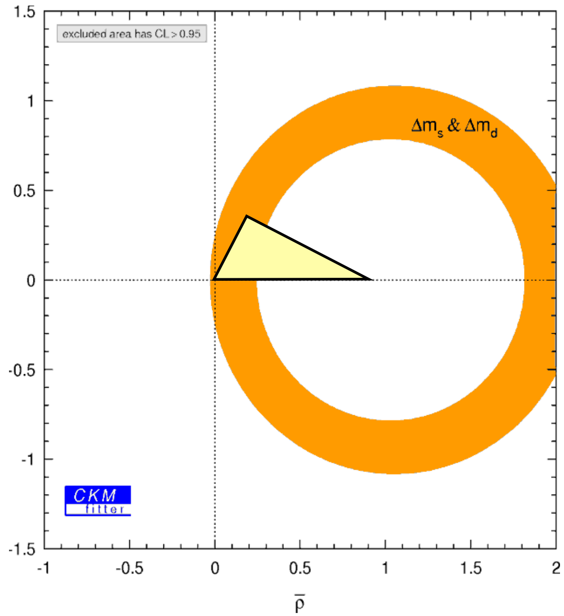
$$\frac{|V_{td}|}{|V_{ts}|} \sim \sqrt{\frac{\Delta m_d}{\Delta m_s}}$$

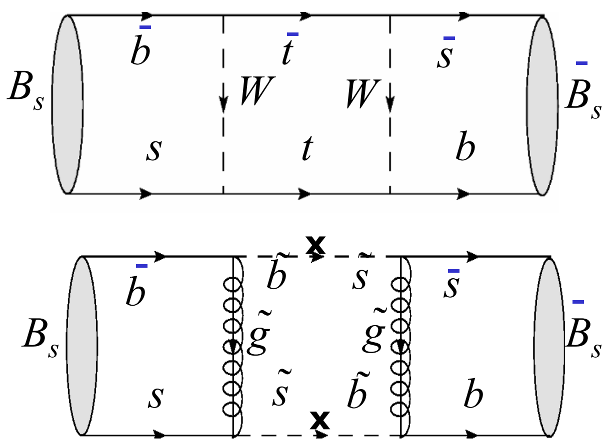
B_s^0 mixing (Δm_s)

$$\frac{N_{B^0 \to B^0}(t) - N_{B^0 \to \bar{B}^0}(t)}{N_{B^0 \to B^0}(t) + N_{B^0 \to \bar{B}^0}(t)} = \cos(\Delta m \cdot t)$$

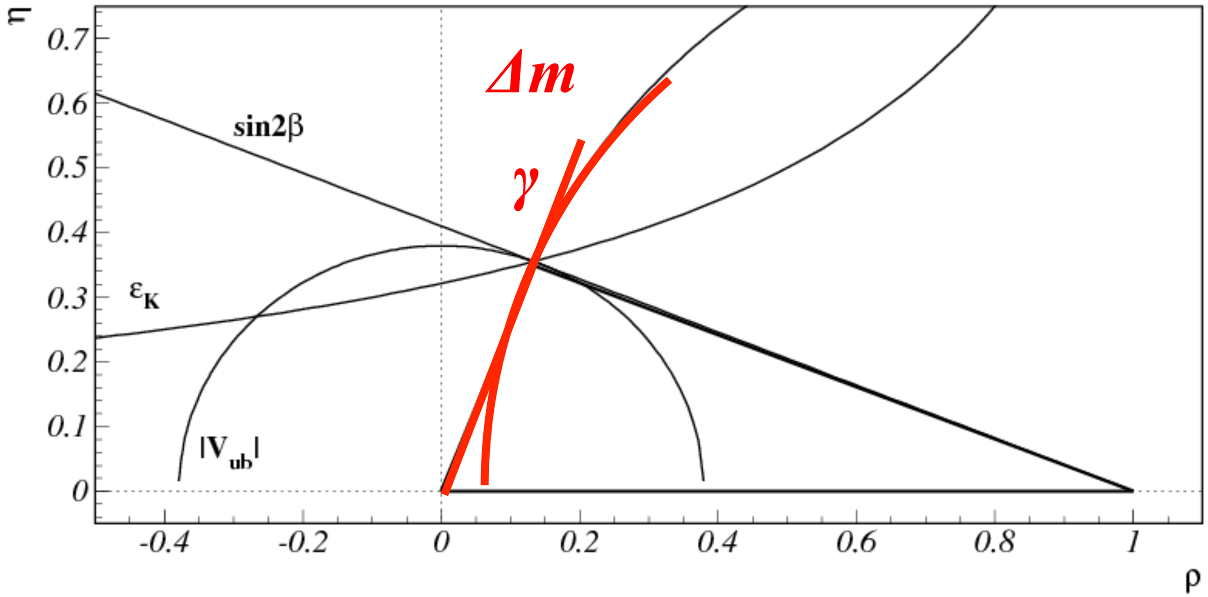
Δm_s=17.77 ±0.10(stat)±0.07(sys) ps^{-1}







