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Abstract

We present results of a computation of the BRST-antibracket cohomology in the space of local functionals of the fields and antifields for a class of $2d$ gravitational theories which are conformally invariant at the classical level. In particular all classical local action functionals, all candidate anomalies and all BRST-invariant functionals depending nontrivially on antifields are given and discussed for these models.

1 Introduction

Conformal invariance plays a crucial role in various two dimensional physical models. Of special interest is the question whether conformal invariance of a classical theory is maintained in the quantum theory or becomes anomalous. In string theory, for instance, vanishing of the conformal anomaly determines the critical dimension and imposes “equations of motion” in target space [1]. Since the work of Wess and Zumino [2] it is well-known that anomalies have to satisfy consistency conditions following from the algebra of the symmetries of the classical theory. The general form of these conditions can be elegantly formulated in the BV-antifield formalism [3] as the vanishing of the antibracket of the proper solution S of the classical master equation and a functional \mathcal{A} of the fields and antifields representing the anomaly [4]:

$$(S, \mathcal{A}) = 0. \tag{1.1}$$

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(1.1) amounts to a cohomological problem since it requires BRST invariance of \mathcal{A} : the BRST operator¹ s is defined on arbitrary functionals \mathcal{F} of fields and antifields through

$$s \mathcal{F} := (S, \mathcal{F}) \quad (1.2)$$

and its nilpotency is implied by the Jacobi identity for the antibracket and by the fact that S solves the classical master equation $(S, S) = 0$:

$$s^2 = 0. \quad (1.3)$$

Let us denote by $H^*(s)$ the BRST cohomology in the relevant space of functionals of the fields and antifields which must be specified in each particular case (usually it is the space of local functionals whose precise definition must be adapted to the problem). Since the BRST operator increases the ghost number (gh) by one unit due to $gh(S) = 0$, $H^*(s)$ can be computed in each subspace of functionals with a definite ghost number g separately where we denote it by $H^g(s)$. Anomalies are represented by cohomology classes of $H^1(s)$, at least if the ghost number is conserved in the quantum theory which holds at tree level due to $gh(S) = 0$. However it is often useful to compute $H^g(s)$ for other values of g as well. In particular $H^0(s)$ is interesting since it contains S itself. This opens the possibility to construct \tilde{S} by computing $H^0(s)$ after fixing the desired field content and gauge invariances of a model. This was actually our starting point for the computation of $H^*(s)$ in a class of two dimensional models which are conformally invariant at the classical level (see next section). A computation of $H^0(s)$ can also be useful for given S since it can provide information about observables or counterterms arising in a theory. Moreover antifield-dependent solutions \mathcal{M} of $(S, \mathcal{M}) = 0$ with ghost number 0 can have interesting interpretations and applications. For instance they are needed for the construction of a functional $\tilde{S} = S + \mathcal{M} + \dots$ satisfying $(\tilde{S}, \tilde{S}) = 0$. If \tilde{S} exists, it provides a nontrivial extension of the theory which is consistent in the sense that it is invariant under suitable extensions of the gauge transformations of the original theory characterized by S . This was pointed out and exemplified in [5]. BRST-invariant functionals \mathcal{M} with ghost number 0 which do not satisfy $(\mathcal{M}, \mathcal{M}) = 0$ may alternatively receive an interpretation as background charges, which can cancel anomalies. This can be implemented in the BV antifield formalism [6] by formally considering $\mathcal{M} = \sqrt{\hbar} \mathcal{M}_{1/2}$ as a contribution to the quantum action W of order $\sqrt{\hbar}$ since $W = S + \sqrt{\hbar} \mathcal{M}_{1/2} + \hbar \mathcal{M}_1 + \dots$ implies $(W, W) = \hbar(\mathcal{M}_{1/2}, \mathcal{M}_{1/2}) + \dots$, i.e. $(\mathcal{M}, \mathcal{M})$ indeed can cancel one-loop anomalies.

¹We call s the BRST operator although this terminology often is used only when it acts on the fields, and not on antifields.

