Classical approximation to quantum cosmological correlation functions

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`arXiv:0707.0842`

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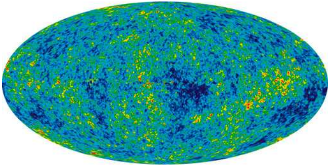
Nikhef, 19th October 2007

Introduction

Inflation

- Insight in physics of inflation from cosmological perturbations



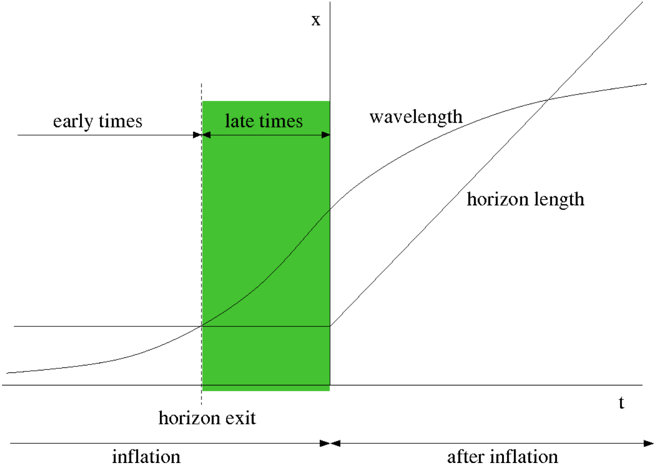
Credit: WMAP/NASA Science Team

- Correlation functions

Power spectrum: $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle = \delta^3(\mathbf{k}_1 + \mathbf{k}_2)\, \frac{2\pi^2}{k_1^3}\, \mathcal{P}_\zeta(k_1)$

Bispectrum: $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, B_\zeta(k_1, k_2, k_3)$

Introduction



Introduction

Evolution after horizon exit

Evolution after horizon exit described by classical physics

- Arguments
  - large particle number
  - decoherence

C. Kiefer e.a. *Class. Quant. Grav.* **24** (2007) 1699, C.P. Burgess e.a., arXiv:astro-ph/0601646
T. Prokopec, G.I. Rigopoulos, arXiv:astro-ph/0612067

- Used to calculate non-Gaussianities in
  - stochastic approach

G.I. Rigopoulos, E.P.S. Shellard and B.J.W. van Tent, *Phys. Rev.* **D73** (2006) 083521, 083522

  - δN formalism

D.H. Lyth and Y. Rodriguez, *Phys. Rev. Lett.* **95**, 121302 (2005)

Quantum corrections?

S. Weinberg, *Phys. Rev.* **D72** (2005) 043514, **D74** (2006) 023508

Introduction

This work

Goals

- Quantum corrections? Size?
- Understanding

Method

Classical approximation for toy model (ϕ^3 theory on dS):

- Formulate quantum theory in suitable way
- Compare with classical theory
- Study how the differences (quantum corrections) behave after horizon exit

Introduction

Notation

Classical approximation for toy model:

- ϕ^3 theory:

$$\mathcal{L}[\phi] = -\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) + \frac{1}{3!}\lambda\phi^3(x) + \frac{1}{2}\delta_m\phi^2\right)$$

- de Sitter background

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$$

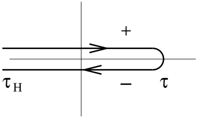
  - conformal time: $\tau = -\int_t^\infty dt'/a(t')$, runs from $-\infty$ to 0
  - scale factor: $a(t) = \exp(Ht)$, or $a(\tau) = -1/H\tau$, with Hubble scale H
  - horizon exit: $|k\tau| = 1$, for comoving wavenumber k
- Consider equal time correlation functions

$$\langle\phi(\tau, \mathbf{x}_1)\ldots\phi(\tau, \mathbf{x}_n)\rangle$$

Quantum theory

Closed Time Path formalism

In non-equilibrium quantum field theory, one calculates the expectation value $\langle Q(\tau) \rangle$ by using the



Closed Time Path formalism

Time integration along Schwinger-Keldysh contour.