Consistency?



Compare eg.:

1) Δm_s with γ

2) $|V_{ub}|$ with sin2β

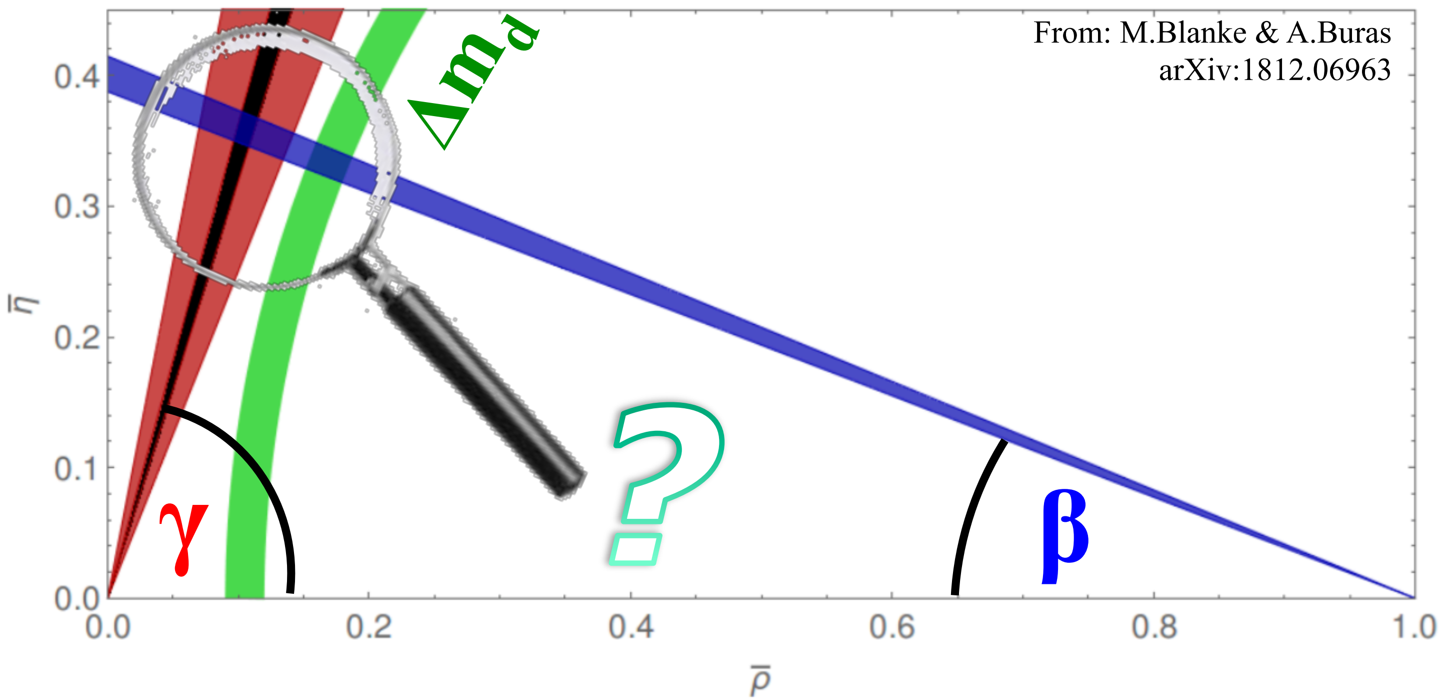


Figure 2: *Constraints on the UT from the angles* γ *(red) and* β *from* $S_{\psi K_S}$ *(blue), and* R_t *from* $\Delta M_d/\Delta M_s$ *(green).*

