

# Particle Physics II – CP violation

(also known as “Physics of Anti-matter”)

## *Lecture 3*

N. Tuning

# Plan

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- 1) Mon 2 Feb: Anti-matter + SM
- 2) Wed 4 Feb: CKM matrix + Unitarity Triangle
- 3) Mon 9 Feb: Mixing + Master eqs. +  $B^0 \rightarrow J/\psi K_s$
- 4) Wed 11 Feb: CP violation in  $B_{(s)}$  decays (I)
- 5) Mon 16 Feb: CP violation in  $B_{(s)}$  decays (II)
- 6) Wed 18 Feb: CP violation in K decays + Overview
- 7) Mon 23 Feb: Exam on part 1 (CP violation)

## ➤ Final Mark:

- if (mark > 5.5) mark = max(exam, 0.8\*exam + 0.2\*homework)
- else mark = exam

## ➤ In parallel: Lectures on Flavour Physics by prof.dr. R. Fleischer

- Tuesday + Thursday

# Plan

- 2 x 45 min

1) Keep track of room!

Periode SEM2 - Hoorcollege (Aanwezigheid verplicht)																		
Groep	Blokweken													Dag	Tijd	Gebouw	Zaal	
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Periode SEM2 - Werkcollege (Aanwezigheid verplicht)																		
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1) Monday + Wednesday:

- Start: 9:00 → 9:15
- End: 11:00
- Werkcollege: 11:00 - ?

# Table of contents

- Introduction** . . . . . 1
- 1 CP Violation in the Standard Model** . . . . . 3
  - 1.1 Parity transformation . . . . . 3
    - 1.1.1 The Wu-experiment:  $^{60}\text{Co}$  decay . . . . . 4
    - 1.1.2 Parity violation . . . . . 5
    - 1.1.3 CPT . . . . . 7
  - 1.2 C, P and T: Discrete symmetries in Maxwell's equations . . . . . 8
  - 1.3 C, P and T: Discrete symmetries in QED . . . . . 9
  - 1.4 CP violation and the Standard Model Lagrangian . . . . . 12
    - 1.4.1 Yukawa couplings and the Origin of Quark Mixing . . . . . 12
    - 1.4.2 CP violation . . . . . 15
- 2 The Cabibbo-Kobayashi-Maskawa Matrix** . . . . . 17
  - 2.1 Unitarity Triangle(s) . . . . . 17
  - 2.2 Size of matrix elements . . . . . 20
  - 2.3 Wolfenstein parameterization . . . . . 23
  - 2.4 Discussion . . . . . 26
- 3 Neutral Meson Decays** . . . . . 27
  - 3.1 Neutral Meson Oscillations . . . . . 27
  - 3.2 The mass and decay matrix . . . . . 27

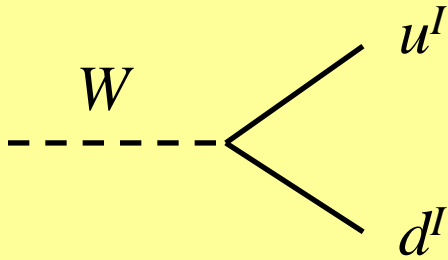
- 3.3 Eigenvalues and -vectors of Mass-decay Matrix . . . . . 29
- 3.4 Time evolution . . . . . 30
- 3.5 The Amplitude of the Box diagram . . . . . 32
- 3.6 Meson Decays . . . . . 35
- 3.7 Classification of CP Violating Effects . . . . . 36
- 4 CP violation in the B-system** . . . . . 39
  - 4.1  $\beta$ : the  $B^0 \rightarrow J/\psi K_S^0$  decay . . . . . 40
  - 4.2  $\beta_s$ : the  $B_s^0 \rightarrow J/\psi \phi$  decay . . . . . 44
  - 4.3  $\gamma$ : the  $B_s^0 \rightarrow D_s^\pm K^\mp$  decay . . . . . 46
  - 4.4 Direct CP violation: the  $B^0 \rightarrow \pi^- K^+$  decay . . . . . 48
  - 4.5 CP violation in mixing: the  $B^0 \rightarrow l^+ \nu X$  decay . . . . . 49
  - 4.6 Penguin diagram: the  $B^0 \rightarrow \phi K_S^0$  decay . . . . . 50
- 5 CP violation in the K-system** . . . . . 51
  - 5.1 CP and pions . . . . . 51
  - 5.2 Description of the K-system . . . . . 53
  - 5.3 The Cronin-Fitch experiment . . . . . 54
    - 5.3.1 Regeneration . . . . . 56
  - 5.4 CP violation in mixing:  $\epsilon$  . . . . . 57
  - 5.5 CP violation in decay:  $\epsilon'$  . . . . . 58
- 6 Experimental Aspects and Present Knowledge of Unitarity Triangle** . . . . . 61
  - 6.1 B-meson production . . . . . 61
  - 6.2 Flavour Tagging . . . . . 65
  - 6.3 Present Knowledge on Unitarity Triangle . . . . . 66
    - 6.3.1 Measurement of  $\sin 2\beta$  . . . . . 67
    - 6.3.2 Measurement of  $\epsilon_K$  . . . . . 67
    - 6.3.3  $|V_{ub}/V_{cb}|$  . . . . . 68
    - 6.3.4 Measurement of  $\Delta m$  . . . . . 68
  - 6.4 Outlook: the LHCb experiment . . . . . 71

# Recap

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$$-\mathcal{L}_{Yuk} = Y_{ij}^d (\bar{u}_L^I, \bar{d}_L^I)_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^I + \dots$$

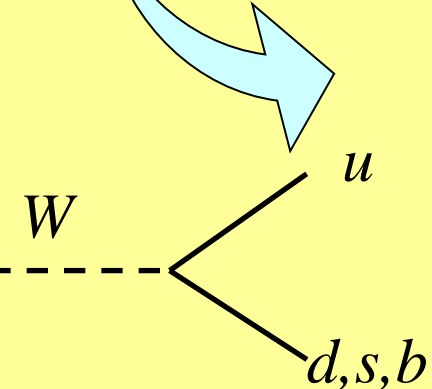
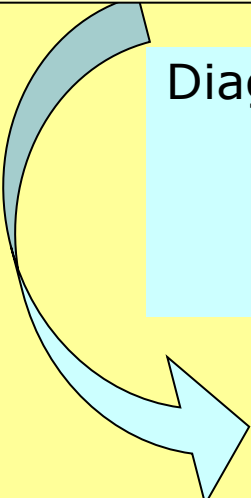
$$\mathcal{L}_{Kinetic} = \frac{g}{\sqrt{2}} \bar{u}_{Li}^I \gamma^\mu W_\mu^- d_{Li}^I + \frac{g}{\sqrt{2}} \bar{d}_{Li}^I \gamma^\mu W_\mu^+ u_{Li}^I + \dots$$



Diagonalize Yukawa matrix  $Y_{ij}$

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



$$-\mathcal{L}_{Mass} = (\bar{d}, \bar{s}, \bar{b})_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + (\bar{u}, \bar{c}, \bar{t})_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \dots$$

$$\mathcal{L}_{CKM} = \frac{g}{\sqrt{2}} \bar{u}_i \gamma^\mu W_\mu^- V_{ij} (1 - \gamma^5) d_j + \frac{g}{\sqrt{2}} \bar{d}_j \gamma^\mu W_\mu^+ V_{ij}^* (1 - \gamma^5) u_i + \dots$$

$$\mathcal{L}_{SM} = \mathcal{L}_{CKM} + \mathcal{L}_{Higgs} + \mathcal{L}_{Mass}$$

# Charged Currents

The charged current term reads:

$$\begin{aligned}
 L_{CC} &= \frac{g}{\sqrt{2}} \bar{u}_{Li}^I \gamma^\mu W_\mu^- d_{Li}^I + \frac{g}{\sqrt{2}} \bar{d}_{Li}^I \gamma^\mu W_\mu^+ u_{Li}^I = J_{CC}^{\mu-} W_\mu^- + J_{CC}^{\mu+} W_\mu^+ \\
 &= \frac{g}{\sqrt{2}} \bar{u}_i \left( \frac{1-\gamma^5}{2} \right) \gamma^\mu W_\mu^- V_{ij} \left( \frac{1-\gamma^5}{2} \right) d_j + \frac{g}{\sqrt{2}} \bar{d}_j \left( \frac{1-\gamma^5}{2} \right) \gamma^\mu W_\mu^+ V_{ji}^\dagger \left( \frac{1-\gamma^5}{2} \right) u_i \\
 &= \frac{g}{\sqrt{2}} \bar{u}_i \gamma^\mu W_\mu^- V_{ij} (1-\gamma^5) d_j + \frac{g}{\sqrt{2}} \bar{d}_j \gamma^\mu W_\mu^+ V_{ij}^* (1-\gamma^5) u_i
 \end{aligned}$$

Under the CP operator this gives:

(Together with (x,t) → (-x,t))

$$L_{CC} \xrightarrow{CP} \frac{g}{\sqrt{2}} \bar{d}_j \gamma^\mu W_\mu^+ V_{ij} (1-\gamma^5) u_i + \frac{g}{\sqrt{2}} \bar{u}_i \gamma^\mu W_\mu^- V_{ij}^* (1-\gamma^5) d_j$$

A comparison shows that CP is conserved only if  $V_{ij} = V_{ij}^*$

In general the charged current term is CP violating

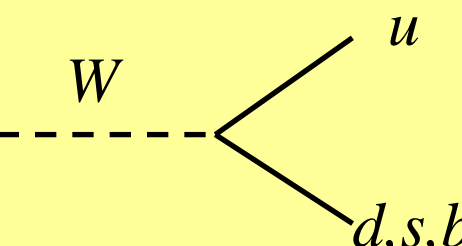
# CKM-matrix: where are the phases?

