

NIKHEF ACADEMIC LECTURES 2006:
THE STANDARD MODEL
LECTURE 4: ADVANCED TOPICS

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OUTLINE

- 1 CUSTODIAL SYMMETRY
 - Custodial symmetry in the Standard Model
- 2 ANOMALY CANCELLATION
 - General chiral anomaly
 - Technicalities
 - Anomaly cancellation
- 3 HIGGS MASS AND FINE-TUNING
 - A toy theory
 - \overline{MS} subtraction
 - Fine tuning
- 4 WHAT IF EW SYMMETRY WERE NOT BROKEN?

CUSTODIAL SYMMETRY

Custos: *guardian*.

At lowest order the W and Z boson masses are related by

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1.$$

This is a consequence of an extra *global* symmetry of just the Higgs + EW part of the Lagrangian

$$\mathcal{L}_H = -(D_\mu \phi)^\dagger D^\mu \phi - V(\phi)$$

with

$$V(\phi) = \mu^2 (\phi^\dagger \phi) + \lambda (\phi^\dagger \phi)^2$$

$$D_\mu = \partial_\mu - i g W_\mu^a \left(\frac{\tau_a}{2} \right) - i g' B_\mu \frac{1}{2} \hat{Y}$$

Let us rewrite this in terms of

$$\Phi = (i\tau_2 \phi^*, \phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix}$$

CUSTODIAL SYMMETRY, CONT'D

The Lagrangian now reads

$$\mathcal{L}_H = -\frac{1}{2} \left\{ \text{Tr} \left[(D_\mu \Phi)^\dagger D^\mu \Phi \right] + \mu^2 \text{Tr} \left[\Phi^\dagger \Phi \right] + \lambda \text{Tr} \left[\Phi^\dagger \Phi \Phi^\dagger \Phi \right] \right\},$$

$$D_\mu \Phi = \partial_\mu \Phi - i g W_\mu^a \frac{1}{2} \tau_a \Phi + i g' B_\mu \Phi \tau_3$$

In the form we can better see the custodial symmetry at work. Usual local $SU_{I_w}(2)$ symmetry acts as $\Phi(x) \rightarrow U(x)\Phi(x)$. Note global symmetry

$$\Phi(x) \rightarrow \Phi(x)V^\dagger, \quad V \text{ is unitary}$$

SSB is effected by

$$\Phi_0 = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

leaving the remaining *global* symmetry

$$\Phi(x) \rightarrow V\Phi(x)V^\dagger$$

This is the **custodial symmetry**

CONSEQUENCES OF CUSTODIAL SYMMETRY

Consider

$$D_\mu \Phi_0 = \underbrace{\partial_\mu \Phi_0}_{=0} - i g W_\mu^a \frac{1}{2} \tau_a \Phi_0 + i g' B_\mu \Phi_0 \tau_3$$

and put $g' = 0 \rightarrow \cos \theta_W = 1$. Then

$$g W_\mu^a \frac{1}{2} \tau_a \Phi_0 = -\frac{g v}{\sqrt{2}} \begin{pmatrix} W_\mu^3 & W_\mu^1 - i W_\mu^2 \\ W_\mu^1 + i W_\mu^2 & -W_\mu^3 \end{pmatrix}$$

Leading to a gauge field mass term

$$-\frac{1}{2} \text{Tr} \left[(D_\mu \Phi)^\dagger D^\mu \Phi \right] = -\frac{1}{4} g^2 v^2 \sum_{a=1}^3 W_\mu^a W^{a \mu} \Rightarrow \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1.$$

The custodial symmetry requires this. Now let $g' \neq 0$. Then

$$g W_\mu^a \frac{1}{2} \tau_a \Phi_0 + g' B_\mu \Phi_0 \tau_3 = -\frac{g v}{\sqrt{2}} \begin{pmatrix} (W_\mu^3 - \frac{g'}{g} B_\mu) & W_\mu^1 - i W_\mu^2 \\ W_\mu^1 + i W_\mu^2 & -(W_\mu^3 - \frac{g'}{g} B_\mu) \end{pmatrix},$$

$$W_\mu^3 - \frac{g'}{g} B_\mu \equiv \frac{1}{\cos \theta_W} Z_\mu$$

CONSEQUENCES OF CUSTODIAL SYMMETRY, CONT'D

The combination $Z_\mu / \cos \theta_W$ acts like W_μ^3 before

$$M_W^2 = \frac{1}{2} g^2 v^2, \quad M_Z^2 = \frac{1}{2 \cos^2 \theta_W} g^2 v^2$$

