

# Prospects for direct measurement of time-integrated $B_s$ mixing

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June 10, 1994

## Abstract

This note investigates the prospects of measuring time-integrated  $B_s$  mixing. Three inclusive decay modes of the  $B_s$  meson are discussed. For each reconstruction mode, the expected number of events and the different background channels are discussed. Estimates are given for the uncertainty on the mixing parameter  $\chi_s$ .

# 1 Introduction

The production of  $B$  mesons at LEP is a mixture of  $B_u^+$ ,  $B_d^0$  and  $B_s^0$  mesons. The time integrated mixing parameter  $\bar{\chi}$  is an average quantity that reflects this mixture of  $B_d^0$  and  $B_s^0$  mesons:

$$\bar{\chi} = f_d \chi_d + f_s \chi_s$$

Here  $\chi_q$  is the probability that the original  $B_q$  meson decays as a  $\bar{B}_q$ , and  $f_d$  and  $f_s$  are the fractions of  $B^0$  and  $B_s$  mesons produced in the fragmentation of the  $b$  quark.

The combined measurements of ALEPH, DELPHI, L3 and OPAL give [4]:

$$\bar{\chi} = 0.119 \pm 0.012$$

From this a value for  $\chi_s$  can be deduced <sup>1</sup>:

$$\chi_s = 0.45 \pm 0.07(\chi_d) \pm 0.10(\bar{\chi}) \pm \Delta(f)$$

The error on the value of  $\chi_s$  is dominated by the uncertainty in the parameters  $f_d$  and  $f_s$ , indicated by  $\Delta(f)$ . These fractions are model dependent and therefore hard to estimate. In this note  $f_d = 0.4$  and  $f_s = 0.12$  are used.

The value of  $\chi_d$  that was used to deduce  $\chi_s$  comes from measurements of CLEO and ARGUS [6, 8] where  $B$  mesons are produced below the production threshold of the  $B_s$ , at the  $\Upsilon(4S)$  resonance:

$$\chi_d = 0.162 \pm 0.021$$

A direct way to measure  $\chi_s$  is to use an enriched sample of  $B_s$  mesons. The ideal signature to study  $B_s$  mixing would be a completely reconstructible and flavor sensitive channel, e.g.  $B_s \rightarrow D_s^- \pi^+$ . However, the number of observed  $D_s^- \pi^+$  decays per million  $Z^0$  events is only of the order of 5 events and can therefore not be used to study mixing with the current statistics at LEP. To increase the statistics we propose to reconstruct the  $B_s$  meson only partially. Unfortunately this does imply a background from other physics channels producing the same particles in the final state.

Three partial reconstruction modes of the  $B_s$  are discussed in this note. The semileptonic decay of the  $B_s$  into  $D_s^- l^+ \nu$  is very clean but also very low in statistics. More statistics can be obtained if the inclusive semileptonic decay of the  $B_s$  into a  $\phi$  and a lepton is used, although there will be a large background from other  $B$  mesons for this decay channel. The third decay channel is the inclusive decay of the  $B_s$  meson into a  $D_s$  meson. This channel is the most abundant indication of a  $B_s$  decay.

To determine the original state of the  $B_s$  meson, the flavor of the other  $b$  quark created in the  $Z^0$  decay is tagged. A high- $p_T$  lepton will be used to tag this flavor. In the mixing measurement each event is therefore divided into two sides. One side consists of the inclusively tagged  $B_s$  meson, while on the other side the flavor of the opposite  $b$  quark is tagged.

For each channel we will discuss the relation between  $\chi_s$  and the number of same and opposite flavor pairs that is measured. To determine the efficiencies of the various kinematic cuts, standard LUND Monte Carlo data on the generation level was used. Estimates of the statistical error on  $\chi_s$  and the expected number of events will be given for each reconstruction mode. Experimental errors are neglected. This means in particular that combinatoric background is not considered. Therefore the error estimates are optimistic. Since the combinatoric

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<sup>1</sup>The world average value presented by the Particle Data Group is:  $\chi_s = 0.53 \pm 0.15$  [41].

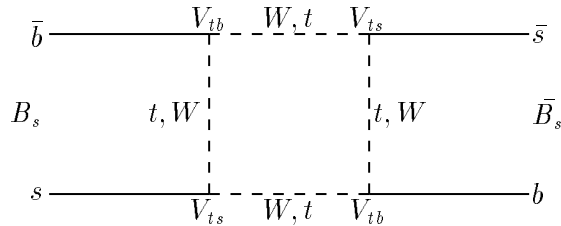


Figure 1: Box diagram for the process  $B_s \rightarrow \bar{B}_s$

background will differ for each individual reconstruction mode, one should be careful in comparing the effectiveness of each mode. The only experimental effect included is a background from misidentified hadrons producing fake leptons.

To estimate the number of expected events several results of LEP experiments are used. An analysis by Stefan Haider [1] uses a data sample of 720,000 hadronic  $Z^0$  events collected by the DELPHI experiment in 1992. The expected number of events will be given per million hadronic  $Z^0$  events with a detection efficiency corresponding to the DELPHI experiment.

The branching ratios used in this note are reviewed in the appendix, based on a mixture of experimental and theoretical predictions. The systematic errors arising from the uncertainties of these branching ratios are not discussed in detail.

## 2 $B_s \bar{B}_s$ mixing

The physically relevant parameter that we aim to measure in  $B_s$  mixing is the oscillation frequency  $x_s$ . In the framework of the Standard Model this oscillation frequency is directly related to the elements  $V_{ts}$  and  $V_{tb}$  of the Cabibbo-Kobayashi-Maskawa matrix. This is illustrated by the dominant box diagrams of  $B_s \bar{B}_s$  oscillations, see figure 1. The ratio of the oscillation frequencies  $x_d$  and  $x_s$  is directly related to two elements of the CKM matrix [13]:

$$\frac{x_d}{x_s} \simeq \left| \frac{V_{ts}}{V_{td}} \right| \quad (1)$$

This note will concentrate on the measurement of the time-integrated mixing parameter  $\chi$ , defined as the probability that a produced  $B^0$  meson decays as a  $\bar{B}^0$ , integrated over all times. Due to the integration no information about the proper time of the  $B$  meson is needed.

As illustrated in figure 2, the relation between  $x_q$  and  $\chi_q$  is given by:

$$\chi = \frac{x^2}{2(1+x^2)} \quad or \quad x^2 = \frac{2\chi}{1-2\chi} \quad (2)$$

The time-integrated method is most sensitive for measuring small oscillation frequencies in the region  $x < 1$ . This is the case for  $x_d$  where  $x \simeq 0.69$ . However, the Standard Model prediction for  $x_s$  is large:  $x_s > 8$  [13]. Unfortunately, the time-integrated method is not very sensitive to such values of  $x$ , because in this case a small error on  $\chi$  leads to large error on  $x$ . This is clearly illustrated by figure 2.

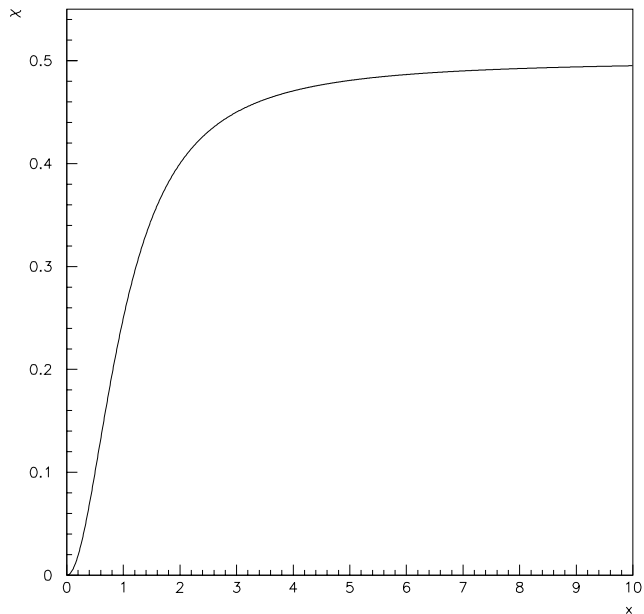


Figure 2: The time-integrated parameter  $\chi$  versus the oscillation frequency  $x$

### 3 Measuring time-integrated $B_s$ mixing

To measure time-integrated mixing, two states of the  $B_s$  meson have to be known: the state of the  $B_s$  meson at the time of its creation and the state of the  $B_s$  meson at the time of its decay.

