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Initial-state QED corrections to four-fermion production in e^+e^- collisions at LEP200 and beyond [a]

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Abstract

The implementation of QED initial-state radiative corrections in the process of four-fermion production at LEP200 and higher-energy e^+e^- colliders is discussed. Because of the presence of charged-current processes, this is a nontrivial problem, and we compare our approach with other existing treatments. We describe the Monte Carlo algorithm used for the generation of 4-fermion events with photon bremsstrahlung. Comparison between our event generator and semi-numerical calculations are presented, as well as predictions for W and Z pair related cross sections.

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1 Introduction

In this letter we discuss the implementation of initial-state QED radiation (ISR) in the reaction

$$e^+e^- \quad \rightarrow \quad 4 \text{ fermions} \ . \tag{1}$$

We shall discuss our method, present a variety of results for the energy regime of LEP200 and slightly above, and, where possible, make comparisons with other existing results. The purpose of this study is to provide an efficient and flexible calculational tool, in the form of a Monte Carlo event generator, `EXCALIBUR`, that incorporates the expected dominant physics effects. Therefore, this tool should be useful for all kinds of physics studies at LEP200 and beyond. We start by outlining our method.

2 The method

In a recent paper [1], we studied all electroweak (EW) e^+e^- processes leading to a final state of four effectively massless fermions, without regard for the effects of photon emission. In the case of four quarks, one would like to add the concomitant QCD production channel, and also the production of a quark pair and two gluons, since both different final states will appear as jets. This QCD-extended treatment is now also availabe [2].

The motivation for our considering 4-fermion final states is that the single or pairwise production of vectorbosons, at LEP200 energies or beyond, typically manifest themselves as 4-fermion production. A priori, one does not know the relative sizes of the production *via* an intermediate step of vector boson production (the ‘signal’), or *via* direct production (the ‘background’), and, moreover, these two alternatives interfere; for the interpretation of future experiments this knowledge is requisite. The results of [1] show that, for a number of final states, background effects can be as large as radiative-correction effects.

In [1], a general procedure was developed in which any 4-fermion final state can be chosen, and (weighted) Monte Carlo events efficiently generated. In this way, every possible distribution or cross section in these processes becomes calculable. The only restriction in [1] is that charged particles in the final state be, experimentally, visible and distinguishable, *i.e.* they must be produced with sufficient energy, and at nonnegligible angle to the beams or to each other. Under this condition, one avoids collinear singularities: therefore, the fermions can be treated as massless, which considerably accelerates the calculation [1]. When one considers only collinearly *convergent* diagrams, we

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[1] Note that signals with an intermediate Higgs therefore fall outside of our scope.

may even omit our visibility requirements. Amongst others, this case occurs for the W pair signal reaction

$$e^+e^- \quad \rightarrow \quad W^+W^- \quad \rightarrow \quad 4 \text{ fermions} . \tag{2}$$

In the calculation of the Born diagrams that are needed in the event generator, a number of input parameters can be chosen independently, to wit, the electroweak couplings parametrized by a (running) α and $\sin^2\theta_w$, the boson masses m_W and m_Z, and the corresponding total widths Γ_W and Γ_Z. For instance, one has the freedom to choose $\cos\theta_w$ different from m_W/m_Z, and to take arbitrary values for the widths, which may also be assigned an energy dependence. This convention, in which α occurs as an overall factor in the matrix element, automatically ensures the unitarity cancellations, so that the value of α may be changed at will.

Although the event generator of [1] is suitable for many signal-versus-background studies, its usefulness is increased considerably when the most important radiative correction effects are incorporated. Let us discuss how this can be achieved in a practical way.

3 Inclusion of radiative corrections

To start, we focus on the reaction

$$e^+e^- \quad \rightarrow \quad W^+W^- \ , \tag{3}$$

with stable W's ($\Gamma_W = 0$), and on the reaction in eq. (2), with decaying W's ($\Gamma_W > 0$). The radiative-correction problem is much more involved than for the, by now familiar, case (see, *e.g.* [3])

$$e^+e^- \quad \rightarrow \quad Z^0 \quad \rightarrow \quad \mu^+\mu^- \ . \tag{4}$$

In the latter reaction, photonic and weak corrections can be separated. Thus, the weak one-loop corrections lead to a modification of the Born cross section into a ‘dressed’ Born cross section. To this latter, the sizeable QED initial-state corrections can be applied by means of the structure-function method. These QED corrections incorporate $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$ leading-log (LL) and subleading terms, while the leading higher-order soft-photon contributions are implemented by exponentiation.