$$\langle 0|\, U^{\dagger}(\tau, \tau_H)\, Q\, U(\tau, \tau_H)\, |0\rangle$$

where $U(\tau_1, \tau_2) = T \exp \left[-i \int_{\tau_1}^{\tau_2} d\tau' \; H(\tau') \right]$ is the time evolution operator

T. Brunier, V.K. Onemli, R.P. Woodard, *Class. Quant. Grav.* **22** (2005) 59
D. Boyanovsky, H.J. de Vega, N.G. Sanchez, *Nucl. Phys.* **B747** (2006) 25
S. Weinberg, *Phys. Rev.* **D72** (2005) 043514, **D74** (2006) 023508

Quantum theory

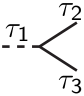
Keldysh basis

Feynman rules in the Keldysh basis

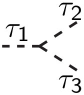
$$\begin{pmatrix} \phi^{(1)} \\ \phi^{(2)} \end{pmatrix} = \begin{pmatrix} (\phi^{+} + \phi^{-})/2 \\ \phi^{+} - \phi^{-} \end{pmatrix}$$

$$= F(k, \tau_1, \tau_2)$$

$$= -iG^{R}(k, \tau_1, \tau_2)$$



$$= -i\lambda\, a^{4}(\tau_1)\delta(\tau_1 - \tau_2)\delta(\tau_1 - \tau_3)$$



$$= -i\frac{\lambda}{4}\, a^{4}(\tau_1)\delta(\tau_1 - \tau_2)\delta(\tau_1 - \tau_3)$$

Example

Two point function

$$\int d^3x\; e^{i\mathbf{k}\cdot\mathbf{x}}\langle\phi(\tau,\mathbf{x})\phi(\tau,\mathbf{0})\rangle$$

Tree level:

$F(k,\tau,\tau)$

One loop level:

Vanishing diagram:

Example

$$\frac{\lambda^2}{36(2\pi)^2k^3}\Bigg\{\frac{7}{9\epsilon}+\frac{392}{27}-\frac{7}{3}\gamma-\frac{17}{18}\pi^2-\frac{4}{3}\ln 2-4\zeta(3)-\ln\frac{2\mu}{H}+\frac{4}{9}\ln(-k\tau_H)+$$
$$\left(\frac{2}{\epsilon}+15-\frac{17}{3}\gamma-\frac{2}{3}\pi^2-\frac{8}{3}\ln 2-3\ln\frac{2\mu}{H}+\frac{8}{3}\ln(-k\tau_H)\right)\ln\frac{\tau}{\tau_H}+$$
$$\left(\frac{2}{\epsilon}+\frac{22}{3}-2\gamma-2\ln 2+4\ln(-k\tau_H)\right)\ln^2\frac{\tau}{\tau_H}+\frac{8}{3}\ln^3\frac{\tau}{\tau_H}+\mathcal{O}(\frac{\tau}{\tau_H})+\mathcal{O}(\epsilon)\Bigg\}$$

with the infrared regulator

$$\epsilon=\frac{m^2}{3H^2}$$

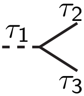
Classical theory

It can be shown that the diagrams in the classical theory form a subset of the Feynman diagrams of the quantum theory

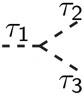
Quantum Feynman rules

$$= F(k, \tau_1, \tau_2)$$

$$= -iG^R(k, \tau_1, \tau_2)$$



$$= -i\lambda\, a^4(\tau_1)\delta(\tau_1 - \tau_2)\delta(\tau_1 - \tau_3)$$



$$= -i\frac{\lambda}{4}\, a^4(\tau_1)\delta(\tau_1 - \tau_2)\delta(\tau_1 - \tau_3)$$

Example

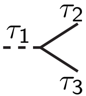
Classical theory

It can be shown that the diagrams in the classical theory form a subset of the Feynman diagrams of the quantum theory

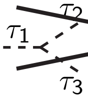
Classical graphical rules

$$= F_{\rm cl}(k,\tau_1,\tau_2)$$

$$= -iG^R(k,\tau_1,\tau_2)$$



$$= -i\lambda\, a^4(\tau_1)\delta(\tau_1-\tau_2)\delta(\tau_1-\tau_3)$$



$$\xcancel{= -i\frac{\lambda}{4}\, a^4(\tau_1)\delta(\tau_1-\tau_2)\delta(\tau_1-\tau_3)}$$

Example

G. Aarts and J. Smit, *Nucl.Phys.* **B511** (1998) 451-478

Classical theory

We obtain the 'best' classical approximation for

$$F_{\mathrm{cl}}(k,\tau_1,\tau_2) = F(k,\tau_1,\tau_2)$$

- With this choice for F_{cl}, the classical theory is ultraviolet divergent
- This can be treated by introducing an ultraviolet cutoff Λ, which should be chosen larger than H.

