Particle Physics II – CP violation

Lecture 2

N. Tuning

Plan

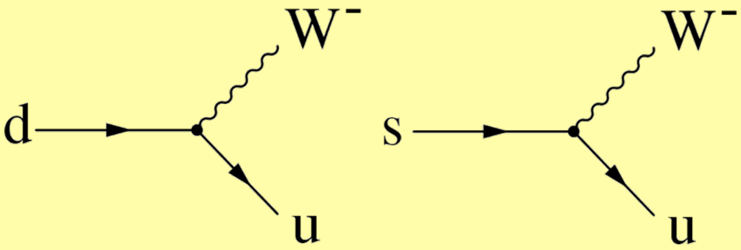
1) Wed 12 Feb: Anti-matter + SM
2) Mon 17 Feb: CKM matrix + Unitarity Triangle
3) Wed 19 Feb: Mixing + Master eqs. + $B^0 \rightarrow J/\psi K_s$
4) Mon 20 Feb: CP violation in $B_{(s)}$ decays (I)
5) Wed 9 Mar: CP violation in $B_{(s)}$ and K decays (II)
6) Mon 16 Mar: Rare decays + Flavour Anomalies
7) Wed 18 Mar: Exam

- Final Mark:
  - if (mark > 5.5) mark = max(exam, 0.85*exam + 0.15*homework)
  - else mark = exam
- In parallel: Lectures on Flavour Physics by prof.dr. R. Fleischer

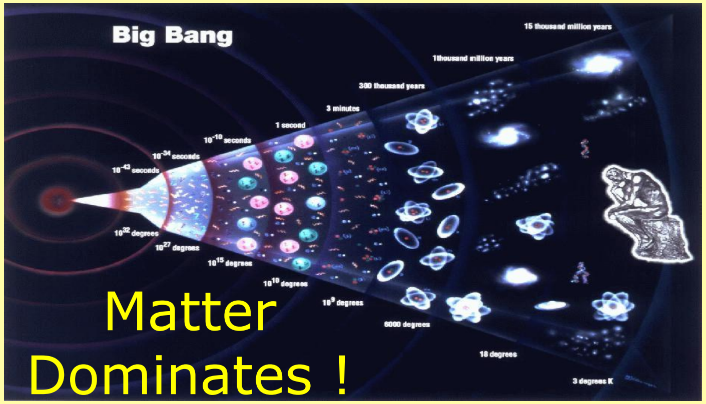
Recap: Motivation

- CP-violation (or flavour physics) is about charged current interactions

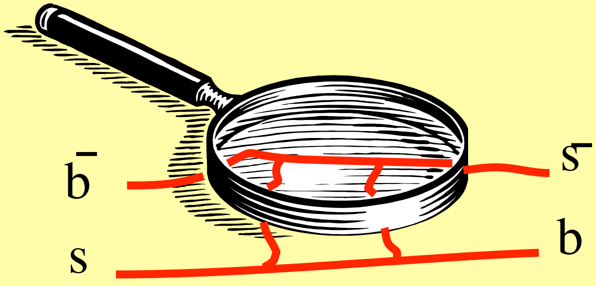
- Interesting because:
  1) Standard Model:
     in the heart of quark interactions



  2) Cosmology:
     related to matter – anti-matter asymetry



  3) Beyond Standard Model:
     measurements are sensitive to new particles



Recap: Anti matter

- Dirac equation (1928)
  - Find linear equation to avoid negative energies
  - and that is relativistically correct

$$H\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\ \psi$$

$$\begin{aligned} (i\gamma^\mu \partial_\mu - m)\ \psi &= 0 \\ \text{with}:\ \gamma^\mu = (\beta, \beta\vec{\alpha}) &\equiv \text{Dirac } \gamma\text{-matrices} \end{aligned}$$

➢ Predict existence of anti-matter

- Positron discovered (1932)
- Anti matter research at CERN very active
  - 1980: 270 GeV anti protons for SppS
  - 1995: 9 anti hydrogen atoms detected
  - 2014: anti hydrogen *beam*
    - ´` detection of 80 antihydrogen atoms 2.7 metres downstream of their production``

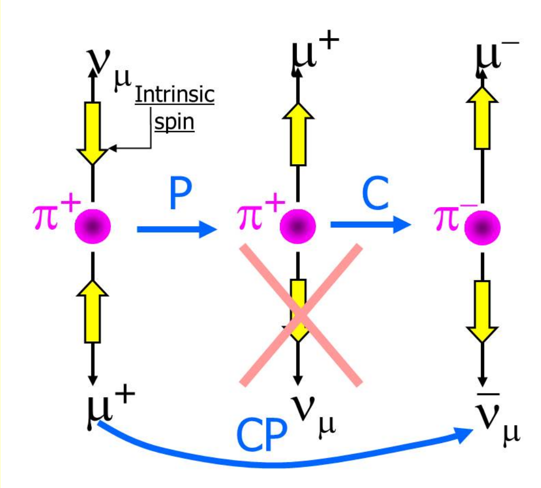
➢ Test CPT invariance: measure hyperfine structure and gravity

Recap: C and P

- C and P maximally violated in weak decays
  - Wu experiment with ^{60}Co
  - Ledermann experiment with pion decay
  - ➢ Neutrino´s are lefthanded!

- C and P conserved in strong and EM interactions
  - C and P conserved quantitites
  - ➢ C and P eigenvalues of particles

- Combined CP conserved?



Fields: Notation

$Y = Q - T_3$

Explicitly:

- The left handed quark doublet :

$$Q^I_{Li}(3,2,1/6) = \begin{pmatrix} u^I_r, u^I_g, u^I_b \\ d^I_r, d^I_g, d^I_b \end{pmatrix}_L, \begin{pmatrix} c^I_r, c^I_g, c^I_b \\ s^I_r, s^I_g, s^I_b \end{pmatrix}_L, \begin{pmatrix} t^I_r, t^I_g, t^I_b \\ b^I_r, b^I_g, b^I_b \end{pmatrix}_L \quad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad (Y = 1/6)$$

- Similarly for the quark singlets:

$$u^I_{Ri}(3,1,\ 2/3) = \left(u^I_r, u^I_r, u^I_r\right)_R, \left(c^I_r, c^I_r, c^I_r\right)_R, \left(t^I_r, t^I_r, t^I_r\right)_R \quad (Y = 2/3)$$

$$d^I_{Ri}(3,1,-1/3) = \left(d^I_r, d^I_r, d^I_r\right)_R, \left(s^I_r, s^I_r, s^I_r\right)_R, \left(b^I_r, b^I_r, b^I_r\right)_R \quad (Y = -1/3)$$

- The left handed leptons: $L^I_{Li}(1,2,-1/2) = \begin{pmatrix} \nu^I_e \\ e^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\mu \\ \mu^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\tau \\ \tau^I \end{pmatrix}_L \quad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad (Y = -1/2)$

- And similarly the (charged) singlets: $l^I_{Ri}(1,1,-1) = e^I_R, \mu^I_R, \tau^I_R \quad (Y = -1)$

Weak interaction: parity violating (and not only for neutrinos!)

$$L_{kinetic} : i\overline{\psi}(\partial^{\mu}\gamma_{\mu})\psi \rightarrow i\overline{\psi}(D^{\mu}\gamma_{\mu})\psi$$

$$with \quad \psi = Q_{Li}^{I}, \quad u_{Ri}^{I}, \quad d_{Ri}^{I}, \quad L_{Li}^{I}, \quad l_{Ri}^{I}$$

For example, the term with Q_{Li}^{I} becomes:

$$L_{kinetic}(Q_{Li}^{I}) = i\overline{Q_{Li}^{I}}\gamma_{\mu}D^{\mu}Q_{Li}^{I}$$

$$= i\overline{Q_{Li}^{I}}\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}g_s G_a^{\mu}\lambda_a + \frac{i}{2}gW_b^{\mu}\tau_b + \frac{i}{6}g'B^{\mu}\right)Q_{Li}^{I}$$

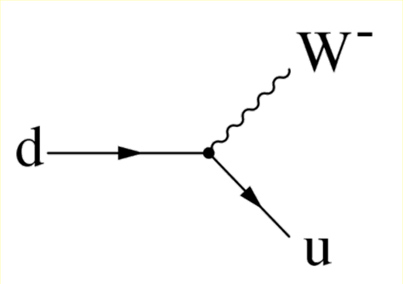
$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Only acts on the left-handed doublet!

Recap: SM Lagrangian

- C and P violation in weak interaction
- How is weak (charged) interaction described in SM?

$$\boxed{L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}}$$



$$L_{kinetic} : i\overline{\psi}(\partial^{\mu}\gamma_{\mu})\psi \rightarrow i\overline{\psi}(D^{\mu}\gamma_{\mu})\psi$$

$$with \quad \psi = Q^{I}_{Li}, \quad u^{I}_{Ri}, \quad d^{I}_{Ri}, \quad L^{I}_{Li}, \quad l^{I}_{Ri}$$

$$L_{Kinetic} = \frac{g}{\sqrt{2}}\overline{u^{I}_{Li}}\gamma^{\mu}W^{-}_{\mu}d^{I}_{Li} + \frac{g}{\sqrt{2}}\overline{d^{I}_{Li}}\gamma^{\mu}W^{+}_{\mu}u^{I}_{Li} + \ldots$$

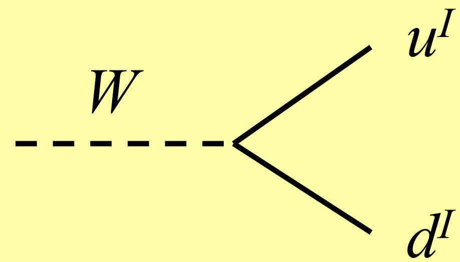
$$-L_{Yuk} = Y^{d}_{ij}\,(\overline{u^{I}_{L}}, \overline{d^{I}_{L}})_{i}\begin{pmatrix}\varphi^{+}\\ \varphi^{0}\end{pmatrix} d^{I}_{Rj} + \ldots$$

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

Recap

$$-L_{Yuk} = Y^d_{ij}\,(\overline{u^I_L}, \overline{d^I_L})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d^I_{Rj} + \ldots$$

$$L_{Kinetic} = \frac{g}{\sqrt{2}} \overline{u^I_{Li}} \gamma^\mu W^-_\mu d^I_{Li} + \frac{g}{\sqrt{2}} \overline{d^I_{Li}} \gamma^\mu W^+_\mu u^I_{Li} + \ldots$$



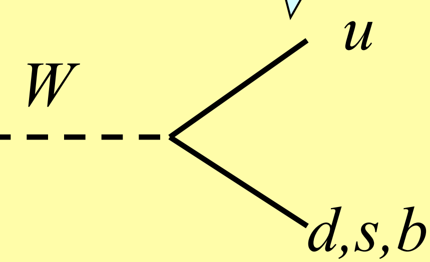
Diagonalize Yukawa matrix Y_{ij}

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$-L_{Mass} = (\bar{d}, \bar{s}, \bar{b})_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + (\bar{u}, \bar{c}, \bar{t})_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \ldots$$

$$L_{CKM} = \frac{g}{\sqrt{2}} \overline{u_i} \gamma^\mu W^-_\mu V_{ij} \left(1 - \gamma^5\right) d_j + \frac{g}{\sqrt{2}} \overline{d_j} \gamma^\mu W^+_\mu V^*_{ij} \left(1 - \gamma^5\right) u_i + \ldots$$



$$L_{SM} = L_{CKM} + L_{Higgs} + L_{Mass}$$

CKM matrix

$$-L_{Mass} = \left(\bar{d},\bar{s},\bar{b}\right)_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\bar{u},\bar{c},\bar{t}\right)_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \ldots$$

$$L_{CKM} = \frac{g}{\sqrt{2}} \overline{u}_i \gamma^\mu W_\mu^- V_{ij} \left(1-\gamma^5\right) d_j + \frac{g}{\sqrt{2}} \overline{d}_j \gamma^\mu W_\mu^+ V_{ij}^* \left(1-\gamma^5\right) u_i + \ldots$$

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- CKM matrix: `rotates` quarks between different bases
- Describes charged current coupling of quarks (mass eigenstates)

- NB: weak interaction responsible for P violation

➢ What are the properties of the CKM matrix?

➢ What are the implications for CP violation?

Ok…. We've got the CKM matrix, now what?