Since two quarks of opposite flavor are produced in the hadronic decay of the  $Z^0$  event, one of which hadronized to form a  $B_s$  meson, the other  $b$  quark on the opposite side of the event can be used to tag the state of the  $B_s$  meson at the time of its creation. The second state is directly determined by the charge of the decay products of the  $B_s$  meson. Each event is therefore divided into two sides. One side consists of the inclusively tagged  $B_s$  meson, while on the other side the flavor of the opposite quark is tagged.

The time-integrated mixing measurement simply consists of counting the number of events containing a same or opposite flavor pair. The parameter used to indicate this counting is called  $X$ . In this section a general relation will be derived between the measured quantity  $X$ , and the physical parameter  $\chi_s$ . The relation will be illustrated using simple examples.

#### 3.1 Flavor tagging

There are several methods to detect the flavor of a  $b$  jet: a high- $p_T$  lepton, jet charge algorithms, or a high momentum kaon. In this note we will only discuss the presence of a high- $p_T$  lepton. These leptons have a high probability of coming directly from the decay of the original  $b$  quark. The other two possibilities are briefly discussed at the end of this note.

### 3.2 The probability for a wrong flavor tag

There is a probability that the high- $p_T$  lepton in a  $b\bar{b}$  event indicates the wrong flavor of the quark. This occurs when the lepton does not come directly from the  $b$  quark, but is created in the cascade decay  $b \rightarrow c \rightarrow l$ . Most of these leptons can be rejected because the  $p_T$  spectrum of these secondary leptons is much softer.

Another possibility for a "wrong" flavor indication occurs if the original  $B$  meson, that was contained in the jet, mixed before the time of its decay.

The probability for a wrong flavor and a right flavor tag are given by  $P_+$  and  $P_-$ :

$$P_+ = (1 - f_{prim})(1 - \bar{\chi}) + f_{prim}\bar{\chi} \quad (3)$$

$$P_- = (1 - f_{prim})\bar{\chi} + f_{prim}(1 - \bar{\chi}) \quad (4)$$

Where:

$$P_- = 1 - P_+ \quad (5)$$

$f_{prim}$  and  $(1 - f_{prim})$  denote the fractions of primary and secondary leptons in all  $b\bar{b}$  events, and  $\bar{\chi}$  is the probability that the original  $B$  meson mixed before the time of its decay.

A useful relation, that will be used later, is:

$$P_- - P_+ = (2f_{prim} - 1)(1 - 2\bar{\chi}) \quad (6)$$

$f_{prim}$  is determined by the momentum cuts used to reject secondary leptons. A typical value for  $f_{prim}$  is 0.90. This value will be used throughout this note.

### 3.3 The relation between $X$ and $\chi_s$

Now we consider both sides of the event and derive a general relation between  $X$  and  $\chi_s$ . As a first step we consider the final event sample, and see that it consists of three different categories:<sup>2</sup>

- $F_{b\bar{b}}$  :  $b\bar{b}$  events
- $F_{c\bar{c}}$  :  $c\bar{c}$  events
- $F_{mis}$  : misidentified hadrons, all events containing a fake lepton

Each category has a different way of contributing to the number of same and opposite flavor pairs,  $N_{++}$  and  $N_{+-}$ . The first and largest category in the final event sample consisting of  $b\bar{b}$  events will be discussed shortly. The second category is the background from  $c\bar{c}$  events. These events always produce an opposite flavor pair because there is no mixing nor are there any secondary leptons in charm events. The third category contains all events where a lepton was falsely identified, and was in fact a misidentified hadron. Because the charge of this hadron is expected to be randomly distributed, this background will contribute equally to the number of same and opposite flavor events.

The parameter  $X$  is used to count the number of events containing an opposite flavor pair relative to the total number of events. If indeed there were no mixing at all, nor any wrong flavor tags, there would not be any events containing a same flavor pair and  $X$  would be equal to one.

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<sup>2</sup>The exact composition between the three different categories depends on the set of cuts that is applied on both the  $B_s$  and the lepton side.

The result is given by the following relations:

$$X = \frac{N_{+-}}{N_{tot}} = (P_+R_+ + P_-R_-)F_{b\bar{b}} + 1/2F_{mis} + F_{c\bar{c}} \quad (7)$$

$$1 - X = \frac{N_{++}}{N_{tot}} = (P_+R_- + P_-R_+)F_{b\bar{b}} + 1/2F_{mis} \quad (8)$$

Where:

$$N_{tot} = N_{++} + N_{+-} \quad (9)$$

The evaluation of the first category, consisting of  $b\bar{b}$  events, is more complicated. We need to divide the  $b\bar{b}$  events into different subcategories, of which the most important is the signal from the  $B_s$  meson. The other subcategories are background channels from other  $B$  meson decays that produce the same particles in the final state <sup>3</sup>.

Each subcategory therefore corresponds to an inclusive decay channel of a  $B_s$  meson or another  $B$  meson. The probabilities for deriving the original state of the  $B$  meson or its opposite, have to be considered separately for all channels. An opposite flavor indication occurs if the original  $B$  meson mixed before the time of its decay (this is precisely the physical quantity we want to measure), but an opposite tag also occurs if the reconstruction mode of the  $B_s$  includes a primary lepton, whereas instead a secondary lepton is detected coming from the decay of another  $B$  meson.

Assume that there is a signal from  $B_s$  mesons, with a rate  $R_s$  w.r.t. the total number of  $b\bar{b}$  events and two background channels from  $B_d$  mesons, with rates  $R_1$  and  $R_2$ . The latter background behaves like the signal, whereas the first always indicates the opposite flavor. The probabilities for an opposite or original flavor tag on the  $B_s$  side are then defined as:

$$R_+ = R_s\chi_s + R_1(1 - \chi_d) + R_2\chi_d \quad (10)$$

$$R_- = R_s(1 - \chi_s) + R_1\chi_d + R_2(1 - \chi_d) \quad (11)$$

Where:

$$R_+ = 1 - R_- \quad (12)$$

The sum of the rates is by definition equal to one:  $R_s + R_1 + R_2 = 1$ . In general there will also be background from non-mixing  $B_u$  mesons, which is not shown here.

The probabilities  $R_+$  and  $R_-$  are now combined with the probabilities  $P_+$  and  $P_-$  from the lepton side to derive the probabilities to produce a same or an opposite flavor pair in a  $b\bar{b}$  event. Four different combinations are possible. For example, the probability for both sides to produce original flavor tags is  $P_-R_-$ . This combination produces an opposite flavor pair.

Using equations (6), (10) and (11) the parameter  $X$  of equation (8) can now be split into two parts, of which one depends on  $\chi_s$ :

$$X = C - D\chi_s \quad (13)$$

Where  $D$  is the dilution factor given by:

$$D = F_{b\bar{b}}R_s(2f_{prim} - 1)(1 - 2\bar{\chi}) \quad (14)$$

An important conclusion is that the dilution is directly proportional to  $R_s$ , the fraction of  $B_s$  mesons and to the purity of the flavor tagging. The fraction  $R_s$  depends on the different

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<sup>3</sup>The contribution of these backgrounds depends strongly on the kinematic cuts that are used to select the signal.

backgrounds from other physics channels and varies strongly for the different reconstruction modes of the  $B_s$  that are analyzed. Before discussing these modes, three simple examples will be discussed to illustrate the use of this general equation: the ideal scenario, the addition of background from secondary leptons and the poor event sample.

### 3.4 The ideal scenario

Imagine a scenario where there is no background from other physics channels, secondary leptons,  $c\bar{c}$  events, or misidentified hadrons. Then the final event sample would be absolutely pure in  $B_s$  mesons on one side and would have a direct, primary lepton on the opposite side. The only remaining dilution factor in this scenario is the fact that the  $B$  meson on the other side of the  $B_s$  does have a probability to mix.

