NIKHEF-H 93–22

Antifield Dependence of Anomalies

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Abstract

It is shown that generally the consistency equation for anomalies of quantum field theories has solutions which depend nontrivially on the sources of the (generalized) BRS-transformations of the fields. Explicit previously unknown examples of such solutions are given for Yang-Mills and super Yang-Mills theories.

*Supported by Deutsche Forschungsgemeinschaft

1 Introduction

Gauge symmetries are of central importance in our present theories of fundamental interactions. If there is no regularization procedure which respects all symmetries of the classical theory it is not guaranteed that these symmetries survive the quantization of the theory, i.e. the theory may turn out to be anomalous. The BRS-formalism [5, 3] allows to characterize anomalies as solutions of the so-called consistency equation which generalizes the Wess-Zumino consistency conditions [17]. This makes possible an algebraic classification of anomaly candidates without referring to a particular regularization scheme. A generalization of the BRS-formalism to symmetries whose algebra does not necessarily close off-shell is given by the BV-antifield-formalism [1] whose forerunner has been formulated in [12]. Surprisingly it took quite a long time until anomalies have been discussed in this formalism [16] although the BV-formulation allows to characterize anomalies analogously as solutions of a consistency equation which follows from the anomalous Slavnov–Taylor–Ward identity for the effective action [14].

The consistency equation is most conveniently written in terms of the fields and antifields and it can be shown that each of its nontrivial solutions contains an antifield independent part which characterizes and determines it almost completely [15]. Nevertheless in general the complete solution depends on the antifields resp. on the sources of the (generalized) BRS-transformations of the fields. The main point I want to make in this paper is to show that this dependence is generally nontrivial, whether the algebra of the classical symmetries closes off-shell or not. Namely by means of an explicit example I show that even in the simple and phenomenologically important case of four dimensional renormalizable Yang-Mills theories there are previously unknown solutions of the consistency equation which depend nontrivially on the sources (antifields), contrary to the common belief and to different and consequently erroneous statements which have been given in the literature [2].

Furthermore I discuss the connection between the BV- and the usual BRS-formulation of theories with symmetries whose algebra can be closed off-shell by means of appropriate auxiliary fields. In particular it is shown that solutions of the consistency equation which do not depend on the antifields in the formulation with auxiliary fields will generally (but not necessarily) depend on the antifields in the formulation without auxiliary fields. This result is exemplified for the case of abelian super-Yang-Mills theories where it yields alternative forms of recently found new anomaly candidates of these theories.

2 The consistency equation

I shall now briefly discuss the consistency equation in the BV-formalism. An anomaly shows up as a violation of the Slavnov-Taylor-Ward identity for the generating functional $\Gamma[\Phi, q, \zeta, b]$ of renormalized 1PI Green functions which depends on the classical fields and the ghosts, denoted collectively by Φ, as well as on the sources q of their generalized BRS-transformations, the antighosts ζ and the Lagrange multiplier fields b. Γ is constructed order by order in a loop expansion

$$\Gamma = \sum_{n\geq 0} \hbar^n \Gamma^{(n)} \tag{2.1}$$

where the tree functional $\Gamma^{(0)}$ contains the invariant classical action as well as gauge fixing and corresponding ghost contributions (see below). It is chosen such that Γ satisfies the Slavnov-Taylor identity in 0th order:

$$\mathcal{B}_{\Gamma^{(0)}} \Gamma^{(0)} = 0 \tag{2.2}$$

where $\mathcal{B}_{\Gamma^{(0)}}$ is the nilpotent operator

$$\mathcal{B}_{\Gamma^{(0)}} = \int d^D x \left(\frac{\delta \Gamma^{(0)}}{\delta q_A} \frac{\delta}{\delta \Phi^A} + \frac{\delta \Gamma^{(0)}}{\delta \Phi^A} \frac{\delta}{\delta q_A} + b^N \frac{\delta}{\delta \zeta^N} \right). \tag{2.3}$$

