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Conserved currents and gauge invariance in Yang-Mills theory

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Abstract

It is shown that in the absence of free abelian gauge fields, the conserved currents of (classical) Yang-Mills gauge models coupled to matter fields can be always redefined so as to be gauge invariant. This is a direct consequence of the general analysis of the Wess-Zumino consistency condition for Yang-Mills theory that we have provided recently.

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It is well known that the standard Noether method [1] for deriving the energy momentum tensor of the Yang-Mills field from the invariance of the Yang-Mills action under space-time translations does not yield a gauge invariant answer if one regards the connection A^a_μ as an ordinary covector,

$$\delta_\xi A^a_\mu = \mathcal{L}_\xi A^a_\mu = a^\rho \partial_\rho A^a_\mu \tag{1}$$

($\xi = a^\rho \partial / \partial x^\rho$). Rather, one gets the Yang-Mills canonical energy momentum tensor

$$\Theta^\lambda_\mu = F^{\lambda\rho}_a \partial_\mu A^a_\rho - \frac{1}{4}\delta^\lambda_\mu F^{\rho\sigma}_a F^a_{\rho\sigma}, \tag{2}$$

which fails to be gauge invariant because of the first term. It has been observed by Jackiw [2], however, that if one "improves" the transformation law (1) by adding to it an appropriately chosen gauge transformation,

$$\delta_\xi A^a_\mu \longrightarrow \delta^{imp}_\xi A^a_\mu = \mathcal{L}_\xi A^a_\mu + D_\mu(-A^a_\rho a^\rho) = a^\rho F^a_{\rho\mu}, \tag{3}$$

then, the standard Noether procedure yields the Yang-Mills energy momentum tensor

$$T^\lambda_\mu = F^{\lambda\rho}_a F^a_{\mu\rho} - \frac{1}{4}\delta^\lambda_\mu F^{\rho\sigma}_a F^a_{\rho\sigma}, \tag{4}$$

which is gauge invariant. The "improved" diffeomorphisms (3) for Yang-Mills gauge fields have been discussed by various authors in different contexts, see e.g. [3]. The conserved quantities (2) and (4) differ on-shell by an identically conserved current and yield the same integrated conserved charges. However, it is only (4) that is gauge invariant and that describes correctly the local coupling of the Yang-Mills field to the gravitational field.

In fact one can improve analogously all infinitesimal conformal transformations of the A^a_μ so as to get corresponding gauge invariant conserved Noether currents [2]. This raises immediately the following question : can the same "improvement" be achieved for *any* global symmetry of the Yang-Mills theory ? We show in this letter that the answer is affirmative provided that there is no free abelian gauge field. Our results are in fact direct consequences of the analysis of the Wess-Zumino consistency condition that we have carried out in [4, 5]. For this reason, we shall closely follow the language and the notations of these references. In particular, we consider only polynomial functions of the fields, the antifields and their derivatives, and we take the spacetime dimension n to be greater than or equal to 3.

The question of the gauge invariance of the conserved currents has indirectly arisen recently in the very interesting work [6], where the symmetries of the Yang-Mills equations have been all systematically derived under simplifying assumptions, one of which was the gauge covariance of the symmetry transformation laws. The analysis of this paper establishes that this assumption is really not necessary for symmetries of the Yang-Mills action, since it can always be fulfilled by redefinitions. Earlier insights into the structure of the conserved currents for Yang-Mills models may be found in [7].

That the conserved currents can be chosen to be gauge invariant is perhaps not surprising since there are probably no non trivial conserved currents besides those associated with the known global symmetries. These can be chosen to be gauge invariant. However, to our knowledge, that result has never been proved completely before.

We shall assume throughout the letter that the Yang-Mills structure Lie algebra is the direct sum of a semi-simple Lie algebra plus abelian ideals ("reductive Lie algebra"). We allow for (minimal) couplings to spin 0 and spin 1/2 matter fields transforming under linear representations of the gauge group. Thus, our analysis covers e.g. globally supersymmetric Yang-Mills models. However, we exclude matter fields carrying a gauge invariance of their own. Remarks concerning the gravitational case are given at the end.

We shall first consider the case when the abelian gauge fields (if any) are coupled to at least one charged matter field. The theorem holds only under that physically natural assumption. We shall next indicate explicitly what happens in the presence of free abelian gauge fields, for which we shall give the complete list of all the currents that cannot be covariantized (in $n \neq 4$ dimensions).