From the definition of  $X$  it follows that:

$$X = (1 - \chi_s)(1 - \bar{\chi}) + \chi_s \bar{\chi} = 1 - \bar{\chi} - (1 - 2\bar{\chi})\chi_s \quad (15)$$

This relation can be derived from the general expression (13) by demanding :

$$F_{b\bar{b}} = f_{prim} = R_S = 1$$

For  $\bar{\chi} = 0.119 \pm 0.012$  the relation becomes:

$$X = 0.88 - 0.76\chi_s \quad (16)$$

For  $\chi_s = \frac{1}{2}$  it follows that  $X = \frac{1}{2}$  and the uncertainty on  $\chi_s$  is given by:

$$\Delta\chi_s = \frac{\Delta X}{1 - 2\bar{\chi}} = \frac{\Delta X}{0.762 \pm 0.024}$$

If one hundred of these ideal events were seen the following uncertainties could be obtained:  $\Delta X = 0.09$  and  $\Delta\chi_s = 0.11$ .

The uncertainty in  $\bar{\chi}$  can be neglected, since this gives a negligible error on  $\chi_s$  of 0.004. We can conclude that 100 ideal events would suffice to obtain a value of  $\chi_s$  with a smaller error than the current measurements and independent of  $f_s$  and  $f_d$ .

### 3.5 Introducing background from secondary leptons

As a first step towards a more realistic scenario we now introduce a background  $(1 - f_{prim})$  from secondary leptons. The signal on the other side remains absolutely pure in  $B_s$  mesons.

The relation between  $X$  and  $\chi_s$  now becomes:

$$X = f_{prim}(1 - \bar{\chi}) + (1 - f_{prim})\bar{\chi} - (2f_{prim} - 1)(1 - 2\bar{\chi})\chi_s \quad (17)$$

This relation can be derived from the general expression (13) by demanding :  $F_{b\bar{b}} = R_S = 1$ .

Using  $f_{prim} = 0.9$  and  $\bar{\chi} = 0.119$ , this leads to :

$$X = 0.80 - 0.61\chi_s \quad (18)$$

As in the ideal scenario  $\chi_s = \frac{1}{2}$  gives  $X = \frac{1}{2}$ . This is a general rule: the value of  $X$  is insensitive to the composition of the sample on the lepton side.

Because of the dilution from secondary leptons, the error on  $\chi_s$  has increased by a factor  $(2f_{prim} - 1)$ , as compared to the error in the ideal scenario :

$$\Delta\chi_s = \frac{\Delta X}{(2f_{prim} - 1)(1 - 2\bar{\chi})} \quad (19)$$

If a total number of 100 of these pure  $B_s$  events were found, the error estimates would be:

$$\Delta X = 0.09, \Delta\chi_s = 0.14$$

These errors are purely caused by physics effects, no detector effects are included. Therefore the value  $\Delta\chi_s = 0.14$  represents the lower limit on the uncertainty in  $\chi_s$  that can be obtained with a sample of 100 events.

### 3.6 The poor event sample

It is very illustrative to study the situation where there is no  $B_s$  enhancement at all. Remember that in this situation there is a large dependence on the branching fractions  $f_s$  and  $f_d$ . This example is therefore only used to illustrate the use of the general expression and is just considered as a generic case.

For the poor event sample  $X$  is equal to:

$$X = \frac{[f_{prim}f_{prim} + (1 - f_{prim})(1 - f_{prim})]}{[(1 - \bar{\chi})(1 - \bar{\chi}) + \bar{\chi}\bar{\chi}]} \quad (20)$$

For  $\bar{\chi} = 0.119$  this results in:

$$\begin{aligned} X &= 0.79 \text{ if } f_{prim} = 1 \\ X &= 0.65 \text{ if } f_{prim} = 0.9. \end{aligned}$$

If we now treat one side of the event as if it were enhanced in  $B_s$  mesons, the fraction of  $B_s$  mesons being  $f_s$ , we can derive the following relation between  $X$  and  $\chi_s$ :

$$X = 0.82 [1 - \bar{\chi} - f_d(1 - 2\bar{\chi})\chi_d - f_s(1 - 2\bar{\chi})\chi_s] \quad (21)$$

Which leads to:

$$X = 0.68 - 0.08\chi_s \quad (22)$$

This example shows that a sample of events without any enhancement can be seen as a special case of the general relation between  $X$  and  $\chi_s$ . It results in a small dilution factor of 0.08, although in reality the dilution factor is higher since both sides are sensitive to  $\chi_s$ .

### 3.7 The statistical errors on $X$ and $\chi_s$

In the experimental situation  $B_s$  enhanced events are observed as a peak in a reconstructed mass spectrum. A fit determines the number of events over the background spectrum. For each different  $B_s$  reconstruction mode that is studied, the combinatoric background is different.

In this note, however, only the purely statistical error is used, i.e. the root of the number of expected events. The effect of the combinatoric background is ignored. As a result all error estimates that will be given for  $\Delta\chi_s$  throughout this note, are always the most optimistic error estimates. The purely statistical error on  $X$  that will be used, is given by :

$$\Delta X = X \sqrt{\left(\frac{\Delta N_{+-}}{N_{+-}}\right)^2 + \left(\frac{\Delta N}{N}\right)^2} \quad (23)$$

The statistical error on  $\chi_s$  is then simply:

$$\Delta\chi_s = \frac{\Delta X}{D} \tag{24}$$

## 4 The $B_s \rightarrow D_s^{-(*)}l^+\nu$ decay

The signal comes from the semileptonic decay of a  $B_s$  meson, as displayed in figure 3(a). The branching ratio for this decay is a factor 10 higher than the completely reconstructible decay of the  $B_s$  into  $D_s^{(*)}\pi$ .

The  $D_s$  meson is completely reconstructed through the decays into  $\phi\pi$  and  $K^{*0}K$ , the only particles that are not reconstructed are the neutrino and possibly a photon coming from the decay of a  $D_s^*$ . This channel includes almost all semileptonic decays of the  $B_s$  meson since approximately 85 % of all semileptonic  $B_s$  decays contain a  $D_s$  meson in the final state.

Event type	Fraction (%)
$F_{c\bar{c}}$	0
$F_{mis}$	15
$F_{b\bar{b}}$	85

Table 1: Main event sample composition for the  $B_s \rightarrow D_s^{-(*)}l^+\nu$  decay channel

figure	decay channel	Br	(%)
3(a)	$R_S = \epsilon_S f_s Br(B_s \rightarrow D_s^- l^+ X)$	$6.4 \cdot 10^{-3}$	85
3(b)	$R_{W1} = \epsilon_W 2 f_d Br(B \rightarrow D_s^+ l^- X)$	$0.7 \cdot 10^{-3}$	9
3(c)	$R_{W2} = 2 \cdot \epsilon_W f_s Br(B_s \rightarrow D_s D_s \rightarrow D_s l)$	$0.2 \cdot 10^{-3}$	3
3(d)	$R_R = \epsilon_R 2 f_d Br(B \rightarrow D_s^- K^+ l^+ X)$	$0.2 \cdot 10^{-3}$	3

Table 2: Composition of  $F_{b\bar{b}}$ , the fraction of  $b\bar{b}$  events

The backgrounds from other  $B$  mesons are reduced considerably by applying cuts on the reconstructed mass of the  $D_s^- l^+$  pair and the momentum of the lepton. Typical cuts are:

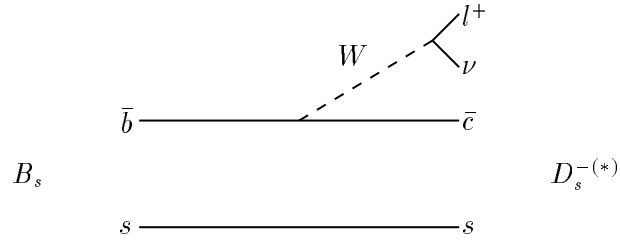
- momentum of the lepton  $> 3 \text{ GeV}/c$
- reconstructed mass of the  $D_s^- l^+$  pair  $> 3 \text{ GeV}/c^2$

The efficiency  $\epsilon_S$  for a signal event to pass these cuts is 0.6.

