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New supersymmetries for spinning particles and black holes[1]

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Abstract

The usual extensions of supersymmetry require the existence of a complex structure and the formulation of the theory on Kähler manifolds. It is shown, that by relaxing the constraints on the algebra of supercharges we can get new supersymmetries whenever a manifold possesses a structure admitting the existence of a Killing-Yano tensor field. Examples of such manifolds are the Kerr-Newman space-times describing spinning black holes in four dimensions.

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1 Introduction

Spinning black holes in four-dimensional space-time, electrically neutral or charged, are described by the Kerr and Kerr-Newman solutions of the pure Einstein and coupled Einstein-Maxwell equations, respectively. An explicit form of these static solutions, of mass M, charge e and angular momentum $J = Ma$, is represented by the line-element

$$\begin{aligned} ds^2 &= -\frac{\Delta}{\rho^2} \left(dt - a \sin^2 \theta d\phi \right)^2 + \frac{\sin^2 \theta}{\rho^2} \left((r^2 + a^2) d\phi - a dt \right)^2 \\ &\quad + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2, \end{aligned} \tag{1}$$

where we have used the abbreviations

$$\rho^2 = r^2 + a^2 \cos^2 \theta, \qquad \Delta = r^2 + a^2 - 2Mr + e^2 > 0. \tag{2}$$

In case of non-vanishing charge e the corresponding electro-magnetic field is described by the Maxwell 2-form

$$\begin{aligned} F &= \frac{e}{\rho^4} \left(r^2 - a^2 \cos^2 \theta \right) dr \wedge \left(dt - a \sin^2 \theta d\phi \right) \\ &\quad + 2 \frac{ear \cos \theta \sin \theta}{\rho^4} d\theta \wedge \left((r^2 + a^2) d\phi - a dt \right). \end{aligned} \tag{3}$$

It was discovered by Carter [1] that the geodesic equations for the four-dimensional space-time geometry described by this metric are completely integrable in the sense of Liouville. Closely connected to this is the existence of a constant of motion Z for a point-particle moving in Kerr-Newman space-time, of the form

$$Z = \frac{1}{2} K_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu, \qquad \dot{Z} = 0. \tag{4}$$

The overdot here denotes proper-time differentiation, and the symmetric tensor field $K_{\mu\nu}$ is a Killing-tensor, implying that its completely symmetrized covariant derivative vanishes:

$$K_{(\mu\nu;\lambda)} = 0. \tag{5}$$

In the co-ordinate system (1) the explicit form of Z is

$$\begin{aligned} Z &= \frac{\Delta \cos^2\theta}{\rho^2} \left(\dot{t} - a \sin^2\theta \dot{\phi} \right)^2 + \frac{r^2 \sin^2\theta}{\rho^2 a^2} \left((r^2 + a^2)\dot{\phi} - a\dot{t} \right)^2 \\ & \quad - \frac{\rho^2 \cos^2\theta}{\Delta} \dot{r}^2 + \frac{\rho^2}{r^2} a^2 \dot{\theta}^2. \end{aligned} \tag{6}$$

Note that the symmetric quadratic differential $K_{\mu\nu} dx^\mu dx^\nu = 2Z d\tau^2$ is *different* from the line-element ds^2; the constant of motion corresponding to the latter is the world-line hamiltonian H.

In correpondence with these classical results is the observation by the authors of [2, 3] that the Klein-Gordon and Dirac equations in the curved space-time (1) are separable. For the Klein-Gordon equation this follows from the conservation of the operator corresponding to Z, eq.(4). In the case of the Dirac equation it was established in [4, 5] that there exists a square root of the Killing tensor $K_{\mu\nu}$:

$$K_{\mu\nu} = \eta_{ab} f_\mu^{\ a} f_\nu^{\ b}, \tag{7}$$

which is anti-symmetric after contraction with the vierbein:

$$f_{\mu\nu} \equiv f_\mu^{\ a} e_{\nu a} = -f_{\nu\mu}, \tag{8}$$

and such that one can construct from it a linear differential operator which anti-commutes with the Dirac operator. This result depends crucially on the complete anti-symmetry of the covariant derivative of $f_{\mu\nu}$:

$$H_{\mu\nu\lambda} = f_{[\mu\nu;\lambda]} = f_{\mu\nu;\lambda}. \tag{9}$$

Anti-symmetric tensors of this kind are known as Killing-Yano tensors; their appearance is not restricted to the spinning black-hole solutions of general relativity. In the following we will see that they are closely connected to the existence of certain new types of supersymmetries, of which the spinning black holes provide only one example, though an interesting one. A systematic analysis of these supersymmetries is presented in [6].

