Particle Physics II – CP violation

Lecture 1

N. Tuning

Plan

1) Wed 12 Feb: Anti-matter + SM
2) Mon 17 Feb: CKM matrix + Unitarity Triangle
3) Wed 19 Feb: Mixing + Master eqs. + $B^0 \rightarrow J/\psi K_s$
4) Mon 20 Feb: CP violation in $B_{(s)}$ decays (I)
5) Wed 9 Mar: CP violation in $B_{(s)}$ and K decays (II)
6) Mon 16 Mar: Rare decays + Flavour Anomalies
7) Wed 18 Mar: Exam

- Final Mark:
  - if (mark > 5.5) mark = max(exam, 0.85*exam + 0.15*homework)
  - else mark = exam

- In parallel: Lectures on Flavour Physics by prof.dr. R. Fleischer

Plan

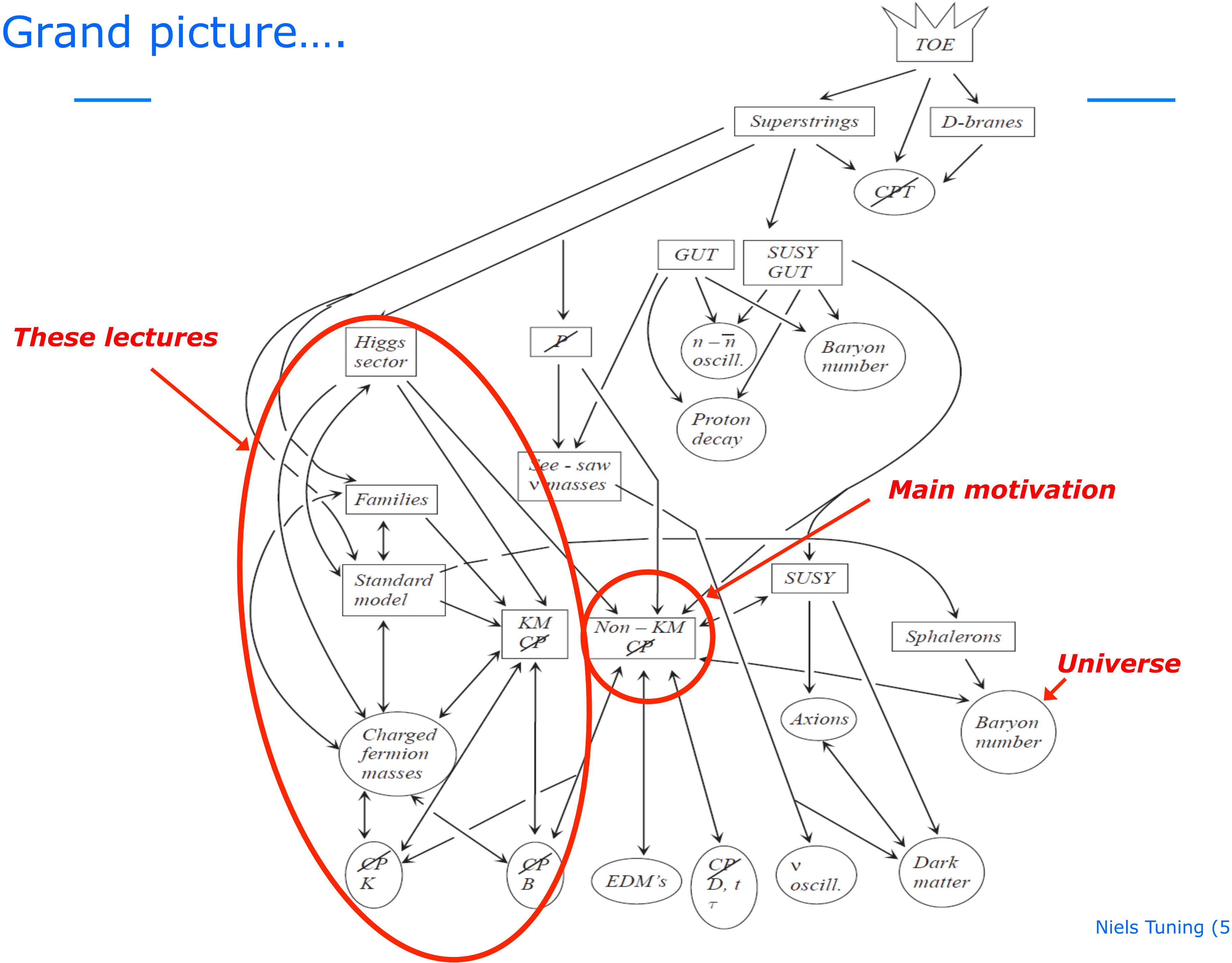
- 2 x 60 min (with break)

- Monday
  - Start: 13:15
  - End: 15:15
  - Werkcollege: 15:15 – 16:15

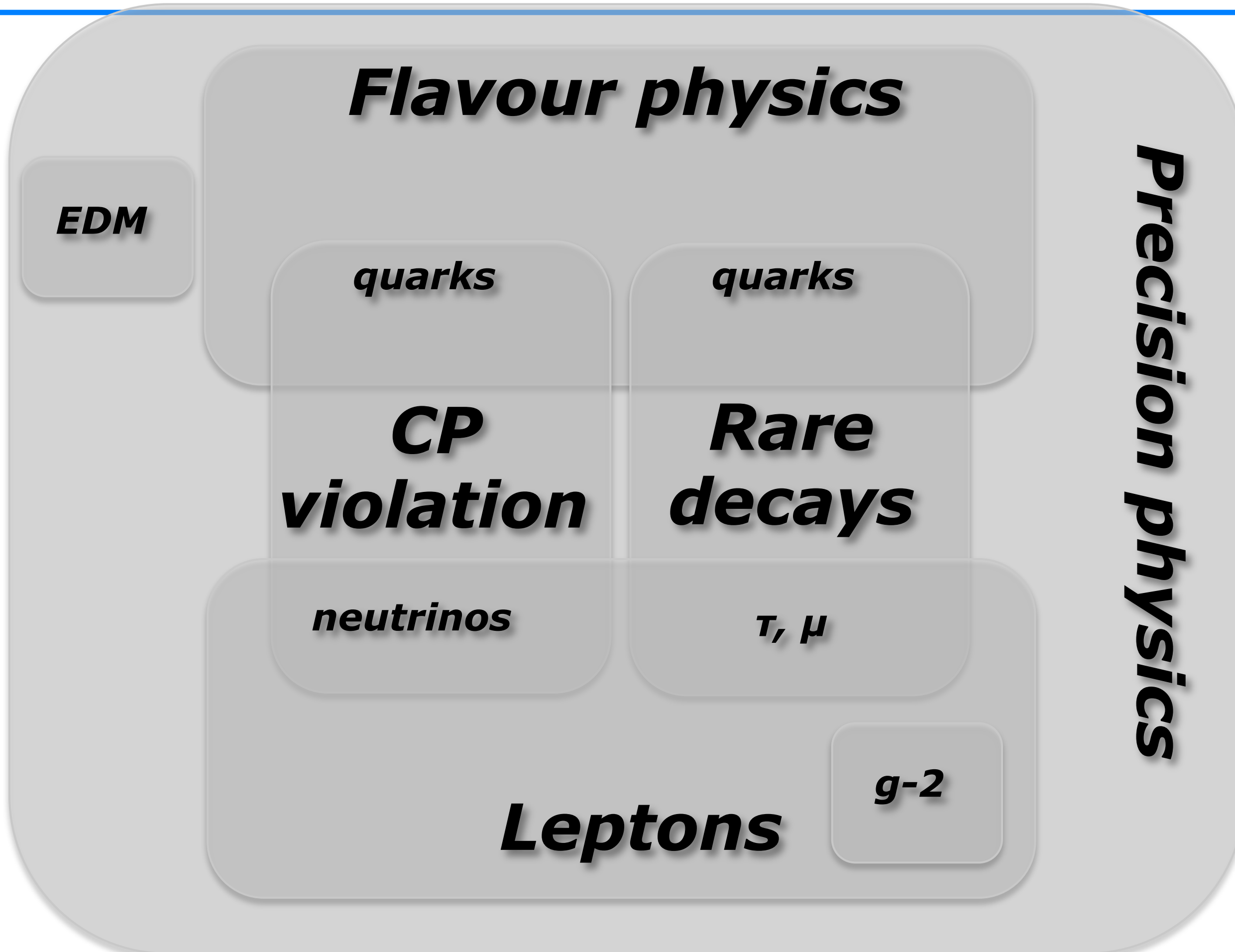
- Wednesday:
  - Start: 9:00
  - End: 11:00
  - Werkcollege: 11:00 – 12:00

Table of contents

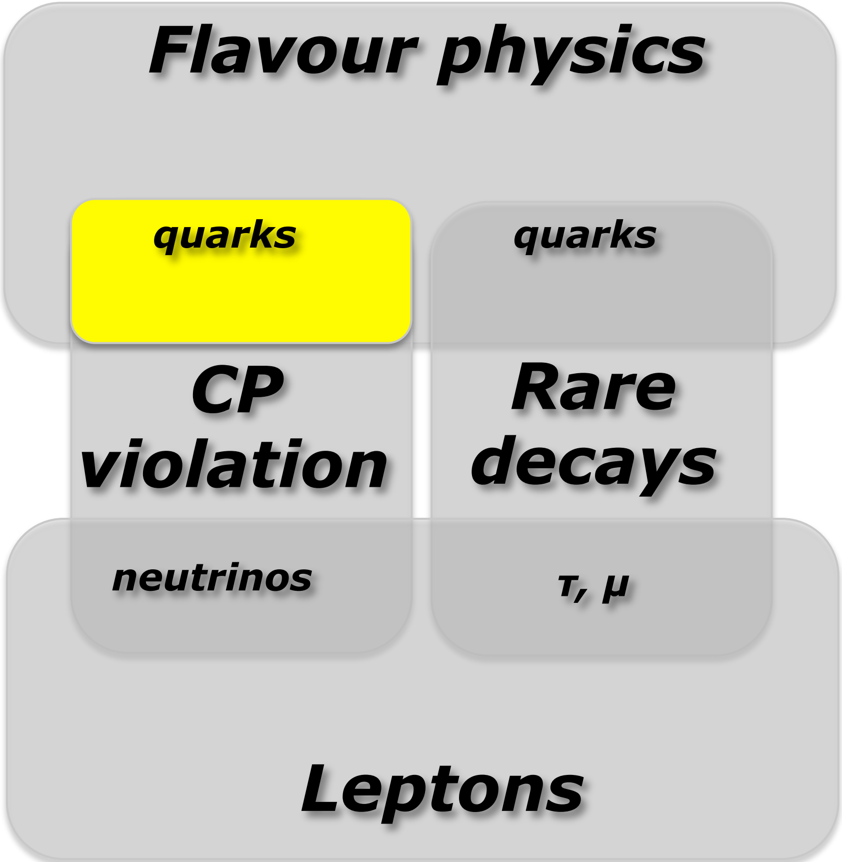
Grand picture….



Jargon


Flavour physics
EDM
quarks
quarks
CP violation
Rare decays
neutrinos
τ, μ
g-2
Leptons
Precision physics

Today



Flavour physics
quarks
quarks
CP violation
Rare decays
neutrinos
τ, μ
Leptons

Flavour physics has a track record…

GIM mechanism in $K^0 \to \mu\mu$

Weak Interactions with Lepton-Hadron Symmetry*

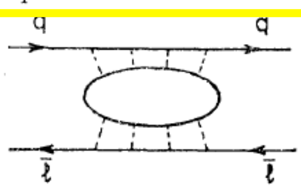
S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Milis theory is discussed.

splitting, beginning at order $G(G\Lambda^2)$, as well as contributions to such unobserved decay modes as $K_2 \to \mu^+ + \mu^-$, $K^+ \to \pi^+ + l + \bar{l}$, etc., involving neutral lepton

We wish to propose a simple model in which the divergences are properly ordered. Our model is founded in a quark model, but one involving four, not three, fundamental fermions; the weak interactions are medi-

new quantum number $\mathcal{C}$ for charm.



Glashow, Iliopoulos, Maiani, Phys.Rev. D2 (1970) 1285

CP violation, $K_L^0 \to \pi\pi$

27 July 1964

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

This Letter reports the results of experimental studies designed to search for the 2π decay of the K_2^0 meson. Several previous experiments have

three-body decays of the K_2^0. The presence of a two-pion decay mode implies that the K_2^0 meson is not a pure eigenstate of CP. Expressed as $K_2^0 = 2^{-1/2}[(K_0 - \bar{K}_0) + \epsilon(K_0 + \bar{K}_0)]$ then $|\epsilon|^2 \cong R_T \tau_1 \tau_2$

Christenson, Cronin, Fitch, Turlay, Phys.Lett. 13 (1964) 138-140

$B^0 \leftrightarrow \bar{B}^0$ mixing

DESY 87-029
April 1987

OBSERVATION OF B^0-$\bar{B}^0$ MIXING

The ARGUS Collaboration

In summary, the combined evidence of the investigation of B^0 meson pairs, lepton pairs and B^0 meson-lepton events on the $\Upsilon(4S)$ leads to the conclusion that B^0-$\bar{B}^0$ mixing has been observed and is substantial.

Parameters	Comments
$r > 0.09$ 90% CL	This experiment
$x > 0.44$	This experiment
$B^{\frac{1}{2}} f_B \approx f_\pi < 160$ MeV	B meson ($\approx$ pion) decay constant
$m_b < 5\,\mathrm{GeV}/c^2$	b-quark mass
$\tau_b < 1.4 \cdot 10^{-12}$ s	B meson lifetime
$\lvert V_{td}\rvert < 0.018$	Kobayashi-Maskawa matrix element
$\eta_{QCD} < 0.86$	QCD correction factor [17]
$m_t > 50\,\mathrm{GeV}/c^2$	t quark mass

ARGUS Coll. Phys.Lett.B192:245,1987

Flavour physics has a track record…

Rare decay implies 2nd up quark "discovery" of charm?

Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed o… fields and interact with a charged massive vector boson. We show, to all orders i… that the leading divergences do not violate any strong-interaction symmetry an… divergences respect all observed weak-interaction selection rules. The model featu… between leptons and quarks. The extension of our model to a complete Yan…

splitting, beginning at order $G(G\Lambda^2)$ …on- tributions to such unobserved de… $K_2 \to \mu^+ + \mu^-$, $K^+ \to \pi^+ + l + \bar{l}$, etc., inv… lepton

We wish to propose a si… which the divergences are properly or… …el is founded in a quark model, but o… …ur, not three, fundamental fermions; … …ctions are medi-

new quantu… …r charm.

CP violation implies 3rd family: "discovery" of bottom?

