The Higgs System

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Plan of the Lectures

1 Introduction
Vacuum expectation value.
Cosmological constant.

2 The original Higgs Model
Unitarity limit, Heavy Higgs.
Equivalence theorem.

3 The σ-model
Symmetries. Higgs Sector Standard Model.
Isopsin. Fermion masses. Parity.

4 Other Higgs systems
Higher represention. Additional doublets.

5 Searching Higgs
Screening. Radiative corrections.
Higgs at high energy (TeV).
Extrapolation to high energy.

Introduction

Perturbative $\longleftrightarrow$ Non-perturbative

Point of view: Feynman rules describe best our understanding.

Feynman rules = perturbation theory.

Considered as included in perturbation theory those effects that can be obtained by some partial summation of the perturbation series (diagrams). Examples:

- bound states such as the H-atom
- unstable particles.

Feynman rules are usually formulated in momentum space.
This is our Universe.

All this, through experiments at LEP, agrees very well with the data. But what about the Higgs ? Various cases:

- Light Higgs. Will be discovered. Very boring possibility.
- Very heavy Higgs. Non-perturbative new physics that can to some extend be guessed.
- Non-existent Higgs. New physics.

In discussing the physics of the Higgs system we must distinguish between physical effects that depend on the symmetry properties of the Higgs system and those that are due to the dynamics of the system.

Some effects will not depend on m (the Higgs mass), others will.

By dynamical effects we will somewhat arbitrarily mean those that do not depend on the precise value of the Higgs mass.

Sofar there is no real evidence on the value of the Higgs mass.

What makes the Higgs particle different from the other particles ? What is so special about the Higgs particle ?

• It is the only spin 0 particle of the Standard Model, and some of us, for vague reasons (releated to gravity), think that there is something wrong with scalar particles.

• The Higgs couples with a strength depending on the mass of the particle to which it couples. Thus it couples with different strength to the particles of different families (such as up and charm quark). Its (non-) discovery may shed light on why there are three families. Weak, e.m. and strong forces are identical for the three families. Actually also gravity couples proportional the the mass of the particle that it is coupled to. What is going on ?

• Another remarkable fact is this. A particle may have mass of its own (example: the electron in the old days, when we did not know about the Higgs), or it may derive its mass from its interaction with the Higgs system. Also a mixture of the two is possible.

It happens that in the Standard Model **all** particles (the Higgs itself excepted) get **all** of their mass from the Higgs system. If there were no Higgs all particles would be massless.

• There is the much older question why parity is violated. This question may find its answer in the Higgs system.

If one insists (as nature seems to do) that **all** particles get their mass from the Higgs system, then it follows that parity must be violated.

• Higgs and the vacuum: things go wrong. No monopoles, axions, strong CP violation.

• Higgs conflicts with the weak spot of gravity, the cosmological constant.

One thing appears to be clear: there is some link between the Higgs and gravitation. Note that gravitation is deeply in trouble, both theoretically and experimentally.

History

History is often rewritten. Here some facts.

The use of a field in the vacuum to generate masses was first published by Schwinger, in 1957. In his article he generated the mass of the muon in that way.

In that same article Schwinger introduced the σ-model, which is actually the Higgs sector of the Standard Model.

Gell-Mann and Levy (1960) linked PCAC to symmetry breaking and the σ-model.

Renormalizability for gauge theories with the vector boson masses generated through a vacuum field was proven in 1971 (V+'t Hooft). We did not know about the work of Higgs-Brout-Englert.

In a separate development, Anderson (1958) discussed massive quantum electrodynamics as perceived in superconductivity. This led Higgs-Brout-Englert (1967) to their work, which is the use of a field in the vacuum to give mass to vector bosons. In 1968 Kibble worked this out in a non-abelian model of vector bosons with a mass due to the Higgs system. He used the wrong group (not SU_2 x U_1) from the point of view of phenomenology. Weinberg (1968), knowing the work of Kibble, used the correct group as proposed by Glashow (1961) in a theory of weak interactions of leptons.

The proof of renormalizability led, in 1971, to credibility of the Weinberg model of leptons. Including the Glashow-Illiopoulos-Maiani (1970) model for quarks (involving charm as introduced by Hara in 1964 but without Higgs system) this was build out to a complete anomaly free model of weak interactions between quarks and leptons.

The development of the theory of strong interactions (quantum chromodynamics) was gradual and complicated.

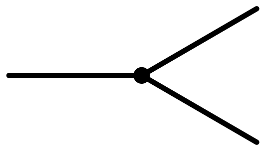
The Vacuum

To exhibit the relevant features consider the simplest possible field theory, the φ^3 theory with tadpole.

$$\mathcal{L} = -\frac{1}{2}\ \partial_\mu \varphi \partial_\mu - \frac{1}{2}\ m^2 \varphi^2 + g\varphi^3 + t\varphi$$

The Feynman rules are:

Propagator:  $\dfrac{1}{p^2 + m^2 - i\epsilon}$

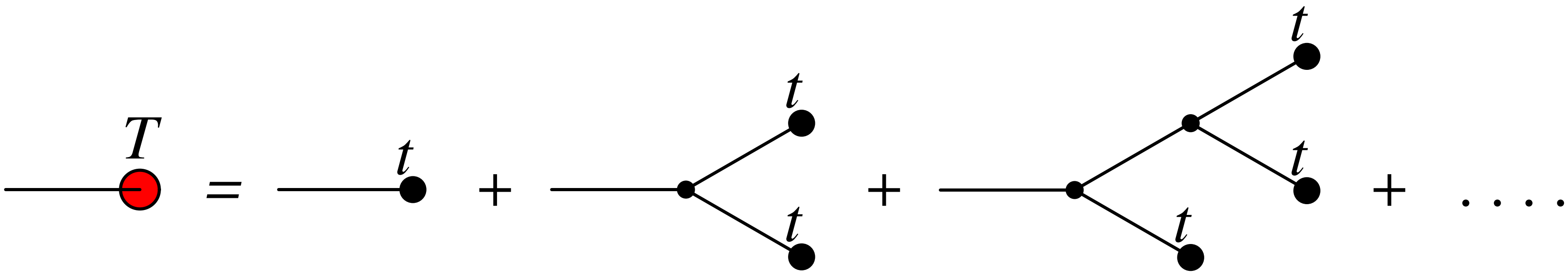
Vertex:  $6g$

Tadpole: 

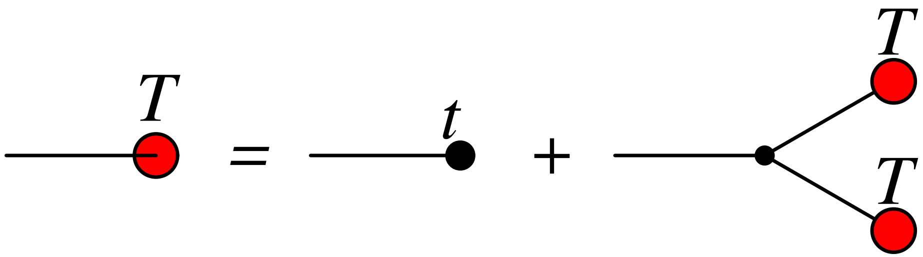
t

This last diagram, the tadpole diagram, shows that a φ particle (of momentum zero) can disappear.

At the tree level a tadpole generates further vacuum diagrams:



This series can be rewritten as an equation:



This is a quadratic equation:

$$T = t + \frac{6}{2}\ \frac{g}{m^4}\ T^2$$

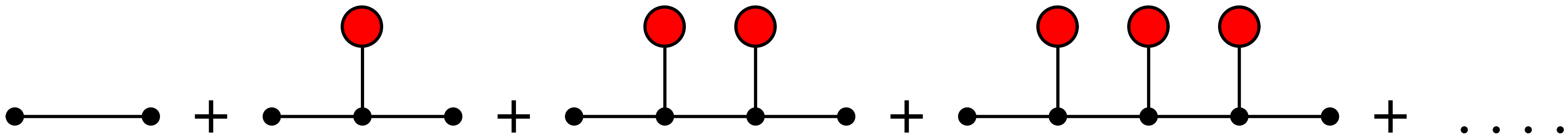
There are two solutions:

$$T_{\pm} = \frac{m^2 \pm \sqrt{m^4 - 12gt}}{6g/m^2}$$

The solution with a + sign makes no sense for $g = 0$.

The solution that can be expanded in a power series is the one with a minus sign, T_-. Does the other solution make sense ?

What are the physical consequences of such a tadpole ? First of all, it influences the particle mass. Consider the propagator of the φ including diagrams with a tadpole:



$$P = P_0 + P_0 \frac{6gT}{m^2} P_0 + P_0 \frac{6gT}{m^2} P_0 \frac{6gT}{m^2} P_0 + P_0 \frac{6gT}{m^2} P_0 \frac{6gT}{m^2} P_0 \frac{6gT}{m^2} P_0 + ...$$

$$P_0 = \frac{1}{k^2 + m^2 - i\epsilon}$$

This is a geometric series:

$$P_0 \left[1 + r + r^2 + r^3 + ... \right] = \frac{1}{k^2 + m^2 - i\epsilon} \; \frac{1}{1 - r} \; ; \quad r = \frac{6gT}{m^2} \; \frac{1}{k^2 + m^2 - i\epsilon}$$

Thus:

$$P = \frac{1}{k^2 + m^2 - 6gT/m^2 - i\epsilon}$$

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Again, partial summation of a perturbation series, where the answer makes sense even if the series diverges, which happens if $|r| \geq 1$, that is

$$\left| \frac{6gT}{m^2} \; \frac{1}{k^2 + m^2 - i\epsilon} \right| \geq 1$$

If $k^2 + m^2 \approx 0$ (particle on mass-shell) the series diverges. Even so, the equation for P makes perfect sense also in that case.

