

General solution of the Wess-Zumino consistency condition for Einstein gravity

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The Wess-Zumino consistency condition for four-dimensional Einstein gravity is investigated in the space of local forms involving the fields, the ghosts, the antifields and their derivatives. Its general solution is constructed for all values of the form degree and of the ghost number. It is shown in particular that the antifields (= sources for the BRST variations) can occur only through cohomologically trivial terms.

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The Wess-Zumino (WZ) consistency condition [1] is a central equation in field theories with a local gauge symmetry. At ghost number zero, this equation characterizes the consistent deformations of the original classical action, i.e., its solutions exhaust all possible local perturbation terms that can be added to the action while maintaining the number (but not necessarily the form nor the algebra) of the original gauge invariances [2]. An example is given by the Yang-Mills cubic vertex, which is contained in a ghost number zero solution of the WZ consistency equation for a system of free abelian gauge fields. At ghost number one - for which it was initially discussed -, the WZ consistency condition constrains severely the form of the candidate anomalies. Finally, as it has been shown in [3], this condition yields at negative ghost number the characteristic cohomology of the field equations [4], which includes the non trivial conserved currents.

We present in this letter the general solution of the WZ consistency condition for pure four-dimensional Einstein gravity, for arbitrary values of the form degree and of the ghost number. Couplings to matter and gravity in other spacetime dimensions can be handled along the same lines by our method [5] but are not treated here for the sake of brevity. We shall just comment on these generalizations at the end of the letter, as well as on the modifications that arise if one adds to the Hilbert-Einstein Lagrangian higher powers of the curvature. For simplicity, we assume that the spacetime manifold  $\mathcal{M}$  has the topology of  $R^4$  and work in global coordinates  $x^\mu$  ( $\mu = 0, 1, 2, 3$ ) throughout. Our approach does not use power counting (which would be meaningless in the case of gravity) and is purely cohomological.

In terms of the BRST operator  $s$  given below, the WZ consistency condition reads explicitly

$$sa + db = 0 \tag{1}$$

where  $a$  and  $b$  are local differential forms and where  $d$

is the spacetime exterior derivative. Trivial solutions of (1) are given by  $a = sm + dn$  and fulfill indeed (1) due to  $s^2 = sd + ds = d^2 = 0$ . The investigation of (1) for gravity has already received considerable attention in the past, particularly at ghost number one (anomalies) [6] but also at ghost number zero (deformations of the Hilbert-Einstein action) [7], as well as for all ghost numbers [8]. Our analysis goes beyond previous works on the subject in that we do not impose any a priori form to the dependence on the antifields. This means, for instance, that we allow in principle for deformations of the Hilbert-Einstein action such that the original off-shell closed gauge algebra is replaced by a new, possibly off-shell open, gauge algebra. It turns out, however - and this is one of our main results - that all solutions of (1) involve the antifields only through trivial terms. Hence, our results justify the simplifying assumptions made in previous studies.

The relevant algebra for the discussion of (1) is the algebra of local differential forms. This algebra is the tensor algebra  $\mathcal{E} = \Omega(\mathcal{M}) \otimes \mathcal{A}$ , where  $\Omega(\mathcal{M})$  is the algebra of exterior forms on the manifold  $\mathcal{M}$  and where  $\mathcal{A}$  is the algebra of local functions. A local function is by definition a function of the (invertible) tetrad  $e_\mu^a$  ( $a =$  local index,  $\mu =$  world index), of the diffeomorphism ghosts  $\xi^\mu$ , of the Lorentz (or  $SO(4)$  in the Euclidean case) ghosts  $C^{ab} = -C^{ba}$ , of the corresponding antifields  $e_a^{*\mu}$ ,  $\xi_\mu^*$  and  $C_{ab}^*$ , and of their derivatives up to some finite order. Thus, here, local functions do not depend on  $x^\mu$ ; the explicit  $x^\mu$ -dependence is included in the factor  $\Omega(\mathcal{M})$  of  $\mathcal{E}$ . As standard in field theory, we shall assume that the local functions in  $\mathcal{A}$  depend polynomially on all their arguments, except on the undifferentiated tetrad, which may appear non polynomially (e.g. the inverse tetrad  $E_a^\mu$  is allowed).