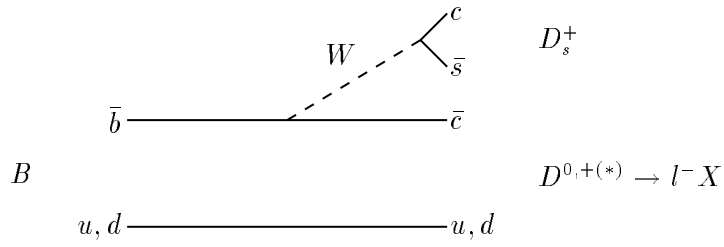
The most significant background comes from other  $B$  mesons that decay into two  $D$  mesons, indicated by  $R_{W1}$ . One of the  $D$  mesons is a  $D_s$  meson which is produced through a virtual  $W$  boson. The other  $D$  meson produces the lepton in the final state, see figure 3(b). The lepton always has the opposite charge to that of a primary lepton. The kinematic cuts help to reduce this background considerably, see figure 4. The efficiency  $\epsilon_W$  for these decays is 0.1. Note that before any kinematical cuts are applied, the number of  $D_s$  lepton pairs coming from  $D_s D$  pairs is of the same order as those coming from  $B_s$  mesons.

The  $B_s$  meson itself can decay in a similar way, producing two  $D_s$  mesons, of which one decays semileptonically, see figure 3(c). Depending on which one of the  $D_s$  mesons decays semileptonically, the lepton will imitate the flavor of a signal event, or indicate the opposite flavor. This special type of background is indicated by  $R_{W2}$ .

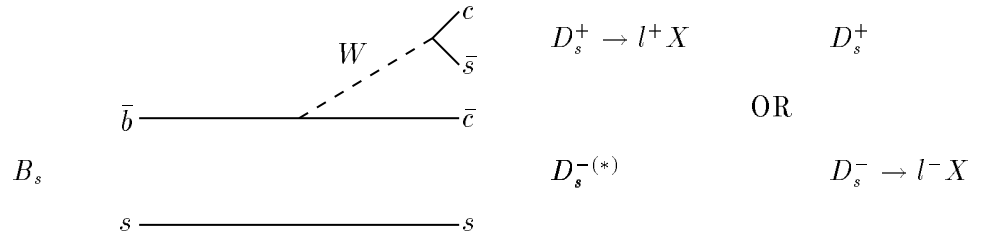
There is another possible background that comes from "rare" semileptonic  $B$  decays into a  $D_s$  meson. The creation of an extra  $s\bar{s}$  pair in the semileptonic decay of the  $B$  meson results in a  $D_s$  meson and a kaon in the final state, see figure 3(d). The branching ratio for this decay mode is expected to be smaller than 0.2 %. The background from these  $B$  decays will



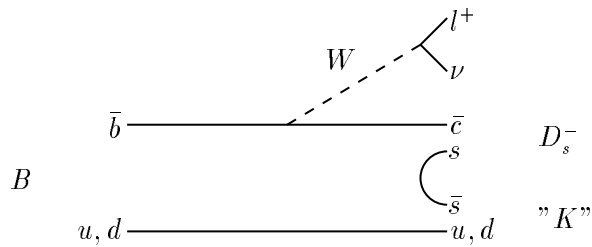
(a)  $B_s \rightarrow D_s^{-(*)} l^+ \nu$



(b)  $B \rightarrow D_s^+ l^- \nu$



(c)  $B_s \rightarrow D_s D_s \rightarrow D_s l \nu X$



(d)  $B \rightarrow D_s^- K l^+ \nu$

Figure 3: Channels producing a  $D_s$  lepton pair

be indicated with  $R_R$ . Fortunately the kinematics for this decay channel is different from the semileptonic  $B_s$  decay. Because one more particle is produced in the final state, the momentum spectra of the decay products are softer. The kinematic cuts therefore result in an efficiency  $\epsilon_R = 0.15$ , which is of the same order as  $\epsilon_W$  [17]. Because this decay channel was not included in the currently used program, this efficiency has not been checked on Monte Carlo data.

Lastly, there is no background contribution from  $c\bar{c}$  events, except a small contribution in the background events containing a misidentified hadron.

The contribution of all these backgrounds leads to the following relation between  $X$  and  $\chi_s$ :

$$X = 0.74 - 0.44\chi_s \quad (25)$$

The value of  $X$  for  $\chi_s = \frac{1}{2}$  is 0.49.

Unfortunately the statistics for this reconstruction mode is low. Let us consider nonetheless the very high number of 100 events. The statistical error estimates would then be:

$$\Delta X = 0.085, \Delta\chi_s = 0.19$$

when  $\Delta X$  and  $\Delta\chi_s$  are calculated using equations (23) and (24).

This decay channel in principle provides a very sensitive measurement of  $\chi_s$  because it is very pure in  $B_s$  mesons. The error on  $\chi_s$  does not differ much from the case where there was only background from secondary leptons.

### The expected number of events

Although low in statistics, the signal is relatively easy to observe. By using mainly kinematic cuts and aided by some particle identification, the combinatoric background can be drastically reduced. Unfortunately only a small fraction of the  $D_s$  decays can be completely reconstructed. The  $D_s$  mesons are reconstructed in their decays to  $\phi\pi$  and  $K^{*0}K$ . Both decays produce  $K^+K^-\pi^+$  in the final state. The total branching ratio for these decays is:

$$Br(D_s \rightarrow K^+K^-\pi^+) = 4\% \quad (26)$$

Using a sample of 720,000 hadronic  $Z^0$  events Stefan Haider records 5 events. The expected number of events can also be deduced from other results in  $B_s$  physics. Several LEP experiments have analyzed this semileptonic decay channel, but without doing a mixing measurement. No additional lepton in the opposite jet was therefore required, which leads to an increase in statistics by a factor 10. A factor 5 comes from the semileptonic branching ratio to either muons or electrons and another factor 2 is added because the leptons would be required to have a high- $p_T$ .

The ALEPH experiment has observed  $33 \pm 6$   $D_s^+l^-$  combinations in 450,000 hadronic  $Z$  decays in both the  $\phi\pi$  and  $K^{*0}K$  channels [17]. This is all the data recorded in 1990 and 1991. OPAL records 22  $D_s^+l^-$  events in 1.26 million hadronic  $Z$  events using the complete 1990–1992 running [20]. The sample has been used to do a lifetime measurement of the  $B_s$  meson. A preliminary analysis by DELPHI using only muons, shows a number of  $19 \pm 5$   $D_s^+l^-$  events using the combined 1991 and 1992 data (270,000 + 750,000 = 1,020,000 hadronic  $Z^0$  events) [21].

Combining all these measurements the expected number of events per million hadronic  $Z^0$  decays is of the order of 4 events.