2 Characterization of the models

Our aim was the computation of $H^*(s)$ for a class of two dimensional models which are conformally invariant at the classical level. To this end we did not characterize these models by specific conformally invariant classical actions but we only specified the field content and the gauge invariances of the classical theory. Then we computed $H^*(s)$ in the space of antifield-independent functionals which for ghost number 0 in particular provides the most general local classical action functional S_0 and thus characterizes more precisely the models to which our results apply. Then we completed the computation of $H^*(s)$ by inclusion of the antifields. This procedure is possible due to the closure of the algebra of gauge symmetries since in this case the BRST transformations of the fields do not depend on the antifields. The BRST transformations of the antifields however involve (functional derivatives of) S_0 and therefore their inclusion requires the knowledge of S_0 .

In detail, the models which we investigated are characterized by

- (i) Field content: S_0 is a local functional of the $2d$ metric $g_{\alpha\beta} = g_{\beta\alpha}$ ($\alpha, \beta \in \{+, -\}$) and a set of bosonic scalar matter fields X^μ ($\mu \in \{1, 2, \dots, D\}$).
- (ii) S_0 is invariant under $2d$ diffeomorphisms and local Weyl transformations of the metric $g_{\alpha\beta}$.
- (iii) S_0 does not possess any nontrivial gauge symmetries apart from those mentioned in (ii).²

In order to make (i) precise we have to add the definition of local functionals we used:

- (iv) A functional of a set of fields Z^A is called local if its integrand is a polynomial in the derivatives of the Z^A (without restriction on the order of derivatives) but may depend nonpolynomially on the undifferentiated fields Z^A and explicitly on the coordinates x^α of the two dimensional base manifold.

(ii) requires

$$s S_0 = 0 \tag{2.1}$$

where s acts on $g_{\alpha\beta}$ and X^μ according to

$$s g_{\alpha\beta} = \xi^\gamma \partial_\gamma g_{\alpha\beta} + g_{\gamma\beta} \partial_\alpha \xi^\gamma + g_{\alpha\gamma} \partial_\beta \xi^\gamma + c g_{\alpha\beta}, \quad s X^\mu = \xi^\alpha \partial_\alpha X^\mu. \tag{2.2}$$

Of course these are just the BRST transformations of $g_{\alpha\beta}$ and X^μ where ξ^α and c are the anticommuting ghosts of diffeomorphisms and local Weyl

²Trivial gauge symmetries of an action S_0 depending on a set of (bosonic) fields φ^i are by definition of the form $\delta_\epsilon \varphi^i = P^{ij} \delta S_0 / \delta \varphi^j$ where $P^{ij} = -P^{ji}$ are arbitrary functions of the φ^i , arbitrary parameters $\epsilon(x)$ and the derivatives of the φ^i and ϵ .

transformations respectively. The BRST transformations of the ghosts are chosen such that (1.3) holds on all fields $g_{\alpha\beta}, X^\mu, \xi^\alpha, c$. This leads to

$$s \xi^\alpha = \xi^\beta \partial_\beta \xi^\alpha, \quad s c = \xi^\alpha \partial_\alpha c. \quad (2.3)$$

(2.1)–(2.3) and requirement (iii) guarantee that the proper solution of the classical master equation is given by

$$S = S_0 - \int d^2x (s\Phi^A)\Phi_A^* \quad (2.4)$$

where we used customary collective notations $\{\Phi^A\} = \{g_{\alpha\beta}, X^\mu, \xi^\alpha, c\}$ and $\{\Phi_A^*\} = \{g^{*\alpha\beta}, X_\mu^*, \xi_\alpha^*, c^*\}$ for fields and antifields. The BRST transformation of Φ_A^* is given by the functional right derivative of S w.r.t. Φ^A (in our conventions the BRST operator acts from the left everywhere)

$$s \Phi_A^* = \frac{\delta_r S}{\delta \Phi^A}. \quad (2.5)$$

Remark:

Although we allow the integrands of local functionals to depend explicitly on x^α according to (iv), it turns out that integrands of BRST-invariant functionals actually do not carry an explicit x -dependence (up to trivial contributions of course). Nevertheless we need this definition of local functionals in order to cancel candidate anomalies as e.g. $\int d^2x \xi^\alpha L$ where L is a Weyl invariant density ($sL = \partial_\alpha(\xi^\alpha L)$). Namely these functionals are BRST invariant but not BRST exact unless we admit counterterms whose integrands depend explicitly (and in fact polynomially) on the x^α . These are well-known features of all gravitational theories (cf. [7, 8, 9]). However, if one takes into account topological properties of the base manifold, the x -independence of the integrands of nontrivial BRST-invariant functionals holds strictly only if the manifold does not allow closed p -forms with $p \neq 0$ which are not exact. The results we present in the next section therefore hold in a strict sense only under this additional assumption (cf. [9] for general remarks on this point).