2 Spinning particles as supersymmetric σ-models

In [7, 8] it was shown how Killing vectors and tensors related to symmetries of manifolds, and their fermionic extensions for spinning manifolds, can be obtained from the geodesic equations of motion for point particles and spinning particles in curved background space-times. The Lie-algebra of the Killing vector and tensor fields is reflected in the algebra of Poisson-Dirac brackets of the constants of motion of these physical systems. The starting point is the lagrangian for a spinning particle in a gravitational field

$$L = \frac{1}{2}\, g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + \frac{i}{2}\, \eta_{ab} \psi^a \frac{D\psi^b}{D\tau}. \tag{10}$$

The action is invariant under the supersymmetry

$$\delta x^\mu = -i\epsilon\psi^\mu, \qquad \delta\psi^\mu = \dot{x}^\mu \epsilon, \tag{11}$$

where we use the notation $\psi^\mu(x,\psi) = e^\mu_{\ a}\psi^a$, and $x^\mu(\tau)$ represents the position variables, whilst the Grassmann-odd variables $\psi^a(\tau)$, transforming as a local Lorentz vector, describe the spin.

The equations of motion obtained from the variation of the lagrangian (10) are

$$\frac{D^2 x^\mu}{D\tau^2} = \ddot{x}^\mu - \Gamma_{\lambda\nu}^{\ \ \mu}\, \dot{x}^\lambda \dot{x}^\nu = -\frac{i}{2}\, \psi^a \psi^b\, R_{ab\ \nu}^{\ \ \mu} \dot{x}^\nu, \tag{12}$$

$$\frac{D\psi^a}{D\tau} = \dot{\psi}^a - \dot{x}^\mu \omega_{\mu\ b}^{\ a}\, \psi^b = 0, \tag{13}$$

with $\omega_{\mu\ b}^{\ a}$ the spin connection and $R_{ab\ \nu}^{\ \ \mu}$ the Riemann tensor. In terms of the spin tensor $S^{ab} = -i\psi^a\psi^b$ (an anti-symmetric local Lorentz tensor) this becomes

$$\frac{D^2 x^\mu}{D\tau^2} = \frac{1}{2}\, S^{ab} R_{ab\ \nu}^{\ \ \mu} \dot{x}^\nu, \qquad \frac{DS^{ab}}{D\tau} = 0. \tag{14}$$

The above theory can be recast in canonical form; we introduce the momentum

$$p_\mu = g_{\mu\nu}\dot{x}^\nu + \omega_\mu, \tag{15}$$

and hamiltonian

$$H = \frac{1}{2}\, g^{\mu\nu} \left(p_{\mu} - \omega_{\mu} \right) \left(p_{\nu} - \omega_{\nu} \right), \tag{16}$$

in which

$$\omega_{\mu} = \frac{1}{2}\, \omega_{\mu ab} S^{ab} \tag{17}$$

is the Grassmann-valued spin connection. In terms of the Poisson-Dirac brackets

$$\left\{ F, G \right\} = \frac{\partial F}{\partial x^{\mu}} \frac{\partial G}{\partial p_{\mu}} - \frac{\partial F}{\partial p_{\mu}} \frac{\partial G}{\partial x^{\mu}} + i (-1)^{a_F}\, \frac{\partial F}{\partial \psi^{a}} \frac{\partial G}{\partial \psi_{a}}, \tag{18}$$

with a_F the Grassmann parity of F, the equations of motion for an arbitrary dynamical quantity F can now be written as

$$\dot{F} = \left\{ F, H \right\}. \tag{19}$$

With this defintion of the brackets it also follows, that the spin tensor satisfies the Lorentz algebra

$$\left\{ S^{ab}, S^{cd} \right\} = \eta^{ad} S^{bc} + \eta^{bc} S^{ad} - \eta^{ac} S^{bd} - \eta^{bd} S^{ac}. \tag{20}$$

This confirms the interpretation of S^{ab} as the spin: it generates the 'internal' part of the Lorentz transformations on the phase space spanned by $(x^{\mu}, p_{\mu}, \psi^{a})$.

