NIKHEF Academic Lectures 2006: The Standard Model

Lecture 3: Confined, broken, hidden and discrete symmetries

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TODAYS OUTLINE

LAST WEEK'S MESSAGES

- Important concept: fields
- Dynamics via path-integral in field space (functional integral)
- Global symmetries of action constrain possible interactions
- Local symmetries lead to new interactions with gauge fields
- Standard Model: NAGT

$$\partial_\mu \to D_\mu = \partial_\mu + ig_s G_\mu^a T_a + igW_\mu^i T_i + ig' B_\mu Y$$

- Local symmetries on chiral fermions $(SU_{I_W}(2) \times U_Y(1))$ [$SU_c(3)$ sees no difference among chiralities]
- Complex scalar doublet, couples to fermions in two possible ways

$$\mathcal{L}_{Yukawa} = y_u^{ij} \; \overline{Q_L^i} \sigma_2 \Phi^* u_R^j + y_d^{ij} \; \overline{Q_L^i} \Phi d_R^j + \ldots$$

- SM: renormalizable, unitary quantum field theory

WHAT HAPPENS TO THE SYMMETRIES?

The Standard Model action has a lot of symmetry, both local and global. Yet so little of it is visible in the real world - only *electromagnetism* is obvious.

- Local color symmetry is confined
- Local Electroweak symmetry is spontaneously broken and hidden in vector bosons
- Global chiral symmetry of quarks is spontaneously broken and hidden in mesons
- Discrete Parity symmetry is broken by weak interactions
- ...

Let us first review what happens to the local symmetries.

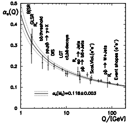
CONFINED COLOR SYMMETRY

Characteristic properties of onbroken local $SU_c(3)$ symmetry:

- “Charged” matter particles
- Massless gauge bosons

Neither are visible in the low-energy particle spectrum, because of confinement:

- free quarks and gluons do not exist, linear potential
- confinement seen in numerical lattice calculations
- consistent with perturbative behavior of coupling



- but precise analytical mechanism not yet clear

Confinement of color is a strictly dynamical phenomenon that does not seem to need any “new physics”.

BROKEN EW SYMMETRY

From data we know

- Gauge bosons of $SU(2)$ bizarrely heavy
- The *interactions* of these gauge bosons look exactly like that of a NAGT

So only the mass spectrum belies that one is dealing with a NAGT.

It is therefore broken spontaneously.

SPONTANEOUS SYMMETRY BREAKING

occurs when, even though all interactions respect the symmetry, the groundstate of the system is not symmetric.

Better: the symmetry is hidden.
We will engineer it to be so in the Standard Model.

BROKEN EW SYMMETRY, CONT'D

The key theorem:

GOLDSTONE'S THEOREM

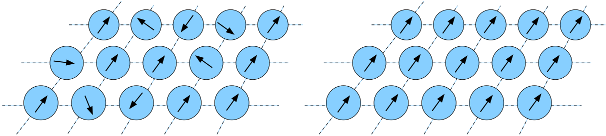
A spontaneously broken continous symmetry implies the existence of massless bosons [Goldstone bosons] whose properties correspond to the generators of the symmetry that are broken.

This holds for both local and global *continuous* symmetries

- If the symmetry is global, the Goldstone bosons should be in the observable spectrum
- If the symmetry is local, the Goldstone bosons are sometimes called would-be Goldstone bosons, and can be "eaten": the Higgs mechanism.

NON-INVARIANT GROUNDSTATES

Have we seen something so weird before? Yes, in condensed matter physics. Global case: Heisenberg ferromagnet.



Goldstone bosons can be identified among excitations.

Local case: Superconductor. Two properties

- Zero resistance due to Bose-condensation of Cooper pairs
- Meissner effect: no B-field inside

NON-INVARIANT GROUNDSTATES, CONT'D

Meissner effect in superconductors is excellent example:

- Groundstate is electrically charged, not invariant under $U_{EM}(1)$
- Electromagnetism spontaneously broken → would-be Goldstone bosons
- Higgs mechanism kicks in: photons obey field equation

$$\nabla^2 \cdot \vec{A} - \underbrace{\kappa|\psi_0|}_{\text{mass term}} \vec{A} = 0$$

- Massive photons, only shortrange B-field.

Here the groundstate is a consequence of interesting dynamics between electrons and background lattice.

SSB IN THE STANDARD MODEL

In the SM, the non-invariant groundstate is imposed via the scalar field. It is a very economical way to implement it. Scalar field can

- furnish the groundstate structure
- give mass to the gauge bosons through its coupling to them
- give mass to the fermions through its coupling to them
- has survived all tests and many competitors (technicolor)

QUIZ [QUIGG]

What would the universe look like if electroweak symmetry were not broken by the Higgs mechanism?