The result for P shows that the mass changes due to the tadpole T:

$$m^2 \to m^2 - \frac{6gT}{m^2} \qquad \text{Recall}: \quad T = T_\pm = \frac{m^2 \pm \sqrt{m^4 - 12gt}}{6g/m^2}$$

Inserting the solutions for T there are then two possible masses:

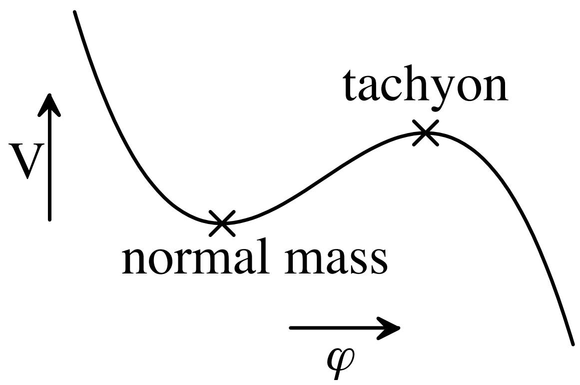
$$m^2 \to \mp\sqrt{m^4 - 12gt}$$

The lower sign makes sense. The upper changes the sign of m^2, which would mean that the particle becomes a tachyon, which is unacceptable.

Now the conventional approach. Usually the Lagrangain is split in a kinetic and potential part:

$$\mathcal{L} = T - V \ , \qquad\qquad V = \tfrac{1}{2}m^2\varphi^2 - g\varphi^3 - t\varphi$$

The equations of motion follow by considering the extrema of $\mathcal{L}$. Consider V as a function of φ. It is a curve of third order which has two extrema.



To find the extrema differentiate V with respect to φ:

$$m^2\varphi - 3g\varphi^2 - t = 0$$

Substituting $\varphi = T/m^2$ we find an equation for T that is identical to the one found before. Thus Feynman rules bring us automatically to the extrema of the Lagrangian. They correspond to the equations of motion of the theory.

How to make life simpler ? By shifting the field φ formulate the theory in such a way that there is no tadpole. Then we are directly in the extremum. The extrema were given by $\varphi = \frac{1}{m^2} T_\pm$. Let us call them $C_\pm$. Now shift φ such that the extremum is at $\varphi = 0$. Thus substitute

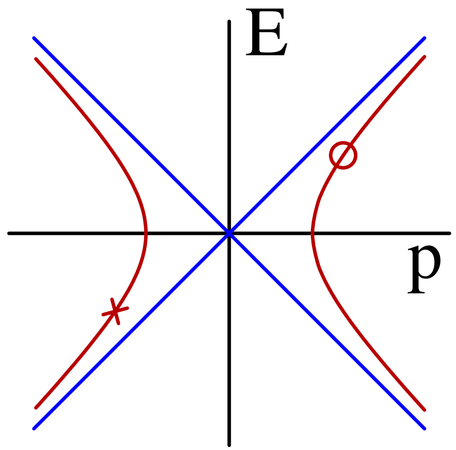
$$\varphi \rightarrow \varphi + C_\pm$$

That changes the potential energy V:

$$\frac{1}{2} m^2 \varphi^2 - g\varphi^3 - \varphi \quad \longrightarrow \quad \pm \frac{1}{2} \sqrt{m^4 - 12gt}\, \varphi^2 - g\varphi^3 - \Lambda$$

with Λ some constant. Taking now the new Lagrangian with the + sign we obtain directly the theory obtained by summing the perturbation series.

Are tachyons physically acceptable particles ? Very likely not, because the vacuum becomes unstable. This is because tachyons can have negative energy, and one could have a tachyon with negative energy and momentum together with a tachyon with the opposite energy and momentum and then have a state with zero energy and momentum. Out of nothing one could create two tachyons. The vacuum would contain an undetermined number of tachyons. Perhaps the theory can be summed, depending on the tachyon interactions, thus resulting in some non-perturbative theory. Not appetizing.



To illustrate the foregoing here some kinematics of tachyons. The energy-momentum relation for a tachyon with mass squared $-M^2$ is

$$E^2 = p^2 - M^2$$

In the figure above the red lines are according to that relation. A Lorentz transformation transforms a point on a red line into a point on that same line. Thus, a Lorentz transformation may transform a tachyon of positive energy (little circle) into a tachyon of negative energy. If Lorentz invariance holds there is no way that tachyons of negative energy can be excluded. The cross represents a tachyon of negative energy. Together with the tachyon represented by the little circle there can be a state of zero energy and momentum with two tachyons.

Cosmological Constant

Theory of gravitation of Einstein as a quantum field theory: massless particles of spin 2.

Massless particles of spin 1 or 2 always have spin states that are physically unacceptable (negative norm states).

Therefore:

For spin 1 or spin 2 particles of mass 0 a symmetry is needed that guarantees that the unphysical degrees of freedom do not couple to matter.

This is achieved bij gauge invariance, abelian for e.m, non-abelian for quantum chromodynamics, invariance funder general coordinate transformations for gravitation. The physical basis for the latter is the principle of equivalence.

In general a non-abelian gauge symmetry leads to the same coupling of the gauge particles to the other particles. Thus gluons couple with identical strenght to all quarks. This in contrast to electromagnetic interactions, where the coupling of the photon to some particle depends on its charge that may differ from particle to particle. For gravitation this leads to one well-defined unique way in which gravitons couple to matter. No choice, or very little.

Coupling of the graviton in the φ^3 theory:

$$\mathcal{L} = \sqrt{\mathrm{Det}}\ \left\{ -\tfrac{1}{2} D_\mu \varphi D_\mu \varphi - \tfrac{1}{2} m^2 \varphi^2 + g\varphi^3 + \Lambda \right\}$$

$$\Lambda = \frac{t^2}{m^2} + \mathcal{O}(g) \quad \text{for the ok solution}$$

Det = determinant of $g_{\mu\nu}$.

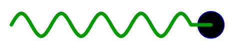
$g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu}$ $\qquad$ $h_{\mu\nu}$ = gravitational field $\qquad$ κ = coupling const.

To first order in the coupling constant κ:

$$\sqrt{\mathrm{Det}} \approx 1 + \frac{\kappa}{2} h_{\mu\mu}$$

Due to Λ there is a gravitational tadpole term:

$$\frac{\kappa}{2} \Lambda h_{\mu\mu}$$

which gives a vertex of a graviton disappearing in the vacuum: 
This is the cosmological term. Observed value is very close (or equal) to zero.

Solving the classical gravitational equations of motion with a cosmological term and interpreting $g_{\mu\nu}$ as the metric tensor leads to a curved universe with a curvature determined by Λ.

The Higgs field normally produces a value for Λ far, far from the observed magnitude. Typically the cosmological constant produced by the Higgs system produces a Universe with a size of about the order of the size of the head of a theorist ($\approx$ 15 cm radius). The observed cosmological constant is about a factor 10^{-55} or less times the value produced by the Higgs system.

In Einstein's theory of gravitation the cosmological constant is a free parameter. It may be chosen such that the addition of such a constant (i.e. Λ) produced by the Higgs system is compensated. So, while it is hard to understand how something like that could happen, it is nonetheless a formal possibility. We have no hard conflict.

The Original Higgs Model

The model is essentially q.e.d. with a massive photon. It is still useful to understand:

- Magnitude cosmological constant
- Unitarity limit
- Equivalence theorem

Unitarity limit. This is the range of validity of the theory as function of the Higgs mass m.

If m exceeds some limit (~ 0.5 TeV) then perturbation theory breaks down.

Equivalence theorem. If the Higgs heavy and thus perturbation theory breaks down then there might still be some way out. The theory might be shown to be analogous to pion physics at low energy. Then experimental results about pion physics may be used to make guesses about vector-boson and Higgs physics.

Note: taking the limit $m \to \infty$ (large Higgs mass) is in a vague sense understood as trying to remove the Higgs from the theory.

Lagrangian and Feynman rules of the simple Higgs model.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - (D_\mu K)^* D_\mu K - \mu K^* K - \frac{1}{2}\lambda(K^* K)^2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad\qquad D_\mu = \partial_\mu - igA_\mu$$

K is a complex field, thus two degrees of freedom, with mass squared μ. This Lagrangan is invariant under gauge transformations involving an arbitrary function Λ:

$$A_\mu \rightarrow A_\mu - \partial_\mu \Lambda \qquad\qquad K \rightarrow e^{-ig\Lambda}K$$

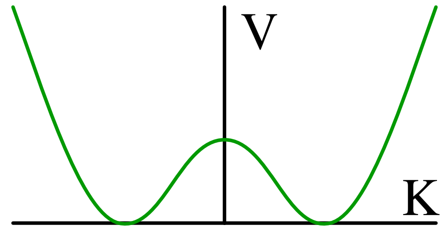
This Lagrangian can be rewritten:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - (D_\mu K)^* D_\mu K - \underline{\underline{\frac{1}{2}\lambda(K^* K - f_0^2)^2}} + \frac{1}{2}\lambda f_0^4$$

with $f_0^2 = -\mu/\lambda$. The doubly underlined piece is the potential (note the - sign in front). If $f_0^2 > 0$ this potential has two minima.

$V = \frac{1}{2}\lambda(K^*K - f_0^2)^2$

Assume $f_0^2 > 0$



There is a minimum for $|K| = f_0$. Actually, since K is complex one should draw the imaginary part along an axis perpendicular to the paper. The figure can then be obtained from the figure here by rotation around the V-axis. The two minima become a valley. The valley is directly a consequence of gauge invariance. If there is a minimum for $K = f_0$ (imaginary part of K zero) then there should alse be a minumum for any gauge transformed of the K. Thus the same minimum should also occur at $K = e^{i\Lambda} f_0$.