In the algebra of local differential forms, the BRST differential  $s$  acting on  $e_\mu^a$ ,  $\xi^\mu$ ,  $C^{ab}$ ,  $e_a^{*\mu}$ ,  $\xi_\mu^*$  and  $C_{ab}^*$  is obtained by taking the antibracket with the solution  $S$  of the master equation [9,10]. In the case of gravity, whose gauge symmetries form a closed algebra, the construction of  $S$  presents no difficulty. One finds that  $S$  is given by the sum of the Einstein action and of integrated terms of the form "antifields" times "BRST variation of the corresponding fields". We leave the details to the reader - the explicit form of  $s$  is given below in terms of a different set of variables.

In order to analyse (1), it turns out indeed to be more convenient to express the local functions in terms of new

variables, which are:

$$\{\mathcal{T}^r\} = \{D_{a_1} \dots D_{a_m} R_{ab}^I : m = 0, 1, \dots\}, \quad (2)$$

$$\{\hat{\xi}^a, \hat{C}^I\}, \quad \hat{\xi}^a = \xi^\mu e_\mu^a, \quad \hat{C}^I = C^I + \xi^\mu \omega_\mu^I, \quad (3)$$

$$\{\mathcal{T}_r^*\} = \{D_{a_1} \dots D_{a_m} \hat{\Phi}_A^* : m = 0, 1, \dots\}, \quad (4)$$

$$\{U_I\} = \{\partial_{(\mu_1 \dots \mu_m} e_{\mu)}^a, \partial_{(\mu_1 \dots \mu_m} \omega_{\mu)}^I : m = 0, 1, \dots\}, \quad (5)$$

$$\{V_I\} = \{sU_I\}. \quad (6)$$

Here (i) the index  $I$  collectively denotes the antisymmetric pair of Lorentz indices  $ab$ ,  $k^I \equiv k^{ab} = -k^{ba}$  (we use the summation convention  $k^I k_I = k^{ab} k_{ab}/2$ ); (ii)  $\omega_\mu^I$  is the standard torsion-free spin connection; (iii) the antifield variables  $\hat{\Phi}_A^* \equiv \hat{e}_a^{*b}, \hat{C}_I^*, \hat{\xi}_a^*$  are defined by

$$\hat{e}_a^{*b} = e_\mu^b e_a^{*\mu}/e, \quad (7)$$

$$\hat{C}_I^* = C_I^*/e, \quad \hat{\xi}_a^* = E_a^\mu (\xi_\mu^* - \omega_\mu^I C_I^*)/e \quad (8)$$

with  $e = \det e_\mu^a$ ; (iv)  $R_{ab}^I = E_a^\mu E_b^\nu R_{\mu\nu}^I$  are the tetrad components of the curvature tensor  $R_{\mu\nu}^I = \partial_\mu \omega_\nu^I - \dots$ ; and (v)  $D_a = E_a^\mu (\partial_\mu - \omega_\mu^I M_I)$ . The symbol  $M_I \equiv M_{ab}$  acting on a variable belonging to a representation of the Lorentz group stands for its Lorentz variation; it fulfills  $[M_I, M_J] = f_{IJ}^K M_K$ , where  $f_{IJ}^K$  are the structure constants of the Lorentz algebra. One easily verifies that each local function of the fields, the antifields and their derivatives can indeed be expressed as a local function of the variables (2)–(6) and vice versa.