Δm_s / Δm_d

- Check ratio

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{Bs}}{m_{Bd}} \frac{f_{Bs}^2 B_{Bs}}{f_{Bd}^2 B_{Bd}} \frac{|V_{ts}|^2}{|V_{td}|^2}$$

- Hadronic parameter much more precise:

$$\xi^2 = \frac{f_{B_s}^2 \hat{B}_{B_s}}{f_B^2 \hat{B}_B}$$

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{Bs}}{m_{Bd}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$

ξ for Δm_s / Δm_d

lattice QCD

- Fermilab Lattice/MILC — Bazavov et al. (2016)
  $\xi = 1.206 \pm 0.019 \succ \gamma = (63.0 \pm 2.1)^\circ$
- RBC/UKQCD — Boyle et al. (2018)
  $\xi = 1.1853 \pm 0.0054^{+0.0116}_{-0.0156} \succ \gamma = (60.7 \pm 1.5)^\circ$

QCD sum rules — King, Lenz, Rauh (2019)

- $\xi = 1.2014^{+0.0065}_{-0.0072} \succ \gamma = (62.5 \pm 0.9)^\circ$

Used by Buras → (Fermilab Lattice/MILC)

Calculated by Lenz → (QCD sum rules)

- Hadronic parameter:

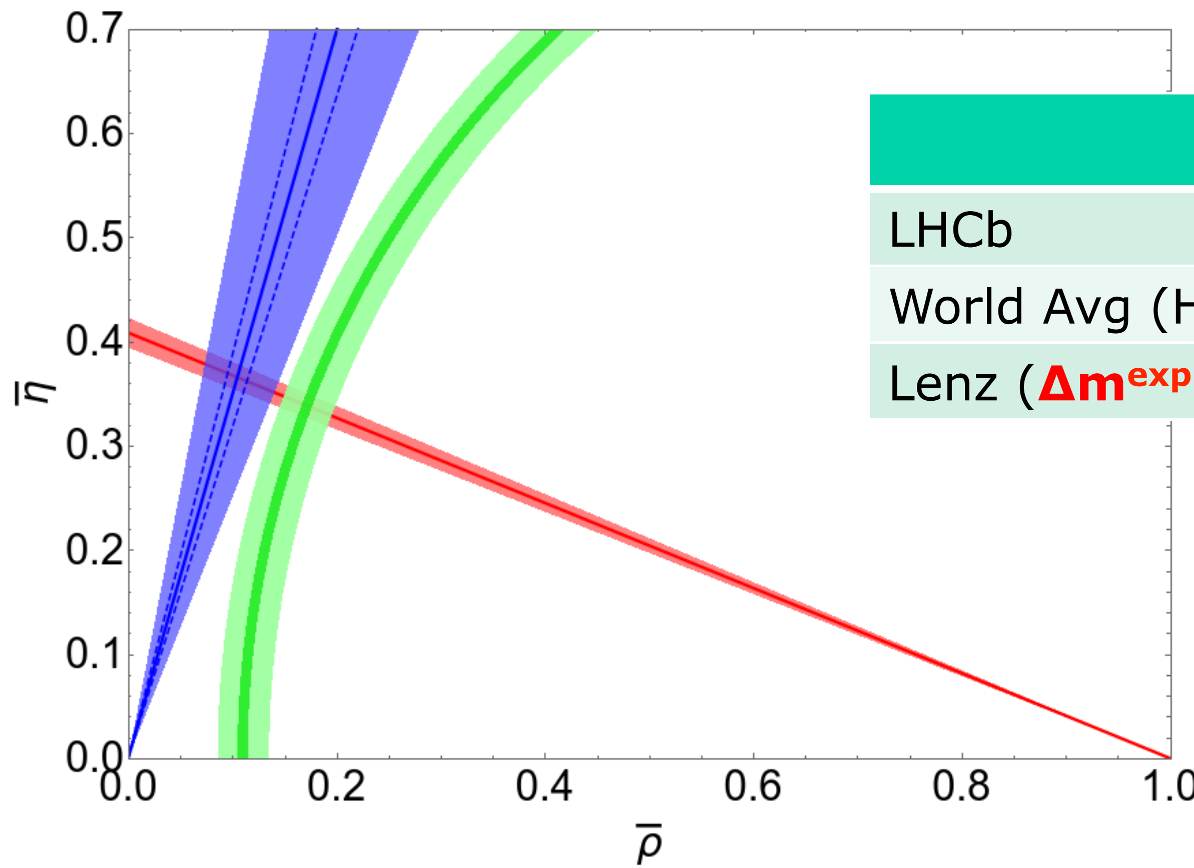
$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{Bs}}{m_{Bd}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$