In the anomalous case (2.2) cannot be extended to all orders and the Slavnov-Taylor identity for Γ is violated by an anomaly $\mathcal{A}$ occurring at some order ℓ which is nonzero due to (2.2) but generally not known in advance:

$$\mathcal{B}_\Gamma \Gamma = \mathcal{A} = \sum_{n\geq \ell} \hbar^n \mathcal{A}^{(n)}, \quad \ell > 0. \tag{2.4}$$

The lowest order contribution $\mathcal{A}^{(\ell)}$ is a local functional and satisfies the consistency equation

$$\mathcal{B}_{\Gamma^{(0)}} \mathcal{A}^{(\ell)} = 0 \tag{2.5}$$

which follows from the identity $\mathcal{B}_\Gamma \mathcal{B}_\Gamma \Gamma = 0$ since the latter and (2.4) imply $\mathcal{B}_\Gamma \mathcal{A} = 0$ whose lowest order contribution is just (2.5). Trivial contributions $\mathcal{B}_{\Gamma^{(0)}} X$ can be removed from $\mathcal{A}^{(\ell)}$ by subtracting the counterterm $\hbar^\ell X$ from Γ. In particular $\mathcal{A}^{(\ell)}$ itself can be assumed to be nontrivial since otherwise the anomaly can be removed up to terms of order $n > \ell$.

In the BV-formalism $\Gamma^{(0)}$ is constructed from the proper solution S of the so-called classical master equation which has the form

$$S = S[\Phi, \Phi^*] = S_{cl}[\phi] + \int d^D x \, \Phi^*_A R^A(\Phi) + O(2) \tag{2.6}$$

where $O(2)$ collects all terms which are at least bilinear in the antifields Φ^*, S_{cl} is the classical invariant action and the R^i generate its symmetries. Here $\{\Phi^A, \Phi^*_A\}$ denote collectively the minimal set of fields and antifields in the sense of [1] which consists of the classical fields ϕ^i, the ghosts C^N and their respective antifields[1]:

$$\{\Phi^A\} = \{\phi^i, C^N\}, \quad \{\Phi^*_A\} = \{\phi^*_i, C^*_N\}. \tag{2.7}$$

In order to construct $\Gamma^{(0)}$ one first adds the term $\zeta^*_N b^N$ to the integrand of S and then one fixes the gauge by means of an appropriate fermionic functional $\Psi[\Phi, \zeta, b]$ with ghost number -1:

$$\Gamma^{(0)} = \left(S[\Phi, \Phi^*] + \int d^D x\, \zeta^*_N b^N \right)\Big|_{\Sigma'}, \tag{2.8}$$

$$\Sigma' : \quad \Phi^*_A = q_A - (-)^{\varepsilon(\Phi^A)} \frac{\delta \Psi}{\delta \Phi^A}, \quad \zeta^*_N = -(-)^{\varepsilon(\zeta^N)} \frac{\delta \Psi}{\delta \zeta^N} \tag{2.9}$$

where $\varepsilon(Z)$ denotes the grading of Z (the signs occur since I use leftderivatives only). Straightforwardly one shows by means of standard methods that solutions of (2.5) depend on the fields ζ^N, b^N only trivially if written in terms of the variables $\{\Phi^A, \Phi^*_A, \zeta^N, b^N\}$ since in these variables the generalized BRS-transformations take the simple form

$$\mathcal{B}_{\Gamma^{(0)}} \Phi^A = \mathcal{B}_S \Phi^A, \quad \mathcal{B}_{\Gamma^{(0)}} \Phi^*_A = \mathcal{B}_S \Phi^*_A, \quad \mathcal{B}_{\Gamma^{(0)}} \zeta^N = b^N, \quad \mathcal{B}_{\Gamma^{(0)}} b^N = 0$$

where $\mathcal{B}_S$ is the operator

$$\mathcal{B}_S = \int d^D x \left(\frac{\delta S}{\delta \Phi^*_A} \frac{\delta}{\delta \Phi^A} + \frac{\delta S}{\delta \Phi^A} \frac{\delta}{\delta \Phi^*_A} \right). \tag{2.10}$$

