NIKHEF-H 93–12
ITP–UH 07/93

Anomaly candidates and invariants of D=4, N=1 supergravity theories

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Abstract

All anomaly candidates and the form of the most general invariant local action are given for old and new minimal supergravity, including the cases where additional Yang–Mills and chiral matter multiplets are present. Furthermore nonminimal supergravity is discussed. In this case local supersymmetry itself may be anomalous and some of the corresponding anomaly candidates are given explicitly. The results are obtained by solving the descent equations which contain the consistency equation satisfied by integrands of anomalies and invariant actions.

*Supported by Deutsche Forschungsgemeinschaft

1 Introduction

For a large class of gauge theories one can construct a nilpotent BRS operator s and use it to characterize invariant actions and anomaly candidates as nontrivially BRS-invariant local functionals of the fields [1], [2], i.e. as solutions $\mathcal{W}^g$ of the so-called consistency equation

$$s\,\mathcal{W}^g = 0, \quad \mathcal{W}^g \neq s\,\Gamma^{g-1} \tag{1.1}$$

where $\mathcal{W}^g$, Γ^{g-1} are local functionals of the fields whose superscript denotes their ghost number. Invariant actions are solutions $\mathcal{W}^0$ with ghost number 0, anomaly candidates are solutions $\mathcal{W}^1$ with ghost number 1 (in the case $g = 1$ (1.1) contains the consistency conditions which anomalies have to satisfy and which have been first derived for Yang–Mills theories [3]).

The present paper contains results of an investigation of (1.1) for old minimal supergravity [5], new minimal supergravity [6] and nonminimal supergravity [7], [8] where in all cases not only pure supergravity is considered but its coupling to Yang–Mills multiplets and chiral matter multiplets is included. I remark that the results for old and new minimal supergravity are complete i.e. in these cases there are no solutions $\mathcal{W}^0$ and $\mathcal{W}^1$ of (1.1) apart from those given here (modulo trivial solutions $s\Gamma^{-1}$ resp. $s\Gamma^0$)[1]. However it is not the subject of this paper to prove this statement since the proof can be performed in more generality [9]. Instead the present paper spells out the general results of [9] explicitly for the supergravity theories mentioned above. This includes the solution of the so-called descent equations which follow from (1.1) and read

$$0 < p \leq 4: \quad s\,\omega_p^{g+4-p} + d\,\omega_{p-1}^{g+5-p} = 0, \quad s\,\omega_0^{g+4} = 0 \tag{1.2}$$

where $d = dx^m\partial_m$ is the exterior derivative, ω_p^{g+4-p} denotes a local p–form with ghost number $g + 4 - p$ and ω_4^g is the integrand of the solution of (1.1):

$$\mathcal{W}^g = \int \omega_4^g. \tag{1.3}$$

In a more compact form (1.2) reads

$$\tilde{s}\,\tilde{\omega}^{g+4} = 0, \quad \tilde{s} = s + d \tag{1.4}$$

where $\tilde{\omega}^{g+4}$ denotes the formal sum of the forms ω_p^{g+4-p}

$$\tilde{\omega}^{g+4} = \sum_{p=0}^{4} \omega_p^{g+4-p}. \tag{1.5}$$

The discussion of (1.2) resp. (1.4) will give insight into the method used to solved (1.1). Namely in fact the solutions of (1.4) have been calculated first and then the solutions of (1.1) have been obtained from them via (1.3). This method takes advantage of the fact that, as one can prove for a large class of gauge theories [10], each solution of (1.4) can be written—up to trivial contributions of the form $\tilde{s}\tilde{\omega}^{g+3}$—entirely in terms of tensor fields and certain variables which generalize the ordinary connection forms. This holds in particular for the supergravity theories discussed here where the variables which generalize the connection forms are given in (2.20) and (3.32) and the set of tensor fields is given in (3.7) where $\{\Psi^l\}$ in the case of minimal supergravity is given by (3.1) and in the case of nonminimal supergravity by the fields listed in (3.40) and the respective complex conjugated fields:

$$\tilde{\omega}^{g+4} = \tilde{\omega}^{g+4}(\tilde{\xi}^A, \tilde{C}^I, \tilde{Q}, \Phi^r). \tag{1.6}$$

The paper is organized as follows: In section 2 the BRS operator for supergravity is given (the BRS transformations given in this paper agree with those derived in [11]), section 3 recalls the basic features

[1] The results given for nonminimal supergravity are incomplete.

and the field content of old, new and nonminimal supergravity and contains the solutions of (1.1) and (1.4) for $g=0,1$, and in section 4 it is verified that $\tilde{s}\tilde{\omega}^{g+4}=0$ holds for the expressions given in section 3 for $\tilde{\omega}^{g+4}$. Finally a short conclusion is given, followed by three appendices.

The conventions concerning grading, complex conjugation, σ–matrices etc. agree with those used in [12] and as there I use exclusively the component formalismus, i.e. none of the fields appearing in this paper denotes a superfield.

2 BRS operator

The BRS operator for supergravity is conveniently constructed by means of the algebra

$$[\mathcal{D}_A,\mathcal{D}_B\}=-T_{AB}{}^C\mathcal{D}_C-F_{AB}{}^I\delta_I,\quad [\delta_I,\mathcal{D}_A]=-g_{IA}{}^B\mathcal{D}_B,\quad [\delta_I,\delta_J]=f_{IJ}{}^K\delta_K \tag{2.1}$$

which is realized on tensor fields. $\{\mathcal{D}_A\}$ consists of the covariant derivatives $\mathcal{D}_a$ and supersymmetry transformations $\mathcal{D}_\alpha$, $\bar{\mathcal{D}}_{\dot\alpha}=(\mathcal{D}_\alpha)^*$. $\{\delta_I\}$ denotes a real basis of the Liealgebra $\mathcal{G}$ of the structure group which in general is the direct sum of the Lorentz algebra $\mathcal{G}_L$, a further semisimple Liealgebra $\mathcal{G}_s$ and abelian part $\mathcal{G}_a$. The Lorentz generators are denoted by $l_{ab}=-l_{ba}$, the elements of $\mathcal{G}_s+\mathcal{G}_a$ by δ_i.

$$\{\mathcal{D}_A\}=\{\mathcal{D}_a,\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\alpha}\},\quad \{\delta_I\}=\{\delta_i,l_{ab}:a>b\},\quad \delta_i\in\mathcal{G}_s+\mathcal{G}_a,\quad l_{ab}\in\mathcal{G}_L.$$

In (2.1) the structure constants of $\mathcal{G}$ are denoted by $f_{IJ}{}^K$ and the $g_{IA}{}^B$ are the entries of the matrices g_I which represent $\mathcal{G}$ on the $\mathcal{D}_A$. The only nonvanishing $g_{IA}{}^B$ occur for $\delta_I\in\{l_{ab},\delta_{(w)},\delta_{(r)}\}$ where $\delta_{(w)}$ and $\delta_{(r)}$ denote the generators of Weyl transformations and of $U(1)$ transformations called R-transformations:

$$\begin{array}{lll}[l_{ab},\mathcal{D}_c]=\eta_{bc}\mathcal{D}_a-\eta_{ac}\mathcal{D}_b, & [l_{ab},\mathcal{D}_\alpha]=-\sigma_{ab\,\alpha}{}^\beta\mathcal{D}_\beta, & [l_{ab},\bar{\mathcal{D}}_{\dot\alpha}]=\bar\sigma_{ab}{}^{\dot\beta}{}_{\dot\alpha}\bar{\mathcal{D}}_{\dot\beta},\\ [\delta_{(r)},\mathcal{D}_a]=0, & [\delta_{(r)},\mathcal{D}_\alpha]=-i\,\mathcal{D}_\alpha, & [\delta_{(r)},\bar{\mathcal{D}}_{\dot\alpha}]=i\,\bar{\mathcal{D}}_{\dot\alpha},\\ [\delta_{(w)},\mathcal{D}_a]=-\mathcal{D}_a, & [\delta_{(w)},\mathcal{D}_\alpha]=-\frac12\,\mathcal{D}_\alpha, & [\delta_{(w)},\bar{\mathcal{D}}_{\dot\alpha}]=-\frac12\,\bar{\mathcal{D}}_{\dot\alpha}.\end{array} \tag{2.2}$$

The gauge fields corresponding to $\mathcal{D}_a$, $\mathcal{D}_\alpha$, $\bar{\mathcal{D}}_{\dot\alpha}$, l_{ab} and δ_i are the components $e_m{}^a$, $\psi_m{}^\alpha$, $\bar\psi_m{}^{\dot\alpha}$, $\omega_m{}^{ab}$, $A_m{}^i$ of the vierbein, the gravitino and its complex conjugate, the spin connection and the gauge fields of $\mathcal{G}_s+\mathcal{G}_a$ respectively. They are used to define the covariant derivatives of a tensor field Φ as in [13] by

$$\mathcal{D}_a\,\Phi=E_a{}^m(\partial_m-\psi_m{}^{\underline{\alpha}}\mathcal{D}_{\underline{\alpha}}-\mathcal{A}_m{}^I\delta_I)\,\Phi\ \Leftrightarrow\ \partial_m\,\Phi=(\mathcal{A}_m{}^A\mathcal{D}_A+\mathcal{A}_m{}^I\delta_I)\,\Phi \tag{2.3}$$

where ∂_m denote the partial derivatives, $E_a{}^m$ are the entries of the inverse vielbein

$$E_a{}^m e_m{}^b=\delta_a^b,\quad e_m{}^a E_a{}^n=\delta_m^n,$$

and the following notation and summation conventions are used in order to simplify the notation:

$$\{\mathcal{A}_m{}^A\}=\{e_m{}^a,\ \psi_m{}^\alpha,\ -\bar\psi_m{}^{\dot\alpha}\},\quad \{\mathcal{A}_m{}^I\}=\{\omega_m{}^{ab},\ A_m{}^i\}, \tag{2.4}$$

$$\mathcal{A}_m{}^A X_A=e_m{}^a X_a+\psi_m{}^{\underline{\alpha}}X_{\underline{\alpha}},\quad \psi_m{}^{\underline{\alpha}}X_{\underline{\alpha}}=\psi_m{}^\alpha X_\alpha-\bar\psi_m{}^{\dot\alpha}X_{\dot\alpha}. \tag{2.5}$$

Requiring $[\partial_m,\partial_n]\Phi=0$ one obtains from (2.3) by means of (2.1)

$$F_{ab}{}^I=2E_a{}^m E_b{}^n(\partial_{[m}\mathcal{A}_{n]}{}^I+\tfrac12 f_{JK}{}^I\mathcal{A}_m{}^J\mathcal{A}_n{}^K+e_{[m}{}^c\psi_{n]}{}^{\underline{\alpha}}F_{\underline{\alpha}c}{}^I+\tfrac12\psi_m{}^{\underline{\alpha}}\psi_n{}^{\underline{\beta}}F_{\underline{\alpha\beta}}{}^I), \tag{2.6}$$

$$T_{ab}{}^A=2E_a{}^m E_b{}^n(\partial_{[m}\mathcal{A}_{n]}{}^A+e_{[m}{}^c\psi_{n]}{}^{\underline{\alpha}}T_{\underline{\alpha}c}{}^A+\mathcal{A}_{[m}{}^B\mathcal{A}_{n]}{}^I g_{IB}{}^A+\tfrac12\psi_m{}^{\underline{\alpha}}\psi_n{}^{\underline{\beta}}T_{\underline{\alpha\beta}}{}^A) \tag{2.7}$$

i.e. the curvatures $F_{ab}{}^I$ and the torsions $T_{ab}{}^A$ are given in terms of the gauge fields, their partial derivatives and the remaining torsions and curvatures. The equation which determines $T_{ab}{}^c$ can also be solved for $\omega_m{}^{ab}$ and yields in particular:

$$\begin{aligned}&T_{ab}{}^c=0\ \Leftrightarrow\ \omega_m{}^{ab}=E^{an}E^{br}(\omega_{[mn]r}-\omega_{[nr]m}+\omega_{[rm]n}),\\ &\omega_{[mn]r}=e_{ra}(\partial_{[m}e_{n]}{}^a+e_{[m}{}^a A_{n]}{}^{(w)}+e_{[m}{}^b\psi_{n]}{}^{\underline{\alpha}}T_{\underline{\alpha}b}{}^a+\tfrac12\psi_m{}^{\underline{\alpha}}\psi_n{}^{\underline{\beta}}T_{\underline{\alpha\beta}}{}^a)\end{aligned} \tag{2.8}$$