$$\xi^2 = \frac{f_{B_s}^2 \hat{B}_{B_s}}{f_B^2 \hat{B}_B}$$

Consistency?

If you believe $\xi = 1.201 \pm 0.007$ then ...

γ has to decrease considerably for consistent CKM picture!



	γ (°)
LHCb	$74.0^{+5.0}_{-5.8}$
World Avg (HFLAV)	$71.1^{+4.6}_{-5.3}$
Lenz (**Δm^{exp}**, ξ^{SR})	63.4 ± 0.9

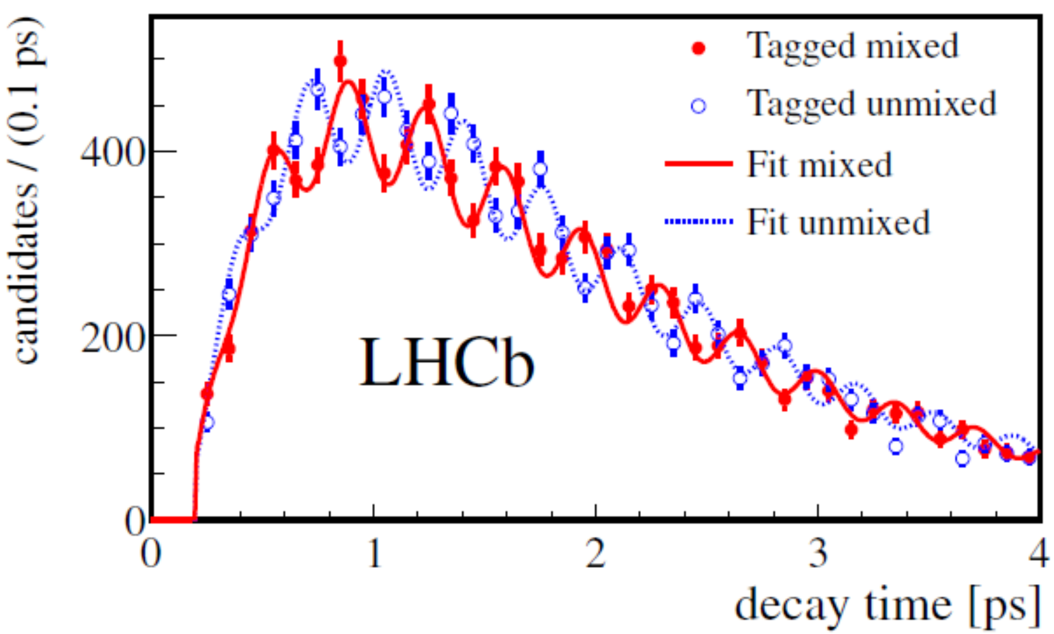
D. King, A. Lenz, Th. Rauh, arXiv:1911.07856, $|V_{cb}|$ *and γ from mixing* (addendum)

B_s^0 mixing (Δm_s): New: LHCb

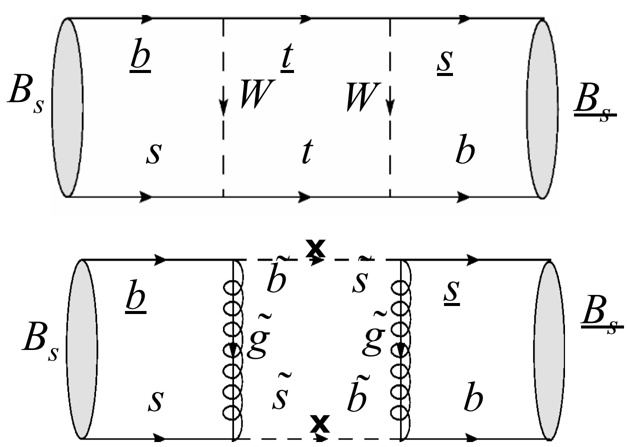
$$\frac{N_{B^0 \to B^0}(t) - N_{B^0 \to \bar{B}^0}(t)}{N_{B^0 \to B^0}(t) + N_{B^0 \to \bar{B}^0}(t)} = \cos(\Delta m \cdot t)$$