Conversely, if we do not demand that a scalar field implements the SSB, but only ask that the custodial $SU(2)$ is present, we will find

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1.$$

- $\rho = 1$ more generally a consequence of custodial symmetry
- Effects from fermions will break the custodial symmetry, but in calculable way

$$\rho = 1 + \Delta\rho(m_t^2, \ln(m_H))$$

WHAT IS AN ANOMALY?

(Major topic in theoretical physics)

An anomaly occurs when quantum corrections do not respect a symmetry of the classical Lagrangian. Is that bad?

For a global symmetry, not necessarily.

- New types of interactions can appear: $\pi^0 \rightarrow \gamma\gamma$ (\sim global $\psi \rightarrow e^{i\alpha\gamma_5}\psi$)
- Explains absence of certain meson states
- Classical scale invariance of QCD broken by anomaly, good for meson masses
- Can give geometric, topological information

For a local symmetry, most definitely:

- loss of unitarity
- loss of renormalizability
- general mayhem

TECHNICALITIES

How does an anomaly manifest itself in a quantum field theory?

- The non-conservation of the corresponding (Noether) current
- The non-invariance of the path-integral integration measure

Focus now on **chiral anomalies**; occur when chiral fermions in the theory

- Chirality involves $\gamma_5 \equiv \gamma_0\gamma_1\gamma_2\gamma_3$, an essentially 4-dimensional “object”
- Loop integrals in Feynman diagrams are often regulated by working in d dimensions \Rightarrow incompatibility

Since we have gauge fields coupling to chiral fermions in the SM, we better pay close attention to this problem.

Gauge fields must couple to conserved currents

$$\delta \left(\int A^\mu J_\mu \right) = \int (\partial_\mu \alpha) J^\mu = - \int \alpha \partial_\mu J^\mu = 0$$

TECHNICALITIES

Consider massless fermion

$$\mathcal{L} = -\bar{\psi}\not{\partial}\psi$$

Two global symmetries

$$\text{I: } \psi \rightarrow e^{i\alpha}\psi, \quad \text{II: } \psi \rightarrow e^{i\beta\gamma_5}\psi$$

Find (Noether) currents by temporarily making $\alpha, \beta \rightarrow \alpha(x), \beta(x)$

$$\text{I: } J_V^\mu = \bar{\psi}\gamma^\mu\psi, \quad \text{II: } J_A^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$$

Classically (tree level)

$$\partial_\mu J_V^\mu = 0, \quad \partial_\mu J_A^\mu = 0$$

After one-loop calculations $\partial_\mu J_\mu^V = 0$, and can add $A_\mu(x)J^\mu(x)$ to Lagrangian \rightarrow massless QED. One then finds

$$\partial_\mu J_A^\mu = \frac{e^2}{16\pi} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

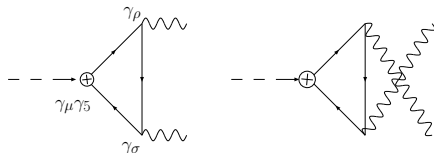
The RHS is the (Adler-Bell-Jackiw) chiral anomaly.

RELEVANT DIAGRAMS FOR ABJ ANOMALY

To see what becomes of $\partial_\mu J_A^\mu$ at quantum level, evaluate

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \partial_\mu J_A^\mu e^{iS_0 + ie \int d^4x A_\mu \bar{\psi} \gamma^\mu \psi}$$

$$\simeq \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \partial_\mu \bar{\psi} \gamma^\mu \gamma_5 \psi \left[\frac{1}{2!} (ie \int d^4x A_\mu \bar{\psi} \gamma^\mu \psi)^2 \right] e^{iS_0}$$



Computing those diagrams gives a non-zero result, if indeed there is 1 axial-vector coupling, and two vector couplings (VVA)