As a result we obtain

$$\mathcal{A}^{(\ell)} = \mathcal{W}^1[\Phi, \Phi^*]|_{\Sigma'} + \mathcal{B}_{\Gamma^{(0)}} X[\Phi, q, \zeta, b] \tag{2.11}$$

where X is a local functional with ghost number 0 and $\mathcal{W}^1$ solves

$$\mathcal{B}_S \mathcal{W}^1[\Phi, \Phi^*] = 0 \tag{2.12}$$

which is the form of the consistency equation discussed in [15]. Thus (2.5) reduces to (2.12) since $\mathcal{B}_{\Gamma^{(0)}} X$ in (2.11) is a trivial contribution to $\mathcal{A}^{(\ell)}$. Furthermore contributions $\mathcal{B}_S Y[\Phi, \Phi^*]$ to solutions of (2.12) obviously correspond to trivial contributions $\mathcal{B}_{\Gamma^{(0)}} \hat{Y}$ to $\mathcal{A}^{(\ell)}$ where $\hat{Y} = Y|_{\Sigma'}$ and solve (2.12) since the master equation implies

[1] For simplicity only gauge theories are considered which are irreducible in the sense of [1] though everything extends straightforwardly to the reducible case as well.

$(\mathcal{B}_S)^2 = 0$. Therefore two solution of (2.12) are called equivalent if they differ by such trivial contributions:

$$\tilde{\mathcal{W}}^1[\Phi,\Phi^*] \cong \mathcal{W}^1[\Phi,\Phi^*] \;\Leftrightarrow\; \tilde{\mathcal{W}}^1[\Phi,\Phi^*] - \mathcal{W}^1[\Phi,\Phi^*] = \mathcal{B}_S Y[\Phi,\Phi^*]. \tag{2.13}$$

It can be shown [15] that each nontrivial solution of (2.12) has a nonvanishing antifield independent part $\mathcal{W}^1_0[\Phi]$ which satisfies

$$\mathcal{B}_1\,\mathcal{W}^1_0[\Phi] \sim 0, \quad \mathcal{W}^1_0[\Phi] \not\sim \mathcal{B}_1 X_0[\phi] \tag{2.14}$$

where $\mathcal{B}_1$ is an operator whose action on the Φ^A is defined by means of the part of (2.6) which is linear in the antifields,

$$\mathcal{B}_1\Phi^A = R^A(\Phi), \tag{2.15}$$

and $\sim$ denotes 'weak equality' defined according to

$$\mathcal{F}[\Phi] \sim \mathcal{G}[\Phi] \quad :\Leftrightarrow \quad \mathcal{F}[\Phi] - \mathcal{G}[\Phi] = \int d^Dx\,\frac{\delta S_{cl}[\phi]}{\delta\phi^i}\,Z^i(\Phi) \tag{2.16}$$

where $Z^i(\Phi)$ are arbitrary local functions of the Φ^A and their derivatives. Notice that (2.14) imposes only on-shell conditions on $\mathcal{W}^1_0$ since $\sim$ requires equality up to contributions which contain the classical equations of motion.

The importance of (2.14) consists in the fact that it represents a necessary and sufficient condition for the existence and nontriviality of the complete solution of (2.12). Namely each solution of (2.14) can be completed to a nontrivial solution

$$\mathcal{W}^1[\Phi,\Phi^*] = \mathcal{W}^1_0[\Phi] + O(1) \tag{2.17}$$

of (2.12) and each nontrivial solution of (2.12) contains a solution $\mathcal{W}^1_0[\Phi]$ of (2.14) [15]. This can be proved by means of a result about the cohomology of the so-called Koszul–Tate differential which holds under appropriate assumptions about the classical action and the gauge transformations [13].

