Lecture notes Particle Physics II

Quantum Chromo Dynamics

3. U(1) Local Gauge Invariance

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November 16, 2013

Electric charge conservation

- In subatomic physics it is customary to express electric charge in units of the **elementary charge** $e = 1.6 \times 10^{-19}$ Coulomb. The electron then has charge -1, the positron $+1$, the up quark $+\frac{2}{3}$, the down quark $-\frac{1}{3}$, *etc.*, see the table on Page 1–5.
- As far as we know, total electric charge is the same in the initial and final state of any elementary reaction, and this **charge conservation** is experimentally verified to great accuracy.
- For instance electron decay

$$\mathrm{e} \to \gamma\, \nu_{\mathrm{e}}$$

is allowed by all known conservation laws but is forbidden by charge conservation and it indeed has never been observed. In fact, the life time of the electron is measured to be larger than 5×10^{26} years.

- We have seen that conserved quantities are related to symmetries in the Hamiltonian, or the Lagrangian, so the question is now which symmetry causes this charge conservation. Charge is obviously an *additive* conserved quantity so that the symmetry transformation must be *continuous*.
- The answer, as we will see, is that a so-called **gauge symmetry** is responsible for the charge conservation. Gauge transformations enter when interactions are described in terms of potentials, instead of forces. A well known example is from classical electrodynamics where we can transform the scalar and vector potentials in such a way that the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields are unaffected.

Gauge transformation in electrodynamics

- In electrodynamics the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields are related to the scalar and vector potentials V and $\boldsymbol{A}$ by

$$\boldsymbol{E} = -\partial \boldsymbol{A}/\partial t - \nabla V \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

- A gauge transformation leaves the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields invariant

$$V' = V - \partial\Lambda/\partial t \qquad \boldsymbol{A}' = \boldsymbol{A} + \nabla\Lambda$$

Here $\Lambda(\boldsymbol{x}, t)$ is an arbitrary function of $\boldsymbol{x}$ and t.

- To this gauge transformation corresponds a unitary operator that transforms the wave function of a particle in an electromagnetic field. We can write this transformation as (see page 2–7)

$$|\psi\rangle' = \exp(i\epsilon G)|\psi\rangle$$

where the generator G is to be identified later. Since Λ is an arbitrary function of $\boldsymbol{x}$ and t we require that ϵ is also an arbitrary function of $\boldsymbol{x}$ and t. Because ϵ can vary in space-time, we speak of a **local gauge transformation**.

- Now consider the Schrödinger equation of a particle in a static electric field before and after our gauge transformation

$$\begin{aligned} i\frac{\partial|\psi\rangle}{\partial t} &= \left(-\frac{\nabla^2}{2m} + q\,V\right)|\psi\rangle \\ i\frac{\partial|\psi\rangle'}{\partial t} &= \left(-\frac{\nabla^2}{2m} + q\,V'\right)|\psi\rangle' \end{aligned}$$

Here q is the charge of the particle.

- Because of **gauge invariance**, both equations should apply and this fixes the generator G, as we will now show.

From local gauge invariance to charge conservation

- Let us work out the transformed Schrödinger equation (for clarity we write ψ instead of $|\psi\rangle$). To simplify the mathematics we will take ϵ to be a function of t only, instead of $\boldsymbol{x}$ and t:

$$\begin{aligned}
i\frac{\partial}{\partial t}\left(e^{i\epsilon G}\,\psi\right) &= \left(-\frac{\nabla^2}{2m} + q\,V - q\frac{\partial\epsilon}{\partial t}\right) e^{i\epsilon G}\,\psi \\
ie^{i\epsilon G}\left(iG\,\psi\,\frac{\partial\epsilon}{\partial t} + \frac{\partial\psi}{\partial t}\right) &= e^{i\epsilon G}\left(-\frac{\nabla^2}{2m} + q\,V\right)\psi - e^{i\epsilon G}\,q\,\psi\,\frac{\partial\epsilon}{\partial t} \\
-e^{i\epsilon G}\,G\,\psi\,\frac{\partial\epsilon}{\partial t} + ie^{i\epsilon G}\,\frac{\partial\psi}{\partial t} &= ie^{i\epsilon G}\,\frac{\partial\psi}{\partial t} - e^{i\epsilon G}\,q\,\psi\,\frac{\partial\epsilon}{\partial t} \\
-e^{i\epsilon G}\,G\,\psi\,\frac{\partial\epsilon}{\partial t} &= -e^{i\epsilon G}\,q\,\psi\,\frac{\partial\epsilon}{\partial t} \\
G\,\psi &= q\,\psi
\end{aligned}$$