3 Results

3.1 Antifield-independent functionals

We found that $H^g(s)$ vanishes for $g > 4$ in the space of local antifield-independent functionals, i.e. each local BRST-invariant functional with ghost number $g > 4$ which does not depend on antifields is the BRST variation of a local functional with ghost number $g - 1$ which also does not depend on antifields. We only spell out the results for $g = 0, 1$. Those for $g = 2, 3, 4$ will be given in [10].

$H^0(s)$ provides the most general classical action S_0 . It is given by

$$S_0 = \int d^2x \left(\frac{1}{2} \sqrt{g} g^{\alpha\beta} G_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu + B_{\mu\nu}(X) \partial_+ X^\mu \partial_- X^\nu \right) \quad (3.1)$$

with $g = |\det(g_{\alpha\beta})|$. $G_{\mu\nu}$ and $B_{\mu\nu}$ are arbitrary functions of the X^μ satisfying

$$G_{\mu\nu} = G_{\nu\mu}, \quad B_{\mu\nu} = -B_{\nu\mu}.$$

$B_{\mu\nu}$ is defined only up to contributions $\partial_\mu B_\nu(X) - \partial_\nu B_\mu(X)$ which yield total derivatives in the integrand of (3.1). Here and henceforth

$$\partial_\mu := \frac{\partial}{\partial X^\mu}$$

denote derivatives w.r.t. matter fields. (3.1) is the most general functional satisfying requirements (i) and (ii) listed in section 2 with the restrictions imposed by (iv). (iii) represents an additional requirement which excludes e.g. functions $G_{\mu\nu}$ and $B_{\mu\nu}$ admitting a nonvanishing solution $g^\mu(X)$ of

$$G_{\mu\nu} g^\nu = \Gamma_{\mu\nu\rho} g^\rho = H_{\mu\nu\rho} g^\rho = 0 \quad (3.2)$$

where

$$\Gamma_{\mu\nu\rho} = \frac{1}{2} (\partial_\mu G_{\nu\rho} + \partial_\nu G_{\mu\rho} - \partial_\rho G_{\mu\nu}), \quad H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}. \quad (3.3)$$

Namely (3.2) implies the invariance of S_0 under $\delta_\epsilon X^\mu = \epsilon(x) g^\mu(X)$ for an arbitrary function $\epsilon(x)$ and thus the presence of an additional gauge invariance which violates requirement (iii).

It is also worth noting that invariance of the theory under target space reparametrizations can be elegantly formulated in the antifield formalism as well (this kind of an invariance must not be confused with invariance of the action functional in the usual sense, of course). Namely any two action functionals $S'_0[X] := S_0[X + \delta X]$ and $S_0[X]$ which are related by an arbitrary infinitesimal target space reparametrization $\delta X^\mu = f^\mu(X)$ differ by the BRST-variation of a local antifield-dependent functional (with BRST transformation of the antifields defined by means of $S_0[X]$). Both $S'_0[X]$ and $S_0[X]$ are of the form (3.1) with functions $G'_{\mu\nu}$ and $G_{\mu\nu}$ resp. $B'_{\mu\nu}$ and $B_{\mu\nu}$ related by³

$$G'_{\mu\nu} = G_{\mu\nu} + 2\partial_{(\mu} f_{\nu)} - \Gamma_{\mu\nu\rho} f^\rho; \quad B'_{\mu\nu} = B_{\mu\nu} + 2\partial_{[\mu} B_{\nu]} + H_{\mu\nu\rho} f^\rho \quad (3.4)$$

where $f_\mu := G_{\mu\nu} f^\nu$. We note that an analogous statement holds for *any* theory characterized by a local action functional $S_0[\phi]$ (where ϕ^i are the fields with ghost number 0). Namely consider infinitesimal (local) field redefinitions $\delta\phi^i = f^i(\phi, \partial\phi, \dots)$ which are chosen such that

$$s (S_0[\phi + \delta\phi] - S_0[\phi]) = 0 \quad (3.5)$$

³(Anti-)Symmetrization of indices is defined by $f_{(\mu\nu)} = \frac{1}{2}(f_{\mu\nu} + f_{\nu\mu})$ etc.

