

NIKHEF ACADEMIC LECTURES 2006:
THE STANDARD MODEL
LECTURE 1: CONCEPTS

Eric Laenen

NIKHEF and Utrecht University

NIKHEF ACADEMIC LECTURES

MISSION STATEMENT

To remind ourselves of the physics behind the NIKHEF science program and beyond.

- LHC start now quite close, APP experiments bearing fruit
- New experiments contemplated
- Steady progress in stress-testing our present knowledge (B_s , WMAP)
- → opportune time
- Modelled on Fermilab AC's
- Twice a year, by NIKHEF-TH staf, and visitors
- 4 times 45 minutes
- No homework, sheets will be copied later.

OUTLINE OF 1ST LECTURE

1 PREFACE

- Introduction

2 FIELDS AND SYMMETRIES

- Fields
- Actions
- Path integral
- Groups of transformations
- Global symmetries

3 LOCAL SYMMETRIES

- Local symmetries
- Covariant derivative
- Interpretation

NOT A DULL DECADE PAST..

We learned about

- Top quark existence
- Neutrino flavor oscillations
- CP violation in B-meson physics
- Direct CP violation in $K \rightarrow \pi\pi$
- QCD dynamics at high energy and low energy
- Electroweak symmetry breaking, already indirectly
- Universe dominated by Dark Matter and Energy

Put together, very impressive! The potential for learning more is high.

PURPOSE OF MY LECTURES

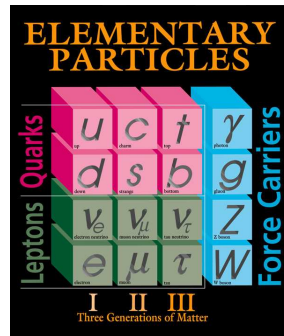
To bring to mind the Standard Model's structure, and some consequences thereof.

Approach: construct the Standard Model ourselves.

For this we need to review some tools

- Groups (of symmetries)
- Covariant derivatives
- Quantum fields
- Path integrals

The SM is much more than a cartoonish table of particles!



Warning: may generate more questions than answers!

APPROACH

Central theme:

Symmetry in quantum field theory.

- Meaning of [invariance; conserved quantities]
- External, internal
- Groups of symmetries
- Whole and broken

Mostly electroweak, QCD topic of separate lectures.

FIELDS

What is a field? Classical examples:

- Gravitational field
- Earth magnetic field

So, *something* that varies with/depends on \vec{x} . Since also time passes, we write, more completely x^μ .

Something: can be a quantity, have also a direction, etc:

- scalar field $\phi(x^\mu)$
- vector field $A_\rho(x^\mu)$
- spin- $\frac{1}{2}$ (spinor) field $\psi_\alpha(x^\mu)$

QUANTUM FIELDS

Moreover we wish to have relativistically correct quantum mechanics.

But then particle **number** cannot be conserved, because E can be converted to $\sum_i m_i c^2$.

A *field* contains a lot of single-particle *modes*.

- Each acts like a harmonic oscillator

$$\phi(x) = \sum_n \phi_n e^{i\omega_n x}$$

- They are coupled

$$\int dx \phi(x)^3 = \sum_{n,m,k} \phi_n \phi_m \phi_k \int d^4x e^{i(\omega_n + \omega_m + \omega_k)x}$$

Their frequencies are related to their momenta $\omega = \sqrt{\mathbf{k}^2 + m^2}$

Ok, so we use fields to represent relativistically correct quantum mechanics. But how to describe the way they interact?

DYNAMICS: HAMILTON OR LAGRANGE?



From QM we are used to Hamiltonians, eigenstates of them, etc. It seems cleaner: just q, p , not q, \dot{q} .