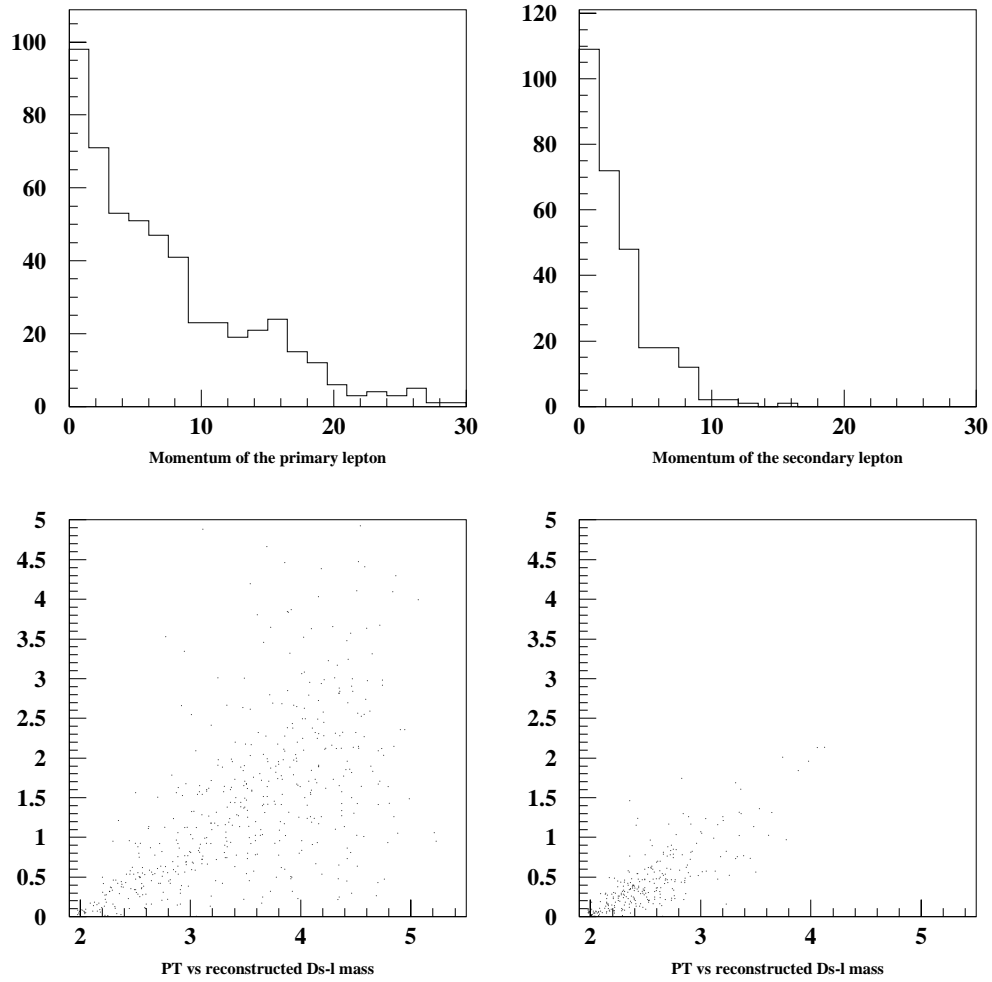


Figure 4: Kinematical cuts for the  $D_s l$  channel. Left side: primary leptons, Right: secondary leptons. All momenta are displayed in  $GeV/c$  and masses in  $GeV/c^2$

## 5 The $B_s \rightarrow \phi l X$ decay channel

To obtain more statistics the inclusive semileptonic decay channel of the  $B_s$  into  $\phi l$  can be studied, since it is expected that about 15 % of all  $D_s$  mesons decay inclusively into a  $\phi$  meson. Apart from the  $\phi$  meson and the primary lepton no other particles from the  $B_s$  decay are reconstructed.

The  $\phi l$  pair can of course originate from one  $D_s$  meson. This occurs for  $D_s$  mesons coming from  $B_s$  meson decays, and for  $D_s$  mesons from  $c\bar{c}$  events. Fortunately, these decays can be strongly suppressed by demanding the reconstructed mass of the  $\phi l$  pair to be larger than  $2 \text{ GeV}/c^2$ .

The composition of the final event sample after all cuts are applied is shown in table 3. Table 4 contains many different contributions to the composition of the  $b\bar{b}$  event sample because  $D_s$  mesons as well as other  $D$  mesons can produce  $\phi$  mesons and in addition the lepton can come from different sources.

Event type	Fraction (%)
$F_{c\bar{c}}$	1
$F_{mis}$	20
$F_{frag}$ ( $b\bar{b}$ event with a $\phi$ from the fragmentation process)	15
$F_{b\bar{b}}$	64

Table 3: Main event sample composition for the  $B_s \rightarrow \phi l X$  decay channel

figure	decay channel	Br	(%)
5(a)	$R_{s11} = \epsilon_{s11} f_s Br_{si}(B_s \rightarrow D_s^{(*)-} l^+ X) Br(D_s \rightarrow \phi X)$	$11 \cdot 10^{-4}$	48
5(b)	$R_{s12} = \epsilon_{s12} 2 f_d Br_{si}(B_{ud} \rightarrow D^{(*)} l^+ X) Br(D \rightarrow \phi X)$	$6 \cdot 10^{-4}$	28
5(c)	$R_{s13} = \epsilon_{s13} 2 f_d Br_{si}(B_{ud} \rightarrow D_s^{(*)-} l^+ X) Br(D_s \rightarrow \phi X)$	$1 \cdot 10^{-4}$	3
6(a)	$R_{W1} = \epsilon_{cascade} 2 f_d Br(B \rightarrow DD_s^+) Br(D_s^+ \rightarrow \phi X) Br(D \rightarrow l^- X)$	$2 \cdot 10^{-4}$	11
6(a)	$R_{W2} = \epsilon_{cascade} 2 f_d Br(B \rightarrow DD_s^+) Br(D_s^+ \rightarrow l^+ X) Br(D \rightarrow \phi X)$	$1 \cdot 10^{-4}$	3
6(b)	$R_{W3} = 2 \cdot \epsilon_{cascade} f_s Br(B_s \rightarrow D_s^- D_s^+) Br(D_s \rightarrow l X) Br(D_s \rightarrow \phi X)$	$2 \cdot 10^{-4}$	7

Table 4: Composition of  $F_{b\bar{b}}$ , the fraction of  $b\bar{b}$  events

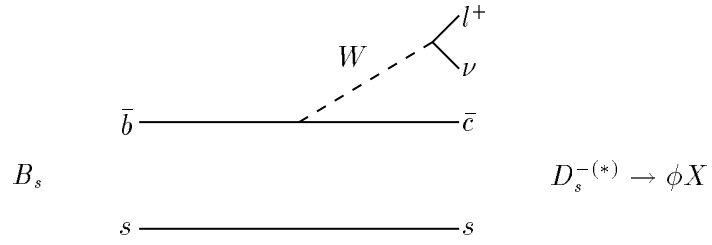
There is a special background for this channel that consists of  $\phi$  mesons created in the fragmentation process. A cut on the momentum of the  $\phi$  meson reduces this background considerably. However, the surviving events still form a large fraction of the final event sample and are distributed over the fractions of charm and bottom events.

Typical cuts are:

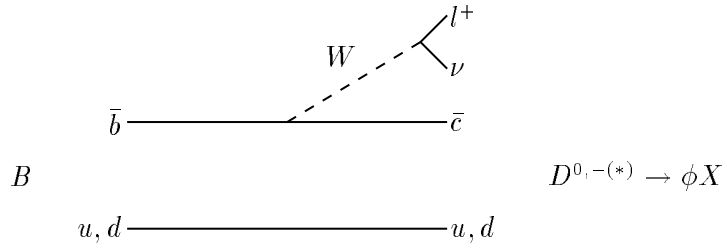
- a momentum of the  $\phi$  meson  $> 3 \text{ GeV}/c$
- a reconstructed mass of the  $\phi l$  pair between 2 and 4  $\text{GeV}/c^2$

After these cuts are applied  $\phi$  mesons from the fragmentation process still constitute 15 % of the final event sample.

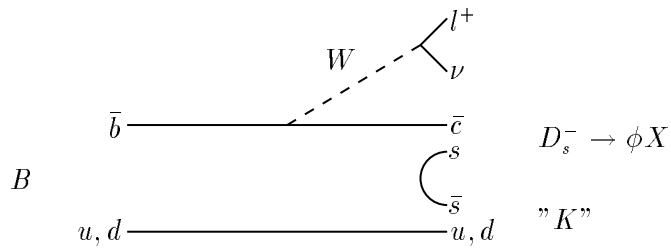
The lepton that accompanies the  $\phi$  meson can come from a semileptonic decay or from a cascade decay. The efficiencies to accept direct or cascade leptons depend on the cuts that



(a)  $B_s \rightarrow D_s^{-(*)} l^+ \nu$  followed by  $D_s \rightarrow \phi X$

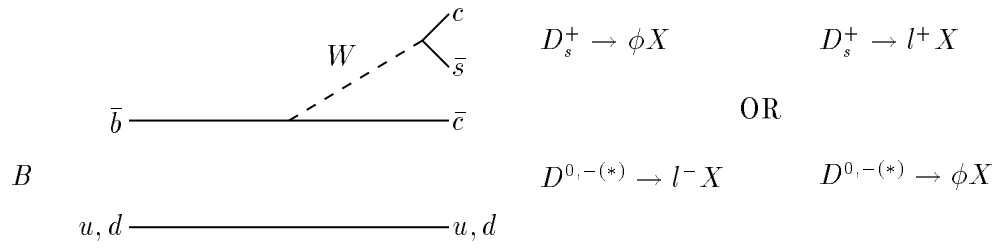


(b)  $B \rightarrow D^{0,-(*)} l^+ \nu$  followed by  $D^{0,-(*)} \rightarrow \phi X$