As it is usual in the physics literature, we call "symmetry" of the theory a transformation of the fields

$$\Delta\phi^i = a^i(x, \phi, \partial\phi, \dots, \partial^k\phi) \tag{5}$$

($(\phi^i) \equiv (A^a_\mu, matter\ fields)$) which leaves the Lagrangian $\mathcal{L}$ invariant up to a total derivative,

$$\frac{\delta\mathcal{L}}{\delta\phi^i}\Delta\phi^i + \partial_\mu j^\mu = 0 \tag{6}$$

("variational symmetry" in the mathematics literature). If one introduces the antifields ϕ^*_i associated with the fields ϕ^i, the equation (6) may be rewritten as

$$\delta a_1 + \partial_\mu j^\mu = 0. \tag{7}$$

Here, $a_1 \equiv \phi^*_i a^i$ and δ is the field theoretical Koszul-Tate differential [8]. Explicitly,

$$\begin{aligned} \delta\phi^*_i &= \frac{\delta^R \mathcal{L}}{\delta\phi^i}, \qquad \delta\phi^i = 0 \\ \delta C^*_a &= -D_\mu A^{*\mu}_a + g T^j_{ai} y^*_j y^i, \qquad \delta C^a = 0 \end{aligned} \tag{8}$$

where the T^j_{ai} denote a basis of generators for the linear representation of the gauge group under which the matter fields y^i transform. As usual, the C^a in (8) stand for the ghosts. They have their own antifields, denoted by C^*_a. The Koszul-Tate differential implements the equations of motion in cohomology, i.e., any on-shell vanishing function may be written as a δ-variation. Equ. (7) expresses simply that the divergence of the current j^μ is zero on-shell.

It follows from (7) that a symmetry defines an element of the cohomological group $H^n_1(\delta|d)$ [4], where n is the form degree and 1 is the antighost number (we are using dual notations; in form-notations, a_1 would be a n-form and j^μ a $(n-1)$-form).

A symmetry is said to be a trivial global symmetry if the corresponding a_1 is in the trivial class of $H^n_1(\delta|d)$, i.e., if it can be written as

$$a_1 = \delta b_2 + \partial_\mu c^\mu \qquad (trivial\ a_1) \tag{9}$$

for some b_2 and c^μ. This happens if and only if the transformation a^i reduces on-shell to a gauge transformation. That a^i reduces on-shell to a gauge symmetry when (9) holds is obvious. That, conversely, any symmetry transformation that reduces on-shell to a gauge symmetry can be written as in (9) follows from the facts (i) that gauge symmetries are of the form (9) ; and (ii) that any symmetry transformation leaving the Lagrangian invariant up to a total derivative and vanishing on-shell is necessarily an antisymmetric combination of the field equations and thus of the form (9) (see [4] sections 7 and 11).

The above redefinition (3) of the translations is of this type, i.e. the solutions a_1 and a^{imp}_1 of (7) corresponding to (1) and (3) satisfy $a_1 - a^{imp}_1 =$

$\delta b_2 + \partial_\mu c^\mu$ for some b_2 and c^μ. For a trivial global symmetry, the conserved current identically fulfills $\partial_\mu (j^\mu + \delta c^\mu) = 0$ from (7). Thus,

$$j^\mu + \delta c^\mu = \partial_\nu S^{\nu\mu}, \quad S^{\nu\mu} = -S^{\mu\nu} \qquad (trivial \quad j^\mu), \tag{10}$$

which implies that j^μ reduces on-shell to an identically conserved current and is "trivial"[1]. Conversely, if the current is trivial, then a_1 is necessarily of the form (9) [4]. This means that a trivial redefinition of the symmetry is completely equivalent to a trivial redefinition of the current (see [9] for related considerations).