Late time behaviour

Late time behaviour (after horizon exit, $|k\tau| \ll 1$)

- Analysis by counting powers of τ
  - $\tau^{n>0}$: decay quickly after horizon exit
  - **τ^0: powers of $\ln\tau \rightarrow$ growing late time contributions**
  - $\tau^{n<0}$: do not occur
- Separately for small ($|p\tau| \ll 1$) and large ($|p\tau| \gtrsim 1$) internal momenta p

Small internal momenta

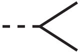
The two point functions can be expanded in $k\tau$ or $p\tau$

$$
\begin{aligned}
F(k,\tau_1,\tau_2) =& \frac{H^2}{2k^3}[1+\mathcal{O}(k^2\tau_i^2)] \\
G^R(k,\tau_1,\tau_2) =& \theta(\tau_1-\tau_2)\frac{H^2}{3k^3}[k^3(\tau_1^3-\tau_2^3)+\mathcal{O}(k^4\tau_i^4)]
\end{aligned}
$$

Counting of powers of τ

- vertex: τ^{-3} (from $d\tau\ a^4(\tau) = d\tau/\tau^4$)
- F propagator: τ^0
- G^R propagator: τ^3

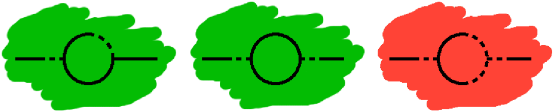
Small internal momenta

Diagrams with only  vertices

- Proportional to $\tau^0 \rightarrow$ **late time contributions!**
- These are exactly the diagrams of the classical theory

Diagrams with at least one  vertex

- Proportional to $\tau^{n>0} \rightarrow$ **no** late time contributions!

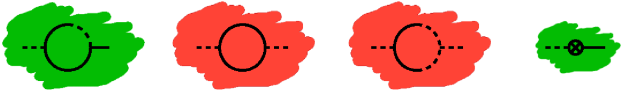
Example: 

Large internal momenta

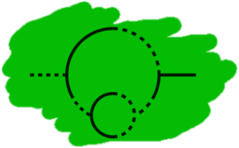
1PI diagrams with one external dashed line lead to late time contributions

Examples

One loop:



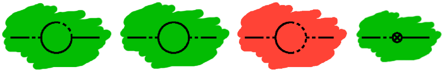
Two loops:



Classical approximation

At 1 loop

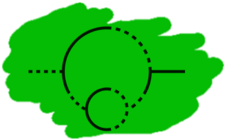
- All late time contributions are contained in classical diagrams:



- Classical approximation is UV divergent → cutoff

At more than 1 loop

- Classical approximation misses contributions



Classical approximation

Summary

$$\mathcal{C}_{\lambda,k}\Bigg\{ f^{(0)}_{\mathrm{cl},<} + \lambda^2 \left(f^{(1)}_{\mathrm{cl},<} + f^{(1)}_{\mathrm{cl},>} \right) + \\ \lambda^4 \left(f^{(2)}_{\mathrm{cl},<} + f^{(2)}_{\mathrm{cl},>} + f^{(2)}_{\mathrm{q}} \right) + \lambda^6 \left(f^{(3)}_{\mathrm{cl},<} + f^{(3)}_{\mathrm{cl},>} + f^{(3)}_{\mathrm{q}} \right) + \ldots \Bigg\}$$

Classical methods that are used in practice

- small internal momenta $f_{cl,<}$: OK
- large internal momenta $f_{cl,>}$: problematic

Conclusions

- There are quantum corrections (i.e. corrections that cannot be reproduced in classical theory)
- They occur from 2 loop order
- Already at one loop order one has to be careful with classical approximation