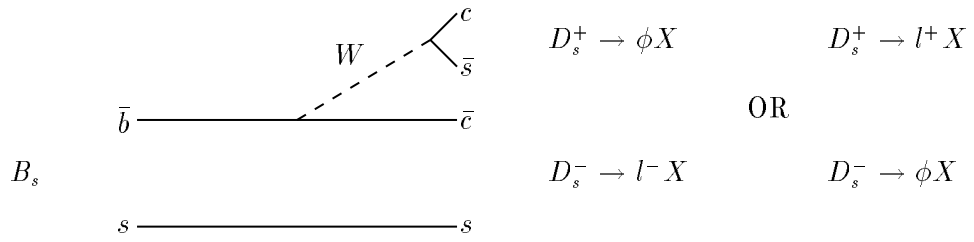


(c)  $B \rightarrow D_s^- K^0 l^+ \nu$  followed by  $D_s \rightarrow \phi X$

Figure 5: Direct semileptonic  $B$  decays producing a  $\phi$  lepton pair



(a)  $B \rightarrow D_s D \rightarrow \phi l X$



(b)  $B \rightarrow D_s D_s \rightarrow \phi l X$

Figure 6: Cascade  $B$  decays producing a  $\phi$  lepton pair

are used on the reconstructed mass of the  $\phi l$  pair, and are studied using Monte Carlo data. Typical results are:  $\epsilon_{sl1} = \epsilon_{sl2} = 0.7$  and  $\epsilon_{cascade} = 0.3$

The signal, defined by  $R_{sl1}$ , is the fraction of semileptonically decaying  $B_s$  mesons, see figure 5(a). The background from other semileptonically decaying  $B$  mesons, is indicated by  $R_{sl2}$ , see figure 5(b).

Rare semileptonic  $B$  decays into  $D_s^- l^+$  also create  $\phi l$  pairs, see figure 5(c), following the same decay channel that was discussed in the previous section. We again assume a branching ratio of 0.2 % for this process. The efficiency for this process has to be extracted from Monte Carlo, we estimate an efficiency  $\epsilon_{sl3} = 0.5$ .

Two cascade decays contribute to the background. In these decays a  $B$  meson decays into two  $D$  mesons. One of these is a  $D_s$  meson, which is produced through a virtual  $W$  boson, and the other a  $D$  meson, see figure 6(a). One of the  $D$  mesons decays inclusively into a  $\phi$  meson, while the other  $D$  meson decays semileptonically. The other cascade decay comes from the  $B_s$  meson itself, decaying into two  $D_s$  mesons, see figure 6(b).

If all these channels are evaluated, the following relation between  $X$  and  $\chi_s$  is obtained:

$$X = 0.58 - 0.17\chi_s$$

The value of  $X$  for  $\chi_s = \frac{1}{2}$  is 0.50.

A total number of 100 events would lead to the following error estimates:

$$\Delta X = 0.09, \Delta\chi_s = 0.51$$

Because the signal from the  $B_s$  constitutes now only half of the final event sample, this decay channel is twice as insensitive as the previous decay channel into  $D_s l$ . The advantage of this channel is the relatively high statistics. Unfortunately the large number of background channels, through the uncertainty of their contributions, will result in a large systematic error.

## The expected number of events

Current measurements at DELPHI indicate that the expected number of events per million hadronic  $Z^0$  events is in the order of 40 events. This number includes all the different background channels. The number of events from semileptonic  $B_s$  decays constitute about half of this total.

To estimate this number the preliminary result of Stefan Haider was used. He measures 41 events over a data sample of 750,000  $Z^0$  events.

Another result comes from A. Stocchi who analyzed the 1991 and 1992 data recorded by the DELPHI detector, and finds  $65 \pm 13$   $\phi l$  events [21]. These events are not required to have an additional lepton in the opposite jet, and only muons were used. Correcting for these two differences, (accounting for respectively a factor 0.2 and 2 difference), the equivalent number of events would be 26.

In this note we will use an expected number of events per million hadronic  $Z^0$  events of 40.

## 6 The $B_s \rightarrow D_s X$ decay channel

Because a large fraction, in the order of 85%, of the  $B_s$  mesons decay inclusively into a  $D_s$  meson this decay channel will have much more statistics compared to the semileptonic  $B_s$  decay channels. In the previously discussed reconstruction modes the charge of a primary lepton was used to tag the state of the  $B_s$  meson at the time of its decay, but in this case the charge of the  $D_s$  meson itself already provides this information.

Unfortunately the increase in statistics has to be paid with a loss of purity. Half of the  $D_s$  mesons originate not from a  $B_s$  meson but from other  $B$  mesons. The majority of these  $D_s$  mesons come from  $B^0$  and  $B^+$  decays through the decay of a virtual  $W$  boson. They are therefore always oppositely signed to  $D_s$  mesons coming from a  $B_s$  decay.

Another disadvantage of this decay channel is the relatively high background from  $c\bar{c}$  events, because only one high- $p_T$  lepton is required in the event. The results are summarized in tables 5 and 6. For this reconstruction mode no cuts can be applied since the kinematic properties of the  $D_s$  are equal for all channels.

Event type	Fraction (%)
$F_{c\bar{c}}$	4
$F_{mis}$	10
$F_{b\bar{b}}$	86

Table 5: Main event sample composition for the  $B_s \rightarrow D_s X$  decay channel

decay channel	Br	(%)
$R_S = f_s Br(B_s \rightarrow D_s^- X)$	0.09	44
$R_{W1} = 2f_d Br(B \rightarrow D_s^+ X)$	0.09	44
$R_{W2} = f_s Br(B_s \rightarrow D_s D_s)$	0.01	5
$R_R = 2f_d Br(B \rightarrow D_s^- X)$	0.02	7

Table 6: Composition of  $F_{b\bar{b}}$ , the fraction of  $b\bar{b}$  events

The signal coming from  $B_s$  decays will be denoted by  $R_S$ . The most important physics background coming from other  $B$  mesons is indicated by  $R_{W1}$ .

A small background is the decay channel of the  $B_s$  meson into two  $D_s$  mesons. It is denoted by  $R_{W2}$ . This decay channel can add to the signal or indicate exactly the opposite state, depending on which  $D_s$  meson is identified.

The last background comes from rare  $B$  decays that produce, through the creation of an  $s\bar{s}$  pair, a  $D_s$  meson with a negative charge. This gives a small contribution to the background.

From tables 5 and 6 we derive:

$$X = 0.52 - 0.23\chi_s \quad (27)$$

The value of  $X$  for  $\chi_s = \frac{1}{2}$  is 0.40.

The possibilities of this signature have been studied before by a special CKM working group at CERN [35]. The quoted result corresponds closely to their result:  $X = 0.57 - 0.32\chi_s$ .

The statistical error estimates for a total of 100 events are:

$$\Delta X = 0.07, \Delta\chi_s = 0.32$$

The dilution, which is mainly caused by other  $B$  mesons, is of the same order as in the  $\phi l X$  decay channel.

### **The expected number of events**

One would expect at least a factor 20 more events in this reconstruction mode as compared to the  $B_s \rightarrow D_s^{(*)} l \nu$  decay. This comes from the branching ratio of the  $B_s$  into  $D_s X$  which is a factor 10 higher, and because other  $B$  meson decays are expected to add an equal amount of events to the total. If for the first reconstruction mode in the order of 4 events are expected then this mode should yield at least 80 events per million hadronic  $Z^0$  decays.

However, the measured number of events for this reconstruction mode depends strongly on the cuts that are used, which in this case have to be tighter to help to reduce combinatoric background. For instance, Stefan Haider uses relatively tight cuts and finds a total of 34 events, again for 720,000 hadronic  $Z^0$  decays.

In this note we will use an expected number of events per million hadronic  $Z^0$  events of 70.

## 7 Summary

reconstruction mode	$X = C - D\chi_s$	$1/D$	$X(\chi_s = \frac{1}{2})$	$\Delta\chi_s(100)$	$N$	$\Delta\chi_s(N)$
ideal scenario	$0.88 - 0.76\chi_s$	1.3	0.50	0.11		
BG from secondary leptons	$0.80 - 0.61\chi_s$	1.6	0.50	0.14		
$D_s l$	$0.71 - 0.44\chi_s$	2.3	0.49	0.19	4	0.97
$\phi l X$	$0.58 - 0.17\chi_s$	5.9	0.50	0.51	40	0.80
$D_s X$	$0.52 - 0.23\chi_s$	4.3	0.40	0.32	70	0.39

Table 7: Summary of all reconstruction modes

The general relation between  $X$  and  $\chi_s$  was demonstrated to be:

$$X = C - D\chi_s \quad (28)$$

Where  $D$  is the dilution factor given by:

$$D = F_{b\bar{b}} R_s (2f_{prim} - 1)(1 - 2\bar{\chi}) \quad (29)$$

The dilution factor  $D$  is directly proportional to  $F_{b\bar{b}}$  the fraction of  $b\bar{b}$  events,  $f_{prim}$  the purity of the flavor tag, and depends in particular on  $R_s$  the ratio of the signal from the  $B_s$  meson compared to the different background channels.