Now, the current is gauge invariant if and only if it is annihilated by the γ piece of the BRST differential $s = \delta + \gamma$,

$$\gamma j^\mu = 0. \tag{11}$$

Explicitly, γ reads [10],

$$\begin{aligned} \gamma A_\mu^a = D_\mu C^a, \ \gamma C^a = \frac{1}{2} g C^a_{bc} C^b C^c, \ \gamma y^i = g T^i_{aj} C^a y^j \\ \gamma A_a^{*\mu} = g C^b C^c_{ab} A_c^{*\mu}, \ \gamma C_a^* = g C^c_{ab} C^b C_c^*, \ \gamma y_i^* = -g T^j_{ai} C^a y_j^*, \end{aligned} \tag{12}$$

where C^a_{bc} are the structure constants of the Lie algebra. The differential γ incorporates the gauge symmetry since the γ-variations of the fields A^a_μ and y^i are obtained by replacing the gauge parameters by the ghosts in the gauge variations of the fields. The γ-variations of the ghosts are such that $\gamma^2 = 0$. The γ-variations of the antifields (sources for the BRST variations of the fields) are also quite standard and express the fact that the antifields $A_a^{*\mu}$ and C_a^* transform in the co-adjoint representation, while the antifields y_i^* transform in the representation dual to that of the y's. Finally, the complete BRST differential s takes both the equations of motion (through δ) and the gauge symmetry (through γ) into account. It is nilpotent because $\delta^2 = \delta\gamma + \gamma\delta = \gamma^2 = 0$.

[1]The terminology is borrowed from the mathematical literature and does not imply that the currents are necessarily physically trivial. The conserved currents associated with gauge symmetries reduce indeed to divergences [1] and the corresponding conserved charges are given by non-vanishing physically relevant surface integrals. Furthermore, on-shell trivial terms must be handled with care inside Green functions in the quantum theory. For the purposes of this letter, however, it is convenient to call a current of the form (10) "trivial".

Condition (11) implies $\gamma\delta a_1 = 0$. Accordingly, a necessary condition for the existence of trivial redefinitions that make j^μ gauge invariant is that there exist trivial redefinitions that make δa_1 invariant. Our first task will be to prove more, namely, that one can always perform redefinitions that make a_1 itself invariant, $\gamma a_1 = 0$. The invariance of a_1 means that the infinitesimal variations $\Delta\phi^i = a^i$ transform in representations contragredient to the representations of the antifields. Thus ΔA^a_μ transforms in the adjoint representation of the gauge group (as it does in (3) above), while Δy^i transforms as y^i for all matter fields. This implies that the global symmetry written in its covariant version commutes with the gauge transformations.

Theorem 1 : *In the absence of uncoupled abelian gauge fields, any cohomological class of $H^n_1(\delta|d)$ contains a γ-invariant representative. That is, by adding to any symmetry $a_1 = \phi^*_i a^i$ a δ-trivial term modulo d,*

$$a_1 \longrightarrow a'_1 = a_1 + \delta b_2 + \partial_\mu c^\mu_1, \tag{13}$$

one can arrange so that the equivalent symmetry a'_1 fulfills $\gamma a'_1 = 0$.

Proof. We start with the equation $\delta a_1 + \partial_\mu j^\mu = 0$. Acting with γ on it, one gets $\delta\gamma a_1 = \partial_\mu(\gamma j^\mu)$, i.e., $[\gamma a_1] \in H^n_1(\delta|d)$. But γa_1 involves the ghosts and thus is δ-exact modulo d [11],

$$\gamma a_1 = -\delta a_2 + \partial_\mu j'^\mu. \tag{14}$$

By acting with γ on this equation, one then finds that γa_2 is also δ-trivial modulo d. This enables one to construct recursively the sum

$$a = a_1 + a_2 + \ldots + a_k \tag{15}$$

such that

$$sa + \partial_\mu k^\mu = 0 \tag{16}$$

for some k^μ. The construction, which follows the standard lines of homological perturbation theory [12, 8] is reviewed in [4, 5], where it is explicitly demonstrated that the sum (15) terminates after a finite number of steps ([5], section 3). Therefore, a defines an element of $H(s|d)$.

The analysis of the cohomological groups $H(s|d)$ made in [5] shows then that one can successively remove a_k, a_{k-1}, $a_{k-2}, \dots, a_2$ from a by adding an appropriate s-boundary modulo d. At the same time, one may redefine k^μ so that it reduces to a single term of antighost number zero, $k^\mu \rightarrow k'^\mu = k^\mu + st^\mu + \partial_\nu S^{\mu\nu}$, $S^{\mu\nu} = -S^{\nu\mu}$, $k'^\mu = k_0'^\mu$. After this is done, a reduces to a term of antighost number 1,

$$\begin{aligned} a \longrightarrow a' = a + sm + \partial_\mu n^\mu, \qquad a' = a'_1 \\ m = m_2 + \dots m_{k+1}, \qquad n^\mu = n_1^\mu + \dots + n_k^\mu. \end{aligned} \tag{17}$$