- Possibility 1: simply 3 ‘rotations’, and put phase on smallest:

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

- Possibility 2: parameterize according to magnitude, in  $O(\lambda)$ :

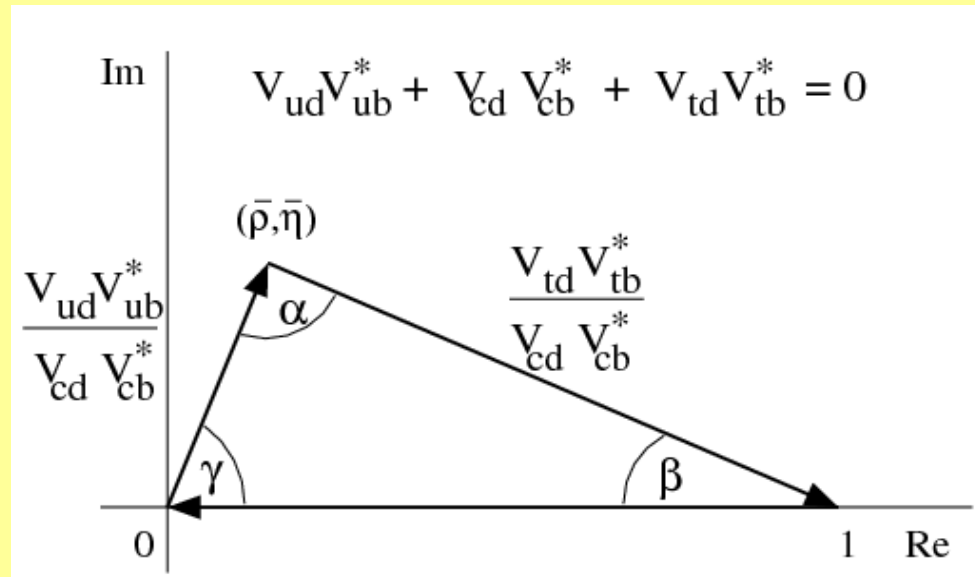


A Feynman diagram showing a dashed line labeled 'W' on the left, which splits into two solid lines on the right. The upper solid line is labeled 'u' and the lower solid line is labeled 'd,s,b'.

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

## This was theory, now comes experiment

- We already saw how the **moduli**  $|V_{ij}|$  are determined
- Now we will work towards the measurement of the **imaginary** part
  - Parameter:  $\eta$
  - Equivalent: angles  $\alpha, \beta, \gamma$ .



- To measure this, we need the formalism of **neutral meson oscillations**...

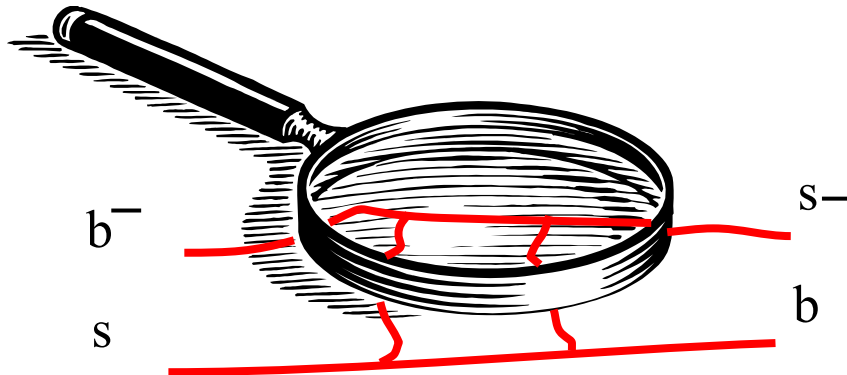


# Neutral Meson Oscillations

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Why?

- Loop diagram: sensitive to new particles
- Provides a second amplitude
  - interference effects in B-decays



# Dynamics of Neutral B (or K) mesons...

Time evolution of  $B^0$  and  $\bar{B}^0$  can be described by an *effective* Hamiltonian:

$$i \frac{\partial}{\partial t} \Psi = H \Psi \quad \Psi(t) = a(t) |B^0\rangle + b(t) |\bar{B}^0\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} \quad \text{No mixing, no decay...}$$

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2} \underbrace{\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}}_{\text{hermitian}} \quad \begin{array}{l} \text{No mixing, but with decays...} \\ \text{(i.e.: H is not Hermitian!)} \end{array}$$

→ With decays included, probability of observing either  $B^0$  or  $\bar{B}^0$  must go down as time goes by:

$$\frac{d}{dt} \left( |a(t)|^2 + |b(t)|^2 \right) = - \begin{pmatrix} a(t)^* & b(t)^* \end{pmatrix} \begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \quad \Rightarrow \Gamma > 0$$

# Describing Mixing...

Time evolution of  $B^0$  and  $\bar{B}^0$  can be described by an *effective* Hamiltonian:

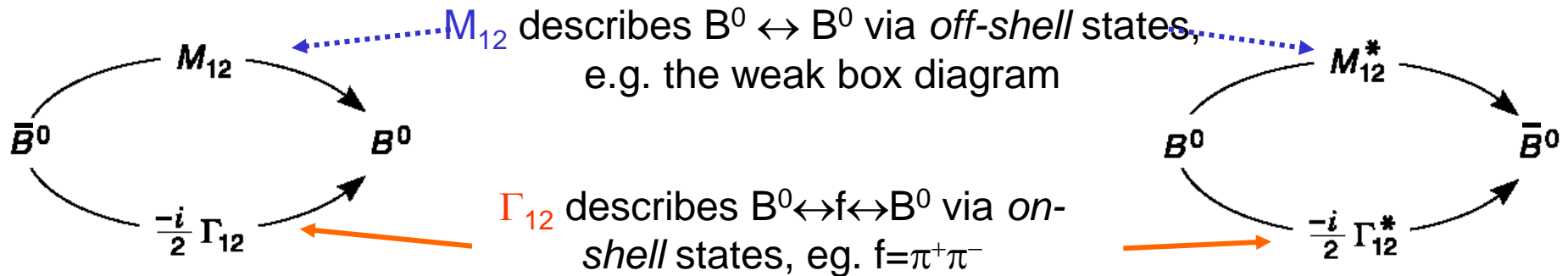
$$i \frac{\partial}{\partial t} \Psi = H \Psi \quad \Psi(t) = a(t) |B^0\rangle + b(t) |\bar{B}^0\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$H = \underbrace{\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2} \underbrace{\begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

Where to put the mixing term?

$$H = \underbrace{\begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}}_{\text{hermitian}} - \frac{i}{2} \underbrace{\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}}_{\text{hermitian}}$$

Now with mixing – but what is the difference between  $M_{12}$  and  $\Gamma_{12}$ ?



# Solving the Schrödinger Equation

$$i \frac{\partial}{\partial t} \psi(t) = \begin{pmatrix} M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma \end{pmatrix} \psi(t)$$

Eigenvalues:

– Mass and lifetime of physical states: mass eigenstates

$$\begin{vmatrix} M - \frac{i}{2} \Gamma - \lambda & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma - \lambda \end{vmatrix} = 0$$

notation  $F = \sqrt{(M_{12} - \frac{i}{2} \Gamma_{12})(M_{12}^* - \frac{i}{2} \Gamma_{12}^*)}$

$$\begin{aligned} m_1 + \frac{i}{2} \Gamma_1 &= M - \Re F - \frac{i}{2} \Gamma - \Im F \\ m_2 + \frac{i}{2} \Gamma_2 &= M + \Re F - \frac{i}{2} \Gamma + \Im F \end{aligned}$$

$$\Delta m = 2 \Re \sqrt{\left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)}$$

$$\Delta \Gamma = 4 \Im \sqrt{\left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)}$$

# Solving the Schrödinger Equation

$$i \frac{\partial}{\partial t} \psi(t) = \begin{pmatrix} M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma \end{pmatrix} \psi(t)$$

Eigenvectors:  
– mass eigenstates

$$|P_1\rangle = p|P^0\rangle - q|\bar{P}^0\rangle$$

$$|P_2\rangle = p|P^0\rangle + q|\bar{P}^0\rangle$$

find  $p$  and  $q$  by solving

$$\begin{pmatrix} M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \lambda_{\pm} \begin{pmatrix} p \\ q \end{pmatrix}$$

$$|B_H\rangle = p|B\rangle + q|\bar{B}\rangle$$

$$|B_L\rangle = p|B\rangle - q|\bar{B}\rangle$$

$$q/p = \sqrt{\left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right) / \left( M_{12} - \frac{i}{2} \Gamma_{12} \right)}$$

## Time evolution

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- With diagonal Hamiltonian, usual time evolution is obtained:

$$\begin{aligned} |P_H(t)\rangle &= e^{-im_H t - \frac{1}{2}\Gamma_H t} |P_H(0)\rangle \\ |P_L(t)\rangle &= e^{-im_L t - \frac{1}{2}\Gamma_L t} |P_L(0)\rangle \end{aligned}$$

$$\begin{aligned} |P^0\rangle &= \frac{1}{2p} [|P_H\rangle + |P_L\rangle] & |P_H\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \\ |\bar{P}^0\rangle &= \frac{1}{2q} [|P_H\rangle - |P_L\rangle] & |P_L\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \end{aligned}$$

$$\begin{aligned} |P^0(t)\rangle &= \frac{1}{2p} \left\{ e^{-im_H t - \frac{1}{2}\Gamma_H t} |P_H(0)\rangle + e^{-im_L t - \frac{1}{2}\Gamma_L t} |P_L(0)\rangle \right\} \\ &= \frac{1}{2p} \left\{ e^{-im_H t - \frac{1}{2}\Gamma_H t} (p|P^0\rangle + q|\bar{P}^0\rangle) + e^{-im_L t - \frac{1}{2}\Gamma_L t} (p|P^0\rangle - q|\bar{P}^0\rangle) \right\} \\ &= \frac{1}{2} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle \\ &= g_+(t) |P^0\rangle + \left( \frac{q}{p} \right) g_-(t) |\bar{P}^0\rangle \end{aligned} \tag{3.6}$$

# B Oscillation Amplitudes

For an initially produced  $B^0$  or a  $\bar{B}^0$  it then follows: (using:  $|B^0\rangle = \frac{1}{2p}(|B_H\rangle + |B_L\rangle)$ )

$$|\bar{B}^0\rangle = \frac{1}{2q}(|B_H\rangle - |B_L\rangle)$$

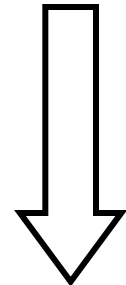
$$|\psi(t)\rangle:$$

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle$$

with  $g_{\pm}(t) = \frac{e^{-i\omega_+t} \pm e^{-i\omega_-t}}{2}$

$$|\bar{B}^0(t)\rangle = g_+(t)|\bar{B}^0\rangle + \frac{p}{q}g_-(t)|B^0\rangle$$

