Constraining Geometries in the AdS / QCD Correspondence

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Overview

Can we verify / discount geometries in the AdS / QCD correspondence?

- The AdS / CFT correspondence...
- ... the AdS / QCD correspondence.
- Calculation of the static interquark potential.
- Back-reacted geometry and friends.
- Comparing geometries using lattice data.
- Conclusions.

AdS / CFT

- String Theory might be a fundamental theory of nature.
- Even if it isn't, it is still useful...
- Maldacena conjecture: String Theory in higher dimensional space $\equiv$ gauge theory on the (lower dim) boundary of this space.
- Originally formulated using type IIB string theory on $\mathrm{AdS}_5 \otimes \mathrm{S}^5$ and $\mathcal{N} = 4$ $U(N)$ Super-Yang-Mills at large N.
- Strongly coupled gauge dynamics $\Leftrightarrow$ Weakly coupled string dynamics.
- Tantalising prospect - can we find a perturbative description of strongly coupled QCD?

Two paths to QCD

AdS / CFT Approach

- Consider alternative string theories, generalising Maldacena conjecture.
- Try to find field theory duals.
- 10 dimensional theories $\rightarrow$ compactification needed.
- This last step is ambiguous.
- (semi)- Rigorous.

AdS / QCD Approach

- Try and *guess* a suitable gravity dual to QCD.
- Work already in 5 dimensions - no compactification to worry about.
- However, clearly lots of options for the bulk geometry.
- Slightly less than rigorous.

AdS / QCD - Justification

- There is no proof that a correspondence should exist between non-supersymmetric gauge theories and string theories.
- Neither is there a proof of the original conjecture, although there are strong foundations.
- However, it may well be that there is a dual field description of some string theory that is approximately QCD-like.
- Or it may turn out to be the case that calculated observables are relatively insensitive to the bulk geometry.
- Have already been successes from the AdS / QCD approach (e.g. in thermal field theory, QGP etc.), so the situation is clearly not hopeless.

But...

- Still don't know which geometry to use.
- Can use theoretical arguments - which are better motivated? Example: back-reaction effects (see later).
- Or can use phenomenological arguments - which better describe "real" observables?

Idea: pick a suitable observable, and compare the predictions of various geometries by examining the goodness of fit.

- Can we tell the difference between geometries in practice? If so, how?

A Suitable Observable

- A complication is that the semi-classical string calculation is expected to break down in some regime (ironically, the weak QCD coupling limit).
- Ideally, want an observable where the perturbative and non-perturbative regimes are cleanly separable.
- Also, an observable that has been measured on the lattice in the quenched approximation (no dynamical quarks $\equiv$ no stringy corrections).
- $\Rightarrow$ Static interquark potential $V(r)$.
- Non-perturbative (QCD) regime: $r \to \infty$.
- Perturbative regime: $r \to 0$.

The Cornell Potential

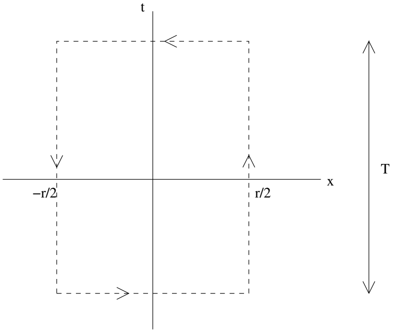
- Measured interquark potential (charmonium spectra, lattice data) found to be consistent with:

$$V(r) = V_0 + \kappa r - \frac{e}{r} + \frac{f}{r^2}.$$

- Last term spurious - helps parameterise lattice data.
- Coulomb + linear.

- Arises in field theory from the expectation value of a Wilson line:

$$< W(\mathcal{C}) > \propto e^{-TV(r)}.$$

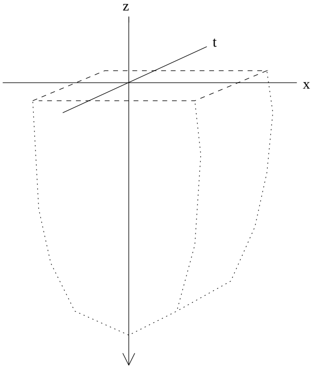


Potential from AdS / QCD

- According to the AdS / QCD conjecture:

$$< W(\mathcal{C}) > \propto e^{-A},$$

where A is area of worldsheet.