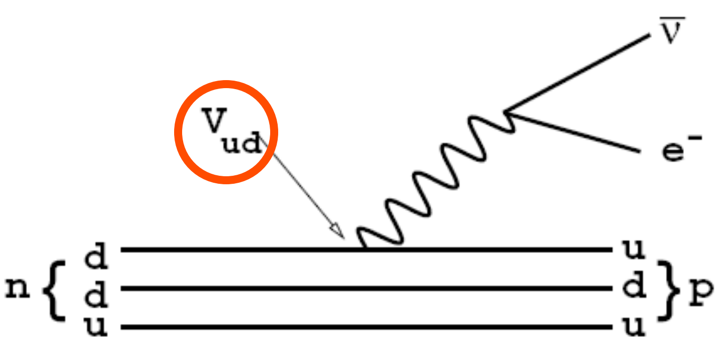
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

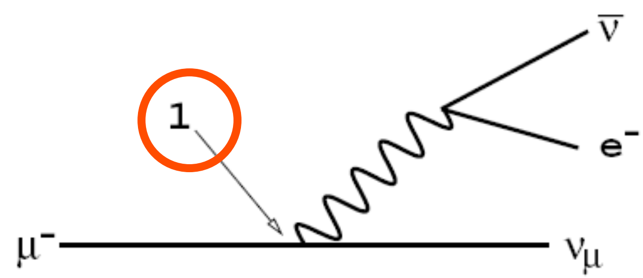
- It's *unitary*
  - *"probabilities add up to 1":*
  - d'=0.97 d + 0.22 s + 0.003 b   ($0.97^2+0.22^2+0.003^2=1$)
- How many free parameters?
  - How many real/complex?
- How do we normally visualize these parameters?

How do you measure those numbers?

- Magnitudes are typically determined from *ratio* of decay rates
- Example 1 – Measurement of V_{ud}
  - Compare decay rates of neutron decay and muon decay
  - Ratio proportional to V_{ud}^2
  - $|V_{ud}| = 0.97420 \pm 0.00021$
  - V_{ud} of order 1

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

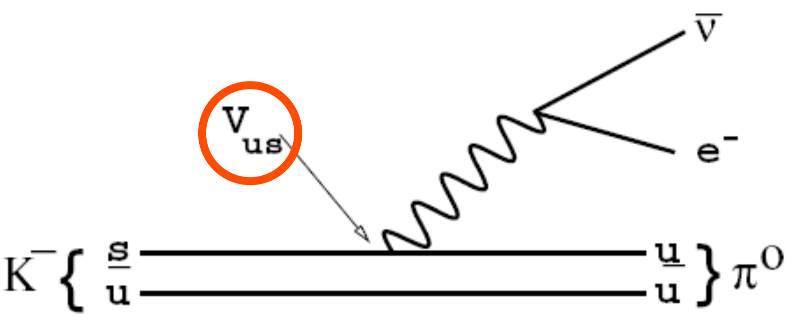


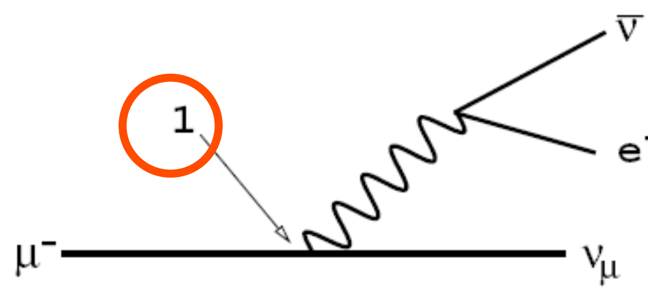


How do you measure those numbers?

- Example 2 – Measurement of V_{us}
  - Compare decay rates of semileptonic K- decay and muon decay
  - Ratio proportional to V_{us}^2
  - $|V_{us}| = 0.2243 \pm 0.0005$
  - $V_{us} \equiv \sin(\theta_c)$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





$$\frac{d\Gamma(\overline{K^0} \to \pi^+ e^- \overline{\nu}_e)}{dx_\pi} = \frac{G_F^2 m_K^5}{192\pi^2} |V_{us}|^2 f(q^2)^2 \left(x_\pi^2 - 4\frac{m_\pi^2}{m_K^2} \right)^{3/2}, x_\pi = \frac{2E_\pi}{m_K}$$

How do you measure those numbers?

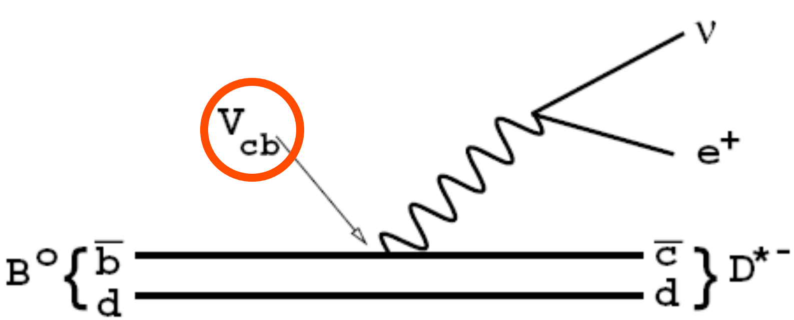
- Example 3 – Measurement of V_{cs}
  - D_s decay: $D_s^+ \rightarrow \mu^+ \nu$
  - Ratio proportional to V_{cs}^2
  - $|V_{cs}| = 0.997 \pm 0.017$
  - $V_{cs} \sim 1$

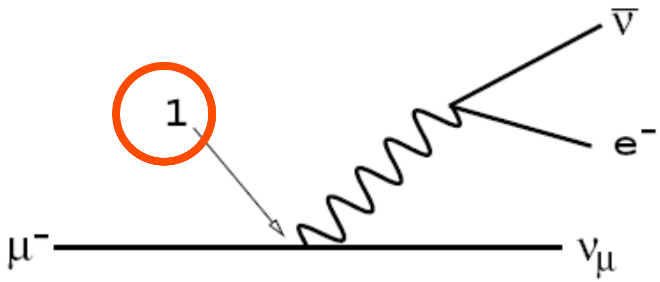
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

How do you measure those numbers?

- Example 4 – Measurement of V_{cb}
  - Compare decay rates of $B^0 \rightarrow D^{*-} l^+ \nu$ and muon decay
  - Ratio proportional to V_{cb}^2
  - $|V_{cb}| = 0.0422 \pm 0.0008$
  - V_{cb} is of order $\sin(\theta_c)^2$ [= 0.0484]

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





$$\frac{d\Gamma(b \rightarrow u_\alpha l^- \overline{\nu_l})}{dx} = \frac{G_F^2 m_b^5}{192\pi^2} |V_{\alpha b}|^2 \left(2x^2 \left(\frac{1-x-\zeta}{1-x} \right)^2 \left(3 - 2x + \zeta + \frac{2\zeta}{1-x} \right) \right)$$

$$\alpha = u, c$$

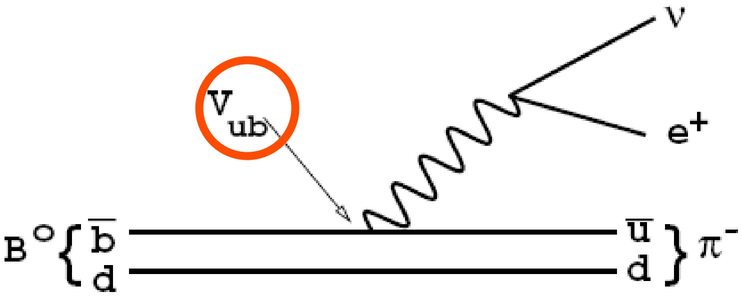
$$\zeta = \frac{m_\alpha^2}{m_b^2}$$

$$x = \frac{2E_l}{m_b}$$

How do you measure those numbers?

- Example 5 – Measurement of V_{ub}
  - Decay rate of $B^0 \rightarrow \pi^- l^+ \nu$
  - Proportional to $(V_{ub})^2$
  - $|V_{ub}| = 0.00394 \pm 0.00036$
  - V_{ub} is of order $\sin(\theta_c)^3$ [= 0.01]

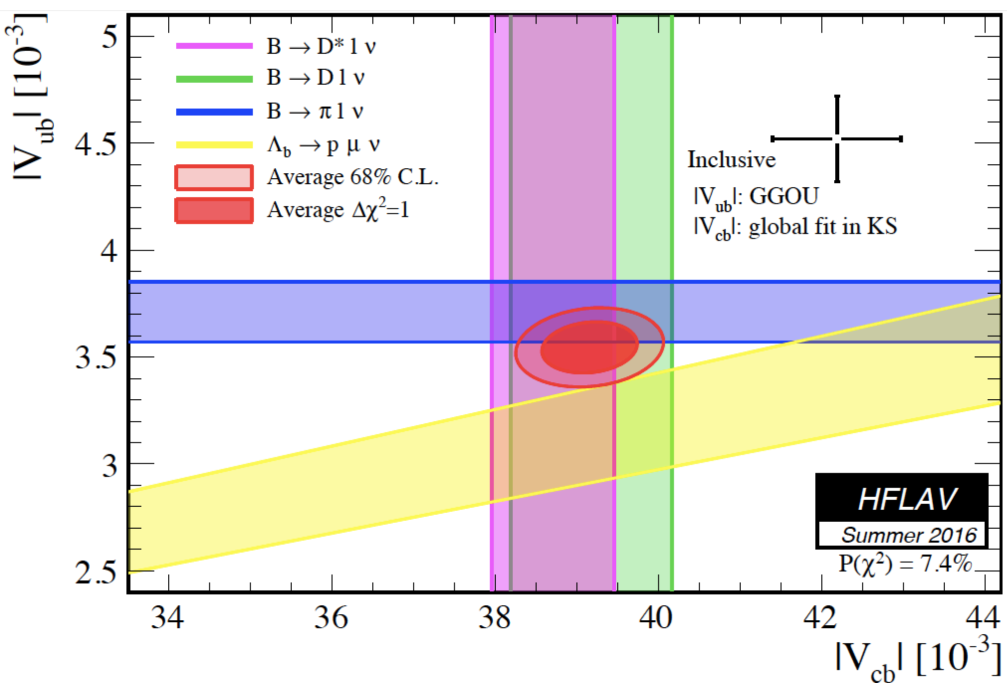
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

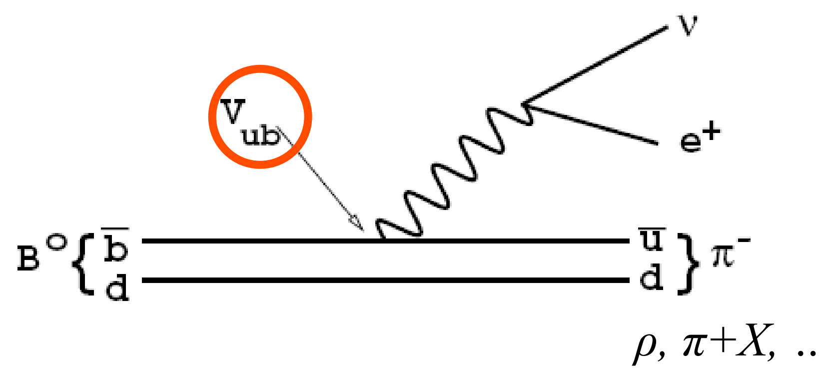


How do you measure those numbers?

- Example 5 – Measurement of V_{ub}
  - Decay rate of $B^0 \rightarrow \pi^- l^+ \nu$
  - Proportional to $(V_{ub})^2$
  - $|V_{ub}| = 0.00394 \pm 0.00036$
  - V_{ub} is of order $\sin(\theta_c)^3$ [= 0.01]
  - Inclusive vs exclusive…?

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

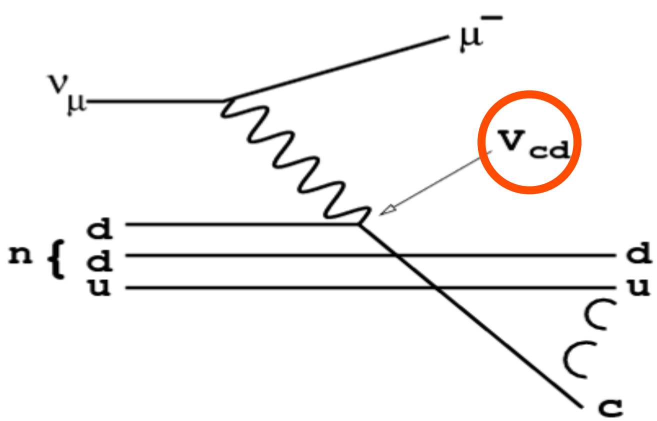




How do you measure those numbers?

- Example 6 – Measurement of V_{cd}
  - Early measurement charm in DIS with neutrinos
  - Rate proportional to V_{cd}^2
  - $|V_{cd}| = 0.218 \pm 0.004$
  - V_{cb} is of order $\sin(\theta_c)$ [= 0.24]

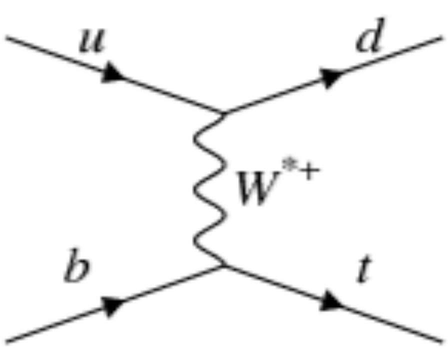
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

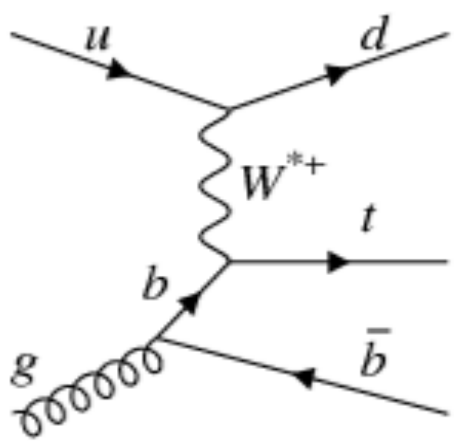


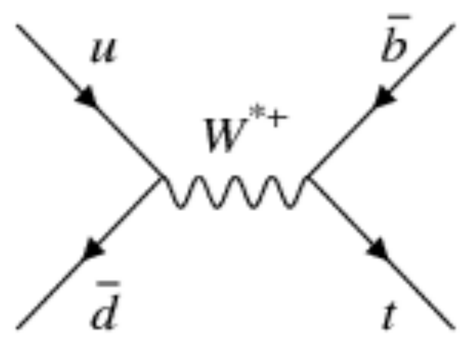
How do you measure those numbers?