3 Covariant phase space

Results and calculations for the spinning particle theory can be simplified by introducing a covariant phase space formulation, obtained by changing to the covariant momentum

$$\Pi_{\mu} = p_{\mu} - \omega_{\mu} = g_{\mu\nu} \dot{x}^{\nu}. \tag{21}$$

In terms of this variable the hamiltonian reads

$$H = \frac{1}{2}\, g^{\mu\nu} \Pi_{\mu} \Pi_{\nu}, \tag{22}$$

whilst the Poisson-Dirac brackets take the form

$$\{F, G\} \,=\, \mathcal{D}_\mu F \frac{\partial G}{\partial \Pi_\mu} \,-\, \frac{\partial F}{\partial \Pi_\mu} \mathcal{D}_\mu G \,+\, R_{\mu\nu} \frac{\partial F}{\partial \Pi_\mu} \frac{\partial G}{\partial \Pi_\nu} \,+\, i(-1)^{a_F}\, \frac{\partial F}{\partial \psi^a} \frac{\partial G}{\partial \psi_a}. \quad (23)$$

Here

$$\mathcal{D}_\mu F \,=\, \partial_\mu F \,+\, \Gamma_{\mu\nu}^{\;\;\;\lambda} \Pi_\lambda \frac{\partial F}{\partial \Pi_\nu} \,+\, \omega_{\mu ab} \psi^b \frac{\partial F}{\partial \psi_a}. \quad (24)$$

For scalar observables F, G the covariant brackets always automatically give covariant results. For the covariant momenta we obtain the classical version of the Ricci identity:

$$\{\Pi_\mu, \Pi_\nu\} \,=\, R_{\mu\nu} \,=\, \frac{1}{2}\, S^{ab} R_{ab\mu\nu}. \quad (25)$$

Since in this formulation it is advantageous to work only with scalar quantities, we observe that these can always be obtained from any tensorial expression by saturating the indices with either covariant momenta Π_μ or spin variables ψ^a, depending on whether one needs to symmetrize or anti-symmetrize. Thus all quantities we consider are of the form

$$F(x, \Pi, \psi) \,=\, \sum_{m,n \geq 0} \frac{i^{\left[\frac{m}{2}\right]}}{m!n!}\, \psi^{a_1} ... \psi^{a_m}\, f_{a_1 ... a_m}^{\mu_1 ... \mu_n}(x)\, \Pi_{\mu_1} ... \Pi_{\mu_n}. \quad (26)$$

One may think of these scalar functions as generalized differential forms on a graded phase space.

4 Symmetries

In the hamiltonian formulation, symmetry transformations are generated by the constants of motion through the Poisson-Dirac brackets. In particular, the supersymmetry transformations (11) are obtained from the conserved supercharge

$$Q = \Pi \cdot \psi = e^\mu_{\;\;a}\, \Pi_\mu \psi^a, \qquad \dot{Q} = 0, \quad (27)$$

by taking the bracket

$$\delta F = i\epsilon \{Q, F\}. \tag{28}$$

That Q is conserved and the super-transformations (28) represent a symmetry follows from the bracket relations

$$\{Q, Q\} = -2iH, \qquad \{Q, H\} = 0. \tag{29}$$

The second relation, which follows from the first by the Jacobi identity, implies at the same time the conservation of Q and the invariance of H under the transformations (28).

After the pattern established for the supercharge Q, we can now investigate the full set of symmetries for a given space-time by solving the equation

$$\{J, H\} = \Pi^\mu \left(\mathcal{D}_\mu J + R_{\mu\nu} \frac{\partial J}{\partial \Pi_\nu} \right) = 0, \tag{30}$$

which give all constants of motion $J(x, \Pi, \psi)$. This equation is the generalization of the usual Killing equation to spinning space [7, 8]. However, unlike the usual case, in which the solutions of the Killing equation are single completely symmetric tensors, here the solutions consist of linear combinations of symmetric tensors of different rank:

$$J(x, \Pi, \psi) = \sum_{n \geq 0} \frac{1}{n!} J^{\mu_1 \dots \mu_n}(x, \psi) \Pi_{\mu_1} \dots \Pi_{\mu_n}, \tag{31}$$

subject to the conditions

$$J_{(\mu_1 \dots \mu_n;\mu_{n+1})} + \omega_{(\mu_{n+1}}^{\;\;ab} \psi_b \frac{\partial J_{\mu_1 \dots \mu_n)}}{\partial \psi^a} = -\frac{i}{2} \psi^a \psi^b R_{ab\,(\mu_{n+1}}^{\;\;\;\;\;\;\;\nu} J_{\mu_1 \dots \mu_n)\nu}. \tag{32}$$