- Thus get (using Nambu-Goto action):

$$V(r) = \frac{1}{2\pi\alpha' T} \int d^2\xi \sqrt{\det g_{nm} \partial_\alpha X^n \partial_\beta X^m}.$$

- Make a general ansatz for the metric:

$$ds^2 = f(z) dx^\mu dx_\mu + \frac{dz^2}{z^2}.$$

- Then get (using method of Andreev & Zakharov):

$$r(z_0) = \int_{-\frac{r}{2}}^{\frac{r}{2}} dx = 2\int_0^{z_0} dz \frac{1}{\sqrt{\tilde{f}}} \left[\frac{z_0^4 \tilde{f}^2}{z^4 \tilde{f}_0^2} - 1\right]^{-\frac{1}{2}}$$

$$V_R(z_0) = \frac{g}{\pi}\left\{-\frac{1}{z_0} + \int_0^{z_0} \frac{dz}{z^2}\left[\sqrt{\tilde{f}}\left(1 - \frac{z^4 \tilde{f}_0^2}{z_0^4 \tilde{f}^2}\right)^{-\frac{1}{2}} - 1\right]\right\},$$

  where energy has been regularised.
- z_0 is value of z at $x = 0$, $f\tilde{(z)} = z^2 f(z)$.
- $r(z_0)$ diverges at a value $z_0 = z_0^*$ given by:

$$z_0^* \left.\frac{d\tilde{f}}{dz}\right|_{z_0^*} = 2\tilde{f}_0^*, \tag{1}$$

  with corresponding value $\tilde{f}(z_0^*) = \tilde{f}_0^*$.

- These values dictate the Cornell behaviour of the potential (Maldacena; Kinar et al.):

$$
\begin{aligned}
V_R(r) &\to -\frac{g}{2\pi\rho^2 r}, \quad r \to 0; \\
&\to \frac{g\tilde{f}_0^*}{2\pi(z_0^*)^2} r, \quad r \to \infty,
\end{aligned}
$$

  where $\rho = \Gamma^2(1/4)/(2\pi)^{3/2}$.
- Can get $V(r)$ at all r by numerical integration of the parametric forms for $r(z_0)$, $V(z_0)$.
- Characterise geometries according to warp factor:

$$
f(z) = e^{2A(z)},
$$

  with $A(z) = -\log z + \dots$

Back-Reacted Geometry

- Holographic hadron model of Erlich et. al. involves a bulk field X dual to the bilinear quark operator in the field theory.
- This field will deform the space it sits in by Einstein's equations.
- Subject to the ansatz:

$$X(z) = \frac{m_q}{2} z + \frac{\sigma}{2} z^3$$

can solve the Einstein equations to get a warp factor (Shock et. al.):

$$A(z) = -\log z + \frac{m_q^2}{48} z^2 - \frac{m_q \sigma}{32} z^4 + \frac{\sigma^2}{48} z^6.$$

- One hopes that this geometry is better motivated theoretically.

Other Geometries

Use for comparison:

- Simple quadratic warp factor (as used by Andreev & Zakharov):

$$A(z) = -\log z + \frac{c}{4} z^2.$$

- A geometry with the same number of parameters as the back-reacted geometry but a different warp factor (III):

$$A(z) = -\log z + k_1 z^2 + k_2 z^4.$$

- A geometry with a polynomial warp factor of the same degree as the back-reacted geometry, but with unconstrained coefficients (IV):

$$A(z) = -\log z + k_1 z^2 + k_2 z^4 + k_3 z^6.$$

- When comparing to data, also let g and an overall constant term vary...