Rather then two minima as in the φ^3 theory whe have now this continuum. Instead of having to choose between $\frac{1}{m^2}T_\pm$ nature must now make a choice for some point somewhere in the valley.

The choice is not in the Feynman rules. Because nature must chose a point from a curve that resulted from gauge invariance, that choice implies a breaking of the gauge invariance. This is called spontaneous symmetry breaking. Actually it is not truly breaking of the symmetry, the invariance remains in a somewhat different form.

Thus the actual physical vacuum is a choice among many. This choice then seemingly breaks the symmetry.

Note however that there is no symmetry breaking in the Lagrangian.

It is quite possible to have mass generation by a field in the vacuum without spontaneous symmetry breakdown. The φ^3 theory is an example. Thus mass generation and spontaneous symmetry brakdown are two different things. Here it happens that they go together.

The Higgs model is a lot more complicated than the φ^3 model, and we will restrict ourselves to the essentials. First, the extremum is reached if $K^*K = f_0^2$. That solves to $K = e^{i\alpha} f_0$ with arbitrary α. A gauge transformation may be performed to eliminate α. Shift to the extremum:

$$K \to K + f_0$$

Next, split the field K in a real and imaginary part:

$$K = \frac{1}{\sqrt{2}}(H + i\varphi)$$

The crucial part comes from the term $-D_\mu K^* D_\mu K$. Remember that $D_\mu = \partial_\mu - igA_\mu$. Thus there is a term $-g^2 A_\mu A_\mu K^* K$ and after the shift a term $-g^2 f_0^2 A_\mu A_\mu$ will arise. That is a mass term for the photon.

Here the Lagrangian after the shift in terms of H and φ:

$$\begin{aligned}
\mathcal{L} = & -\tfrac{1}{2}(\partial_\mu A_\nu)^2 - \tfrac{1}{2}M^2 A_\mu^2 \\
& -\tfrac{1}{2}(\partial_\mu \varphi)^2 - \tfrac{1}{2}M^2\varphi^2 - \tfrac{1}{2}(\partial_\mu H)^2 - \tfrac{1}{2}m^2 H^2 \\
& - gA_\mu(\varphi\partial_\mu H - H\partial_\mu\varphi) - \tfrac{1}{2}g^2 A_\mu^2(H^2+\varphi^2) - gMA_\mu^2 H \\
& - \frac{m^2 g^2}{8M^2}(\varphi^2 + H^2) - \frac{m^2 g}{2M}H(\varphi^2+H^2) \\
& + \frac{m^2 M^2}{8g^2} \\
& + \text{Faddeev} - \text{Popov part.}
\end{aligned}$$

with $M = gf_0\sqrt{2}$ and $m^2 = 2\lambda f_0^2$ (remember $f_0^2 = -\mu/\lambda$).

The last line of the Lagrangian contains field-theoretical things that we do not need to know in detail. The gauge fixing term $\frac{1}{2}(\partial_\mu A_\mu)^2$ is included, cancelling against a part of the term $-\frac{1}{4}F_{\mu\nu}F_{\mu\nu}$. The constant on the last but one line will become the cosmological constant when gravity is included.

Gauge invariance

Even if the K field was shifted that does not ruin gauge invariance. It just takes a different form.

Before the shift we had invariance under the transformation

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda \qquad\qquad K \rightarrow e^{-ig\Lambda} K$$

with an arbitrary function Λ. For infinitesimal Λ and in terms of the fields H and φ (remember, $K = \frac{1}{\sqrt{2}}(H + i\varphi)$):

$$K \rightarrow (1 - ig\Lambda)K \qquad \text{or} \qquad H \rightarrow H + g\Lambda\varphi \quad \text{and} \quad \varphi \rightarrow \varphi - g\Lambda H$$

After the shift $K \rightarrow K + f_0$, that is $H \rightarrow H + f_0\sqrt{2}$ and φ unchanged, the gauge transformation that leaves the Lagrangian invariant is:

$$H \rightarrow H + g\Lambda\varphi \qquad \varphi \rightarrow \varphi - g\Lambda H - g\Lambda f_0\sqrt{2}$$

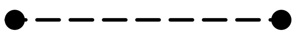
Thus the fields H and φ transform among themselves but in addition a constant is added to φ. That marks φ as a ghost, because by a suitable choice of Λ it can effectively be transformed away. Detailed study of the Feynman rules and the Ward identities following from this gauge invariance confirm this.

Feynman rules.

Propagators:

$$\frac{\delta_{\mu\nu}}{k^2 + M^2 - i\epsilon}$$

Vector particle (massive photon)



$$\frac{1}{k^2 + M^2 - i\epsilon}$$

Higgs ghost φ

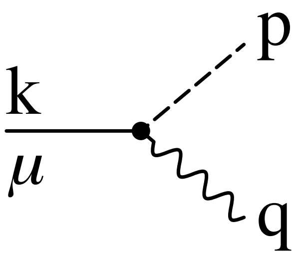
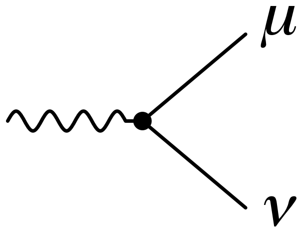
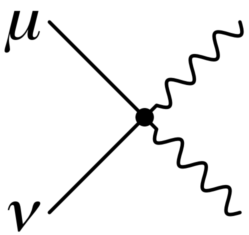
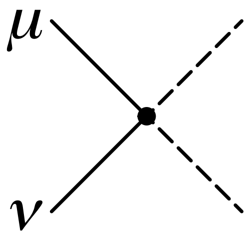
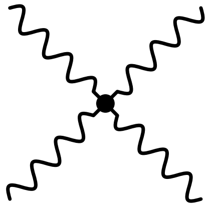
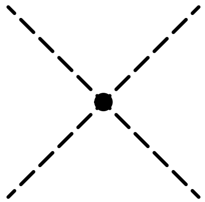
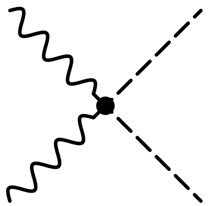
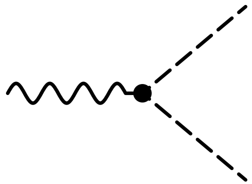
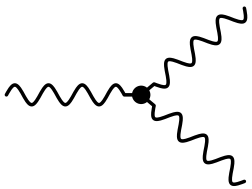
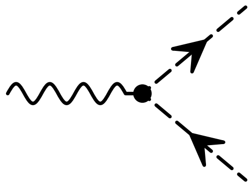
$$\frac{1}{k^2 + m^2 - i\epsilon}$$

Higgs particle H

$$\frac{1}{k^2 + M^2 - i\epsilon}$$

Faddeev – Popov ghost

Even for this simple theory there are quite a number of vertices.

	$ig(p-q)_\mu$		$-2gM\delta_{\mu\nu}$
	$-2g^2\delta_{\mu\nu}$		$-2g^2\delta_{\mu\nu}$
	$-3\dfrac{m^2g^2}{M^2}$		$-3\dfrac{m^2g^2}{M^2}$
	$-\dfrac{m^2g^2}{M^2}$		$-\dfrac{m^2g}{M}$
	$-3\dfrac{m^2g}{M}$		$-gM$
×	$\dfrac{m^2M^2}{8g^2}$ Cosmological Constant		

There is an interesting point that may be mentioned. We have noted that the φ field is a ghost field. That was concluded on the basis of the gauge invariance of the theory:

$$H \rightarrow H + g\Lambda\varphi \qquad \varphi \rightarrow \varphi - g\Lambda H - g\Lambda f_0\sqrt{2}$$

showing that the field φ can be shifted by an arbitrary amount determined by the function $\Lambda(x)$. The degree of freedom that is thus lost here comes back as a degree of freedom of the photon field, because a massless vector field has two degrees of freedom, a massive one three. One may ask: how is this if there had been no vector boson ? Where would that degree of freedom have gone ? The answer is that the symmetry is lost. The invariance under a space-time dependent gauge transformation (space-time dependent Λ) needs the vector boson. If there is no vector boson one has no D_μ and the theory is only invariant under gauge transformations with space-time independent Λ. Then φ is no more a ghost.

Conclusions Higgs Model

1. Cosmological Constant
2. Unitarity Limit, Heavy Higgs
3. Equivalence Theorem

Cosmological Constant

The Feynman rules show a cosmological constant:

$$C \equiv \frac{m^2 M^2}{8g^2} (\approx 100 \text{ GeV})^4 \quad + \text{rad. corr.} + \text{initial value}$$

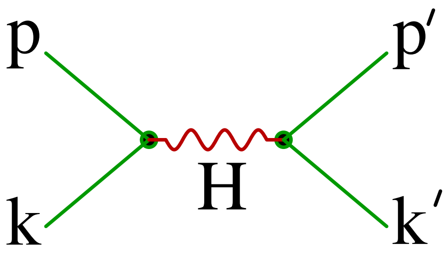
From astronomy:

$$C < (1.23 \times 10^{-9} \text{MeV})^4$$

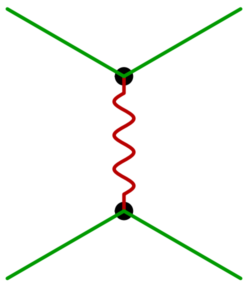
This is outrageously different. However, since we do not know the initial value there is no hard confrontation.