A sufficient, though not necessary, condition for a solution of the generalized Killing equations is superinvariance of a dynamical variable:

$$\{J, Q\} = \psi \cdot \mathcal{D} J + i\Pi \cdot \frac{\partial J}{\partial \psi} = 0. \tag{33}$$

This equation may be considered as a kind of square root of the generalized Killing equation. The new supersymmetries we present later satisfy this superinvariance condition. A noteworthy exception is the supercharge Q

itself; according to (29) its bracket with Q gives the hamiltonian, which generates proper-time translations.

The solutions of the generalized Killing equation (30) are of two distinct types: *generic* ones, which exist for any spinning particle model (10), and *non-generic* ones, which depend on the specific background space-time considered. To the first class belong supersymmetry and proper-time translations, generated by the supercharge and hamiltonian, respectively. In addition there also is a 'chiral' symmetry, generated by the conserved charge

$$\Gamma_* \,=\, -\frac{i^{\left[\frac{d}{2}\right]}}{d!}\, \varepsilon_{a_1 \ldots a_d}\, \psi^{a_1} \ldots \psi^{a_d}, \tag{34}$$

and a dual supersymmetry generated by

$$Q^* \,=\, i\, \{Q, \Gamma_*\} \,=\, -\frac{i^{\left[\frac{d}{2}\right]}}{(d-1)!}\, e^{\mu a_1}\, \Pi_\mu\, \varepsilon_{a_1 \ldots a_d}\, \psi^{a_2} \ldots \psi^{a_d}. \tag{35}$$

Note that Q^* is Grassmann odd in even-dimensional space-times and Grassmann even in odd-dimensional space-times. In the special case $d = 2$ dual supersymmetry is a real supersymmetry, in the sense that the bracket of Q^* with itself closes on the hamiltonian. For all $d > 2$ this bracket vanishes identically.

5 New supersymmetries

The existence of non-generic symmetries depends by definition on the background space-time considered. We now ask, what are the necessary conditions for the existence of new supersymmetries such that

$$\delta x^\mu \,=\, -i\epsilon\, f^\mu_{\ a}(x) \psi^a, \tag{36}$$

with $f^\mu_{\ a}$ some vector not equal to the vierbein $e^\mu_{\ a}$. It is straightforward to establish that the solution to this problem is the existence of a constant of motion

$$Q_f \,=\, f^\mu_{\ a}\, \Pi_\mu \psi^a \,+\, \frac{i}{3!}\, c_{abc}\, \psi^a \psi^b \psi^c, \tag{37}$$

with the tensorial quantities $f^\mu_{\ a}$ and c_{abc} subject to

$$
\begin{aligned}
D_\mu f_\nu{}^a + D_\nu f_\mu{}^a &= 0, \\
D_\mu c_{abc} + R_{\mu\nu ab} f^\nu{}_c + R_{\mu\nu bc} f^\nu{}_a + R_{\mu\nu ca} f^\nu{}_b &= 0.
\end{aligned}
\tag{38}
$$

These conditions express the contents of the generalized Killing equation for Q_f. The existence of a new supersymmetry of this kind then implies automatically the existene of a new Grassmann-even constant of motion Z, defined by the bracket of Q_f with itself:

$$
\{Q_f, Q_f\} = -2iZ. \tag{39}
$$

The explicit form of Z is

$$
Z = \frac{1}{2} K^{\mu\nu} \Pi_\mu \Pi_\nu + I^\mu \Pi_\mu + G, \tag{40}
$$

with

$$
\begin{aligned}
K^{\mu\nu} &= K^{\nu\mu} = \eta_{ab} f^{\mu a} f^{\nu b}, \\
I^\mu &= \frac{i}{2} \psi^a \psi^b \left(2 f^\nu{}_b D_\nu f^\mu{}_a + f^{\mu c} c_{abc} \right), \\
G &= -\frac{1}{4} \psi^a \psi^b \psi^c \psi^d \left(R_{\mu\nu ab} f^\mu{}_c f^\nu{}_d + \frac{1}{2} c_{ab}{}^e c_{cde} \right).
\end{aligned}
\tag{41}
$$