Despite many attempts, a simple dressed Born cross section for the reaction of eq. (3) has not be found, so that the above procedure for the Z cannot be applied: the weak corrections do not decouple from the the QED ones in W pair production. A cognate difficulty is that a division of photon emission into initial- and final-state radiation is meaningful for reaction

	Final state		
RC treatment	W's on-shell	4 fermions from W's	4 fermions all diags.
$\mathcal{O}(\alpha)$ EW + 1 soft γ	[5]-[8]	[14]	[14]
$\mathcal{O}(\alpha)$ EW + 1 hard γ	[9]	[14]	[14]
same, but event generator	[10, 11]		
LL QED ISR + exponentiation	[12]	[4]	
Full EW $\mathcal{O}(\alpha)$ + high.ord. LL ISR	[9, 13]		
LL QED ISR + exp. event generator		[15], and this paper	this paper

Table 1: A summary of references to radiative-correction studies of W pair or 4-fermion production. Either electroweak or LL QED ISR have been considered. Event-generator approaches are mentioned explicitly. All references contain numerical results, except ref. [14] which devises a strategy for the most complete treatment.

(4) but not for (3), since the two sets of Feynman diagrams are not separately gauge-invariant. A priori, then, it is meaningless to consider those for initial-state radiation in isolation. An elegant trick has been proposed in [4] to introduce a gauge-invariant definition of initial- and final-state terms, based on adding and subtracting extra terms in the radiative matrix element. Although solving, in a sense, the problem of gauge invariance, there is as yet no proof that the initial-state radiation terms thus obtained yield a quantitatively good description. Since the LL terms in the cross section are anyhow gauge-invariant, the method of [4] is correct for these terms, but the subleading ISR terms now contain some arbitrariness.

In order to facilitate the discussion that follows, we find it useful to summarize in table 1 what has been done in the literature, and what some authors hope to achieve. We also indicate the position of the present paper in this welter of possibilities.

For stable W's, complete one-loop EW effects have been calculated in a number of papers [5, 6, 7, 8]. The results of refs. [7] and [8] are in perfect agreement. To obtain the complete $\mathcal{O}(\alpha)$ correction, the effect of emission of a single hard photon has to be included [9]. An event generator based on this calculation also exists [10], whereas recently another event generator has been constructed where, for collinear photons, $\mathcal{O}(\alpha^2)$ QED corrections are also considered [11].

In order to asses the importance of higher-order photonic effects, a LL calculation up to $\mathcal{O}(\alpha^2)$, including exponentiated soft-photon effects, was already carried out in an early for the total cross section and for an angular distribution [12]. Somewhat later, the $\mathcal{O}(\alpha)$ term of such a QED LL treatment was replaced by a full $\mathcal{O}(\alpha)$ EW calculation [9, 13]. The difference between these two approaches amounts to less than 1% in the LEP200 energy range.

For decaying W's, the radiative corrections are far less known. The full $\mathcal{O}(\alpha)$ EW corrections have not yet been computed. Questions of strategy, and theoretical issues like gauge invariance, have been addressed in the literature, which may form the basis for a conclusive calculation [14]. What *has* been done, are ISR calculations to the 4-fermion final state produced by the WW signal. In ref. [4], the LL plus exponentiated soft-photon contributions are evaluated semi-analytically. For the total cross section, a 3-dimensional integral has to be done numerically, after a number of analytic integrals have been performed. Thus, only very specific distributions can be obtained. The influence of subleading terms in the ISR are studied, but, since the above-mentioned trick has been used in [4] in order to give a gauge-invariant definition of ISR, the *quantitative* meaning of precisely these terms is obscure until a full calculation, including final-state radiation (and interference) has been achieved. Another approach uses LL QED corrections to the signal diagrams, leading to an event generator for the three signal diagrams [15].

In the present paper, we want to be able to study *any* experimental distribution [2] and all background effects, and we want to incorporate the dominant radiative correction effects. This leads to an event generator that can handle all diagrams leading to a specified 4-fermion final state (with, of course, the option of a restriction to the signal diagrams), and that incorporates the LL $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$ ISR, with exponentiation of the remaining soft-photon effects. We shall now discuss how the initial-state radiation effects are incorporated in our Monte Carlo event generator [1].

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[2] We except the transverse-momentum distribution of the bremsstrahlung.