- Example 7 – Measurement of V_{tb}
  - Very recent measurement: March '09!
  - Single top production at Tevatron
  - CDF+D0+LHC: $|V_{tb}| = 1.019 \pm 0.025$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



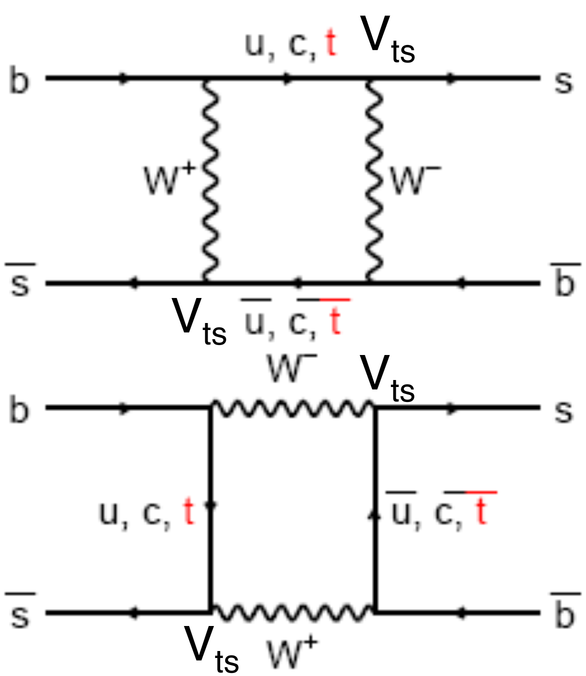




How do you measure those numbers?

- Example 8 – Measurement of V_{td}, V_{ts}
  - Cannot be measured from top-decay…
  - Indirect from loop diagram
  - $|V_{td}| = 0.0081 \pm 0.0005$
  - $|V_{ts}| = 0.0394 \pm 0.0023$
  - $|V_{td}/V_{ts}| = 0.210 \pm 0.008$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Ratio of frequencies for B^0 and B_s

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{Bs}}{m_{Bd}} \frac{f_{Bs}^2 B_{Bs}}{f_{Bd}^2 B_{Bd}} \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{m_{Bs}}{m_{Bd}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$

$V_{ts} \sim \lambda^2$

$V_{td} \sim \lambda^3$ $\rightarrow \Delta m_s \sim (1/\lambda^2)\Delta m_d \sim 25\, \Delta m_d$

$\xi = 1.239 \pm 0.046$ from lattice QCD

What do we know about the CKM matrix?

- Magnitudes of elements have been measured over time
  - Result of a *large* number of measurements and calculations

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.04214 \\ 0.00896 & 0.04133 & 0.99911 \end{pmatrix} \pm \begin{pmatrix} 0.00010 & 0.00044 & 0.00012 \\ 0.00044 & 0.00011 & 0.00076 \\ 0.00024 & 0.00974 & 0.00003 \end{pmatrix}$$

Magnitude of elements shown only, no information of phase

What do we know about the CKM matrix?

- Magnitudes of elements have been measured over time
  - Result of a *large* number of measurements and calculations

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

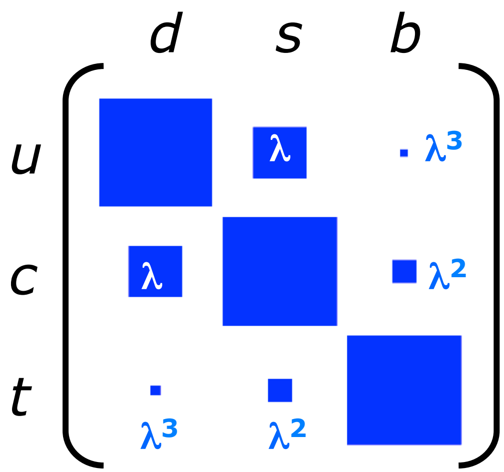
$$\lambda \approx \sin\theta_C = \sin\theta_{12} \approx 0.24$$

Magnitude of elements shown only, no information of phase

Approximately diagonal form

- Values are strongly ranked:
  - Transition within generation favored
  - Transition from 1st to 2nd generation suppressed by $\sin(\theta_c)$
  - Transition from 2nd to 3rd generation suppressed bu $\sin^2(\theta_c)$
  - Transition from 1st to 3rd generation suppressed by $\sin^3(\theta_c)$

CKM magnitudes



$\lambda = \sin(\theta_c) = 0.23$

Why the ranking?
We don't know (yet)!

*If you figure this out, you **will** win the nobel prize*

Intermezzo: How about the leptons?

- We now know that neutrinos also have flavour oscillations
  - Neutrinos have mass
  - Diagonalizing Y^l_{ij} doesn't come for free any longer

$$\mathcal{L}_{Yukawa} = Y_{ij}\overline{\psi_{Li}}\,\phi\,\psi_{Rj} + h.c.$$
$$= Y^d_{ij}\overline{Q^I_{Li}}\,\phi\,d^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\,\tilde{\phi}\,u^I_{Rj} + Y^l_{ij}\overline{L^I_{Li}}\,\phi\,l^I_{Rj}$$

- thus there is the equivalent of a CKM matrix for them:
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \begin{bmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

vs

$$\begin{bmatrix}|d'\rangle\\ |s'\rangle\\ |b'\rangle\end{bmatrix} = \begin{bmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{bmatrix}\begin{bmatrix}|d\rangle\\ |s\rangle\\ |b\rangle\end{bmatrix}.$$

Intermezzo: How about the leptons?

- the equivalent of the CKM matrix
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

vs

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} .$$

- a completely different hierarchy!

$$U_{MNSP} \approx \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.37 & 0.57 & 0.70 \\ 0.39 & 0.59 & 0.69 \end{pmatrix}$$

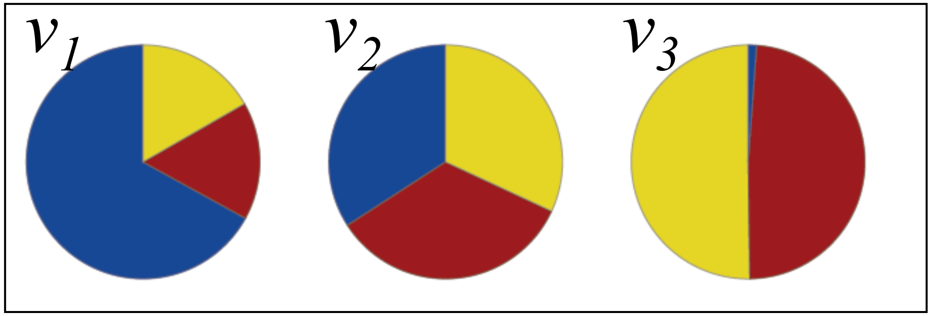
$$V_{CKM} = \begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.04214 \\ 0.00896 & 0.04133 & 0.99911 \end{pmatrix}$$

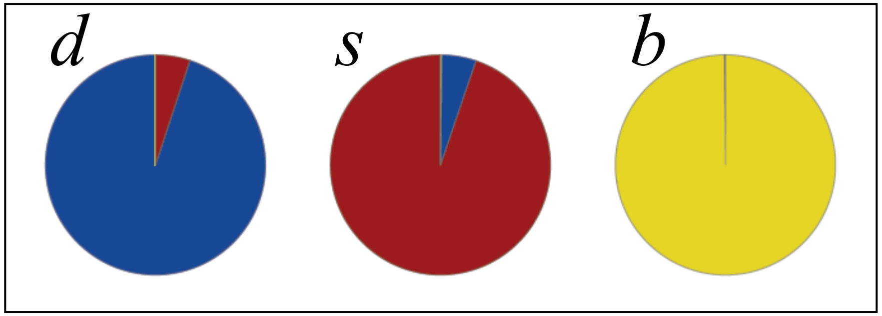
Intermezzo: How about the leptons?

- the equivalent of the CKM matrix
  - *Pontecorvo-Maki-Nakagawa-Sakata matrix*

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \begin{bmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} \quad \textbf{vs} \quad \begin{bmatrix}|d'\rangle\\ |s'\rangle\\ |b'\rangle\end{bmatrix} = \begin{bmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{bmatrix}\begin{bmatrix}|d\rangle\\ |s\rangle\\ |b\rangle\end{bmatrix}.$$

- a completely different $\begin{pmatrix}|U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2\\ |U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2\\ |U_{\tau1}|^2 & |U_{\tau2}|^2 & |U_{\tau3}|^2\end{pmatrix} \approx \begin{pmatrix}\frac{2}{3} & \frac{1}{3} & 0\\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2}\\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2}\end{pmatrix}$



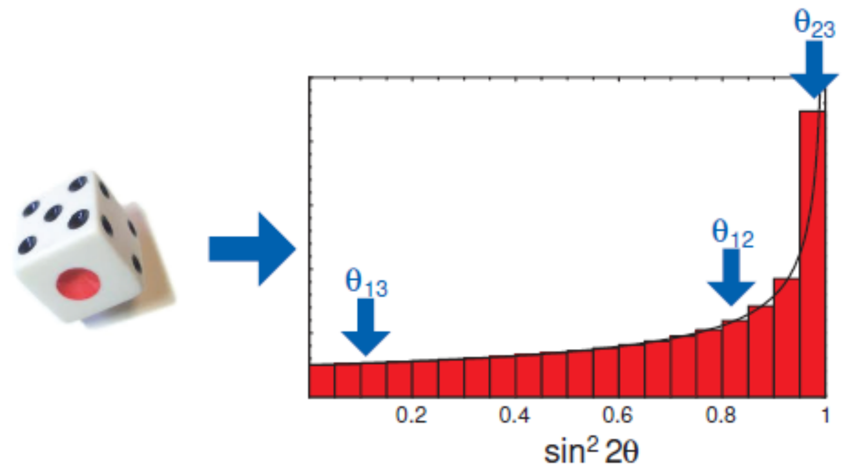


See eg. PhD thesis R. de Adelhart Toorop

Intermezzo: what does the size tell us?

H.Murayama, 6 Jan 2014, arXiv:1401.0966

- Neutrino mixing due to ´anarchy´:
  - `quite typical of the ones obtained by randomly drawing a mixing matrix from an unbiased distribution of unitary 3x3 matrices´



and found that it is 47% probable [21]! So we learned indeed that the neutrino masses and mixings do not require any deeper symmetries or new quantum numbers. On the other hand, quarks clearly do need additional input, which is yet to be understood.

Harrison, Perkins, Scott, Phys.Lett. B530 (2002) 167, hep-ph/0202074

- Neutrino mixing due to underlying symmetry:

$$U_l = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{\bar{\omega}}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\bar{\omega}}{\sqrt{3}} \end{pmatrix} \ (\text{columns } e,\ \mu,\ \tau) \qquad U_\nu = \begin{pmatrix} \sqrt{\frac{1}{2}} & 0 & -\sqrt{\frac{1}{2}} \\ 0 & 1 & 0 \\ \sqrt{\frac{1}{2}} & 0 & \sqrt{\frac{1}{2}} \end{pmatrix} \ (\text{columns } \nu_1,\ \nu_2,\ \nu_3) \qquad (4)$$

i.e. $U_l^\dagger M_l^2 U_l = \text{diag}\ (m_e^2,\ m_\mu^2,\ m_\tau^2)$ and $U_\nu^\dagger M_\nu^2 U_\nu = \text{diag}\ (m_1^2,\ m_2^2,\ m_3^2)$, so that the lepton mixing matrix (or MNS matrix) $U = U_l^\dagger U_\nu$ is given by:

$$\begin{matrix} e \\ \mu \\ \tau \end{matrix} \begin{pmatrix} \frac{1}{\sqrt{3}} & \sqrt{\frac{1}{3}} & \frac{1}{\sqrt{3}} \\ \frac{\omega}{\sqrt{3}} & \sqrt{\frac{1}{3}} & \frac{\bar{\omega}}{\sqrt{3}} \\ \frac{\bar{\omega}}{\sqrt{3}} & \sqrt{\frac{1}{3}} & \frac{\omega}{\sqrt{3}} \end{pmatrix} \overset{\nu_1 \quad \nu_2 \quad \nu_3}{\begin{pmatrix} \sqrt{\frac{1}{2}} & 0 & -\sqrt{\frac{1}{2}} \\ 0 & 1 & 0 \\ \sqrt{\frac{1}{2}} & 0 & \sqrt{\frac{1}{2}} \end{pmatrix}} = \begin{matrix} e \\ \mu \\ \tau \end{matrix} \overset{\nu_1 \quad \nu_2 \quad \nu_3}{\begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\frac{i}{\sqrt{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \frac{i}{\sqrt{2}} \end{pmatrix}} \qquad (5)$$

Back to business: quarks

We discussed magnitude.