3 Symmetries whose algebra closes off-shell

It is well-known that the BV-formalism reduces to the usual BRS-formalism for theories with symmetries whose algebra closes off-shell. Let us briefly recall this fact. In the case of an off-shell closing algebra one can define a BRS-operator s which is off-shell nilpotent on the classical fields and the ghosts. The solution of the master equation then takes the simple form

$$S = S_{cl}[\phi] + \int d^Dx\,\Phi^*_A\,s\Phi^A \tag{3.1}$$

and (2.8) can be written in the familiar form

$$\Gamma^{(0)} = S_{cl}[\phi] + s\,\Psi[\Phi,\zeta,b] + \int d^Dx\, q_A\, s\Phi^A \tag{3.2}$$

where s acts on ζ^N and b^N according to $s\zeta^N = b^N$, $sb^N = 0$. Due to (3.2) both $\mathcal{B}_S\Phi^A$ and $\mathcal{B}_1\Phi^A$ agree with the usual nilpotent BRS-transformations:

$$\mathcal{B}_S\Phi^A = \mathcal{B}_1\Phi^A = s\Phi^A, \quad s^2\Phi^A = 0. \tag{3.3}$$

This implies in particular that each BRS-invariant functional $\mathcal{F}[\Phi]$ with ghost number 1 solves (2.12) and thus represents an anomaly candidate which does not depend on the antifields resp. on the sources of the BRS-transformations of the fields:

$$s\,\mathcal{W}^1[\Phi] = 0 \quad \Leftrightarrow \quad \mathcal{B}_S\mathcal{W}^1[\Phi] = 0. \tag{3.4}$$

(3.4) has been investigated extensively for various theories in the literature. Complete results have been derived for instance in [7] for Yang-Mills and Einstein-Yang-Mills theories and in [8, 9, 10] for a class of globally and locally supersymmetric theories by means of methods which can be generalized to a large class of gauge theories [11].

However it is still an open question in which cases the solutions of (3.4) cover already the complete space of solutions of (2.12). In order to show that this is generally not the case I give an explicit example of a source dependent solution of (2.12) in Yang–Mills theory which is not equivalent to a solution of (3.4).

Example: I consider a four dimensional renormalizable abelian Yang–Mills theory defined by the following integrand of a solution of the master equation:

$$\mathcal{L} = \sum_I(-\tfrac{1}{4}\,F_{ab}{}^I F^{abI} + A_I^{*a}\partial_a C^I) + i\sum_j \overline{\Psi}^j\gamma^a\mathcal{D}_a\Psi^j + \sum_{jI} C^I(\Psi_j^*\delta_I\Psi^j + \overline{\Psi}_j^*\delta_I\overline{\Psi}^j) \tag{3.5}$$

where $F_{ab}{}^I = \partial_a A_b{}^I - \partial_b A_a{}^I$ are the abelian field strengths, C^I are the abelian ghost fields and $\{\Psi^j\}$ is a set of fermions in Dirac bi-spinor notation ($\overline{\Psi} = \Psi^\dagger\gamma^0$). δ_I denotes the generator of the Ith $U(1)$-factor and $\mathcal{D}_a$ denotes the covariant derivatives:

$$\delta_I\Psi^j = ig_I^j\Psi^j, \quad \delta_I\overline{\Psi}^j = -ig_I^j\overline{\Psi}^j, \quad \mathcal{D}_a = \partial_a - \sum_I A_a{}^I\delta_I$$

where g_I^j is the charge of Ψ^j under δ_I. One may check that the following functional solves (2.12) and is not equivalent to a solution of (3.4):

$$\mathcal{W}^1 = \sum_{jIJ} k_{IJ}\int d^4x\,\left(C^I A_a{}^J\overline{\Psi}^j\gamma^a\gamma_5\Psi^j + \tfrac{i}{2}\,C^I C^J(\Psi_j^*\gamma_5\Psi^j + \overline{\Psi}^j\gamma_5\overline{\Psi}_j^*)\right) \tag{3.6}$$

where k_{IJ} are antisymmetric constants:

$$k_{IJ} = -k_{JI}. \tag{3.7}$$

Of course the example can be extended by coupling the Ψ to nonabelian gauge fields as well. Notice that due to (3.7) the anomaly candidates (3.6) occur only if the gauge group contains at least two abelian factors. I remark that the nontrivial dependence of (3.6) on the antifields originates in the occurrence of γ_5 in (3.6). Since $\mathcal{L}$ does not contain γ_5-dependent contributions one of course does not expect the presence of an anomaly which corresponds to (3.6) in this simple model but it is not excluded that similar anomaly candidates exist in more complicated theories.