Unitarity Limit, Heavy Higgs

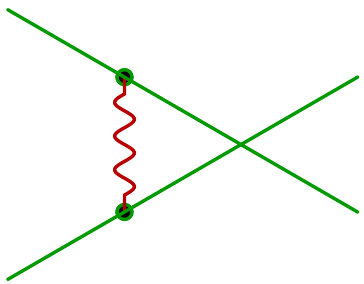
Consider longitudinally polarized photon - photon scattering in this model (massive photons) in lowest order.



$$\frac{g^2}{-s+m^2}\left(\frac{s^2}{M^2}-4s+4M^2\right)$$

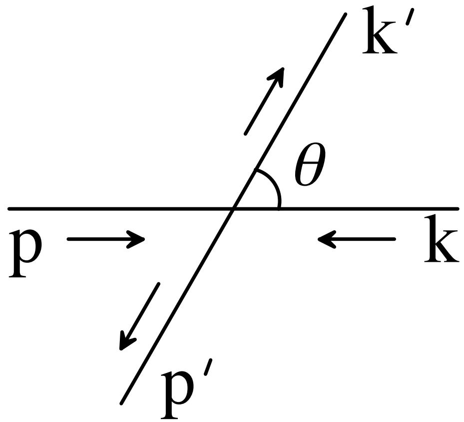


$$\frac{g^2}{-t+m^2}\left(\frac{t'^2}{M^2}-4t'+4M^2\right)$$



$$\frac{g^2}{-u+m^2}\left(\frac{u'^2}{M^2}-4u'+4M^2\right)$$

In here s is the center of mass energy squared, and t' and u' are related to the momentum transfer. They are related to the Mandelstam variables.



The Mandelstam variables are:

$$s = -(p+k)^2 \qquad t = -(p-p')^2 \qquad u = -(k-p')^2$$

These variables are not independent, $s+t+u = 4M^2$.
Instead of t we used the variables t' and u', with

$$t' = \frac{1}{2}(1-\cos\theta)s = -t + \mathcal{O}(M^2)$$

$$u' = \frac{1}{2}(1+\cos\theta)s = -u + \mathcal{O}(M^2)$$

The first diagram behaves for large s proportional to s. Why is this ? That is because of the polarization vectors. The polarization vector associated with a longitudinal photon of momentum k is:

$$e(k) = \frac{1}{M}\left(0, 0, k_0, i|\overrightarrow{k}|\right)$$

and similarly for the others. The components behave like $k_0 \approx \sqrt{s}$, and four polarization vectors and a propagator behaving like $1/s$ give then together a behaviour proportional to s.

Special case: $m \gg M$, $s \to \infty$, forward direction $(t = 0, u = -s + 4M^2)$. If $M^2 \ll m^2 << s$ then the sum of the three diagrams is $A = 2g^2m^2/M^2$.

Clearly, if g^2m^2/M^2 of order 1 or larger then perturbation theory breaks down. The lowest order contribution becomes of the same order as the no-scattering case.

This is usually presented in a somewhat different form, and one says that the Unitarity limit is exceeded. It should be emphasized that Unitarity (conservation of probability) is not violated, but what happens is that higher order contributions (also of order 1 or more) must cancel out the excess. In conclusion:

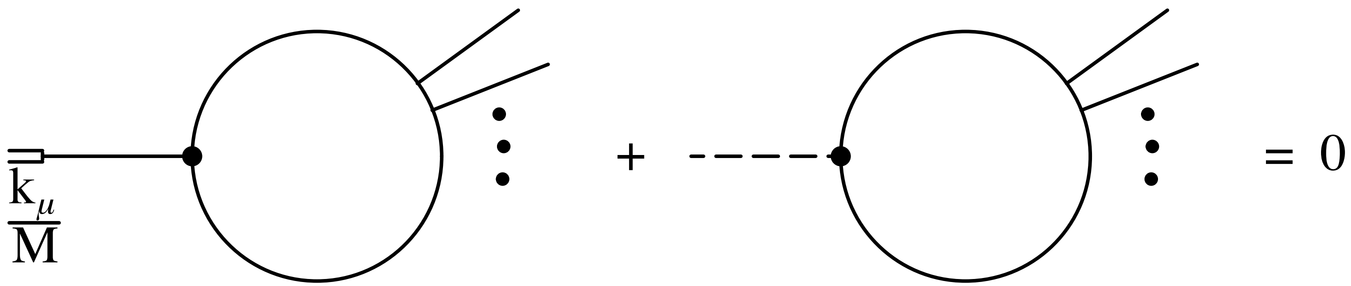
Unless $g^2m^2/M^2 < 1$ or $m^2 < M^2/g^2$ no perturbation theory. Using the values of g and M of the Standard Model gives as limit for the validity of perturbation theory:

$$m^2 < (80 \text{ GeV})^2 \times 30 \approx (500 \text{ GeV})^2$$

Equivalence Theorem

At high energy longitudinally polarized vector bosons behave like the Higgs ghost.

Here high energy means $E \gg M$. The "theorem" is in general of questionable value because "high energy" is a relative statement. In any case, it applies to the case of WW scattering at high energy. The theorem follows from a Ward identity. The original gauge invariance of the Higgs model translates into Ward identities, i.e. equations between diagrams. In our case, with our choice for the gauge breaking terms the Feynman rules as given before lead to a simple Ward identity, namely $-\partial_\mu A_\mu + M\varphi = 0$. In terms of diagrams:



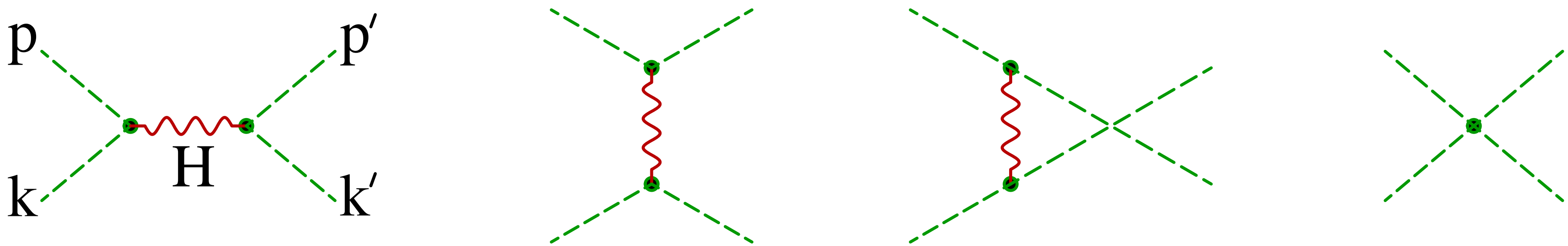
where the two blobs have, apart from the photon and φ lines, the same external lines. Similar identities hold for any number of photon lines with a momentum factor.

Now the polarization vector for a photon of momentum k is:

$$e(k) = \frac{1}{M}\left(0, 0, k_0, i|\overrightarrow{k}|\right) \qquad k_0 = \sqrt{|\overrightarrow{k}|^2 + M^2}$$

It follows that in the limit of large energy k_0 the quantities k_0 and $|\overrightarrow{k}|$ are approximately equal and then $e_\mu \approx k_\mu / M$. The Ward identities may be used with respect to the external longitudinal high energy photon lines.

For longitudinal $W - W$ scattering at high energy the theorem may be used. The amplitude for this process becomes equal to that of $\varphi - \varphi$ scattering. In lowest order the diagrams are:



$$\frac{m^4 g^2}{M^2} \frac{1}{-s + m^2} \qquad \frac{m^4 g^2}{M^2} \frac{1}{-t + m^2} \qquad \frac{m^4 g^2}{M^2} \frac{1}{-u + m^2} \qquad -3\frac{m^2 g^2}{M^2}$$

Compared to the vector scattering diagrams the behaviour as function of the energy is simpler. A cancellation between the three vector boson diagrams is now build-in.

The advantage of the equivalence theorem is that the calculation with φ lines is usually much simpler. Furthermore, only vertices of the Higgs sector are involved. In addition, in the Standard Model, the Higgs ghost amplitudes may be related, by suitable assumptions, to $\pi - \pi$ scattering at low energy (with a scaling from the vector boson mass M to the pion mass). Then much depends on our understanding of low energy $\pi - \pi$ scattering, which is far from perfect. That is a quite complicated subject, with much uncertainty. Things like the ρ resonance must be considered, which translates into a resonance in the $W - W$ channel. The relation between the mass of the ρ-resonance and the $W - W$ resonance depends on the Higgs mass. We will consider that issue later.

Now the Standard Model. The Higgs sector of the Standard Model is Schwingers σ-model. At the same time the σ-model is thought to be a good model to describe pion - pion scattering. Therefore it is very useful to study the σ-model without yet including the weak and e.m. interactions.

The σ-model

Deceptively simple. Take four scalar (spinless) fields φ_i, $i = 1, .., 4$. The Lagrangian is:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi_i\partial_\mu\varphi_i - \frac{1}{2}\mu\varphi_i^2 - \frac{1}{8}\lambda(\varphi_i^2)^2$$

The index i is to be summed over, thus $\varphi^2 = \varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2$. If one decides on four scalar fields this is about the simplest model one can think of. There are also models with with two, three, five fields or whatever. We consider only the case of four scalar fields and speak of the *sigma*-model.

There are very interesting symmetries, visible to the naked eye.

The 4-field model has an O(4) symmetry (orthogonal transformations in four dimensions). That is like Lorentz transformations without the i for the fourth component.

The O(4) symmetry is a 6 parameter symmetry (just like the Lorentz transformations).

That is indeed the complete symmetry content of the theory.

The model can be rewritten in a number of interesting ways, advantegeous in different situations.

As with Lorentz transformations, separate the first three and the fourth component:

$$\varphi_i \to \varphi_i \qquad i = 1, 2, 3$$
$$\varphi_4 \to \sigma$$

The Lagrangian becomes:

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu \sigma)^2 - \tfrac{1}{2}\partial_\mu \varphi_i \partial_\mu \varphi_i - \tfrac{1}{2}\mu(\sigma^2 + \varphi_i^2) - \tfrac{1}{8}\lambda(\sigma^2 + \varphi_i^2)^2$$

This is what one usually does. The three φ correspond to the three pions and the σ corresponds to the σ resonance (?)