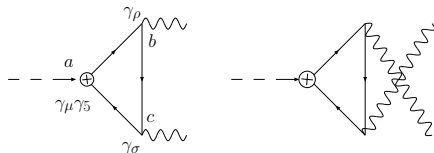
$$\partial_\mu J_A^\mu = \frac{e^2}{16\pi} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

NON-ABELIAN CASE

In the SM we have the couplings

$$W_\mu^a(x) \bar{\psi}_i [t_a]_{ij} (1 + \gamma_5) \psi_j$$

i.e. the gauge fields couple to chiral fermions. But do they couple to conserved currents? Calculate



Result

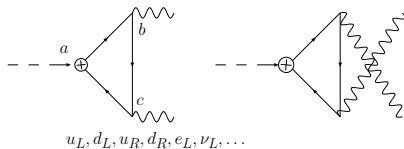
$$\partial_\mu J_{A,a}^\mu = \frac{g^2}{16\pi} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^b G_{\rho\sigma}^c \underbrace{[t_a, \{t_b, t_c\}]}_{D_{abc}}$$

where the t_a are the generators on the corners of the graph.

Serious! The right-hand side is non-zero, the Standard Model *seems a sick theory*.

ANOMALY CANCELLATION

There is a possible way out. Because there are many fermions, that can run around in the loop, we must sum over all allowed types.



Perhaps there will be cancellations? We must examine many cases however for the three generators

- All three $SU_c(3)$
- 2 $SU_c(3)$, 1 $SU_L(2)$
- 3 $SU_L(2)$
- 2 $SU_L(2)$, 1 $U_Y(1)$
- 3 $U_Y(1)$
- ...

We will examine a few cases.

ANOMALY CANCELLATION, CONT'D

- 1 $SU(2) - SU(2) - SU(2)$

$$D_{abc} = \text{Tr}[t_a, \{t_b, t_c\}] = \frac{1}{2}\delta_{bc} \text{Tr}[t_a] = 0$$

- 2 $SU(2) - SU(2) - U(1)$

$$D_{abc} \rightarrow \frac{1}{2}\delta_{bc} \text{Tr}[Y] = \sum_{LH} y = \frac{1}{2}\delta_{bc} (-1 + 3 \cdot \frac{1}{3}) = 0$$

- 3 $SU(3) - SU(3) - U(1)$

$$D_{abc} \rightarrow \frac{1}{2}\delta_{bc} \text{Tr}[Y] = \sum_{quarks} y = \frac{1}{2}\delta_{bc} (2 \cdot \frac{1}{3} - (\frac{4}{3} - \frac{2}{3})) = 0$$

- 4 $U(1) - U(1) - U(1)$

$$D_{abc} \propto \text{Tr}[Y^3] = (2 \cdot (-1)^3 + 3 \cdot 2 \cdot (\frac{1}{3})^3) - ((-2)^3 + 3 \cdot (\frac{4}{3})^3 + 3 \cdot (\frac{-2}{3})^3) = 0$$

Amazingly, it works! The SM is anomaly free, thanks to the quantum numbers of the chiral fermions in a family.

IMPLICATIONS OF ANOMALY CANCELLATION

- We're safe. (To one loop, but also higher [Adler-Bardeen])
- Quarks and leptons need to cooperate - a sort of unification

Are there gauge groups for which D_{abc} is automatically zero? Yes, quite a few ($SO(2n+1)$, G_2 ..).

Here's an intriguing one: $SO(10)$, specifically the 16×16 spinor representation. If the SM gauge group were nothing but a remnant of an $SO(10)$ GUT group, anomaly cancellation would be simply a consequence of group theory, not of (anthropic?) miracles.

DECOUPLING

Degrees of freedom should correspond to relevant length scales

- Do not need QM to describe soccer ball
- Top quark was not noticeable in early low energy experiments

Heavy particles should not be visible, **decouple**, from processes at lower energies. Consider toy scalar field theory

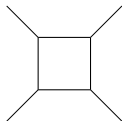
$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\Phi^2 - \frac{1}{6}\phi^3 - \frac{1}{2}g_2\phi\Phi^2$$

- Light field ϕ , mass m
- Heavy field Φ , mass M

DECOUPLING, CONT'D

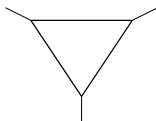
Tree graphs with H external Φ lines $\propto M^{-2H}$