Goodness of Fit Statistics

- Define the usual χ^2 variable:

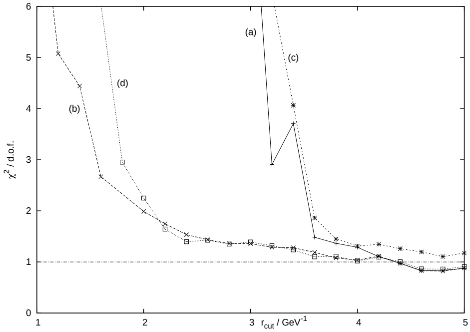
$$\chi^2 = \sum_{i=1}^{N_{\text{data}}} \frac{(V_i^{\text{AdS/QCD}} - V_i^{\text{latt.}})^2}{\sigma_i^2},$$

and number of degrees of freedom:

$$N_{\text{d.o.f}} = N_{\text{data}} - N_{\text{param}}.$$

- $\chi^2/N_{\text{d.o.f.}} \simeq 1$ for a good fit.
- However, we know that the string calculation breaks down in some moderately small r regime...
- $\Rightarrow$ introduce a cut r_{cut} on the lattice data and see how the minimum χ^2 varies as a function of this.

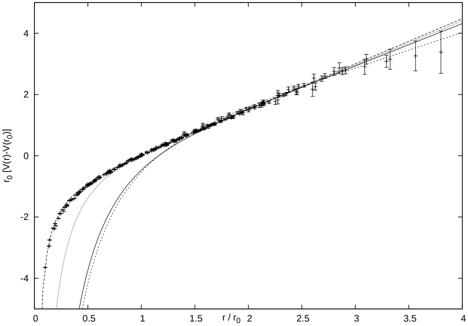
Cutting the Lattice Data



- If $\chi^2/N_{\text{d.o.f.}} = 1$ taken very seriously, can not distinguish geometries.
- However, goodness of fit clearly worse for back-reacted and III geometries.
- Nevertheless, back-reaction effects seem to model physics slightly better.
- But geometry IV and quadratic warp factor geometries are much better (with similar goodness of fit).

- Coefficient of z^4 in geometry IV likes to be positive.
- In back-reacted geometry this is fixed to be negative - may explain why it does not perform so well.
- For each geometry, there is a value of r_{cut} at which the minimum $\chi^2/N_{\mathrm{d.o.f.}} \simeq 1$.
- Define the "best fit" potential for each geometry by the values of the parameters after fitting at this value of r_{cut}.

Results

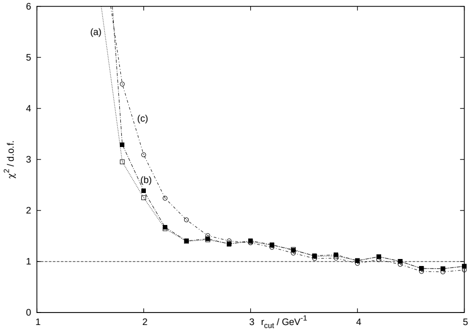


- Back-reacted geometry falls off from lattice data quicker at lower r, but is better than III.
- An extra parameter does better (geometry IV).

- Quadratic warp factor result seems to lie on top of lattice data even at small r - appearances are deceptive, however.

Discussion

- Geometry IV and quadratic warp factor are not distinguishable from χ^2 values, so must consider difference between potentials as a measure of theoretical uncertainty.
- It is not necessarily correct to let all parameters vary unconstrained - may run into conflict with other observables, or enter unphysical regions of parameter space.
- Should really extend the definition of the χ^2 to include other quantities such as meson masses, decay constants etc. This is easily achieved, but the separation of non-perturbative and perturbative regimes is not present.
- Can investigate the issue of negativity of the z^4 coefficient by imposing this constraint in geometry IV.
- Also, can investigate where the linear regime is expected to break down.



- Imposing negativity of the z^4 term does not change the goodness of fit of geometry IV by very much.
- The linear regime seems to break down at a lower r value than the back-reacted geometry can manage to describe...
- However, back-reacted geometry used here is not unique. Higher order ansätze for the bilinear quark field $X(z)$ will result in higher order geometries that may give a better description.

Conclusions

- Have used lattice data to try and constrain AdS / QCD geometries.
- The interquark potential as measured on the lattice is a good observable, as (non)-perturbative regimes clearly separated.
- Lattice data is able to decide between different models.
- Have compared the back-reacted geometry of Shock et. al with other test cases.
- Back-reaction effects seem to model more physics, but description is perhaps incomplete.
- Can extend project to include other observables and / or geometries.