$$\Delta m_s = 17.768 \pm 0.023\ (\text{stat}) \pm 0.006\ (\text{syst})\ \text{ps}^{-1}$$

($\Delta m_s = 17.77 \pm 0.10(\text{stat}) \pm 0.07(\text{sys})\ ps^{-1}$ CDF, 2006 [2])



LHCb, arXiv:1304.4741



B_s^0 mixing (Δm_s): New: LHCb

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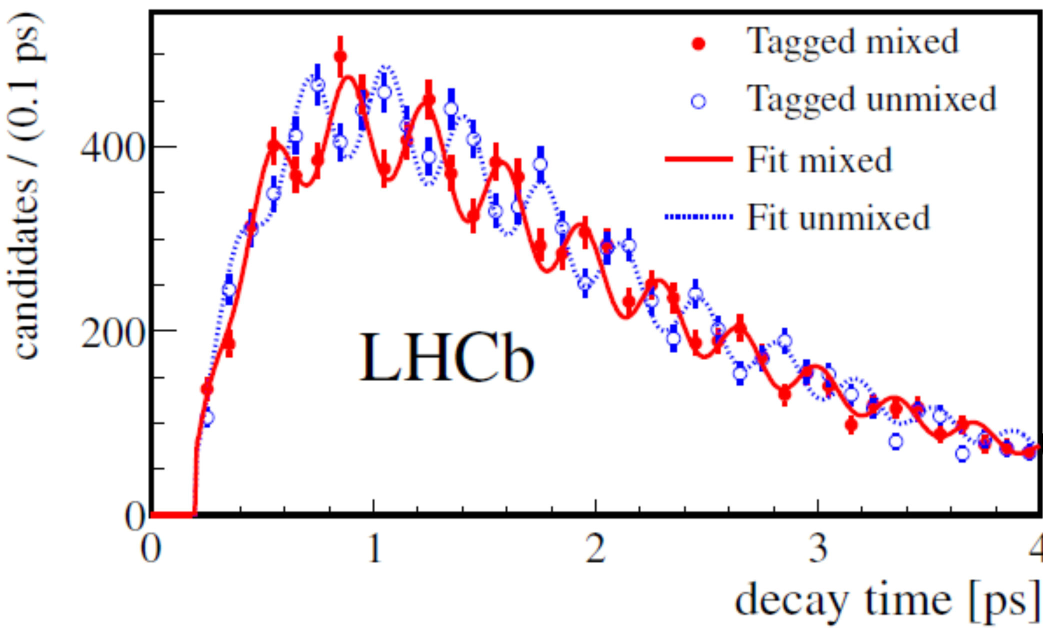