To see how it works, first write the scalar field in an unusual way

$$\Phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix} = U(x) \begin{pmatrix} 0 \\ \rho(x)/\sqrt{2} \end{pmatrix}, \quad U(x) = \exp\left[i\xi^a(x) t_a^F\right]$$

Think of

$$z = x + iy = re^{i\phi}$$

SYMMETRY BREAKING

Characterize the groundstate by

$$\Phi(x) = \Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \text{all other fields} = 0$$

Now, when it is said: “The groundstate breaks $SU_{I_W}(2) \times U_Y(1)$ down to $U_{EM}(1)$”, how does that work, precisely?
Let

$$R = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} e^{iy/2\alpha} & 0 \\ 0 & e^{iy/2\alpha} \end{pmatrix}, \quad U = \exp(i\xi^a \sigma_a/2)$$

be an $SU(2) \times U(1)$ transformation in the 2-D representation. An invariant groundstate would mean ($y = 1$ for the scalar doublet)

$$\Phi_0 = R\Phi_0 \simeq (1 + i/2(\xi^a \sigma_a + \alpha \mathbf{1}_2) + \ldots)\Phi_0$$

In other words, the generators must annihilate the groundstate

$$\{\sigma_a, \mathbf{1}_2\}\Phi_0 = 0$$

ANNIHILATING THE GROUNDSTATE

Let us try if that is so

$$\sigma_1\Phi_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0,$$

etc. Only the combination $T_3 + Y$ works:

$$2Q = (\sigma_3 + \cdot\mathbf{1}_2)\Phi_0 = 4\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = 0!$$

So only the generator that represents electric charge leaves the groundstate invariant. That symmetry is therefore not broken, and should be a local symmetry of the final action.

ANOTHER ARGUMENT FOR SSB

You might still feel that the whole SSB + Higgs mechanism is *fake elegance*. Could one not achieve the whole scheme with a few clever non-symmetric terms in Lagrangian? No.

- Standard Model before SSB was shown to be renormalizable
- *It still is after SSB, without need to do anything extra*!

So the whole EW symmetry breaking with scalars fits like a glove.

ABELIAN HIGGS MODEL

To show how a massless gauge boson can "eat" a Goldstone boson to become massive, we look at simpler case.

$$\mathcal{L} = -|\partial_\mu \phi|^2 - \mu^2 |\phi|^2 - \lambda \left(|\phi|^2\right)^2 ., \quad \phi(x) = \frac{1}{\sqrt{2}} \rho(x) \exp\left(i \frac{\theta(x)}{v}\right) ,$$

with phase invariance $\phi(x) \to e^{iq\xi} \phi(x)$. Make that local

$$D_\mu = \partial_\mu - iq A_\mu, \quad A_\mu \to A_\mu + \partial_\mu \xi(x), \quad \theta \to \theta + qv\xi(x)$$

Modify groundstate by taking $\mu^2 < 0$, and so

$$\phi(x) = \frac{1}{\sqrt{2}} (v + h(x)) \exp\left(i \frac{\theta(x)}{v}\right) , \quad \phi_0 = \frac{1}{\sqrt{2}} v$$

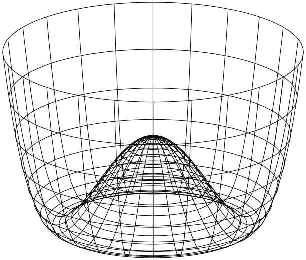
Now *define*

$$B_\mu = A_\mu - \frac{1}{qv} \partial_\mu \theta$$

is actually gauge invariant The massless gauge boson $A_\mu(x)$ has just eaten the Goldstone boson $\theta(x)$! What happens now?

ABELIAN HIGGS MODEL

Potential form for $\mu^2 < 0$



- θ field has action $(\partial_\mu \theta)^2$, no mass term
- Mass term for B_μ field from $(D_\mu \phi_0)^2$: find $M = qv$
- Degrees of freedom: before (2+2), after (3+1)
- In the SM the same mechanism works, except that 1 gauge field stays massless, the photon field.
- The field $h(x)$ in $\rho(x) = v + h(x)$ is the (massive) Higgs boson, a real particle, with a mass $2v^2\lambda$.

QUARK MASS, HEURISTICS

We saw that because quark mass terms combine LH and RH fermions, we could not build them easily into the SM.

The basic idea, aside from interesting subtleties

$$y\phi(x)\bar{\psi}(x)\psi(x) \to y(v + h(x))\bar{\psi}(x)\psi(x),$$

so that

$$\mathrm{mass} = \mathrm{Yukawa} \times \mathrm{V.e.v}$$

and the interaction of the fermions with the Higgs i

$$y\, h(x)\bar{\psi}(x)\psi(x) = \frac{m}{v} h(x)\bar{\psi}(x)\psi(x)$$

Higgs boson coupling strength to fermions proportional to mass.