where ${A_m}^{(w)}$ denote the gauge fields of Weyl transformations. The BRS operator is constructed by means of ghost fields associated with diffeomorphisms, supersymmetry and structure group transformations and denoted by C^m, ξ^α, $\bar\xi^{\dot\alpha}$ and C^I respectively (C^m, C^I are anticommuting and ξ^α, $\bar\xi^{\dot\alpha}$ are commuting ghosts). The BRS transformations read

$$\begin{aligned}
s\,\Phi &= (C^n\partial_n + \xi^\alpha\mathcal{D}_\alpha + \bar\xi^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha} + C^I\delta_I)\,\Phi, && (2.9)\\
s\,{e_m}^a &= C^n\partial_n {e_m}^a + (\partial_m C^n){e_n}^a + C^I {g_{Ib}}^a {e_m}^b - \xi^{\underline{\alpha}}{\mathcal{A}_m}^B {T_{B\underline{\alpha}}}^a, && (2.10)\\
s\,{\psi_m}^\alpha &= C^n\partial_n {\psi_m}^\alpha + (\partial_m C^n){\psi_n}^\alpha + D_m\xi^\alpha + C^I {g_{I\beta}}^\alpha {\psi_m}^\beta - \xi^{\underline{\beta}}{\mathcal{A}_m}^B {T_{B\underline{\beta}}}^\alpha, && (2.11)\\
s\,{\mathcal{A}_m}^I &= C^n\partial_n {\mathcal{A}_m}^I + (\partial_m C^n){\mathcal{A}_n}^I + D_m C^I - \xi^{\underline{\alpha}}{\mathcal{A}_m}^B {F_{B\underline{\alpha}}}^I, && (2.12)\\
s\,C^m &= C^n\partial_n C^m + \tfrac12 \xi^{\underline{\gamma}}\xi^{\underline{\beta}} {T_{\underline{\beta\gamma}}}^a {E_a}^m, && (2.13)\\
s\,\xi^\alpha &= C^n\partial_n \xi^\alpha + C^I {g_{I\beta}}^\alpha \xi^\beta + \tfrac12 \xi^{\underline{\gamma}}\xi^{\underline{\beta}} ({T_{\underline{\beta\gamma}}}^\alpha - {T_{\underline{\beta\gamma}}}^a {E_a}^m {\psi_m}^\alpha), && (2.14)\\
s\,C^I &= C^n\partial_n C^I + \tfrac12 {f_{KJ}}^I C^J C^K + \tfrac12 \xi^{\underline{\gamma}}\xi^{\underline{\beta}} ({F_{\underline{\beta\gamma}}}^I - {T_{\underline{\beta\gamma}}}^a {E_a}^m {\mathcal{A}_m}^I) && (2.15)
\end{aligned}$$

where $D_m\xi^\alpha$ and $D_m C^I$ are defined by

$$D_m\xi^\alpha = \partial_m\xi^\alpha - {\mathcal{A}_m}^I {g_{I\beta}}^\alpha\xi^\beta, \quad D_m C^I = \partial_m C^I + {\mathcal{A}_m}^J {f_{JK}}^I C^K$$

and the following summation convention is used (notice the different signs in (2.5) and (2.16))

$$\xi^{\underline{\alpha}} X_{\underline{\alpha}} = \xi^\alpha X_\alpha + \bar\xi^{\dot\alpha} X_{\dot\alpha}. \qquad (2.16)$$

(2.3), (2.6), (2.7) and (2.9)–(2.15) can be written in the following compact form

$$\begin{aligned}
\tilde s\,\Phi &= (\tilde\xi^A\mathcal{D}_A + \tilde C^I\delta_I)\,\Phi, && (2.17)\\
\tilde s\,\tilde\xi^A &= \tilde C^I {g_{IB}}^A \tilde\xi^B - \tfrac12(-)^{|B|}\tilde\xi^B\tilde\xi^C {T_{CB}}^A, && (2.18)\\
\tilde s\,\tilde C^I &= \tfrac12 {f_{KJ}}^I \tilde C^J \tilde C^K - \tfrac12(-)^{|A|}\tilde\xi^A\tilde\xi^B {F_{BA}}^I && (2.19)
\end{aligned}$$

where $|A|$ denotes the grading of $\mathcal{D}_A$ ($|a|=0$, $|\alpha|=|\dot\alpha|=1$), and $\tilde\xi^A$, $\tilde C^I$ are the generalized connection 'forms' mentioned in the introduction and defined by

$$\tilde\xi^a = \hat C^n {e_n}^a, \quad \tilde\xi^\alpha = \xi^\alpha + \hat C^n {\psi_n}^\alpha, \quad \tilde\xi^{\dot\alpha} = \bar\xi^{\dot\alpha} - \hat C^n {\bar\psi_n}{}^{\dot\alpha} \quad \tilde C^I = C^I + \hat C^n {\mathcal{A}_n}^I \qquad (2.20)$$

where $\hat C^n$ denotes the sum of the differential dx^n and the ghost C^n of diffeomorphisms:

$$\hat C^n = dx^n + C^n. \qquad (2.21)$$

Using (2.17)–(2.19) one may verify that

$$\tilde s^2 = 0 \quad \Leftrightarrow \quad s^2 = [s,\partial_m] = [\partial_m,\partial_n] = 0 \qquad (2.22)$$

holds by virtue of the algebra (2.1) and its Bianchi identities which have been discussed in [14], [8], [15] and read

$$\begin{aligned}
\sum_{ABC}\!\!\!\!\!\!\!\!\circlearrowright\;\; (-)^{|A|\,|C|}(\mathcal{D}_A {T_{BC}}^D + {T_{AB}}^E {T_{EC}}^D + {F_{AB}}^I {g_{IC}}^D) &= 0, && (2.23)\\
\sum_{ABC}\!\!\!\!\!\!\!\!\circlearrowright\;\; (-)^{|A|\,|C|}(\mathcal{D}_A {F_{BC}}^I + {T_{AB}}^D {F_{DC}}^I) &= 0 && (2.24)
\end{aligned}$$

where $\sum\!\!\!\!\!\circlearrowright$ denotes the cyclic sum.

In order to construct a gauge fixed action one introduces in addition an antighost field ζ and a 'Lagrange multiplier field' b for each gauge field and defines $s\zeta = b$, $sb = 0$ which implies that these fields contribute only trivially to solutions of (1.1). This completes the construction of the BRS algebra for supergravity theories whose field content consists of tensor fields, gauge fields ${\mathcal{A}_m}^A$, ${\mathcal{A}_m}^I$ and the corresponding ghosts, antighosts and b–fields. For supergravity theories containing additional fields one has to specify their BRS transformations as well. This will be done in subsection 3.2 for new minimal supergravity which contains antisymmetric gauge potentials, corresponding ghosts and a ghost for these ghosts.

I stress that the BRS operator is defined and nilpotent on all fields unlike the algebra (2.1) which is assumed to be realized only on tensor fields but not on the gauge fields or the ghosts.

3 Results

3.1 Old minimal supergravity

Apart from the gauge, ghost, antighost and b fields, the field content of old minimal supergravity with Yang–Mills and chiral matter multiplets consists of elementary tensor fields

$$\{\Psi^l\} = \{M, \bar{M}, B^a, \lambda^i_\alpha, \bar{\lambda}^i_{\dot{\alpha}}, D^i, \phi^s, \bar{\phi}^s, \chi^s_\alpha, \bar{\chi}^s_{\dot{\alpha}}, F^s, \bar{F}^s\} \tag{3.1}$$

where M,ϕ^s,F^s are complex Lorentz scalar fields ($\bar{M}$,$\bar{\phi}^s$,$\bar{F}^s$ denote the complex conjugated fields), B is a real Lorentz vector field, D^i are real Lorentz scalar fields and λ^i and χ^s are complex spinors. λ^i and D^i are the gauginos and auxiliary fields of the super Yang–Mills multiplets of $\mathcal{G}_s + \mathcal{G}_a$, ϕ^s,χ^s,F^s denote the elementary component fields of the sth chiral matter multiplet, M and B are the auxiliary fields of the supergravity multiplet. In the following all formulas are given for the case in which $\mathcal{G}_a$ contains $\delta_{(w)}$ and $\delta_{(r)}$. The results for the cases in which Weyl or R-transformations are not contained in the gauge group are simply obtained by setting to zero the corresponding fields everywhere.

Except for ${F_{ab}}^I$ and ${T_{ab}}^\alpha$ which are obtained from (2.6) and (2.7) and up to complex conjugation the torsions and curvatures are given by

$$\begin{aligned}
&{T_{ab}}^c = {T_{\alpha\beta}}^a = {T_{\underline{\alpha\beta}}}^{\underline{\gamma}} = {T_{\underline{\alpha}a}}^b = {F_{\underline{\alpha\beta}}}^i = 0, \quad {F_{\dot{\alpha}a}}^i = i\lambda^{i\,\alpha}\sigma_{a\,\alpha\dot{\alpha}},\\
&{T_{\alpha\dot{\alpha}}}^a = 2i{\sigma^a}_{\alpha\dot{\alpha}}, \quad {T_{\alpha a}}^{\dot{\alpha}} = -\tfrac{i}{8}\bar{M}{\sigma_{a\,\alpha}}^{\dot{\alpha}}, \quad {T_{\alpha a}}^\beta = -i\,(\delta^\beta_\alpha B_a + B^b{\sigma_{ab\,\alpha}}^\beta),\\
&{F_{\alpha\beta}}^{ab} = -\bar{M}{\sigma^{ab}}_{\alpha\beta}, \quad {F_{\alpha\dot{\alpha}}}^{ab} = 2i\,\varepsilon^{abcd}\sigma_{c\,\alpha\dot{\alpha}}B_d,\\
&{F_{\dot{\alpha}b}}^{cd} = i\,(T^{cd\alpha}\sigma_{b\,\alpha\dot{\alpha}} - 2{\sigma^{[c}}_{\alpha\dot{\alpha}}{{T^{d]}}_b}^\alpha - 2{\delta_b}^{[c}{\sigma^{d]}}_{\alpha\dot{\alpha}}\lambda^{(w)\,\alpha}).
\end{aligned} \tag{3.2}$$

${\omega_m}^{ab}$ is obtained from (2.8). The BRS transformations of the ghosts and gauge fields are obtained from (2.10)–(2.15), those of the fields (3.1) from (2.9) using table 1 where S, U, G^i, $\bar{S}$, $\bar{U}$, $\bar{G}^i$ denote $SL(2,\mathbf{C})$ irreducible tensors constructed of the ${F_{ab}}^i$ and ${T_{ab}}^{\underline{\alpha}}$:

$$\begin{aligned}
&{G_{\alpha\beta}}^i = -{F_{ab}}^i{\sigma^{ab}}_{\alpha\beta}, \quad S_\alpha = {T_{ab}}^\beta{\sigma^{ab}}_{\beta\alpha}, \quad {U_{\dot{\alpha}\dot{\beta}}}^\gamma = {T_{ab}}^\gamma{\bar{\sigma}^{ab}}{}_{\dot{\alpha}\dot{\beta}}, \quad W_{\alpha\beta\gamma} = T_{ab(\alpha}{\sigma^{ab}}_{\beta\gamma)}\\
\Leftrightarrow\ \ &T_{\alpha\dot{\alpha}\,\beta\dot{\beta}\,\gamma} = \varepsilon_{\alpha\beta}U_{\dot{\alpha}\dot{\beta}\gamma} + \varepsilon_{\dot{\beta}\dot{\alpha}}(W_{\alpha\beta\gamma} + \tfrac{2}{3}\varepsilon_{\gamma(\alpha}S_{\beta)}), \quad {F_{\alpha\dot{\alpha}\,\beta\dot{\beta}}}^i = \varepsilon_{\alpha\beta}{\bar{G}_{\dot{\alpha}\dot{\beta}}}{}^i + \varepsilon_{\dot{\alpha}\dot{\beta}}{G_{\alpha\beta}}^i.
\end{aligned} \tag{3.3}$$

Table 1:

Ψ	$\mathcal{D}_\alpha\Psi$	$\bar{\mathcal{D}}_{\dot{\alpha}}\Psi$
M	$\frac{16}{3}(S_\alpha - i\lambda^{(r)}_\alpha - \frac{3}{2}\lambda^{(w)}_\alpha)$	0
$B_{\beta\dot{\beta}}$	$\frac{1}{3}\varepsilon_{\beta\alpha}(\bar{S}_{\dot{\beta}} + 4i\bar{\lambda}^{(r)}_{\dot{\beta}}) - \bar{U}_{\alpha\beta\dot{\beta}}$	$\frac{1}{3}\varepsilon_{\dot{\beta}\dot{\alpha}}(S_\beta - 4i\lambda^{(r)}_\beta) - U_{\dot{\alpha}\dot{\beta}\beta}$
λ^i_β	$i\varepsilon_{\alpha\beta}D^i + {G_{\alpha\beta}}^i$	0
D^i	$\mathcal{D}_{\alpha\dot{\alpha}}\bar{\lambda}^{i\dot{\alpha}} + \frac{3i}{2}B_{\alpha\dot{\alpha}}\bar{\lambda}^{i\dot{\alpha}}$	$\mathcal{D}_{\alpha\dot{\alpha}}\lambda^{i\alpha} - \frac{3i}{2}B_{\alpha\dot{\alpha}}\lambda^{i\alpha}$
ϕ	χ_α	0
χ_β	$\varepsilon_{\beta\alpha}F$	$-2i\mathcal{D}_{\beta\dot{\alpha}}\phi$
F	$-\frac{1}{2}\bar{M}\chi_\alpha$	$-2i\mathcal{D}_{\alpha\dot{\alpha}}\chi^\alpha - 4\bar{\lambda}^i_{\dot{\alpha}}\delta_i\phi + B_{\alpha\dot{\alpha}}\chi^\alpha$

Let me now describe the results of the investigation of (1.1). Up to contributions of the form $s\Gamma^{-1}$ each real local solution $\mathcal{W}^0$ of (1.1) can be written in the form[2]

$$\mathcal{W}^0_{old} = \int d^4x\, e\, \mathcal{P}\, \Omega + c.c., \quad e = det(e_m{}^a) \tag{3.4}$$

where $\mathcal{P}$ is the operator

$$\mathcal{P} = \bar{\mathcal{D}}^2 - 4i\psi_a\sigma^a\bar{\mathcal{D}} - 3M + 16\psi_a\sigma^{ab}\psi_b, \quad \bar{\mathcal{D}}^2 = \bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}, \quad \psi_a{}^\alpha = E_a{}^m\psi_m{}^\alpha \tag{3.5}$$

and Ω is an antichiral function of the tensor fields given by

$$\Omega = H(\bar M, \bar W, \bar\lambda, \bar\phi) + (\mathcal{D}^2 - \bar M)\, G(\Phi), \quad \mathcal{D}^2 = \mathcal{D}^\alpha\mathcal{D}_\alpha \tag{3.6}$$

where H depends only on the (undifferentiated) fields $\bar M, \bar W_{\dot\alpha\dot\beta\dot\gamma}, \bar\lambda^i_{\dot\alpha}, \bar\phi$ while G depends on all variables

$$\{\Phi^r\} = \{\mathcal{D}_{a_1}\dots\mathcal{D}_{a_n}\Psi^l,\ \mathcal{D}_{a_1}\dots\mathcal{D}_{a_n}F_{ab}{}^I,\ \mathcal{D}_{a_1}\dots\mathcal{D}_{a_n}T_{ab}{}^{\underline{\alpha}}:\ n\geq 0\} \tag{3.7}$$

with $\{\Psi^l\}$ as in (3.1). H and G are additionally restricted by

$$\begin{aligned} &\delta_I H = \delta_I G = 0 \quad \forall\, \delta_I \notin \{\delta_{(r)}, \delta_{(w)}\}, \\ &\delta_{(w)}H = -3H, \quad \delta_{(w)}G = -2G, \quad \delta_{(r)}H = -2iH, \quad \delta_{(r)}G = 0 \end{aligned} \tag{3.8}$$

where the conditions imposed by $\delta_{(w)}$ or $\delta_{(r)}$ of course are absent if $\mathcal{G}$ does not contain the respective generator. I remark that $\mathcal{W}^0_{old}$ contains only one Fayet–Iliopoulos contribution [16], namely $\int d^4x\, e\, a\, D^{(r)}$ where a is a real constant. This Fayet–Iliopoulos contribution arises from the contribution $\frac{3a}{64}\bar M$ to Ω which gives the supersymmetric version of the Einstein–Hilbert action due to

$$\bar{\mathcal{D}}^2\bar M = \tfrac{16}{3}(\tfrac{1}{2}R + 2D^{(r)} + \tfrac{3}{8}M\bar M - 3B^aB_a + 3iD^{(w)} - 3i\mathcal{D}_aB^a), \quad R = F_{ab}{}^{ba}. \tag{3.9}$$

Of course (3.9) is not Weyl-invariant and thus contributes to $\mathcal{W}^0_{old}$ only in the case $\delta_{(w)} \notin \mathcal{G}$. Using table 1 one can check that further Fayet–Iliopoulos contributions indeed do not arise from (3.4).

Each real solution $\mathcal{W}^1$ of (1.1) is in the case of old minimal supergravity of the form

$$\mathcal{W}^1 = \mathcal{W}^1_{abel} + \mathcal{W}^1_{nonabel} + c.c. \tag{3.10}$$

where $\mathcal{W}^1_{abel}$ and $\mathcal{W}^1_{nonabel}$ are complex solutions of (1.1) collecting the candidates for 'abelian' and 'nonabelian' anomalies respectively. $\mathcal{W}^1_{abel}$ is given by

$$\mathcal{W}^1_{abel} = \int d^4x\, e \sum_j{}' \left\{C^j\mathcal{P} + \xi^\alpha\mathcal{P}^j_\alpha\right\}\Omega_j \tag{3.11}$$

where $\sum_j'$ runs only over the abelian factors of the gauge group, $\mathcal{P}$ is the operator (3.5), each Ω_j is of the form (3.6) with H_j and G_j restricted by (3.8) and $\mathcal{P}^j_\alpha$ is the operator

$$\mathcal{P}^j_\alpha = 4iA_a{}^j\sigma^a{}_{\alpha\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha} - 32A_a{}^j\sigma^{ab}{}_\alpha{}^\beta\psi_{b\beta} + 8\lambda^j_\alpha, \quad A_a{}^j = E_a{}^mA_m{}^j. \tag{3.12}$$

In particular (3.11) contains the supersymmetric abelian chiral anomalies which arise from contributions $g_{kl}\bar\lambda^k\bar\lambda^l$ and $a\bar W_{\dot\alpha\dot\beta\dot\gamma}\bar W^{\dot\alpha\dot\beta\dot\gamma}$ to Ω_j where g_{kl} are purely imaginary constant tensors of $\mathcal{G}_a + \mathcal{G}_s$ and a are complex constants. Notice that if $\mathcal{G}$ contains neither $\delta_{(w)}$ nor $\delta_{(r)}$ then Ω_j contains a constant contribution $\frac{a_j}{8}$ which gives rise to the following simple solutions of (1.1):

$$\int d^4x\, e \sum_j{}' a_j\, (\xi\lambda^j - 4A_a{}^j\xi\sigma^{ab}\psi_b + 2C^j\psi_a\sigma^{ab}\psi_b) + c.c. \tag{3.13}$$

[2] Due to the reality of s each complex solution of (1.1) is of the form $\mathcal{W}^g = \mathcal{W}^g_1 + i\mathcal{W}^g_2$ where $\mathcal{W}^g_1$ and $\mathcal{W}^g_2$ are real solutions of (1.1).

$\mathcal{W}^1_{nonabel}$ is a linear combination of the independent supersymmetric nonabelian chiral anomalies $\Delta^\tau_{nonabel}$ which are labeled by τ (there are as many of them as $\mathcal{G}_s$ has independent Casimir operators of third order):

$$\begin{aligned} \mathcal{W}^1_{nonabel} &= \sum_\tau a_\tau\, \Delta^\tau_{nonabel}, \quad a_\tau = const., &(3.14)\\ \Delta^\tau_{nonabel} &= \int Tr\Big\{Cd(AdA+\tfrac{1}{2}A^3)+(LA+AL)dA+\tfrac{3}{2}LA^3 \\ &\qquad -3\,d^4x\,e\,(\bar\xi\Lambda^\dagger\Lambda\Lambda+\xi\Lambda\,\Lambda^\dagger\Lambda^\dagger)\Big\} &(3.15)\end{aligned}$$

where C, A, Λ_α, $\Lambda^\dagger_{\dot\alpha}$, L denote the matrices

$$C=C^iT_i,\ A=dx^mA_m{}^iT_i,\ \Lambda_\alpha=i\lambda^j_\alpha T_j,\ \Lambda^\dagger_{\dot\alpha}=i\bar\lambda^j_{\dot\alpha}T_j,\ L=dx^me_m{}^a(\xi\sigma_a\Lambda^\dagger-\Lambda\sigma_a\bar\xi) \tag{3.16}$$

defined by means of a suitably chosen matrix representation $\{T_i\}$ of $\mathcal{G}_s$ satisfying

$$[T_i,T_j]=f_{ij}{}^kT_k$$

(in general one of course has to choose different representations $\{T_i\}$ for different values of τ). Notice that (3.15) depends on the vielbein only via the 1–forms $e^a=dx^me_m{}^a$ due to

$$d^4x\,e=-\tfrac{1}{24}\,\varepsilon_{abcd}\,e^ae^be^ce^d$$

i.e. the integrand of Δ^τ can be written completely in terms of the forms $A^i=dx^mA_m{}^i$, dA^i, e^a and the fields C^i, ξ^α, $\bar\xi^{\dot\alpha}$, λ^i_α and $\bar\lambda^i_{\dot\alpha}$. In flat space where one can choose $e_m{}^a=\delta^a_m$ and identify e^a with a constant differential dx^a, (3.15) agrees completely with the form of globally supersymmetric nonabelian chiral anomalies derived in [17], i.e. the expressions derived in [17] are in fact not only globally but also locally supersymmetric after replacing dx^a with e^a, without adding any gravitino dependent terms to these expressions. In fact the independence on the gravitino does not only hold for $\Delta^\tau_{nonabel}$ but for the complete solution (3.21) of the descent equations (1.4) arising from it as is shown explicitly in appendix B.

Notice that the contribution $\int Tr\{Cd(AdA+\frac{1}{2}A^3)\}$ to (3.15) is the well-known form of a nonabelian chiral anomaly in the nonsupersymmetric case (see e.g. [18]) and the remaining terms in (3.15) represent its supersymmetric completion.

I remark that the abelian chiral anomalies arising from contributions $g_{kl}\bar\lambda^k\bar\lambda^l$ to Ω_j in (3.11) (with g_{kl} purely imaginary) can be written in a similar form as (3.15). Namely by adding a suitable counterterm $s\Gamma^0$ to (3.11) they become linear combinations of solutions Δ^τ_{abel} of (1.1) given by

$$\begin{aligned}\Delta^\tau_{abel} &= \int\Big\{C^jTr(F^2)+2A^jTr(FL)-i\,d^4x\,e\,\Big[\xi\lambda^jTr(\Lambda^\dagger\Lambda^\dagger)+\bar\xi\bar\lambda^jTr(\Lambda\Lambda)\\ &\qquad +2\bar\lambda^j_{\dot\alpha}Tr(\Lambda^{\dagger\,\dot\alpha}\xi\Lambda)+2\lambda^{j\,\alpha}Tr(\Lambda_\alpha\bar\xi\Lambda^\dagger)\Big]\Big\},\quad F=dA+A^2 \qquad (3.17)\end{aligned}$$

where C^j, A^j and λ^j denote the ghost, connection 1–form and gaugino of an abelian factor of the gauge group.