The BRST transformations of the  $U_I$  and  $V_I$  are extremely simple since they just read  $sU_I = V_I$ ,  $sV_I = 0$ . Accordingly, the  $U_I$  and  $V_I$  belong to the contractible part of the algebra and do not contribute to the BRST cohomology. The BRST transformations of  $\hat{\xi}^a, \hat{C}^I, \mathcal{T}^r$  and  $\mathcal{T}_r^*$  are slightly more complicated. To display them, we decompose the BRST operator according to

$$s = \delta + \gamma \quad (9)$$

where  $\delta$  is the Koszul-Tate differential which has antighost number  $-1$  and  $\gamma$  has antighost number  $0$ . This decomposition is by now quite standard [11,12]. The differential  $\delta$  plays a crucial role below and acts on  $\hat{\xi}^a, \hat{C}^I, \mathcal{T}^r, \mathcal{T}_r^*$  according to

$$\begin{aligned} \delta \mathcal{T}^r &= \delta \hat{C}^I = \delta \hat{\xi}^a = 0, \quad \delta \hat{C}_{ab}^* = -2\hat{e}_{[ab]}^*, \\ \delta \hat{e}_a^{*b} &= -R_a^b + \frac{1}{2}\delta_a^b R, \quad \delta \hat{\xi}_a^* = -D_b \hat{e}_a^{*b}, \\ \delta D_{a_1} \dots D_{a_n} \hat{\Phi}_A^* &= D_{a_1} \dots D_{a_n} \delta \hat{\Phi}_A^* \end{aligned} \quad (10)$$

where  $R_{ab} = R_{acb}^c$  is the Ricci tensor and  $R$  is the scalar curvature. The differential  $\gamma$  acts on  $\hat{\xi}^a, \hat{C}^I, \mathcal{T}^r, \mathcal{T}_r^*$  according to

$$\begin{aligned} \gamma \mathcal{T}^r &= (\hat{\xi}^a D_a + \hat{C}^I M_I) \mathcal{T}^r, \quad \gamma \mathcal{T}_r^* = (\hat{\xi}^a D_a + \hat{C}^I M_I) \mathcal{T}_r^*, \\ \gamma \hat{\xi}^a &= \hat{C}^I M_I \hat{\xi}^a, \quad \gamma \hat{C}^I = \frac{1}{2} \hat{C}^J \hat{C}^K f_{KJ}^I + \hat{R}^I \end{aligned} \quad (11)$$

where we have set  $\hat{C}^I M_I \hat{\xi}^a = \hat{C}_b^a \hat{\xi}^b$  and  $\hat{R}^I = (1/2) \hat{\xi}^a \hat{\xi}^b R_{ab}^I$ . The variables  $\mathcal{T}^r$  and  $\mathcal{T}_r^*$ , whose BRST variations do not involve derivatives of the ghosts, have been called “tensor fields” in [13].

Our first result is :

**Theorem 1 :** *The general solution of the Wess-Zumino consistency condition involves the antifields only through trivial contributions.*

In order to prove this theorem, we shall need two crucial properties of the BRST differential. The first is the fact that the Koszul-Tate differential provides a resolution of the algebra of on-shell functions. It is actually a generic feature of the antifield formalism [11,12]. The second is peculiar to theories with diffeomorphism invariance and relates in a simple way the cohomology  $H(s/d)$  of  $s$  modulo  $d$  to the cohomology  $H(s)$  of  $s$  itself (BRST invariance condition  $sa = 0$ ) [8,13]. We shall describe these two features in turn. We shall then provide the demonstration of the theorem.

Antifields were introduced by Zinn-Justin in order to control how the non linear BRST symmetry survives renormalization. They were known in that context as sources for the BRST variations. It turns out that a different interpretation of the antifields is of greater interest for cohomological investigations. It is that the antifields provide a *resolution* of the algebra of functions on the stationary surface [11,12], even if one takes locality into account [14]. This property means : (i) that an antifield independent local function can be written as a  $\delta$ -variation if and only if it vanishes on-shell; and (ii) that the homology of  $\delta$  is trivial at strictly positive antighost number,  $H_k(\delta) = 0$  for  $k > 0$ . While the first property is well known and somewhat trivial, the second property is more subtle and holds because the antifields associated with the ghosts properly take care of the relations among the equations of motion (Noether identities).