holds with the BRST operator encoding the gauge symmetries of $S_0[\phi]$. Then there is a (local) functional Γ^{-1} with ghost number -1 such that

$$S_0[\phi + \delta\phi] - S_0[\phi] = s \Gamma^{-1}[\Phi, \Phi^*]. \quad (3.6)$$

This statement holds also for theories without gauge invariances.⁴ In this case s reduces to the Koszul-Tate differential $\delta_{KT} = \int d^2x (\delta S_0[\phi]/\delta\phi^i) \delta/\delta\phi_i^*$ and (3.6) holds obviously for arbitrary field redefinitions $\delta\phi^i$.

$H^1(s)$ provides the antifield-independent candidate anomalies. They are given by

$$\mathcal{A} = \mathcal{H}_+ + \mathcal{H}_- + \mathcal{X}_+ + \mathcal{X}_-, \quad (3.7)$$

$$\mathcal{H}_\pm = a_\pm \int d^2x c^\pm (\partial_\pm)^3 h_{\mp\mp}, \quad (3.8)$$

$$\mathcal{X}_\pm = \int d^2x \frac{1}{1-y} (\partial_\pm \xi^\pm + h_{\mp\mp} \partial_\pm \xi^\mp) \nabla_+ X^\mu \nabla_- X^\nu f_{\mu\nu}^\pm(X) \quad (3.9)$$

where a_+, a_- are constants, $f_{\mu\nu}^+, f_{\mu\nu}^-$ are arbitrary functions of the X^μ and

$$\begin{aligned} h_{\pm\pm} &= g_{\pm\pm}/(g_{+-} + \sqrt{g}), & y &= h_{++}h_{--}, \\ \nabla_\pm X^\mu &= (\partial_\pm - h_{\pm\pm} \partial_\mp) X^\mu, & c^\pm &= \xi^\pm + h_{\mp\mp} \xi^\mp. \end{aligned} \quad (3.10)$$

Using the original components of the metric, \mathcal{X}_\pm read

$$\begin{aligned} \mathcal{X}_\pm = \int d^2x (\partial_\pm \xi^\pm + h_{\mp\mp} \partial_\pm \xi^\mp) & \left(\frac{1}{2} \sqrt{g} g^{\alpha\beta} f_{(\mu\nu)}^\pm(X) \partial_\alpha X^\mu \partial_\beta X^\nu \right. \\ & \left. + f_{[\mu\nu]}^\pm(X) \partial_+ X^\mu \partial_- X^\nu \right). \end{aligned}$$

In fact the parts of \mathcal{X}_\pm containing the symmetric and antisymmetric parts of $f_{\mu\nu}^\pm$ are separately BRST invariant. They are also nontrivial and inequivalent in the space of antifield-independent functionals. We remark however that those functionals $\mathcal{X}_+, \mathcal{X}_-$ which arise from contributions

$$2\partial_{(\mu} H^{\pm}_{\nu)} \mp 2\partial_{[\mu} H^{\pm}_{\nu]} + (H_{\mu\nu\rho} - 2\Gamma_{\mu\nu\rho}) H^{\pm\rho} \quad (\text{with } H^\pm_\mu := G_{\mu\nu} H^{\pm\nu}) \quad (3.11)$$

to $f_{\mu\nu}^\pm$ are trivial in the space of local functionals of the fields and antifields where $H^{\pm\mu}(X)$ are arbitrary functions of the X^μ and the upper (lower) sign refers to contributions to $f_{\mu\nu}^+$ ($f_{\mu\nu}^-$).