The solutions $\tilde\omega^4$ and $\tilde\omega^5$ of the descent equations (1.4) arising from (3.4) and (3.11) can be written in the following remarkably simple form

$$\tilde\omega^4_{old}=(\tilde{\mathcal{D}}_{\dot\alpha}\tilde{\mathcal{D}}^{\dot\alpha}-M)(\Xi\,\Omega),\quad \tilde\omega^5_{abel}=\sum_i{}'(\tilde{\mathcal{D}}_{\dot\alpha}\tilde{\mathcal{D}}^{\dot\alpha}-M)(\Xi\,\tilde C^i\Omega_i) \tag{3.18}$$

where Ξ denotes the product of all $\tilde\xi^a$ and $\tilde{\mathcal{D}}_{\dot\alpha}$ is an extension of $\bar{\mathcal{D}}_{\dot\alpha}$ suitably defined on the variables (2.20):

$$\Xi=-\tfrac{1}{24}\,\varepsilon_{abcd}\,\tilde\xi^a\tilde\xi^b\tilde\xi^c\tilde\xi^d,\quad \tilde{\mathcal{D}}_{\dot\alpha}\Phi^r=\bar{\mathcal{D}}_{\dot\alpha}\Phi^r,\quad \tilde{\mathcal{D}}_{\dot\alpha}\tilde\xi^A=\tilde\xi^BT_{B\dot\alpha}{}^A,\quad \tilde{\mathcal{D}}_{\dot\alpha}\tilde C^i=\tilde\xi^AF_{A\dot\alpha}{}^i. \tag{3.19}$$

Using the explicit expressions (3.2) one obtains in particular

$$\tilde{\mathcal{D}}_{\dot\alpha}\tilde{\xi}_{\dot\beta\beta} = 4i\,\varepsilon_{\dot\beta\dot\alpha}\,\tilde{\xi}_\beta, \quad \tilde{\mathcal{D}}_{\dot\alpha}\tilde{\xi}_\alpha = -\tfrac{i}{8}\,\tilde{\xi}_{\alpha\dot\alpha}M, \quad \tilde{\mathcal{D}}_{\dot\alpha}\tilde{C}^j = -i\,\tilde{\xi}_{\alpha\dot\alpha}\lambda^{j\,\alpha}. \tag{3.20}$$

The solution of (1.4) arising from (3.15) reads

$$\tilde{\omega}^{5,\tau}_{nonabel} = Tr\left\{\tilde{C}\tilde{\mathcal{F}}^2 - \tfrac{1}{2}\,\tilde{C}^3\tilde{\mathcal{F}} + \tfrac{1}{10}\,\tilde{C}^5 - 3\,\Xi\,(\tilde{\xi}^\alpha\Lambda_\alpha\,\Lambda^\dagger\Lambda^\dagger + \tilde{\xi}_{\dot\alpha}\Lambda^{\dagger\,\dot\alpha}\,\Lambda\Lambda)\right\} \tag{3.21}$$

with Λ_α and $\Lambda^\dagger_{\dot\alpha}$ as in (3.16) and

$$\tilde{C} = \tilde{C}^iT_i, \quad \tilde{\mathcal{F}} = -\tfrac{1}{2}(-)^{|A|}\tilde{\xi}^A\tilde{\xi}^B F_{BA}{}^iT_i. \tag{3.22}$$

The solution of (1.4) arising from an abelian chiral anomaly (3.17) has a similar form:

$$\begin{aligned}\tilde{\omega}^{5,\tau}_{abel,chir} = \tilde{C}^jTr(\tilde{\mathcal{F}}^2) - i\,\Xi\,\Big\{&\tilde{\xi}^\alpha\lambda^j_\alpha Tr(\Lambda^\dagger\Lambda^\dagger) + \tilde{\xi}_{\dot\alpha}\bar{\lambda}^{\dot\alpha\,j}Tr(\Lambda\Lambda)\\ &+2\bar{\lambda}^j_{\dot\alpha}Tr(\Lambda^{\dagger\,\dot\alpha}\tilde{\xi}^\alpha\Lambda_\alpha) + 2\lambda^{j\,\alpha}Tr(\Lambda_\alpha\tilde{\xi}_{\dot\alpha}\Lambda^{\dagger\,\dot\alpha})\Big\}.\end{aligned} \tag{3.23}$$

3.2 New minimal supergravity

New minimal supergravity differs from old minimal supergravity with regard to its field content. Namely M, $\lambda^{(r)}_\alpha$, $D^{(r)}$ and B^a are not elementary fields any more. Instead new minimal supergravity contains a new set of elementary fields consisting of the components $t_{mn} = -t_{nm}$ of a 2–form gauge potential, corresponding ghosts Q_m and a ghost Q for these ghosts[3] which have the following reality properties, ghost numbers and gradings:

$$\begin{array}{lll} t_{mn} = (t_{mn})^*, & gh(t_{mn}) = 0, & |t_{mn}| = 0,\\ Q_m = (Q_m)^*, & gh(Q_m) = 1, & |Q_m| = 1,\\ Q = -Q^*, & gh(Q) = 2, & |Q| = 0. \end{array} \tag{3.24}$$

This field content arises from setting to zero the field M and the superfield arising from it in old minimal supergravity with gauged R-transformations (but without Weyl transformations). This leads to the identifications

$$M \equiv 0, \quad \lambda^{(r)}_\alpha \equiv -iS_\alpha, \quad D^{(r)} \equiv -\tfrac{1}{4}R + \tfrac{3}{2}B^aB_a, \tag{3.25}$$

$$B^a \equiv e_m{}^a\,\varepsilon^{mnkl}\,(\tfrac{1}{2}\partial_n t_{kl} + i\psi_n\sigma_k\bar{\psi}_l) \tag{3.26}$$

where $\varepsilon^{mnkl} = E_a{}^mE_b{}^nE_c{}^kE_d{}^l\varepsilon^{abcd} \sim 1/e$ and $\sigma_m = e_m{}^a\sigma_a$. (3.25) and (3.26) are required by $M = \mathcal{D}_\alpha M = \mathcal{D}^\alpha\mathcal{D}_\alpha M = 0$, cf. table 1 and (3.9). In particular (3.26) which has been given already in the second ref. [6] 'solves' $\mathcal{D}_aB^a = 0 \Leftrightarrow \mathcal{D}^\alpha\mathcal{D}_\alpha M - c.c. = 0$ identically in terms of elementary fields which is a rather involved condition since it reads more explicitly

$$0 = \mathcal{D}_aB^a = E_a{}^m(\partial_m - \tfrac{1}{2}\omega_m{}^{bc}(e,\psi)l_{bc} - \psi_m{}^\alpha\mathcal{D}_\alpha + \bar{\psi}_m{}^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha})B^a \tag{3.27}$$

where $\bar{\mathcal{D}}_{\dot\alpha}B^a$ and $\mathcal{D}_\alpha B^a$ are obtained from table 1 using the second identity (3.25):

$$\bar{\mathcal{D}}_{\dot\alpha}B_{\beta\dot\beta} = \varepsilon_{\dot\alpha\dot\beta}S_\beta - U_{\dot\alpha\dot\beta\beta} \;\Leftrightarrow\; \bar{\mathcal{D}}_{\dot\alpha}B^a = -\tfrac{i}{2}\,\varepsilon^{abcd}\sigma_{b\,\alpha\dot\alpha}T_{cd}{}^\alpha. \tag{3.28}$$

The nilpotent BRS transformations of t_{mn}, Q_m and Q read

$$\begin{aligned} s\,t_{mn} &= C^k\partial_k t_{mn} + (\partial_mC^k)\,t_{kn} + (\partial_nC^k)\,t_{mk} - \partial_mQ_n + \partial_nQ_m\\ &\quad -i(\xi\sigma_m\bar{\psi}_n - \xi\sigma_n\bar{\psi}_m + \psi_m\sigma_n\bar{\xi} - \psi_n\sigma_m\bar{\xi}), \qquad (3.29)\\ s\,Q_m &= C^n\partial_nQ_m + (\partial_mC^n)Q_n + \partial_mQ + 2i\,\xi\sigma^n\bar{\xi}\,t_{nm} - i\,\xi\sigma_m\bar{\xi}, \qquad (3.30)\\ s\,Q &= C^m\partial_mQ - 2i\,\xi\sigma^m\bar{\xi}\,Q_m \qquad (3.31)\end{aligned}$$

[3] In order to fix a gauge for t_{mn} and Q_m one introduces additional fields whose role is analogous to that of the fields ζ and b. As the latter these additional fields contribute only trivially to solutions of (1.1).

where $\sigma^m = E_a{}^m \sigma^a$. (3.26) and (3.29)–(3.31) can be written in a more compact notation which is analogous to (2.18) and (2.19) and reads (with $\tilde\xi^A$ and $\hat C^m$ as in (2.20), (2.21)):

$$\tilde{\mathcal{H}} = \tilde s\, \tilde Q, \quad \tilde{\mathcal{H}} = \tfrac{1}{6}\, \tilde\xi^a \tilde\xi^b \tilde\xi^c \varepsilon_{abcd} B^d + i\, \tilde\xi^\alpha \tilde\xi_{\alpha\dot\alpha} \tilde\xi^{\dot\alpha}, \quad \tilde Q = \tfrac{1}{2} \hat C^m \hat C^n\, t_{mn} + \hat C^n Q_n + Q. \tag{3.32}$$

The BRS transformations of the remaining fields can be obtained from their counterparts in old minimal supergravity using (3.25) and (3.26). Therefore one obtains solutions of (1.1) for new minimal supergravity simply by making the identifications (3.25), (3.26) in the solutions $\mathcal{W}^0_{old}$, $\mathcal{W}^1_{abel}$ and $\mathcal{W}^1_{nonabel}$ obtained for old minimal supergravity. However the presence of the fields t_{mn}, Q_m and Q is responsible for the fact that this does not give *all* solutions of (1.1) with $g = 0, 1$ for new minimal supergravity. Namely there are a few additional solutions which cannot be obtained in this way. I denote these additional solutions by $\mathcal{W}_{FI}$ in order to point out that $\mathcal{W}^0_{FI}$ are locally supersymmetric versions of Fayet–Iliopoulos contributions to the action and $\mathcal{W}^1_{FI}$ are anomaly candidates of a similar form. They are given by

$$\mathcal{W}^0_{FI} = \int d^4x\, e \sum_i{}' a_i \left\{ D^i - 2B^a A_a{}^i + \lambda^i \sigma^m \bar\psi_m + \psi_m \sigma^m \bar\lambda^i \right\}, \tag{3.33}$$

$$\begin{aligned}\mathcal{W}^1_{FI} = \int d^4x\, e \sum_{i,j}{}' a_{ij} \Big\{ & C^i (D^j + \varepsilon^{mnkl} A_m{}^j \partial_n t_{kl} + \lambda^j \sigma^m \bar\psi_m + \psi_m \sigma^m \bar\lambda^j) \\ & - A_m{}^i (\xi \sigma^m \bar\lambda^j + \lambda^j \sigma^m \bar\xi) - i\varepsilon^{mnkl} A_m{}^i A_n{}^j (\xi \sigma_k \bar\psi_l + \psi_k \sigma_l \bar\xi) \Big\}, \quad a_{ij} = -a_{ji}\end{aligned} \tag{3.34}$$

where $\sum_i'$ and $\sum_{i,j}'$ run only over abelian factors as in (3.11) and the constants a_i, a_{ij} must be chosen real in order to get real solutions of (1.1). Notice that the supersymmetrized Einstein–Hilbert action $\mathcal{W}^0_{EH}$ is contained in (3.33) since $D^{(r)}$ which contributes to (3.33) has to be identified with $-\frac{1}{4}R + \frac{3}{2}B^a B_a$ according to (3.25). The form of $\mathcal{W}^0_{EH}$ obtained from (3.33) agrees with that given in the second ref. [6] and in [15]. Thus we obtain the result that in the case of new minimal supergravity the action contains a Fayet–Iliopoulos contribution for each abelian factor apart from the one corresponding to the R-transformations which becomes a contribution $\int d^4x e(-\frac{1}{4}R + \frac{3}{2}B^a B_a)$ to $\mathcal{W}^0_{EH}$.

I stress again that apart from $\mathcal{W}^0_{EH}$ the solutions $\mathcal{W}^g_{FI}$ do not have counterparts in old minimal supergravity. This holds in particular for the anomaly candidates (3.34). Notice that they are present only if the structure group contains at least two abelian factors, i.e. at least one abelian factor in addition to the R-transformation, since the a_{ij} are antisymmetric.