For  $B^0$ , expect:  
 $\Delta\Gamma \sim 0,$   
 $|q/p|=1$

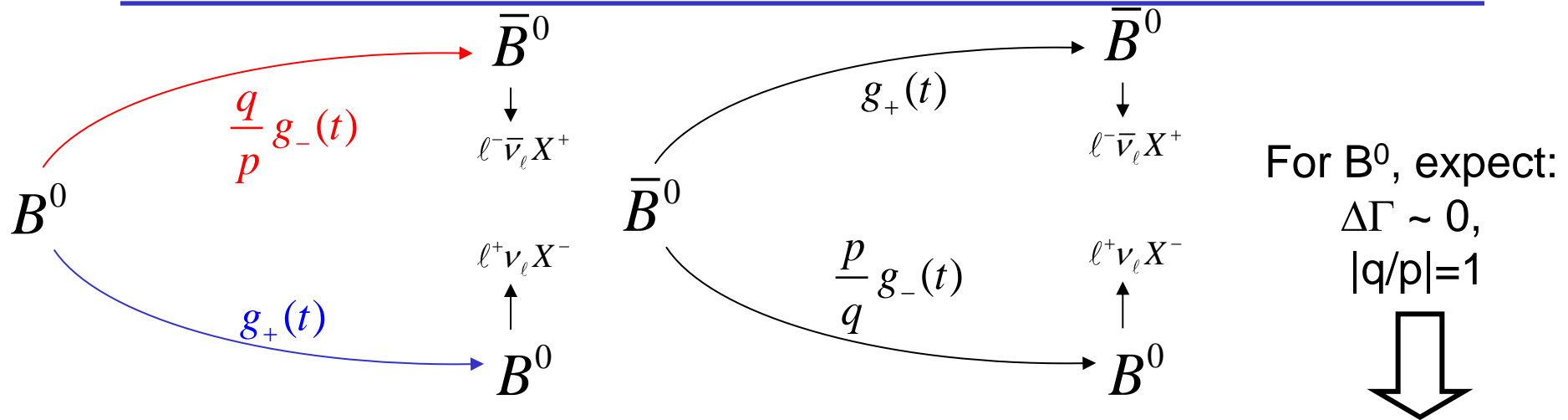


$$g_+(t) = e^{-imt} e^{-\Gamma t/2} \cos \frac{\Delta mt}{2}$$

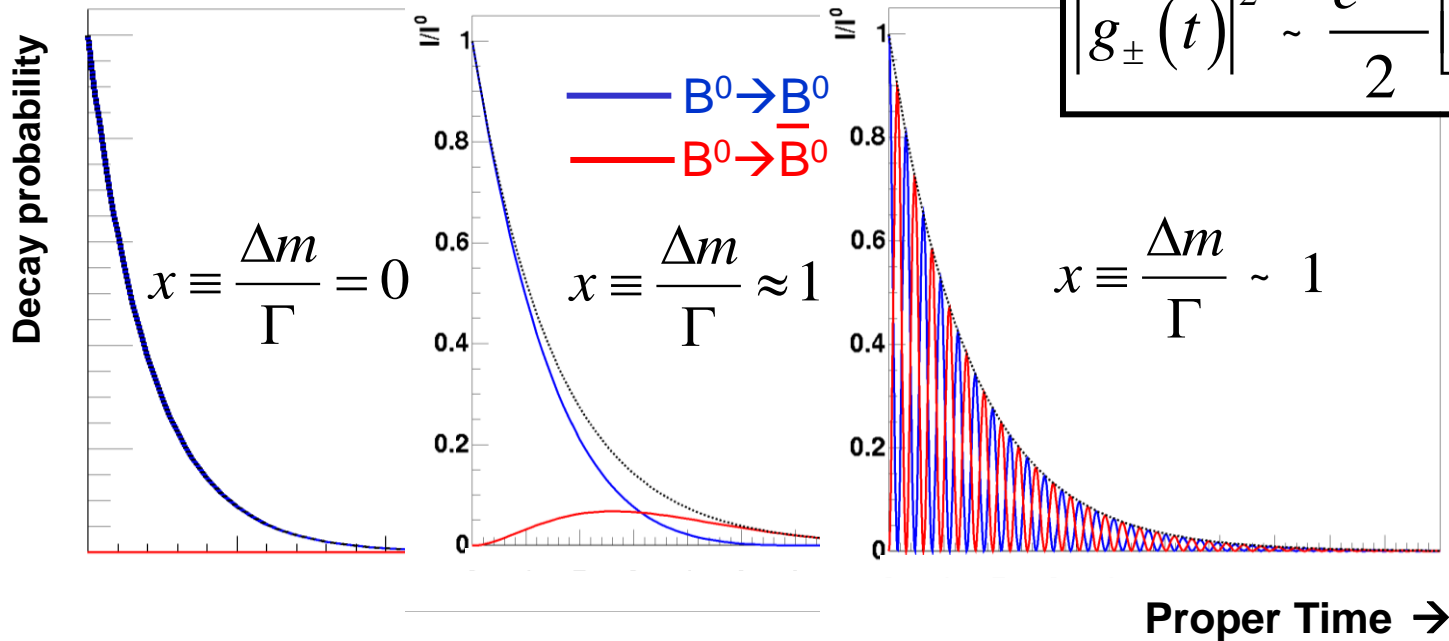
$$g_-(t) = e^{-imt} e^{-\Gamma t/2} i \sin \frac{\Delta mt}{2}$$

$$g_{\pm}(t) \sim e^{-imt} e^{-\Gamma t/2} \left[ \frac{e^{-\frac{1}{2}i\Delta mt} \pm e^{+\frac{1}{2}i\Delta mt}}{2} \right]$$

# Measuring B Oscillations



Examples:





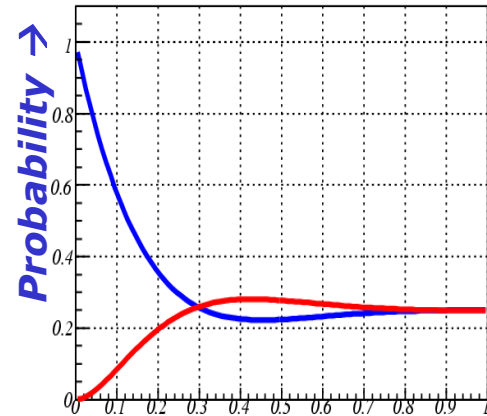
# Compare the mesons:

Probability to measure P or  $\bar{P}$ , when we start with 100% P

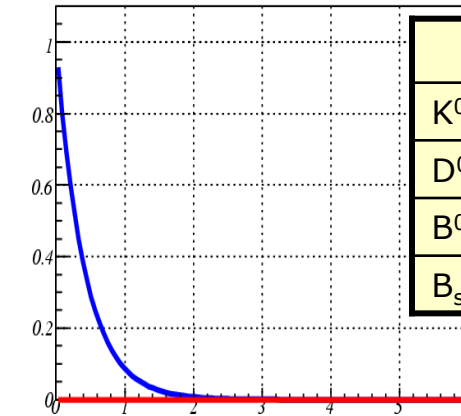
—  $P^0 \rightarrow P^0$

—  $P^0 \rightarrow \bar{P}^0$

K0 (ns)



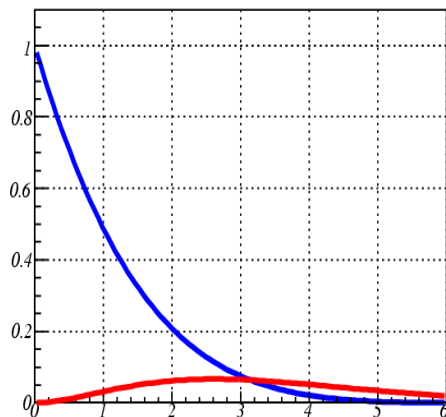
D0 (ps)



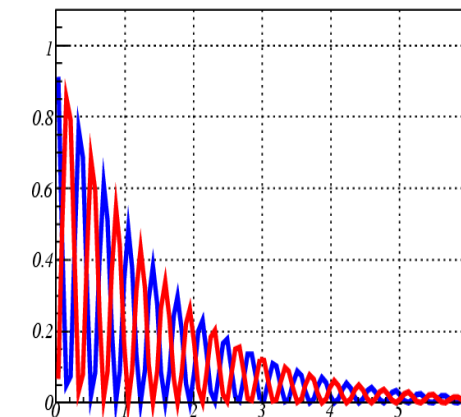
	$\langle \tau \rangle$	$\Delta m$	$x = \Delta m / \Gamma$	$y = \Delta \Gamma / 2\Gamma$
$K^0$	$2.6 \cdot 10^{-8} \text{ s}$	$5.29 \text{ ns}^{-1}$	$\Delta m / \Gamma_S = 0.49$	$\sim 1$
$D^0$	$0.41 \cdot 10^{-12} \text{ s}$	$0.001 \text{ fs}^{-1}$	$\sim 0$	0.01
$B^0$	$1.53 \cdot 10^{-12} \text{ s}$	$0.507 \text{ ps}^{-1}$	0.78	$\sim 0$
$B_s^0$	$1.47 \cdot 10^{-12} \text{ s}$	$17.8 \text{ ps}^{-1}$	12.1	$\sim 0.05$

By the way,  
 $\hbar = 6.58 \cdot 10^{-22} \text{ MeVs}$

B0 (ps)



Bs (ps)



Time →

$x = \Delta m / \Gamma$ : avg nr of oscillations before decay

# Summary (1)

- Start with Schrodinger equation:

$$i\frac{\partial\psi}{\partial t} = H\psi = (M - \frac{i}{2}\Gamma)\psi = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \psi$$

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

(2-component state in  $P^0$  and  $\bar{P}^0$  subspace)

- Find eigenvalue:

$$\begin{vmatrix} M - \frac{i}{2}\Gamma - \lambda & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma - \lambda \end{vmatrix} = 0$$

- Solve eigenstates:

$$\psi_{\pm} = \begin{pmatrix} p \\ \pm q \end{pmatrix}$$

$$\begin{aligned} |P_1\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \\ |P_2\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \end{aligned}$$

we find  $p$  and  $q$  by solving

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \lambda_{\pm} \begin{pmatrix} p \\ q \end{pmatrix} \longrightarrow \frac{q}{p} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}$$