27 Ju…

EVIDENCE FOR THE 2π DECAY OF THE K_2^0

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch…
Princeton University, Princeton, Ne…
(Received 10 July 1964)

This Letter reports the re… …imental studies designed to search … …ay of the K_2^0 meson. Several pre… …nts have

three-body dec… The presence of a two-pion deca… …s that the K_2^0 meson is not a pur… …CP. Expressed as $K_2^0 = 2^{-1/2}[$… $+\bar{K}_0)]$ then $|\epsilon|^2 \cong R_T \tau_1 \tau_2$

Mixing implies heavy quark: "discovery" of top?

…Y 87-029
…pril 1987

OBSERVATION OF B^0…

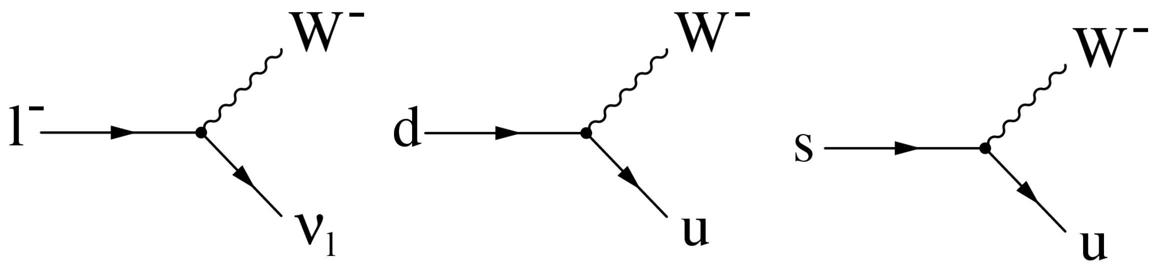
The ARGUS Co…

In summary, the combined evidence of the investigatio… …pairs and B^0 meson-lepton events on the $\Upsilon(4S)$ leads to th… …ixing has been observed and is substantial.

Parameters	
$r > 0.09$ 90%CL	…ment
$x > 0.44$	…eriment
$B^{\frac{1}{2}} f_B \approx f_\pi < 160$ Me…	…on (≈ pion) decay constant
$m_b < 5\mathrm{GeV}/c^2$	… quark mass
$\tau_b < 1.4 \cdot 10^{-12}$	B meson lifetime
$\lvert V_{td} \rvert < 0.01$…	Kobayashi-Maskawa matrix element
$\eta_{QCD} <$ …	QCD correction factor [17]
$m_t >$ …	t quark mass

Introduction: it's all about the charged current

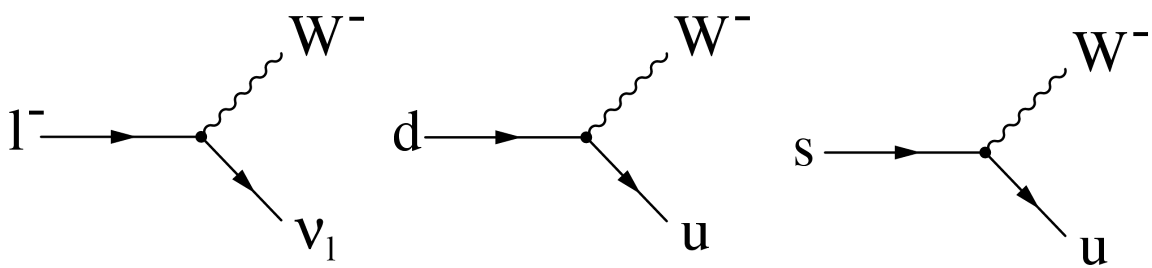
- "CP violation" is about the <u>weak interactions,</u>
- In particular, the charged current interactions:



- The interesting stuff happens in the interaction with quarks
- Therefore, people also refer to this field as "<u>flavour physics</u>"

Motivation 1: Understanding the Standard Model

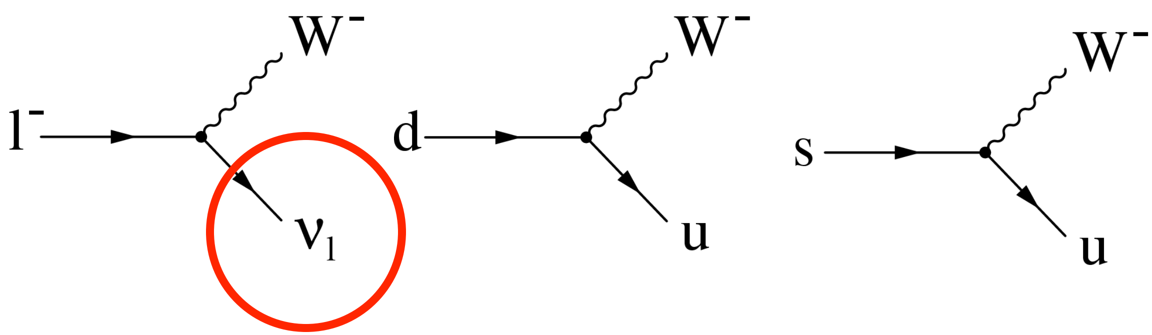
- "CP violation" is about the <u>weak interactions,</u>
- In particular, the charged current interactions:



- Quarks can only change flavour through charged current interactions

Introduction: it's all about the charged current

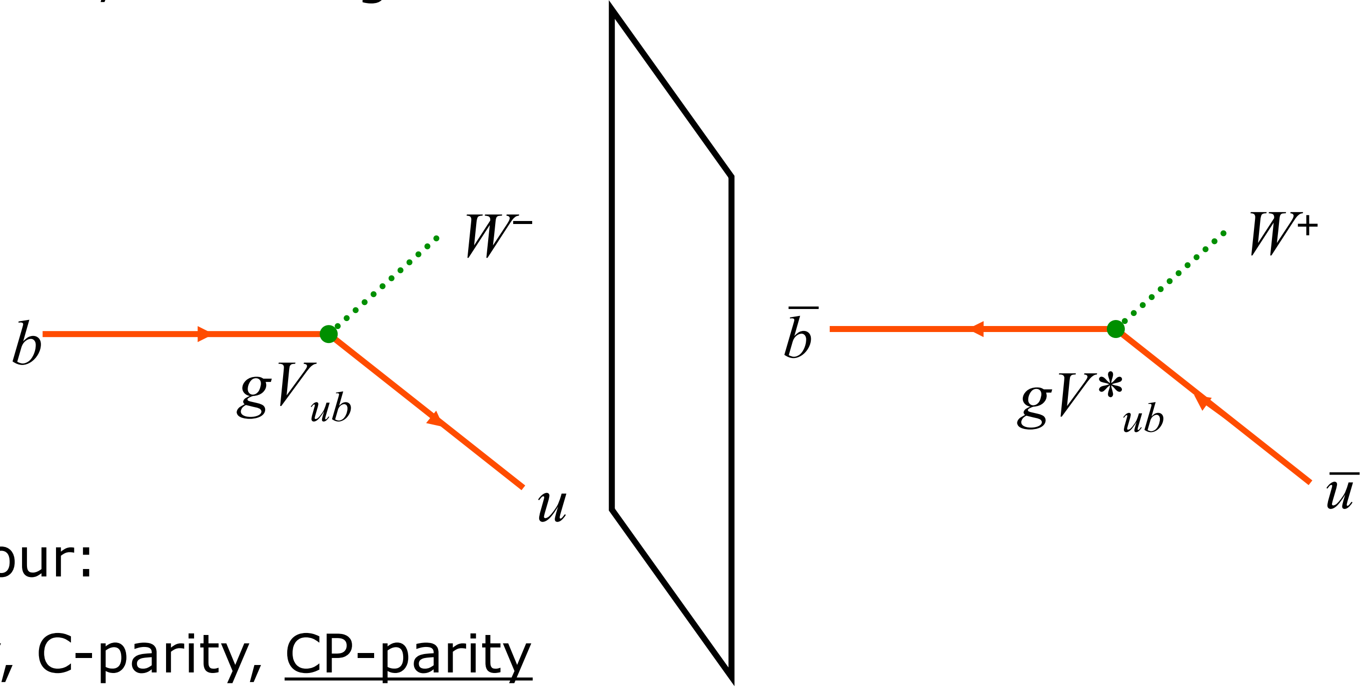
- "CP violation" is about the weak interactions,
- In particular, the charged current interactions:



- In 1st hour:
- P-parity, C-parity, CP-parity
- → the neutrino shows that P-parity is maximally violated

Introduction: it's all about the charged current

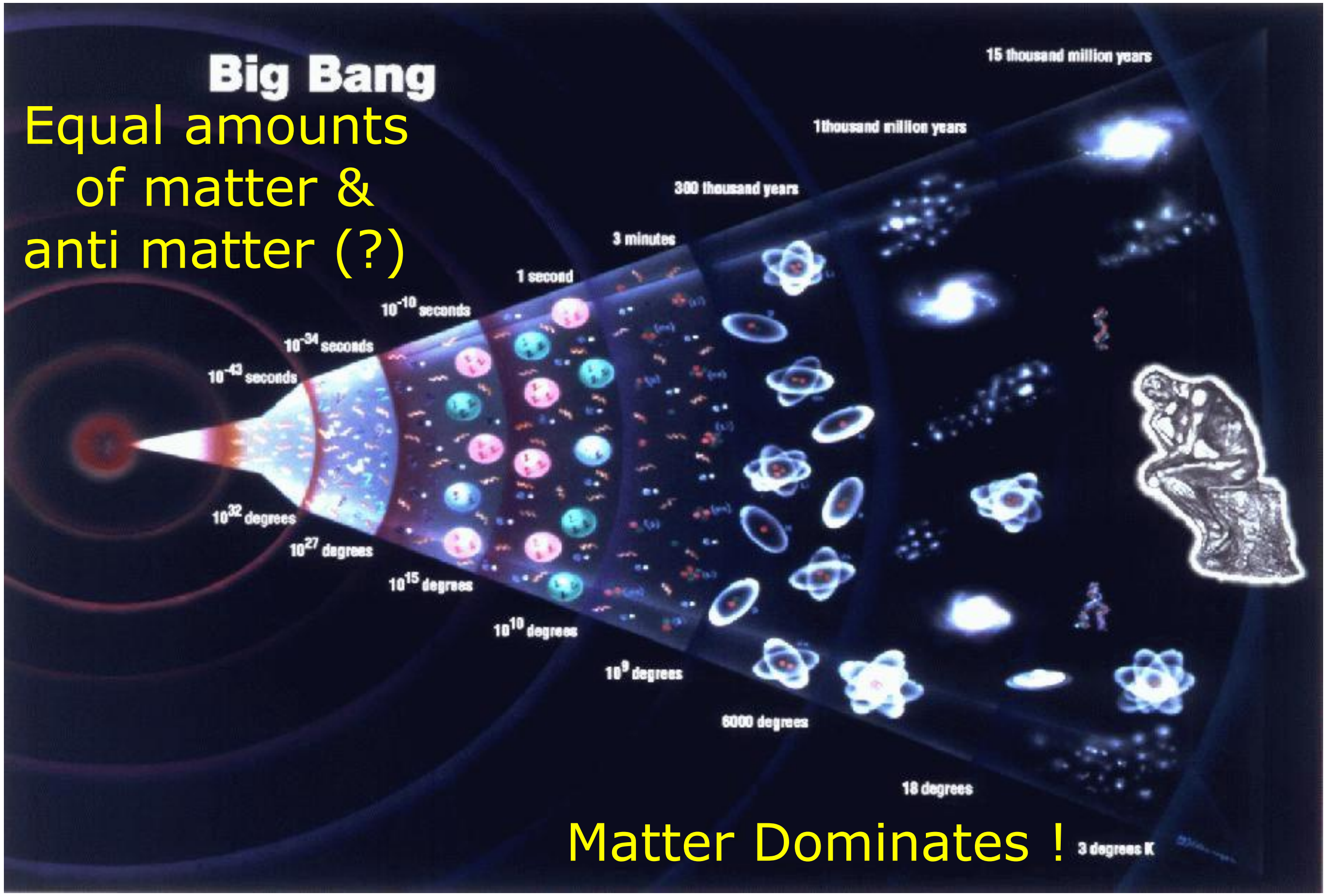
- "CP violation" is about the <u>weak interactions,</u>
- In particular, the charged current interactions:



- In 1st hour:
- P-parity, C-parity, <u>CP-parity</u>
- → Symmetry related to particle – anti-particle

Motivation 2: Understanding the universe

- It's about differences in matter and anti-matter
  - Why would they be different in the first place?
  - We *see* they are different: our universe is matter dominated



Where and how do we generate the Baryon asymmetry?

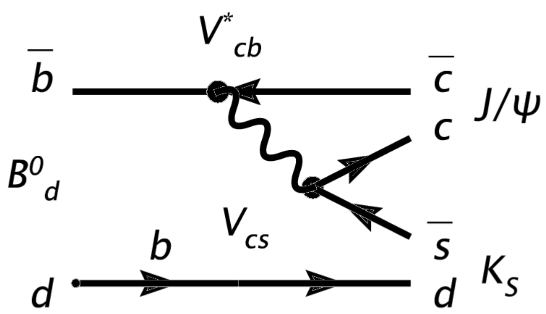
- No definitive answer to this question yet!