Next is the imaginary part !

Quark field re-phasing

Under a quark phase transformation:

$$u_{Li} \rightarrow e^{i\phi_{ui}} u_{Li} \qquad d_{Li} \rightarrow e^{i\phi_{di}} d_{Li}$$

and a simultaneous rephasing of the CKM matrix:

$$V \rightarrow \begin{pmatrix} e^{-\phi_u} & & \\ & e^{-\phi_c} & \\ & & e^{-\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-\phi_d} & & \\ & e^{-\phi_s} & \\ & & e^{-\phi_b} \end{pmatrix}$$

or $V_{\alpha j} \rightarrow \exp\left(i\left(\phi_j - \phi_\alpha\right)\right) V_{\alpha j}$

the charged current $J^{\mu}_{CC} = \overline{u_{Li}} \gamma^{\mu} V_{ij} d_{Lj}$ is left invariant.

Degrees of freedom in V_{CKM} in	3	N	generations
Number of real parameters:	9	+ N²	
Number of imaginary parameters:	9	+ N²	
Number of constraints ($VV^{\dagger} = 1$):	-9	- N²	
Number of relative quark phases:	-5	- (2N-1)	

Total degrees of freedom:	4	(N-1)²	
Number of Euler angles:	3	N (N-1) / 2	
Number of CP phases:	1	(N-1) (N-2) / 2	

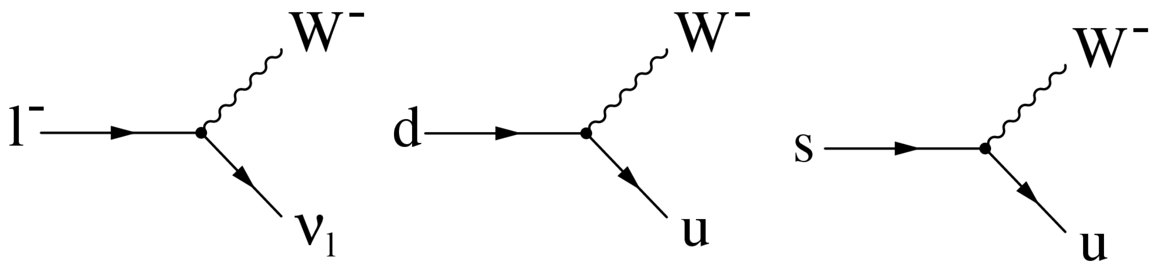
2 generations:

$$V_{CKM} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

No CP violation in SM!
This is the reason Kobayashi and Maskawa *first* suggested a 3rd family of fermions!

First some history…

- Cabibbos theory successfully correlated many decay rates by counting the number of $\cos\theta_c$ and $\sin\theta_c$ terms in their decay diagram



g $\qquad g\cos\theta_C$ $\qquad g\sin\theta_C$

$\Gamma\left(\mu^- \to e^-\bar{\nu}_e\nu_\mu\right) \propto g^4$ purely leptonic

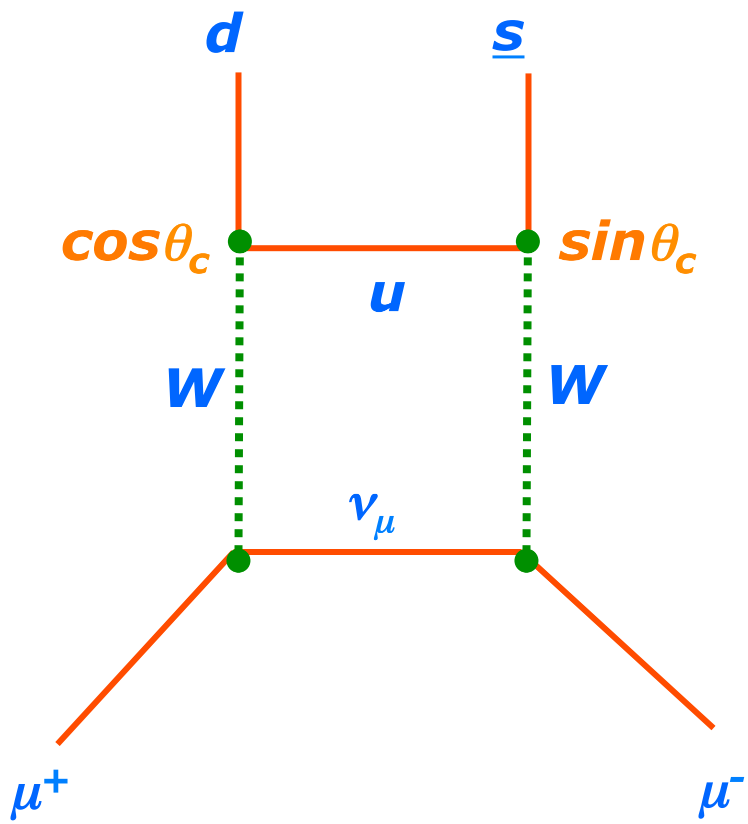
$\Gamma\left(n \to pe^-\bar{\nu}_e\right) \propto g^4\cos^2\theta_C$ semi-leptonic, $\Delta S = 0$

$\Gamma\left(\Lambda^0 \to pe^-\bar{\nu}_e\right) \propto g^4\sin^2\theta_C$ semi-leptonic, $\Delta S = 1$

$$\Gamma = \left|\sum_i A_i\right|^2$$

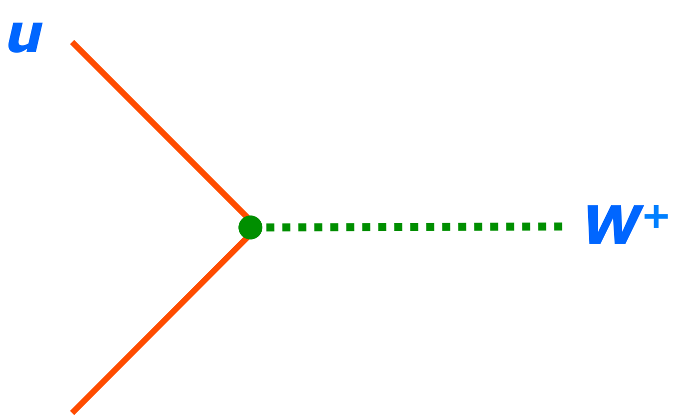
Cabibbos theory successfully correlated many decay rates

- There was however one major exception which Cabibbo could *not* describe: $K^0 \rightarrow \mu^+ \mu^-$
  - *Observed rate **much** lower than expected from Cabibbos rate correlations (expected rate $\propto g^8 \sin^2\theta_c \cos^2\theta_c$)*

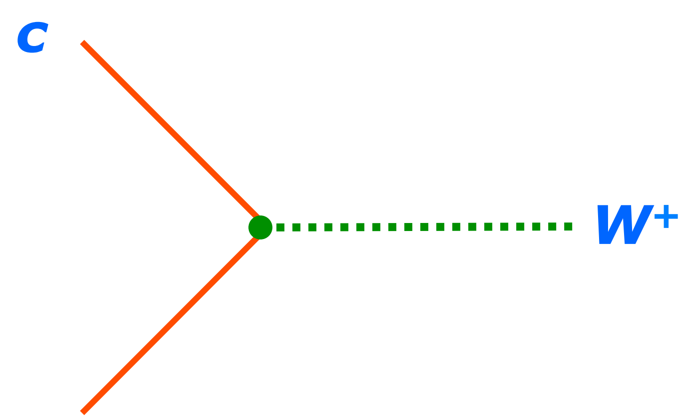


The Cabibbo-GIM mechanism

- Solution to K^0 decay problem in 1970 by Glashow, Iliopoulos and Maiani → postulate existence of 4th quark
  - Two 'up-type' quarks decay into rotated 'down-type' states
  - Appealing symmetry between generations



$d' = \cos(\theta_c) d + \sin(\theta_c) s$



$s' = -\sin(\theta_c) d + \cos(\theta_c) s$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

The Cabibbo-GIM mechanism

Phys.Rev.D2,1285,1970

Weak Interactions with Lepton-Hadron Symmetry*

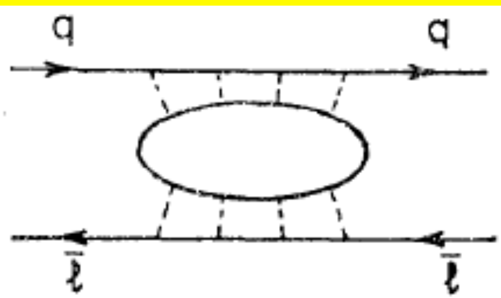
S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Mills theory is discussed.

splitting, beginning at order $G(G\Lambda^2)$, as well as contributions to such unobserved decay modes as $K_2 \to \mu^+ + \mu^-$, $K^+ \to \pi^+ + l + \bar{l}$, etc., involving neutral lepton

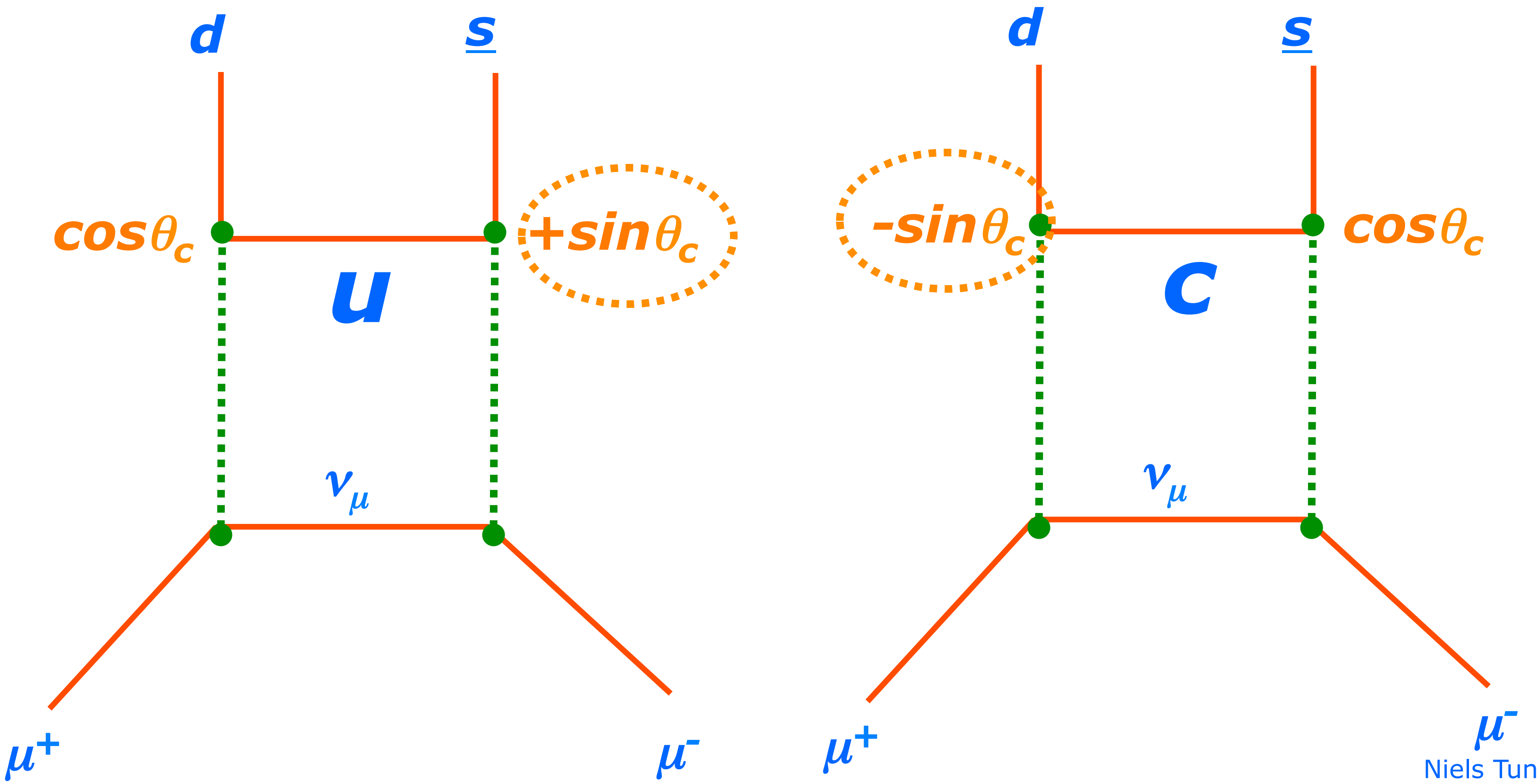
We wish to propose a simple model in which the divergences are properly ordered. Our model is founded in a quark model, but one involving four, not three, fundamental fermions; the weak interactions are medi-

new quantum number $\mathcal{C}$ for charm.