- Eigenstates have diagonal Hamiltonian: **mass eigenstates!**

## Summary (2)

---

- Two mass eigenstates

$$|P_H\rangle = p|P^0\rangle + q|\bar{P}^0\rangle$$

$$|P_L\rangle = p|P^0\rangle - q|\bar{P}^0\rangle$$

- Time evolution:

$$|P_H(t)\rangle = e^{-im_H t - \frac{1}{2}\Gamma_H t} |P_H(0)\rangle$$

$$|P_L(t)\rangle = e^{-im_L t - \frac{1}{2}\Gamma_L t} |P_L(0)\rangle$$

$$|P^0\rangle = \frac{1}{2p} [|P_H\rangle + |P_L\rangle]$$

$$|\bar{P}^0\rangle = \frac{1}{2q} [|P_H\rangle - |P_L\rangle]$$

$$|P^0(t)\rangle = \frac{1}{2} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle$$

- Probability for  $|P^0\rangle \rightarrow |\bar{P}^0\rangle$  !
- Express in  $M=m_H+m_L$  and  $\Delta m=m_H-m_L \rightarrow \Delta m$  dependence

# Summary

---

- $p, q$ :  $|B_H\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$

$$|B_L\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

- $\Delta m, \Delta\Gamma$ :  $\Delta m = 2\Re\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$

$$\Delta\Gamma = 4\Im\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$$

$q, p, M_{ij}, \Gamma_{ij}$  related through:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}$$

- $x, y$ : mixing often quoted in *scaled* parameters:

$$x = \frac{\Delta m}{\Gamma} \quad y = \frac{\Delta\Gamma}{2\Gamma}$$

$$\cos(\Delta m t) = \cos\left(\frac{\Delta m}{\Gamma} \frac{t}{\tau}\right) = \cos\left(x \frac{t}{\tau}\right)$$

## Time dependence (if $\Delta\Gamma \sim 0$ , like for $B^0$ ):

---

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle$$

with

$$g_+(t) = e^{-imt} e^{-\Gamma t/2} \cos\frac{\Delta m t}{2}$$

$$|\bar{B}^0(t)\rangle = g_+(t)|\bar{B}^0\rangle + \frac{p}{q}g_-(t)|B^0\rangle$$

$$g_-(t) = e^{-imt} e^{-\Gamma t/2} i \sin\frac{\Delta m t}{2}$$

# Personal impression:

- People think it is a complicated part of the Standard Model (me too:-). Why?

## 1) Non-intuitive concepts?

- *Imaginary phase* in transition amplitude,  $T \sim e^{i\phi}$
- *Different bases* to express quark states,  $d' = 0.97 d + 0.22 s + 0.003 b$
- *Oscillations* (mixing) of mesons:  $|K^0\rangle \leftrightarrow |\bar{K}^0\rangle$

## 2) Complicated calculations?

$$\Gamma(B^0 \rightarrow f) \propto |A_f|^2 \left[ |g_+(t)|^2 + |\lambda|^2 |g_-(t)|^2 + 2\Re(\lambda g_+^*(t) g_-(t)) \right]$$

$$\Gamma(\bar{B}^0 \rightarrow f) \propto |\bar{A}_f|^2 \left[ |g_+(t)|^2 + \frac{1}{|\lambda|^2} |g_-(t)|^2 + \frac{2}{|\lambda|^2} \Re(\lambda^* g_+^*(t) g_-(t)) \right]$$

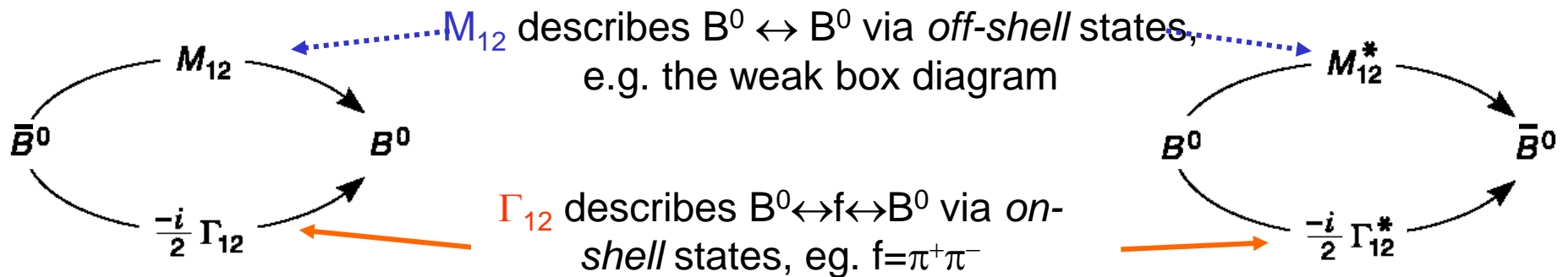
## 3) Many decay modes? “Beetopaipaigamma...”

- PDG reports 347 decay modes of the  $B^0$ -meson:
  - $\Gamma_1 \neq \nu_l \text{ anything} \quad (10.33 \pm 0.28) \times 10^{-2}$
  - $\Gamma_{347} \nu \nu \gamma \quad < 4.7 \times 10^{-5} \quad CL=90\%$
- And for one decay there are often more than one decay *amplitudes*...

# Describing Mixing

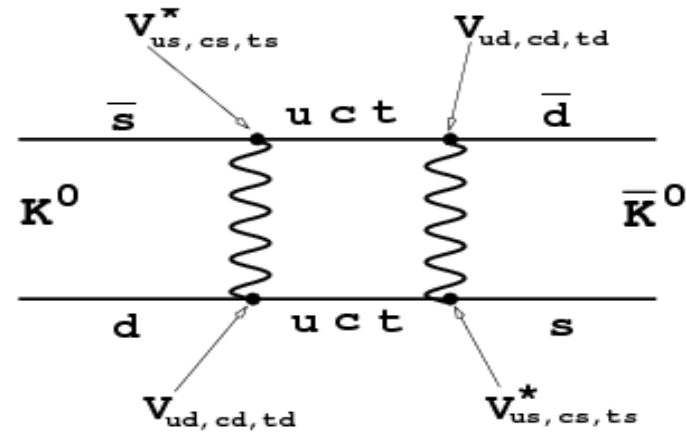
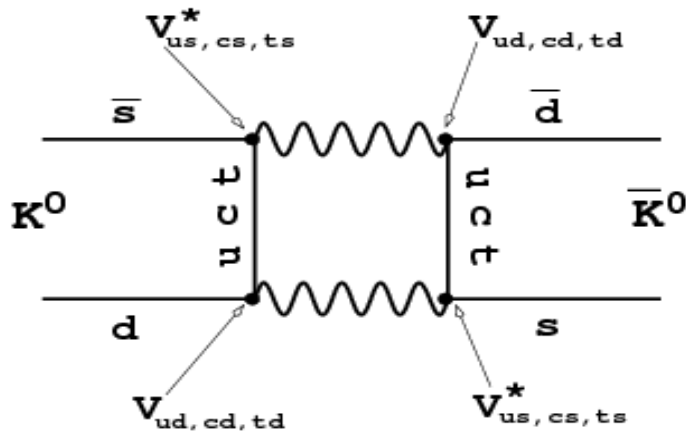
Time evolution of  $B^0$  and  $\bar{B}^0$  can be described by an *effective* Hamiltonian:

$$i\frac{\partial\psi}{\partial t} = H\psi = (M - \frac{i}{2}\Gamma)\psi = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \psi$$



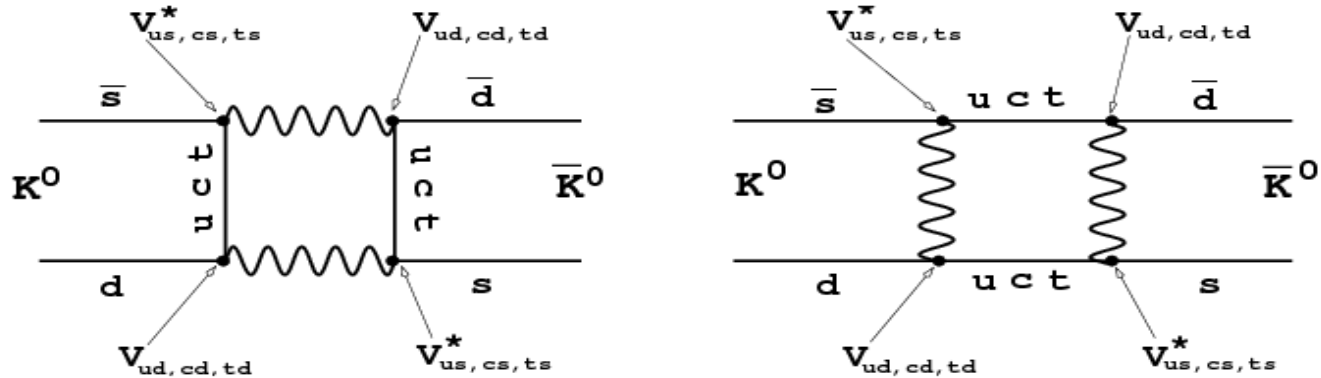
# Box diagram and $\Delta m$

$$\Delta m = m_{P_H^0} - m_{P_L^0} = \langle P_H^0 | H | P_H^0 \rangle - \langle P_L^0 | H | P_L^0 \rangle$$