For pure gravity, the acyclicity of  $\delta$  extends to the cohomology of  $\delta$  modulo  $d$ , which has been shown in [3] to be isomorphic to the characteristic cohomology associated with the equations of motion. Thanks to general theorems valid for generic gauge systems [4,3],  $H_k(\delta/d)$  vanishes for  $k > 2$  in the case of gravity. It was also shown in [3] that  $H_2(\delta/d)$  vanishes for gravity because there is no global reducibility identity among the gauge transformations. Finally,  $H_1(\delta/d)$ , which is isomorphic to the space of non trivial conserved currents, vanishes in pure gravity because of a remarkable result due to Anderson and Torre [15]. Thus, one has also  $H_k(\delta/d) = 0$  for  $k > 0$ . We shall need below a Lorentz invariant refinement of this result, namely, that if  $\delta \mathcal{L} + \partial_\mu j^\mu = 0$  where  $\mathcal{L} = eL(\mathcal{T}, \mathcal{T}^*)$  is a Lorentz invariant density of antighost number  $k > 0$ , then  $\mathcal{L} = \delta m + \partial_\mu k^\mu$ , where  $m$  has the same covariance properties. This refinement is fully proved in the more complete version [5] of this paper.

The second property needed for the proof of the theorem relates solutions of  $sa + db = 0$  to solutions  $\alpha \in \mathcal{A}$

of  $s\alpha = 0$ . Consider a non trivial cohomological class of  $s$  in  $\mathcal{A}$ , i.e., a zero-form  $\alpha \neq s\beta$  solving  $s\alpha = 0$  (with  $\alpha \sim \alpha + s\beta$ ). Without loss of generality, one may assume that  $\alpha$  depends only on the tensor fields  $\mathcal{T}$  and  $\mathcal{T}^*$ , and on the undifferentiated ghosts  $\hat{\xi}, \hat{C}$ . One can construct from  $\alpha$  solutions of  $sa + db = 0$  in two different ways.

(a) Because  $sx^\mu = sdx^\mu = 0$ , any form  $\omega \in \Omega(\mathcal{M})$  times  $\alpha$  fulfills  $s(\omega\alpha) = 0$ , and thus, is a solution of (1) with  $b = 0$ . This solution is trivial if  $\omega = d\eta$ .

(b) Let  $\tilde{s} = s + d$ ,  $\tilde{\xi}^a = \hat{\xi}^a + e_\mu^a dx^\mu$  and  $\tilde{C}^I = \hat{C}^I + \omega_\mu^I dx^\mu$ . It is easy to verify that the action of  $\tilde{s}$  on  $\mathcal{T}, \mathcal{T}^*, \tilde{\xi}$  and  $\tilde{C}$  is exactly the same as the action of  $s$  on  $\mathcal{T}, \mathcal{T}^*, \hat{\xi}$  and  $\hat{C}$ . Thus,  $\tilde{s}\tilde{\alpha} = 0$ , where  $\tilde{\alpha}$  is the multi-form obtained by replacing  $\tilde{\xi}$  for  $\hat{\xi}$  and  $\tilde{C}$  for  $\hat{C}$  in  $\alpha$ . Each component of  $\tilde{\alpha}$  of definite form degree  $k$  ( $= 0, 1, 2, 3, 4$ ) is a solution of (1) which is not trivial because  $\alpha$  itself is non trivial.

As shown in [8,13], (a) and (b) yield the most general solution of (1). That is, any solution of (1) is a linear combination of solutions of type (a) and of solutions of type (b). Therefore, in order to solve (1), it is enough to compute the cohomology of  $s$  in  $\mathcal{A}$ . Notice that all non trivial 4-forms solving (1) are of type (b), since any volume form  $\omega \in \Omega(\mathcal{M})$  is exact. Therefore the descent equations [16] in gravity go all the way from form degree 4 to form degree 0 [17].