The Cabibbo-GIM mechanism

- How does it solve the $K^0 \rightarrow \mu^+\mu^-$ problem?
  - Second decay amplitude added that is almost identical to original one, *but has relative minus sign* → Almost fully destructive interference
  - Cancellation not perfect because u, c mass different



Quark field re-phasing

Under a quark phase transformation:

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and a simultaneous rephasing of the CKM matrix:

$$V \rightarrow \begin{pmatrix} e^{-\phi_u} & & \\ & e^{-\phi_c} & \\ & & e^{-\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-\phi_d} & & \\ & e^{-\phi_s} & \\ & & e^{-\phi_b} \end{pmatrix}$$ or $$V_{\alpha j} \rightarrow \exp\left(i\left(\phi_j - \phi_\alpha\right)\right) V_{\alpha j}$$

In other words:

$$\overline{(u,\ c,\ t)}_L \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L ,$$

$$=$$

$$\overline{(u,\ c,\ t)}_L \begin{pmatrix} V_{ud}\, e^{-i\phi} & V_{us} & V_{ub} \\ V_{cd}\, e^{-i\phi} & V_{cs} & V_{cb} \\ V_{td}\, e^{-i\phi} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d\, e^{i\phi} \\ s \\ b \end{pmatrix}_L$$

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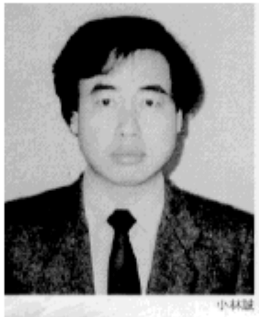
No CP violation in SM!
This is the reason Kobayashi and Maskawa *first* suggested a **3rd** family of fermions!

Intermezzo: Kobayashi & Maskawa



Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

Next we consider a 6-plet model, another interesting model of CP-violation. Suppose that 6-plet with charges $(Q, Q, Q, Q-1, Q-1, Q-1)$ is decomposed into $SU_{\text{weak}}(2)$ multiplets as $2+2+2$ and $1+1+1+1+1+1$ for left and right components, respectively. Just as the case of (A, C), we have a similar expression for the charged weak current with a 3×3 instead of 2×2 unitary matrix in Eq. (5). As was pointed out, in this case we cannot absorb all phases of matrix elements into the phase convention and can take, for example, the following expression:

Timeline:

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

- Timeline:
  - Sep 1972: Kobayashi & Maskawa predict ***3*** generations
  - Nov 1974: Richter, Ting discover J/ψ: fill ***2nd*** generation
  - July 1977: Ledermann discovers Υ: discovery of ***3rd*** generation

From 2 to 3 generations

- 2 generations: d'=0.97 d + 0.22 s (θ_c=13°)

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- 3 generations: d'=0.97 d + 0.22 s + 0.003 b

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- NB: probabilities have to add up to 1: $0.97^2+0.22^2+0.003^2=1$
  - ➔ "Unitarity" !

From 2 to 3 generations

- 2 generations: d'=0.97 d + 0.22 s (θ_c=13°)

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- 3 generations: d'=0.97 d + 0.22 s + 0.003 b

Parameterization used by Particle Data Group (3 Euler angles, 1 phase):

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

Possible forms of 3 generation mixing matrix

- 'General' 4-parameter form (Particle Data Group) with three rotations θ_{12},θ_{13},θ_{23} and one complex phase δ_{13}
  - $c_{12} = \cos(\theta_{12})$, $s_{12} = \sin(\theta_{12})$ etc… $\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

- Another form (Kobayashi & Maskawa's original)
  - Different but equivalent

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- Physics is independent of choice of parameterization!
  - But for any choice there will be ***complex-valued*** elements

Possible forms of 3 generation mixing matrix

→ Different parametrizations! It's about phase *differences!*

Re-phasing V:

$$\overline{(u,\ c,\ t)}_L \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L ,$$

$$=$$

$$\overline{(u,\ c,\ t)}_L \begin{pmatrix} V_{ud}\, e^{-i\phi} & V_{us} & V_{ub} \\ V_{cd}\, e^{-i\phi} & V_{cs} & V_{cb} \\ V_{td}\, e^{-i\phi} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d\, e^{i\phi} \\ s \\ b \end{pmatrix}_L$$

Parametrization		Useful relations
P1: $V = R_{12}(\theta)\ R_{23}(\sigma,\varphi)\ R_{12}^{-1}(\theta')$ $\begin{pmatrix} s_\theta s_{\theta'} c_\sigma + c_\theta c_{\theta'} e^{-i\varphi} & s_\theta c_{\theta'} c_\sigma - c_\theta s_{\theta'} e^{-i\varphi} & s_\theta s_\sigma \\ c_\theta s_{\theta'} c_\sigma - s_\theta c_{\theta'} e^{-i\varphi} & c_\theta c_{\theta'} c_\sigma + s_\theta s_{\theta'} e^{-i\varphi} & c_\theta s_\sigma \\ -s_{\theta'} s_\sigma & -c_{\theta'} s_\sigma & c_\sigma \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta s_{\theta'} c_{\theta'} s_\sigma^2 c_\sigma \sin\varphi$; $\tan\theta = \lvert V_{ub}/V_{cb}\rvert$; $\tan\theta' = \lvert V_{td}/V_{ts}\rvert$; $\cos\sigma = \lvert V_{tb}\rvert$
P2: $V = R_{23}(\sigma)\ R_{12}(\theta,\varphi)\ R_{23}^{-1}(\sigma')$ $\begin{pmatrix} c_\theta & s_\theta c_{\sigma'} & -s_\theta s_{\sigma'} \\ -s_\theta c_\sigma & c_\theta c_\sigma c_{\sigma'} + s_\sigma s_{\sigma'} e^{-i\varphi} & -c_\theta c_\sigma s_{\sigma'} + s_\sigma c_{\sigma'} e^{-i\varphi} \\ s_\theta s_\sigma & -c_\theta s_\sigma c_{\sigma'} + c_\sigma s_{\sigma'} e^{-i\varphi} & c_\theta s_\sigma s_{\sigma'} + c_\sigma c_{\sigma'} e^{-i\varphi} \end{pmatrix}$	**KM**	$\mathcal{J} = s_\theta^2 c_\theta s_\sigma c_\sigma s_{\sigma'} c_{\sigma'} \sin\varphi$; $\cos\theta = \lvert V_{ud}\rvert$; $\tan\sigma = \lvert V_{td}/V_{cd}\rvert$; $\tan\sigma' = \lvert V_{ub}/V_{us}\rvert$
P3: $V = R_{23}(\sigma)\ R_{31}(\tau,\varphi)\ R_{12}(\theta)$ $\begin{pmatrix} c_\theta c_\tau & s_\theta c_\tau & s_\tau \\ -c_\theta s_\sigma s_\tau - s_\theta c_\sigma e^{-i\varphi} & -s_\theta s_\sigma s_\tau + c_\theta c_\sigma e^{-i\varphi} & s_\sigma c_\tau \\ -c_\theta c_\sigma s_\tau + s_\theta s_\sigma e^{-i\varphi} & -s_\theta c_\sigma s_\tau - c_\theta s_\sigma e^{-i\varphi} & c_\sigma c_\tau \end{pmatrix}$	**PDG**	$\mathcal{J} = s_\theta c_\theta s_\sigma c_\sigma s_\tau c_\tau^2 \sin\varphi$; $\tan\theta = \lvert V_{us}/V_{ud}\rvert$; $\tan\sigma = \lvert V_{cb}/V_{tb}\rvert$; $\sin\tau = \lvert V_{ub}\rvert$
P4: $V = R_{12}(\theta)\ R_{31}(\tau,\varphi)\ R_{23}^{-1}(\sigma)$ $\begin{pmatrix} c_\theta c_\tau & c_\theta s_\sigma s_\tau + s_\theta c_\sigma e^{-i\varphi} & c_\theta c_\sigma s_\tau - s_\theta s_\sigma e^{-i\varphi} \\ -s_\theta c_\tau & -s_\theta s_\sigma s_\tau + c_\theta c_\sigma e^{-i\varphi} & -s_\theta c_\sigma s_\tau - c_\theta s_\sigma e^{-i\varphi} \\ -s_\tau & s_\sigma c_\tau & c_\sigma c_\tau \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta s_\sigma c_\sigma s_\tau c_\tau^2 \sin\varphi$; $\tan\theta = \lvert V_{cd}/V_{ud}\rvert$; $\tan\sigma = \lvert V_{ts}/V_{tb}\rvert$; $\sin\tau = \lvert V_{td}\rvert$
P5: $V = R_{31}(\tau)\ R_{12}(\theta,\varphi)\ R_{31}^{-1}(\tau')$ $\begin{pmatrix} c_\theta c_\tau c_{\tau'} + s_\tau s_{\tau'} e^{-i\varphi} & s_\theta c_\tau & -c_\theta c_\tau s_{\tau'} + s_\tau c_{\tau'} e^{-i\varphi} \\ -s_\theta c_{\tau'} & c_\theta & s_\theta s_{\tau'} \\ -c_\theta s_\tau c_{\tau'} + c_\tau s_{\tau'} e^{-i\varphi} & -s_\theta s_\tau & c_\theta s_\tau s_{\tau'} + c_\tau c_{\tau'} e^{-i\varphi} \end{pmatrix}$		$\mathcal{J} = s_\theta^2 c_\theta s_\tau c_\tau s_{\tau'} c_{\tau'} \sin\varphi$; $\cos\theta = \lvert V_{cs}\rvert$; $\tan\tau = \lvert V_{ts}/V_{us}\rvert$; $\tan\tau' = \lvert V_{cb}/V_{cd}\rvert$
P6: $V = R_{12}(\theta)\ R_{23}(\sigma,\varphi)\ R_{31}(\tau)$ $\begin{pmatrix} -s_\theta s_\sigma s_\tau + c_\theta c_\tau e^{-i\varphi} & s_\theta c_\sigma & s_\theta s_\sigma c_\tau + c_\theta s_\tau e^{-i\varphi} \\ -c_\theta s_\sigma s_\tau - s_\theta c_\tau e^{-i\varphi} & c_\theta c_\sigma & c_\theta s_\sigma c_\tau - s_\theta s_\tau e^{-i\varphi} \\ -c_\sigma s_\tau & -s_\sigma & c_\sigma c_\tau \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta s_\sigma c_\sigma^2 s_\tau c_\tau \sin\varphi$; $\tan\theta = \lvert V_{us}/V_{cs}\rvert$; $\sin\sigma = \lvert V_{ts}\rvert$; $\tan\tau = \lvert V_{td}/V_{tb}\rvert$
P7: $V = R_{23}(\sigma)\ R_{12}(\theta,\varphi)\ R_{31}^{-1}(\tau)$ $\begin{pmatrix} c_\theta c_\tau & s_\theta & -c_\theta s_\tau \\ -s_\theta c_\sigma c_\tau + s_\sigma s_\tau e^{-i\varphi} & c_\theta c_\sigma & s_\theta c_\sigma s_\tau + s_\sigma c_\tau e^{-i\varphi} \\ s_\theta s_\sigma c_\tau + c_\sigma s_\tau e^{-i\varphi} & -c_\theta s_\sigma & -s_\theta s_\sigma s_\tau + c_\sigma c_\tau e^{-i\varphi} \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta^2 s_\sigma c_\sigma s_\tau c_\tau \sin\varphi$; $\sin\theta = \lvert V_{us}\rvert$; $\tan\sigma = \lvert V_{ts}/V_{cs}\rvert$; $\tan\tau = \lvert V_{ub}/V_{ud}\rvert$
P8: $V = R_{31}(\tau)\ R_{12}(\theta,\varphi)\ R_{23}(\sigma)$ $\begin{pmatrix} c_\theta c_\tau & s_\theta c_\sigma c_\tau - s_\sigma s_\tau e^{-i\varphi} & s_\theta s_\sigma c_\tau + c_\sigma s_\tau e^{-i\varphi} \\ -s_\theta & c_\theta c_\sigma & c_\theta s_\sigma \\ -c_\theta s_\tau & -s_\theta c_\sigma s_\tau - s_\sigma c_\tau e^{-i\varphi} & -s_\theta s_\sigma s_\tau + c_\sigma c_\tau e^{-i\varphi} \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta^2 s_\sigma c_\sigma s_\tau c_\tau \sin\varphi$; $\sin\theta = \lvert V_{cd}\rvert$; $\tan\sigma = \lvert V_{cb}/V_{cs}\rvert$; $\tan\tau = \lvert V_{td}/V_{ud}\rvert$
P9: $V = R_{31}(\tau)\ R_{23}(\sigma,\varphi)\ R_{12}^{-1}(\theta)$ $\begin{pmatrix} -s_\theta s_\sigma s_\tau + c_\theta c_\tau e^{-i\varphi} & -c_\theta s_\sigma s_\tau - s_\theta c_\tau e^{-i\varphi} & c_\sigma s_\tau \\ s_\theta c_\sigma & c_\theta c_\sigma & s_\sigma \end{pmatrix}$		$\mathcal{J} = s_\theta c_\theta s_\sigma c_\sigma^2 s_\tau c_\tau \sin\varphi$; $\tan\theta = \lvert V_{cd}/V_{cs}\rvert$; $\sin\sigma = \lvert V_{cb}\rvert$