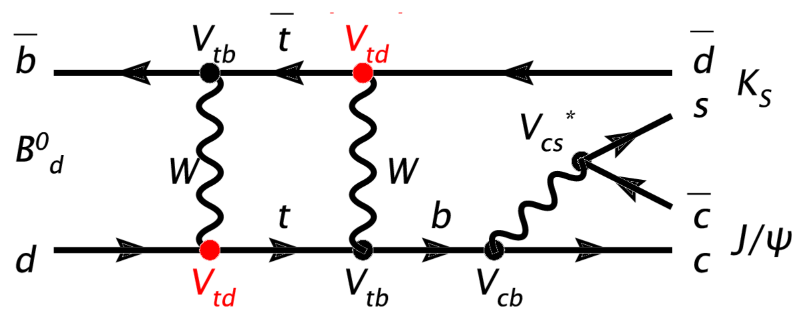
- In 1967 A. Sacharov formulated a set of general conditions that any such mechanism has to meet
  1) You need a process that violates the baryon number B: (Baryon number of matter=1, of anti-matter = -1)
  2) Both C and CP symmetries should be violated
  3) Conditions 1) and 2) should occur during a phase in which there is *no* thermal equilibrium

- In these lectures we will focus on 2): CP violation
- Apart from cosmological considerations, I will convince you that there are more interesting aspects in CP violation

Introduction: it's all about the charged current

- "CP violation" is about the <u>weak interactions,</u>
- In particular, the charged current interactions:



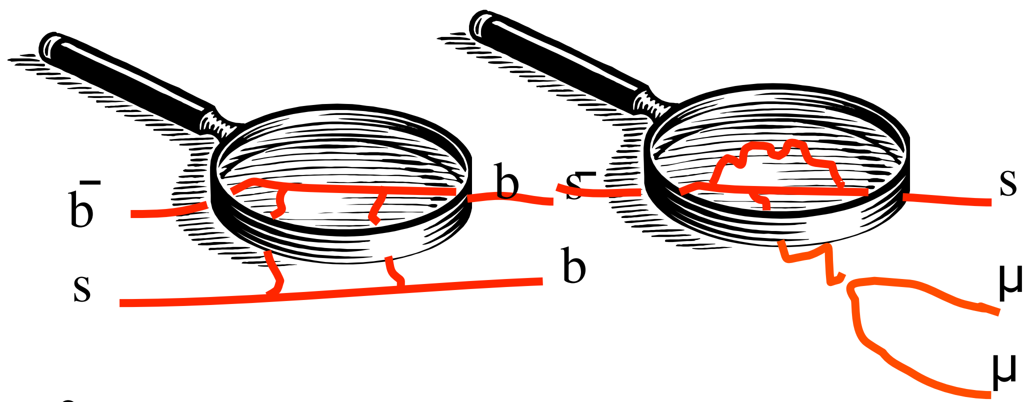


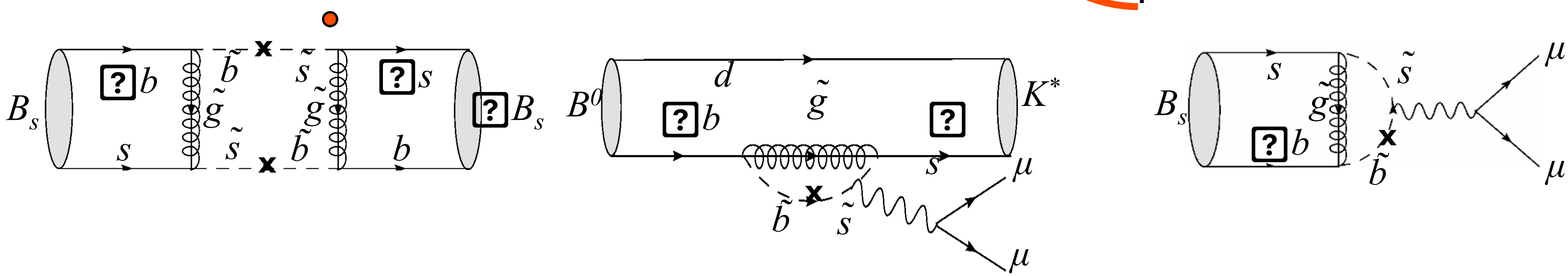
- Same initial and final state
- Look at interference between $B^0 \rightarrow f_{CP}$ and $B^0 \rightarrow \overline{B^0} \rightarrow f_{CP}$

Motivation 3: Sensitive to find new physics

- "CP violation" is about the weak interactions,
- In particular, the charged current interactions:

"Box" diagram: ΔB=2 *"Penguin" diagram: ΔB=1*

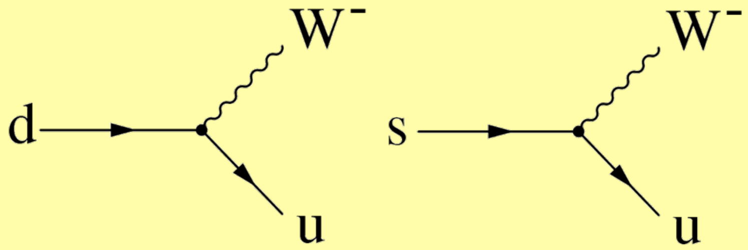




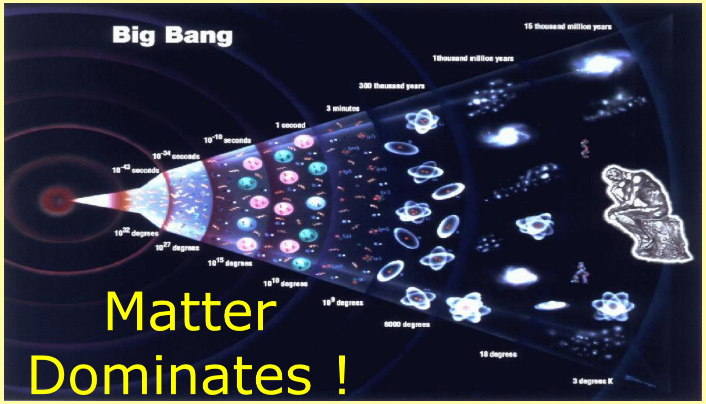
- Are heavy particles running around in loops?

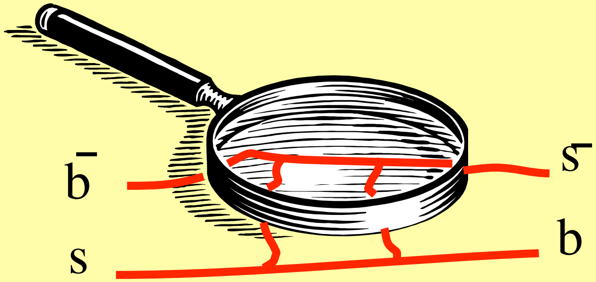
Recap:

- CP-violation (or flavour physics) is about charged current interactions

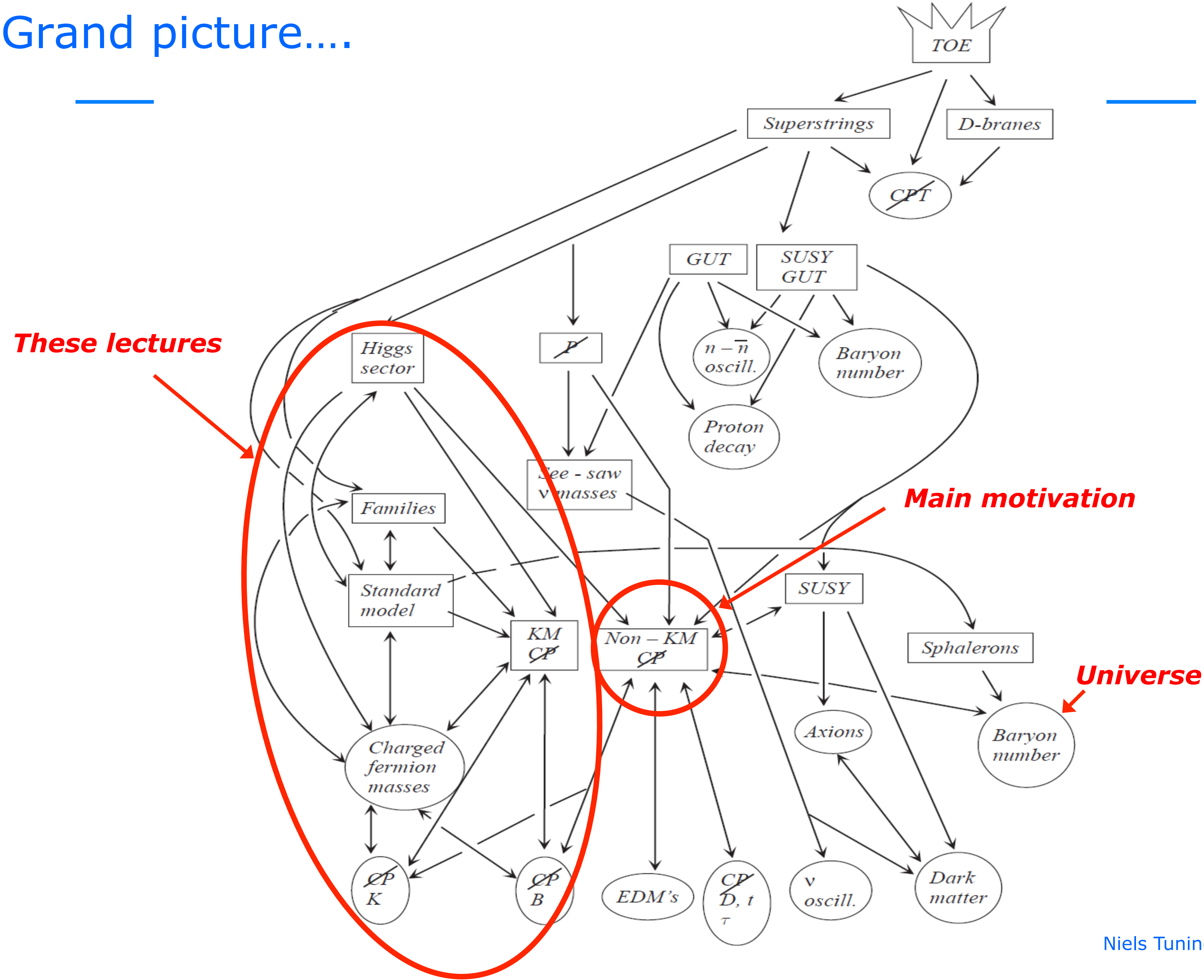


- Interesting because:
  1) <u>Standard Model:</u>
     in the heart of quark interactions
  2) <u>Cosmology:</u>
     related to matter – anti-matter asymetry
  3) <u>Beyond Standard Model:</u>
     measurements are sensitive to new particles





Grand picture….



Personal impression:

- People think it is a complicated part of the Standard Model (me too:-). Why?

1) Non-intuitive concepts?

   - *Imaginary phase* in transition amplitude, T ~ $e^{i\varphi}$
   - *Different bases* to express quark states, d' =0.97 d + 0.22 s + 0.003 b
   - *Oscillations* (mixing) of mesons: $|K^0\rangle \leftrightarrow |\boxed{?}K^0\rangle$

2) Complicated calculations?

$$\Gamma\left(B^0 \rightarrow f\right) \propto \left|A_f\right|^2 \left[\left|g_+(t)\right|^2 + |\lambda|^2 \left|g_-(t)\right|^2 + 2\Re\left(\lambda g_+^*(t) g_-(t)\right)\right]$$

$$\Gamma\left(\overline{B}^0 \rightarrow f\right) \propto \left|\overline{A}_f\right|^2 \left[\left|g_+(t)\right|^2 + \frac{1}{|\lambda|^2} \left|g_-(t)\right|^2 + \frac{2}{|\lambda|^2}\Re\left(\lambda^* g_+^*(t) g_-(t)\right)\right]$$

3) Many decay modes? *"Beetopaipaigamma…"*

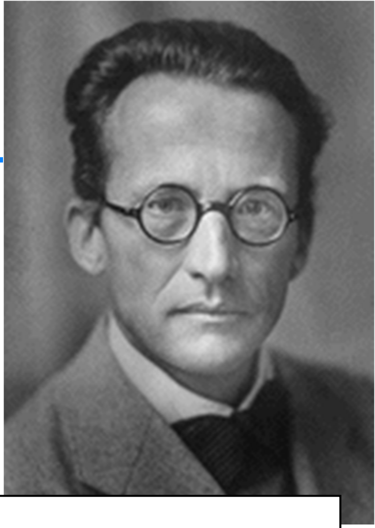
   – PDG reports 347 decay modes of the B^0-meson:
      - Γ_1 $l^+ \nu_l$ *anything* (10.33 ± 0.28) × 10^{-2}
      - Γ_{347} $\nu \nu \gamma$ $<4.7 \times 10^{-5}$ *CL=90%*

   – And for one decay there are often more than one decay *amplitudes*…

Anti-matter

- Dirac (1928): Prediction
- Anderson (1932): Discovery
- Present-day experiments

Schrödinger



Classic relation between E and p:

$$E = \frac{\vec{p}^2}{2m}$$

Quantum mechanical substitution:

(operator acting on wave function ψ)

$$E \to i\frac{\partial}{\partial t} \qquad \text{and} \qquad \vec{p} \to -i\vec{\nabla}$$

Schrodinger equation:

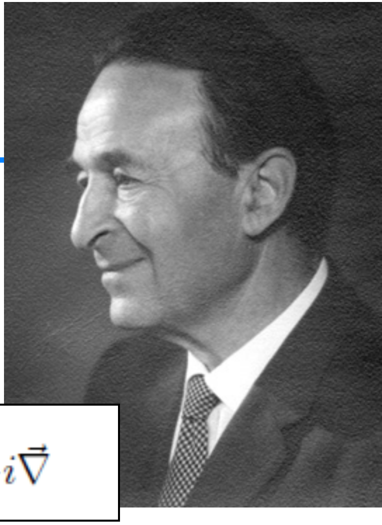
$$i\frac{\partial}{\partial t}\psi = \frac{-1}{2m}\nabla^2\psi$$

Solution:

$$\psi = N\, e^{i(\vec{p}\vec{x} - Et)}$$

(show it is a solution)

Klein-Gordon



Relativistic relation between E and p: $E^2 = \vec{p}^2 + m^2$

Quantum mechanical substitution:
(operator acting on wave function ψ)

$$E \rightarrow i\frac{\partial}{\partial t} \qquad \text{and} \qquad \vec{p} \rightarrow -i\vec{\nabla}$$

Klein-Gordon equation:

$$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\,\phi$$

$$\text{or} : \quad \left(\Box + m^2\right)\,\phi(x) = 0$$

$$\text{or} : \quad \left(\partial_\mu\partial^\mu + m^2\right)\,\phi(x) = 0$$

Solution: $\phi(x) = N\,e^{-ip_\mu x^\mu}$ with eigenvalues: $E^2 = \vec{p}^2 + m^2$

But! Negative energy solution?

$$E = \pm\sqrt{\vec{p}^2 + m^2}$$

Dirac

Paul Dirac tried to find an equation that was

- relativistically correct,
- but *linear* in d/dt to avoid negative energies
- (and linear in d/dx (or ∇) for Lorentz covariance)

He found an equation that

- turned out to describe spin-1/2 particles and
- **predicted the existence of anti-particles**

Dirac

➢ How to find that relativistic, linear equation ??

Write Hamiltonian in general form, $$\boxed{H\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\ \psi}$$

but when squared, it must satisfy: $$\boxed{H^2\psi = \left(\vec{p}^2 + m^2\right)\ \psi}$$

Let's find α_i and β !

$$\boxed{\begin{aligned} H^2\psi &= (\alpha_i p_i + \beta m)^2\ \psi \qquad\qquad \text{with}:\ \ i = 1,2,3 \\ &= \left(\underbrace{\alpha_i^2}_{=1}\ p_i^2 + \underbrace{(\alpha_i\alpha_j + \alpha_j\alpha_i)}_{=0\quad i>j}\ p_i p_j + \underbrace{(\alpha_i\beta + \beta\alpha_i)}_{=0}\ p_i m + \underbrace{\beta^2}_{=1}\ m^2\right)\ \psi \end{aligned}}$$

So, α_i and β must satisfy:

- $\alpha_1{}^2 = \alpha_2{}^2 = \alpha_3{}^2 = \beta^2$
- $\alpha_1, \alpha_2, \alpha_3, \beta$ anti-commute with each other
- (not a unique choice!)

$$H\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\ \psi$$

Dirac

➢ What are α and β ??

The lowest dimensional matrix that has the desired behaviour is **4x4** !?

Often used

Pauli-Dirac representation:

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \quad ; \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

with:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ; \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad ; \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

So, α_i and β must satisfy:

- $\alpha_1{}^2 = \alpha_2{}^2 = \alpha_3{}^2 = \beta^2$
- $\alpha_1, \alpha_2, \alpha_3, \beta$ anti-commute with each other
- (not a unique choice!)