In order to compute  $H(s)$ , we decompose  $s$  according to its degree in the Lorentz ghosts  $\hat{C}^I$ ,  $s = s_{-1} + s_0 + s_1$ , with

$$s_1 \hat{C}^I = \frac{1}{2} \hat{C}^J \hat{C}^K f_{KJ}^I, \quad s_1 Y = \hat{C}^I M_I Y, \quad (12)$$

$$s_0 = \delta + \mathcal{D}, \quad (13)$$

$$s_{-1} \hat{C}^I = \hat{R}^I, \quad s_{-1} Y = 0. \quad (14)$$

Here, we have set  $\mathcal{D}\hat{\xi}^a = \mathcal{D}\hat{C}^I = 0$ ,  $\mathcal{D}\mathcal{T}^r = \hat{\xi}^a D_a \mathcal{T}^r$ ,  $\mathcal{D}\mathcal{T}_r^* = \hat{\xi}^a D_a \mathcal{T}_r^*$  and  $Y \equiv (\mathcal{T}, \mathcal{T}^*, \hat{\xi})$ . The differential  $s_1$  increases the number of Lorentz ghosts by one unit and is just the standard coboundary operator for the Lie algebra cohomology of the Lorentz algebra  $so(3,1)$ , whose cohomology is well known. The differential  $s_0$  does not modify the number of Lorentz ghosts, while the differential  $s_{-1}$  decreases it by one unit.

Let  $\alpha$  be a solution of  $s\alpha = 0$ . We may also decompose it according to its powers in the Lorentz ghosts,  $\alpha = \alpha_0 + \dots + \alpha_N$ . From  $s\alpha = 0$ , one gets  $s_1\alpha_N = 0$ , i.e.,  $\alpha_N$  is a cocycle of the Lie algebra cohomology. Up to  $s_1$ -trivial terms that can be absorbed by redefinitions, the most general  $s_1$ -cocycle is given by  $\alpha_N = \sum P_\tau(\mathcal{T}, \mathcal{T}^*, \hat{\xi}) \omega^\tau(\hat{C})$ , where the polynomials  $P_\tau$  are Lorentz-invariant functions of their arguments ( $M_I P_\tau = 0$ ) and where the  $\omega^\tau(\hat{C})$  belong to the basis of the Lie algebra cohomology of  $so(3,1)$  explicitly given by  $\{\omega^\tau\} = \{1, \hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_1 \hat{\theta}_2\}$ , where  $\hat{\theta}_1 = \eta^{ac} \eta^{bd} \eta^{ef} \hat{C}_{ab} \hat{C}_{ce} \hat{C}_{df}$  and  $\hat{\theta}_2 = \epsilon^{abcd} \eta^{ef} \hat{C}_{ab} \hat{C}_{ce} \hat{C}_{df}$ . Thus,  $N = 0, 3$  or  $6$ .

We start with the case  $N = 6$ . If one inserts the expression  $\alpha_6 = P(\mathcal{T}, \mathcal{T}^*, \hat{\xi}) \hat{\theta}_1 \hat{\theta}_2$  in the condition  $s\alpha = 0$ , one gets at  $\hat{C}$ -degree 6 the equation  $(\delta + \mathcal{D})P \hat{\theta}_1 \hat{\theta}_2 =$