Since Q_f satisfies the generalized Killing equation, and therefore

$$
\{Q_f, H\} = 0, \tag{42}
$$

the bracket relation (39) in combination with the Jacobi identity imply the conservation of Z:

$$
\{Z, H\} = 0. \tag{43}
$$

Therefore Z is a solution of the generalized Killing equations as well, and its components satisfy

$$
\begin{aligned}
K_{(\mu\nu;\lambda)} &= 0, \\
D_{(\mu} I_{\nu)ab} &= R_{ab\lambda(\mu} K_{\nu)}^{\;\;\lambda}, \\
D_\mu G_{abcd} &= R_{\lambda\mu[ab}\, I_{cd]}^{\;\;\lambda},
\end{aligned}
\tag{44}
$$

where the square brackets in the last expression denote anti-symmetrization over the latin indices enclosed.

6 Killing-Yano tensors

We now impose the requirement of the independence of the new supersymmetry by requiring it to anti-commute with ordinary supersymmetry, or

$$
\{Q, Q_f\} = 0. \tag{45}
$$

As a direct consequence, we find that the second-rank tensor $f^{\mu\nu} = f^{\mu a} e^{\nu}_{\;\;a}$ is anti-symmetric:

$$
f^{\mu\nu} + f^{\nu\mu} = 0. \tag{46}
$$

In addition, the first Killing equation (38) implies that

$$
f_{\mu\nu;\lambda} + f_{\lambda\nu;\mu} = 0. \tag{47}
$$

Combining the two results yields the complete anti-symmetry of the covariant derivative:

$$
\begin{aligned}
f_{\mu\nu;\lambda} &= \frac{1}{3} \left(f_{\mu\nu;\lambda} + f_{\nu\lambda;\mu} + f_{\lambda\mu;\nu} \right) \\
&= H_{\mu\nu\lambda},
\end{aligned}
\tag{48}
$$

the field strength of the anti-symmetric tensor $f_{\mu\nu}$. It follows that $f_{\mu\nu}$ is a Killing-Yano tensor.

Further differentiation of $H_{\mu\nu\lambda}$ then leads to the result

$$
H_{\mu\nu\lambda;\kappa} = \frac{1}{2} \left(R_{\nu\lambda\mu}^{\quad\;\sigma} f_{\sigma\kappa} + R_{\lambda\kappa\mu}^{\quad\;\sigma} f_{\sigma\nu} + R_{\kappa\nu\mu}^{\quad\;\sigma} f_{\sigma\lambda} \right). \tag{49}
$$

Comparing with the second Killing equation (38) we conclude, that it is solved by taking

$$c_{abc} = -\frac{1}{2} H_{abc}, \tag{50}$$

the local Lorentz 3-form corresponding to the field strength tensor. Thus, given a Killing-Yano tensor $f_{\mu\nu}$, such an anti-symmetric 3rd rank tensor always exists.

We therefore conclude that the existence of a Killing-Yano tensor is both a necessary and a sufficient condition for the existence of a new supersymmetry of the type (37), obeying the condition (45).

7 Kerr-Newman geometry

In the introduction we presented the uniformly rotating Kerr-Newman solution of the Einstein equations, and mentioned the existence of a Kiling-Yano tensor in this space-time; its explicit components are

$$\begin{aligned} f_{\mu}^{\ 0}\, dx^{\mu} &= \frac{\rho}{\Delta} \cos\theta\, dr, \\ f_{\mu}^{\ 1}\, dx^{\mu} &= -\frac{\sqrt{\Delta}}{\rho} \cos\theta \left(dt - a \sin^2\theta\, d\phi \right), \\ f_{\mu}^{\ 2}\, dx^{\mu} &= -\frac{r \sin\theta}{a\rho} \left((r^2 + a^2) d\phi - a dt \right), \\ f_{\mu}^{\ 3}\, dx^{\mu} &= -\frac{r\rho}{a}\, d\theta. \end{aligned} \tag{51}$$