Dirac

$$H\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\ \psi$$

Usual substitution: $H \to i\frac{\partial}{\partial t},\ \vec{p} \to -i\vec{\nabla}$

Leads to: $$i\frac{\partial}{\partial t}\psi = \left(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m\right)\ \psi$$

Multiply by β: $$\left(i\beta\frac{\partial}{\partial t}\psi \quad + \quad i\beta\alpha_1\frac{\partial}{\partial x} + i\beta\alpha_2\frac{\partial}{\partial y} + i\beta\alpha_3\frac{\partial}{\partial z}\right)\ \psi \overset{(\beta^2=\mathbf{1})}{-} m\psi = 0$$

Gives the famous Dirac equation:

$$\begin{aligned} (i\gamma^\mu\partial_\mu - m)\ \psi \quad &= \quad 0 \\ \text{with}:\ \gamma^\mu = (\beta, \beta\vec{\alpha}) \quad &\equiv \quad \text{Dirac } \gamma\text{−matrices} \end{aligned}$$

$$\begin{matrix}\text{for each} \\ \text{j=1,2,3,4}\end{matrix} \quad : \quad \sum_{k=1}^{4}\left[\sum_{\mu=0}^{3} i\,(\gamma^\mu)_{jk}\ \partial_\mu - m\delta_{jk}\right](\psi_k) = 0$$

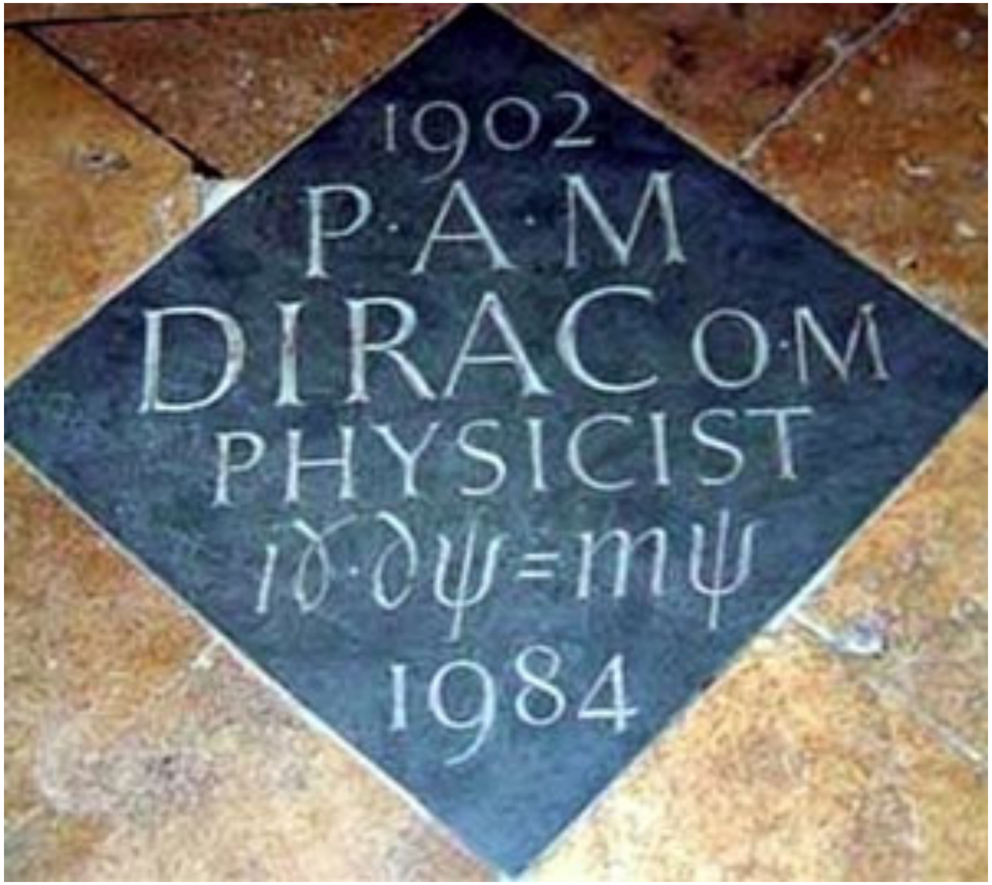
Dirac

$$H\psi = (\vec{\alpha} \cdot \vec{p} + \beta m)\ \psi$$

The famous Dirac equation:

$$\begin{aligned} (i\gamma^\mu \partial_\mu - m)\ \psi &= 0 \\ \text{with}: \quad \gamma^\mu = (\beta, \beta\vec{\alpha}) &\equiv \text{Dirac } \gamma\text{–matrices} \end{aligned}$$

R.I.P. :



$$H\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\ \psi$$

Dirac

The famous Dirac equation:

$$\begin{aligned}(i\gamma^\mu \partial_\mu - m)\ \psi &= 0 \\ \text{with}:\ \gamma^\mu = (\beta, \beta\vec{\alpha}) &\equiv \text{Dirac } \gamma\text{−matrices}\end{aligned}$$

Remember!

- μ : <u>Lorentz index</u>
- 4x4 γ matrix: <u>Dirac index</u>

Less compact notation:

$$\text{for each j=1,2,3,4} \quad : \quad \sum_{k=1}^{4}\left[\sum_{\mu=0}^{3} i\,(\gamma^\mu)_{jk}\ \partial_\mu - m\delta_{jk}\right](\psi_k) = 0$$

Even less compact… :

$$\left[\begin{pmatrix} 1\!\!1 & 0 \\ 0 & -1\!\!1 \end{pmatrix}\frac{i\partial}{\partial t} + \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}\frac{i\partial}{\partial x} + \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}\frac{i\partial}{\partial y} + \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}\frac{i\partial}{\partial z} - \begin{pmatrix} 1\!\!1 & 0 \\ 0 & 1\!\!1 \end{pmatrix} m\right]\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

➢ What are the solutions for ψ ??

Dirac

$$H\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\ \psi$$

The famous Dirac equation:

$$\begin{aligned} (i\gamma^\mu \partial_\mu - m)\ \psi &= 0 \\ \text{with} :\ \gamma^\mu = (\beta, \beta\vec{\alpha}) &\equiv \text{Dirac } \gamma\text{−matrices} \end{aligned}$$

Solutions to the Dirac equation?

Try plane wave: $\psi(x) = u(p)\, e^{-ipx}$ →

$$\begin{aligned} (\gamma^\mu p_\mu - m)\ u(p) &= 0 \\ \text{or} :\ (\not{p} - m)\ u(p) &= 0 \end{aligned}$$

Linear set of eq:

$$\left[\begin{pmatrix} 1\!\!1 & 0 \\ 0 & -1\!\!1 \end{pmatrix} E - \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} p^i - \begin{pmatrix} 1\!\!1 & 0 \\ 0 & 1\!\!1 \end{pmatrix} m \right] \begin{pmatrix} u_A \\ u_B \end{pmatrix} = 0$$

- 2 coupled equations:

$$\begin{cases} (\vec{\sigma}\cdot\vec{p})\ u_B = (E - m)\, u_A \\ (\vec{\sigma}\cdot\vec{p})\ u_A = (E + m)\, u_B \end{cases}$$

If p=0:

$$u^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},\quad u^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},\quad u^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},\quad u^{(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Dirac

$$H\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\ \psi$$

The famous Dirac equation:

$$\begin{aligned} (i\gamma^\mu\partial_\mu - m)\ \psi &= 0 \\ \text{with}:\ \gamma^\mu = (\beta, \beta\vec{\alpha}) &\equiv \text{Dirac } \gamma\text{-matrices} \end{aligned}$$

Solutions to the Dirac equation?

Try plane wave: $\psi(x) = u(p)\, e^{-ipx}$ →

$$\begin{aligned} (\gamma^\mu p_\mu - m)\ u(p) &= 0 \\ \text{or}:\ (\not{p} - m)\ u(p) &= 0 \end{aligned}$$

- 2 coupled equations:

$$\begin{cases} (\vec{\sigma}\cdot\vec{p})\ u_B = (E-m)\, u_A \\ (\vec{\sigma}\cdot\vec{p})\ u_A = (E+m)\, u_B \end{cases}$$

If p≠0:

Two solutions for E>0:

(and two for E<0)

$$u^{(1)} = \begin{pmatrix} u_A^{(1)} \\ u_B^{(1)} \end{pmatrix} \quad , \quad u^{(2)} = \begin{pmatrix} u_A^{(2)} \\ u_B^{(2)} \end{pmatrix}$$

with:

$$u_A^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad u_A^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$u_B^{(1)} = \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\, u_A^{(1)} = \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad u_B^{(2)} = \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\, u_A^{(2)} = \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Dirac

$$H\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\ \psi$$

The famous Dirac equation:

$$\begin{aligned} (i\gamma^\mu \partial_\mu - m)\ \psi &= 0 \\ \text{with}:\ \ \gamma^\mu = (\beta, \beta\vec{\alpha}) &\equiv \text{Dirac } \gamma\text{−matrices} \end{aligned}$$

Solutions to the Dirac equation?

Try plane wave: $\psi(x) = u(p)\, e^{-ipx}$ →

$$\begin{aligned} (\gamma^\mu p_\mu - m)\ u(p) &= 0 \\ \text{or}:\ \ (\not{p} - m)\ u(p) &= 0 \end{aligned}$$

- 2 coupled equations:

$$\begin{cases} (\vec{\sigma}\cdot\vec{p})\ u_B = (E-m)\, u_A \\ (\vec{\sigma}\cdot\vec{p})\ u_A = (E+m)\, u_B \end{cases}$$

If p≠0:

Two solutions for E>0:

(and two for E<0)

$$u^{(1)} = \begin{pmatrix} u_A^{(1)} \\ u_B^{(1)} \end{pmatrix} \quad , \quad u^{(2)} = \begin{pmatrix} u_A^{(2)} \\ u_B^{(2)} \end{pmatrix}$$

$$u^{(1)} = \begin{pmatrix} 1 \\ 0 \\ \vec{\sigma}\bullet\vec{p}/(E+m) \\ 0 \end{pmatrix} \qquad u^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vec{\sigma}\bullet\vec{p}/(E+m) \end{pmatrix}$$

$$H\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\ \psi$$

Dirac

The famous Dirac equation:

$$\begin{array}{rcl} (i\gamma^\mu \partial_\mu - m)\ \psi & = & 0 \\ \text{with}: \ \gamma^\mu = (\beta, \beta\vec{\alpha}) & \equiv & \text{Dirac } \gamma\text{–matrices} \end{array}$$

ψ is 4-component spinor

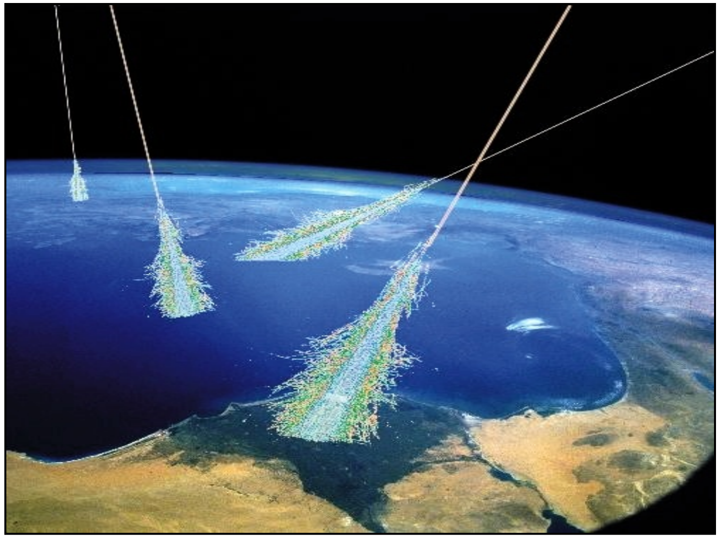
4 solutions correspond to fermions and anti-fermions with spin+1/2 and -1/2

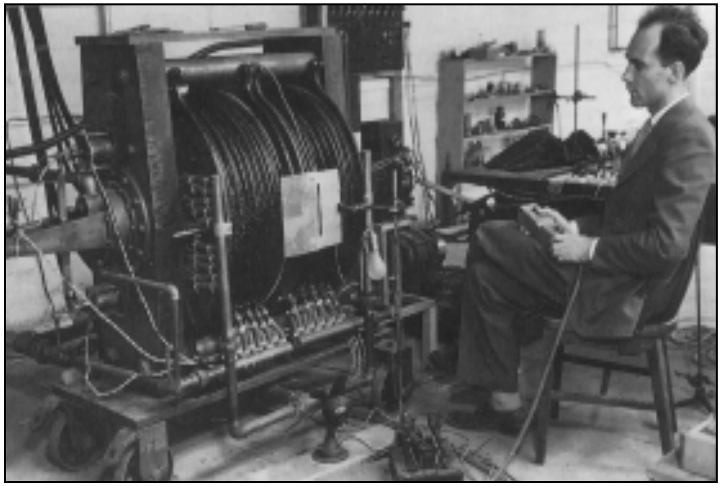
Two solutions for E>0:

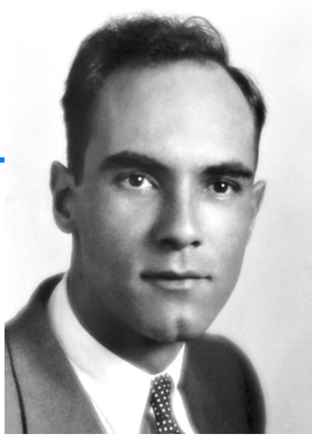
(and two for E<0)

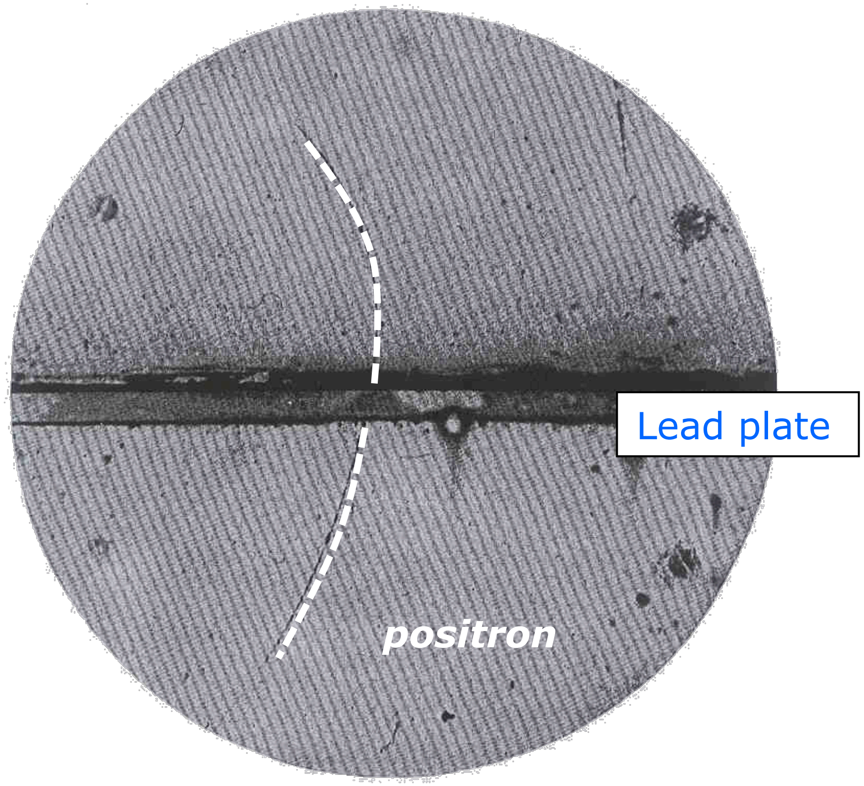
$$u^{(1)} = \begin{pmatrix} 1 \\ 0 \\ \vec{\sigma} \bullet \vec{p} / (E+m) \\ 0 \end{pmatrix} \qquad u^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vec{\sigma} \bullet \vec{p} / (E+m) \end{pmatrix}$$

Discovery of anti-matter









Nobelprize 1936

P and C violation

- What is the link between anti-matter and discrete symmetries?