4 Generating initial-state radiation

In order to upgrade the event generator of ref. [1] with QED ISR, the following description of the radiation is used. Each of the incoming fermions is assumed to have its energy degraded by an amount of bremsstrahlung. Under the assumption that the bremsstrahlung photons are emitted parallel to the radiating beam, the energy distribution of the fermion after radiation is described by the 'structure' function

$$\begin{aligned}\Phi(x) &= \frac{\exp\left(-\beta\gamma_E + 3\alpha L/4\pi\right)}{\Gamma(1+\beta)}\beta(1-x)^{\beta-1} - \frac{\alpha}{2\pi}(1+x)L \\ &\quad -\frac{\alpha^2}{8\pi^2}\left[\frac{1+3x^2}{1-x}\log x + 4(1+x)\log(1-x) + 5 + x\right]L^2 \ , \\ \beta &= \frac{\alpha}{\pi}(L-1) \quad , \quad L = \log\left(\frac{Q^2}{m_e^2}\right) \quad , \end{aligned} \tag{5}$$

where x is the fermion's energy in units of the beam energy, m_e is the electron mass, γ_E is Euler's constant, and Q^2 is some appropriate energy scale. This is a LL $\mathcal{O}\left(\alpha^2\right)$ structure function with exponentiated soft-photon effects. Subleading logarithmic terms are not considered, since one needs for this the non leading logarithmic terms from the one loop EW corrections to the Born cross section of reaction in eq. (1). Another important point is the choice of Q^2. Formally, a change in Q^2 is a subleading effect, but its precise value is of course a matter of numerical concern. We use $Q^2 = s$, the total energy squared, which is known to be acceptable [13]. It is a unique advantage of the Monte Carlo approach that, if we wish, the Q^2 can even be determined on an event-by-event basis. Finally, the structure functions $\Phi(x_1)$ for the incoming e^+ (with original momentum p_1, degraded to x_1p_1) and $\Phi(x_2)$ for the incoming e^- (with momentum p_2, degraded to x_2p_2), can be convoluted: with

$$s' \equiv (x_1p_1 + x_2p_2)^2 = x_1x_2s \ , \tag{6}$$

we arrive at the 'flux function'

$$G(s'/s) = \int\limits_0^1\int\limits_0^1 dx_1\, dx_2\ \Phi(x_1)\ \Phi(x_2)\ \delta(x_1x_2 - s'/s) \tag{7}$$

which enables one to write the total radiative cross section as

$$\sigma(s) = \int\limits_0^1 dz\ G(z)\sigma_0(zs) \ , \tag{8}$$

where σ_0 is the nonradiative cross section. Incidentally, the form of $G(x)$ is, to given order in α, quite close to that of $\Phi(x)$, where α is replaced by

2α [13]. In LL approximation the flux function G_D of ref. [3] is related to the choice made in eq. (5). We have opted for the use of structure functions rather than that of a flux function for the following reason. Assuming that the four-momentum lost to radiation is lightlike (as one usually does in problems such as this one), the total *energy loss* of the beams must be equal, in the flux-function formalism, to $(1-s'/s)\sqrt{s}/2$; for instance, this identification is made in [4]. The energy loss can, however, be quite different in the structure-function formalism, where the lost momentum is the sum of two lightlike vectors; and this is the more realistic approach, since radiation from the two beams tends to be contained in two narrow, back-to-back cones. Another way to appreciate the difference is to note that, when all the lost momentum is lumped into a single lightlike vector, events with small s' will *always* be boosted away from the lab frame, whereas in the structure-function formalism they can easily be at rest in the lab frame.

We are now in a position to decribe the radiation algorithm. To start, two values for x_1 and x_2 are generated, each with a probability distribution

$$\begin{aligned}\Phi_1(x) &= \frac{1}{\beta_1}(1-x)^{\beta_1-1} \quad ,\\ \beta_1 &= \frac{\alpha}{\pi}\left(\log\frac{Q_1^2}{m_e^2}-1\right) \quad ,\end{aligned} \tag{9}$$

where we have introduced yet another scale Q_1^2. We then compute x_1p_1 and x_2p_2, so that the total momentum of the 4-fermion system is determined. This allows us to move over to the centre-of-mass frame of the 4-fermion system; here, the total energy is $\sqrt{s'}$ rather than $\sqrt{s}$, and, importantly, the beam directions are the same as in the lab frame. This is due to the assumption that the bremsstrahlung is strictly collinear: relaxing this assumption would lead to, as yet nearly insuperable, complications in the Monte Carlo.