- We find that G is the charge operator Q! This is due to the cancellations that occur because ϵ is *local* (*i.e.* a function of t in our derivation); all this would not work if ϵ would be a constant.
- Clearly if H and Q commute, then it follows that the expectation value $\langle Q\rangle$ is conserved, in other words, charge is conserved.
- It is straight-forward to extend the derivation above to local transformations that depend on both $\boldsymbol{x}$ and t, instead of on t alone, but we will not do this here since it brings a lot of additional algebra and is not very illuminating.
- The family of phase transformations $U(\alpha) \equiv e^{i\alpha}$, with real α, forms a unitary Abelian group called U(1). Phase invariance is therefore also known as U(1) invariance.

Lagrangian formalism

- Gauge theories, or field theories in general, are usually defined in terms of a **Lagrangian**. This is a well-known concept from classical mechanics; a brief summary can be found on page 0–7.
- In classical mechanics the Lagrangian is the difference between the kinetic and potential energy and is written as the function $L(\boldsymbol{q}, \dot{\boldsymbol{q}})$ of a set of N coordinates q_i and velocities $\dot{q}_i$ that fully describe the system at any instant t. N is called the number of degrees of freedom of the system.
- The **action** is defined by

$$S[\text{path}] = \int_{t_1}^{t_2} \mathrm{d}t \; L(\boldsymbol{q}, \dot{\boldsymbol{q}})$$

  where the integral is taken along some path from $\boldsymbol{q}(t_1)$ to $\boldsymbol{q}(t_2)$.
- The **principle of least action** states that the system will evolve along the path that minimises the action. The equations of motion then follow from the **Euler-Lagrange** equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \qquad i = 1, \ldots, N$$

- Example: Mass m in a central potential $V(r)$

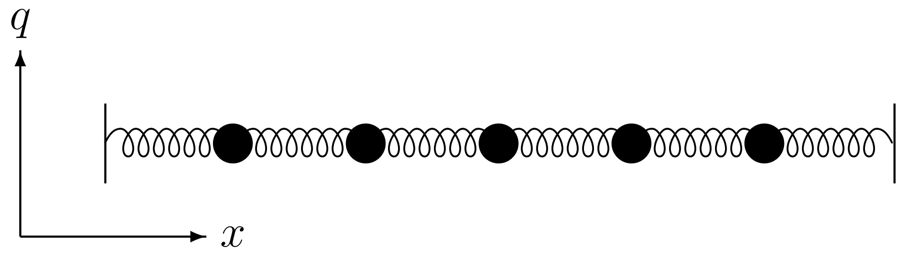
$$L(\boldsymbol{r}, \dot{\boldsymbol{r}}) = \tfrac{1}{2}m\dot{\boldsymbol{r}}^2 - V(r) \quad \rightarrow \quad m\ddot{\boldsymbol{r}} = -\nabla V(r)$$

- Example: Harmonic oscillator

$$L(x, \dot{x}) = \tfrac{1}{2}m\dot{x}^2 - \tfrac{1}{2}kx^2 \quad \rightarrow \quad m\ddot{x} = -kx$$

Infinite number of degrees of freedom

- Consider small *transverse* vibrations of a system of N masses m connected by springs.



The state of this system is described by the vertical deviations $q_1(t), \ldots, q_N(t)$ from the equilibrium position.