3.2 Antifield-dependent functionals

The existence and explicit form of antifield-dependent BRST-invariant functionals depends of course on the specific form of S_0 , i.e. on the specific choice of the functions $G_{\mu\nu}$ and $B_{\mu\nu}$ in (3.1) since they enter in the BRST transformation of X_μ^* and $g^{*\mu\nu}$, see (2.5). Nevertheless one can classify all antifield-dependent BRST-invariant functionals as follows:

⁴Notice that transformations $\delta\phi^i$ satisfying (3.5) are more general transformations than symmetry transformations of S_0 since the latter satisfy the much stronger condition $S_0[\phi + \delta\phi] = S_0[\phi]$.

a) $g \notin \{-1, 0, 1\}$:

There are no antifield-dependent cohomology classes in these cases, independently of the specific form of S_0 . More precisely: If W^g is an antifield-dependent local BRST-invariant functional with ghost number $g \notin \{-1, 0, 1\}$ then there is a local functional W^{g-1} such that $\tilde{W}^g := W^g - sW^{g-1}$ does not depend on antifields anymore.

b) $g = -1$:

BRST-invariant local functionals with ghost number -1 exist if and only if $G_{\mu\nu}$ and $B_{\mu\nu}$ admit a nonvanishing solution $f^\mu(X)$ of

$$\partial_\mu f_\nu + \partial_\nu f_\mu - 2\Gamma_{\mu\nu\rho} f^\rho = 0, \quad H_{\mu\nu\rho} f^\rho = \partial_\mu H_\nu - \partial_\nu H_\mu \quad (3.12)$$

for some arbitrary functions $H_\mu(X)$. (3.12) identifies $f_\mu := G_{\mu\nu} f^\nu$ as the components of a Killing vector in target space. Any solution of (3.12) generates a continuous *global* symmetry of S_0 through

$$\delta_\epsilon X^\mu = \epsilon f^\mu(X), \quad \epsilon = \text{const.} \quad (3.13)$$

In other words: BRST-invariant local functionals with ghost number -1 correspond one-to-one to these global symmetries of S_0 . They are given by

$$W^{-1} = \int d^2x X_\mu^* f^\mu(X). \quad (3.14)$$

c) $g = 0$:

BRST-invariant local functionals with ghost number 0 depending non-trivially on the antifields exist if and only if $G_{\mu\nu}$ and $B_{\mu\nu}$ admit a nonvanishing solution $f^\mu(X)$ of

$$0 = \partial_\mu f_\nu + \partial_\nu f_\mu - 2\Gamma_{\mu\nu\rho} f^\rho, \quad 0 = \partial_\mu f_\nu - \partial_\nu f_\mu \mp H_{\mu\nu\rho} f^\rho \quad (3.15)$$

where the second condition must be satisfied either with the $+$ or the $-$ sign for a particular solution f^μ . Comparing (3.15) and (3.12) we conclude that (3.15) requires that S_0 possesses a global symmetry (3.13) with the additional restriction imposed by the second condition (3.15). The BRST-invariant functionals with ghost number 0 arising from a solution of (3.15) with a minus sign in front of $H_{\mu\nu\rho} f^\rho$ are given by

$$\mathcal{M}_+ = \int d^2x \left[X_\mu^* (\partial_+ \xi^+ + h_{--} \partial_+ \xi^-) - \frac{2}{1-y} \nabla_+ X^\nu \partial_+ h_{--} G_{\mu\nu} \right] f^\mu \quad (3.16)$$

and the functionals \mathcal{M}_- arising from a solution of (3.15) with a plus sign in front of $H_{\mu\nu\rho} f^\rho$ are obtained from (3.16) by exchanging all $+$ and $-$ indices. These solutions do not satisfy $(\mathcal{M}, \mathcal{M}) = 0$ and can thus not be added to the extended action without breaking $(S, S) = 0$, however they can be used to introduce background charges as explained

in the introduction. Namely, taking $h_{++} = 0$, dropping the corresponding ξ^- ghost, and specialising to $G_{\mu\nu} = \delta_{\mu\nu}$, (3.16) becomes $\int d^2x (X_\mu^* \partial_+ \xi^+ - 2\partial_+ X_\mu \partial_+ h_{--}) f^\mu$ in which one recognises the so-called background charge terms (see [6] for their inclusion in the BV formalism). Therefore, (3.16) constitutes the generalization of this chiral gauge treatment.

d) $g = 1$:

In this case we obtain (3.2) as necessary and sufficient conditions for the existence of BRST-invariant local functionals with ghost number 1 depending nontrivially on the antifields. As discussed above, (3.2) implies that S_0 possesses an additional gauge symmetry which violates requirement (iii) and thus has to be excluded. Namely in presence of additional gauge symmetries, (2.4) is not a proper solution of the classical master equation anymore. To construct a proper solution one must introduce a ghost and its antifield for each additional gauge symmetry. In the extended space of functionals depending also on these additional fields, the antifield-dependent functionals arising from solutions g^μ of (3.2) indeed are trivial. Nevertheless one of course has to reexamine the whole investigation of $H^*(s)$ in the case of a higher gauge symmetry and therefore our results do not apply to this case.