4 Elimination of auxiliary fields

Often one can close an only on-shell closing algebra also off-shell by means of an appropriate set of auxiliary fields. However on the one hand auxiliary fields enlarge the field content unnecessarily in the BV-formalism and on the other hand it is in practice often difficult to find a set of auxiliary fields. Therefore it is instructive to compare the formulations of a theory with and without auxiliary fields. To this end we denote by Φ^A the classical fields and ghosts which occur in the BV-formulation without auxiliary fields and denote the latter by H^r. In the formulation with auxiliary fields the solution of the master equation has the form (3.1):

$$\hat{S}[\Phi,\Phi^*,H,H^*] = \hat{S}_{cl}[\Phi,H] + \int d^Dx\, (\Phi^*_A\, s\Phi^A + H^*_r\, sH^r) \tag{4.1}$$

where some of the nilpotent BRS-transformations $s\Phi^A$ of course depend on the auxiliary fields. As the defining property of the auxiliary fields we require that the 'equations of motion' for the H^r which follow from (4.1) after setting to zero H^*_r have an algebraic solution $\hat{H}^r(\Phi,\Phi^*)$:

$$0 = \frac{\delta\hat{S}[\Phi,\Phi^*,H,0]}{\delta H^r} \quad\Leftrightarrow\quad H^r = \hat{H}^r(\Phi,\Phi^*). \tag{4.2}$$

Notice that this definition of auxiliary fields differs slightly from the usual one which requires only that $\delta\hat{S}_{cl}[\Phi,H]/\delta H^r$ can be solved algebraically for the H^r. Our definition is motivated by the fact that the elimination of the auxiliary fields from (4.1) by means of (4.2) provides directly the BV-formulation of the theory since

$$S[\Phi,\Phi^*] := \hat{S}[\Phi,\Phi^*,\hat{H}(\Phi,\Phi^*),0] \tag{4.3}$$

solves the master equation. This result is contained in the following lemma:

Auxiliary lemma: If a local functional $\hat{\mathcal{F}}[\Phi, \Phi^*, H, H^*]$ is $\mathcal{B}_{\hat{S}}$-invariant then the functional which arises from it for $H^r = \hat{H}^r(\Phi, \Phi^*)$, $H^*_r = 0$ is $\mathcal{B}_S$-invariant:

$$\mathcal{B}_{\hat{S}}\hat{\mathcal{F}}[\Phi, \Phi^*, H, H^*] = 0 \quad \Rightarrow \quad \mathcal{B}_S\hat{\mathcal{F}}[\Phi, \Phi^*, \hat{H}(\Phi, \Phi^*), 0] = 0 \tag{4.4}$$

where $\mathcal{B}_S$ denotes the operator (2.10) arising from (4.3) and $\mathcal{B}_{\hat{S}}$ denotes the analogous operator arising from (4.1) ($\mathcal{B}_{\hat{S}}$ contains functional derivatives with respect to H^r and H^*_r).