We now generate a weighted Monte Carlo 4-fermion event by the procedure described in [1]. The resultant momenta are then boosted back to the lab frame. The final event weight is then the product of the '4-fermion weight' defined in [1], and the 'radiation weight'

$$w_{\mathrm{rad}} = \frac{\Phi(x_1)}{\Phi_1(x_1)}\frac{\Phi(x_2)}{\Phi_1(x_2)} \quad . \tag{10}$$

As usual, the Monte Carlo cross section and its estimated error are then extracted from the mean and the variance of the weight distribution. We want to stress that the adoption of a structure function different from that of eq. (5) is trivial in our approach, since it only entails modifying the definition of w_{rad}: in this way, we can, for instance, perform delicate checks with the results of ref. [4].

A few remarks are in order here. In the first place, note that the use of two structure functions with terms up to, say $\mathcal{O}(\alpha)$, is *not* equivalent to a calculation based on a flux function to $\mathcal{O}(\alpha)$, since the product of the two structure functions contains some $\mathcal{O}(\alpha^2)$ terms. Of course, with the above algorithm we may also settle for generating only a single value for $z = x_1 x_2$, and proceeding with the generation of the 4-fermion final state using the reduced energy $s' = zs$. Since, however, the Lorentz boost is then not determined, we can only compute the total cross section in that case, and no differential ones. Nevertheless, we have applied this procedure in order to compare our event generator with the semi-analytical results of ref. [4].

Secondly, we have to discuss the choice of Q_1^2. In principle, it may be chosen at will, since the Monte Carlo cross section does, formally, not depend on it. However, it is easily seen that w_{rad} will diverge as x_1 or x_2 approache one, unless $Q_1^2 \leq Q^2$. We therefore must, in the generation of events, choose Q_1^2 to be the minimum possible value for Q^2. If one desires to use a fixed Q^2 scale (as we have done in this paper) this poses no problem, and one simply puts $Q_1^2 = Q^2$; but for a study where Q^2 depends on the particular event generated, some care must be taken.

Finally, it should be realized that the various kinematical cuts described above must also be boosted to the 4-fermion rest frame. For invariant-mass cuts this is no problem, but the energy and angular cuts require some attention. Given two values x_1 and x_2, we know the relativisitc velocity of the Lorentz boost to the 4-fermion centre-of-mass frame (CMF):

$$\beta = (x_1 - x_2)/(x_1 + x_2) \quad . \tag{11}$$

If the scattering angle θ of a (massless) particle in the lab frame is restricted between

$$c_0 \; < \; \cos\theta \; < \; c_1 \quad , \tag{12}$$

we may then compute the bounds in the CMF on the CMF scattering angle θ':

$$\frac{c_0 - \beta}{1 - c_0\beta} \; < \cos\theta' \; < \frac{c_1 - \beta}{1 - c_1\beta} \quad . \tag{13}$$

Similarly, suppose its energy E in the lab is restricted by

$$E \; > \; E_{\mathrm{min}} \quad . \tag{14}$$

Its energy E' in the CMF now depends on both the energy and the angle in the lab frame, so that a CMF energy cut is complicated; we replace it by its lower bound (the minimum over all scattering angles):

$$E' \; > \; \frac{E_{\rm min}}{\sqrt{1-\beta^2}}\Big(1-\max(c_0\beta, c_1\beta)\Big) \quad . \tag{15}$$

Since this cut is somewhat looser, some particles may end up with an energy lower than $E_{\rm min}$; this means that an additional number of generated events has to be rejected. It must be stressed that these cuts would become impractically complicated if the bremsstrahlung were also to have a transverse momentum component.

5 Results and conclusions

We shall now discuss a number of results from `EXCALIBUR`. In the first place, we have to establish agreement, where possible, with the results of ref. [4]. To this end, we must of course make sure to use the same electroweak input parameters. In the structure functions and the flux functions we use the low-energy value

$$\alpha_{\rm str.f.,\ flux} = (137.036)^{-1} \quad . \tag{16}$$

In the rest of the calculation, we take the input parameters such that important electroweak corrections to the reaction (3) are effectively incorporated. In the notation of ref. [13], the amplitude M for reaction (3) can be divided into two gauge-invariant parts:

$$M = \frac{e^2}{2\sin^2\theta_w} M_I + e^2 M_Q \quad , \tag{17}$$

where M_I is the purely $V-A$ part, and M_Q a purely vector-like part. The number $e^2 = 4\pi\alpha$ we choose such that the running value of α at LEP200 energies is obtained, and $\sin^2\theta_w$ we fix by

$$\frac{\alpha}{2\sin^2\theta_w} = \frac{G_\mu m_W^2}{\pi\sqrt{2}} \quad . \tag{18}$$