The corresponding components of the Lorentz 3-form c_{abc} obtained from the field strength are:

$$
\begin{aligned}
c_{012} &= \frac{2\sin\theta}{\rho}, \\
c_{123} &= -\frac{2\sqrt{\Delta}}{a\rho}, \qquad (52)\\
c_{013} &= c_{023} = 0.
\end{aligned}
$$

Hence the orbit of a spinning particle moving in a Kerr-Newman space-time is characterized by a new supersymmetry of the type (37). Its physical content is, that the projection of the spin in the direction obtained by rotating the four velocity $\dot{x}^\mu$ by $f_\mu{}^a(x)$ is constant.

8 N = 2 supersymmetry

As another check on the previous results we can reproduce the well-known conditions for the existence of a conventional $N = 2$ supersymmetry, which is an independent supersymmetry generated by a charge $\tilde{Q}$ for which the algebra closes on the hamiltonian:

$$
\left\{Q, \tilde{Q}\right\} = 0, \qquad \left\{\tilde{Q}, \tilde{Q}\right\} = -2iH. \qquad (53)
$$

Since the bosonic constant of motion Z now coincides with H, we have

$$
K^{\mu\nu} = g^{\mu\nu}, \qquad (54)
$$

and therefore

$$
f^\mu{}_a f_\nu{}^a = \delta^\mu_\nu. \qquad (55)
$$

The anti-commutativity of the two independend supercharges requires the anti-symmetry of $f^{\mu\nu} = f^{\mu a} e^\nu{}_a$ as before. As a result we can rewrite eq.(55) in the form

$$
f^\mu{}_\lambda f^\lambda{}_\nu = -\delta^\mu_\nu. \qquad (56)
$$

Moreover, since the covariant hamiltonian contains no explicit ψ^4-terms, there is no ψ^3-term in $\tilde{Q}$:

$$c_{abc} = 0, \qquad \Rightarrow \qquad f^{\mu b} f_{\nu\ ;\mu}^{\ a} = f^{\mu b} f_{\nu\ ;\mu}^{\ a}. \tag{57}$$

Thus the existence of $N = 2$ supersymmetry requires an (almost) complex structure $f^{\mu\nu}$, and this restricts the manifolds on which the models are defined to be of Kähler type.

9 Quantum theory

Up until now we have presented the discussion of new supersymmetries in the context of (pseudo) classical mechanics and geometry. The construction can be carried over straightforwardly to the case of quantum mechanics, by the usual replacement of phase-space co-ordinates by operators and Poisson-Dirac brackets by (anti) commutators. In particular, we have

$$\begin{aligned} \psi^a &\rightarrow \sqrt{\frac{2}{\hbar}}\, \gamma^a, \\ \Pi_\mu &\rightarrow -i\hbar D_\mu = -i\hbar \left(\partial_\mu - \frac{1}{2} \omega_{\mu ab} \sigma^{ab} \right). \end{aligned} \tag{58}$$

Here γ^a are the usual Dirac γ-matrices, and σ^{ab} 1/4 times their commutators. In terms of these operators the supercharge is replaced by the Dirac operator

$$\sqrt{\frac{2}{\hbar}}\, Q \rightarrow -i\hbar \gamma \cdot D, \tag{59}$$

the dot denoting contraction with a vierbein, and the Hamiltonian becomes a Laplacian, essentially the square of the Dirac operator.

The new supersymmetry charge is now replaced similarly by the operator

$$\sqrt{\frac{2}{\hbar}}\, Q_f \rightarrow -i\hbar \gamma^a \left(f^{\mu}_{\ a} D_\mu - \frac{1}{3!}\, c_{abc} \sigma^{bc} \right) \equiv -i\hbar \gamma \cdot F. \tag{60}$$

The independence of the symmetry transformations generated by these operators on the spinor fields is expressed by the condition

$$\left[\gamma \cdot D, \gamma \cdot F \right]_+ = 0. \tag{61}$$

The anti-commutator can be rewritten as a commutator by multiplication with γ_5. Therefore $\gamma \cdot F$ corresponds to an operator which can be diagonalized on solutions of the Dirac equation. This is the origin of the separability of the Dirac equation in Kerr-Newman geometry observed in [3].

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