The solutions of (1.4) arising from (3.33) and (3.34) are

$$\tilde\omega^4_{FI} = \sum_i{}' a_i \left\{ 2\tilde C^i \tilde{\mathcal{H}} + \bar\lambda^i \bar\eta + \eta \lambda^i + \Xi\, D^i \right\}, \tag{3.35}$$

$$\tilde\omega^5_{FI} = \sum_{i,j}{}' a_{ij} \left\{ \tilde C^i \tilde C^j \tilde{\mathcal{H}} + \tilde C^i \bar\lambda^j \bar\eta + \eta \lambda^j \tilde C^i + \Xi\, \tilde C^i D^j \right\} \tag{3.36}$$

with $\tilde{\mathcal{H}}$ as in (3.32) and

$$\bar\eta^{\dot\alpha} = \tfrac{i}{6}\, \tilde\xi^{\dot\alpha\alpha} \tilde\xi_{\alpha\dot\beta} \tilde\xi^{\dot\beta\beta} \tilde\xi_\beta, \quad \eta^\alpha = -\tfrac{i}{6}\, \tilde\xi_{\dot\beta} \tilde\xi^{\dot\beta\beta} \tilde\xi_{\beta\dot\alpha} \tilde\xi^{\dot\alpha\alpha}. \tag{3.37}$$

3.3 Nonminimal supergravity

The nonminimal supergravity theories discussed in the following are as in [8] parametrized by the real number n occurring in the torsions

$$T_{\alpha a}{}^b = 2n\, \delta^b_a\, T_\alpha, \quad T_{\alpha\beta}{}^\gamma = (n+1)(\delta^\gamma_\alpha T_\beta + \delta^\gamma_\beta T_\alpha), \quad T_{\alpha\dot\alpha}{}^\beta = (n-1)\, \delta^\beta_\alpha\, \bar T_{\dot\alpha} \tag{3.38}$$

and are in addition characterized by the constraints

$$T_{\alpha\dot\alpha}{}^a = 2i\sigma^a{}_{\alpha\dot\alpha}, \quad T_{ab}{}^c = T_{\alpha\beta}{}^a = F_{\underline{\alpha\beta}}{}^i = F_{\alpha\beta}{}^{cd} = 0. \tag{3.39}$$

A detailed discussion of (3.38) and (3.39) and their implications can be found in [8]. I consider the case in which $\mathcal{G}$ contains neither Weyl nor R-transformations. Apart from the special cases $n=0$ and $n=-\frac{1}{3}$ which are excluded for simplicity the set of elementary tensor fields is given by

$$T_\alpha,\ \mathcal{J},\ v^a,\ G^a,\ \lambda_\alpha,\ \lambda^i_\alpha,\ D^i,\ \phi^s,\ \chi^s_\alpha,\ F^s \tag{3.40}$$

and the complex conjugated fields where $\mathcal{J}$, v^a and G^a denote a complex scalar field, a complex and a real vector field which in [8] are denoted by S, c^a+id^a and b^a. λ_α (denoted as in [8]) must not be confused with one of the gauginos λ^i_α of $\mathcal{G}_s+\mathcal{G}_a$ which appear as in minimal supergravity in the curvatures $F_{\dot\alpha a}{}^i = i\lambda^{i\alpha}\sigma_{a\,\alpha\dot\alpha}$. The field content is completed by the gauge fields, ghosts, antighosts and the corresponding Lagrange multiplier fields where again due to $T_{ab}{}^c=0$ the spin connection is not an elementary field but determined by (2.8). The supersymmetry transformations of the fields (3.40) can be found in [8] or obtained from table 1 (by replacing there all fields and operators by their primed counterparts, see below). In particular one has

$$\mathcal{D}_\alpha T_\beta = \tfrac{1}{2}\varepsilon_{\alpha\beta}\left[\mathcal{J}+(n+1)T^\gamma T_\gamma\right],\quad \bar{\mathcal{D}}_{\dot\alpha}T_\alpha = v_{\alpha\dot\alpha},\quad \mathcal{D}_\alpha\mathcal{J}=0,\quad \bar{\mathcal{D}}_{\dot\alpha}\mathcal{J}=\bar\lambda_{\dot\alpha}. \tag{3.41}$$

The BRS transformations of the fields (3.40) are obtained from (2.9), those of the ghosts and gauge fields by (2.10)–(2.15). However it will be useful to rewrite the BRS algebra in terms of redefined ghosts, gauge fields and supersymmetry transformations marked by primes and defined in terms of the original (unprimed) ghosts, gauge fields and supersymmetry transformations such that $\mathcal{D}'_\alpha$, $\bar{\mathcal{D}}'_{\dot\alpha}$ and $\mathcal{D}'_a$ satisfy the algebra

$$[\mathcal{D}'_A,\mathcal{D}'_B\} = -T'_{AB}{}^C\mathcal{D}'_C - F'_{AB}{}^I\delta_I$$

where the $T'_{AB}{}^C$ and $F'_{AB}{}^I$ satisfy the constraints of minimal supergravity with R-transformations (in fact such redefinitions have been used also in [8]). This is achieved by

$$\mathcal{D}'_\alpha = \mathcal{D}_\alpha + 4n\,T^\beta l_{\alpha\beta} + i\,(3n+1)\,T_\alpha\delta_{(r)},\quad \bar{\mathcal{D}}'_{\dot\alpha} = \bar{\mathcal{D}}_{\dot\alpha} + 4n\,\bar T^{\dot\beta} l_{\dot\alpha\dot\beta} - i\,(3n+1)\,\bar T_{\dot\alpha}\delta_{(r)} \tag{3.42}$$

where $l_{\alpha\beta}$ and $l_{\dot\alpha\dot\beta}=(l_{\alpha\beta})^*$ denote the generators of $SL(2,\mathbf{C})$ transformations of undotted and dotted indices defined by

$$\begin{aligned} &l_{\alpha\beta}=\tfrac{1}{2}\sigma^{ab}{}_{\alpha\beta}l_{ab},\quad l_{\alpha\beta}X_\gamma = -\varepsilon_{\gamma(\alpha}X_{\beta)},\quad l_{\alpha\beta}X_{\dot\gamma}=0,\\ &l_{\dot\alpha\dot\beta}=-\tfrac{1}{2}\bar\sigma^{ab}{}_{\dot\alpha\dot\beta}l_{ab},\quad l_{\dot\alpha\dot\beta}X_{\dot\gamma} = -\varepsilon_{\dot\gamma(\dot\alpha}X_{\dot\beta)},\quad l_{\dot\alpha\dot\beta}X_\gamma=0 \end{aligned} \tag{3.43}$$

and $\delta_{(r)}$ which originally was not contained in $\mathcal{G}$ is defined according table 2 where the R-charge of the lowest component field ϕ^s of the sth chiral multiplet is denoted by the real number k^s (k^s is not fixed by requiring (2.2) unlike the R-charges of the remaining fields).

Table 2:

Ψ	T	$\mathcal{J}$	v	B	λ	λ^j	D^j	ϕ^s	χ^s	F^s
$\delta_{(r)}\Psi$	$-iT$	$-2i\mathcal{J}$	0	0	$i\lambda$	$i\lambda^j$	0	$-ik^s\phi^s$	$-2ik^s\chi^s$	$-3ik^sF^s$

The redefined covariant derivatives read

$$\mathcal{D}'_a = E_a{}^m(\partial_m - \psi'_m{}^\alpha\mathcal{D}'_\alpha + \bar\psi'_m{}^{\dot\alpha}\bar{\mathcal{D}}'_{\dot\alpha} - \tfrac{1}{2}\omega'_m{}^{ab}l_{ab} - A'_m{}^i\delta_i) \tag{3.44}$$

where the sum over i contains the R-transformation and the redefined gauge fields are given by

$$e'_m{}^a = e_m{}^a, \tag{3.45}$$

$$\psi'_m{}^\alpha = \psi_m{}^\alpha + in\,\bar T_{\dot\alpha}\bar\sigma_m{}^{\dot\alpha\alpha},\quad \bar\psi'_m{}^{\dot\alpha} = \bar\psi_m{}^{\dot\alpha} - in\,\bar\sigma_m{}^{\dot\alpha\alpha}T_\alpha, \tag{3.46}$$

$$A'_m{}^i = A_m{}^i \quad \forall\, i\neq(r), \tag{3.47}$$

$$A'_m{}^{(r)} = (3n+1)\{\tfrac{1}{2}\,(v+\bar v)_m - \tfrac{1}{2}(n-1)T\sigma_m\bar T + i\bar\psi_m\bar T - i\psi_m T\}, \tag{3.48}$$

$$\omega'_m{}^{ab} = E^{an}E^{br}(\omega'_{[mn]r} - \omega'_{[nr]m} + \omega'_{[rm]n}),$$

$$\omega'_{[mn]r} = e_{ra}\partial_{[m}e_{n]}{}^a - i\psi'_m\sigma_r\bar\psi'_n + i\psi'_n\sigma_r\bar\psi'_m \tag{3.49}$$

with $v_m = e_m{}^a v_a$. One can check that $T'_{AB}{}^C$ and $F'_{AB}{}^I$ indeed are given by (3.2) where of course now unprimed fields have to be replaced by primed fields. In particular one finds

$$
\begin{aligned}
M' &= -4n\left[\bar{\mathcal{J}} + 2(3n+1)\bar{T}\bar{T}\right], && (3.50)\\
\lambda'^{\,i}_{\alpha} &= \lambda^i_\alpha \quad \forall\, i \neq (r), && (3.51)\\
\lambda'^{\,(r)}_{\alpha} &= -i\,S'_\alpha + 3ni\left[-\tfrac{1}{4}\,\lambda_\alpha + (3n+1)\bar{v}_{\alpha\dot{\alpha}}\bar{T}^{\dot{\alpha}} + \tfrac{3n+1}{2}\bar{\mathcal{J}}T_\alpha + (3n+1)^2 T_\alpha\bar{T}\bar{T}\right] && (3.52)
\end{aligned}
$$

where S'_α is obtained from (2.7) using primed gauge fields and torsions:

$$S'_\alpha = T'_{ab}{}^\beta(e,\psi',\omega',A'^{(r)},M',B')\,\sigma^{ab}{}_{\beta\alpha}.$$

Now we choose redefined ghosts such that

$$s = C^m\partial_m + \xi^\alpha\mathcal{D}_\alpha + \bar{\xi}^{\dot{\alpha}}\bar{\mathcal{D}}_{\dot{\alpha}} + C^I\delta_I = C'^{\,m}\partial_m + \xi'^{\,\alpha}\mathcal{D}'_\alpha + \bar{\xi}'^{\,\dot{\alpha}}\bar{\mathcal{D}}'_{\dot{\alpha}} + C'^{\,I}\delta_I \tag{3.53}$$

holds identically in the fields where $C'^I\delta_I$ contains $\delta_{(r)}$ unlike $C^I\delta_I$, i.e. in particular all terms containing $\delta_{(r)}$ must cancel on the r.h.s. of (3.53). One easily verifies that this leads to

$$
\begin{aligned}
&C'^{\,m} = C^m, \quad \xi'^{\,\alpha} = \xi^\alpha, \quad \bar{\xi}'^{\,\dot{\alpha}} = \bar{\xi}^{\dot{\alpha}}, \quad C'^{\,i} = C^i \quad \forall\, i \neq (r), && (3.54)\\
&C'^{\,(r)} = -i\,(3n+1)(\xi T + \bar{\xi}\bar{T}), && (3.55)\\
&C'^{\,ab} = C^{ab} + 4n(\xi\sigma^{ab}T - \bar{\xi}\bar{\sigma}^{ab}\bar{T}). && (3.56)
\end{aligned}
$$