4 Sketch of the computation

In the first step of the computation, the BRST cohomology in the space of local functionals $W = \int d^2x f$ is related to the BRST cohomology in the space of local functions by means of the descent equations following from $sW = 0$:

$$s\omega_2 + d\omega_1 = 0, \quad s\omega_1 + d\omega_0 = 0, \quad s\omega_0 = 0 \quad (4.1)$$

where $\omega_2 = d^2x f$ is the integrand of W written as a 2-form and ω_1 and ω_0 are local 1- and 0-forms. It is well-known that the descent equations terminate in gravitational theories always with a nontrivial 0-form ω_0 (contrary to the Yang–Mills case)⁵ and that their “integration” is trivial:

$$\omega_1 = b\omega_0, \quad \omega_2 = \frac{1}{2}bb\omega_0, \quad b = dx^\alpha \frac{\partial}{\partial \xi^\alpha}. \quad (4.2)$$

According to these statements which were first proved and applied in [8] (for arbitrary dimensions) it is sufficient to determine the general solution of

$$s\omega_0 = 0 \quad (4.3)$$

⁵This statement holds in a strict sense only in absence of closed p -forms ($p \neq 0$) which are not exact, cf. [9].

in the space of local functions of the fields and their derivatives. The BRST-invariant functionals resp. their integrands are then obtained via (4.2) from the solutions of (4.3).

The investigation of (4.3) is considerably simplified by performing it in an appropriate new basis of variables substituting the fields, antifields and their derivatives. The construction of this new basis is the second and crucial step within the computation. The new basis contains in particular the following variables $T_{m,n}^\mu$ substituting one-by-one the partial derivatives $(\partial_+)^m(\partial_-)^n X^\mu$ of the matter fields:

$$T_{m,n}^\mu = \left(\frac{\partial}{\partial c^+} s \right)^m \left(\frac{\partial}{\partial c^-} s \right)^n X^\mu \quad (4.4)$$

where c^+ and c^- are the ghost variables defined in (3.10) and it is understood that the BRST transformations occurring in (4.4) are expressed in terms of these ghosts. The first few (and most important) T 's are given by

$$T_{0,0}^\mu = X^\mu, \quad T_{1,0}^\mu = \frac{1}{1-y} \nabla_+ X^\mu, \quad T_{0,1}^\mu = \frac{1}{1-y} \nabla_- X^\mu$$

with $\nabla_\pm X^\mu$ as in (3.10). The most important ghost variables are

$$c_+^n = \frac{1}{(n+1)!} (\partial_+)^{n+1} c^+, \quad c_-^n = \frac{1}{(n+1)!} (\partial_-)^{n+1} c^-, \quad n \geq -1. \quad (4.5)$$

The remarkable property of the $T_{m,n}^\mu$ is that they span the representation space for two copies of the ‘‘Virasoro algebra’’ (without central extension) whose associated ghosts are just the variables (4.5). Namely the BRST transformations of $T_{m,n}^\mu$ and c_\pm^n can be written as

$$s T_{m,n}^\mu = \sum_{k \geq -1} (c_+^k L_k^+ + c_-^k L_k^-) T_{m,n}^\mu, \quad s c_\pm^n = \frac{1}{2} f_{mn}{}^k c_\pm^m c_\pm^n \quad (4.6)$$

where L_n^+ and L_n^- represent on the $T_{m,n}^\mu$ the Virasoro algebra according to

$$[L_m^\pm, L_n^\pm] = f_{mn}{}^k L_k^\pm, \quad [L_m^+, L_n^-] = 0, \quad f_{mn}{}^k = (m-n) \delta_{m+n}^k \quad (m, n, k \geq -1). \quad (4.7)$$

$L_k^\pm T_{m,n}^\mu$ can be evaluated using (4.7) and

$$T_{m,n}^\mu = \left(L_{-1}^+ \right)^m \left(L_{-1}^- \right)^n X^\mu, \quad L_n^\pm X^\mu = 0 \quad \forall n \geq 0. \quad (4.8)$$