3 parameters: θ, τ, σ

1 phase: φ

Wolfenstein parameterization

$$\boxed{\sin\theta_{12} = \lambda} \tag{2.7}$$

$$\sin\theta_{23} = A\lambda^2 \tag{2.8}$$

$$\sin\theta_{13}e^{-i\delta_{13}} = A\lambda^3(\rho - i\eta) \tag{2.9}$$

where A, ρ and η are numbers of order unity. The CKM matrix then becomes $\mathcal{O}(\lambda^3)$:

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \delta V \tag{2.10}$$

3 real parameters: A, λ, ρ

1 imaginary parameter: η

Wolfenstein parameterization

$$\begin{aligned}
\sin\theta_{12} &= \lambda && (2.7)\\
\sin\theta_{23} &= A\lambda^2 && (2.8)\\
\sin\theta_{13}e^{-i\delta_{13}} &= A\lambda^3(\rho - i\eta) && (2.9)
\end{aligned}$$

where A, ρ and η are numbers of order unity. The CKM matrix then becomes $\mathcal{O}(\lambda^3)$:

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \delta V \qquad (2.10)$$

The higher order terms in the Wolfenstein parametrization are of particular importance for the B_s-system, as we will see in chapter 4, because the phase in $|V_{ts}|$ is only apparent at $\mathcal{O}(\lambda^4)$:

$$\delta V = \begin{pmatrix} -\frac{1}{8}\lambda^4 & 0 & 0 \\ \frac{1}{2}A^2\lambda^5(1-2(\rho + i\eta)) & -\frac{1}{8}\lambda^4(1+4A^2) & 0 \\ \frac{1}{2}A\lambda^5(\rho + i\eta) & \frac{1}{2}A\lambda^4(1-2(\rho + i\eta)) & -\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6) \qquad (2.11)$$

3 real parameters: A, λ, ρ

1 imaginary parameter: η

Exploit apparent ranking for a convenient parameterization

- Given current experimental precision on CKM element values, we usually drop λ^4 and λ^5 terms as well
  - Effect of order 0.2%...

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

- Deviation of ranking of 1st and 2nd generation (λ vs λ^2) parameterized in *A* parameter
- Deviation of ranking between 1st and 3rd generation, parameterized through $|\rho-i\eta|$
- Complex phase parameterized in $\arg(\rho-i\eta)$

~1995 What do we know about A, λ, ρ and η?

- Fit all known V_{ij} values to Wolfenstein parameterization and extract A, λ, ρ and η

$$V_{CKM} = \begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.04214 \\ 0.00896 & 0.04133 & 0.99911 \end{pmatrix} \pm \begin{pmatrix} 0.00010 & 0.00044 & 0.00012 \\ 0.00044 & 0.00011 & 0.00076 \\ 0.00024 & 0.00974 & 0.00003 \end{pmatrix}$$

- Results for A and λ most precise (but don't tell us much about CPV)
  - A = 0.83, λ = 0.227
- Results for ρ,η are usually shown in complex plane of ρ-iη for easier interpretation

Deriving the triangle interpretation

- Starting point: the 9 unitarity constraints on the CKM matrix

$$V^+V = \begin{pmatrix} V^*_{ud} & V^*_{cd} & V^*_{td} \\ V^*_{us} & V^*_{cs} & V^*_{ts} \\ V^*_{ub} & V^*_{cb} & V^*_{tb} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Pick (arbitrarily) orthogonality condition with (i,j)=(3,1)

$$V^*_{ub}V_{ud} + V^*_{cb}V_{cd} + V^*_{tb}V_{td} = 0$$

Deriving the triangle interpretation

- Starting point: the 9 unitarity constraints on the CKM matrix
  - 3 orthogonality relations

$$V^{+}V = \begin{pmatrix} V^{*}_{ud} & V^{*}_{cd} & V^{*}_{td} \\ V^{*}_{us} & V^{*}_{cs} & V^{*}_{ts} \\ V^{*}_{ub} & V^{*}_{cb} & V^{*}_{tb} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Pick (arbitrarily) orthogonality condition with (i,j)=(3,1)

$$V^{*}_{ub}V_{ud} + V^{*}_{cb}V_{cd} + V^{*}_{tb}V_{td} = 0$$

$$V^{*}_{ud}V_{ub} + V^{*}_{cd}V_{cb} + V^{*}_{td}V_{tb} = 0$$

Deriving the triangle interpretation

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- Pick (arbitrarily) orthogonality condition with (i,j)=(3,1)

$$V^{*}_{ub}V_{ud}+V^{*}_{cb}V_{cd}+V^{*}_{tb}V_{td}=0$$

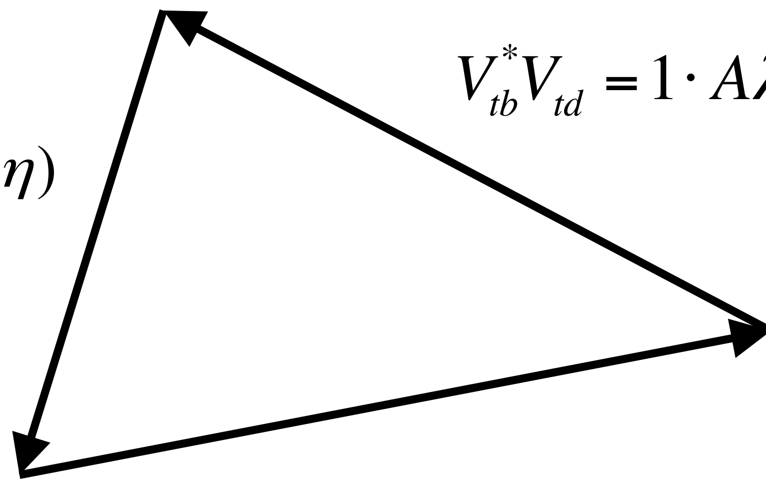
Visualizing the unitarity constraint

- Sum of three complex vectors is zero → Form triangle when put head to tail

(Wolfenstein params to order λ^4)

$$V_{tb}^* V_{td} = 1 \cdot A\lambda^3(1-\rho-i\eta)$$

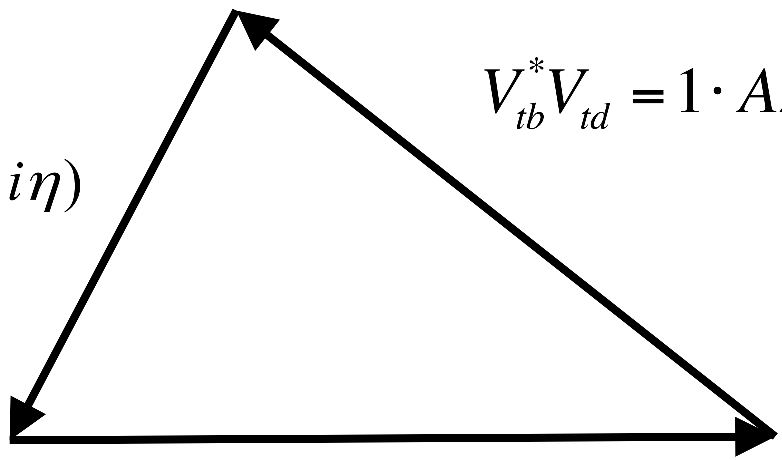
$$V_{ub}^* V_{ud} = A\lambda^3(\rho+i\eta)$$



$$V_{cb}^* V_{cd} = A\lambda^2 \cdot (-\lambda)$$

Visualizing the unitarity constraint

- Phase of 'base' is zero → Aligns with 'real' axis,



$$V_{ub}^* V_{ud} = A\lambda^3(\rho + i\eta)$$

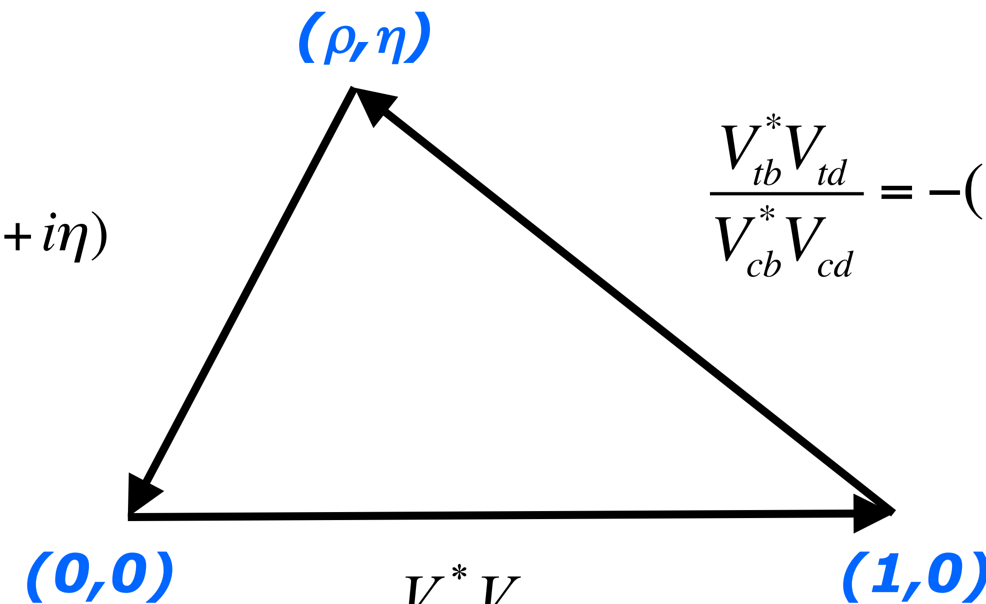
$$V_{tb}^* V_{td} = 1 \cdot A\lambda^3(1 - \rho - i\eta)$$

$$V_{cb}^* V_{cd} = A\lambda^2 \cdot (-\lambda)$$

Visualizing the unitarity constraint

- Divide all sides by length of base

$$\frac{V_{ub}^{*}V_{ud}}{V_{cb}^{*}V_{cd}} = -(\rho + i\eta)$$



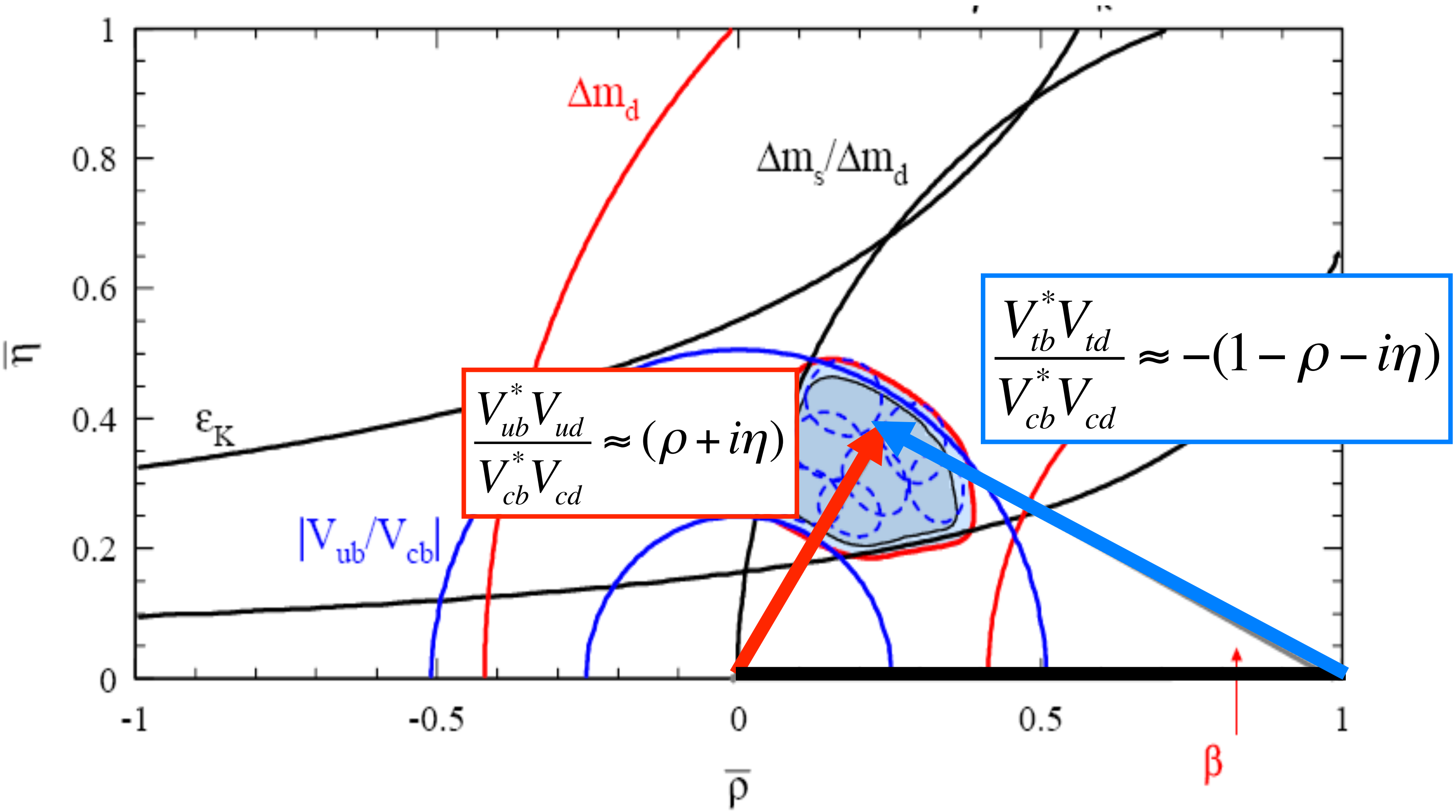
$$\frac{V_{tb}^{*}V_{td}}{V_{cb}^{*}V_{cd}} = -(1 - \rho - i\eta)$$

$$\frac{V_{cb}^{*}V_{cd}}{V_{cb}^{*}V_{cd}} \equiv 1$$

- Constructed a triangle with apex (ρ,η)