The (infinitesimal) invariances are separated correspondingly:

$$\varphi_i \to \varphi + \epsilon_{ijk}\Lambda_j \varphi_k \qquad \text{(Orthogonal rotat. in 3 dim. } \varphi - \text{space}$$
$$\sigma \to \sigma \qquad \text{(Isospin!)}$$

and

$$\varphi_i \to \varphi - \sigma\overline{\Lambda}_i \qquad \text{("True" Lorentz transf.)}$$
$$\sigma \to \sigma + \varphi\overline{\Lambda}_i$$

The Λ and $\overline{\Lambda}$ are infinitesimal. The first transformation, with the Λ_i, are three dimensional rotations, just like isospin. In this form the model is used for describing pions and the σ resonance.

Another way of writing the model is advantageous for the Standard Model. A two component complex field K is used:

$$\mathcal{L} = -\partial_\mu K^\dagger \partial_\mu K - \mu K^\dagger K - \frac{1}{4}\lambda(K^\dagger K)^2$$

$$K = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma + i\varphi_3 \\ -\varphi_2 + i\varphi_1 \end{pmatrix}$$

This Lagrangian is manifest invariant for complex rotations in two dimensions, i.e. SU_2:

$$K \to UK, \text{ thus } K^\dagger \to K^\dagger U^\dagger \quad \text{with} \quad U^\dagger U = 1$$

The two by two matrices U can be written in exponential form. Any two by two matrix can be written as a linear combination of the Pauli matrices:

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The matrices U can then be written as:

$$U = e^{-\frac{i}{2}(\Lambda_i^L \tau_i)}$$

This involves three real parameters Λ_i^L. But we know that the model has a six-parameter symmetry. Where are the other three parameters ?

There is also a manifest U_1 symmetry:

$$K \to e^{-i\Lambda_0} K$$

This is just a phase factor. There is still a hidden two parameter symmetry. Thus we have explicitly a $SU_2 \times U_1$ symmetry, the symmetry of the Standard Model. Using the model in this notation for the Higgs sector of the Standard Model there is thus a hidden symmetry.

This hidden symmetry gives rise to a relation between the W and Z_0 masses:

$$\rho \equiv \frac{M^2}{M_0^2 \cos^2(\theta)} = 1 + \text{rad. corr.}$$

This ρ-parameter has played an important role, because the radiative corrections turn out to be dependent on the quark masses, in practice mainly on the top quark mass. By a careful measurement of ρ at LEP the top quark mass could then be predicted. It was subsequently found at Fermilab with a mass value agreeing with this prediction.

We how have written the σ model in terms of

- four fields φ displaying a manifest O_4 symmetry
- three fields φ and a field σ showing a manifest isospin symmetry
- a complex two-component field K showing a manifest $SU_2 \times U_1$ symmetry.

Another rewrite of the σ model shows explicitly an $SU_2 \times SU_2$ symmetry.

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}(\partial_\mu \Phi^\dagger \partial_\mu \Phi) - \frac{1}{2}\mu \left[\frac{1}{2}\mathrm{Tr}(\Phi^\dagger \Phi)\right] - \frac{1}{8}\lambda \left[\frac{1}{2}\mathrm{Tr}(\Phi^\dagger \Phi)\right]^2$$

The complex two by two matrix Φ is given by (τ^0 is the 2×2 unit matrix):

$$\Phi = \sigma\tau^0 + i\varphi_j\tau^j = \begin{pmatrix} \sigma + i\varphi_3 & i\varphi_1 + \varphi_2 \\ i\varphi_1 - \varphi_2 & \sigma - i\varphi_3 \end{pmatrix}$$

with real fields σ and φ_i. The basic quantity is the trace $\mathrm{Tr}(\Phi^\dagger\Phi)$. This trace has two invariances:

$$\mathrm{SU}_2 \text{ left :} \qquad \Phi \rightarrow e^{\frac{i}{2}(\Lambda_j^L \tau^j)}\,\Phi \quad \text{and} \quad \Phi^\dagger \rightarrow \Phi^\dagger e^{-\frac{i}{2}(\Lambda_j^L \tau^j)}$$

$$\mathrm{SU}_2 \text{ right :} \qquad \Phi \rightarrow \Phi\, e^{\frac{i}{2}(\Lambda_j^R \tau^j)} \quad \text{and} \quad \Phi^\dagger \rightarrow e^{-\frac{i}{2}(\Lambda_j^R \tau^j)}\Phi^\dagger$$

There are now 6 parameters (three Λ^L and three Λ^R) and we have manifestly the full symmetry content of the model. Mathematicians say: $\mathrm{O}_4 = \mathrm{SU}_2 \times \mathrm{SU}_2 : \mathrm{Z}_2$. In here : means division. The discrete group Z_2 (elements 1 and -1) is of no interest to us here.

This way of writing of the σ model is probably the best one in connection with the Standard Model. The full symmetry is explicit, and it is also useful in connection with invariances of quantum chromodynamics. Remarkably, left and right symmetry become symmetries of right and left handed quarks.

The Standard Model

Vector boson part:

$$\mathcal{L}_{\rm vb} = -\tfrac{1}{2}{\rm Tr}(b_{\mu\nu}b_{\mu\nu}) - \tfrac{1}{2}{\rm Tr}(c_{\mu\nu}c_{\mu\nu})$$

$$b_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu + g[b_\mu, b_\nu] \quad ([,]\ \text{indicates commutator})$$

$$c_{\mu\nu} = \partial_\mu c_\nu - \partial_\nu c_\mu$$

$$b_\mu = -\frac{i}{2}\begin{pmatrix} B^3_\mu & B^1_\mu - iB^2_\mu \\ B^1_\mu + iB^2_\mu & -B^3_\mu \end{pmatrix}$$

$$c_\mu = -\frac{i}{2}\begin{pmatrix} \alpha B^0_\mu & 0 \\ 0 & \beta B^0 \end{pmatrix} \qquad \alpha^2 + \beta^2 = 1$$

The B-fields are related to the W, Z_0 and A fields. In fact B^3 and B^0 are certain mixtures of Z_0 and A and $W^\pm = \frac{1}{\sqrt{2}}(B^1 \mp iB^2)$.

From the point of view of the Higgs sector (the σ-model) Nature could have used **two** vector boson triplets, one with SU_2^L, the other triplet with SU_2^R. In that case there would have been 6 vector bosons of which 3 massless.

However, Nature uses SU_2^L and only a U_1 part of SU_2^R (only $\Lambda_3^R \neq 0$). There are 4 vector bosons of which one massless.

Technical detail: the replacement to make in the σ-model to make the global $SU_2 \times U_1$ invariance (space-time independent Λ) to a local one (space-time dependent Λ) is:

$$D_\mu \Phi = \partial_\mu \Phi + g b_\mu \Phi - g' \Phi \, c_\mu \qquad \text{with in } c \quad \beta = -\alpha$$

Now c is a multiple of τ^3. Note that g' is an arbitrary new constant. For a U_1 type symmetry such as in q.e.d. the coupling constant (e in the case of q.e.d) may have different values for different fields.

Mass term

Everything depends on $\Phi^\dagger \Phi$. The shift

$$\Phi \to \Phi + f_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

generates the vector boson masses. Let now $g' \equiv g \frac{s}{c}$, with $s = \sin \theta_w$ and $c = \cos \theta_w$. Then the mass part of the Lagrangian is obtained from $D_\mu \Phi^\dagger D_\mu \Phi$, keeping only the f_0-part of Φ:

$$\mathcal{L}_M = -\frac{1}{8} f_0^2 g^2 (c B_\mu^a - s R_\mu^a)^2, \quad a = 1, 2, 3$$

$$R_\mu^3 = B_\mu^0, \; R_\mu^1 \text{ and } R_\mu^2 = 0$$

Three field combinations get a mass, namely B^1 and B^2 and $cB^3 - sB^0$. The combination $sB^3 + cB^0$ remains massless.

$$\begin{aligned}\mathcal{L}_M &= -\tfrac{1}{2}M^2(B^1_\mu)^2 - \tfrac{1}{2}M^2(B^2_\mu)^2 - \tfrac{1}{2}M_0^2(cB^3_\mu - sB^0_\mu)^2 \\ &= -\tfrac{1}{2}M^2(W^+_\mu)^2 - \tfrac{1}{2}M^2(W^-_\mu)^2 - \tfrac{1}{2}M_0^2(Z^0_\mu)^2\end{aligned}$$

$$M^2 = \tfrac{1}{4}g^2 f_o^2, \quad M_0^2 = \frac{M^2}{c^2}$$

Remember, $W^\pm_\mu = \frac{1}{\sqrt{2}}(B^1_\mu \mp iB^2_\mu)$

Even if there had been 6 fields (two more R fields) there would still have been only three mass terms. The σ-model cannot generate more than 3 masses.

The general rule is this. If the Higgs system has N fields (N degrees of freedom) than it can generate no more than $N - 1$ masses. This is because at least one degree of freedom of the Higgs system must remain physical. Else one could have a renormalizable model with massive vector particles without any Higgs particles; such models have been proven to be non-renormalizable.

Isospin invariance

Ignore electromagnetisme. Then we have SU_2^L with its vector bosons, but no photon. The Z_0 mass would be the same as the W mass. Even after spontaneous symmetry breakdown, i.e. after making the shift:

$$\Phi \longrightarrow \Phi + f_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

something remains. Consider the Higgs sector (the σ-model). Under SU_2^L and SU_2^R this Φ transforms as

$$\Phi \longrightarrow e^{\frac{i}{2}(\Lambda_j^L \tau^j)}\ \Phi\ e^{\frac{i}{2}(\Lambda_j^R \tau^j)}$$

If we take simultaneously SU^L and SU^R and choose $\Lambda^R = -\Lambda^L$ then also the shift remains invariant.