Nevertheless, we now go with Lagrange:

- No special treatment of time, symmetries manifest
- Direct link to Feynman rules
- Natural in path integral, through action

ACTION

What determines how particles, as field excitations, behave in a certain situation? Classically:

$$m \frac{d^2}{dt^2} x(t) = F = \frac{\partial V(x(t))}{\partial x(t)}$$

The equivalent for a scalar field is

$$\frac{d^2}{dx_\mu dx^\mu} \phi(x) - m^2 \phi(x) = J(x)$$

where $J(x)$ is a source field. One can derive these **equations of motion** through a variational principle

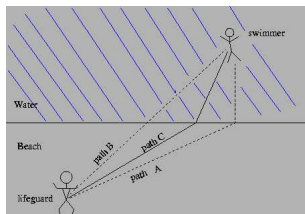
$$\delta S[x(t)] = 0, \quad \text{or} \quad \delta S[\phi] = 0$$

where S is the action, and its variation is induced by the variation in its argument.

(This is confusingly called Hamilton's principle)

ACTION

Hamilton's principle:



Path chosen is one with minimum action. The action for a particle is:

$$S[q] = \int dt L(q(t), \dot{q}(t))$$

For fields it is the spacetime integral of the **Lagrangian**

$$S[\phi] = \int d^4x \mathcal{L}(\phi, \partial_\mu \phi)$$

FROM ACTION TO EQUATIONS OF MOTION

A.k.a. Euler-Lagrange equations. Particle case:

$$L(q(t), \dot{q}(t)) = \frac{1}{2} m \dot{q}(t)^2 - V(q(t)), \quad K - V$$

$$\delta S = \delta \left(\int dt L(q(t), \dot{q}(t)) \right) = 0 \quad \Rightarrow \quad \frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = 0$$

$$\frac{\partial V}{\partial q} - m \ddot{q} = 0$$

(Real scalar) field:

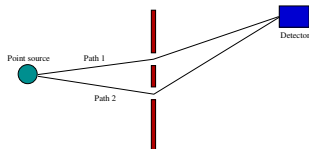
$$\mathcal{L}(\phi, \partial_\mu \phi) = \frac{1}{2} \underbrace{(\partial_\mu \phi(x))(\partial^\mu \phi(x))}_{(\partial_\mu \phi)^2} - \frac{1}{2} m^2 \phi^2(x)$$

$$\delta S = \delta \left(\int d^4 x \mathcal{L}(\phi, \partial_\mu \phi) \right) = 0 \quad \Rightarrow \quad \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} = 0$$

I have suppressed a partial integration step.

PATH INTEGRATION (FEYNMAN)

Hamilton's principle can be shown to follow from a **path integral**, a.k.a. “functional integral” approach. Double slit experiment:



Amplitude, using superposition

$$A = \sum_{\text{all paths}} e^{i(\text{phase}[\text{path}])} \equiv \int \mathcal{D}x(t) e^{i(\text{phase}[x(t)])}$$

What to choose for phase? If we wish to do integral by *stationary phase method*, i.e.

$$\frac{\delta}{\delta x(t)} (\text{phase}[x(t)])_{x_{cl}} = 0$$

then very natural is

$$\text{Phase}[\textit{path}] \propto \text{Action}[\textit{path}]$$

PATH INTEGRAL FOR FIELDS

For fields we have then the somewhat formal expression

$$A(\phi_a(0, \mathbf{x}_a) \rightarrow \phi_b(T, \mathbf{x}_b)) = \int \mathcal{D}\phi \exp \left[(i/\hbar) \int_0^T d^4x \mathcal{L} \right]$$

For our purposes, we introduce a source term in the Lagrangian, and consider the vacuum-to-vacuum amplitude in the presence of a source

$$Z[J(x)] = \int \mathcal{D}\phi \exp \left[\frac{i}{\hbar} \int d^4x \mathcal{L} + \frac{i}{\hbar} \int d^4x J(x)\phi(x) \right]$$

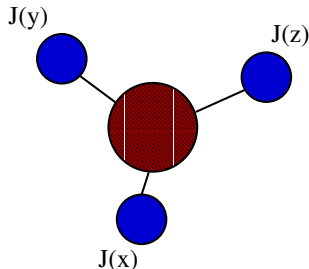
This is mostly used to *generate* Green's/correlation functions. E.g. the two-point correlation function (=propagator):

$$\Delta(y, z) = \int \mathcal{D}\phi \phi(y)\phi(z) \exp \left[\frac{i}{\hbar} \int d^4x \mathcal{L} \right] = \frac{\delta}{\delta J(y)} \frac{\delta}{\delta J(z)} Z[J] \Big|_{J=0}$$

PATH INTEGRALS AND FEYNMAN DIAGRAMS

We just saw how to derive the 2-point function from the *generating functional* $Z[J]$.