Loop graphs: 4-point function



$$\Gamma_4 = i(2\pi)^6 \frac{g_2^4}{384\pi^3 M^2}$$

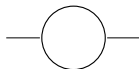
3-point function



$$\Gamma_3 = i(2\pi)^n \left(\frac{\mu^\epsilon g_2^3}{128\pi^3} \frac{1}{\epsilon} - \frac{\mu^\epsilon g_2^3}{128\pi^3} \left[\gamma_E + \ln \left(\frac{M^2}{4\pi\mu^2} \right) \right] \right).$$

DECOUPLING, CONT'D

2-point function



$$\Gamma_2 = i(2\pi)^n \left(-\frac{g_2^2}{128\pi^3} \frac{1}{\epsilon} \left(M^2 + \frac{1}{6} p^2 \right) + \frac{g_2^2}{128\pi^3} \left[M^2 \left(\gamma_E - 1 + \ln \left(\frac{M^2}{4\pi\mu^2} \right) \right) + \frac{p^2}{6} \left(\gamma_E + \ln \left(\frac{M^2}{4\pi\mu^2} \right) \right) \right] \right).$$

$\overline{\text{MS}}$ RENORMALIZATION

The following $\overline{\text{MS}}$ renormalizations cancel the divergences

$$\begin{aligned}
-\frac{1}{2}(\partial\phi)^2 &= -\frac{1}{2}Z_\phi(\partial\phi_R)^2, & Z_\phi &= 1 - \frac{1}{\epsilon} \frac{g_2^2}{768\pi^3} \\
-\frac{1}{2}m^2\phi^2 &= -\frac{1}{2}m_R^2\phi_R^2, & m_R^2\phi_R^2 &= m^2\phi^2 - \frac{1}{\epsilon} \frac{g_2^2 M^2}{128\pi^3}\phi^2 \\
g_1\phi^2 &= \mu^\epsilon Z_1 g_{1,R}\phi_R^3, & Z_1 g_{1,R} &= g_{1,R} - \frac{1}{\epsilon} \frac{g_2^3}{128\pi^3}
\end{aligned}$$

The quantities with subscript are finite. Now drop label R , and arrive at effective lagrangian

$$\begin{aligned}
\mathcal{L} = -\frac{1}{2}(\partial_\mu\phi)^2 &\left\{ 1 - \frac{g_2^2}{768\pi^3}[\gamma_E + L] \right\} - \frac{1}{2}\phi^2 \left\{ m^2 - \frac{g_2^2 M^2}{128\pi^3}[\gamma_E - 1 + L] \right\} \\
&- \frac{1}{6}\phi^3 \left\{ g_1 - \frac{g_2^3}{128\pi^3}[\gamma_E + L] \right\}
\end{aligned}$$

where $L = \ln(M^2/4\pi\mu^2)$. Effect of heavy particles only in coefficients. But notice **fine-tuning** of mass!

FINE TUNING

We saw that somehow m must be very close M such

$$m^2 - \frac{g_2^2 M^2}{128\pi^3} [\gamma_E - 1 + L]$$

is small compared to M^2 . Not impossible, just unnatural.

If ϕ is the Higgs boson, M is the scale of new physics, (e.g. GUT scale 10^{19} GeV), the fine tuning is very severe. Solutions:

- SUSY
- Extra dimensions (GUT scale is Near!)
- Little Higgs (elegant postponement of problem)

WHAT IF EW SYMMETRY WERE NOT BROKEN AS IN SM?

- Chiral symmetry breaking by QCD would break $SU_L(2)!!$
- W and Z would get mass, just only 30 MeV
- Quarks and leptons massless
- Mesons and baryons *would* form.
- Protons heavier than neutrons! Rapid beta decay
- No atoms, chemistry, us...

OUTREACH ADVICE

When the Higgs is found, don't say it is just a particle. It is (the first step) towards a radically new view of our Universe.

SUMMARY

As regards the physics

- Reviewed structure of Standard Model
- Local and global symmetry determine structure
- Quantum aspects further restrict its structure
- Good(?) reasons to expect more

As regards these lectures

- Keep reviewing, keep discussing
- More in the fall..