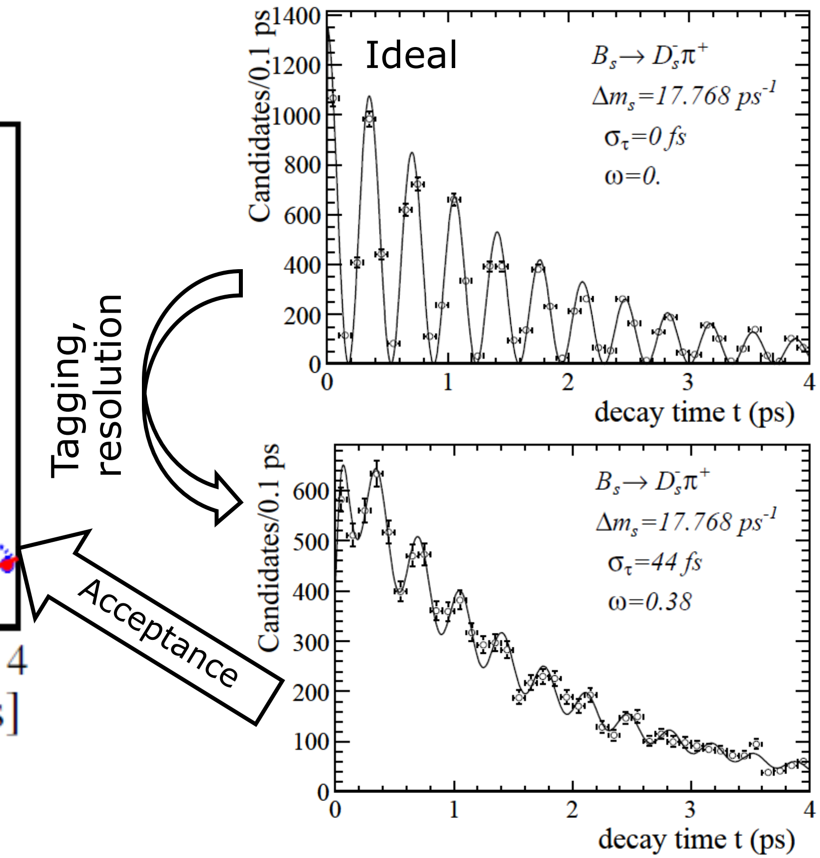

LHCb, arXiv:1304.4741

Mixing → CP violation?

- NB: Just mixing is not necessarily CP violation!
- However, by studying certain ***decays with and without mixing***, CP violation is observed

- Next: Measuring CP violation… Finally

Meson Decays

- Formalism of meson *oscillations*:

$$\left|P^0(t)\right\rangle = \frac{1}{2}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}+e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)|P^0\rangle + \frac{q}{2p}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}-e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)|\bar{P}^0\rangle$$

$$|\langle\bar{P}^0(t)|P^0\rangle|^2 = |g_-(t)|^2\left(\frac{p}{q}\right)^2$$

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t \pm \cos\Delta m t\right)$$

- Subsequent: *decay*

$$P^0 \longrightarrow f$$

Notation: Define A_f and λ_f

$$P^0 \longrightarrow f$$

$$\begin{aligned} A(f) &= \langle f|T|P^0\rangle & \bar{A}(f) &= \langle f|T|\bar{P}^0\rangle \\ A(\bar{f}) &= \langle \bar{f}|T|P^0\rangle & \bar{A}(\bar{f}) &= \langle \bar{f}|T|\bar{P}^0\rangle \end{aligned}$$

and define the complex parameter λ_f (not be confused with the Wolfenstein parameter λ !):

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}, \qquad \bar{\lambda}_f = \frac{1}{\lambda_f}, \qquad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}, \qquad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}} \tag{3.14}$$

The general expression for the time dependent decay rates, $\Gamma_{P^0\to f}(t) = |\langle f|T|P^0(t)\rangle|^2$,

Some algebra for the decay $P^0 \to f$

$$\left|P^0(t)\right\rangle = \frac{1}{2}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}+e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)|P^0\rangle + \frac{q}{2p}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}-e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)|\bar{P}^0\rangle$$

$$\Gamma_{P^0\to f}(t) = |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2|g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)]\right)$$

$$A(f) = \langle f|T|P^0\rangle$$

$$\bar{A}(f) = \langle f|T|\bar{P}^0\rangle$$

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

Interference

$P^0 \to f$

$P^0 \to \bar{P}^0 \to f$

Some algebra for the decay $P^0 \rightarrow f$

$$
\begin{aligned}
\Gamma_{P^0\to f}(t) &= |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2|g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t)g_-(t)]\right)\\
\Gamma_{P^0\to \bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left|\frac{q}{p}\right|^2 \left(|g_-(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2|g_+(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+(t)g_-^*(t)]\right)\\
\Gamma_{\bar{P}^0\to f}(t) &= |A_f|^2 \left|\frac{p}{q}\right|^2 \left(|g_-(t)|^2 + |\lambda_f|^2|g_+(t)|^2 + 2\Re[\lambda_f g_+(t)g_-^*(t)]\right)\\
\Gamma_{\bar{P}^0\to \bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left(|g_+(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2|g_-(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+^*(t)g_-(t)]\right) \qquad (3.15)
\end{aligned}
$$

$$
\begin{aligned}
|g_\pm(t)|^2 &= \frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t \pm \cos\Delta m t\right)\\
g_+^*(t)g_-(t) &= \frac{e^{-\Gamma t}}{2}\left(\sinh\frac{1}{2}\Delta\Gamma t + i\sin\Delta m t\right)\\
g_+(t)g_-^*(t) &= \frac{e^{-\Gamma t}}{2}\left(\sinh\frac{1}{2}\Delta\Gamma t - i\sin\Delta m t\right) \qquad (3.16)
\end{aligned}
$$