➢ C operator changes matter into anti-matter

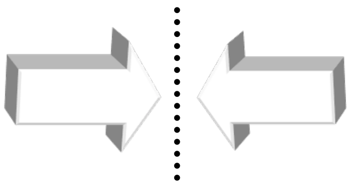
- 2 more discrete symmetries: P and T

Continuous vs discrete symmetries

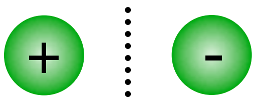
- Space, time translation & orientation symmetries are all *continuous* symmetries
  - Each symmetry operation associated with one ore more continuous parameter
- There are also *discrete* symmetries
  - Charge sign flip (Q → -Q) : **C parity**
  - Spatial sign flip (x,y,z → -x,-y,-z) : **P parity**
  - Time sign flip (t → -t) : **T parity**
- Are these discrete symmetries *exact* symmetries that are observed by all physics in nature?
  - Key issue of this course

Three Discrete Symmetries

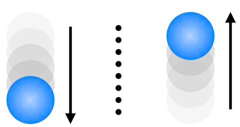
- Parity, *P*
  - Parity reflects a system through the origin. Converts right-handed coordinate systems to left-handed ones.
  - Vectors change sign but axial vectors remain unchanged
    - $\vec{x} \rightarrow -\vec{x}$, $\vec{p} \rightarrow -\vec{p}$, but $\vec{L} = \vec{x} \times \vec{p} \rightarrow \vec{L}$



- Charge Conjugation, *C*
  - Charge conjugation turns a particle into its anti-particle
    - $e^+ \rightarrow e^-$, $K^- \rightarrow K^+$



- Time Reversal, *T*
  - Changes, for example, the direction of motion of particles
    - $t \rightarrow -t$



Example: People believe in symmetry…

<u>Instruction for Abel Tasman, explorer of Australia (1642):</u>

- "Since many rich mines and other treasures have been found in countries north of the equator between 15^{o} and 40^{o} latitude, there is no doubt that countries alike exist south of the equator.

  The provinces in Peru and Chili rich of gold and silver, all positioned south of the equator, are revealing proofs hereof."

Example: People believe in symmetry…

Award Ceremony Speech Nobel Prize (1957):

- *"it was assumed almost tacitly, that elementary particle reactions are symmetric with respect to right and left."*
- *"In fact, most of us were inclined to regard the symmetry of elementary particles with respect to right and left as a necessary consequence of the general principle of right-left symmetry of Nature."*
- *"… only Lee and Yang … asked themselves what kind of experimental support there was for the assumption that all elementary particle processes are symmetric with respect to right and left. "*

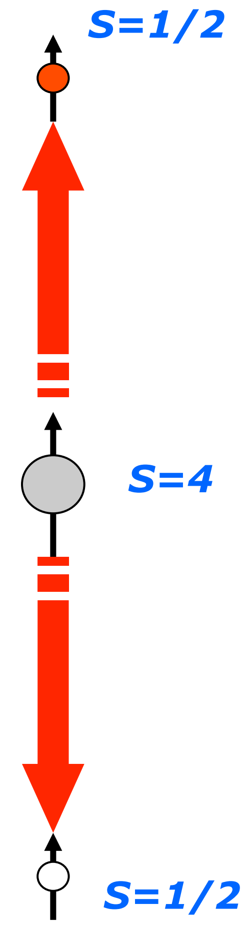
Chen Ning Yang
Prize share: 1/2

Tsung-Dao (T.D.) Lee
Prize share: 1/2

A realistic experiment: the Wu experiment (1956)

- Observe radioactive decay of Cobalt-60 nuclei
  - The process involved: ${}^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$
  - **${}^{60}_{27}\mathrm{Co}$ is spin-5 and ${}^{60}_{28}\mathrm{Ni}$ is spin-4, both e- and ν_e are spin-½**
  - If you start with fully polarized Co (S_Z=5) the experiment is essentially the same (i.e. there is only one spin solution for the decay)

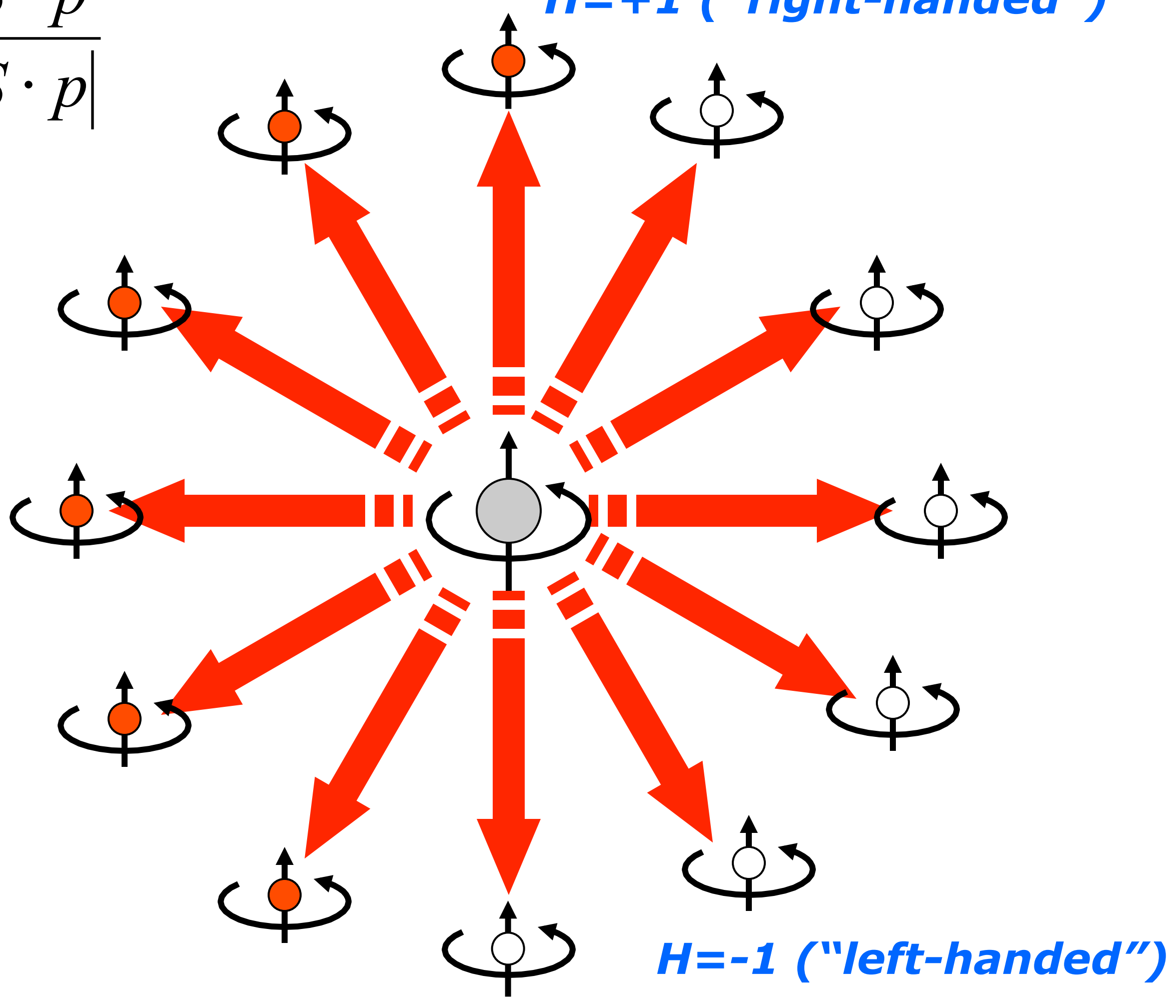
|5,+5> → |4,+4> + |½ ,+½> + |½,+½>



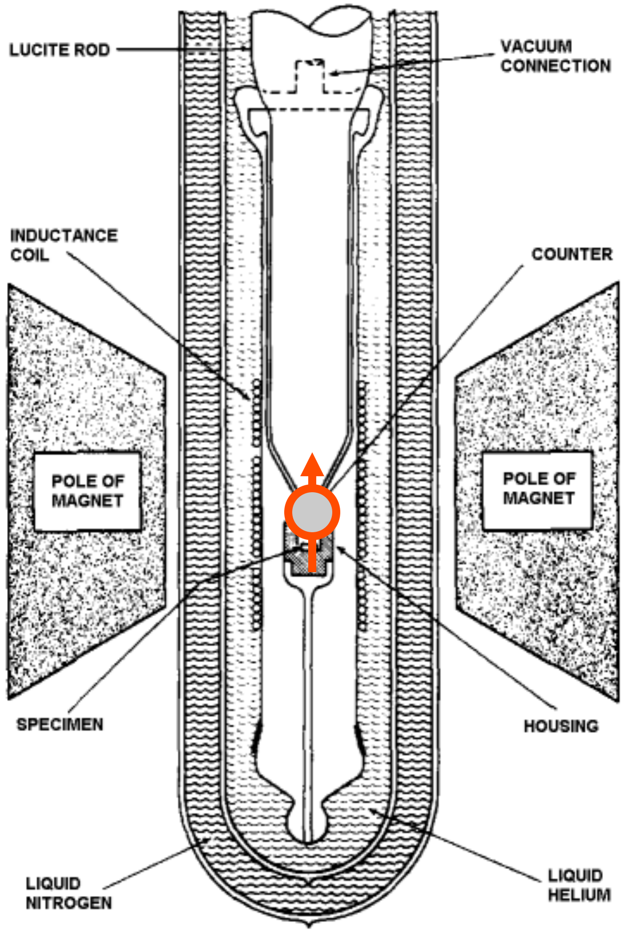
Intermezzo: Spin and Parity and Helicity

- We introduce a new quantity: Helicity = the projection of the spin on the direction of flight of a particle

$$H \equiv \frac{S \cdot p}{|S \cdot p|}$$



The Wu experiment – 1956

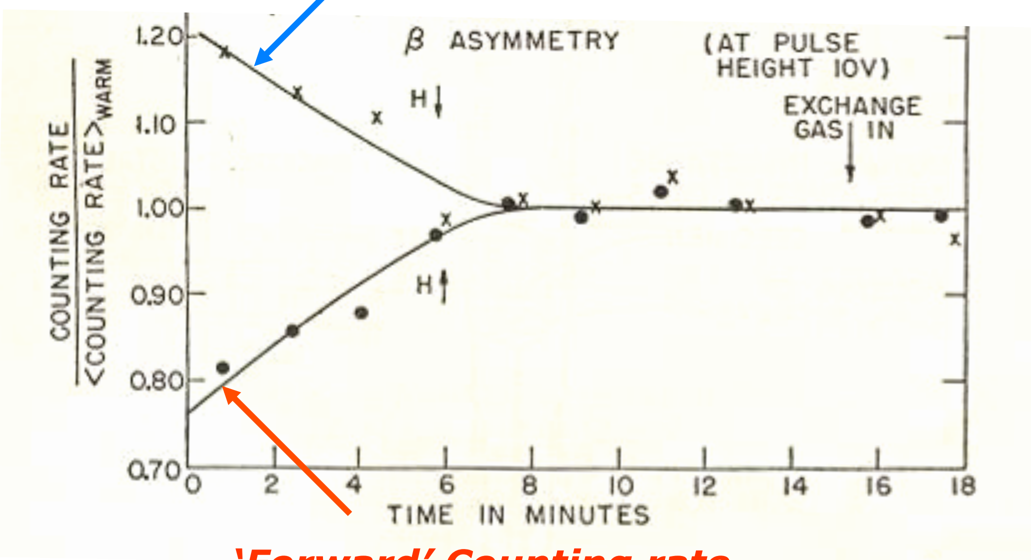


- Experimental challenge: how do you obtain a sample of Co(60) where the spins are aligned in one direction
  - Wu's solution: adiabatic demagnetization of Co(60) in magnetic fields at very low temperatures (~1/100 K!). Extremely challenging in 1956.

The Wu experiment – 1956

- The surprising result: the counting rate is different
  - **Electrons are preferentially emitted in direction opposite of ^{60}Co spin!**
  - Careful analysis of results shows that experimental data is consistent with emission of left-handed (H=-1) electrons only at any angle!!

'Backward' Counting rate w.r.t unpolarized rate



'Forward' Counting rate w.r.t unpolarized rate

^{60}Co polarization decreases as function of time

Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

AND

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *National Bureau of Standards, Washington, D. C.*

(Received January 15, 1957)

The Wu experiment – 1956

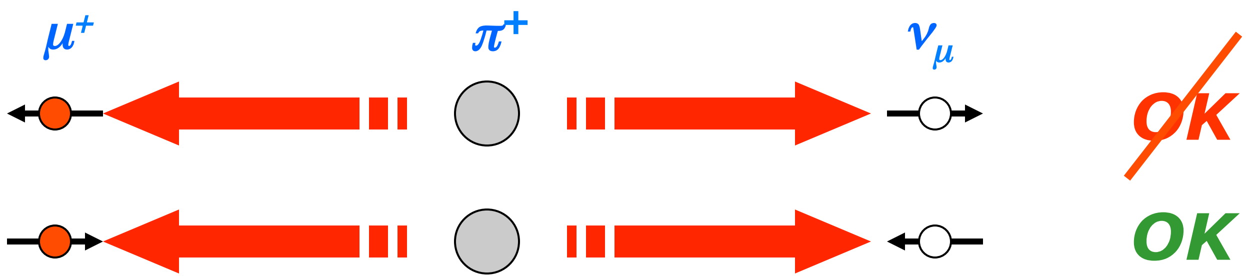
- Physics conclusion:
  - Angular distribution of electrons shows that only pairs of left-handed electrons / right-handed anti-neutrinos are emitted regardless of the emission angle
  - Since right-handed electrons are known to exist (for electrons H is not Lorentz-invariant anyway), this means **no left-handed anti-neutrinos are produced in weak decay**

- **Parity is violated in weak processes**
  - **Not just a little bit but 100%**

- How can you see that ^{60}Co violates parity symmetry?
  - If there is parity symmetry there should exist no measurement that can distinguish our universe from a parity-flipped universe, but we can!