The generic case, where the event sample is absolutely pure in  $B_s$  mesons and the only background comes from secondary leptons on the other side, shows that the minimal achievable uncertainty on  $\chi_s$  that can possibly be obtained is of the order of 0.14 for one hundred of these events.

Although a measurement of time-integrated  $B_s$  mixing will of course include all three decay modes, it is nevertheless clarifying to discuss their differences, as summarized in table 7. The fifth column shows  $\Delta\chi_s$ , which is the minimal statistical uncertainty on  $\chi_s$  for a total of 100 events. The most sensitive channel is clearly the  $D_s l$  decay, because it is very pure in  $B_s$  mesons. In both other reconstruction modes the background from other  $B$  meson decays is of the same order as the signal from  $B_s$  mesons. Consequently the dilution factors are about a factor two larger.

The effectiveness of each reconstruction mode in measuring  $\chi_s$  depends not only on the dilution factor but also on the expected number of events. In the 6th column the total number of events  $N$  is shown expected from a million hadronic  $Z^0$  decays. The last column shows the minimal statistical uncertainty on  $\chi_s$  corresponding to this number of events. A good reconstruction mode will have a low value of  $\Delta\chi_s(N)$ . In this respect the  $D_s X$  reconstruction mode has apparently the best possibilities. In theory one million  $Z^0$  events could result in a statistical error on  $\chi_s$  of 0.39.

One should be careful in drawing conclusions from these tables, because in reality the combinatoric background increases the statistical error. So far we have neglected this experimental combinatoric background which will be present under the peak. The reconstruction mode  $D_s X$  suffers from this in particular. In this respect the  $D_s l$  channel has a clear advantage, since it is experimentally a very clean signal.

Furthermore, uncertainties in the branching ratios lead to systematic errors which vary for the different reconstruction modes of the  $B_s$ . Many have to be deducted from several different

measurements. Some are even completely unknown and have to be estimated through theoretical assumptions. The many different branching ratios which have to be used to evaluate the  $\phi l X$  channel e.g. will lead to a relatively large systematic error. A complete study of the systematic errors as well as a more accurate determination of the efficiencies is in progress.

To increase the statistics two other methods can be used to tag the flavor of the opposite quark: jet charge or a high momentum kaon. Jet charge algorithms have already been used [27, 28], they allow in principal every event to be included in the measurement, because the requirement of at least one high- $p_T$  lepton is no longer needed. This increase in statistics is accompanied by a loss in the purity of the flavor tagging. If every event is used,  $f_{prim}$  will be of the order of 0.6. Note that for the  $D_s X$  reconstruction mode the use of a jet charge algorithm means that there will be a large background from charm events.

Summarizing, a complete framework for measuring time-integrated  $B_s$  mixing was presented, giving a clear view of the effect of the different dilution factors, and a good indication of the minimal statistical errors that can be obtained. It can be concluded that the best statistical uncertainty on  $\chi_s$ , using several million  $Z^0$  decays, will be of the order of 0.15. This would allow to set a minimum limit on the oscillation frequency  $x_s$  of approximately 2.

## Acknowledgements

I would like to thank Bert Koene, Achille Stocchi, Stefan Haider and Werner Ruckstuhl for their invaluable contributions to this work.

1	$Br(Z \rightarrow hadrons)$	$69.80 \pm 0.33\%$	PDG
2	$Br(Z \rightarrow \nu\bar{\nu})$	$20.2 \pm 0.4\%$	PDG
3	$Br(Z \rightarrow b\bar{b})$	$15.2 \pm 1.0\%$	PDG
4	$Br(Z \rightarrow c\bar{c})$	$12.6 \pm 2.1\%$	PDG
5	$f_s$	12%	[25, 26]
6		$10 \pm 2.5\%$	[24]
7	$f_d$	0.4	

Table 8: Branching ratios of the  $Z^0$ .

## Appendix

This appendix contains extensive lists of branching ratios that are of general importance in  $B_s$  physics. Some of the important branching ratios are discussed in more detail.

PDG indicates that the value is taken from the Particle Data Group [41]. The letters MC indicate Monte Carlo values as used in the standard LUND generation, and the letter E indicates that the value is an estimation based on certain assumptions.

**The semileptonic decay of the  $B_s$**  The branching ratio of the semileptonic decay  $B_s \rightarrow D_s l^- \nu$  is derived with the assumption that the production of the  $D_s^{**}$  is similar to the production of  $D^{**}$  mesons in semileptonic decays of non-strange  $B$  mesons. The latter branching ratio is measured [15]:

$$Br(B \rightarrow D^{**} l \nu) / Br_{sl} = 17 \pm 4\%$$

A small percentage of the  $D_s^{**}$  decays do result in a  $D_s$  meson. This is estimated to be less than 4 % [21]. From this it follows that the fraction of semileptonic  $B_s$  decays containing a  $D_s$  meson is expected to be:

$$Br_{sl}(B_s \rightarrow D_s l \nu X) / Br_{sl} = 85 \pm 5\%$$

**$Br(B \rightarrow D_s^{+(*)} D^{(*)})$**  One of the most important backgrounds in  $B_s$  physics is the decay of the  $B$  meson into two  $D$  mesons of which one is a  $D_s$  meson. ARGUS has measured this branching ratio to be :

$$Br(B \rightarrow D_s^{(*)} D^{(*)}) = 8.4 \pm 3\%$$

The branching ratio of the  $B_s$  meson into two  $D_s$  mesons is assumed to be equal to this branching ratio, although theoretical predictions are a bit lower, in the order of 5 %.

**$Br(B \rightarrow D_s^+ l^- X)$**  In this decay the lepton comes from the cascade decay of the  $D$  meson. The ratio  $B \rightarrow D_s^+ l^-$  is deduced from two measurements by ARGUS [9]. One is the branching ratio of the  $B$  into  $D_s D$  as given above and the other is the inclusive branching ratio:

$$Br(B \rightarrow D_s X) = 10 \pm 2 \pm 4\%$$

Using the semileptonic branching ratio of the  $D$  meson this results in:

$$Br(B \rightarrow D_s^+ l^- X) = 1\%$$

1	$Br(B \rightarrow l^\pm \nu hadrons)$	$10.5 \pm 0.7\%$	PDG
2	$Br(B \rightarrow D^\pm X)$	$22.7 \pm 3.3\%$	PDG
3		22%	MC
4	$Br(B \rightarrow D^0/\bar{D}^0 X)$	$46 \pm 5\%$	PDG
5	$Br(B \rightarrow D^0 X)$	66%	MC
6	$Br(B \rightarrow D_s^\pm X)$	$11.5 \pm 2.8\%$	PDG
7		10%	MC
8	$Br(B \rightarrow \phi X)$	$2.3 \pm 0.8\%$	PDG
9		3.3%	MC
10	$Br_{sl}(B \rightarrow \phi \mu X)$	0.2%	MC
11	$Br(B \rightarrow D_s^{(*)} D)$	$6.5 \pm 1.9\%$	PDG
12		8%	MC
13		$8.4 \pm 4.0\%$	[9]
14	$Br(B \rightarrow D_s^+ X) Br(D_s^+ \rightarrow \phi \pi^+)$	$(2.92 \pm 0.50) \cdot 10^{-3}$	[9]
15			[11]
16	$Br(B \rightarrow D^{**} l \nu) / Br_{sl}$	$17.4 \pm 4\%$	[15]
17	$Br(B^+ \rightarrow \bar{D}^{0*} l^+ \nu)$	$4.6 \pm 1.0\%$	PDG
18	$Br(B^+ \rightarrow \bar{D}^0 l^+ \nu)$	$1.6 \pm 0.7\%$	PDG
19	$Br(B^0 \rightarrow D^{-*} l^+ \nu)$	$4.9 \pm 0.5\%$	PDG
20	$Br(B^0 \rightarrow \bar{D}^- l^+ \nu)$	$1.8 \pm 0.5\%$	PDG
21	$N(D^0)/N(D^+)$ in $b\bar{b}$ events	3.0	MC
22	$Br(c \rightarrow D_s) Br(D_s \rightarrow \phi \pi)$	$0.300 \pm 0.045\%$	[31, 12]

Table 9: Branching ratios of the  $B^0$  and  $B^+$  mesons.