The 'master equations'

('direct') Decay — *Interference*

$$
\begin{aligned}
\Gamma_{P^0\to f}(t) &= |A_f|^2 \frac{e^{-\Gamma t}}{2} \\
&\left((1+|\lambda_f|^2)\cosh\frac{1}{2}\Delta\Gamma t + 2\Re\lambda_f \sinh\frac{1}{2}\Delta\Gamma t + (1-|\lambda_f|^2)\cos\Delta m t - 2\Im\lambda_f \sin\Delta m t \right) \\
\Gamma_{\bar{P}^0\to f}(t) &= |A_f|^2 \left|\frac{p}{q}\right|^2 \frac{e^{-\Gamma t}}{2} \qquad (3.17) \\
&\left((1+|\lambda_f|^2)\cosh\frac{1}{2}\Delta\Gamma t + 2\Re\lambda_f \sinh\frac{1}{2}\Delta\Gamma t - (1-|\lambda_f|^2)\cos\Delta m t + 2\Im\lambda_f \sin\Delta m t \right)
\end{aligned}
$$

The sinh- and sin-terms are associated to the interference between the decays with and without oscillation. Commonly, the master equations are expressed as:

The 'master equations'

(*'direct'*) *Decay* — ***Interference***

$$\Gamma_{P^0\to f}(t) = |A_f|^2 \frac{e^{-\Gamma t}}{2} \left((1+|\lambda_f|^2)\cosh\frac{1}{2}\Delta\Gamma t + 2\Re\lambda_f \sinh\frac{1}{2}\Delta\Gamma t + (1-|\lambda_f|^2)\cos\Delta m t - 2\Im\lambda_f \sin\Delta m t \right)$$

$$\Gamma_{\bar{P}^0\to f}(t) = |A_f|^2 \left|\frac{p}{q}\right|^2 \frac{e^{-\Gamma t}}{2} \left((1+|\lambda_f|^2)\cosh\frac{1}{2}\Delta\Gamma t + 2\Re\lambda_f \sinh\frac{1}{2}\Delta\Gamma t - (1-|\lambda_f|^2)\cos\Delta m t + 2\Im\lambda_f \sin\Delta m t \right) \quad (3.17)$$

The sinh- and sin-terms are associated to the interference between the decays with and without oscillation. Commonly, the master equations are expressed as:

$$\Gamma_{P^0\to f}(t) = |A_f|^2 (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2} \left(\cosh\frac{1}{2}\Delta\Gamma t + D_f \sinh\frac{1}{2}\Delta\Gamma t + C_f \cos\Delta m t - S_f \sin\Delta m t \right)$$

$$\Gamma_{\bar{P}^0\to f}(t) = |A_f|^2 \left|\frac{p}{q}\right|^2 (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2} \left(\cosh\frac{1}{2}\Delta\Gamma t + D_f \sinh\frac{1}{2}\Delta\Gamma t - C_f \cos\Delta m t + S_f \sin\Delta m t \right) \quad (3.18)$$

with

$$D_f = \frac{2\Re\lambda_f}{1+|\lambda_f|^2} \qquad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \qquad S_f = \frac{2\Im\lambda_f}{1+|\lambda_f|^2}. \quad (3.19)$$

Classification of CP Violating effects

1. CP violation in decay

$$\Gamma(P^0 \to f) \neq \Gamma(\bar{P}^0 \to \bar{f})$$

This is obviously satisfied (see Eq. (3.15)) when

$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1.$$

2. CP violation in mixing

$$\mathrm{Prob}(P^0 \to \bar{P}^0) \neq \mathrm{Prob}(\bar{P}^0 \to P^0)$$

$$\left|\frac{q}{p}\right| \neq 1.$$

3. CP violation in interference

$$\Gamma(P^0{}_{(\leadsto \bar{P}^0)} \to f)(t) \neq \Gamma(\bar{P}^0{}_{(\leadsto P^0)} \to f)(t)$$

$$\Im \lambda_f = \Im\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

What's the time?