Proof: In order to prove this lemma we introduce the notation

$$G[\Phi, \Phi^*, H, H^*]| := G[\Phi, \Phi^*, \hat{H}(\Phi, \Phi^*), 0]$$

where G denotes an arbitrary functional of the Φ, Φ^*,H,H^*. (4.4) is proved as follows:

$$\begin{aligned} 0 &= \int d^Dx \left((\mathcal{B}_{\hat{S}}\Phi^A)\frac{\delta\hat{\mathcal{F}}}{\delta\Phi^A} + (\mathcal{B}_{\hat{S}}\Phi^*_A)\frac{\delta\hat{\mathcal{F}}}{\delta\Phi^*_A} + (\mathcal{B}_{\hat{S}}H^r)\frac{\delta\hat{\mathcal{F}}}{\delta H^r} + (\mathcal{B}_{\hat{S}}H^*_r)\frac{\delta\hat{\mathcal{F}}}{\delta H^*_r}\right)\Bigg| \\ &= \int d^Dx \left((\mathcal{B}_S\Phi^A)\frac{\delta\hat{\mathcal{F}}}{\delta\Phi^A} + (\mathcal{B}_S\Phi^*_A)\frac{\delta\hat{\mathcal{F}}}{\delta\Phi^*_A} + (\mathcal{B}_S\hat{H}^r)\frac{\delta\hat{\mathcal{F}}}{\delta H^r}\right)\Bigg| = \mathcal{B}_S(\hat{\mathcal{F}}|) \end{aligned}$$

where the first row is obtained from writing out $\mathcal{B}_{\hat{S}}\hat{\mathcal{F}}[\Phi, \Phi^*, H, H^*] = 0$ explicitly and the second row follows from the first due to

$$(\mathcal{B}_{\hat{S}}\Phi^A)| = \mathcal{B}_S\Phi^A, \quad (\mathcal{B}_{\hat{S}}\Phi^*_A)| = \mathcal{B}_S\Phi^*_A, \quad (\mathcal{B}_{\hat{S}}H^r)| = \mathcal{B}_S H^r(\Phi, \Phi^*), \quad (\mathcal{B}_{\hat{S}}H^*_r)| = 0 \tag{4.5}$$

which hold due to (4.2). □

As mentioned above, the lemma implies in particular that (4.3) solves the master equation since $\hat{S}$ is $\mathcal{B}_{\hat{S}}$-invariant by assumption and the master equation can be written in the form

$$\mathcal{B}_S S = 0. \tag{4.6}$$

(4.3) and (4.5) are often useful in themselves since they facilitate the construction of the solution of the master equation and the generalized BRS-transformations considerably if a formulation of the theory with auxiliary fields is known. For instance by means of (4.3) and (4.5) one can easily reproduce the results given in [4] for N=1, D=4 supergravity and super-Yang-Mills theories (analogously to the example discussed below).

Example: The outlined procedure is now exemplified for abelian D=4, N=1 super-Yang-Mills theories whose classical Lagrangian reads in the formulation with auxiliary fields

$$\begin{aligned}\hat{\mathcal{L}}_{cl} &= \sum_I(-\tfrac{1}{4}\,F_{ab}{}^I F^{abI} - i\,\lambda^I\sigma^a\partial_a\bar{\lambda}^I + \tfrac{1}{2}\,D^I D^I) \\ &\quad + \sum_j(-\bar{\varphi}^j\mathcal{D}_a\mathcal{D}^a\varphi^j - i\,\chi^j\sigma^a\mathcal{D}_a\bar{\chi}^j + F^j\bar{F}^j) + V(\varphi,\chi,F,\bar{\varphi},\bar{\chi},\bar{F}) \\ &\quad + \sum_{jI}(-iD^I\bar{\varphi}^j\delta_I\varphi^j + \sqrt{2}\,\lambda^I\chi^j\delta_I\bar{\varphi}^j + \sqrt{2}\,\bar{\lambda}^I\bar{\chi}^j\delta_I\varphi^j)\end{aligned} \tag{4.7}$$

where $F_{ab}{}^I$ denote as in the previous section abelian field strengths, λ^I_α denote the gauginos, D^I are the real auxiliary fields of the super-Yang-Mills multipletts, φ^j and χ^j_α are component fields of chiral matter multipletts and F^j are the corresponding complex auxiliary fields. Contrary to the previous section, a two component Weyl spinor notation is used in (4.7) in which λ and $\bar{\lambda}$ are related just by complex conjugation[2]. Again δ_I denotes the generator of the Ith $U(1)$-factor and $\mathcal{D}_a$ denotes the covariant derivatives

$$\delta_I\varphi^j = ig^j_I\varphi^j, \quad \delta_I\bar{\varphi}^j = -ig^j_I\bar{\varphi}^j, \quad \mathcal{D}_a = \partial_a - \sum_I A_a{}^I\delta_I$$