Visualizing arg(V_{ub}) and arg(V_{td}) in the (ρ,η) plane

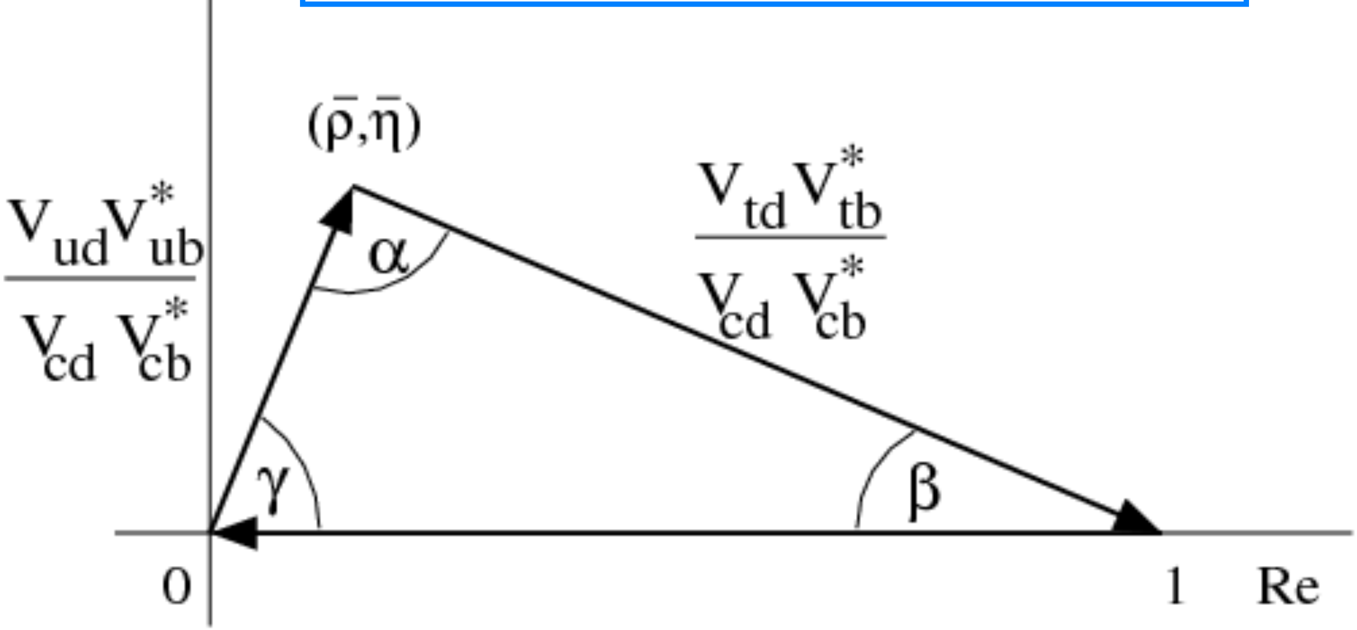
- We can now put this triangle in the (ρ,η) plane



“The” Unitarity triangle

- We can visualize the CKM-constraints in (ρ,η) plane

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

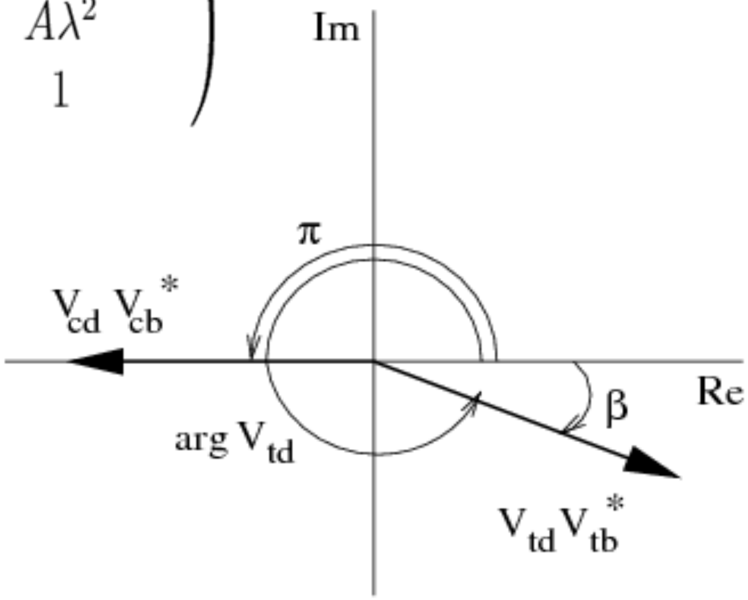


β

- We can correlate the angles β and γ to CKM elements:

$$\beta = \arg\left[-\frac{V_{cb}^{*}V_{cd}}{V_{tb}^{*}V_{td}}\right] = \pi + \arg\left[V_{cb}^{*}V_{cd}\right] - \arg\left[V_{tb}^{*}V_{td}\right] = 2\pi - \arg\left[V_{td}\right]$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



Deriving the triangle interpretation

- Another 3 orthogonality relations

$$VV^\dagger = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} V^*_{ud} & V^*_{cd} & V^*_{td} \\ V^*_{us} & V^*_{cs} & V^*_{ts} \\ V^*_{ub} & V^*_{cb} & V^*_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

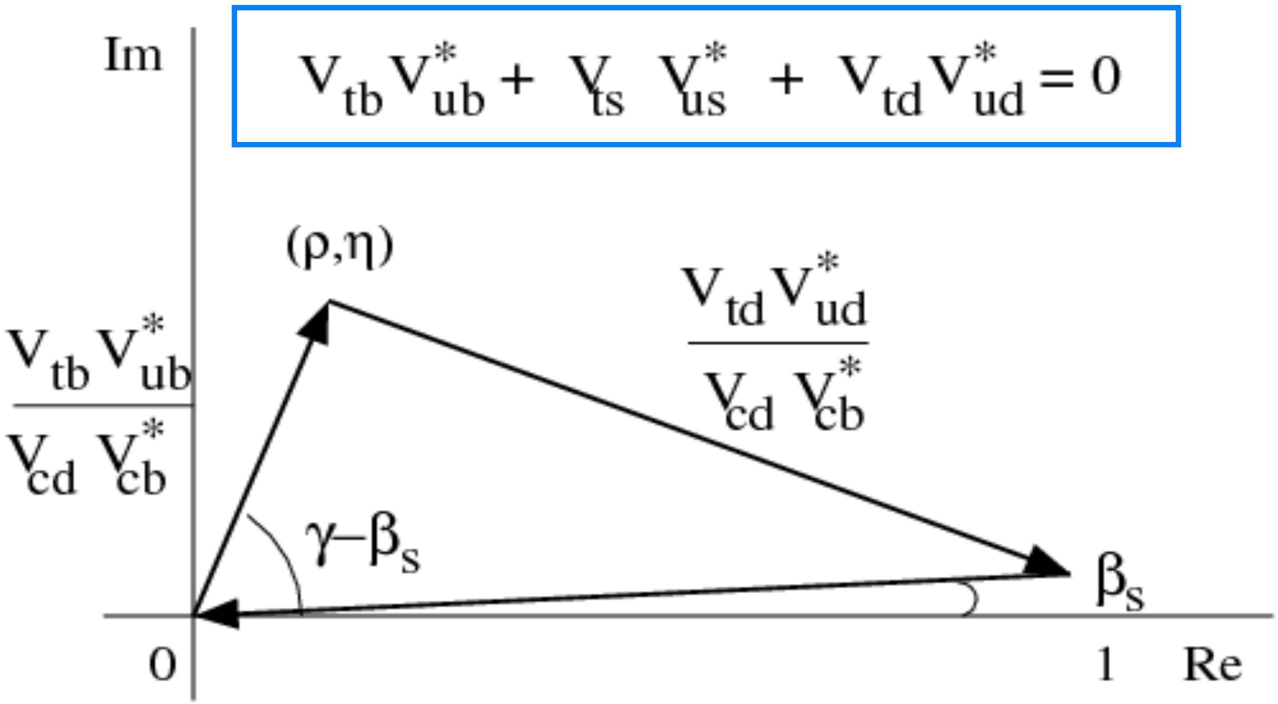
- Pick (arbitrarily) orthogonality condition with (i,j)=(3,1)

$$V^*_{ud} V_{td} + V^*_{us} V_{ts} + V^*_{ub} V_{tb} = 0$$

The “other” Unitarity triangle

- Two of the six unitarity triangles have equal sides in O(λ)

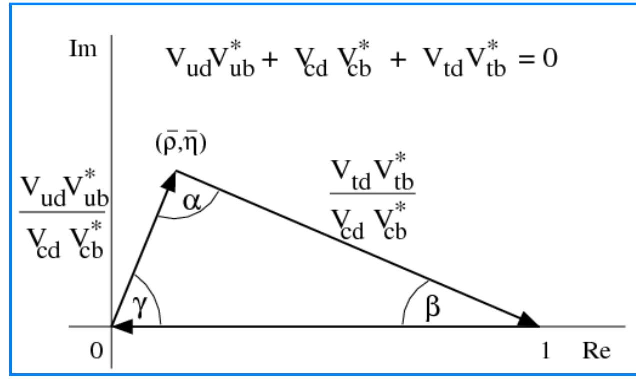
$$V_{tb}V_{ub}^* + V_{ts}V_{us}^* + V_{td}V_{ud}^* = 0$$

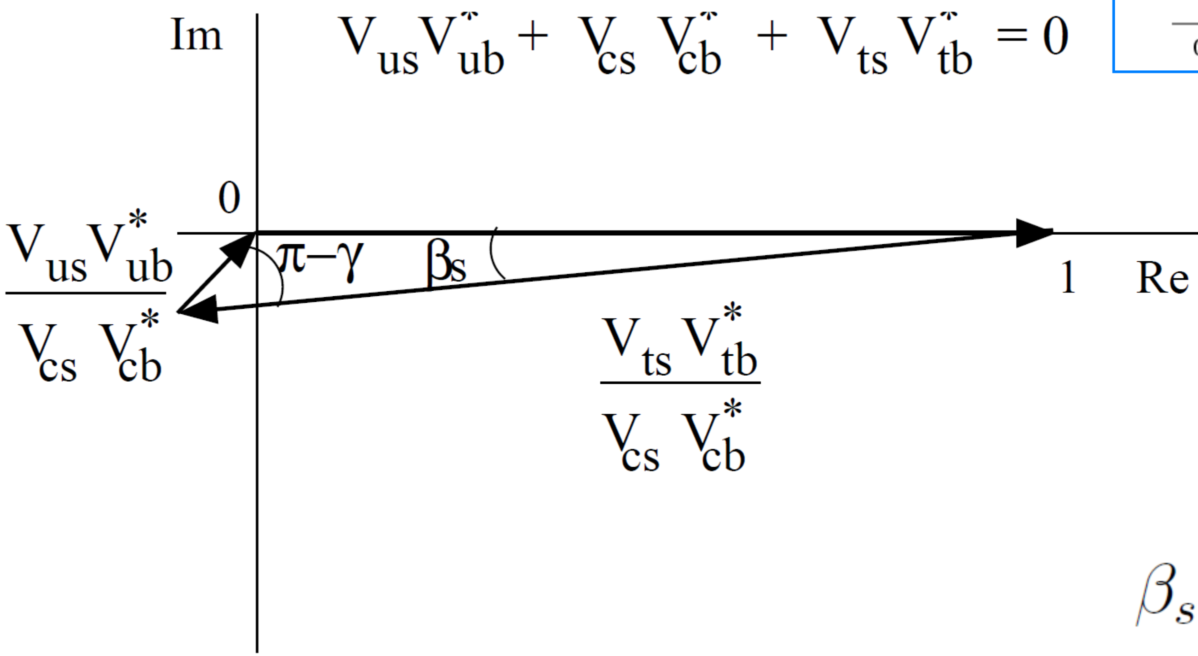


- NB: angle β_s introduced. But… not phase invariant definition!?