So P is violated, what's next?

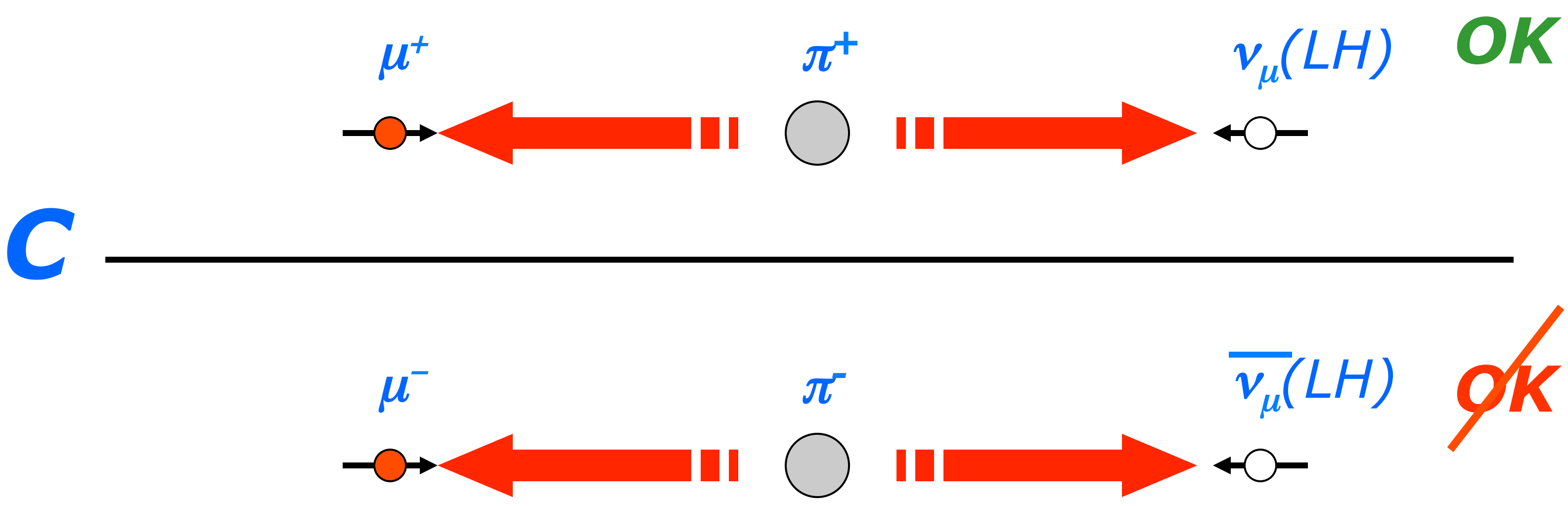
- Wu's experiment was shortly followed by another clever experiment by L. Lederman: Look at decay $\pi^+ \rightarrow \mu^+ \nu_\mu$
  - Pion has spin 0, μ,ν_μ both have spin ½
    → spin of decay products must be oppositely aligned
    → Helicity of muon is same as that of neutrino.



- Nice feature: can also measure polarization of both neutrino (π^+ decay) and anti-neutrino (π^- decay)
- Ledermans result: All neutrinos are left-handed and all anti-neutrinos are right-handed

Charge conjugation symmetry

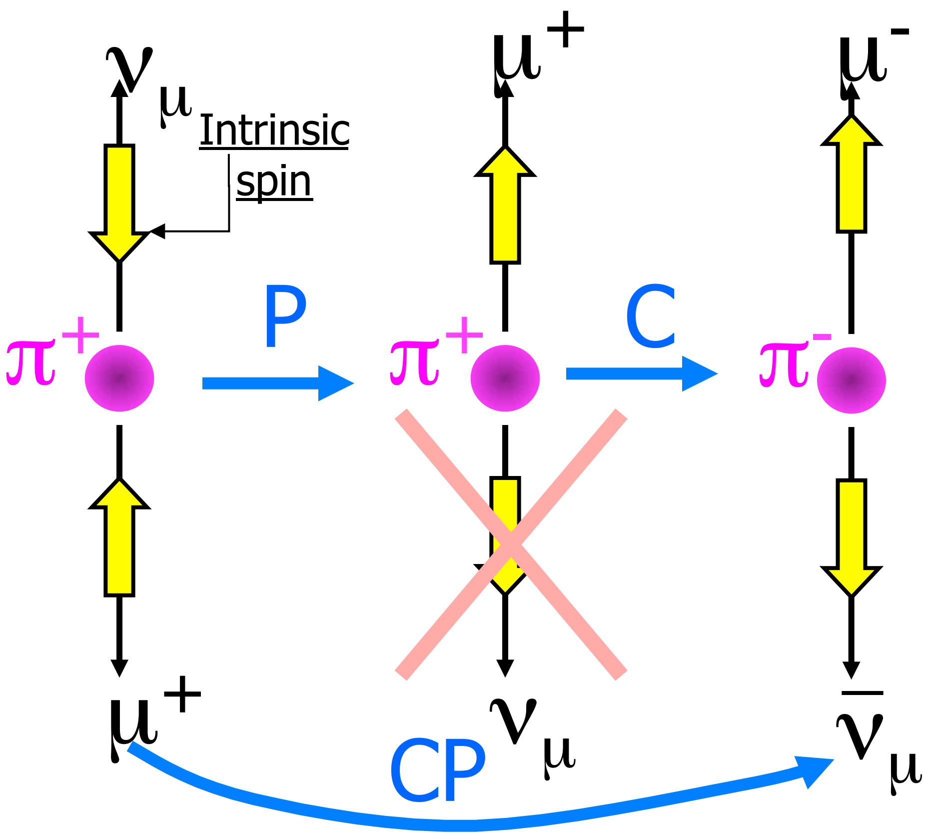
- Introducing C-symmetry
  - The C(harge) conjugation is the operation which exchanges **particles and anti-particles** (not just electric charge)
  - It is a discrete symmetry, just like P, i.e. $C^2 = 1$



- C symmetry is broken by the weak interaction,
  - just like P

The Weak force and C,P parity violation

- What about C+P ≡ CP symmetry?
  - CP symmetry is parity conjugation (x,y,z → -x,-y,z) followed by charge conjugation ($X \rightarrow \overline{X}$)



CP appears to be preserved in weak interaction!

CPT theorem

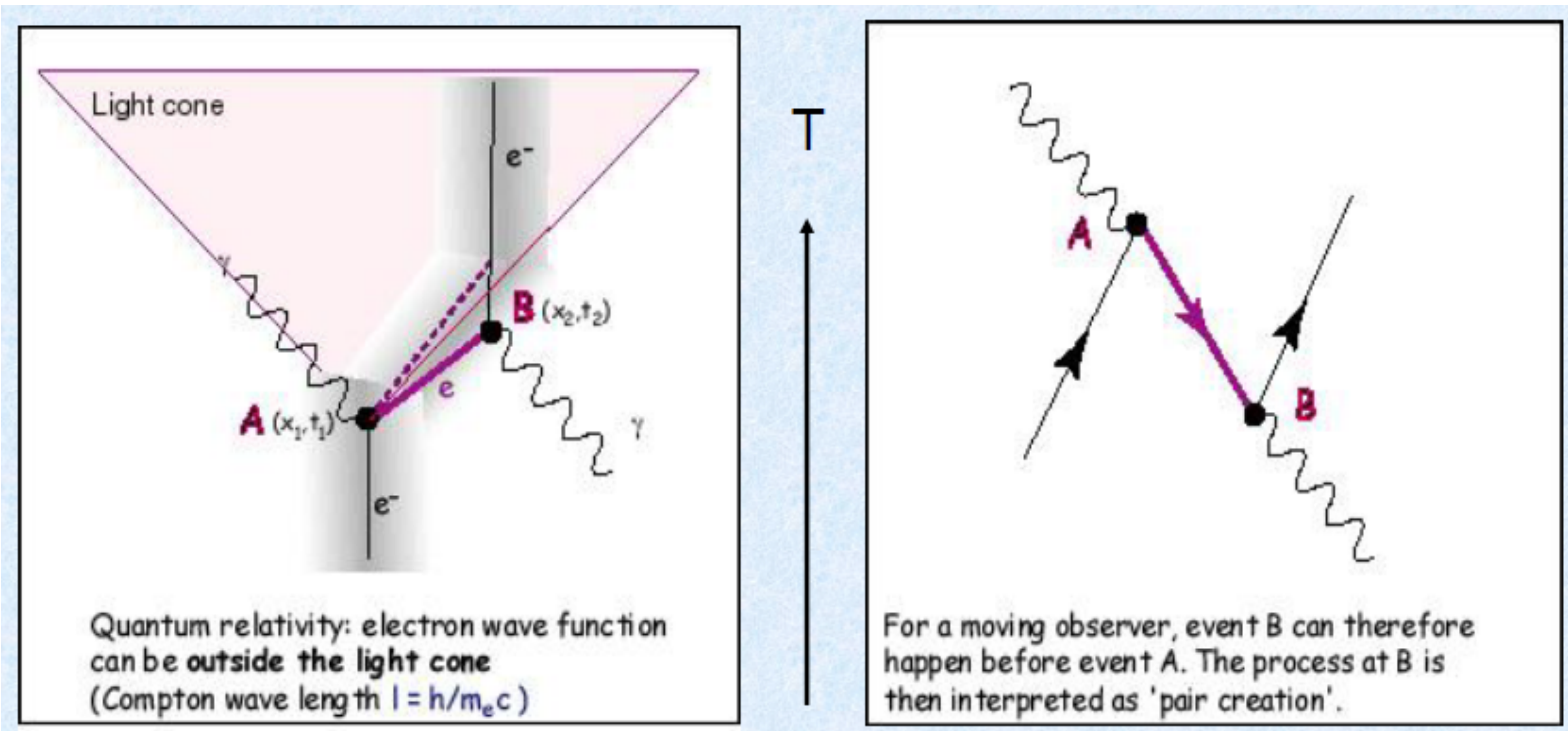
- CPT transformation:
  - C: interchange particles and anti-particles
  - P: reverse space-coordinates
  - T: Reverse time-coordinate

- CPT transformation closely related to Lorentz-boost

➢ CPT invariance implies
  - Particles and anti-particles have same mass and lifetime
  - Lorentz invariance

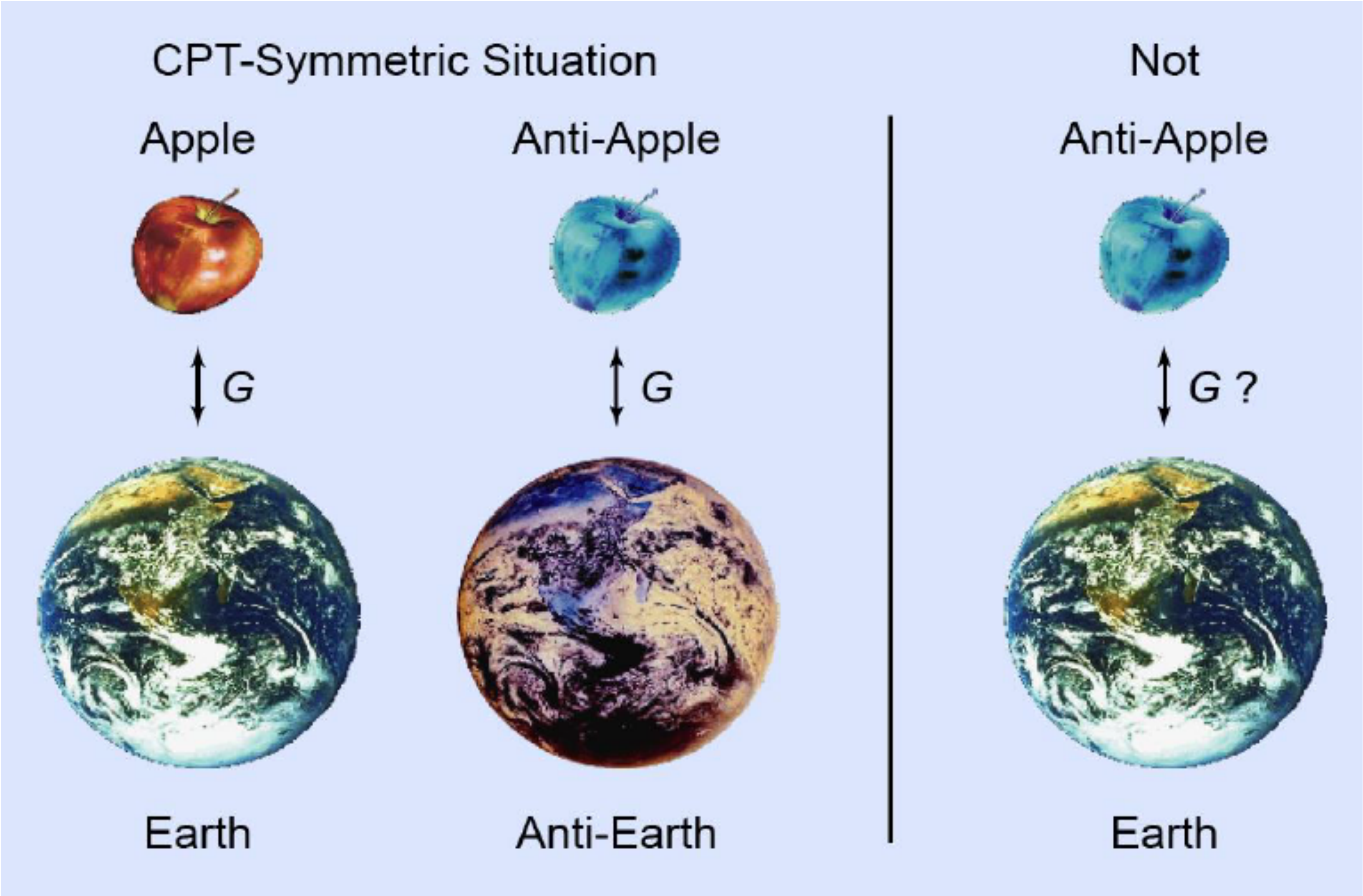
Why anti-matter must exist!

- *"Feynman-Stueckelberg interpretation"*



- "One observer's electron is the other observer's positron"

CPT is conserved, but does anti-matter fall down?



C, P, T

- C, P, T transformation:
  - C: interchange particles and anti-particles
  - P: reverse space-coordinates
  - T: Reverse time-coordinate

- CPT we discussed briefly …

- After the break we deal with P and CP…

  … violation!

What do we know now?