- We can let $N \to \infty$ in such a way that we obtain a vibrating string that can be described by a function $q(x,t)$.
- Such a function is called a **field**, a *displacement field* in this case.
- For our field, the Lagrangian is a function of q, $\dot{q}$, and the gradient $\mathrm{d}q/\mathrm{d}x$, and is written as the integral of a **Lagrangian density**

$$L(q, \dot{q}, \mathrm{d}q/\mathrm{d}x) = \int \mathrm{d}x \; \mathcal{L}(q, \dot{q}, \mathrm{d}q/\mathrm{d}x)$$

Generalising to 3 dimensions, the action integral reads

$$S[\text{path}] = \int_{t_1}^{t_2} \mathrm{d}t \int \mathrm{d}^3x \; \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \nabla \boldsymbol{q})$$

- In 4-vector notation this gives for the action integral of a field $\phi(x^\mu)$

$$S[\text{path}] = \int \mathrm{d}^4x \; \mathcal{L}(\phi, \partial_\mu \phi)$$

- In this notation the Euler-Lagrange equation reads

$$\boxed{\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \phi \right)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0}$$

Exercise 3.1:

The Lagrangian of a vibrating string is:

$$\mathcal{L} = (\partial\phi/\partial t)^2 - (\partial\phi/\partial x)^2.$$

(a) [0.25] Write this Lagrangian in 4-vector notation.

(b) [0.25] Now use the Euler-Lagrange equation

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \phi \right)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

to derive the wave equation of a vibrating string.

Remark: When you have to derive a field equation from a Lagrangian but do not feel confident in manipulating upper and lower Lorentz indices to keep track of the signs, you can always resort to writing it all out into the components (t, x, y, z). This is elaborate, but it works. Here is the conversion of the derivative indices

$$(\partial_0, \partial_1, \partial_2, \partial_3) = (\partial^0, -\partial^1, -\partial^2, -\partial^3) = (\partial_t, \partial_x, \partial_y, \partial_z)$$

And here is that of four-vector fields A, if present

$$(A^0, A^1, A^2, A^3) = (A_0, -A_1, -A_2, -A_3) = (A_t, A_x, A_y, A_z)$$

You may find it useful to also make conversion tables for $F^{\mu\nu}$ and $F_{\mu\nu}$.

A few Lagrangians ...

- Here are a few well-known Lagrangians that yield—via the E-L equations—several field equations of interest.

- Klein-Gordon Lagrangian for a real scalar field (spin 0).

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 \quad \underset{\text{E-L}}{\longrightarrow} \quad \partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

- KG for a complex scalar field (take ϕ and ϕ^* as independent).

$$\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) - m^2 \phi^* \phi \quad \underset{\text{E-L}}{\longrightarrow} \quad \begin{cases} \partial_\mu \partial^\mu \phi + m^2 \phi = 0 \\ \partial_\mu \partial^\mu \phi^* + m^2 \phi^* = 0 \end{cases}$$

- Dirac Lagrangian for a spin $\frac{1}{2}$ spinor field (ψ and $\overline{\psi}$ independent).

$$\mathcal{L} = i\overline{\psi}\gamma^\mu \partial_\mu \psi - m\overline{\psi}\psi \quad \underset{\text{E-L}}{\longrightarrow} \quad \begin{cases} (i\gamma^\mu \partial_\mu - m)\psi = 0 \\ (i\gamma^\mu \partial_\mu + m)\overline{\psi} = 0 \end{cases}$$

- Proca Lagrangian for a vector field (spin 1).

$$\mathcal{L} = -\frac{1}{4}(F^{\mu\nu})(F_{\mu\nu}) + \frac{1}{2}m^2 A^\nu A_\nu \quad \underset{\text{E-L}}{\longrightarrow} \quad \partial_\mu F^{\mu\nu} + m^2 A^\nu = 0,$$

  where $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$.

  For massless fields we recover the Maxwell equations in empty space (no sources or currents)

$$\partial_\mu F^{\mu\nu} = 0.$$

Exercise 3.2:

(a) [1.0] Derive the field equations from the KG, complex and Dirac Lagrangians given on page 3–9.

(b) [1.0] The Proca Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}(F^{\mu\nu})(F_{\mu\nu}) + \tfrac{1}{2}m^2 A^\nu A_\nu$$

The field tensor is defined by $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$.

- Show that

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = -(\partial^\mu A^\nu - \partial^\nu A^\mu) = -F^{\mu\nu}$$

  Hint: Work this out for two components, $(\mu = 0, \nu = 1)$ and $(\mu = 1, \nu = 2)$, for instance, and then generalise to the result above.[22] Remember that $\partial^\mu = (\partial_t, -\nabla)$ and $\partial_\mu = (\partial_t, +\nabla)$.