The "B_s-triangle": β_s

- Replace *d* by *s*:





$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right]$$

The phases in the Wolfenstein parameterization

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right] \qquad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] \qquad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right] \qquad \beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right]$$

$$\begin{aligned}
\beta &\approx \quad \pi + \arg(V_{cd}V_{cb}^*) - \arg(V_{td}V_{tb}^*) = \pi + \pi - \arg(V_{td}) &&= -\arg(V_{td}) \\
\gamma &\approx \quad \pi + \arg(V_{ud}V_{ub}^*) - \arg(V_{cd}V_{cb}^*) = \pi - \arg(V_{ub}) - \pi &&= -\arg(V_{ub}) \\
\beta_s &\approx \quad \pi + \arg(V_{ts}V_{tb}^*) - \arg(V_{cs}V_{cb}^*) = \pi + \arg(V_{ts}) - 0 &&= \arg(V_{ts}) + \pi
\end{aligned}$$

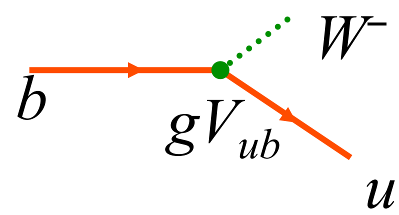
Alternatively, the Wolfenstein phase convention in the CKM-matrix elements can be shown as:

$$V_{CKM,\mathrm{Wolfenstein}} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5) \qquad (2.16)$$

The CKM matrix

- *Couplings* of the charged current:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



- Wolfenstein *parametrization*:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

- Magnitude:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97428 & 0.2253 & 0.00347 \\ 0.2252 & 0.97345 & 0.0410 \\ 0.00862 & 0.0403 & 0.999152 \end{pmatrix}$$

- Complex phases:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$

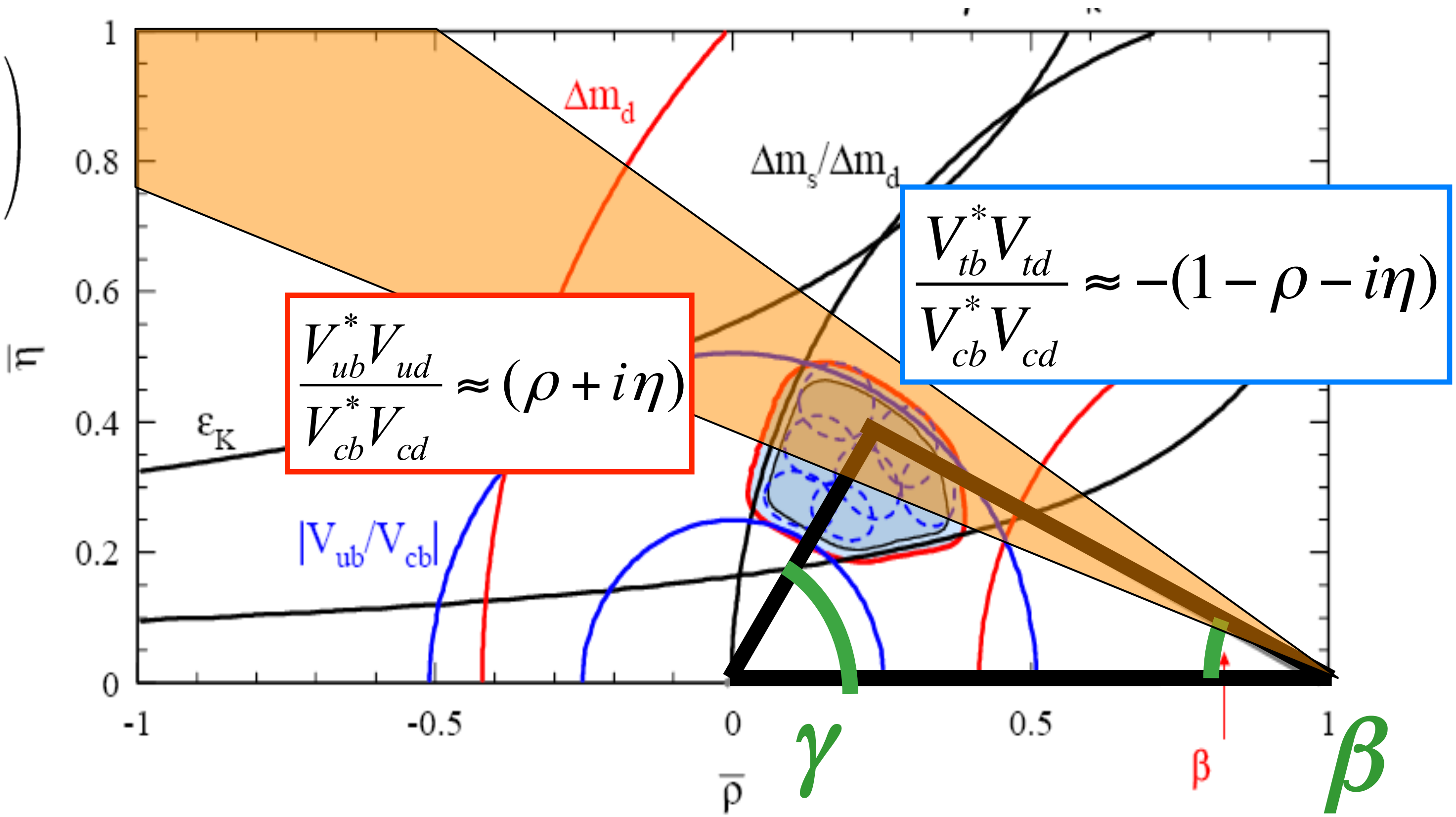
Back to finding new measurements

- Next order of business: Devise an experiment that measures $\arg(V_{td}) \equiv \beta$ and $\arg(V_{ub}) \equiv \gamma$.
  - What will such a measurement look like in the (ρ,η) plane?

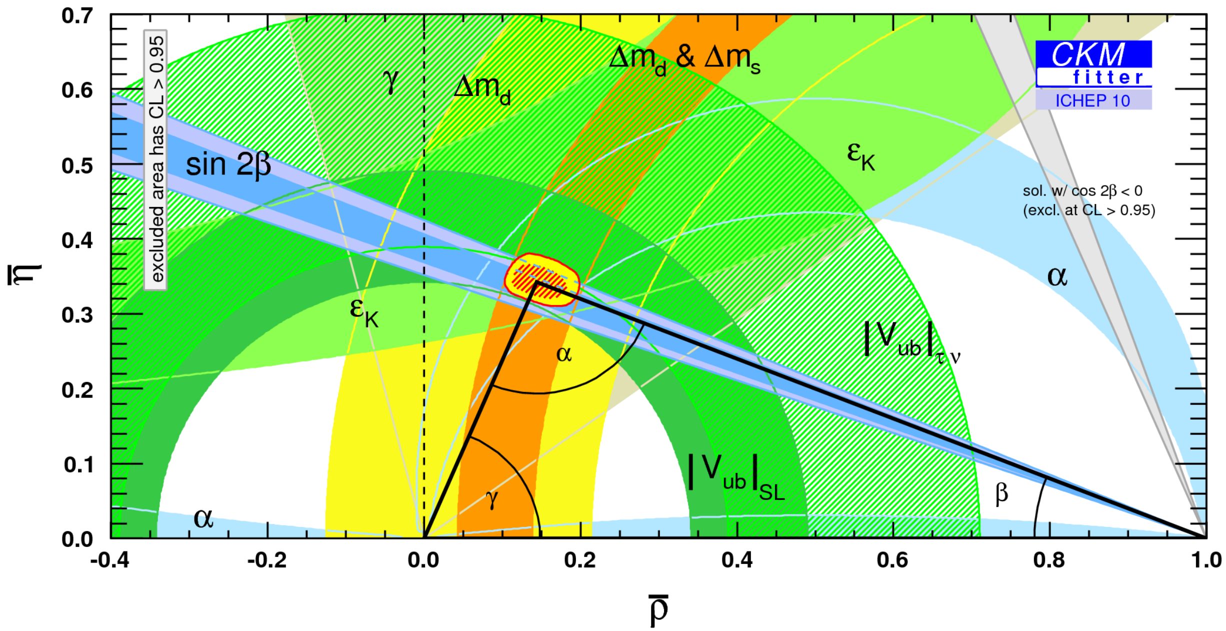
CKM phases

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$

Fictitious measurement of β consistent with CKM model

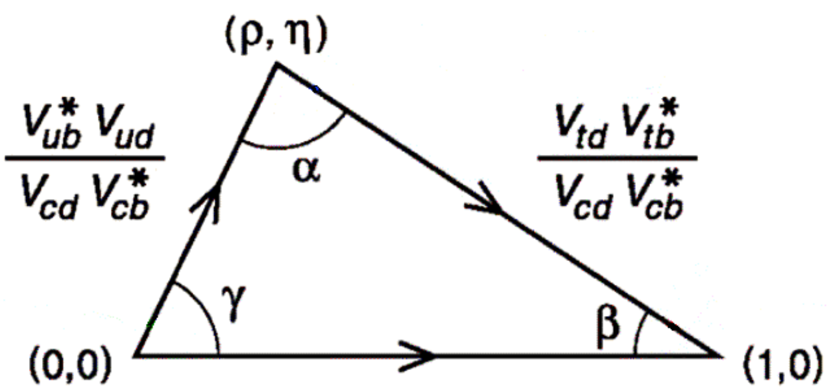


Consistency with other measurements in (ρ,η) plane

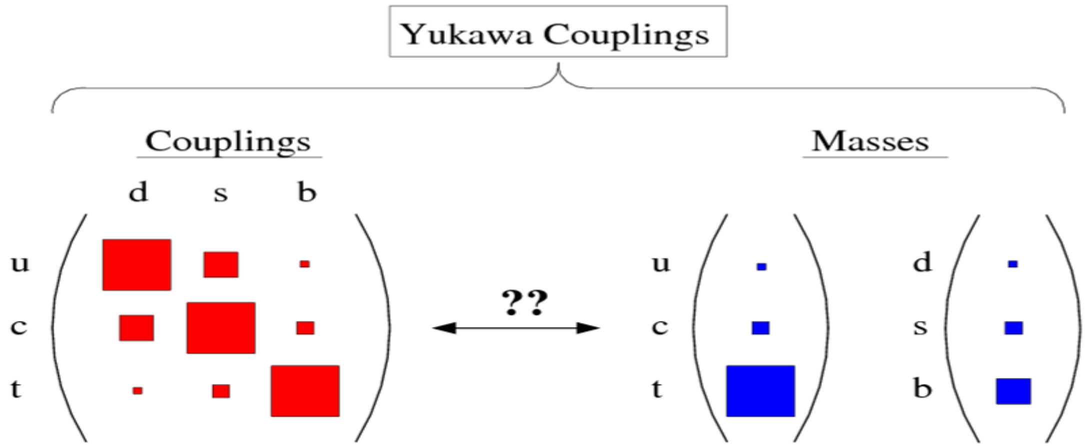


Precise measurement of sin(2β) agrees perfectly with other measurements and CKM model assumptions

The CKM model of CP violation experimentally confirmed with high precision!



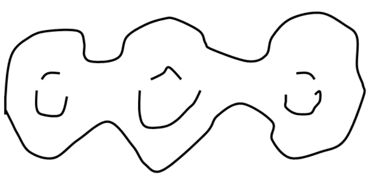
What's going on??



- ??? Edward Witten, 17 Feb 2009…

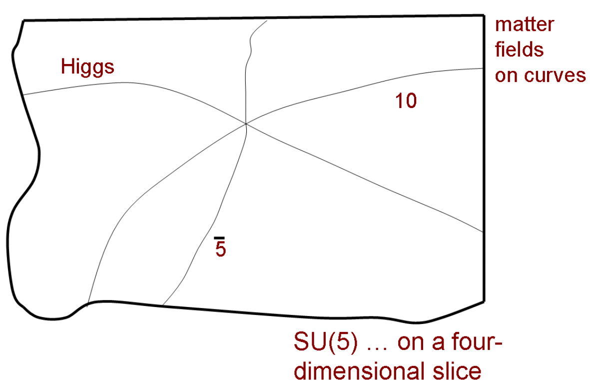
In 2004, Time magazine stated that Witten was widely thought to be the world's greatest living theoretical physicist.

In this approach, the ordinary Higgs field is a wavefunction on K, as are the quark and lepton fields



Quark and lepton masses and the CKM matrix are determined by the overlaps of these wavefunctions.

The picture is a little like this:



Higgs fields and quarks and leptons are supported on the three curves, and the Yukawa couplings that gives masses to down quarks and charged leptons come from the intersection drawn. (Up quark masses come from a similar intersection.)

In the leading approximation, only one particle of each type (i.e. the third generation particles – top, bottom, tau) get masses. The others have wavefunctions that vanish at the intersection point.

- See "*From F-Theory GUT's to the LHC*" by Heckman and Vafa (arXiv:0809.3452)