The theory is invariant, including the vacuum.

This symmetry is denoted as $\mathrm{SU}_2^L + \mathrm{SU}_2^R$.

It is a global symmetry, because due to the absence of vector bosons for the SU_2^R part there is only invariance for space-time independent Λ^R. It is in fact ordinary isospin.

The isospin symmetry is broken if we re-introduce the U_1 part, which is the Λ_3^R part. There will be corrections proportional to g', or rather $\sin\theta_w$.

If there were no e.m. interactions then isospin would be strictly valid, and therefore the Z_0 mass would be equal to the W mass. After re-introduction of e.m related terms it follows

$$M^2 = M_0^2 + \mathcal{O}(\sin\theta_w)$$

Actual calculation shows the earlier mentioned result for the ρ-parameter:

$$\rho \equiv \frac{M^2}{M_0^2\cos^2(\theta)} = 1 + \text{rad. corr.}$$

While it is easy to understand why ρ equals 1 if $\sin\theta_w = 0$, the precise form for ρ as shown here can be obtained only by actual calculation, not by any simple symmetry argument. See above, where we considered the vector boson mass term in the Lagrangian, with the result $M_0 = M/\cos\theta$. For more complicated systems the relation is no more true, although sometimes limits can be established.

Further breaking of isospin will affect the ρ-parameter. For example, mass differences between the quarks must be considered as isospin breaking. Through radiative corrections involving those quarks as internal particles the ρ-parameter will be affected. Thus, the mass difference between top and bottom quark produces a measurable addition to the ρ-parameter. This radiative correction is a most peculiar one: it grows with the square of the top mass. No other correction is known that increases as the mass of some virtual particle increases. It produces a window on the mass spectrum beyond the directly accessible region. Thus LEP could produce a value for the top-mass from a precise measurement of the ρ-parameter.

Also the Higgs system contains isopsin breaking, and produces a change in the ρ-parameter, although not as spectacular as the top quark. Here is the relevant equation:

$$\rho = 1 + \frac{3G_F}{8\pi^2\sqrt{2}} \left(m_t^2 - \frac{M^2 s^2}{c^2} \ln \frac{m^2}{M^2} \right)$$

where m is the Higgs mass. The bottom mass has been taken to be small with respect to the top mass m_t. Note the proportionality to $\sin^2\theta_w$ in the second term. This second term is the basis of all estimates of the Higgs mass from LEP data plus the value of the top-mass (178±4.3 GeV).

If the $m = 2M \sim 160$ GeV then the logarithmic term gives the same as subtracting 7.3 GeV from the top mass. 1.5 $M = 120$ GeV gives 4.3 GeV.

Fermion masses

Consider fermion doublet such as the u-d quark combination. Define left and right handed fields:

$$\psi_a^L = \tfrac{1}{2}(1+\gamma^5)\psi_a \qquad a = \text{u or d}$$

$$\psi_a^R = \tfrac{1}{2}(1-\gamma^5)\psi_a$$

In the Lagrangian a mass term is of the form

$$m_f\overline{\psi}\psi = m_f(\overline{\psi}^L\psi^R + \overline{\psi}^R\psi^L)$$

Such a term may also be generated by the Higgs field Ψ:

$$\mathcal{L}_{fm} = -g_f\,\overline{\psi}_a^L\Phi_{ab}\psi_b^R + h.c.$$

with g_f to be chosen such that the right mass results. The above expression respects both SU_2^L **and** SU_2^R. After the shift the f_0 zero part becomes a mass term with mass $g_f\ f_0$, and the above term gives equal mass to the two quarks. This is of course not true experimentally, isospin is broken and the two quark masses are different. If however we require only $\mathrm{SU}_2{\times}\mathrm{U}_1$ symmetry the following term involving an arbitrary constant η is allowed as well:

$$-g_f\eta\,\overline{\psi}_a^L\Phi_{ab}\,\tau^3\psi_b^R + h.c.$$

Now SU_2^R is no longer respected, only the U_1 part corresponding to a non-zero Λ_3^R, which is precisely the part used in the Standard Model. The above term gives the opposite contribution to up and down quark mass, and by suitably chosing g_f and η any mass value can be generated.

Thus a general fermion mass generating term is of the form:

$$-g_f \, \overline{\psi}_a^L \Phi_{ab} \, (1 + \eta \tau^3) \psi_b^R + h.c.$$

A non-zero η gives isospin breaking. The quantities g_f and η differ from fermion multiplet to fermion multiplet.

Parity

The Higgs field Φ_{ab} transforms as an isospin spinor (transformation of the index a under SU_2^L and a U_1 singlet (transformation of the index b. To construct a fermion mass term we must make an $\mathrm{SU}_2 \times \mathrm{U}_1$ invariant. Thus out of $\overline{\psi}^L$ and ψ^R together with Φ we must make an invariant. Think of a situation where a spin $\frac{1}{2}$ particle (the Ψ field) decays in two other partiles. The possibilities are (i) ine spin $\frac{1}{2}$ particle and one scalar or (ii) one spin $\frac{1}{2}$ particle and one spin 1 particle. Here, we have that either (i) $\overline{\psi}^L$ is a doublet and ψ^R a singlet or (ii) $\overline{\psi}^L$ is a doublet and ψ^R an isovector. In any case, unavaidable, $\overline{\psi}^L$ and ψ^R transform differently under SU_2^L. Therefore they couple differently to the vectorbosons and **parity is violated**. The standard solution is the one sketched before:

$$\overline{\psi}_a^L \Phi_{ab} \, (1 + \eta \tau^3) \psi_b^R$$

where the index a transforms as a doublet under SU_2^L. ψ^R is invariant under SU_2^L, but it transform under the U_1 transformation, as does the Ψ through the index b. Altogether this mass term is an $\mathrm{SU}_2^L \times \mathrm{U}_1$ invariant.

Parity violation is a consequence of the generation of fermion masses by means of the Higgs field.

Given then that ψ^L and ψ^R behave differently under $\mathrm{SU}_2^L \times \mathrm{U}_1$ it follows that an ordinary type mass term, of the form:

$$m_f \; \overline{\psi}_a^L \psi_b^R a$$

must be excluded. The reverse statement holds true as well.

If parity is violated then the fermion masses must be generated using the Higgs field.

In conclusion: all masses, those of the vector bosons as well as those of the fermions are generated by the Higgs field.

Other Higgs systems

General remarks: by using sufficiently many different Higgs systems any amount and type of symmetry breaking may be achieved. In particular, any value of the Z_0 mass can be generated. Therefore, a priory, any amount of neutral currents can be accomodated.

Specific consequences of the use of the σ-model as Higgs system.

- There are three massive and one massless vector boson.
  The zero mass of the photon is a consequence of this σ-model.
- There is a relation between the vector boson masses:

$$\rho = \frac{M^2}{M_0^2 cos^2 \theta_W} = 1 + \text{rad. corr.}$$

- Parity is violated (not specific to the σ-model) whenever fermion masses are generated by the Higgs field.
- (not discussed) Parity is conserved in e.m. interactions.

Experiment agrees very well with the model. The Higgs system appears very much to be according to the σ-model.

If we want to keep the ρ-parameter prediction then the use of other than σ-model type Higgs systems is excluded. One could however have two Higgs systems, two σ-models. For example, with two Higgs systems one of them could generate the up-down masses, the other the charm-strange masses and the top-bottom masses.

This in fact was done by Peccei and Quinn in order to cure a problem of the Standard Model (the strongly interacting piece, QCD) namely strong CP violation. However, the use of two Higgs systems introduces some problems.

- Vacuum alignment problem.
  The photon will not automatically have zero mass. A prediction is lost.
- Not all Higgs particles become ghosts. In the case of two σ-models there are originally 8 Higgs fields of which 3 will become ghosts, the remaining 5 will be ordinary spin 0 particles.

In addition, with such multiple Higgs systems often one of the physical scalar particles will be massless. This is also the case for the Peccei-Quinn model: **the axion.** Has not been seen.

Also supersymmetric models use more than one Higgs system. So they have to explain why the photon has zero mass. In the minimal supersymmetric model the two vacua align and the photon is massless.

Searching for the Higgs

Screening

The theory without the Higgs is non-renormalizable. That implies infinities that cannot be absorbed in the free parameters of the theory, and are therefore observable. By making the Higgs heavy, thus removing the Higgs from the system, these infinities should come up. It follows that the non-renormalizable infinities correspond to effects that become large if the Higgs mass becomes large. From the absence of such (observable) terms it should then be possible to deduce an upper limit to the Higgs mass.

The case of the top-mass is an example of this type of reasoning. Without the top quark (but with a bottom quark) the theory is non-renormalizable. The correction to the ρ-parameter, depending quadratically on the top mass, is an example of things that may happen. We now must look for analogous things involving the Higgs.

The first one has already been mentioned: the Higgs mass dependent correction to the ρ-parameter. That becomes large with large Higgs mass, but only logarithmically.

It could have been quadratic as well, like for the top mass, but through some cancellation the quadratic term drops out and the logarithmic one remains.

There is a statement proven long ago that without a Higgs the theory is one-loop renormalizable. This means that contributions that blow up with the Higgs mass are at most quadratic in that mass for one loop self-energy diagrams, linear for three-point diagrams and logarithmic for four-point diagrams. Linear becomes in practice logarithmic, because the theory never produces anything depending linearly on the Higgs mass, always quadratically. In fact, the Lagrangian contains only m^2. The possibly quadratic divergence for self-energy diagrams is in principle unobservable as it can be absorbed in the W and Z_0 mass. Only if there is a relation between those masses could such a term be observable, That is precisely the situation with the ρ-parameter, but there it does not appear. The sum total of this reasoning is that the only observable dependencies on the Higgs mass in the limit of large Higgs mass will be logarithmic in that mass. This statement, far from trivial, is called the **"screening theorem"**. It means that nature has constructed the theory such that it is difficult to see if the Higgs is there, to find an upper limit to the Higgs mass.