- C.S. Wu discovered from ^{60}Co decays that the weak interaction is 100% asymmetric in P-conjugation
  - We can distinguish our universe from a parity flipped universe by examining ^{60}Co decays

- L. Lederman et al. discovered from π^+ decays that the weak interaction is 100% asymmetric in C-conjugation as well, but that CP-symmetry appears to be preserved
  - First important ingredient towards understanding matter/anti-matter asymmetry of the universe:
    weak force violates matter/anti-matter(=C) symmetry!
  - C violation is a required ingredient, but not enough as we will learn later

Conserved properties associated with C and P

- C and P are still good symmetries in any reaction not involving the weak interaction
  - Can associate a conserved value with them (Noether Theorem)
- Each hadron has a conserved P and C quantum number
  - What are the values of the quantum numbers
  - Evaluate the eigenvalue of the P and C operators on each hadron
    $\mathbf{P}|\psi\rangle = p|\psi\rangle$
- What values of C and P are possible for hadrons?
  - Symmetry operation squared gives unity so eigenvalue squared must be 1
  - Possible C and P values are +1 and -1.
- Meaning of P quantum number
  - If P=1 then $P|\psi\rangle = +1|\psi\rangle$ (wave function symmetric in space)
    if P=-1 then $P|\psi\rangle = -1\,|\psi\rangle$ (wave function anti-symmetric in space)

Figuring out P eigenvalues for hadrons

- QFT rules for particle vs. anti-particles
  - Parity of particle and anti-particle must be opposite for fermions (spin-N+1/2)
  - Parity of bosons (spin N) is same for particle and anti-particle
- Definition of convention (i.e. arbitrary choice in def. of q vs $\overline{q}$)
  - **Quarks have positive parity** → Anti-quarks have negative parity
  - e^- has positive parity as well.
  - (Can define other way around: Notation different, physics same)
- Parity is a *multiplicative* quantum number for composites
  - For composite AB the parity is P(A)*P(B), Thus:
  - Baryons have P=1*1*1=1, anti-baryons have P=-1*-1*-1=-1
  - (Anti-)mesons have P=1*-1 = -1
- Excited states (with orbital angular momentum)
  - Get an extra factor $(-1)^{l}$ where l is the orbital L quantum number
  - Note that parity formalism is parallel to total angular momentum J=L+S formalism, it has an *intrinsic* component and an *orbital* component
- NB: Photon is spin-1 particle has intrinsic P of -1

Parity eigenvalues for selected hadrons

- The π^+ meson
  - Quark and anti-quark composite: intrinsic P = (1)*(-1) = -1
  - Orbital ground state → no extra term
  - **P(π^+)=-1**

Meaning: P|π^+> = -1|π^+>

Experimental proof: J.Steinberger (1954)
$\pi d \rightarrow nn$
▪n are fermions, so (nn) anti-symmetric
▪ S_d=1, S_π=0 → L_{nn}=1
1) final state:P|nn> = $(-1)^L$|nn> = -1 |nn>
2) init state: P|d> = P |pn> = $(+1)^2$|pn> = +1 |d>
➔To conserve parity: **P|π> = -1 |π>**

- The neutron
  - Three quark composite: intrinsic P = (1)*(1)*(1) = 1
  - Orbital ground state → no extra term
  - **P(n) = +1**

- The K_1(1270)
  - Quark anti-quark composite: intrinsic P = (1)*(-1) = -1
  - Orbital excitation with L=1 → extra term $(-1)^1$
  - **P(K_1) = +1**

Figuring out C eigenvalues for hadrons

- Only particles that are their own anti-particles are C eigenstates because $C|x\rangle \equiv \overline{|x\rangle} = c|x\rangle$
  - E.g. $\pi^0,\eta,\eta',\rho^0,\phi,\omega,\psi$ and photon

- C eigenvalues of quark-anti-quark pairs is determined by L and S angular momenta: $C = (-1)^{L+S}$
  - Rule applies to all above mesons

- C eigenvalue of photon is -1
  - Since photon is carrier of EM force, which obviously changes sign under C conjugation

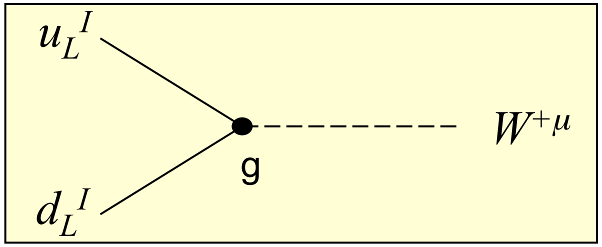
- Example of C conservation:
  - Process $\pi^0 \rightarrow \gamma\,\gamma$ $\quad$ C=+1(π^0 has spin 0) → (-1)*(-1)
  - Process $\pi^0 \rightarrow \gamma\,\gamma\,\gamma$ does not occur (and would violate C conservation)

Experimental proof of C-invariance:
$BR(\pi^0 \rightarrow \gamma\gamma\gamma) < 3.1\ 10^{-5}$

- This was an introduction to P and C
- Let’s change gear…

CP violation in the SM Lagrangian

- Focus on charged current interaction ($W^{\pm}$): let's trace it



The Standard Model Lagrangian

$$\boxed{\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}}$$

- $\mathcal{L}_{Kinetic}$: • Introduce the massless fermion fields
  • Require local gauge invariance ➔ gives rise to existence of gauge bosons

- $\mathcal{L}_{Higgs}$: • Introduce Higgs potential with $<\phi> \neq 0$
  • Spontaneous symmetry breaking

  $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$

  The W^+, W^-,Z^0 bosons acquire a mass

- $\mathcal{L}_{Yukawa}$: • Ad hoc interactions between Higgs field & fermions

Fields: Notation

$Y = Q - T_3$

Fermions: $\psi_L = \left(\frac{1-\gamma_5}{2}\right)\psi \quad ; \quad \psi_R = \left(\frac{1+\gamma_5}{2}\right)\psi$ with $\psi = Q_L, u_R, d_R, L_L, l_R, \nu_R$

Quarks:

Under SU2:
Left handed doublets
Right hander singlets

- $\begin{pmatrix} u^I(3,2,1/6) \\ d^I(3,2,1/6) \end{pmatrix}_{Li} \equiv Q^I_{Li}(3,2,1/6)$

Interaction rep.
Left-handed
generation index
SU(3)$_C$
SU(2)$_L$
Hypercharge Y
(=avg el.charge in multiplet)

- $u^I_{Ri}(3,1,2/3)$
- $d^I_{Ri}(3,1,-1/3)$

Leptons:

- $\begin{pmatrix} \nu^I(1,2,-1/2) \\ l^I(1,2,-1/2) \end{pmatrix}_{Li} \equiv L^I_{Li}(1,2,-1/2)$
- $l^I_{Ri}(1,1,-1)$
- $\left(\nu^I_{Ri}\right)$

Scalar field:

- $\phi(1,2,1/2) = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$

Note:
Interaction representation: standard model interaction is independent of generation number

Fields: Notation

$Y = Q - T_3$

Explicitly:

• The left handed quark doublet :

$$Q^I_{Li}(3,2,1/6) = \begin{pmatrix} u^I_r, u^I_g, u^I_b \\ d^I_r, d^I_g, d^I_b \end{pmatrix}_L, \begin{pmatrix} c^I_r, c^I_g, c^I_b \\ s^I_r, s^I_g, s^I_b \end{pmatrix}_L, \begin{pmatrix} t^I_r, t^I_g, t^I_b \\ b^I_r, b^I_g, b^I_b \end{pmatrix}_L \qquad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \qquad (Y = 1/6)$$

• Similarly for the quark singlets:

$$u^I_{Ri}(3,1,\ 2/3) = \left(u^I_r, u^I_r, u^I_r\right)_R, \left(c^I_r, c^I_r, c^I_r\right)_R, \left(t^I_r, t^I_r, t^I_r\right)_R \qquad (Y = 2/3)$$

$$d^I_{Ri}(3,1,-1/3) = \left(d^I_r, d^I_r, d^I_r\right)_R, \left(s^I_r, s^I_r, s^I_r\right)_R, \left(b^I_r, b^I_r, b^I_r\right)_R \qquad (Y = -1/3)$$

• The left handed leptons: $L^I_{Li}(1,2,-1/2) = \begin{pmatrix} \nu^I_e \\ e^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\mu \\ \mu^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\tau \\ \tau^I \end{pmatrix}_L \qquad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \qquad (Y = -1/2)$

• And similarly the (charged) singlets: $l^I_{Ri}(1,1,-1) = e^I_R, \mu^I_R, \tau^I_R \qquad (Y = -1)$

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

:The Kinetic Part

$\mathcal{L}_{Kinetic}$: Fermions + gauge bosons + interactions

Procedure:
Introduce the Fermion fields and *__demand__* *that the theory is local gauge invariant under* $SU(3)_C x SU(2)_L x U(1)_Y$ *transformations.*

Start with the Dirac Lagrangian: $\mathcal{L} = i\overline{\psi}(\partial^\mu \gamma_\mu)\psi$

Replace: $\partial^\mu \to D^\mu \equiv \partial^\mu + ig_s G_a^\mu L_a + igW_b^\mu T_b + ig' B^\mu Y$

Fields:
- $G_a{}^\mu$: 8 gluons
- $W_b{}^\mu$: weak bosons: W_1, W_2, W_3
- B^μ : hypercharge boson

Generators:

L_a	: Gell-Mann matrices:	½ λ_a	(3x3)	$SU(3)_C$
T_b	: Pauli Matrices:	½ τ_b	(2x2)	$SU(2)_L$
Y	: Hypercharge:			$U(1)_Y$

For the remainder we only consider Electroweak: $SU(2)_L \times U(1)_Y$

$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$: The Kinetic Part

$$\mathcal{L}_{kinetic} : i\overline{\psi}(\partial^{\mu}\gamma_{\mu})\psi \rightarrow i\overline{\psi}(D^{\mu}\gamma_{\mu})\psi$$

$$with \quad \psi = Q^{I}_{Li}, \quad u^{I}_{Ri}, \quad d^{I}_{Ri}, \quad L^{I}_{Li}, \quad l^{I}_{Ri}$$

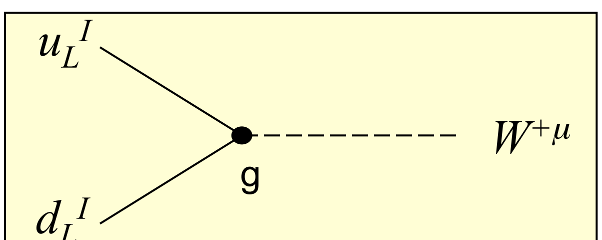
For example, the term with $Q_{Li}{}^{I}$ becomes:

$$\begin{aligned} \mathcal{L}_{kinetic}(Q^{I}_{Li}) &= i\overline{Q^{I}_{Li}}\gamma_{\mu}D^{\mu}Q^{I}_{Li} \\ &= i\overline{Q^{I}_{Li}}\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}g_{s}G^{\mu}_{a}\lambda_{a} + \frac{i}{2}gW^{\mu}_{b}\tau_{b} + \frac{i}{6}g'B^{\mu}\right)Q^{I}_{Li} \end{aligned}$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Writing out only the weak part for the quarks:

$$\begin{aligned} \mathcal{L}^{Weak}_{kinetic}(u,d)^{I}_{L} &= i\overline{(u,d)^{I}_{L}}\gamma_{\mu}\left(\partial^{\mu} + \frac{i}{2}g\left(W_{1}^{\mu}\tau_{1} + W_{2}^{\mu}\tau_{2} + W_{3}^{\mu}\tau_{3}\right)\right)\begin{pmatrix} u \\ d \end{pmatrix}^{I}_{L} \\ &= i\overline{u^{I}_{L}}\gamma_{\mu}\partial^{\mu}u^{I}_{L} + i\overline{d^{I}_{L}}\gamma_{\mu}\partial^{\mu}d^{I}_{L} - \frac{g}{\sqrt{2}}\overline{u^{I}_{L}}\gamma_{\mu}W^{-\mu}d^{I}_{L} - \frac{g}{\sqrt{2}}\overline{d^{I}_{L}}\gamma_{\mu}W^{+\mu}u^{I}_{L} - \ldots \end{aligned}$$



$\mathcal{L} = J_{\mu}W^{\mu}$

$W^{+} = (1/\sqrt{2})\,(W_1 + i\,W_2)$

$W^{-} = (1/\sqrt{2})\,(W_1 - i\,W_2)$

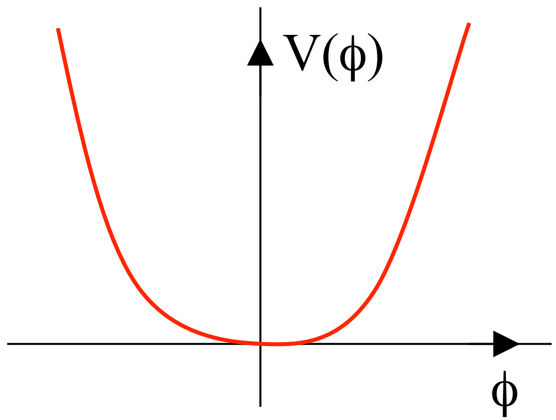
$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$: The Higgs Potential

$$\mathcal{L}_{Higgs} = D_\mu \phi^\dagger D^\mu \phi - V_{Higgs} \qquad V_{Higgs} = \frac{1}{2}\mu^2\left(\phi^\dagger\phi\right) + |\lambda|\left(\phi^\dagger\phi\right)^2$$

Symmetry

$\mu^2 > 0:$

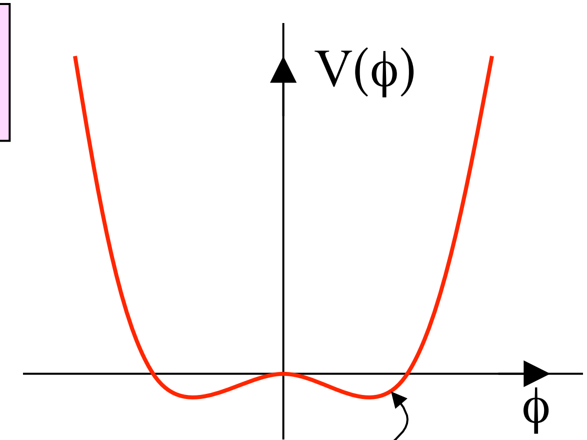
$<\varphi>= 0$



Broken Symmetry

$\mu^2 < 0:$

$$<\varphi>= \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$



$v = \sqrt{-\mu^2/|\lambda|} \sim 246$ GeV

Spontaneous Symmetry Breaking: The Higgs field adopts a non-zero vacuum expectation value

Procedure: $$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \Re e\,\varphi^+ + i\Im m\,\phi^+ \\ \Re e\,\varphi^0 + i\Im m\,\phi^0 \end{pmatrix}$$

Substitute: $$\Re e\,\varphi^0 = \frac{v + H^0}{\sqrt{2}}$$

And rewrite the Lagrangian (tedious):

(The other 3 Higgs fields are “eaten” by the W, Z bosons)

1. $G_{SM} : \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right) \rightarrow \left(SU(3)_C \times U(1)_{EM}\right)$
2. The W^+, W^-, Z^0 bosons acquire mass
3. The Higgs boson H appears

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Yukawa Part

Since we have a Higgs field we can (should?) add (ad-hoc) interactions between ϕ and the fermions in a gauge invariant way.