This gives us the parameters also used in [4]. We also adopt the values used there for the boson masses and widths. Numerically, we then have

$$\begin{aligned} &\alpha = (127.29)^{-1} \quad , \quad \sin^2\theta_w = 0.2325 \quad , \quad G_\mu = 1.16635\ 10^{-5}\ \mathrm{GeV}^{-2} \quad , \\ &m_Z = 91.173\ \mathrm{GeV} \quad , \quad \Gamma_Z = 2.4971\ \mathrm{GeV} \quad , \\ &m_W = 80.220\ \mathrm{GeV} \quad , \quad \Gamma_W = 2.033\ \mathrm{GeV} \quad . \end{aligned} \tag{19}$$

We use, moreover, boson propagators with energy-dependent widths; their denominators have the typical form $s - m^2 + is\Gamma/m$. In [13], a discussion of an 'improved Born approximation' is given. In our approach, where we have both signal and background effects, we can retain of this discussion only the above-mentioned tuning of α and $\sin^2\theta_w$. In particular, the effects of the Coulomb singularity in the WW system [16, 17], is left out.

We are now ready to present some saliant results. We have chosen the explicit process

$$e^+e^- \quad \rightarrow \quad e^-\bar{\nu}_e u\bar{d} \ , \tag{20}$$

which contains the WW pair production signal, as well as a nonnegligible background. In the first place, we must reproduce the results of [4, 18] where we can. These are the Born cross section σ_0; the total cross section with ISR from ref. [18] and the average 'energy loss', which in [4] is defined as $\frac{\sqrt{s}}{2}(1-x_1x_2)$, which we denote by $\bar{\epsilon}$. This is (see above) not the *real* energy loss $\frac{\sqrt{s}}{2}(2-x_1-x_2)$, denoted by ϵ; but which of the two quantities is actually the most relevant in the measurement of the W mass is, of course, determined by the prospective data analysis.

We have performed a number of different Monte Carlo studies, under different strategies, which we now list.

- **WW,f,1**: the WW signal diagrams only, with the flux-function apraoch to $\mathcal{O}(\alpha)$; no cuts.

- **WW,s,1,a**: the WW signal diagrams only, with structure functions that are simply the flux function of [4] to $\mathcal{O}(\alpha)$, in which 2α is replaced by α; no cuts.

- **WW,s,1,b**: the WW signal diagrams only, with the structure functions of eq. (5) where the last (L^2) terms are left out; no cuts.

- **WW,f,2**: the WW signal diagrams only, with the complete $\mathcal{O}(\alpha^2)$ flux function of ref. [4] (which, unfortunately, is not given explicitly); no cuts.

- **WW,s,2**: the WW signal diagrams only, with structure functions as given in eq. (5); no cuts.

- **WW,cuts**: like the previous case, except that now we also impose the following cuts:
  $E_{e^-,\ u,\ \bar{d}} > 20\ GeV$, $|\cos\theta_{e^-,\ u,\ \bar{d}}| < 0.9$, $|\cos_{(u\bar{d})}| < 0.9$, $m_{(u\bar{d})} > 10\ GeV$.

- **all,cuts**: like the previous case, except that now also all the background Feynman diagrams are taken into account.

The various results are given in table 2, where the numbers taken over from ref. [4, 18] are indicated by the subscript 4. The cross sections are given in picobarns, and the energy losses in GeV.

Similarly in table 3 results are given for Z-pair production. The choice of the second energy was determined by the condition that the velocities of the Z's would be the same as for the W's at $\sqrt{s} = 190$ GeV, when there is no radiation present. The cuts used are $E_{(all\ particles)} > 20\ GeV$, $|\cos\theta_{(all\ particles)}| < 0.9$, $m_{(e^+e^-)}$ and $m_{(u\bar{u})} > 10$ GeV and $|\cos_{(u\bar{u})}| < 0.9$.