$s_1(\text{something})$ , which is possible only if the right and left hand sides vanish separately ( $(\delta + \mathcal{D})P$  is a Lorentz invariant polynomial, and no  $s_1$ -cocycle  $\sum P_\tau \omega^\tau$  of the above form is  $s_1$ -exact unless  $P_\tau = 0$  for all  $\tau$ 's). Thus,  $(\delta + \mathcal{D})P = 0$ . We analyse this equation by decomposing  $P$  according to the antighost number,  $P = P_0 + \dots + P_t$ . The condition  $(\delta + \mathcal{D})P = 0$  implies  $\mathcal{D}P_t = 0$ . Assume  $t \neq 0$ . Since  $\mathcal{D}$  acting on invariant polynomials is just the exterior derivative operator (after the substitution  $\xi^\mu \rightarrow dx^\mu$ ), the covariant Poincaré lemma [8] implies that  $P_t$  is  $\mathcal{D}$ -trivial in the space of invariant polynomials in  $\mathcal{T}, \mathcal{T}^*, \hat{\xi}$ , unless it contains the ‘‘volume form’’  $\hat{\Theta} \equiv \hat{\xi}^0 \hat{\xi}^1 \hat{\xi}^2 \hat{\xi}^3$  (the substitution  $\xi^\mu \rightarrow dx^\mu$  yields  $\hat{\Theta} \rightarrow d^4 x e$ ). However, even in that case, the next condition  $\delta P_t + \mathcal{D}P_{t-1} = 0$  (with  $t \neq 0$ ) implies that  $P_t$  is  $\delta$ -trivial modulo  $\mathcal{D}$  (see above discussion on the invariant homology  $H(\delta/d)$ ) and can thus be absorbed by allowed redefinitions. Accordingly, we may assume  $t = 0$ , i.e.,  $P$  does not contain the antifields. The condition  $\mathcal{D}P = 0$  implies then that  $P$  has the form  $P = a + \hat{\Theta} m(\mathcal{T}^r)$  where  $a$  is a constant (covariant Poincaré lemma again;  $\mathcal{D}$ -exact pieces can be absorbed by redefinitions).

If  $a \neq 0$ ,  $\alpha_6$  cannot be completed to a BRST cocycle. Indeed, the obstructions for doing this are the characteristic classes  $f_1 = \hat{R}^{ab} \hat{R}_{ab}$  and  $f_2 = \epsilon^{abcd} \hat{R}_{ab} \hat{R}_{cd}$ , which do not vanish even on-shell and can thus not be written as  $\delta$ -variations. Accordingly, we must take  $a = 0$ , in which case  $\alpha_6 = \hat{\Theta} m(\mathcal{T}^r) \hat{\theta}_1 \hat{\theta}_2$  is a BRST cocycle.

One repeats the discussion in exactly the same way for the terms of lower order in the Lorentz ghosts, which must fulfill the BRST cocycle condition independently of the term of order 6, since this one fulfills that condition by itself. For  $N = 0$ , one gets in addition the constant solutions. Since  $\hat{\Theta}$  has maximal degree in the  $\xi^\mu$ , one has actually  $\hat{\Theta} \hat{\theta}_i = \hat{\Theta} \theta_i$ , where  $\theta_i$  is obtained from  $\hat{\theta}_i$  by replacing  $\hat{C}^I$  by the ordinary Lorentz ghosts  $C^I$ . We can thus conclude:

**Theorem 2 :** *Up to  $s$ -trivial terms, the general solution  $\alpha$  of the BRST cocycle condition  $s\alpha = 0$  does not depend on the antifields and is given by*

$$\alpha = a + \hat{\Theta} L(\mathcal{T}^r, \theta_i), \quad \hat{\Theta} = \hat{\xi}^0 \hat{\xi}^1 \hat{\xi}^2 \hat{\xi}^3 \quad (15)$$

where  $a$  is a constant and  $L$  is a Lorentz invariant polynomial.

Theorem 1 follows from Theorem 2 since the process by which one constructs  $H(s/d)$  from  $H(s)$  does not introduce any antifield dependence.