(the charges of χ^j and F^j are also given by g^j_I). V denotes contributions obtained from a δ_I-invariant superpotential $f(\varphi)$ according to

$$V(\varphi,\chi,F,\bar{\varphi},\bar{\chi},\bar{F}) = \mathcal{D}^\alpha\mathcal{D}_\alpha f(\varphi) + \bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}}\bar{f}(\bar{\varphi}) \tag{4.8}$$

where $\mathcal{D}_\alpha$ and $\bar{\mathcal{D}}_{\dot{\alpha}}$ are spinor transformations defined by

$$\begin{array}{lll}\mathcal{D}_\alpha\varphi^j = \sqrt{2}\,\chi^j_\alpha, & \mathcal{D}_\alpha\chi^j_\beta = \sqrt{2}\,\varepsilon_{\beta\alpha}F^j, & \mathcal{D}_\alpha F^j = 0, \\ \bar{\mathcal{D}}_{\dot{\alpha}}\bar{\varphi}^j = \sqrt{2}\,\bar{\chi}^j_{\dot{\alpha}}, & \bar{\mathcal{D}}_{\dot{\alpha}}\bar{\chi}^j_{\dot{\beta}} = \sqrt{2}\,\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{F}^j, & \bar{\mathcal{D}}_{\dot{\alpha}}\bar{F}^j = 0.\end{array} \tag{4.9}$$

The nilpotent BRS-transformations under which $\hat{S}_{cl} = \int d^4x\hat{\mathcal{L}}_{cl}$ is invariant read

$$sA_a{}^I = \partial_a C^I + i\lambda^I\sigma\bar{\xi} - i\xi\sigma_a\bar{\lambda}^I + C^b\partial_b A_a{}^I, \tag{4.10}$$

$$s\lambda^I_\alpha = -i\xi_\alpha D^I + \sigma^{ab}{}_\alpha{}^\beta\xi_\beta F_{ab}{}^I + C^a\partial_a\lambda^I_\alpha, \tag{4.11}$$

$$sD^I = \partial_a\lambda^I\sigma^a\bar{\xi} + \xi\sigma^a\partial_a\bar{\lambda}^I + C^a\partial_a D^I, \tag{4.12}$$

$$s\varphi^j = \sqrt{2}\,\xi\chi^j + C^I\delta_I\varphi^j + C^a\partial_a\varphi^j, \tag{4.13}$$

$$s\chi^j_\alpha = \sqrt{2}\,\xi_\alpha F^j - \sqrt{2}\,i\bar{\xi}^{\dot{\alpha}}\mathcal{D}_{\alpha\dot{\alpha}}\varphi^j + C^I\delta_I\chi^j_\alpha + C^a\partial_a\chi^j_\alpha, \tag{4.14}$$

[2]The conventions are the same as in [9].

$$
\begin{aligned}
sF^j &= -\sqrt{2}\, i\mathcal{D}_a\chi^j\sigma^a\bar{\xi} + 2\bar{\xi}\bar{\lambda}^I\delta_I\varphi^j + C^I\delta_I F^j + C^a\partial_a F^j, & (4.15)\\
sC^I &= -2i\xi\sigma^a\bar{\xi}A_a{}^I + C^a\partial_a C^I, & (4.16)\\
sC^a &= 2i\xi\sigma^a\bar{\xi}, & (4.17)\\
s\xi^\alpha &= 0 & (4.18)
\end{aligned}
$$

where C^I are anticommuting Yang-Mills ghosts, C^a denote constant anticommuting ghosts of translations and ξ^α, $\bar{\xi}^{\dot{\alpha}}$ are constant commuting supersymmetry ghosts. Nontrivial solutions of (3.4) are given by