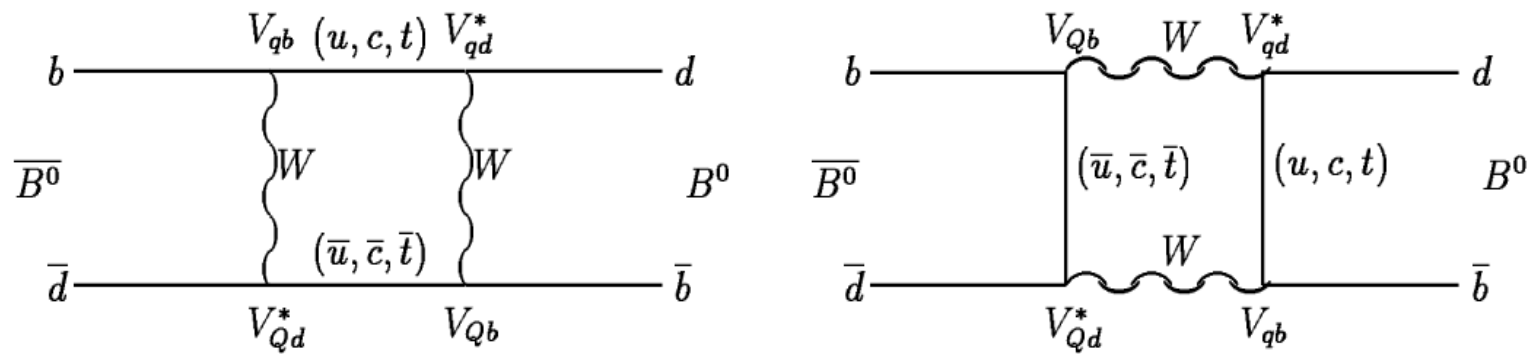
$$\begin{aligned} \mathcal{M}_{uu} = & i \left( \frac{-ig_w}{2\sqrt{2}} \right)^4 (V_{us}^* V_{ud} V_{us}^* V_{ud}) \\ & \int \frac{d^4 k}{(2\pi)^4} \left( \frac{-ig^{\lambda\sigma} - k^\lambda k^\sigma / m_W^2}{k^2 - m_W^2} \right) \left( \frac{-ig^{\alpha\rho} - k^\alpha k^\rho / m_W^2}{k^2 - m_W^2} \right) \\ & \left[ \bar{u}_s \gamma_\lambda (1 - \gamma^5) \frac{\not{k} + m_u}{k^2 - m_u^2} \gamma_\rho (1 - \gamma^5) u_d \right] \left[ \bar{v}_s \gamma_\alpha (1 - \gamma^5) \frac{\not{k} + m_u}{k^2 - m_u^2} \gamma_\sigma (1 - \gamma^5) v_d \right] \end{aligned}$$

# Box diagram and $\Delta m$



$$\Delta m_K = \frac{G_F^2 m_W^2}{6\pi^2} \eta_{QCD} B_K f_K^2 m_K \left[ S_0(m_c^2/m_W^2) |V_{cd} V_{cs}|^2 \right]$$

$$\Delta m_B = \frac{G_F^2 m_W^2}{6\pi^2} \eta_{QCD} B_B f_B^2 m_B \left[ S_0(m_t^2/m_W^2) |V_{td} V_{tb}|^2 \right]$$





# Box diagram and $\Delta m$ : Inami-Lim

- K-mixing

Table 2: Factors entering the matrix element, which is proportional to the product of the Inami-Lim function and the CKM term.

Internal quarks	I-L factor	$B^0$ CKM	$B_s^0$ CKM	$K^0$ CKM
<b>c,c</b>	$3.5 \times 10^{-4}$	$A^2 \lambda^6$ ( $7.4 \times 10^{-5}$ )	$A^2 \lambda^4$ ( $1.4 \times 10^{-3}$ )	$\lambda^2$ ( $2.7 \times 10^{-2}$ )
<b>c,t</b>	$3.0 \times 10^{-3}$	$A^2 \lambda^6  1 - \rho - i\eta $ ( $7.3 \times 10^{-5}$ )	$A^2 \lambda^4$ ( $1.5 \times 10^{-3}$ )	$A^2 \lambda^6  1 - \rho - i\eta $ ( $8.8 \times 10^{-6}$ )
<b>t,t</b>	2.5	$A^2 \lambda^6  1 - \rho - i\eta ^2$ ( $7.2 \times 10^{-5}$ )	$A^2 \lambda^4$ ( $1.5 \times 10^{-3}$ )	$A^4 \lambda^{10}  1 - \rho - i\eta ^2$ ( $1.1 \times 10^{-7}$ )

# Box diagram and $\Delta m$ : Inami-Lim

- $B^0$ -mixing

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# Box diagram and $\Delta m$ : Inami-Lim

- $B_s^0$ -mixing

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# Next: measurements of oscillations

---

## 1. $B^0$ mixing:

- 1987: Argus, first
- 2001: Babar/Belle, precise

## 2. $B_s^0$ mixing:

- 2006: CDF: first
- 2010: D0: anomalous ??

# $B^0$ mixing

---

# B<sup>0</sup> mixing

---

- What is the probability to observe a B<sup>0</sup>/ $\bar{B}^0$  at time t, when it was produced as a B<sup>0</sup> at t=0?
  - Calculate observable probability  $\Psi^*\Psi(t)$

$$\text{prob}(B^0(t) | B^0) \propto \frac{e^{-t/\tau}}{2} (1 + \cos(\Delta m t))$$

$$\text{prob}(\bar{B}^0(t) | B^0) \propto \frac{e^{-t/\tau}}{2} (1 - \cos(\Delta m t))$$

- A simple B<sup>0</sup> decay experiment.
  - Given a source B<sup>0</sup> mesons produced in a flavor eigenstate |B<sup>0</sup>>
  - You measure the decay time of each meson that decays into a flavor eigenstate (either B<sup>0</sup> or  $\bar{B}^0$ ) you will find that

$$\frac{N_{B^0 \rightarrow B^0}(t) - N_{B^0 \rightarrow \bar{B}^0}(t)}{N_{B^0 \rightarrow B^0}(t) + N_{B^0 \rightarrow \bar{B}^0}(t)} = \cos(\Delta m \cdot t)$$

# B<sup>0</sup> mixing: 1987 Argus

B<sup>0</sup> oscillations:

- First evidence of heavy top
- →  $m_{\text{top}} > 50 \text{ GeV}$
- Needed to break GIM cancellations

**NB: loops can reveal heavy particles!**

DESY 87-029 April 1987		Phys.Lett.B192:245,1987
<b>OBSERVATION OF B<sup>0</sup> - <math>\bar{B}^0</math> MIXING</b> <i>The ARGUS Collaboration</i>		
In summary, the combined evidence of the investigation of B <sup>0</sup> meson pairs, lepton pairs and B <sup>0</sup> meson-lepton events on the Υ(4S) leads to the conclusion that B <sup>0</sup> - $\bar{B}^0$ mixing has been observed and is substantial.		
Parameters	Comments	
$r > 0.09 \text{ } 90\%CL$	This experiment	
$x > 0.44$	This experiment	
$B \frac{1}{2} f_B \approx f_\pi < 160 \text{ MeV}$	B meson ( $\approx$ pion) decay constant	
$m_b < 5 \text{ GeV}/c^2$	b-quark mass	
$\tau_b < 1.4 \cdot 10^{-12} \text{ s}$	B meson lifetime	
$ V_{td}  < 0.018$	Kobayashi-Maskawa matrix element	
$\eta_{\text{QCD}} < 0.86$	QCD correction factor [17]	
$m_t > 50 \text{ GeV}/c^2$	t quark mass	

# B<sup>0</sup> mixing: t quark

# GIM: c quark

B<sup>0</sup> mixing pointed to the **top** quark:

K<sup>0</sup> → μμ pointed to the **charm** quark:

ARGUS Coll, Phys.Lett.B192:245,1987

GIM, Phys.Rev.D2,1285,1970

DESY 87-029  
April 1987

## OBSERVATION OF B<sup>0</sup> - B<sup>0</sup> MIXING

The ARGUS Collaboration

In summary, the combined evidence of the investigation of B<sup>0</sup> meson pairs, lepton pairs and B<sup>0</sup> meson-lepton events on the Υ(4S) leads to the conclusion that B<sup>0</sup>-B<sup>0</sup> mixing has been observed and is substantial.

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## Weak Interactions with Lepton-Hadron Symmetry\*

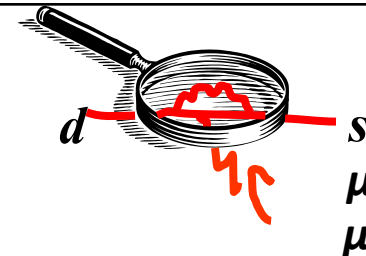
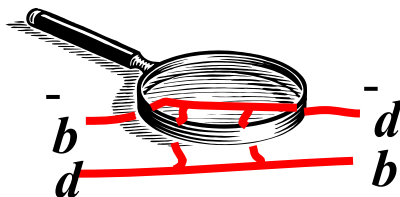
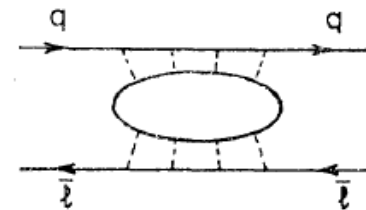
S. L. GLASHOW, J. ILIOPoulos, AND L. MAIANI†  
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139  
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Mills theory is discussed.

splitting, beginning at order  $G(GA^2)$ , as well as contributions to such unobserved decay modes as  $K_2 \rightarrow \mu^+ + \mu^-$ ,  $K^+ \rightarrow \pi^+ + l + \bar{l}$ , etc., involving neutral lepton

We wish to propose a simple model in which the divergences are properly ordered. Our model is founded in a quark model, but one involving four, not three, fundamental fermions; the weak interactions are medi-

new quantum number C for charm.



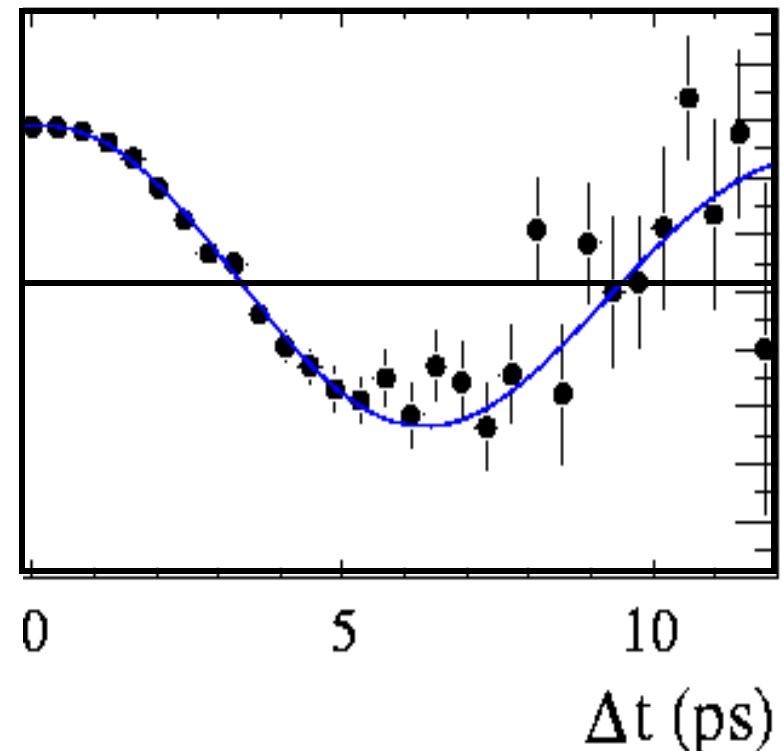


# B<sup>0</sup> mixing: 2001 B-factories

- You can really see this because (amazingly)  
B<sup>0</sup> mixing has same time scale as decay
  - $\tau=1.54$  ps
  - $\Delta m=0.5$  ps<sup>-1</sup>
  - 50/50 point at  $\pi\Delta m \approx \tau$
  - Maximal oscillation at  $2\pi\Delta m \approx 2\tau$