Notice that $C'^{\,(r)}$ does not have a part which is linear in the elementary fields as a consequence of the fact that $\mathcal{G}$ does not contain $\delta_{(r)}$. I remark that analogously to (3.53)

$$
\begin{aligned}
&\partial_m = \mathcal{A}_m{}^A\mathcal{D}_A + \mathcal{A}_m{}^I\delta_I = \mathcal{A}'_m{}^A\mathcal{D}'_A + \mathcal{A}'_m{}^I\delta_I, && (3.57)\\
&\tilde{s} = \tilde{\xi}^A\mathcal{D}_A + \tilde{C}^I\delta_I = \tilde{\xi}'^{\,A}\mathcal{D}'_A + \tilde{C}'^{\,I}\delta_I, && (3.58)\\
&\tilde{\xi}'^{\,a} = \hat{C}^n e_n{}^a, \quad \tilde{\xi}'^{\,\underline{\alpha}} = \xi'^{\,\underline{\alpha}} + \hat{C}^n\psi'^{\,\underline{\alpha}}_n, \quad \tilde{C}'^{\,I} = C'^{\,I} + \hat{C}^n\mathcal{A}'^{\,I}_n && (3.59)
\end{aligned}
$$

hold identically in the fields (all contributions containing $\delta_{(r)}$ cancel on the r.h.s. of (3.57) and (3.58)) with $\hat{C}^n = C^n + dx^n$. $\tilde{s}^2 = 0$ implies that (2.10)–(2.15) hold for the primed ghosts and gauge fields (with primed torsions and curvatures), including $C'^{(r)}$ and $A'_m{}^{(r)}$ as one can verify explicitly:

$$
\begin{aligned}
s\,A'_m{}^{(r)} &= C^n\partial_n A'_m{}^{(r)} + (\partial_m C^n)A'_n{}^{(r)} + \partial_m C'^{(r)} + i(\lambda'^{(r)}\sigma_m\bar{\xi} - \xi\sigma_m\bar{\lambda}'^{(r)}), && (3.60)\\
s\,C'^{(r)} &= C^n\partial_n C'^{(r)} - 2i\xi\sigma^m\bar{\xi}\,A'_m{}^{(r)}. && (3.61)
\end{aligned}
$$

Thus on primed fields the BRS operator has the same form as in minimal supergravity and therefore one obtains solutions of $s\mathcal{W}^g = 0$ for nonminimal supergravity from those obtained for old minimal supergravity by replacing in the expressions for the latter all operators and fields with primed ones. For instance one obtains from (3.5) invariant contributions to the action for nonminimal supergravity given by

$$\int d^4x\, e\,\mathcal{P}'\,\Omega' + c.c., \quad \Omega' = H(\bar{W}',\bar{\lambda}',\bar{M}',\bar{\phi}) + (\mathcal{D}'^2 - \bar{M}')G(\Phi') \tag{3.62}$$

where $\mathcal{P}'$ is obtained from (3.5):

$$
\begin{aligned}
\mathcal{P}' &= \mathcal{P}(\bar{\mathcal{D}}',\psi',M') = \bar{\mathcal{D}}'^2 - 4i\psi'_a\sigma^a\bar{\mathcal{D}}' - 3M' + 16\psi'_a\sigma^{ab}\psi'_b && (3.63)\\
&= \bar{\mathcal{D}}^2 - 4i\psi_a\sigma^a\bar{\mathcal{D}} + 16\psi_a\sigma^{ab}\psi_b + (13n-3)\bar{T}\bar{\mathcal{D}} + 2(3n-1)(\bar{\mathcal{J}} + 8n\bar{T}\bar{T} - 4i\psi_a\sigma^a\bar{T}).
\end{aligned}
$$

I stress that H and G have to satisfy (3.8) where of course the condition imposed by $\delta_{(w)}$ is absent unlike the condition imposed by $\delta_{(r)}$ which must hold (otherwise (3.62) is not invariant except for the case $n = -\frac{1}{3}$). Contributions to H are therefore e.g.

$$H(\bar{W}',\bar{\lambda}',\bar{M}',\bar{\phi}) = a\bar{M}' + b\,\bar{W}'_{\dot{\alpha}\dot{\beta}\dot{\gamma}}\bar{W}'^{\dot{\alpha}\dot{\beta}\dot{\gamma}} + g_{ij}\bar{\lambda}'^i\bar{\lambda}'^j + V(\bar{\phi}) + \dots \tag{3.64}$$

where a and b are constants, g_{ij} are invariant symmetric tensors of $\mathcal{G}$ and the superpotential $V(\bar\phi)$ has to satisfy

$$V(\bar\phi)=\sum_{r\geq 1}\sum_{s_1\ldots s_r} d_{s_1\ldots s_r}\bar\phi^{s_1}\ldots\bar\phi^{s_r},\quad \delta_I V(\bar\phi)=0\quad \forall\,\delta_I\neq\delta_{(r)},\quad \delta_{(r)}V(\bar\phi)=-2iV(\bar\phi). \tag{3.65}$$

Notice that this requires that a monomial $\bar\phi^{s_1}\ldots\bar\phi^{s_r}$ contributing to V satisfies

$$\sum_{s=s_1}^{s_r} k^s=-2 \tag{3.66}$$

which cannot be fulfilled if one imposes e.g. the chirality condition $\bar{\mathcal{D}}_{\dot\alpha}\phi^s=0$ on each of the chiral multiplets. Thus one has to admit non zero k^s in order to allow for solutions of (3.66). This corresponds to a relaxed chirality condition imposed on the chiral multiplets:

$$\bar{\mathcal{D}}'_{\dot\alpha}\,\phi^s=0\quad\Leftrightarrow\quad \bar{\mathcal{D}}_{\dot\alpha}\,\phi^s=(3n+1)\,k^s\,\bar T_{\dot\alpha}\,\phi^s. \tag{3.67}$$

Replacing unprimed fields and operators by primed ones in (3.11), (3.15) and (3.17) one obtains solutions of $s\mathcal{W}^1=0$ for nonminimal supergravity. Of course it is not guaranteed that these solutions are nontrivial since nonminimal supergravity has an extended field content and thus allows for counterterms which do not exist in minimal supergravity. The nonabelian chiral anomalies stay nontrivial. Notice that they keep their explicit form (3.15) because all fields contributing to (3.15) agree with their primed counterparts. Among the solutions of $s\mathcal{W}^1=0$ obtained from (3.11) there are in particular those arising from $\int d^4x e\,(C^{(r)}\mathcal{P}+\xi^\alpha\mathcal{P}^{(r)}_\alpha)\Omega_{(r)}$. They are special since they do not depend on the ghosts of the structure group due to (3.55). Most of these solutions turn out to be trivial but there are also nontrivial ones among them. For instance the following (complex) solutions can be shown to be nontrivial:

$$\mathcal{W}^1_{susy}=\int d^4x\,e\,\{(3n+1)(\xi T+\bar\xi\bar T)\mathcal{P}'+i\xi^\alpha\mathcal{P}'^{(r)}_\alpha\}\{g_{jk}\bar\lambda^j\bar\lambda^k+V(\bar\phi)\} \tag{3.68}$$

with V as in (3.65), $\mathcal{P}'$ as in (3.63) and

$$\mathcal{P}'^{(r)}_\alpha=4iA'^{(r)}_a\sigma^a{}_{\alpha\dot\alpha}\bar{\mathcal{D}}'^{\dot\alpha}-32A'^{(r)}_a\sigma^{ab}{}_\alpha{}^\beta\psi'_{b\beta}+8\lambda'^{(r)}_\alpha. \tag{3.69}$$

The solution of (1.4) arising from (3.68) is obtained from (3.18) and reads

$$\tilde\omega^5_{susy}=(\tilde{\mathcal{D}}'^2-M')\left(\Xi\,\tilde C'^{(r)}\{g_{jk}\bar\lambda^j\bar\lambda^k+V(\bar\phi)\}\right). \tag{3.70}$$

4 Verification of $\tilde s\,\tilde\omega^{g+4}=0$

In order to verify that (3.18), (3.21), (3.23), (3.35), (3.36) and (3.70) indeed solve (1.4) it is more convenient to use a decomposition of $\tilde\omega^{g+4}$ into parts $\hat\omega_p$ of definite degree in the $\tilde\xi^a$ than the decomposition (1.5):

$$\tilde\omega^{g+4}=\sum_{p=p_0}^{4}\hat\omega_p,\quad \hat\omega_p=\tfrac{1}{p!}\,\tilde\xi^{a_1}\ldots\tilde\xi^{a_p}\omega_{a_1\ldots a_p}(\tilde\xi^\alpha,\tilde\xi^{\dot\alpha},\tilde C^I,\Phi^r). \tag{4.1}$$

As an example the decomposition of (3.21) is given in appendix B. The decomposition (4.1) starts at a degree p_0 which in general is different from zero (unlike the decomposition (1.5) which always starts at degree 0) and the part $\hat\omega_{p_0}$ solves

$$\delta_-\hat\omega_{p_0}=0 \tag{4.2}$$

where δ_- is the operator

$$\delta_-=4i\,\tilde\xi^\alpha\tilde\xi^{\dot\alpha}\frac{\partial}{\partial\tilde\xi^{\dot\alpha\alpha}}=2i\,\tilde\xi^\alpha\sigma^a{}_{\alpha\dot\alpha}\tilde\xi^{\dot\alpha}\frac{\partial}{\partial\tilde\xi^a}. \tag{4.3}$$

(4.2) follows from the fact that δ_- is the only part of $\tilde{s}$ which decreases the degree in the $\tilde{\xi}^a$ (notice that this is due to the constraints $T_{\alpha\beta}{}^a = 0$ and $T_{\alpha\dot{\alpha}}{}^a = 2i\sigma^a{}_{\alpha\dot{\alpha}}$). The knowledge of the cohomology of δ_- will be very useful in the following. It has been computed in the first ref. [9] (this derivation is given in appendix A) and recently in [19]. The result is

Cohomology of δ_-:

$$\delta_- f(\tilde{\xi}^A) = 0 \ \Leftrightarrow\ f(\tilde{\xi}^A) = P(\bar{\vartheta}^{\dot{\alpha}}, \tilde{\xi}^{\alpha}) + Q(\vartheta^{\alpha}, \tilde{\xi}^{\dot{\alpha}}) + k\,\Theta + \delta_- g(\tilde{\xi}^A) \tag{4.4}$$

where k does not depend on the $\tilde{\xi}^A$, and ϑ^α, $\bar{\vartheta}^{\dot{\alpha}}$ and Θ are given by

$$\vartheta^\alpha = \tilde{\xi}_{\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}\alpha}, \quad \bar{\vartheta}^{\dot{\alpha}} = \tilde{\xi}^{\dot{\alpha}\alpha}\tilde{\xi}_{\alpha}, \quad \Theta = \tilde{\xi}_{\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}\alpha}\tilde{\xi}_{\alpha}. \tag{4.5}$$

Notice that the ϑ's anticommute. Therefore the δ_--cohomology does not contain nontrivial elements with larger degree than 2 in the $\tilde{\xi}^a$. This implies the following two useful lemmas:

Lemma 1: Each $\tilde{s}$-invariant 4–form in the $\tilde{\xi}^a$ vanishes:

$$\tilde{s}\,\hat{\omega}_4 = 0 \quad \Leftrightarrow \quad \hat{\omega}_4 = 0. \tag{4.6}$$

Proof: $\tilde{s}\hat{\omega}_4 = 0$ implies $\delta_-\hat{\omega}_4 = 0$, cf. (4.2). This implies $\hat{\omega}_4 = \tilde{\delta}_-\hat{\eta}$ for some $\hat{\eta}$ according to (4.4). However this requires that $\hat{\eta}$ has degree 5 in the $\tilde{\xi}^a$ and thus vanishes.

Lemma 2: Each solution of (1.4) whose decomposition (4.1) reads $\tilde{\omega} = \hat{\omega}_3 + \hat{\omega}_4$ can be written as $\tilde{s}\hat{\eta}_4$ where $\hat{\eta}_4$ is a 4–form in the $\tilde{\xi}^a$. $\hat{\eta}_4$ is unique and the solution of $\hat{\delta}_-\hat{\eta}_4 = \hat{\omega}_3$.

$$\tilde{s}\,(\hat{\omega}_3 + \hat{\omega}_4) = 0 \quad \Leftrightarrow \quad \exists\,\hat{\eta}_4: \quad \tilde{s}\,\hat{\eta}_4 = \hat{\omega}_3 + \hat{\omega}_4, \quad \delta_-\,\hat{\eta}_4 = \hat{\omega}_3. \tag{4.7}$$

Proof: (i) Existence of $\hat{\eta}_4$: $\tilde{s}(\hat{\omega}_3 + \hat{\omega}_4) = 0$ implies $\delta_-\hat{\omega}_3 = 0$ according to (4.2). By means of (4.4) one concludes the existence of $\hat{\eta}_4$ such that $\delta_-\hat{\eta}_4 = \hat{\omega}_3$. $\tilde{s}\hat{\eta}_4 = \hat{\omega}_3 + \hat{\omega}_4$ follows from the fact that $\tilde{\omega}' := \hat{\omega}_3 + \hat{\omega}_4 - \tilde{s}\hat{\eta}_4$ is a $\tilde{s}$-invariant 4–form in the $\tilde{\xi}^a$ and thus vanishes according to lemma 1. (ii) Uniqueness of $\hat{\eta}_4$: Assume that $\hat{\eta}_4$ and $\hat{\eta}_4'$ are two 4–forms in the $\tilde{\xi}^a$ whose $\tilde{s}$ transformations equal $\hat{\omega}_3 + \hat{\omega}_4$. Then the difference $\hat{\eta}_4 - \hat{\eta}_4'$ is a $\tilde{s}$-invariant 4–form and thus vanishes according to lemma 1.