$$
\begin{aligned}
\hat{\mathcal{W}}^1_{chir} &= \sum_{IJK} d_{IJK}\int d^4x\, \Big(\varepsilon^{abcd}\{C^I F_{ab}{}^J F_{cd}{}^K + 2iA_a{}^I F_{bc}{}^J[\xi\sigma_d\bar{\lambda}^K - \lambda^K\sigma_d\bar{\xi}]\}\\
&\qquad + 3i\,\{\xi\lambda^I\bar{\lambda}^J\bar{\lambda}^K + \bar{\xi}\bar{\lambda}^I\lambda^J\lambda^K\}\Big), & (4.19)\\
\hat{\mathcal{W}}^1_{fi} &= \sum_{IJ} k_{IJ}\int d^4x\, \Big(C^I D^J + \xi\sigma^a\bar{\lambda}^I A_a{}^J + \lambda^I\sigma^a\bar{\xi}A_a{}^J\Big) & (4.20)
\end{aligned}
$$

where the coefficients d_{IJK} in (4.19) are totally symmetric and the coefficients k_{IJ} in (4.20) are antisymmetric:

$$
d_{IJK} = d_{(IJK)}, \quad k_{IJ} = -k_{JI}. \tag{4.21}
$$

(4.19) are supersymmetric versions of abelian chiral anomalies which have been derived in this or a similar form for instance in [6, 8, 10]. The solutions (4.20) have been found in [8]. Notice that as the example of the previous section their presence requires at least two abelian factors due to the antisymmetry of k_{IJ}.

By means of the procedure outlined above one easily constructs the BV-formulation of the theory without antifields. To this end one eliminates the auxiliary fields D^I and F^j according to (4.2) which yields in this case:

$$
\hat{D}^I = \sum_j i\,\bar{\varphi}^j\delta_I\varphi^j + i\,\xi\lambda^*_I + i\,\bar{\xi}\bar{\lambda}^*_I, \quad \hat{F}^j = 4\,\frac{\partial\bar{f}(\bar{\varphi})}{\partial\bar{\varphi}^j} + \sqrt{2}\,\bar{\xi}\bar{\chi}^*_j \tag{4.22}
$$

where $\lambda^*_{I\alpha}$ and $\bar{\lambda}^{*\dot{\alpha}}_I$ are the antifields of $\lambda^{I\alpha}$ and $\bar{\lambda}^I_{\dot{\alpha}}$ ($\lambda^{*\alpha}_I$ and $\bar{\lambda}^{*\dot{\alpha}}_I$ are related by complex conjugation) and $\bar{\chi}^{*\dot{\alpha}}_j$ is the antifield of $\bar{\chi}^j_{\dot{\alpha}}$. Notice that the chiral anomalies (4.19) do not depend on auxiliary fields at all. They therefore keep their form and in particular do not depend on antifields even in the BV-formulation without auxiliary fields. This is different in the case of the anomaly candidates (4.20) since they depend on the auxiliary fields D^I and thus give rise to the following antifield dependent solution of (2.12) in the formulation without auxiliary fields:

$$
\mathcal{W}^1_{fi} = \int d^4x \sum_{IJ} k_{IJ}(\xi\sigma^a\bar{\lambda}^I A_a{}^J + \lambda^I\sigma^a\bar{\xi}A_a{}^J + i\,C^I\sum_j\bar{\varphi}^j\delta_J\varphi^j + i\,C^I\xi\lambda^*_J + i\,C^I\bar{\xi}\bar{\lambda}^*_J). \tag{4.23}
$$

One can check the nontriviality of (4.23) by making shure that there is no local counterterm $X[\phi]$ such that $\mathcal{B}_1 X \sim \mathcal{W}^1_{fi}|_{\lambda^*=\bar{\lambda}^*=0}$. I remark that both (4.19) and (4.20) have locally supersymmetric extensions which therefore provide anomaly candidates of supergravity [10].

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