One can be even more explicit and list all the solutions of (1). By the first procedure (a) described above, one generates from (15) solutions of the form  $\omega(x^\mu, dx^\mu) \hat{\Theta} L(\mathcal{T}^r, \theta_i)$  at ghost numbers 4, 7 and 10 and form degrees 0, 1, 2, 3. The multiform  $\tilde{\alpha}$  obtained by the second procedure (b) is simply  $\hat{\xi}^0 \hat{\xi}^1 \hat{\xi}^2 \hat{\xi}^3 L(\mathcal{T}^r, \theta_i)$  and thus the solutions of type (b) read  $(1/k!) dx^{\mu_1} \dots dx^{\mu_k} \xi^{\mu_{k+1}} \dots \xi^{\mu_4} \epsilon_{\mu_1 \dots \mu_4} e L(\mathcal{T}^r, \theta_i)$  at

respective form degrees  $k = 0, 1, 2, 3, 4$  where  $\epsilon_{0123} = 1$ ,  $dx^\mu dx^\nu = -dx^\nu dx^\mu \equiv dx^\mu \wedge dx^\nu$ . This establishes:

**Theorem 3** : *Up to trivial solutions, the general solution of the Wess-Zumino consistency condition is a linear combination of solutions of the form*

$$\omega(x^\mu, dx^\mu)[a + \hat{\Theta}L(\mathcal{T}^r, \theta_i)] \quad (16)$$

and of solutions of the form

$$\frac{1}{k!} dx^{\mu_1} \dots dx^{\mu_k} \xi^{\mu_{k+1}} \dots \xi^{\mu_4} \epsilon_{\mu_1 \dots \mu_4} eL(\mathcal{T}^r, \theta_i) \quad (17)$$

where  $\omega \in \Omega(\mathcal{M})$  is not exact,  $k = 0, 1, 2, 3, 4$  and  $L(\mathcal{T}^r, \theta_i)$  is a Lorentz invariant polynomial in its arguments. In particular, the most general local 4-form solving (1) is given by  $d^4x eL(\mathcal{T}^r, \theta_i) + sm + dn$  with  $d^4x = dx^0 dx^1 dx^2 dx^3$ .

Thus, the only solutions with ghost number zero have form degree equal to four and are given by Lorentz-invariant polynomials in the curvature components  $R_{abcd}$  and their successive covariant derivatives  $D_{a_1} \dots D_{a_m} R_{abcd}$  times  $ed^4x$ . These exhaust all the consistent perturbations of the Hilbert-Einstein action. Similarly, there is no solution at ghost number 1 and form degree 4, i.e., no gravitational anomaly (in four dimensions), and no solution at negative ghost number.

In this letter, we have given the explicit list of all the non trivial solutions of the WZ consistency condition for pure Einstein gravity in four dimensions. We have shown in particular that the dependence on the antifields can always be removed by adding trivial terms. Our method applies also if one includes matter couplings or if one modifies the Hilbert-Einstein Lagrangian by adding higher powers of the curvature, but there may be then modifications in the final results. Indeed, Yang-Mills fields bring in non trivial solutions of their own [19]. Furthermore, there may now exist non trivial conserved currents (associated e.g. with baryon number conservation). In that case,  $H^{-1}(s/d) \sim H_1(\delta/d)$  does not vanish and moreover, non trivial antifield-dependent solutions can be constructed, but only at ghost number 2 or higher if the Yang-Mills gauge group is semi-simple. The discussion follows closely the pattern developed in [20] and the details will be reported elsewhere [5]. [We shall discuss also in that paper the extension to all spacetime dimensions  $\geq 3$ , where there are more antifield-independent solutions - Chern-Simons terms in odd dimensions and Lorentz anomalies of the Adler-Bardeen type in 2 mod 4 dimensions]