In discussing the results, let us first notice that the cross sections from the event generator and the semi-analytical approach [4, 18] agree. For σ_0 it is clear when one takes the numbers .60078 and .67930 from [18], for σ it follows from the flux function comparisons in table 1. These comparisons agree within 1 standard deviation. For $\bar{\epsilon}$ and $\bar{\epsilon}_4$ there are differences up to five standard deviations. The error on the results for $\bar{\epsilon}_4$ of ref. [4] is however not available [18], so that no conclusion can be drawn.

The structure function method differs slightly from the flux function method as comparisons between the first and second rows of table 1 shows. The $\mathcal{O}(\alpha^2)$ results from `EXCALIBUR` and ref. [4] differ at the 2% level for the energy losses, but this may be due to the not specified form of $\mathcal{O}(\alpha^2)$ corrections in [4].

The inclusion of cuts and the inclusion of more diagrams affect both cross sections and energy losses.

Since the proposed direct reconstruction method for the W mass suffers from a shift in M_W due to the radiated energy [19], a precise knowledge of $\bar{\epsilon}$ and ϵ is warranted. Our results show that the precise treatment of ISR, the choice of cuts and the neglect of diagrams all affect the energy losses. The first effect was also found in ref. [4], where also the influence of the Coulomb singularity is discussed. The two other effects show that a Monte Carlo treatment allowing for cuts and being able to include all diagrams is indispensable.

The results for Z-pair production again show the effects of cuts and the inclusion of background diagrams. Although the second energy is tuned in a way that comparisons to W-pair production may make sense, the energy losses divided by the total energy are different for these two processes.

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strategy	σ_0	σ_4	σ	$\bar{\epsilon}_4$	$\bar{\epsilon}$	ϵ
$\sqrt{s}$ =176 GeV						
WW,f,1	.60111 .00032	.50504	.50490 .00032	1.168	1.162 0.002	–
WW,s,1,a	”	–	.50484 .00033	–	1.172 0.002	1.175 0.002
WW,s,1,b	”	–	.50175 .00098	–	1.167 0.006	1.170 0.006
WW,f,2	”	.50315	–	1.200	–	–
WW,s,2	”	–	.50258 .00097	–	1.178 0.006	1.181 0.006
WW,cuts	.44651 .00092	–	.37737 .00091	–	1.192 0.007	1.195 0.007
all,cuts	.45011 .00097	–	.37926 .00095	–	1.149 0.007	1.152 0.007
$\sqrt{s}$ =190 GeV						
WW,f,1	.67911 .00038	.60733	.60750 .00039	2.108	2.092 0.003	–
WW,s,1,a	”	–	.60706 .00039	–	2.111 0.003	2.118 0.003
WW,s,1,b	”	–	.60330 .00117	–	2.098 0.010	2.106 0.010
WW,f,2	”	.60696	–	2.171	–	–
WW,s,2	”	–	.60630 .00117	–	2.124 0.010	2.132 0.010
WW,cuts	.48289 .00107	–	.43124 .00106	–	2.188 0.013	2.196 0.013
all,cuts	.49316 .00115	–	.44164 .00114	–	2.136 0.013	2.144 0.013

Table 2: Results on radiatively corrected cross sections and average energy losses, under various calculational strategies, for the process $e^+e^- \to e^-\bar{\nu}_e u\bar{d}$. The second line in each entry is the estimated Monte Carlo error.

strategy	σ_0	σ	$\bar{\epsilon}$	ϵ
$\sqrt{s}$ =190 GeV				
ZZ,s,2	.70549 10^{-2} .00040	.54632 10^{-2} .00060	0.744 0.002	0.745 0.002
ZZ,cuts	.52192 10^{-2} .00060	.40742 10^{-2} .00065	0.732 0.003	0.734 0.003
all,cuts	.92509 10^{-2} .00182	.78393 10^{-2} .00182	1.929 0.016	1.940 0.016
$\sqrt{s}$ =215.942 GeV				
ZZ,s,2	.93245 10^{-2} .00044	.83903 10^{-2} .00067	2.614 0.008	2.624 0.008
ZZ,cuts	.62890 10^{-2} .00079	.57262 10^{-2} .00084	2.699 0.011	2.709 0.011
all,cuts	.99837 10^{-2} .00194	.92763 10^{-2} .00198	3.503 0.021	3.526 0.021

Table 3: Results on radiatively corrected cross sections and average energy losses, under various calculational strategies, for the process $e^+e^- \to e^+e^-u\bar{u}$. The second line in each entry is the estimated Monte Carlo error.