I use these lemmas in the following in order to prove that $\tilde{\omega}^5_{abel}$, $\tilde{\omega}^{5,\tau}_{nonabel}$ and $\tilde{\omega}^4_{FI}$ solve (1.4). The $\tilde{s}$-invariance of $\tilde{\omega}^4_{old}$, $\tilde{\omega}^{5,\tau}_{abel,chir}$ and $\tilde{\omega}^5_{FI}$ can be proved completely analogously, the $\tilde{s}$-invariance of (3.70) is a special case of $\tilde{s}\tilde{\omega}^5_{abel} = 0$ after replacing unprimed quantities by their primed counterparts.

4.1 Proof of $\tilde{s}\,\tilde{\omega}^5_{abel} = 0$

I first note that the decomposition (4.1) of $\tilde{\omega}^5_{abel}$ reads

$$\tilde{\omega}^5_{abel} = \hat{\omega}_2 + \hat{\omega}_3 + \hat{\omega}_4, \quad \hat{\omega}_2 = -4i{\sum_j}'\vartheta_{\dot{\alpha}}\bar{\vartheta}^{\dot{\alpha}}\tilde{C}^j\,\Omega_j \tag{4.8}$$

where $\hat{\omega}_2$ has been evaluated by means of (C.7). Furthermore one easily verifies by means of (3.20) that $\tilde{\omega}^5_{abel}$ does not depend on $\tilde{\xi}^{\dot{\alpha}}$:

$$\tilde{\omega}^5_{abel} = \tilde{\omega}^5_{abel}(\tilde{\xi}^a, \tilde{\xi}^\alpha, \tilde{C}^i, \Phi^r). \tag{4.9}$$

Using (2.17)–(2.19) one can check that

$$\text{on } \Phi^r,\ \tilde{\xi}^A \text{ and abelian } \tilde{C}^i\text{:} \quad \tilde{s} = \tilde{\xi}^A\tilde{\mathcal{D}}_A + \tilde{C}^I\delta_I, \quad \delta_I\tilde{\xi}^A = g_{IB}{}^A\tilde{\xi}^B \tag{4.10}$$

with $\tilde{\mathcal{D}}_{\dot{\alpha}}$ given by (3.20) and $\tilde{\mathcal{D}}_a$ and $\tilde{\mathcal{D}}_\alpha$ defined by

$$\begin{array}{llll} \tilde{\mathcal{D}}_a\Phi^r = \mathcal{D}_a\Phi^r, & \tilde{\mathcal{D}}_a\tilde{\xi}^b = 0, & \tilde{\mathcal{D}}_a\tilde{\xi}^{\underline{\alpha}} = \frac{1}{2}\tilde{\xi}^b T_{ab}{}^{\underline{\alpha}} + \tilde{\xi}^\beta T_{a\beta}{}^{\underline{\alpha}}, & \tilde{\mathcal{D}}_a\tilde{C}^i = \frac{1}{2}\tilde{\xi}^b F_{ab}{}^i + \tilde{\xi}^\alpha F_{a\alpha}{}^i, \\ \tilde{\mathcal{D}}_\alpha\Phi^r = \mathcal{D}_\alpha\Phi^r, & \tilde{\mathcal{D}}_\alpha\tilde{\xi}^a = 0, & \tilde{\mathcal{D}}_\alpha\tilde{\xi}^{\underline{\alpha}} = 0, & \tilde{\mathcal{D}}_\alpha\tilde{C}^i = 0. \end{array} \tag{4.11}$$

An important property of the operators $\tilde{\mathcal{D}}_{\dot\alpha}$ is that they can be used to construct a 'chiral projector' $(\tilde{\mathcal{D}}_{\dot\alpha}\tilde{\mathcal{D}}^{\dot\alpha}-M)$ for functions which do not carry dotted spinor indices and do not depend on $\tilde\xi^{\dot\alpha}$:

$$l_{\dot\alpha\dot\beta}\, f(\tilde\xi^a,\tilde\xi^\alpha,\tilde C^i,\Phi^r)=0\ \Rightarrow\ \tilde{\mathcal{D}}_{\dot\alpha}(\tilde{\mathcal{D}}_{\dot\beta}\tilde{\mathcal{D}}^{\dot\beta}-M)f(\tilde\xi^a,\tilde\xi^\alpha,\tilde C^i,\Phi^r)=0 \tag{4.12}$$

where $l_{\dot\alpha\dot\beta}=(l_{\alpha\beta})^*$ generates the $SL(2,\mathbf{C})$ transformations of dotted spinor indices, cf. (3.43). (4.12) follows from the fact that the $\tilde{\mathcal{D}}_{\dot\alpha}$ satisfy on the variables Φ^r, $\tilde\xi^a$, $\tilde\xi^\alpha$ and $\tilde C^i$ the same algebra as $\bar{\mathcal{D}}_{\dot\alpha}$ on the Φ^r, namely $\{\tilde{\mathcal{D}}_{\dot\alpha},\tilde{\mathcal{D}}_{\dot\beta}\}=-Ml_{\dot\alpha\dot\beta}$ as one easily checks by means of (3.20). Using $\delta_I\tilde\omega^5_{abel}=0$ one concludes by means of (4.10) and (4.12) that $\tilde s\tilde\omega^5_{abel}$ is given by

$$\tilde X:=\tilde s\,\tilde\omega^5_{abel}=(\tilde\xi^a\tilde{\mathcal{D}}_a+\tilde\xi^\alpha\tilde{\mathcal{D}}_\alpha)\,\tilde\omega^5_{abel}. \tag{4.13}$$

Now I prove that each term occurring in the decomposition (4.1) of $\tilde X$ vanishes. (4.8), (4.11) and (4.13) imply that this decomposition reads $\tilde X=\hat X_2+\hat X_3+\hat X_4$. First one verifies

$$\hat X_2=\tilde\xi^\alpha\tilde{\mathcal{D}}_\alpha\hat\omega_2=4i\,\bar\vartheta_{\dot\alpha}\bar\vartheta^{\dot\alpha}{\sum_j}'\tilde C^j\,\tilde\xi^\alpha\mathcal{D}_\alpha\Omega_j=0 \tag{4.14}$$

which holds due to $\mathcal{D}_\alpha\Omega_j=0$ and implies $\tilde X=\hat X_3+\hat X_4$. Since $\tilde X$ is $\tilde s$-invariant by construction one concludes by means of lemma 2 that there is a 4–form $\hat\eta_4$ such that $\delta_-\hat\eta_4=\hat X_3$. On the other hand one easily verifies that $\tilde X$ does not depend on $\tilde\xi^{\dot\alpha}$ since this holds already for $\tilde\omega^5_{abel}$, cf. (4.9), and since none of the $\tilde{\mathcal{D}}_A$ transformations (4.11) depends on $\tilde\xi^{\dot\alpha}$. This however contradicts $\delta_-\hat\eta_4=\hat X_3$ unless $\hat X_3=0$ since $\delta_-\hat\eta_4$ of course depends both on $\tilde\xi^\alpha$ and $\tilde\xi^{\dot\alpha}$ unless it vanishes. One concludes $\tilde X=\hat X_4=0$ by means of lemma 1 which proves $\tilde s\tilde\omega^5_{abel}=0$.

4.2 Proof of $\tilde s\,\tilde\omega^{5,\tau}_{nonabel}=0$

In order to prove that (3.15) solves (1.4) I show that

$$\tilde s\,\tilde q=Tr(\tilde{\mathcal{F}}^3),\quad \tilde q=Tr(\tilde C\tilde{\mathcal{F}}^2-\tfrac12\,\tilde C^3\tilde{\mathcal{F}}+\tfrac1{10}\,\tilde C^5), \tag{4.15}$$
$$\tilde s\,\tilde p=Tr(\tilde{\mathcal{F}}^3),\quad \tilde p=3\,\Xi\,Tr(\tilde\xi^\alpha\Lambda_\alpha\,\Lambda^\dagger\Lambda^\dagger+\tilde\xi_{\dot\alpha}\Lambda^{\dagger\,\dot\alpha}\,\Lambda\Lambda) \tag{4.16}$$

which implies $\tilde s\,\tilde\omega^{5,\tau}_{nonabel}=0$ due to $\tilde\omega^{5,\tau}_{nonabel}=\tilde q-\tilde p$. (4.15) is easily verified by means of

$$\tilde s\,\tilde C=-\tilde C^2+\tilde{\mathcal{F}},\quad \tilde s\,\tilde{\mathcal{F}}=\tilde{\mathcal{F}}\tilde C-\tilde C\tilde{\mathcal{F}} \tag{4.17}$$

which follows immediately from (2.19) and $\tilde s^2=0$. (4.16) is verified using

$$\tilde{\mathcal{F}}=\tfrac12\,\tilde\xi^a\tilde\xi^bF_{ab}{}^iT_i-\vartheta^\alpha\Lambda_\alpha-\bar\vartheta_{\dot\alpha}\Lambda^{\dagger\,\dot\alpha} \tag{4.18}$$

which is obtained by inserting the explicit expressions for $F_{AB}{}^i$ given in (3.2) into (3.22). Using $\vartheta^\alpha\vartheta^\beta\vartheta^\gamma=\bar\vartheta^{\dot\alpha}\bar\vartheta^{\dot\beta}\bar\vartheta^{\dot\gamma}=0$ one easily verifies that the decomposition (4.1) of $Tr(\tilde{\mathcal{F}}^3)$ reads

$$Tr(\tilde{\mathcal{F}}^3)=\hat\omega_3+\hat\omega_4,\quad \hat\omega_3=\tfrac32\left\{\vartheta^\alpha\vartheta_\alpha\bar\vartheta_{\dot\alpha}Tr(\Lambda^{\dagger\,\dot\alpha}\,\Lambda\Lambda)+\bar\vartheta_{\dot\alpha}\bar\vartheta^{\dot\alpha}\vartheta^\alpha Tr(\Lambda_\alpha\,\Lambda^\dagger\Lambda^\dagger)\right\}. \tag{4.19}$$

(Notice that $\tilde{\mathcal{F}}$ does not contain a part of degree 0 in the $\tilde\xi^a$ due to the contraints $F_{\underline{\alpha\beta}}{}^i=0$). Due to $\tilde s\,Tr(\tilde{\mathcal{F}}^3)=0$ one concludes by means of lemma 2 that in order to prove (4.16) one only has to verify $\delta_-\tilde p=\hat\omega_3$ which holds due to (C.5).

4.3 Proof of $\tilde{s}\,\tilde{\omega}^4_{FI}=0$

In order to calculate $\tilde{s}\,\tilde{\omega}^4_{FI}$ one uses

$$\begin{aligned} s\,\tilde{C}^j &= \tilde{\mathcal{F}}^j = \tfrac{1}{2}\,\tilde{\xi}^a\tilde{\xi}^b F_{ab}{}^j - i\vartheta^\alpha\lambda^j_\alpha - i\bar{\vartheta}_{\dot\alpha}\bar{\lambda}^{j\,\dot\alpha} \\ \Rightarrow\quad \tilde{s}\,(\tilde{C}^j\tilde{\mathcal{H}}) &= \tilde{\mathcal{F}}^j\tilde{\mathcal{H}} = i\tilde{\mathcal{F}}^j\Theta + (4\text{–form in } \tilde{\xi}^a) \end{aligned}$$

which holds for abelian $\tilde{C}^j$ due to (2.19) and (3.32). Using in addition (C.4) one easily checks that the decomposition (4.1) of $\tilde{s}\,\tilde{\omega}^4_{FI}$ is given by

$$\begin{aligned} \tilde{Y} &:= \tilde{s}\,\tilde{\omega}^4_{FI} = \sum_j{}' a_j\,(\hat{Y}^j_2+\hat{Y}^j_3)+\hat{Y}_4, \\ \hat{Y}^j_2+\hat{Y}^j_3 &= 2i\tilde{\mathcal{F}}^j\Theta - 2\bar{\lambda}^j\bar{\vartheta}\Theta - 2\vartheta\lambda^j\Theta + \tilde{\xi}^{\underline{\alpha}}(\mathcal{D}_{\underline{\alpha}}\bar{\lambda}^j_{\dot\beta})\bar{\eta}^{\dot\beta} - \eta^\beta\tilde{\xi}^{\underline{\alpha}}(\mathcal{D}_{\underline{\alpha}}\bar{\lambda}^j_\beta) - \tfrac{1}{3}\,\vartheta^\alpha\tilde{\xi}_{\alpha\dot\alpha}\bar{\vartheta}^{\dot\alpha}D^j. \end{aligned}$$

By means of the explicit form for $\tilde{\mathcal{F}}^j$ given above and the $\mathcal{D}_{\underline{\alpha}}$ transformations of λ^j given in table 1 one verifies $\hat{Y}^j_2+\hat{Y}^j_3=0$ which implies $\tilde{Y}=\hat{Y}_4$. By means of lemma 1 one finally concludes $\tilde{Y}=\tilde{s}\,\tilde{\omega}^4_{FI}=0$.