- Actual measurement of B<sup>0</sup>/  $\bar{B}^0$  oscillation
  - Also precision measurement of  $\Delta m$ !

$$\frac{N_{B^0 \rightarrow B^0}(t) - N_{B^0 \rightarrow \bar{B}^0}(t)}{N_{B^0 \rightarrow B^0}(t) + N_{B^0 \rightarrow \bar{B}^0}(t)} = \cos(\Delta m \cdot t)$$



# $B_s^0$ mixing

---

# $B_s^0$ mixing: 2006

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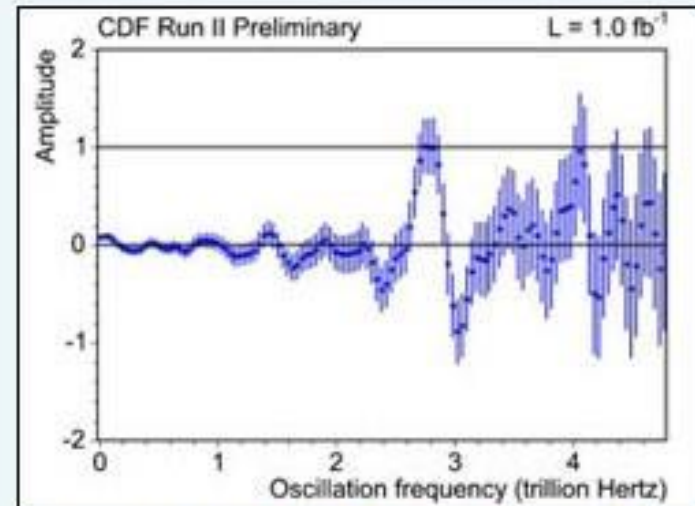
DOE/Fermi National Accelerator Laboratory

12.04.2006

## Fermilab's CDF scientists present a precision measurement of a subtle dance between matter and antimatter

Scientists of the CDF collaboration at the Department of Energy's Fermi National Accelerator Laboratory announced today (April 11, 2006) the precision measurement of extremely rapid transitions between matter and antimatter. As amazing as it may seem, it has been known for 50 years that very special species of subatomic particles can make spontaneous transitions between matter and antimatter. In this exciting new result, CDF physicists measured the rate of the matter-antimatter transitions for the  $B_s$  (pronounced "B sub s") meson, which consists of the heavy bottom quark bound by the strong nuclear interaction to a strange anti-quark, a staggering rate that challenges the imagination - 3 trillion times per second.

Dr. Raymond Orbach, Director of the DOE Office of Science, congratulated the CDF collaboration on "this important and fascinating new result" from the experiment.



The figure shows the CDF measurement of the  $B_s$  oscillation frequency at 2.8 trillion times per second. The analysis is designed such that possible oscillation frequencies have an amplitude consistent with 1.0 while those not present in the data will have an amplitude consistent with zero. Image courtesy CDF collaboration.

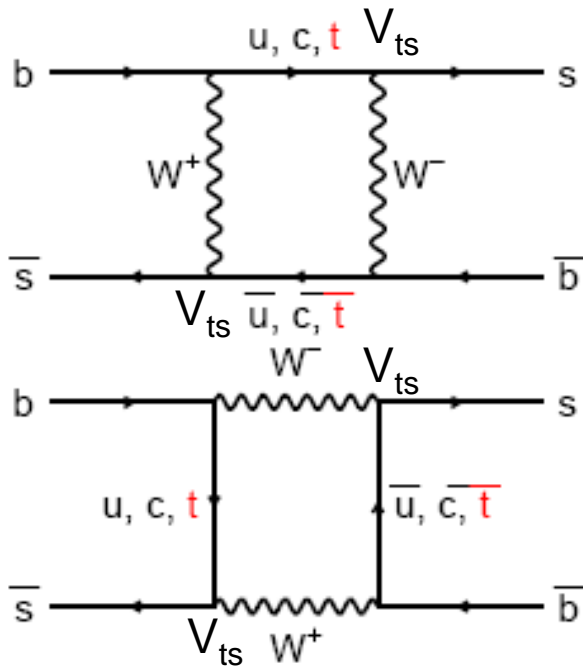
# $B_s^0$ mixing ( $\Delta m_s$ ): SM Prediction

## CKM Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

## Wolfenstein parameterization

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$



## Ratio of frequencies for $B^0$ and $B_s$

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}} \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$

$$V_{ts} \sim \lambda^2$$

$$V_{td} \sim \lambda^3$$

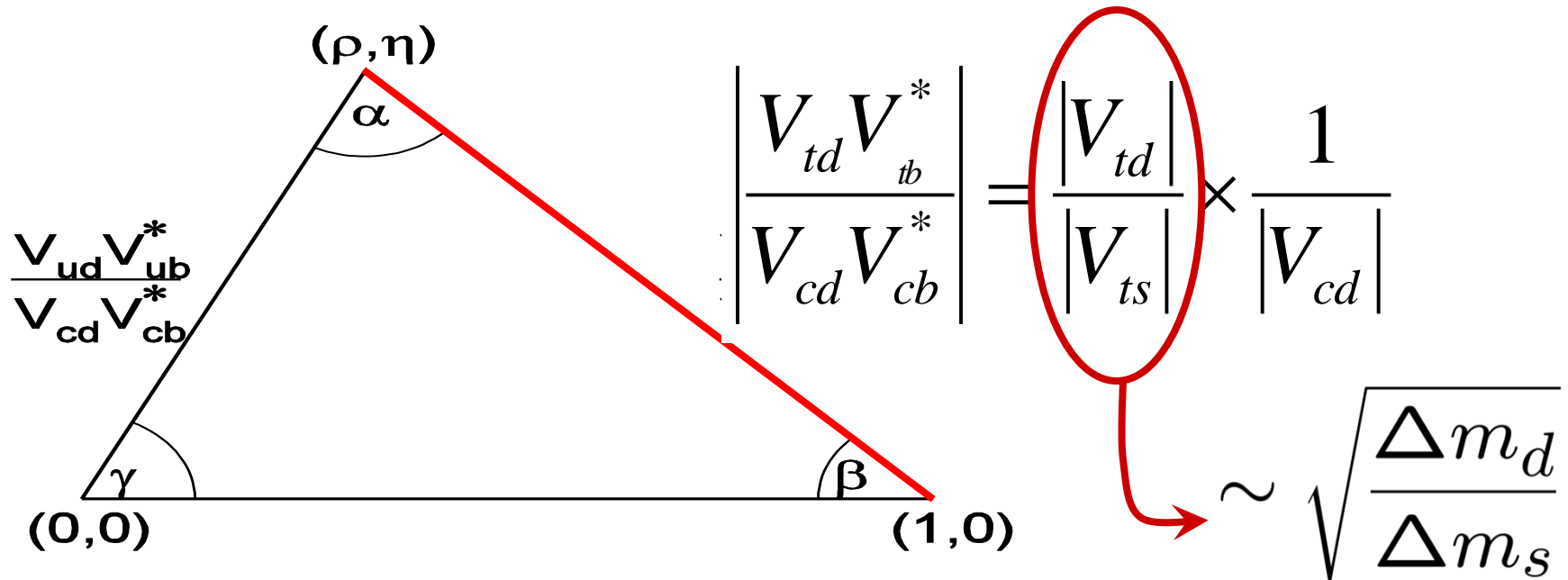
$$\rightarrow \Delta m_s \sim (1/\lambda^2) \Delta m_d \sim 25 \Delta m_d$$

$$\xi = 1.210^{+0.047}_{-0.035} \text{ from lattice QCD}$$

# $B_s^0$ mixing ( $\Delta m_s$ ): Unitarity Triangle

## CKM Matrix Unitarity Condition

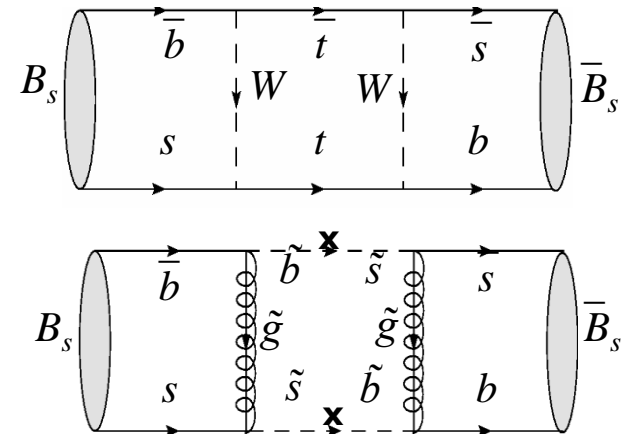
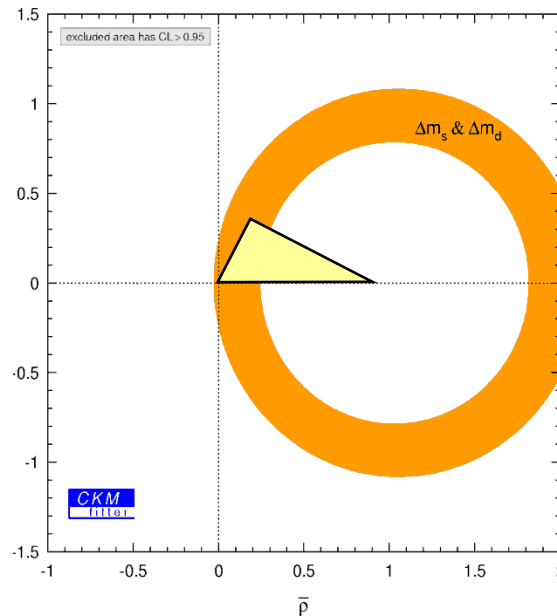
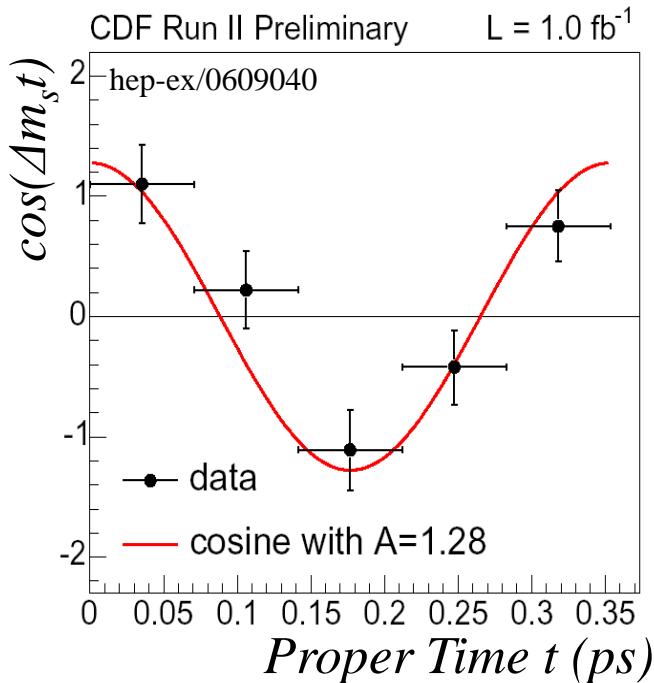
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