The equivalence of (4.4) and the first relation (4.8) can be verified using $s^2 = 0$ and the following representation of L_n^\pm on $T_{m,n}^\mu$ which is implied by (4.6):

$$L_n^+ = \left\{ s, \frac{\partial}{\partial c_+^n} \right\}, \quad L_n^- = \left\{ s, \frac{\partial}{\partial c_-^n} \right\}, \quad n \geq -1. \quad (4.9)$$

The $T_{m,n}^\mu$ are called tensor fields. In fact one can extend the definition of tensor fields to the antifields. Of particular importance are those tensor fields which substitute X_μ^* . They are given by

$$\hat{X}_\mu^* = \frac{1}{1-y} X_\mu^*. \quad (4.10)$$

In the third step one proves by means of standard methods that nontrivial contributions to solutions of (4.3), written in terms of the new basis, depend on the fields, antifields and their derivatives only via the c_{\pm}^n and the tensor fields constructed of the matter fields and the antifields since all other variables group into trivial systems of the form (a, sa) and do not enter in (4.6).

In step four we take advantage of the fact that L_0^+ , L_0^- are diagonal on all tensor fields and on the ghosts (4.5) on which these generators are defined by means of (4.9). Namely one has

$$L_0^+ T_{m,n}^{\mu} = m T_{m,n}^{\mu}, \quad L_0^- T_{m,n}^{\mu} = n T_{m,n}^{\mu}, \quad L_0^{\pm} c_{\pm}^n = n c_{\pm}^n, \quad L_0^{\pm} c_{\mp}^n = 0 \quad (4.11)$$

and similar relations for the tensor fields constructed of the antifields (e.g. \hat{X}_{μ}^* has (L_0^+, L_0^-) weights $(1, 1)$). By means of standard arguments one concludes that solutions of (4.3) can be assumed to have total weight $(0, 0)$ (all other contributions to ω_0 are trivial).

The fifth and final step consists in the investigation of (4.3) in the space of those local functions of the ghosts (4.5) and the tensor fields which have total weight zero under both L_0^+ and L_0^- . It turns out that $c_+^{-1} = c^+$ and $c_-^{-1} = c^-$ are the only variables having negative weights under L_0^+ or L_0^- . In fact they have weights $(-1, 0)$ and $(0, -1)$ respectively. Since the ghosts anticommute, there are only few possibilities to construct local functions with total weight $(0, 0)$ at all. In fact the whole computation reduces to the investigation of functions of the following quantities:

$$\begin{aligned} c_{\pm}^0, \quad \tilde{c}_{\pm} &\equiv 2c_{\pm}^{-1}c_{\pm}^1, \quad X^{\mu} = T_{0,0}^{\mu}, \quad T_+^{\mu} \equiv c_+^{-1}T_{1,0}^{\mu}, \quad T_-^{\mu} \equiv c_-^{-1}T_{0,1}^{\mu}, \\ T_{+-}^{\mu} &\equiv c_-^{-1}c_+^{-1}T_{1,1}^{\mu}, \quad T_{\mu}^* \equiv c_+^{-1}c_-^{-1}\hat{X}_{\mu}^* \end{aligned} \quad (4.12)$$

on which s acts according to

$$\begin{aligned} s c_{\pm}^0 &= \tilde{c}_{\pm}, \quad s X^{\mu} = T_+^{\mu} + T_-^{\mu}, \quad s T_+^{\mu} = T_{+-}^{\mu}, \quad s T_-^{\mu} = -T_{+-}^{\mu}, \\ s T_{\mu}^* &= 2G_{\mu\nu}T_{+-}^{\nu} + (H_{\nu\rho\mu} - 2\Gamma_{\nu\rho\mu})T_+^{\nu}T_-^{\rho} \end{aligned} \quad (4.13)$$

where sT_{μ}^* follows from (2.5) and thus of course requires the knowledge of (3.1) which is obtained from the solution of the antifield independent problem. Taking into account the algebraic identities relating the quantities (4.12) as a consequence of the odd grading of c^{\pm} , like $\tilde{c}_+\tilde{c}_+ = T_+^{\mu}\tilde{c}_+ = T_+^{\mu}T_+^{\nu} = 0$ etc., one sees that the space of nonvanishing functions of these quantities is rather small apart from the occurrence of arbitrary functions of $T_{0,0}^{\mu} = X^{\mu}$. This allows ultimately to solve (4.3) completely. The solution of (4.3) which yields (3.1) is for instance given by $T_+^{\mu}T_-^{\nu}K_{\mu\nu}(X)$ where the symmetric and antisymmetric parts of $K_{\mu\nu}$ are just $G_{\mu\nu}$ resp. $B_{\mu\nu}$.