For 3-point, 4-point functions, expand $\ln(Z[J])$ in source J .



This allows

- extraction of Feynman rules (propagators, vertices, how to combine)
- derivation of pesky combinatorial factors

In short, when computing Feynman diagrams we are, in a sense, computing the path-integral in perturbation theory

PATH INTEGRALS, PRO'S AND CONS

Advantages (w.r.t. operator formalism)

- Straight from covariant Lagrangian to Feynman rules, no Hamiltonian stuff in between
- Symmetries remain manifest
- (This was key to 't Hooft's derivation of Feynman rules from spontaneously broken gauge theories such as Standard Model, using unusual gauge)
- "Straightforward" inclusion of non-perturbative gauge field configurations (Instantons)
- Numerical evaluation on lattice seems to work very well
- Closer connection with statistical physics (partition function)

Disadvantages

- Unitarity of S-matrix ($S^\dagger S = 1$) not obvious
- Mathematical rigour of integrals difficult
- It just seems like magic (cf. double slit experiment)

INTERMEDIATE SUMMARY

We just went flying by a lot of formalism and concepts. Let's stop and recap.

- The important quantities (degree of freedom) are fields
- They have properties (spin,..) corresponding to corresponding particles
- Dynamics defined through least action for fields
- In turn a consequence of defining quantum amplitudes through path integrals
- PI's: formal, but nice! Can derive Feynman rules, symmetries are "easy" to keep in formulation.

We will make conceptual leap to extend the types of symmetries we wish to have in our QFT's. But first, some mathematical toolery.

GROUPS

To discuss the Standard Model at our level of seriousness, we must be acquainted with the notion of a *group*.

Symmetries as invariance: change something and it looks the same. Do two changes in a row, should still look the same.

Group is a set of elements $\{g_i\}$

- with a composition $g_1 \cdot g_2 = g_3$
- that is associative $(g_1 \cdot g_2) \cdot g_3 = g_1 \cdot (g_2 \cdot g_3)$
- with a unit 1: $1 \cdot g_i = g_i \cdot 1 = g_i$ for each i
- and an inverse $g_i^{-1} \cdot g_i = g_i \cdot g_i^{-1} = 1$ for each i

If $g_i \cdot g_j = g_j \cdot g_i$, group is *abelian*.

That is rather abstract. Let's look at examples

- The set $Z_2 = \{-1, 1\}$
- The rotations that leave a cube invariant (24 elements)
- The integers \mathcal{Z} under addition
- Lorentz transformations!

TRANSFORMATION GROUPS

We shall deal with groups of transformations that leave a system invariant. Examples

- Rotations leaving a sphere invariant
- Rotations leaving an inner product of two vectors invariant
- Rotating sets of fields in such a way as that the action is invariant

Representations: $g \rightarrow D(g)$ matrices, such that

$$g_1 g_2 \rightarrow D(g_1) D(g_2) = D(g_1 g_2)$$

Example from QM: $(2l + 1) \times (2l + 1)$ matrices representing the rotation group $SO(3)$.

INTRINSIC BENEFIT OF GROUPS

Do we really need the whole mathematical machinery of groups to describe symmetries? Yes.

- Groups are quite natural, nothing exotic about their definition
- Allow separate discussion of transformation and “what it acts on” (representations - the same rotation can be represented in different ways)
- Gives precise rule about which representation can interact in a QFT.
- Gives precise rules about multiplicities e.g.
- ...