# $B_s^0$ mixing ( $\Delta m_s$ )

$$\frac{N_{B^0 \rightarrow B^0}(t) - N_{B^0 \rightarrow \bar{B}^0}(t)}{N_{B^0 \rightarrow B^0}(t) + N_{B^0 \rightarrow \bar{B}^0}(t)} = \cos(\Delta m \cdot t)$$

$$\Delta m_s = 17.77 \pm 0.10(\text{stat}) \pm 0.07(\text{sys}) \text{ ps}^{-1}$$

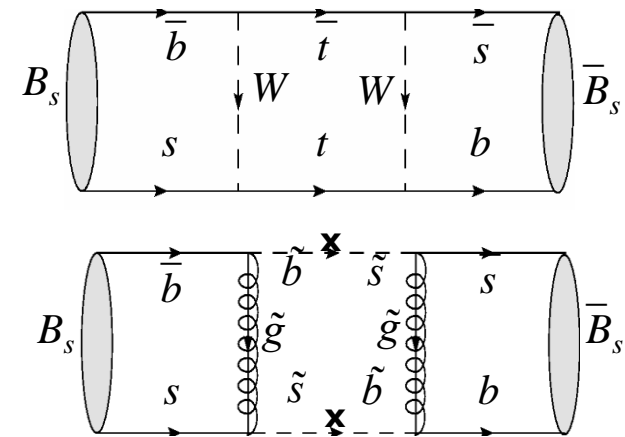
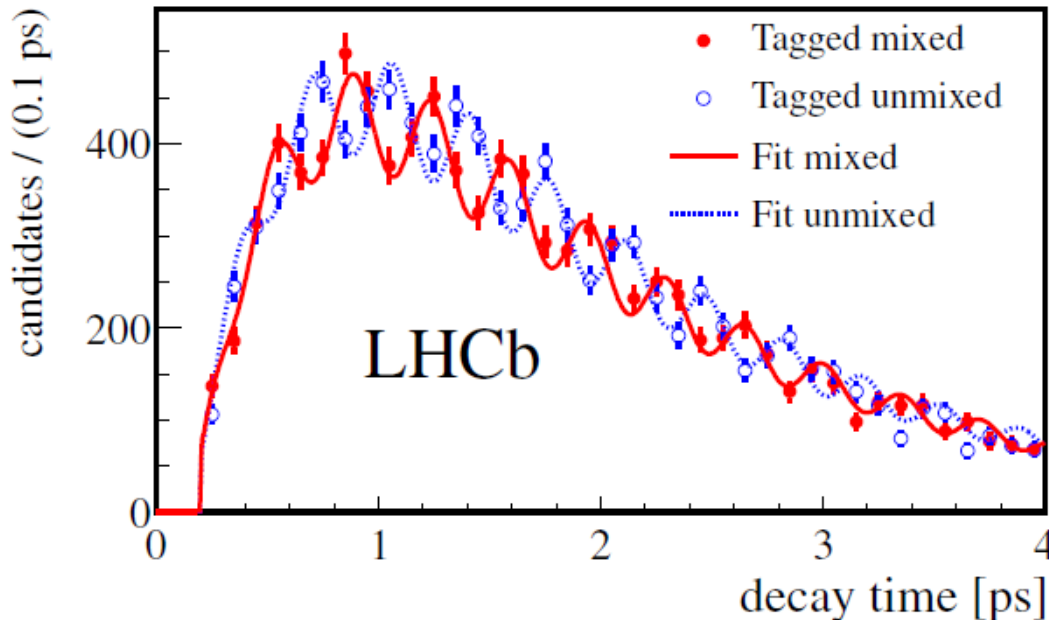


# $B_s^0$ mixing ( $\Delta m_s$ ): New: LHCb

$$\frac{N_{B^0 \rightarrow B^0}(t) - N_{B^0 \rightarrow \bar{B}^0}(t)}{N_{B^0 \rightarrow B^0}(t) + N_{B^0 \rightarrow \bar{B}^0}(t)} = \cos(\Delta m \cdot t)$$

$$\Delta m_s = 17.768 \pm 0.023 \text{ (stat)} \pm 0.006 \text{ (syst)} \text{ ps}^{-1}$$

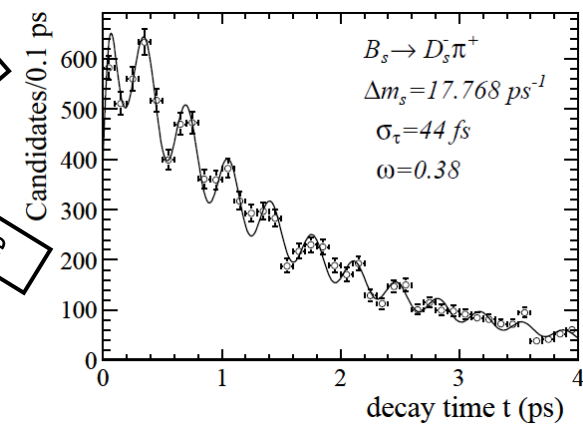
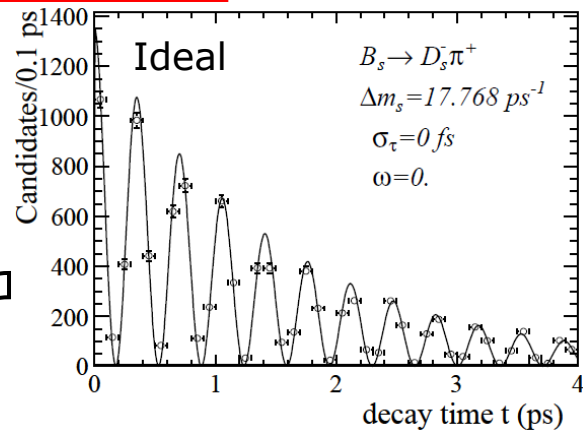
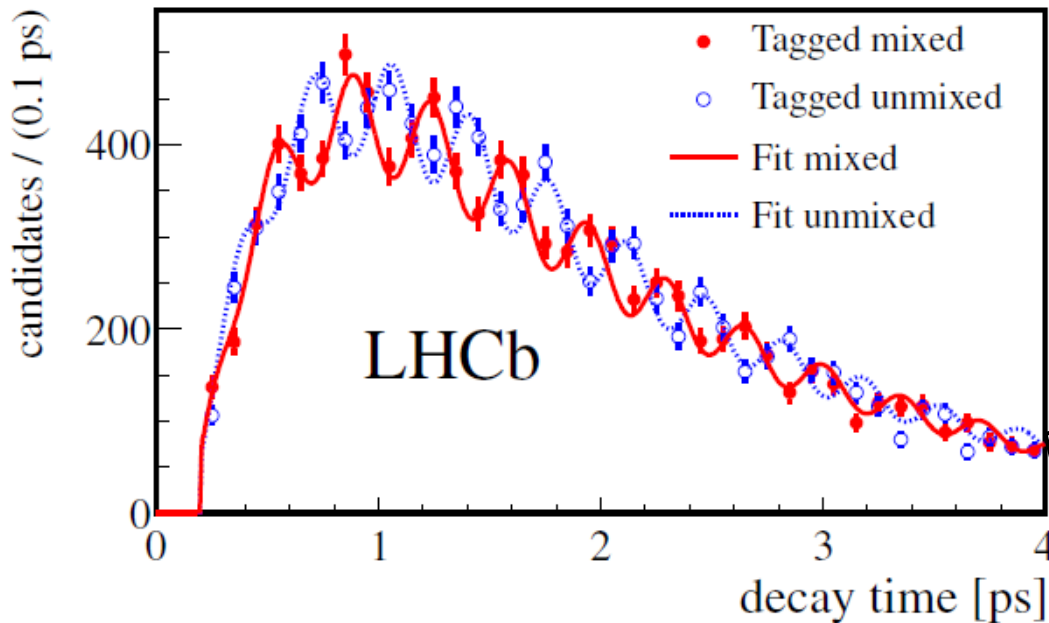
( $\Delta m_s = 17.77 \pm 0.10 \text{ (stat)} \pm 0.07 \text{ (sys)} \text{ ps}^{-1}$  CDF, 2006 [2])



# $B_s^0$ mixing ( $\Delta m_s$ ): New: LHCb

$$\frac{N_{B^0 \rightarrow B^0}(t) - N_{B^0 \rightarrow \bar{B}^0}(t)}{N_{B^0 \rightarrow B^0}(t) + N_{B^0 \rightarrow \bar{B}^0}(t)} = \cos(\Delta m \cdot t)$$

$$\Delta m_s = 17.768 \pm 0.023 \text{ (stat)} \pm 0.006 \text{ (syst)} \text{ ps}^{-1}$$



Tagging,  
resolution  
Acceptance



# Mixing $\rightarrow$ CP violation?

---

- NB: Just mixing is not necessarily CP violation!
- However, by studying certain decays with and without mixing, CP violation is observed
  