Now: Im(λ_f)

1. CP violation in decay

$$\Gamma(P^0 \to f) \neq \Gamma(\bar{P}^0 \to \bar{f})$$

This is obviously satisfied (see Eq. (3.15)) when

$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1.$$

2. CP violation in mixing

$$\mathrm{Prob}(P^0 \to \bar{P}^0) \neq \mathrm{Prob}(\bar{P}^0 \to P^0)$$

$$\left|\frac{q}{p}\right| \neq 1.$$

3. CP violation in interference

$$\Gamma(P^0{}_{(\leadsto \bar{P}^0)} \to f)(t) \neq \Gamma(\bar{P}^0{}_{(\leadsto P^0)} \to f)(t)$$

We will investigate λ_f for various final states f

$$\Im \lambda_f = \Im \left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

CP violation: type 3

$$\Im\lambda_f = \Im\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

$$\boxed{\Gamma(P^0{}_{(\rightsquigarrow\bar{P}^0)} \rightarrow f)(t) \neq \Gamma(\bar{P}^0{}_{(\rightsquigarrow P^0)} \rightarrow f)(t)}$$

$$A_{CP}(t) = \frac{\Gamma_{P^0(t)\to f} - \Gamma_{\bar{P}^0(t)\to f}}{\Gamma_{P^0(t)\to f} + \Gamma_{\bar{P}^0(t)\to f}}$$

$$\begin{aligned}
\Gamma_{P^0\to f}(t) &= |A_f|^2 \quad (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t + D_f \sinh\frac{1}{2}\Delta\Gamma t + C_f\cos\Delta m t - S_f \sin\Delta m t\right) \\
\Gamma_{\bar{P}^0\to f}(t) &= |A_f|^2\left|\frac{p}{q}\right|^2(1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t + D_f \sinh\frac{1}{2}\Delta\Gamma t - C_f\cos\Delta m t + S_f \sin\Delta m t\right)
\end{aligned}$$

$$D_f = \frac{2\Re\lambda_f}{1+|\lambda_f|^2} \qquad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \qquad S_f = \frac{2\Im\lambda_f}{1+|\lambda_f|^2}.$$

$$\boxed{A_{CP}(t) = \frac{\Gamma_{P^0(t)\to f} - \Gamma_{\bar{P}^0(t)\to f}}{\Gamma_{P^0(t)\to f} + \Gamma_{\bar{P}^0(t)\to f}} = \frac{2C_f\cos\Delta m t - 2S_f\sin\Delta m t}{2\cosh\frac{1}{2}\Delta\Gamma t + 2D_f\sinh\frac{1}{2}\Delta\Gamma t}}$$

Classification of CP Violating effects - Nr. 3:

Consider $f=\bar{f}$:

$$A_{CP}(t) = \frac{\Gamma_{P^0(t)\to f} - \Gamma_{\bar{P}^0(t)\to f}}{\Gamma_{P^0(t)\to f} + \Gamma_{\bar{P}^0(t)\to f}} = \frac{2C_f \cos\Delta m t - 2S_f \sin\Delta m t}{2\cosh\frac{1}{2}\Delta\Gamma t + 2D_f \sinh\frac{1}{2}\Delta\Gamma t}$$

If one amplitude dominates the decay, then $A_f = \bar{A}_f$

$$A_{CP}(t) = \frac{-\Im\lambda_f \sin\Delta m t}{\cosh\frac{1}{2}\Delta\Gamma t + \Re\lambda_f \sinh\frac{1}{2}\Delta\Gamma t}$$

3. CP violation in interference

$$\Gamma(P^0{}_{(\rightsquigarrow\bar{P}^0)} \to f)(t) \neq \Gamma(\bar{P}^0{}_{(\rightsquigarrow P^0)} \to f)(t)$$

$$\Im\lambda_f = \Im\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

CP violation: a famous example

- The golden decay $B^0 \rightarrow J/\Psi K_s$

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