BUILDING ACTIONS FOR FIELDS

On the way to building the Standard Model, we first practice on prototypes: bits of action for various fields.

Important: symmetry

$$S[\phi'] = S[\phi]$$

In

$$\int \mathcal{D}\phi e^{\frac{i}{\hbar} S[\phi]}$$

field configurations related by $\phi \rightarrow \phi'$ have precisely the same weight.

Moreover, demanding certain symmetries puts strong constraints on what we can build.

LORENTZ SYMMETRY

Example: Lorentz invariance of scalar field

$$\phi'(x'^{\mu}) \equiv \phi'(\Lambda^{\mu}_{\nu} x^{\nu}) \stackrel{\text{symmetry}}{=} \phi(x^{\mu})$$

Is the “bit of action” $\int d^4x \phi^2(x)$ Lorentz invariant?

$$\int d^4x \phi^2(x) = \int d^4x \phi'^2(x'(x)) = \int \underbrace{\frac{d^4x'}{\det(\Lambda)}}_1 \phi^2(x')$$

For spin-1 fields

$$A'^{\mu}(x') = \Lambda^{\mu}_{\nu} A^{\nu}(x)$$

LORENTZ SYMMETRY, CONT'D

For spin-1/2 fields

$$\psi_\alpha(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix}, \quad \psi'_\alpha(x') = \Lambda_{\alpha\beta}^{(S)} \psi_\beta(x), \quad \Lambda_{\alpha\beta}^{(S)} = \left[\exp \left(\frac{1}{8} \omega^{\mu\nu} [\gamma_\mu, \gamma_\nu] \right) \right]_{\alpha\beta}$$

and one can construct Lorentz-invariant bits of action like

$$\int d^4x A_\mu(x) A^\mu(x), \quad \int d^4x \psi(x)^\dagger_\alpha [\gamma^0]_{\alpha\beta} \psi_\beta(x)$$

Examples of representations! Same Lorentz transformation represented differently on vector fields A_μ and spinor field ψ_α .

This is the first symmetry we should incorporate. It disallows terms like

$$\int d^4x \frac{1}{2m} \vec{\nabla}^2 \phi(x)$$

MORE FIELDS

What if we wish to include more fields? E.g. $\phi_i(x)$, $i = 1, \dots, N$. There would be nothing wrong with a term

$$g \int d^4x \phi_1(x) \phi_5(x) \phi_{17}(x)$$

The number of possible terms can be much reduced if we demand the symmetry *in field-identity space*:

$$\vec{\phi}(x) \equiv \begin{pmatrix} \phi_1(x) \\ \vdots \\ \phi_N(x) \end{pmatrix} = O \vec{\phi}(x), \quad O^T = O^{-1}$$

Then

$$\vec{\phi}'^T \cdot \vec{\phi}' = \vec{\phi}^T O^{-1} \cdot O \vec{\phi} = \vec{\phi}^T \cdot \vec{\phi}$$

is invariant! $O \in SO(N)$. So only

$$\int d^4x \vec{\phi}^T \cdot \vec{\phi}, \quad \int d^4x (\partial_\mu \vec{\phi})^T \cdot (\partial^\mu \vec{\phi}), \quad \int d^4x (\vec{\phi}^T \cdot \vec{\phi})^2$$

are allowed.

MORE FIELDS, CONTINUED

Note: the “rotation” only acts on field identities, it does not muck around with Lorentz indices. So we can also ask

$$\vec{\psi}(x) \equiv \begin{pmatrix} \psi_1(x) \\ \vdots \\ \psi_N(x) \end{pmatrix} \rightarrow U \vec{\psi}(x) = \begin{pmatrix} U_{11} & \dots & U_{1N} \\ \vdots & & \vdots \\ U_{N1} & \dots & U_{NN} \end{pmatrix} \begin{pmatrix} \psi_1(x) \\ \vdots \\ \psi_N(x) \end{pmatrix}, \quad U = (U^{-1})^\dagger \in SU$$

Here entry $\psi_j(x)$ in $\vec{\psi}$ has 4 spinor components, but they all get multiplied by U_{ij} .