But adding to a such a s-boundary modulo d amounts to adding to a_1 a δ-boundary modulo d,

$$a_1 \longrightarrow a'_1 = a_1 + \delta m_2 + \partial_\mu n_1^\mu, \tag{18}$$

Furthermore, the condition $sa' + \partial_\mu k'^\mu = 0$ with $a' = a'_1$ and $k'^\mu = k_0'^\mu$ implies precisely

$$\delta a'_1 + \partial_\mu t_0^\mu = 0 \tag{19}$$

and

$$\gamma a'_1 = 0. \tag{20}$$

This proves the theorem. $\Box$.

The removal of a_k, a_{k-1}, $a_{k-2}, \dots, a_2$ from a by adding s-coboundaries modulo d can be performed because the (invariant) cohomological groups $H_i(\delta|d)$ vanish for $i \geq 2$ [5]. If there were free abelian gauge fields, which we denote by A_μ^α, then $H_2(\delta|d)$ would not vanish. The best that can be done then is to remove all the terms up to a_3 included, $a \longrightarrow a' = a'_1 + a'_2$. The term a'_2 cannot be removed in general and is of the form worked out in section 8 of [5] ("solution of class I_b"). The representatives of $H^{-1}(s|d)$ arising from the non trivial elements of $H_2(\delta|d)$ fall into two categories : those that do not depend on the spacetime coordinates, and those that do. A complete list of representatives not involving x is given by

$$a = a_1 + a_2 \tag{21}$$

with

$$a_1 = f_{\alpha\beta} A_\mu^\alpha A^{*\beta\mu}, \ a_2 = f_{\alpha\beta} C^\alpha C^{*\beta}, \ f_{\alpha\beta} = -f_{\beta\alpha}. \tag{22}$$

The corresponding symmetries are

$$\Delta A^\alpha_\mu = f^\alpha_\beta A^\beta_\mu \tag{23}$$

(Δ (other fields)$= 0$) and simply rotate the free abelian fields among themselves. Here, we lower and raise the indices α, β, ... labeling the free abelian gauge fields with the metric $\delta_{\alpha\beta}$ ($\delta^{\alpha\beta}$). The conserved currents associated with (23) are

$$j^\mu_{\alpha\beta} = F^{\mu\nu}_{[\alpha} A_{\beta]\nu} \tag{24}$$

and cannot be redefined so as to be gauge invariant. Hence, theorem 1 does indeed not hold in the presence of free abelian gauge fields.

We note that the solutions (21) have counterparts with ghost number zero,

$$a = a_0 + a_1 + a_2 \tag{25}$$

with

$$a_0 = \frac{1}{2} f_{[\alpha\beta\gamma]} A^\alpha_\mu A^\beta_\nu F^{\nu\mu\gamma} \tag{26}$$

and

$$a_1 = f_{[\alpha\beta\gamma]} C^\alpha A^\beta_\mu A^{*\mu\gamma},\ a_2 = \frac{1}{2} f_{\alpha\beta\gamma} C^\alpha C^\beta C^{*\gamma},\ f_{\alpha\beta\gamma} = f_{[\alpha\beta\gamma]}. \tag{27}$$

These give rise to the Yang-Mills cubic vertex and the non abelian deformation of the gauge symmetries.

We now turn to the x-dependent solutions and consider only the pure, free abelian case. Preliminary results appear to indicate that in four dimensions, there are no x-dependent non trivial solutions of (16) whose part a_2 cannot be removed. By contrast, flat n-dimensional space-time admits for $n \neq 4$ further x-dependent solutions of (16) with non trivial a_2, for which it is accordingly impossible to covariantize the corresponding symmetries. They are given by

$$\Delta A^\alpha_\mu = f^\alpha_\beta (x^\nu F^\beta_{\nu\mu} + \frac{n-4}{2} A^\beta_\mu), \qquad f_{\alpha\beta} = f_{\beta\alpha} \tag{28}$$

with currents $j^\mu = f^\alpha_\beta j_\alpha{}^{\beta\mu}$ given by

$$j^\mu = f^\alpha_\beta \left(F^{\mu\rho}_\alpha F^\beta_{\nu\rho} x^\nu - \frac{1}{4} F^{\nu\rho}_\alpha F^\beta_{\nu\rho} x^\mu + \frac{n-4}{2} F^{\mu\rho}_\alpha A^\beta_\rho \right). \tag{29}$$