- Next: Measuring CP violation... **Finally**

# Meson Decays

---

- Formalism of meson oscillations:

$$|P^0(t)\rangle = \frac{1}{2} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle$$

$$|\langle \bar{P}^0(t) | P^0 \rangle|^2 = |g_-(t)|^2 \left( \frac{p}{q} \right)^2$$

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{1}{2} \Delta\Gamma t \pm \cos \Delta m t \right)$$

- Subsequent: decay

$$P^0 \rightarrow f$$

## Notation: Define $A_f$ and $\lambda_f$

---

$$\begin{aligned} A(f) &= \langle f|T|P^0\rangle & \bar{A}(f) &= \langle f|T|\bar{P}^0\rangle \\ A(\bar{f}) &= \langle \bar{f}|T|P^0\rangle & \bar{A}(\bar{f}) &= \langle \bar{f}|T|\bar{P}^0\rangle \end{aligned}$$

and define the complex parameter  $\lambda_f$  (not be confused with the Wolfenstein parameter  $\lambda$  !):

$$\lambda_f = \frac{q \bar{A}_f}{p A_f}, \quad \bar{\lambda}_f = \frac{1}{\lambda_f}, \quad \lambda_{\bar{f}} = \frac{q \bar{A}_{\bar{f}}}{p A_{\bar{f}}}, \quad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}} \quad (3.14)$$

The general expression for the time dependent decay rates,  $\Gamma_{P^0 \rightarrow f}(t) = |\langle f|T|P^0(t)\rangle|^2$ ,

# Some algebra for the decay $P^0 \rightarrow f$

$$|P^0(t)\rangle = \frac{1}{2} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} + e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |P^0\rangle + \frac{q}{2p} \left( e^{-im_H t - \frac{1}{2}\Gamma_H t} - e^{-im_L t - \frac{1}{2}\Gamma_L t} \right) |\bar{P}^0\rangle$$

$$\Gamma_{P^0 \rightarrow f}(t) = |A_f|^2 \left( |g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)] \right)$$

$$A(f) = \langle f|T|P^0\rangle$$

$$\bar{A}(f) = \langle f|T|\bar{P}^0\rangle$$

$$\lambda_f = \frac{q \bar{A}_f}{p A_f}$$

**Interference**

—  $P^0 \rightarrow f$

—  $P^0 \rightarrow \bar{P}^0 \rightarrow f$

## Some algebra for the decay $P^0 \rightarrow f$

---

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 \left( |g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\Re[\lambda_f g_+^*(t) g_-(t)] \right) \\
 \Gamma_{P^0 \rightarrow \bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left| \frac{q}{p} \right|^2 \left( |g_-(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2 |g_+(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+(t) g_-^*(t)] \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 \left( |g_-(t)|^2 + |\lambda_f|^2 |g_+(t)|^2 + 2\Re[\lambda_f g_+(t) g_-^*(t)] \right) \\
 \Gamma_{\bar{P}^0 \rightarrow \bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left( |g_+(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2 |g_-(t)|^2 + 2\Re[\bar{\lambda}_{\bar{f}} g_+^*(t) g_-(t)] \right) \quad (3.15)
 \end{aligned}$$

$$\begin{aligned}
 |g_{\pm}(t)|^2 &= \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{1}{2} \Delta\Gamma t \pm \cos \Delta m t \right) \\
 g_+^*(t) g_-(t) &= \frac{e^{-\Gamma t}}{2} \left( \sinh \frac{1}{2} \Delta\Gamma t + i \sin \Delta m t \right) \\
 g_+(t) g_-^*(t) &= \frac{e^{-\Gamma t}}{2} \left( \sinh \frac{1}{2} \Delta\Gamma t - i \sin \Delta m t \right) \quad (3.16)
 \end{aligned}$$

# The 'master equations'

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 \frac{e^{-\Gamma t}}{2} \left( (1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t + (1 - |\lambda_f|^2) \cos \Delta mt - 2\Im\lambda_f \sin \Delta mt \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 \frac{e^{-\Gamma t}}{2} \left( (1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t + 2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t - (1 - |\lambda_f|^2) \cos \Delta mt + 2\Im\lambda_f \sin \Delta mt \right)
 \end{aligned} \tag{3.17}$$

The diagram highlights the terms in the equations:
 

- ( 'direct' ) Decay**: Points to the  $(1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t$  term in the first equation and  $(1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta\Gamma t$  in the second.
- Interference**: Points to the  $2\Re\lambda_f \sinh \frac{1}{2} \Delta\Gamma t$  and  $2\Im\lambda_f \sin \Delta mt$  terms in both equations.

The sinh- and sin-terms are associated to the interference between the decays with and without oscillation. Commonly, the master equations are expressed as:

# The 'master equations'

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 \frac{e^{-\Gamma t}}{2} \left( (1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta \Gamma t + 2\Re \lambda_f \sinh \frac{1}{2} \Delta \Gamma t + (1 - |\lambda_f|^2) \cos \Delta m t - 2\Im \lambda_f \sin \Delta m t \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 \frac{e^{-\Gamma t}}{2} \left( (1 + |\lambda_f|^2) \cosh \frac{1}{2} \Delta \Gamma t + 2\Re \lambda_f \sinh \frac{1}{2} \Delta \Gamma t - (1 - |\lambda_f|^2) \cos \Delta m t + 2\Im \lambda_f \sin \Delta m t \right)
 \end{aligned} \tag{3.17}$$

('direct') Decay      Interference

The sinh- and sin-terms are associated to the interference between the decays with and without oscillation. Commonly, the master equations are expressed as:

$$\begin{aligned}
 \Gamma_{P^0 \rightarrow f}(t) &= |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{1}{2} \Delta \Gamma t + D_f \sinh \frac{1}{2} \Delta \Gamma t + C_f \cos \Delta m t - S_f \sin \Delta m t \right) \\
 \Gamma_{\bar{P}^0 \rightarrow f}(t) &= |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{1}{2} \Delta \Gamma t + D_f \sinh \frac{1}{2} \Delta \Gamma t - C_f \cos \Delta m t + S_f \sin \Delta m t \right)
 \end{aligned} \tag{3.18}$$

with

$$D_f = \frac{2\Re \lambda_f}{1 + |\lambda_f|^2} \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad S_f = \frac{2\Im \lambda_f}{1 + |\lambda_f|^2}. \tag{3.19}$$

# Classification of CP Violating effects

---

1. CP violation in decay

$$\Gamma(P^0 \rightarrow f) \neq \Gamma(\bar{P}^0 \rightarrow \bar{f})$$

This is obviously satisfied (see Eq. (3.15)) when

$$\left| \frac{\bar{A}_f}{A_f} \right| \neq 1.$$

2. CP violation in mixing

$$\text{Prob}(P^0 \rightarrow \bar{P}^0) \neq \text{Prob}(\bar{P}^0 \rightarrow P^0)$$

$$\left| \frac{q}{p} \right| \neq 1.$$

3. CP violation in interference

$$\Gamma(P^0(\rightsquigarrow \bar{P}^0) \rightarrow f)(t) \neq \Gamma(\bar{P}^0(\rightsquigarrow P^0) \rightarrow f)(t)$$

$$\Im \lambda_f = \Im \left( \frac{q \bar{A}_f}{p A_f} \right) \neq 0$$



# What's the time?

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# Table of contents

**Introduction** . . . . . 1

**1 CP Violation in the Standard Model** . . . . . 3

1.1 Parity transformation . . . . . 3

1.1.1 The Wu-experiment:  $^{60}\text{Co}$  decay . . . . . 4

1.1.2 Parity violation . . . . . 5

1.1.3 CPT . . . . . 7

1.2 C, P and T: Discrete symmetries in Maxwell's equations . . . . . 8

1.3 C, P and T: Discrete symmetries in QED . . . . . 9

1.4 CP violation and the Standard Model Lagrangian . . . . . 12

1.4.1 Yukawa couplings and the Origin of Quark Mixing . . . . . 12

1.4.2 CP violation . . . . . 15

**2 The Cabibbo-Kobayashi-Maskawa Matrix** . . . . . 17

2.1 Unitarity Triangle(s) . . . . . 17

2.2 Size of matrix elements . . . . . 20

2.3 Wolfenstein parameterization . . . . . 23

2.4 Discussion . . . . . 26

**3 Neutral Meson Decays** . . . . . 27

3.1 Neutral Meson Oscillations . . . . . 27

3.2 The mass and decay matrix . . . . . 27

3.3 Eigenvalues and -vectors of Mass-decay Matrix . . . . . 29

3.4 Time evolution . . . . . 30

3.5 The Amplitude of the Box diagram . . . . . 32

3.6 Meson Decays . . . . . 35

3.7 Classification of CP Violating Effects . . . . . 36

**4 CP violation in the B-system** . . . . . 39

4.1  $\beta$ : the  $B^0 \rightarrow J/\psi K_S^0$  decay . . . . . 40

4.2  $\beta_s$ : the  $B_s^0 \rightarrow J/\psi \phi$  decay . . . . . 44

4.3  $\gamma$ : the  $B_s^0 \rightarrow D_s^\pm K^\mp$  decay . . . . . 46

4.4 Direct CP violation: the  $B^0 \rightarrow \pi^- K^+$  decay . . . . . 48

4.5 CP violation in mixing: the  $B^0 \rightarrow l^+ \nu X$  decay . . . . . 49

4.6 Penguin diagram: the  $B^0 \rightarrow \phi K_S^0$  decay . . . . . 50

**5 CP violation in the K-system** . . . . . 51

5.1 CP and pions . . . . . 51

5.2 Description of the K-system . . . . . 53

5.3 The Cronin-Fitch experiment . . . . . 54

5.3.1 Regeneration . . . . . 56

5.4 CP violation in mixing:  $\epsilon$  . . . . . 57

5.5 CP violation in decay:  $\epsilon'$  . . . . . 58

**6 Experimental Aspects and Present Knowledge of Unitarity Triangle** . . . . . 61

6.1 B-meson production . . . . . 61

6.2 Flavour Tagging . . . . . 65

6.3 Present Knowledge on Unitarity Triangle . . . . . 66

6.3.1 Measurement of  $\sin 2\beta$  . . . . . 67

6.3.2 Measurement of  $\epsilon_K$  . . . . . 67

6.3.3  $|V_{ub}/V_{cb}|$  . . . . . 68

6.3.4 Measurement of  $\Delta m$  . . . . . 68

6.4 Outlook: the LHCb experiment . . . . . 71