Let us put this somewhat more precisely. At the one-loop level there will be quadratic radiative correcions to the W and Z_0 mass:

$$M^2 \to M^2 + a\, m^2 + \mathcal{O}(\ln m^2)$$

$$M_0^2 \to M_0^2 + b\, m^2 + \mathcal{O}(\ln m^2)$$

The coefficients a and b must be found by explicit calculation. However, the masses M and M_0 are free parameters of the theory and therefore these radiative corrections are invisible. Only if there is a relation between M and M_0 the coefficients a, b might be observable. In the case of the simplest Higgs system, the σ-model, we have that in lowest order the ρ-parameter equals one, thus $M^2 = M_0^2 \cos^2\theta_w$.

Unfortunately, explicit calculation shows that also $a = b\cos^2\theta_w$ and the ρ-parameter contains no m^2 terms.

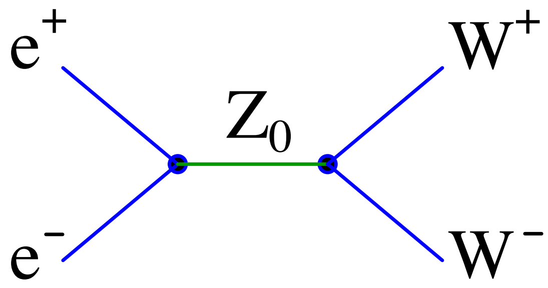
At the two-loop level there will be observable quadratic corrections, but those are diminished by the associated factors of the coupling constant. For example, there is a two-loop contribution proportional to the Higgs mass squared to the ρ-parameter that has been worked out. A Higgs mass of 140 times the vector boson mass is needed before this contribution is of the same order as the one-loop (logarithmic) one. And to make things worse, it has the opposite sign and cancels the lowest order term. However, other effects than radiative corrections become important for much lower Higgs mass ($m > 500$ GeV).

In general one has that two loop radiative corrections are proportional to m^2, three loop to m^4 etc. If the Higgs mass becomes large these corrections become as large as the one loop correction, and one can no more see if the observed correction is the lowest order one or the sum of a large number of terms with unknown coefficients. There is however one way in which this could be established: comparing radiative corrections for different processes. Consider again the ρ-parameter.

From the known one-loop radiative correction to the ρ-parameter one deduces a limit on the Higgs mass **without knowing if the higher order corrections are relevant**. That is something one should keep in mind when considering the limit on the Higgs mass from present day eperiments. **It might be a fake, the lowest order equation may be wrong**. One could say: the limit on the Higgs mass is 120 GeV unless the Higgs is strongly interacting.

There is however one way to check this. If there is another process with logarithmic corrections than one can check if, using the Higgs mass obtained from the ρ-parameter, that other radiative correction gives the correct, observed, result. So, seeing that the lowest order expressions work well in two different places we may infer that very likely they suffer no higher order effects (which would be different for the two cases).

As an example, a potential candidate for such a second process is W pair production at an electron-positron machine. Here is the relevant lowest order diagram.



It involves a $Z_0W^+W^-$ vertex which will suffer radiative corrections proportional to $\ln m^2$. Knowing $\ln m^2$ from a measurement of the ρ-parameter this value can be inserted in the one-loop calculation of the radiative corrections to the three vertex.

If that agrees with experiment than there is reason to have some confidence in the value of the Higgs mass so obtained.

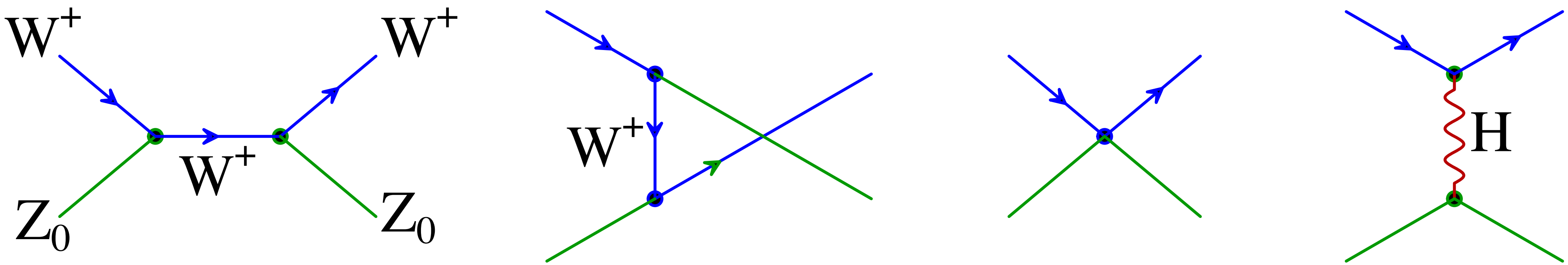
If the Higgs mass is larger than 500 GeV then there are other effects which will be discussed now. Note that for a heavy Higgs there might be low lying resonances produced by the strongly interacting Higgs that might be confused with the Higgs itself. In fact, we have no idea what the experimentally observed Higgs mass would be since it suffers strong corrections.

Higgs at high energy

Consider again the simple Higgs model. The Feynman rules for that model have been given before. There are many vertices with a factor m/M. It follows that perturbation theory breaks down if this gives rise to amplitudes of the order 1 or larger. The relevant quantity is $g^2 m^2 / 4\pi M^2$ .We can conclude that perturbation theory breaks down if $m^2 \approx 4\pi M^2/g^2$. This happens if $m > 500$ GeV. That is also true in the Standard Model.

Perturbation theory breaks down if the Higgs mass is larger than 500 GeV.

A process where the breakdown of perturbation theory can be seen directly is WW scattering at high energy. We are interested in the behaviour with respect to the c.m. energy for large energy. Lowest order diagrams:



Energy: E^4 E^4 E^4 E^2

First three diagrams together: E^2. Including fourth: constant.

The cancellation of the E^4 behaviour is due to gauge invariance. The addition of the Higgs completes the job: a renormalizable theory requires behaviour as a constant.

However, the last diagram contributes significantly only if the energy is larger than the Higgs mass. Therefore the cancellation becomes effective only if the energy is above the Higgs mass.

It is necessary to state here clearly that the above is for longitudinally polarized vector bosons. This point is important, because it is not easy to produce experimentally longitudinally polarized vector bosons.

If the Higgs mass is large then the first three diagrams may give a contribution of order one, and then perturbation theory breaks down here. No one knows what will happen. For low energy there is no problem, even for large Higgs mass, and perturbation theory holds. The important question is this: is there any non-perturbative way that we can extrapolate to high energy starting from a low energy calculation ?

At this point we re-introduce the equivalence theorem. Scattering of longitudinally polarized vector bosons is equivalent to scattering of the Higgs ghost. The relevant vertices are then all contained in the σ-model. The problem becomes identical to the description of pion scattering using the σ-model. Instead of longitudinally polarized vector bosons we have pions. The Higgs is then the σ-particle.

Pion scattering

The σ-model conserves isospin, and it is advantageous to use the appropriate notation. Rather then $\pi^\pm$ and π^0 we will use π^1, π^2 and π^3 with $\pi^\pm = \frac{1}{\sqrt{2}}(\pi^1 \mp i\pi^2)$ and $\pi^0 = \pi^3$. Exhibiting isospin the amplitude for pion-pion scattering can be written as:

$$A(\pi^a\pi^b \to \pi^c\pi^d) = \delta_{ab}\delta_{cd}F(s,t,u) + \delta_{ac}\delta_{bd}F(u,t,s) + \delta_{ad}\delta_{bc}F(t,s,u)$$

There is one function F depending on the Mandelstam variables s, t, u. This function can be worked out easily up to one loop. The result is:

$$F(s,t,u) = \frac{s}{v^2}$$
$$-\frac{1}{96\pi^2 v^4}\Big(2s^2(\ln s - \beta_1) + t(t-u)(\ln t - \beta_2) + u(u-t)(\ln u - \beta_2)\Big)$$

The first line gives the tree contribution, the second the one-loop result. The quantities β_1 and β_2 depend on the σ-mass, they contain $\ln m^2$. From comparison with experiments on $\pi - \pi$ scattering the vacuum expectation value v, usually denoted by F_π, is found to be 98 MeV; in the Standard Model we have $f_0 = 250$ GeV, from the vector boson masses.

That is the scale factor (2551) going from pions to vector-boson scattering.

With that scale factor the ρ-meson would be a resonance in WW scattering of 1977 GeV $\approx$ 2 TeV.

However, whether there is a resonance depends on the values of $\beta_!$ and β_2. Some theoretical understanding is necessary in order to determine what may happen in the Standard Model.

For pion-pion scattering there exist an extrapolation method (Lehmann) to go from low energy (where the equation written above is supposedly valid) to higher energies.

Lehmann analysis

- Assume that the amplitude, also for high energy, is well described using a partial wave expansion (in terms of angular momentum), and keeping only low angular momentum states (spin 0, 1, 2 but no more).
- Fit this amplitude for low s to the one-loop equation given above.

The result is as follows.

Concentrate on the isospin 1 channel as there the ρ occurs. Also, the result will then depend only on $\beta \equiv \beta_2 - \beta_1$ which happens to be independent of $\ln m^2$. In fact, $T(1) = F(t,s,u) - F(u.t.s)$ whwre T(1) is the isospin 1 amplitude.

The partial wave expansion is:

$$T(I) = 32\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) t_l^I(s)$$

$$t_l^I = \frac{1}{\cot \delta_l^I(s) - i}$$

The use of the form shown for t_l^I guarantees unitarity.