The transformations (28) contain pure scale transformations ($f^\alpha_\beta = \lambda\delta^\alpha_\beta$) and exist in any number of dimensions, but it is only in four dimensions that they admit a covariant expression. Furthermore, they cease to be symmetries on a generic curved background, in contrast to (23). The clash between conformal invariance and gauge invariance for p-form gauge fields off their critical dimension has been analysed in depth in [13]. We remark that given a symmetry for a model with free abelian gauge fields, one can always subtract from it a symmetry of the type (23) or (28) ($n \neq 4$) so that a_2 can be made to vanish. The remaining symmetry admits a gauge invariant conserved current.

We are now in a position to formulate and prove the main theorem of this letter :

Theorem 2 : *In Yang-Mills theory without uncoupled abelian gauge fields, one can always redefine the conserved currents by the addition of trivial terms,*

$$j^\mu \longrightarrow j'^\mu \approx j^\mu + \partial_\nu S^{\nu\mu},\ S^{\nu\mu} = -S^{\mu\nu}, \tag{30}$$

in such a way that the equivalent currents j'^μ are gauge invariant ($\approx$ denotes equality up to terms that vanish on-shell).

Proof. We have shown that the symmetry tranformation may be assumed to be gauge invariant,

$$\delta a_1 + \partial_\mu j^\mu = 0$$
$$\gamma a_1 = 0.$$

But if a_1 - which does not involve the ghosts - is annihilated by γ, then it is an invariant polynomial in the field strengths, the matter fields, the antifields and their covariant derivatives (linear in ϕ^*_i). Then δa_1 itself is also an invariant polynomial in the field strengths, the matter fields and their covariant derivatives. This invariant polynomial is a divergence, and the question is whether the current j^μ of which it is the divergence may also be assumed to be an invariant polynomial. The obstructions to taking j^μ invariant have been studied in the literature and are given by the invariant polynomials in the undifferentiated curvature 2-forms $F^a = \frac{1}{2}F^a_{\mu\nu}dx^\mu dx^\nu$ [14, 15]. [The invariant cohomology of d has been studied for gravity in [16] and recently from a general point of view in [17]]. Since the Yang-Mills equations involve the differentiated curvatures, the obstructions for taking j^μ invariant are absent and thus, one may assume $\gamma j^\mu = 0$. This proves the theorem. $\square$.

Note that we have explicitly used here the fact that the Lagrangian is the Yang-Mills one. One can easily construct gauge invariant Lagrangians different from the Yang-Mills one that possess covariant global symmetries whose corresponding current is not gauge invariant. For example, the Lagrangian

$$\mathcal{L} = \lambda tr F^2 \tag{31}$$

with Lagrange multiplier λ leads to the equation $trF^2 = 0$. The characteristic polynomial trF^2 can be written as a divergence, $trF^2 = dQ$, but one cannot choose the conserved current Q to be gauge invariant, although the corresponding symmetry is covariant (it is simply given by $\Delta\lambda = const$, $\Delta A^a_\mu = 0$).

We conclude this letter by indicating how our results extend to other theories with a gauge freedom. The crucial tool by which we have controlled the covariance of the symmetry transformations and the invariance of the currents is the vanishing of the (gauge invariant) cohomology groups $H^n_i(\delta|d)$ for $i \geq 2$. These groups are isomorphic to the groups $H^{n-i}_0(d|\delta)$ of the characteristic cohomology of [18]. It is thus the vanishing of the characteristic cohomology in form degree $< n-1$ that controls the gauge invariance of the currents for Yang-Mills models. One expects similar results for any theory in which $H^{n-i}_0(d|\delta) \simeq H^n_i(\delta|d) = 0$ for $i \geq 2$. And indeed, in the case of Einstein gravity for which this property holds, one may also choose the conserved currents j^μ to behave properly as vector densities. This will be explicitly proved in a separate publication [19]. Similarly, if one modifies the Yang-Mills action by adding terms that (i) preserve the triviality of $H^n_i(\delta|d) = 0$ for $i \geq 2$; and (ii) do not make any of the characteristic classes vanish on-shell, then, one can still assume that the conserved currents are gauge invariant.

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