- Show that

$$\frac{\partial \mathcal{L}}{\partial A_\nu} = m^2 A^\nu$$

- Now write down the field equation.

(c) [0.5] The Maxwell Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu$$

- Show that the Euler-Lagrangian equation leads to the Maxwell equations (see page 0–6 for the Maxwell equations in 4-vector notation):

$$\partial_\mu F^{\mu\nu} = j^\nu$$

- Show that the current is conserved: $\partial_\mu j^\mu = 0$.

[22] For a shorter (but more tricky) derivation see H&M, comment on Exercise 14.3 and 14.4, page 374.

Global phase invariance of the Dirac Lagrangian

- The Dirac Lagrangian $i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi$ is manifestly invariant under a global phase change $\psi' = e^{i\alpha}\,\psi$ and $\overline{\psi}' = e^{-i\alpha}\,\overline{\psi}$.
- According to **Noether's theorem** this implies the existence of a conserved quantity. To find out what this is, consider the infinitesimal transformation

$$\begin{aligned} \psi' &= (1+i\alpha)\psi \quad &\to \quad &\delta\psi = +i\alpha\,\psi \\ \overline{\psi}' &= (1-i\alpha)\overline{\psi} \quad &\to \quad &\delta\overline{\psi} = -i\alpha\,\overline{\psi} \end{aligned}$$

- The variation in $\mathcal{L}$ is

$$\begin{aligned} \delta\mathcal{L} &= \tfrac{\partial\mathcal{L}}{\partial\psi}\,\delta\psi + \tfrac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\,\delta\partial_\mu\psi + \tfrac{\partial\mathcal{L}}{\partial\overline{\psi}}\,\delta\overline{\psi} + \tfrac{\partial\mathcal{L}}{\partial\partial_\mu\overline{\psi}}\,\delta\partial_\mu\overline{\psi} \\ &= i\alpha\left[\tfrac{\partial\mathcal{L}}{\partial\psi}\,\psi + \tfrac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\,\partial_\mu\psi - \tfrac{\partial\mathcal{L}}{\partial\overline{\psi}}\,\overline{\psi} - \tfrac{\partial\mathcal{L}}{\partial\partial_\mu\overline{\psi}}\,\partial_\mu\overline{\psi}\right] \\ &= i\alpha\left[\left(\tfrac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\,\tfrac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\right)\psi + \left(\partial_\mu\,\tfrac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\right)\psi + \tfrac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\,\partial_\mu\psi - \cdots\right] \end{aligned}$$

  Now the first term in brackets is zero (Euler-Lagrange) and the next two terms combine into

$$\left(\partial_\mu\,\tfrac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\right)\psi + \tfrac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\,\partial_\mu\psi = \partial_\mu\left(\tfrac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\,\psi\right)$$

  The same is true for the $\overline{\psi}$ terms so that we obtain

$$\delta\mathcal{L} = i\alpha\,\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi}\,\psi - \overline{\psi}\,\frac{\partial\mathcal{L}}{\partial\partial_\mu\overline{\psi}}\right) \propto \partial_\mu\left(\overline{\psi}\gamma^\mu\psi\right) \underset{\text{Iwant}}{=} 0$$

- Phase invariance leads to (electric) current conservation!

$$\partial_\mu j^\mu = 0 \quad \text{with} \quad j^\mu = q\,\overline{\psi}\gamma^\mu\psi \qquad (q \text{ is the electric charge})$$

Local charge conservation

- We have seen that global phase invariance leads to the continuity equation $\partial_\mu j^\mu = 0$ which reads in 3-vector notation

$$\frac{\partial \rho}{\partial t} = -\nabla \boldsymbol{j}$$

- The meaning of this continuity equation becomes clear after integration over a volume V

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \int_V \rho \, \mathrm{d}V = -\int_V \nabla \boldsymbol{j} \, \mathrm{d}V = -\int_S \boldsymbol{j} \cdot \hat{\boldsymbol{n}} \, \mathrm{d}S$$

  which tells us that the change of charge in some volume should be accounted for by the net flow of charge in or out of that volume. However, we can make this volume as small as we please because we know that charge is really *locally* conserved. Indeed, as we have already mentioned on page 3–3, the decay

$$\mathrm{e} \to \gamma \, \nu_{\mathrm{e}}$$

  has never been observed since it violates charge conservation. The electron is a point charge, so we cannot get more local than this!