QUARK MASS, CONT'D

The interesting subtleties involves how this works with LH doublets and RH singlets. Two Yukawa terms

$$\mathrm{I}: \quad y_d \overline{d_R}\ \left[(\Phi^a)^* \psi_L^b \delta_{ab}\right] \quad \mathrm{II}: \quad y_u \overline{u_R}\ \left[\Phi^a \psi_L^b \epsilon_{ab}\right] + h.c.$$

With $\Phi = (0, v)$ and $\psi_L = (u_L, d_L)$, one gets

$$\mathrm{I}: \quad y_d v \overline{d_R}\ [d_L] \quad \mathrm{II}: \quad y_u v \overline{u_R}\ [u_L] + h.c.$$

So nice way to give masses to up- and down-type fermions.
By the way, the form

$$\overline{u_R}\ \left[(\Phi^a)^* \psi_L^b \delta_{ab}\right]$$

is not allowed due to hypercharge conservation.

QUARK MIXING

Until now the gauge interactions with left- and righthanded quarks are clean and well-defined. We wrote them as so-called *flavor-eigenstates*.

The Yukawa interactions are actually more messy than we just wrote down

$$\mathcal{L}_{Yukawa} = y_u^{ij}\ \overline{Q_L^i}\sigma_2\Phi^* u_R^j + y_d^{ij}\ \overline{Q_L^i}\Phi d_R^j + \ldots$$

where y_u and y_d are in general complex matrices. Clearly it will be difficult to identify a clean mass-term after. Therefore, we now redefine the quark fields to make the mass terms nice.

After SSB, the mass matrices read

$$m_u^{ij} = v\, y_u^{ij}, \quad m_d^{ij} = v, y_d^{ij}$$

For a general matrix

$$Y = A \cdot M \cdot B, \quad \mathrm{A, B\ unitary} \quad \mathrm{M\ real,\ diagonal}$$

QUARK MIXING, CONT'D

So find

$$U_L m_u U_R^\dagger = \mathrm{diag}(m_u, m_c, ..) = D_u, \quad U_L' m_d U_R'^\dagger = \mathrm{diag}(m_d, m_s, ..) = D_d,$$

Then

$$\bar{q}_{L\,i} m_u^{ij} q_{R,j} \to \overline{(U_L q)} D_u (U_R q_R)$$

etc. The masses are now fine. Turns out: also the interactions with the neutral gauge bosons A_μ, Z_μ are diagonal (no FCNC's).
Charged current terms:

$$W_\mu^+ \bar{u}_L \gamma^\mu d_L \to W_\mu^+ \bar{u}_L \gamma^\mu \underbrace{(U_L U_L'^\dagger)}_{CKM \text{ Matrix}} d_L \to$$

After this, one may still redine the phases of the individual fields. This leads then to the CKM matrix having 1 complex phase.

CHIRAL SYMMETRY BREAKING IN QCD

Only a few words on this topic

- Massless QCD, a SM sector, with n_f quark flavors has global chiral $U_L(n_f) \times U_R(n_f)$ symmetry
- Non-perturbatively, a condensate forms in the groundstate $< \bar{Q}_L \cdot Q_R > = < \bar{u}_L u_R + \bar{d}_L d_R + .. >$, breaking part of chiral symmetry. Only

  $$Q_L \to U Q_L, \quad Q_R \to U Q_R,$$

  with same U is symmetry
- Goldstone bosons form: pseudoscalar mesons!
- Not quite massless because there are also terms that *explicitly* break the global chiral symmetry.
- Pions, kaons, ..: Pseudo-Goldstone bosons
- (Similar mechanism to keep Higgs boson light: Little Higgs theories)

DISCRETE SYMMETRIES

I will be very brief about discrete symmetries, CP violation etc.
Symmetries: P, C, T, CP, CPT.

- CPT is conserved for every relativistically invariant local field theory
- P is badly violated by the weak interactions, by having LH and RH fermions with different couplings
- C is violated similarly
- CP violation is small, well-known

P,C,CP

Parity

- Vector ($\bar{\psi}\gamma^\mu\psi$)/axial vector ($\bar{\psi}\gamma^\mu\gamma_5\psi$) change/don't change under parity.
- The SM treats each differently, parity is not symmetry in SM lagrangian.

Charge conjugation

- not a symmetry of $\mathcal{L}_{SM}$
- reason similar as for parity

CP

- Much better, *C* and *P* problems cancel, except...
- There is a complex number in the charged current terms in $\mathcal{L}_{SM}$, therefore also *CP* is not a good symmetry.