At low energy one has in the I-spin 1, angular momentum 1 channel:

$$t_1^1(s) = sA_1^1(s)(1 + sB_1^1(s))$$

From this follows:

$$\cot\delta(s) = \frac{1}{A} - \frac{B}{A}$$

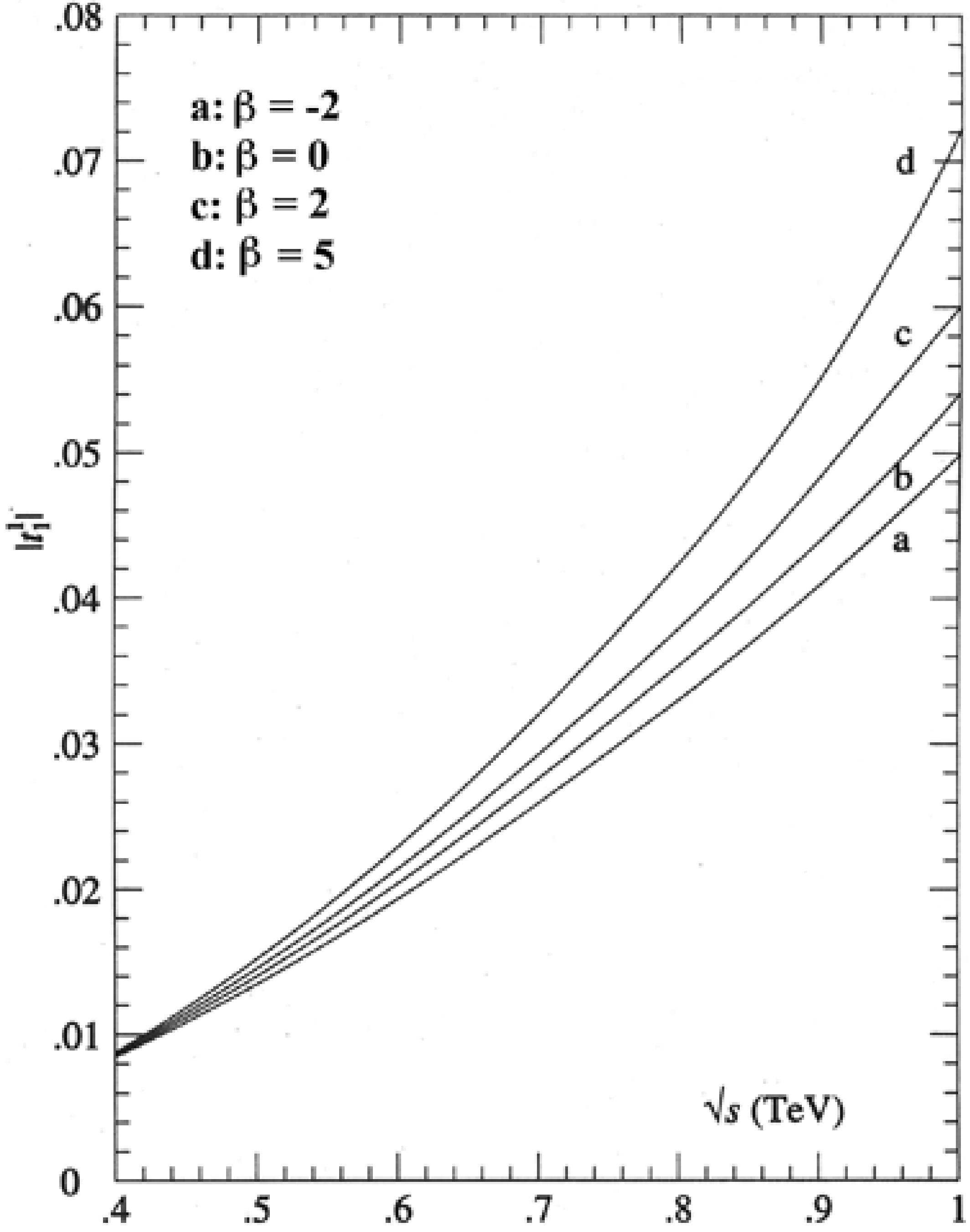
If $\cot\delta = 0$ for some value of s then we have a resonance at thet s.

Comparing the low energy expression for $t_1^1(s)$ with $F(s,t,u)$ gives:

$$A = \frac{1}{96\pi F_\pi^2} \qquad (\mathrm{I-spin\ 1})$$

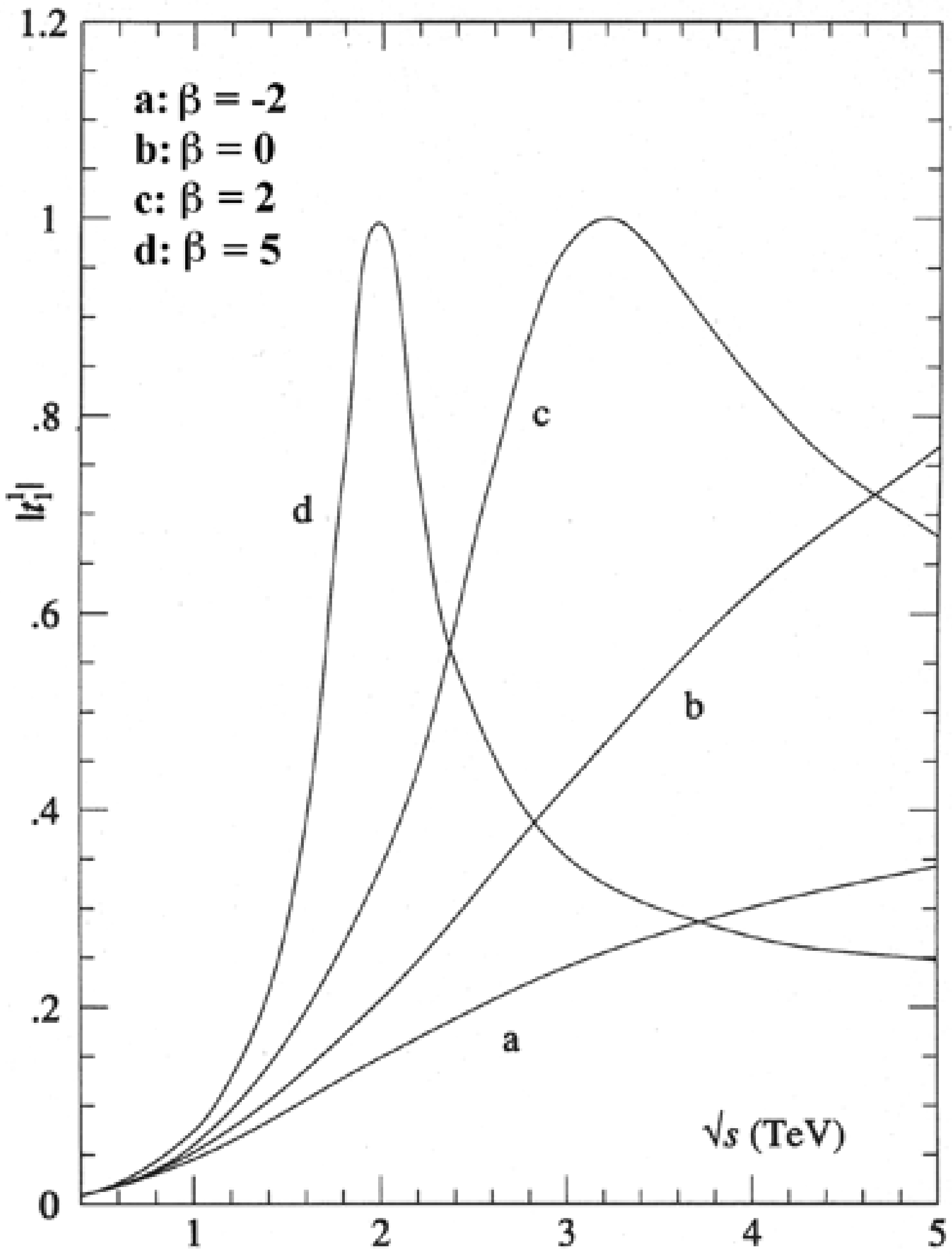
B may be determined from the one-loop calculation. The result will depend on $\beta = \beta_2 - \beta_1$. The Higgs secor of the Standard Model as well as the σ-model for pions give $\beta = \frac{1}{3}$. But in reality that may be different.

What does this mean for the amplitude t_1^1 ? Consider t_1^1 as function of β. The result is shown in the figures.



Low energy behaviour of the $\pi - \pi$ scattering amplitude. This can be calculated using perturbation theory.

The scattering amplitude is shown as a function of the c.m. energy.

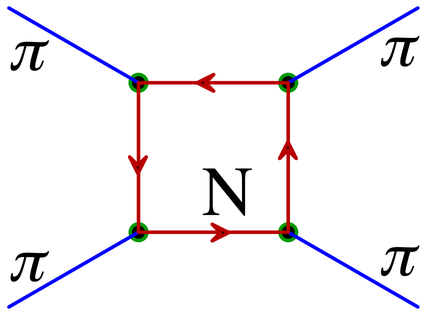


Calculation using the Lehmann technique for extrapolating from low to high energy.

The scattering amplitude is shown as a function of the c.m. energy. A sharp resonance (the ρ) arises only if $\beta > 2$.

H. Veltman and M.V.
Acta Phys. Pol. B22 (1991) 669.

The figures show that there is no resonance for $\beta = \frac{1}{3}$. Yet for pion-pion scattering there is a resonance, the ρ-resonance at 775 MeV. How can this be understood ? Lehmann invokes a further interaction, namely the pion-nucleon interaction. Here is a possible diagram.



This works. Will there be something analogous in the Standard Model ? Continuing the parallel π - W one has introduced technicolor, a scaled-up version of QCD.

But that theory is in trouble, mainly through the ρ-parameter.

Experiment must give the answer.