- Local charge conservation suggests that the Lagrangian should not only be invariant under *global* phase transformations but also under *local* ones:

$$\psi' = e^{i\alpha(x)} \, \psi$$

- On Page 3–4 we have already investigated local phase invariance of the Schrödinger equation of a particle in a static electric field, but let us now investigate what happens when this local invariance is imposed on the Dirac Lagrangian.

Local phase invariance

- Take the Dirac Lagrangian

$$\mathcal{L} = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi$$

and consider a *local* transformation

$$\psi'(x) = e^{-ig_e\alpha(x)}\,\psi(x)$$

where we have introduced a strength parameter g_e (the electromagnetic coupling constant).

- The second term in $\mathcal{L}$ is clearly invariant but not the first term. This is because $\partial_\mu\psi$ depends on the infinitesimal neighbourhood of x where, by construction, ψ transforms differently than at x itself.

- This effect is seen in

$$\partial_\mu\psi' = \partial_\mu e^{-ig_e\alpha}\psi = e^{-ig_e\alpha}\left[\partial_\mu - ig_e(\partial_\mu\alpha)\right]\psi \neq e^{-ig_e\alpha}\,\partial_\mu\psi$$

- To restore local gauge invariance we can construct a **covariant derivative** which has the desired transformation property

$$D_\mu\psi \to D'_\mu\psi' \underset{\text{I want}}{=} e^{-ig_e\alpha}\,D_\mu\psi$$

- We can get this by introducing a **gauge field** A^μ such that

$$D_\mu\psi = (\partial_\mu + ig_eA_\mu)\,\psi.$$

- Indeed, provided that A_μ transforms as

$$A'_\mu = A_\mu + \partial_\mu\alpha$$

we find that, as you can easily check,

$$D'_\mu\psi' = (\partial_\mu + ig_eA'_\mu)\,e^{-ig_e\alpha}\,\psi = e^{-ig_e\alpha}\,(\partial_\mu + ig_eA_\mu)\,\psi = e^{-ig_e\alpha}\,D_\mu\psi$$

Exercise 3.3: [×] Well, please check it.

Locally invariant Dirac Lagrangian

- So we can now propose, as a first step, the Lagrangian

$$\mathcal{L} = i\overline{\psi}\gamma^\mu D_\mu\psi - m\overline{\psi}\psi = \underbrace{i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi}_{\text{free term}} - \underbrace{g_\mathrm{e}(\overline{\psi}\gamma^\mu\psi)A_\mu}_{\text{interaction term}}$$

  which is invariant under local phase transformations and has acquired an interaction term $j^\mu A_\mu$ in addition to the free Lagrangian.

- We have a free term for the Dirac field, which suggests that we should add a free term (Proca Lagrangian) for the gauge field A_μ

$$\mathcal{L} = -\tfrac{1}{4}(F^{\mu\nu})(F_{\mu\nu}) + \tfrac{1}{2}m^2 A^\nu A_\nu$$

- **Exercise 3.4**: [0.5] Check that the first term is invariant under the gauge transformation $A'_\mu = A_\mu + \partial_\mu\alpha$ but not the second term.

- To maintain gauge invariance we are thus forced to set $m = 0$ and consider only a massless gauge field which, of course, turns out to be the electromagnetic (photon) field.

- We have, in fact, found here a restriction that also applies to the SU(2) and SU(3) gauge invariant Lagrangians that we will consider later on:

> To maintain gauge invariance, the gauge field must be massless

The Lagrangian of QED

- We now can write-down the QED Lagrangian describing the interaction of Dirac particles with the electromagnetic field

$$
\boxed{\begin{aligned}
\mathcal{L}_{\mathrm{QED}} &= \overline{\psi}(i\not{D} - m)\psi - \tfrac{1}{4}(F^{\mu\nu})(F_{\mu\nu}) \\
&= \overline{\psi}(i\not{\partial} - m)\psi - g_{\mathrm{e}}(\overline{\psi}\gamma^{\mu}\psi)A_{\mu} - \tfrac{1}{4}(F^{\mu\nu})(F_{\mu\nu})
\end{aligned}}
$$

  In the expression above, we have introduced the usual shorthands $\not{\partial} \equiv \gamma^{\mu}\partial_{\mu} = \gamma_{\mu}\partial^{\mu}$ and $\not{D} \equiv \gamma^{\mu}D_{\mu} = \gamma_{\mu}D^{\mu}$.