5 Summary

We have determined the complete BRST-antibracket cohomology in the space of local functionals for theories satisfying the assumptions (i)–(iv) listed in section 2. We found that nontrivial cohomology classes exist only for ghost numbers $g = -1, \dots, 4$. The representatives of the cohomology classes with $g = 1, \dots, 4$ can be chosen such that they do not depend on antifields at all. Due to their special importance we summarize and comment only the results for $g = -1, 0, 1$ in detail.

The cohomology classes with $g = -1$ correspond one-to-one to the independent solutions $f^\mu(X)$ of (3.12) which can be interpreted as the Killing vectors in target space. Each of them generates a global symmetry of S_0 according to (3.13). The resulting BRST-invariant functionals are given by (3.14). This result is not surprising since it has been shown in [11] that the BRST cohomology classes with ghost number -1 correspond one-to-one to the independent nontrivial continuous global symmetries of the classical action which is part of a cohomological reformulation of Noether's theorem.

For $g = 0$ there are two types of cohomology classes. Those of the first type are represented by antifield-independent functionals and provide the most general classical action for models characterized by (i)–(iv). It is given by (3.1), with the understanding that two such actions are equivalent if they are related by a target space reparametrization (3.4). Representatives of cohomology classes of the second type depend nontrivially on the antifields. They correspond one-to-one to those Killing vectors $f^\mu(X)$ which satisfy (3.15). The corresponding BRST-invariant functionals are given by (3.16) (and an analogous expression for \mathcal{M}_-). They correspond to so-called background charges and might provide BRST-invariant functionals $\mathcal{M} = \mathcal{M}_+ + \mathcal{M}_-$ which, as remarked in the introduction, can be used in order to look for a consistent extension of the models or investigate an anomaly cancellation through background charges (these applications would require appropriate choices of $G_{\mu\nu}$, $B_{\mu\nu}$ and f^μ).

The cohomology classes with $g = 1$ represent candidate anomalies. One can distinguish two types of them. Representatives of the first type can be chosen to be independent of the matter fields. In fact there are precisely two inequivalent cohomology classes of this type, represented by the matter field independent functionals \mathcal{H}_+ and \mathcal{H}_- given in eq. (3.8) (contrary to slightly misleading formulations in [12] which give the impression that there is only one cohomology class represented by a special linear combination of \mathcal{H}_+ and \mathcal{H}_-). Candidate anomalies of the second type depend nontrivially on the matter fields and are represented by the functionals (3.9). A functional (3.9) is cohomologically trivial if and only if $f_{\mu\nu}^\pm$ have the form (3.11). All other functionals (3.9) are BRST invariant and cohomological nontrivial in the complete space of local functionals of fields and antifields. This result

corrects a statement given in [13] where the authors claim that matter field dependent contributions to BRST-invariant functionals with ghost number 1 can be always removed by adding trivial contributions. It is worth noting in this context that in fact both types of anomalies arise in a generic model. The requirement that the matter field dependent anomalies vanish at the one-loop level imposes the target space “equations of motion” for $G_{\mu\nu}$ and $B_{\mu\nu}$, vanishing of the matter field independent anomalies fixes the target space dimension to $D = 26$, as discussed e.g. in [1] (the quantities corresponding to the symmetric and antisymmetric parts of $f_{\mu\nu}^{\pm}$ and to a_{\pm} occurring in the sum $\mathcal{H}_+ + \mathcal{H}_-$ for $a_+ = a_-$ are in the second ref. [1] denoted by $\beta_{\mu\nu}^G$, $\beta_{\mu\nu}^B$ and β^{Φ} respectively).

Finally we point out that the absence of anomaly candidates depending nontrivially on the antifields is a general feature of all models characterized by (i)–(iv) and represents a remarkable difference to the situation in Yang–Mills and Einstein–Yang–Mills theories with a gauge group containing at least two abelian factors if the classical action has at least one nontrivial global symmetry [14].

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