The result is:

$$-L_{Yukawa} = Y_{ij}\left(\underbrace{\overline{\psi}_{Li}\ \phi}_{\text{doublets}}\right)\underbrace{\psi_{Rj}}_{\text{singlet}} + h.c.$$

$$= Y_{ij}^{d}\left(\overline{Q_{Li}^{I}}\ \phi\right)d_{Rj}^{I} + Y_{ij}^{u}\left(\overline{Q_{Li}^{I}}\ \tilde{\phi}\right)u_{Rj}^{I} + Y_{ij}^{l}\left(\overline{L_{Li}^{I}}\ \phi\right)l_{Rj}^{I} + h.c.$$

i, j : indices for the 3 generations!

With: $\tilde{\phi} = i\sigma_2\,\phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}$

(The CP conjugate of ϕ
To be manifestly invariant under SU(2))

$$\boxed{Y_{ij}^{d}\ ,\quad Y_{ij}^{u}\ ,\quad Y_{ij}^{l}}$$

are arbitrary complex matrices which operate in family space (3x3)
➔ Flavour physics!

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Yukawa Part

Writing the first term explicitly:

$$Y^d_{ij}\,(\overline{u^I_L},\overline{d^I_L})_i \begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} d^I_{Rj} \quad =$$

$$\begin{pmatrix}
Y^d_{11}\left(\overline{u^I_L},\overline{d^I_L}\right)\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} & Y^d_{12}\left(\overline{u^I_L},\overline{d^I_L}\right)\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} & Y^d_{13}\left(\overline{u^I_L},\overline{d^I_L}\right)\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} \\
Y^d_{21}\left(\overline{c^I_L},\overline{s^I_L}\right)\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} & Y^d_{22}\left(\overline{c^I_L},\overline{s^I_L}\right)\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} & Y^d_{13}\left(\overline{c^I_L},\overline{s^I_L}\right)\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} \\
Y^d_{31}\left(\overline{t^I_L},\overline{b^I_L}\right)\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} & Y^d_{32}\left(\overline{t^I_L},\overline{b^I_L}\right)\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix} & Y^d_{33}\left(\overline{t^I_L},\overline{b^I_L}\right)\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix}
\end{pmatrix} \bullet \begin{pmatrix} d^I_R \\ s^I_R \\ b^I_R \end{pmatrix}$$

$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$: The Yukawa Part

There are 3 Yukawa matrices (in the case of massless neutrino's):

$$Y_{ij}^{d} \quad , \quad Y_{ij}^{u} \quad , \quad Y_{ij}^{l}$$

Each matrix is 3x3 complex:
- 27 real parameters
- 27 imaginary parameters ("phases")

➢ many of the parameters are equivalent, since the physics described by one set of couplings is the same as another

➢ It can be shown (see ref. [Nir]) that the independent parameters are:
  - 12 real parameters
  - 1 imaginary phase

➢This single phase is the source of all CP violation in the Standard Model

……Revisit later

$\mathcal{L}_{Yukawa} \xrightarrow{\text{S.S.B}} \mathcal{L}_{Mass}$: The Fermion Masses

Start with the Yukawa Lagrangian

$$-L_{Yuk} = Y_{ij}^{d}\,(\overline{u_L^I}, \overline{d_L^I})_i \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix} d_{Rj}^{I} + Y_{ij}^{u}(\ldots) + Y_{ij}^{l}(\ldots)$$

$$S.S.B. \;:\; \Re e\left(\varphi^{0}\right) \rightarrow \frac{v+H}{\sqrt{2}}$$

After which the following mass term emerges:

$$-L_{Yuk} \rightarrow -L_{Mass} = \overline{d_{Li}^{I}}\, M_{ij}^{d}\, d_{Rj}^{I} + \overline{u_{Li}^{I}}\, M_{ij}^{u}\, u_{Rj}^{I} + \overline{l_{Li}^{I}}\, M_{ij}^{l}\, l_{Rj}^{I} + h.c.$$

with $M_{ij}^{d} \equiv \frac{v}{\sqrt{2}} Y_{ij}^{d}$, $M_{ij}^{u} \equiv \frac{v}{\sqrt{2}} Y_{ij}^{u}$, $M_{ij}^{l} \equiv \frac{v}{\sqrt{2}} Y_{ij}^{l}$

$\mathcal{L}_{Mass}$ is CP violating in a similar way as $\mathcal{L}_{Yuk}$

$\mathcal{L}_{Yukawa} \xrightarrow{\text{S.S.B}} \mathcal{L}_{Mass}$: The Fermion Masses

Writing in an explicit form:

$$-\mathcal{L}_{Mass} = \left(\overline{d^I}, \overline{s^I}, \overline{b^I}\right)_L \left(M^d \right) \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R + \left(\overline{u^I}, \overline{c^I}, \overline{t^I}\right)_L \left(M^u \right) \begin{pmatrix} u^I \\ c^I \\ t^I \end{pmatrix}_R + \left(\overline{e^I}, \overline{\mu^I}, \overline{\tau^I}\right)_L \left(M^l \right) \begin{pmatrix} e^I \\ \mu^I \\ \tau^I \end{pmatrix}_R + h.c.$$

The matrices M can always be diagonalised by unitary matrices V_L^f and V_R^f such that:

$$V_L^f \, M^f \, V_R^{f\dagger} = M^f_{diagonal}$$

$$\left[\left(\overline{d^I}, \overline{s^I}, \overline{b^I}\right)_L V_L^{f\dagger} V_L^f \, M^f \, V_R^{f\dagger} V_R^f \begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix}_R \right]$$

Then the real fermion mass eigenstates are given by:

$$d_{Li} = \left(V_L^d\right)_{ij} \cdot d^I_{Lj} \qquad d_{Ri} = \left(V_R^d\right)_{ij} \cdot d^I_{Rj}$$

$$u_{Li} = \left(V_L^u\right)_{ij} \cdot u^I_{Lj} \qquad u_{Ri} = \left(V_R^u\right)_{ij} \cdot u^I_{Rj}$$

$$l_{Li} = \left(V_L^l\right)_{ij} \cdot l^I_{Lj} \qquad l_{Ri} = \left(V_R^l\right)_{ij} \cdot l^I_{Rj}$$

d_L^I, u_L^I, l_L^I are the weak interaction eigenstates

d_L, u_L, l_L are the mass eigenstates ("physical particles")

$\mathcal{L}_{Yukawa} \xrightarrow{\text{S.S.B}} \mathcal{L}_{Mass}$: The Fermion Masses

In terms of the mass eigenstates:

$$-\mathcal{L}_{Mass} = \left(\bar{d},\bar{s},\bar{b}\right)_L \cdot \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\bar{u},\bar{c},\bar{t}\right)_L \cdot \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \cdot \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$

$$+ \left(\bar{e},\bar{\mu},\bar{\tau}\right)_L \cdot \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} \cdot \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R + h.c.$$

$$-\mathcal{L}_{Mass} = m_u \bar{u}u + m_c \bar{c}c + m_t \bar{t}t$$
$$+ m_d \bar{d}d + m_s \bar{s}s + m_b \bar{b}b$$
$$+ m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau$$

In flavour space one can choose:

Weak basis: The gauge currents are diagonal in flavour space, but the flavour mass matrices are non-diagonal

Mass basis: The fermion masses are diagonal, but some gauge currents (charged weak interactions) are not diagonal in flavour space

In the weak basis: $\mathcal{L}_{Yukawa}$ = CP violating

In the mass basis: $\mathcal{L}_{Yukawa} \rightarrow \mathcal{L}_{Mass}$ = CP conserving

➔ What happened to the charged current interactions (in $\mathcal{L}_{Kinetic}$) ?

$L_W \rightarrow L_{CKM}$: The Charged Current

The charged current interaction for quarks in the ***interaction*** basis is:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \overline{u_{Li}^I} \gamma^\mu d_{Li}^I W_\mu^+$$

The charged current interaction for quarks in the ***mass*** basis is:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \overline{u_{Li}} V_L^u \gamma^\mu V_L^{d\dagger} d_{Li} W_\mu^+$$

The unitary matrix: $V_{CKM} = \left(V_L^u \cdot V_L^{d\dagger}\right)$ With: $V_{CKM} \cdot V_{CKM}^\dagger = 1$

is the Cabibbo Kobayashi Maskawa mixing matrix:

$$-L_{W^+} = \frac{g}{\sqrt{2}} \left(\overline{u}, \overline{c}, \overline{t}\right)_L \left(V_{CKM}\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \gamma^\mu W_\mu^+$$

Lepton sector: similarly $V_{MNS} = \left(V_L^\nu \cdot V_L^{l\dagger}\right)$

However, for massless neutrino's: $V_L{}^\nu$ = arbitrary. Choose it such that $V_{MNS} = 1$

➔ There is no mixing in the lepton sector

Charged Currents

The charged current term reads:

$$L_{CC} = \frac{g}{\sqrt{2}} \overline{u^I_{Li}} \gamma^\mu W^-_\mu d^I_{Li} \quad + \quad \frac{g}{\sqrt{2}} \overline{d^I_{Li}} \gamma^\mu W^+_\mu u^I_{Li} \quad = \quad J^{\mu-}_{CC} W^-_\mu + J^{\mu+}_{CC} W^+_\mu$$

$$= \frac{g}{\sqrt{2}} \overline{u_i} \left(\frac{1-\gamma^5}{2}\right) \gamma^\mu W^-_\mu V_{ij} \left(\frac{1-\gamma^5}{2}\right) d_j \quad + \quad \frac{g}{\sqrt{2}} \overline{d_j} \left(\frac{1-\gamma^5}{2}\right) \gamma^\mu W^+_\mu V^\dagger_{ji} \left(\frac{1-\gamma^5}{2}\right) u_i$$

$$= \frac{g}{\sqrt{2}} \overline{u_i} \gamma^\mu W^-_\mu V_{ij} \left(1-\gamma^5\right) d_j \quad + \quad \frac{g}{\sqrt{2}} \overline{d_j} \gamma^\mu W^+_\mu V^*_{ij} \left(1-\gamma^5\right) u_i$$

Under the CP operator this gives:

(Together with (x,t) -> (-x,t))

$$L_{CC} \xrightarrow{CP} \frac{g}{\sqrt{2}} \overline{d_j} \gamma^\mu W^+_\mu V_{ij} \left(1-\gamma^5\right) u_i \quad + \quad \frac{g}{\sqrt{2}} \overline{u_i} \gamma^\mu W^i_\mu V^*_{ij} \left(1-\gamma^5\right) d_j$$

A comparison shows that CP is conserved only if $V_{ij} = V_{ij}^*$

In general the charged current term is CP violating

The Standard Model Lagrangian (recap)

$$\boxed{\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}}$$

- $\mathcal{L}_{Kinetic}$: •Introduce the massless fermion fields
  •Require local gauge invariance → gives rise to existence of gauge bosons
  ➔ CP Conserving

- $\mathcal{L}_{Higgs}$: •Introduce Higgs potential with $<\phi> \neq 0$
  •Spontaneous symmetry breaking
  $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$
  The W^+, W^-,Z^0 bosons acquire a mass
  ➔ CP Conserving

- $\mathcal{L}_{Yukawa}$: •Ad hoc interactions between Higgs field & fermions
  ➔ CP violating with a single phase

- $\mathcal{L}_{Yukawa} \rightarrow \mathcal{L}_{mass}$:
  - fermion weak eigenstates:
    - mass matrix is (3x3) non-diagonal
    ➔ CP-violating
  - fermion mass eigenstates:
    - mass matrix is (3x3) diagonal
    ➔ CP-conserving!

- $\mathcal{L}_{Kinetic}$ in mass eigenstates: CKM – matrix ➔ CP violating with a single phase

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

Recap

$$-L_{Yuk} = Y^d_{ij}\,(\overline{u^I_L}, \overline{d^I_L})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d^I_{Rj} + \ldots$$

$$L_{Kinetic} = \frac{g}{\sqrt{2}} \overline{u^I_{Li}} \gamma^\mu W^-_\mu d^I_{Li} + \frac{g}{\sqrt{2}} \overline{d^I_{Li}} \gamma^\mu W^+_\mu u^I_{Li} + \ldots$$

Diagonalize Yukawa matrix Y_{ij}

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$-L_{Mass} = \left(\bar{d}, \bar{s}, \bar{b}\right)_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\bar{u}, \bar{c}, \bar{t}\right)_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \ldots$$

$$L_{CKM} = \frac{g}{\sqrt{2}} \overline{u_i} \gamma^\mu W^-_\mu V_{ij} \left(1-\gamma^5\right) d_j + \frac{g}{\sqrt{2}} \overline{d_j} \gamma^\mu W^+_\mu V^*_{ij} \left(1-\gamma^5\right) u_i + \ldots$$

$$L_{SM} = L_{CKM} + L_{Higgs} + L_{Mass}$$

Ok…. We've got the CKM matrix, now what?

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- It's *unitary*
  - *"probabilities add up to 1":*
  - d'=0.97 d + 0.22 s + 0.003 b   ($0.97^2+0.22^2+0.003^2=1$)
- How many free parameters?
  - How many real/complex?
- How do we normally visualize these parameters?

Personal impression:

- People think it is a complicated part of the Standard Model (me too:-). Why?

1) Non-intuitive concepts?
   - *Imaginary phase* in transition amplitude, T ~ $e^{i\varphi}$
   - *Different bases* to express quark states, d' =0.97 d + 0.22 s + 0.003 b
   - *Oscillations* (mixing) of mesons: $|K^0> \leftrightarrow |\boxed{?}K^0>$

2) Complicated calculations?

$$\Gamma\left(B^0 \to f\right) \propto \left|A_f\right|^2 \left[\left|g_+(t)\right|^2 + |\lambda|^2 \left|g_-(t)\right|^2 + 2\Re\left(\lambda g_+^*(t) g_-(t)\right)\right]$$

$$\Gamma\left(\bar{B}^0 \to f\right) \propto \left|\bar{A}_f\right|^2 \left[\left|g_+(t)\right|^2 + \frac{1}{|\lambda|^2} \left|g_-(t)\right|^2 + \frac{2}{|\lambda|^2}\Re\left(\lambda^* g_+^*(t) g_-(t)\right)\right]$$

3) Many decay modes? *"Beetopaipaigamma…"*
   - PDG reports 347 decay modes of the B^0-meson:
     - Γ_1 $l^+ \nu_l$ *anything* (10.33 ± 0.28) × 10^{-2}
     - Γ_{347} $\nu\, \nu\, \gamma$ $<4.7 \times 10^{-5}$ *CL=90%*
   - And for one decay there are often more than one decay *amplitudes*…

Table of contents