- Note that the last two terms in the QED Lagrangian correspond to Maxwell Lagrangian

$$\mathcal{L}_{\mathrm{Maxwell}} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^{\mu}A_{\mu}$$

- This Lagrangian leads to the Maxwell equations (see Exercise 3.2)

$$\partial_{\mu}F^{\mu\nu} = j^{\nu}$$

  with j^{μ} the Dirac current $g_{\mathrm{e}}(\overline{\psi}\gamma^{\mu}\psi)$.

From Lagrangian to Feynman rules

- The Lagrangians we have thus far considered may describe *classical* as well as *quantum* fields. Field quantisation is the realm of **quantum field theory** which is outside the scope of these lectures. In QFT, particles emerge as quanta of the associated fields; photons are then the quanta of the electromagnetic field A^μ, leptons and quarks are the quanta of the Dirac field ψ, and gluons are the quanta of an $\mathrm{SU}(3)_\mathrm{c}$ gauge field, as we will see. Field quantisation does not require a modification of the Lagrangian or the field equations, which stay formally the same.

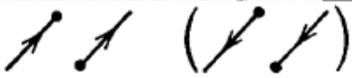
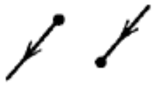
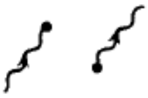
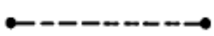
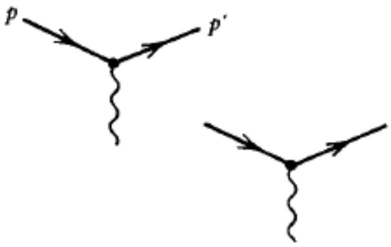
- To each Lagrangian corresponds a particular set of **Feynman rules**. The derivation of these rules is part of QFT and beyond the scope of these lectures. We just mention at this point that the QED Lagrangian contains two types of terms, as we have seen: free terms for the participating fields, and interaction terms that were generated through local gauge invariance. In general, we have the following correspondence:

$$\begin{aligned} \text{Free Lagrangian} &\rightarrow \text{propagator} \\ \text{Interaction term} &\rightarrow \text{vertex factor} \end{aligned}$$

- For the Feynman rules of QED, you can have a look at PP-I section 8, Griffiths section 7.5 and appendix D, or H&M section 6.17 (reproduced on the next page).

Feynman rules for QED

TABLE 6.2
Feynman Rules for $-i\mathfrak{M}$

		Multiplicative Factor
• **External Lines**		
Spin 0 boson (or antiboson)		1
Spin $\frac{1}{2}$ fermion (in, out)		$u, \bar{u}$
antifermion (in, out)		$\bar{v}, v$
Spin 1 photon (in, out)		$\varepsilon_\mu, \varepsilon_\mu^*$
• **Internal Lines—Propagators (need $+i\varepsilon$ prescription)**		
Spin 0 boson		$\dfrac{i}{p^2 - m^2}$
Spin $\frac{1}{2}$ fermion		$\dfrac{i(\not{p} + m)}{p^2 - m^2}$
Massive spin 1 boson		$\dfrac{-i\left(g_{\mu\nu} - p_\mu p_\nu / M^2\right)}{p^2 - M^2}$
Massless spin 1 photon (Feynman gauge)		$\dfrac{-ig_{\mu\nu}}{p^2}$
• **Vertex Factors**		
Photon—spin 0 (charge $-e$)		$ie(p + p')^\mu$
Photon—spin $\frac{1}{2}$ (charge $-e$)		$ie\gamma^\mu$

Loops: $\int d^4k/(2\pi)^4$ over loop momentum; include -1 if fermion loop and take the trace of associated γ-matrices
Identical Fermions: -1 between diagrams which differ only in $e^- \leftrightarrow e^-$ or initial $e^- \leftrightarrow$ final e^+