**Rare  $B$  meson decays producing a  $D_s^-$  meson** The creation of an  $s\bar{s}$  pair in the semileptonic decay of the  $B$  meson results in a  $D_s$  meson of negative charge and a kaon. The ARGUS collaboration has measured an upper limit for this process of 1.2 % [10]. However, the branching ratio is expected to be much lower than this limit.

The semileptonic decays of the  $B$  meson into  $D^{(*)} l^+ \nu$  have a total branching ratio of 6.5 % [41]. The total semileptonic branching ratio of the  $B$  meson is 10 %. This means that the missing 3.5 % of the semileptonic decays do not produce a  $D$  or  $D^*$  meson in the decay. A large part of these are decays into  $D^{**}$ , but there is also a fraction in which an extra  $q\bar{q}$  pair is created. Assuming that the probability for this quark pair to be an  $s\bar{s}$  pair is equal to  $f_s$ , we estimate :

$$Br(B \rightarrow D_s^- K^+ l^+ X) < 0.2\%$$

The creation of an  $s\bar{s}$  pair also occurs in non-semileptonic decays, contributing to the background in the  $B_s \rightarrow D_s^- X$  channel. The maximum value is taken to be:

$$Br(B \rightarrow D_s^- X) < 2\%$$

In this note these conservative values of 0.2 % and 2 % are used.

**Inclusive branching ratios of the  $D^0$ ,  $D^+$  and  $D_s$  into  $\phi$  mesons** In the analysis of the  $B_s \rightarrow \phi l$  decay it is crucial to know the inclusive branching fractions of the different  $D$

1	$Br(B_s \rightarrow D_s^+ X)$	$(86_{-13}^{+8})$	[31]
2	$Br(B \rightarrow D_s^+ l^- X)$	$< 1.2\%$	[10]
3	$Br(B_s \rightarrow D_s^{(*)+} l^- \nu) / Br_{sl}(B_s)$	85%	E [21]p71
4	$Br(B_s \rightarrow D_s^{*-} l^+ \nu \rightarrow D_s X) / Br_{sl}(B_s)$	$< 4\%$	E [21]p75
5	$Br(B_s \rightarrow D_s^- l^+ \nu X) / Br_{sl}(B_s)$	$85 \pm 5\%$	E [21]p75
6		84%	MC
7	$Br(B_s \rightarrow \phi X)$	10%	MC
8	$Br_{sl}(B_s \rightarrow \phi \mu X)$	1%	MC
9	$Br(B_s \rightarrow D_s^- \pi^+)$	$2.8 \cdot 10^{-3}$	[37]
10		$5.5 \cdot 10^{-3}$	[36]
11		$5 \cdot 10^{-3}$	[38]
12	$Br(B_s \rightarrow D_s^{*-} \pi^+)$	$2.8 \cdot 10^{-3}$	[37]
13		$4.1 \cdot 10^{-3}$	[36]
14		$2 \cdot 10^{-3}$	[38]
15	$Br(B_s \rightarrow J/\Psi \phi)$	$(1.4 \pm 0.4) \cdot 10^{-3}$	[37]
16		$1.3 \cdot 10^{-3}$	[36]
17	$Br(B_s \rightarrow D_s^{(*)-} D_s^{(*)+})$	$2.1 \cdot 10^{-3}$	[37]
18		$2.2 \cdot 10^{-3}$	[36]
19		$1.9 \cdot 10^{-3}$	[38]
20	$Br(B_s \rightarrow D_s^{(*)-} D_s^{(*)+})$	$5.4 \cdot 10^{-2}$	[37]
21		$4.1 \cdot 10^{-2}$	[36]
22		$3.8 \cdot 10^{-2}$	[38]

Table 10: Branching ratios of the  $B_s$  meson.

1	$Br(D_s \rightarrow \phi\pi^+)$	$2.8 \pm 0.5\%$	PDG
2		3.4%	MC
3	$Br(D_s \rightarrow K^{*0}K)$	$2.6 \pm 0.5\%$	PDG
4	$Br(D_s \rightarrow K^+K^-\pi^+)$	$3.9 \pm 0.4\%$	PDG
5	$Br(D_s \rightarrow \phi\pi^+\pi^+\pi^-)$	$1.2 \pm 0.4\%$	PDG
6	$Br(D_s \rightarrow \phi\pi^+\pi^0)$	$6.7 \pm 3.3\%$	PDG
7	$Br(D_s \rightarrow \phi l\nu)$	$1.4 \pm 0.5\%$	PDG
8	$Br(D_s \rightarrow \phi X)$	$12 \pm 3\%$	PDG
9		$14.5 \pm 5.0\%$	[22]
10		13%	MC
11	$Br(D_s \rightarrow \mu X)$	11%	MC
12	$Br(D_s \rightarrow e X)$	11%	MC
13		$< 20\%$	PDG
14	$Br(\phi \rightarrow K^+K^-)$	$49.1 \pm 0.8\%$	PDG

Table 11: Decays of the  $D_s$  meson

mesons into  $\phi$  mesons. Most of these branching ratios have not been, or are very poorly measured. To shed some light upon this subject P. Roudeau and A. Stocchi have done a study, estimating these branching ratios through a combination of theoretical arguments and measurements [22]. Their results are used in this note:

$$Br(D^0 \rightarrow \phi X) = 1.8 \pm 0.3\%$$

$$Br(D^+ \rightarrow \phi X) = 1.7 \pm 0.3\%$$

$$Br(D_s^+ \rightarrow \phi X) = 14.5 \pm 5.0\%$$

These results show that the inclusive branching ratios of the  $D^0$  and  $D^+$  into  $\phi$  mesons are equal in the first approximation. This is convenient because  $D^0$  and  $D^+$  mesons are produced in different ratios in  $b\bar{b}$  events.

Nevertheless this production ratio has to be evaluated, because the *semileptonic* branching ratios of the  $D^0$  and the  $D^+$  meson are considerably different. The ratio of  $D^0$  and  $D^+$  production in  $b\bar{b}$  events was extracted from Monte Carlo to be:

$$\frac{N(D^0)}{N(D^+)} = 3.0 \pm 0.5$$

1	$Br(D^+ \rightarrow \phi\pi^+)$	$0.60 \pm 0.08\%$	PDG
2	$Br(D^+ \rightarrow K^{*0}K)$	$0.47 \pm 0.09\%$	PDG
3	$Br(D^+ \rightarrow K^+K^-\pi^+)$	$1.01 \pm 0.13\%$	PDG
4	$Br(D^+ \rightarrow \phi\pi^+\pi^+\pi^-)$	$< 0.2\%$	PDG
5	$Br(D^+ \rightarrow \phi\pi^+\pi^0)$	$2.4_{-0.9}^{+1.1}\%$	PDG
6	$Br(D^+ \rightarrow \phi X)$	$0.60 \pm 0.08\%$	PDG
7		$1.7 \pm 0.3\%$	[22]
8		2.8%	MC
9	$Br(D^+ \rightarrow \mu X)$	20%	MC
10	$Br(D^+ \rightarrow e X)$	25%	MC
11		$17.2 \pm 1.9\%$	PDG

Table 12: Decays of the  $D^+$  meson

1	$Br(D^0 \rightarrow \phi\pi^+\pi^-)$	$0.24 \pm 0.08\%$	PDG
2	$Br(D^0 \rightarrow K^+K^-\pi^+\pi^-)$	$0.24 \pm 0.04\%$	PDG
3	$Br(D^0 \rightarrow \bar{K}^0\phi)$	$0.88 \pm 0.12\%$	PDG
4	$Br(D^0 \rightarrow \phi X)$	$1.12 \pm 0.14\%$	PDG
5		$1.8 \pm 0.2\%$	[22]
6		1.6%	MC
7	$Br(D^0 \rightarrow \mu X)$	$8.8 \pm 2.5\%$	PDG
8		13%	MC
9	$Br(D^0 \rightarrow e X)$	$7.7 \pm 1.2\%$	PDG
10		11%	MC

Table 13: Decays of the  $D^0$  meson

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