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- [1] J. Wess and B. Zumino, *Phys. Lett.* **B37** (1971) 95.
  - [2] G. Barnich and M. Henneaux, *Phys. Lett.* **B 311** (1993) 123.
  - [3] G. Barnich, F. Brandt and M. Henneaux, preprint ULB-TH-94/06, NIKHEF-H 94-13, hep-th/9405109.
  - [4] R.L. Bryant and P.A. Griffiths, *Characteristic Cohomology of Differential Systems (I): General Theory*, Duke University Mathematics Preprint Series, volume 1993 n<sup>o</sup>1 (January 1993).
  - [5] G. Barnich, F. Brandt and M. Henneaux, *in preparation*.
  - [6] L. Alvarez-Gaumé and E. Witten, *Nucl. Phys.* **B234** (1984) 269; W. Bardeen and B. Zumino, *Nucl. Phys.* **B244** (1984) 421; F. Langouche, T. Schücker and R. Stora, *Phys. Lett.* **B145** (1984) 342; L. Alvarez-Gaumé and P. Ginsparg, *Ann. Phys.* **161** (1985) 423; L. Bonora, P. Pasti and M. Tonin, *J. Math. Phys.* **27** (1986) 2259; T. Baulieu and M. Bellon, *Nucl. Phys.* **B266** (1986) 75; T. Schücker, *Comm. Math. Phys.* **109** (1987) 167.
  - [7] S. Deser, *Gen. Rel. Grav.* **1** (1970) 9 ; R. Wald, *Phys. Rev.* **D 33** (1986) 3613 and references therein.
  - [8] F. Brandt, N. Dragon and M. Kreuzer, *Nucl. Phys.* **B340** (1990) 187.
  - [9] B. de Wit and J.W. van Holten, *Phys. Lett.* **B79** (1978) 389; I.A. Batalin and G.A. Vilkovisky, *Phys. Lett.* **B 102** (1982) 27 ; **B 120** (1983) 166 ; *Phys. Rev.* **D 28** (1983) 2567. For a review, see [10].
  - [10] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems*, Princeton University Press (Princeton: New Jersey 1992).
  - [11] J.M.L. Fisch and M. Henneaux, *Commun. Math. Phys.* **128** (1990) 627.
  - [12] M. Henneaux, *Nucl. Phys. B (Proc. Suppl.)* **18A** (1990) 47.
  - [13] F. Brandt, preprint NIKHEF-H 93-21, hep-th/9310123.
  - [14] M. Henneaux, *Commun. Math. Phys.* **140** (1991) 1.
  - [15] C.G. Torre and I.M. Anderson, *Phys. Rev. Lett.* **70** (1993) 3525 ; *Classification of generalized symmetries for the vacuum Einstein equations I*, preprint gr-qc 9404030.
  - [16] B. Zumino, in: *Relativity, Groups and Topology II* (B.S. deWitt and R. Stora, Eds.), North Holland (Amsterdam 1984).
  - [17] This is in sharp contrast with theories for which the diffeomorphisms are not gauge symmetries. For such theories, one may have (i) nontrivial 4-form solutions of (1) that do not descend to non trivial 0-forms; and (ii) vice-versa, non-constant BRST-invariant 0-forms that cannot be lifted to solutions of (1) with higher form degree. When this happens,  $d$  is not the zero operator in  $H^*(s)$  and one cannot write it as a BRST commutator, although this can be done in peculiar subalgebras [18]. For instance, in Maxwell theory with charged matter fields, the Lagrangian (times  $d^4x$ ) provides an example of (i). Similarly, the individual components  $F_{\alpha\beta}$  of the field strength exemplify (ii) since they are solutions of  $sF_{\alpha\beta} = 0$ , but  $dF_{\alpha\beta} = dx^\mu \partial_\mu F_{\alpha\beta} \neq s(\text{something})$ .

- [18] S. P. Sorella, *Commun. Math. Phys.* **157** (1993) 231.
- [19] F. Brandt, N. Dragon and M. Kreuzer, *Nucl. Phys.* **B332** (1990) 224, 250; M. Dubois-Violette, M. Henneaux, M. Talon and C.M. Viallet, *Phys. Lett.* **B267** (1991) 81.
- [20] G. Barnich, F. Brandt and M. Henneaux, preprint ULB-TH-94/07, NIKHEF-H 94-15, hep-th/9405194.