GLOBAL SYMMETRY

We have just seen an example of global *internal* symmetry.

- Global: U, O matrices do not depend on x^μ
- Internal: symmetry acts in identity space, not in real spacetime.

Examples of global symmetries:

- Lorentz invariance
- Translation invariance
- Lepton, baryon number
- Flavor symmetry

GLOBAL SYMMETRIES

Constrain the types of interactions allowed. Reflected in physical observables (multiplets).

GLOBAL SYMMETRY AND CONSERVED QUANTITIES

We used symmetry so far as: **invariance of the action under transformations.**

There is another, related, manifestation of symmetry: conserved quantities.

- Lorentz-invariance: mass, spin
- Translation invariance: momentum
- Lepton number: same
- Flavor symmetry: isospin

The link is formed by

NOETHER'S THEOREM

For every global symmetry of the action there is conserved current \rightarrow conserved quantity

We will however mostly “use” the first viewpoint.

LOCAL (“GAUGE”) SYMMETRIES

Consider a simple theory with a complex scalar field

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

It has a global phase invariance

$$\phi(x) \rightarrow e^{iq\alpha} \phi(x), \quad q = \text{charge}$$

Radical demand: can we let symmetry hold *locally*?

$$\phi(x) \rightarrow e^{iq\alpha(x)} \phi(x)$$

Ok for 2nd and 3rd term, but not for first, because

$$\partial_\mu \phi(x) \rightarrow e^{iq\alpha(x)} \partial_\mu \phi(x) + iq(\partial_\mu \alpha(x)) e^{iq\alpha(x)}$$

Can we define better, **covariant** derivative D_μ such that

$$D_\mu \phi(x) \rightarrow e^{iq\alpha(x)} D_\mu \phi(x)$$

COVARIANT DERIVATIVE

This will be major building tool for the Standard Model Lagrangian.

To make covariant derivative work so nicely, use new field A_μ

$$D_\mu = \partial_\mu - iqA_\mu, \quad A'_\mu(x) = A_\mu(x) + \partial_\mu\alpha(x)$$

Then all we have to do: $\partial_\mu \rightarrow D_\mu$ in \mathcal{L}

$$\mathcal{L} = D_\mu\phi^* D^\mu\phi - m^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

Interactions! Also

$$[D_\mu, D_\nu]\phi = -iqF_{\mu\nu}\phi, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

so also $F_{\mu\nu}$ is locally covariant, here even *invariant*. Covariant derivative adjusts itself to what it acts on. E.g. Leibniz rule

$$D_\mu(\psi_1\psi_2) = \partial_\mu(\psi_1\psi_2) - i(q_1 + q_2)A_\mu(\psi_1\psi_2)$$

DO WE NEED LOCAL SYMMETRIES?

We can construct QED in the same way. Start with

$$\mathcal{L} = \bar{\psi}(x)(\gamma^\mu \partial_\mu - m)\psi(x)$$

and replace ∂_μ with D_μ to get local phase invariance and

$$\mathcal{L} = \bar{\psi}(x)(\gamma^\mu D_\mu - m)\psi(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

We can also express this in terms of gauge-invariant fields a_i, ψ', ϕ

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu a_i)^2 - \frac{1}{2}(\partial_i \phi)^2 - \bar{\psi}'(\not{\partial} + m)\psi' + iq\bar{\psi}'(\gamma_i a_i + \gamma^0 \phi)\psi' \quad , \quad i = 1, 2$$

But

- No manifest Lorentz invariance
- Instantaneous Coulomb interaction

INTERPRETATION

Global symmetry: certain degrees of freedom behave similarly.

Local symmetry: certain degrees of freedom are absent.

A_μ , the electromagnetic vector potential, communicates phase changes from point to point.

DEEPEST PRINCIPLE OF STANDARD MODEL

Fundamental forces follow from local symmetries