5 Conclusion

The result of the investigation of (1.1) for $g=0$ contains the statement that in the case of old minimal supergravity the action contains at the most one Fayet–Iliopoulos contribution, namely the one which corresponds to the R-transformations, while in the case of new minimal supergravity the action contains a Fayet–Iliopoulos contribution for each abelian factor of the gauge group apart from the one which corresponds to the R-transformations.

Among the solutions of (1.1) with $g=1$ there are in the case of new minimal supergravity up-to-now unknown anomaly candidates (3.34) which do not have counterparts in old minimal supergravity and are present if the gauge group contains at least two abelian factors. Furthermore the result contains the statement that local supersymmetry itself is not anomalous in the cases of old and new minimal supergravity since there are no solutions of (1.1) for $g=1$ in these cases which depend only on the supersymmetry ghosts and the "classical fields" but not on the ghosts C^I of the structure group. I remark that this result follows from the QDS structure [12], [9] of the theories discussed in subsections 3.1 and 3.2 and may become invalid for instance if one drops the assumption that all matter multiplets are chiral multiplets and allows for matter multiplets which do not have QDS structure.

Of course not only the matter multiplets must have QDS structure in order to guarantee that supersymmetry itself is not anomalous but this must hold also for the supersymmetry multiplets containing the torsions $T_{AB}{}^C$ and the curvatures $F_{AB}{}^I$. Examples for supergravity theories where these multiplets do not have QDS-structure are the nonminimal theories discussed in subsection 3.3. They indeed allow for solutions of (1.1) which do not depend on the C^I and which therefore provide candidates for anomalies of local supersymmetry. Some of these solutions have been given explicitly in (3.68). They seem to have close relationships to candidates for anomalies of R-transformations present in minimal supergravity as their construction given in subsection 3.3 suggests.

A Cohomology of δ_-

In order to prove (4.4) we first determine those δ_--invariant functions which do not depend on $\tilde{\xi}^\alpha$ (notice that they are nontrivial since each nonvanishing function of the form $\delta_- Y$ depends both on $\tilde{\xi}^\alpha$ and $\tilde{\xi}^{\dot\alpha}$). The result is:

$$\delta_- f(\tilde{\xi}^{\dot\alpha\alpha},\tilde{\xi}^{\dot\alpha})=0 \quad\Leftrightarrow\quad f=Q(\vartheta^\alpha,\tilde{\xi}^{\dot\alpha}). \tag{A.1}$$

In order to prove (A.1) we choose a basis for the functions $f(\tilde{\xi}^{\dot{\alpha}\alpha},\tilde{\xi}^{\dot{\alpha}})$ consisting of the following mutually distinct subsets $f_{i,n}$, $i=1,\ldots,10$, $n=0,1,\ldots$:

$$
\begin{aligned}
&f_{1,n}=\{\tilde{\xi}^{\dot{\alpha}_1}\ldots\tilde{\xi}^{\dot{\alpha}_n}\},\\
&f_{2,n}=\{\tilde{\xi}^{(\dot{\alpha}_1}\ldots\tilde{\xi}^{\dot{\alpha}_n}\tilde{\xi}^{\dot{\beta})\beta}\},\quad f_{3,n}=\{\tilde{\xi}^{\dot{\alpha}_1}\ldots\tilde{\xi}^{\dot{\alpha}_n}\tilde{\xi}_{\dot{\beta}}\tilde{\xi}^{\dot{\beta}\beta}\},\\
&f_{4,n}=\{\tilde{\xi}^{\dot{\alpha}_1}\ldots\tilde{\xi}^{\dot{\alpha}_n}\tilde{\xi}^{\alpha}{}_{\dot{\beta}}\tilde{\xi}^{\dot{\beta}\beta}\},\quad f_{5,n}=\{\tilde{\xi}^{(\dot{\alpha}_1}\ldots\tilde{\xi}^{\dot{\alpha}_n}\tilde{\xi}^{\dot{\beta}}{}_{\alpha}\tilde{\xi}^{\dot{\gamma})\alpha}\},\\
&f_{6,n}=\{\tilde{\xi}^{(\dot{\alpha}_1}\ldots\tilde{\xi}^{\dot{\alpha}_n}\tilde{\xi}^{\dot{\beta})}{}_{\alpha}\tilde{\xi}_{\dot{\gamma}}\tilde{\xi}^{\dot{\gamma}\alpha}\},\quad f_{7,n}=\{\tilde{\xi}^{\dot{\alpha}_1}\ldots\tilde{\xi}^{\dot{\alpha}_n}\tilde{\xi}_{\dot{\gamma}}\tilde{\xi}^{\dot{\gamma}\alpha}\tilde{\xi}_{\dot{\beta}}\tilde{\xi}_{\alpha}{}^{\dot{\beta}}\},\\
&f_{8,n}=\{\tilde{\xi}^{(\dot{\alpha}_1}\ldots\tilde{\xi}^{\dot{\alpha}_n}\tilde{\xi}^{\dot{\beta})\gamma}\tilde{\xi}_{\gamma\dot{\gamma}}\tilde{\xi}^{\dot{\gamma}\alpha}\},\quad f_{9,n}=\{\tilde{\xi}^{\dot{\alpha}_1}\ldots\tilde{\xi}^{\dot{\alpha}_n}\tilde{\xi}_{\dot{\beta}}\tilde{\xi}^{\dot{\beta}\gamma}\tilde{\xi}_{\gamma\dot{\gamma}}\tilde{\xi}^{\dot{\gamma}\alpha}\},\\
&f_{10,n}=\{\tilde{\xi}^{\dot{\alpha}_1}\ldots\tilde{\xi}^{\dot{\alpha}_n}\tilde{\xi}^{\dot{\beta}\beta}\tilde{\xi}^{\dot{\gamma}\gamma}\tilde{\xi}_{\beta\dot{\gamma}}\tilde{\xi}_{\gamma\dot{\beta}}\}
\end{aligned}
$$

where each $f_{i,n}$ denotes a complete irreducible $SL(2,\mathbf{C})$ multiplet, e.g. $f_{1,1}=\{\tilde{\xi}^{\dot{1}},\tilde{\xi}^{\dot{2}}\}$. Since δ_- commutes with the $SL(2,\mathbf{C})$ generators and changes the degree in the $\tilde{\xi}^{\dot{\alpha}\alpha}$ resp. $\tilde{\xi}^{\dot{\alpha}}$ by 1 resp. -1, one can investigate without loss of generality each subspace $f_{i,n}$ separately in order to determine the δ_--invariant functions $f(\tilde{\xi}^{\dot{\alpha}\alpha},\tilde{\xi}^{\dot{\alpha}})$. One easily verifies that only the functions contained in $f_{1,n}$, $f_{3,n}$ and $f_{7,n}$ are δ_--invariant and that they are those which are of the form $Q(\vartheta^{\alpha},\tilde{\xi}^{\dot{\alpha}})$. This proves (A.1). Analogously one can show that the δ_--invariant functions which do not depend on $\tilde{\xi}^{\dot{\alpha}}$ are of the form $P(\bar{\vartheta}^{\dot{\alpha}},\tilde{\xi}^{\alpha})$.

Now we investigate those functions $f(\tilde{\xi}^A)$ which vanish for $\tilde{\xi}^{\alpha}=0$ and for $\tilde{\xi}^{\dot{\alpha}}=0$:

$$f=\sum_{q,r,s,t}a_{qrst}(\tilde{\xi}^{\dot{\alpha}\alpha})\,(\tilde{\xi}^1)^q(\tilde{\xi}^2)^r(\tilde{\xi}^{\dot{1}})^s(\tilde{\xi}^{\dot{2}})^t,\quad a_{00st}=a_{qr00}=0. \tag{A.2}$$

Without loss of generality one can assume that $f(\tilde{\xi}^A)$ has definite ghost number and thus can be written as a polynomial in $\tilde{\xi}^1$ with coefficients which depend on $\tilde{\xi}^2$, $\tilde{\xi}^{\dot{\alpha}\alpha}$ and $\tilde{\xi}^{\dot{\alpha}}$:

$$f(\tilde{\xi}^A)=\sum_{n=0}^{\overline{n}}(\tilde{\xi}^1)^n f_n(\tilde{\xi}^2,\tilde{\xi}^{\dot{\alpha}\alpha},\tilde{\xi}^{\dot{\alpha}}). \tag{A.3}$$

δ_- is decomposed into

$$\delta_-=\tilde{\xi}^{\alpha}\tilde{D}_{\alpha},\quad \tilde{D}_{\alpha}=4i\,\tilde{\xi}^{\dot{\alpha}}\frac{\partial}{\partial\tilde{\xi}^{\dot{\alpha}\alpha}}. \tag{A.4}$$

$\delta_- f=0$ implies in particular

$$\tilde{D}_1 f_{\overline{n}}=0. \tag{A.5}$$

In order to solve (A.5) we define an operator r whose anticommutator with $\tilde{D}_1$ is the counting operator $\mathcal{N}$ for the variables $\tilde{\xi}^{\dot{\alpha}1}$ and $\tilde{\xi}^{\dot{\alpha}}$, $\dot{\alpha}=\dot{1},\dot{2}$:

$$r=-\tfrac{i}{4}\,\tilde{\xi}^{\dot{\alpha}1}\frac{\partial}{\partial\tilde{\xi}^{\dot{\alpha}}}\;\Rightarrow\;\{r,\tilde{D}_1\}=\tilde{\xi}^{\dot{\alpha}1}\frac{\partial}{\partial\tilde{\xi}^{\dot{\alpha}1}}+\tilde{\xi}^{\dot{\alpha}}\frac{\partial}{\partial\tilde{\xi}^{\dot{\alpha}}}=:\mathcal{N}. \tag{A.6}$$

Due to (A.2) f does not contain a zero mode of $\mathcal{N}$. Therefore one concludes by means of standard arguments (for instance by means of the Basic lemma [20]) that $f_{\overline{n}}$ is a trivial solution of (A.5) (notice that $(\tilde{D}_1)^2=0$):

$$f_{\overline{n}}=\tilde{D}_1 g_{\overline{n}}(\tilde{\xi}^2,\tilde{\xi}^{\dot{\alpha}\alpha},\tilde{\xi}^{\dot{\alpha}}). \tag{A.7}$$

This implies due to $\tilde{\xi}^1\tilde{D}_1=\delta_--\tilde{\xi}^2\tilde{D}_2$:

$$\overline{n}\neq 0:\quad f=\delta_- g+f',\quad f'=\sum_{n=0}^{\overline{n}-1}(\tilde{\xi}^1)^n f'_n(\tilde{\xi}^2,\tilde{\xi}^{\dot{\alpha}\alpha},\tilde{\xi}^{\dot{\alpha}}) \tag{A.8}$$

where

$$g=(\tilde\xi^1)^{\overline{n}-1}g_{\overline{n}},\quad f'_{\overline{n}-1}=f_{\overline{n}-1}-\tilde\xi^2\tilde D_2 g_{\overline{n}},\quad n<\overline{n}-1:\quad f'_n=f_n.$$

Notice that f' is a polynomial which has lower degree in $\tilde\xi^1$ than f and differs from f only by a trivial contribution. Therefore one can iterate the argument leading to (A.8) and conclude that a δ_--invariant polynomial of $\tilde\xi^1$ is trivial up to a part F which does not depend on $\tilde\xi^1$ at all. F is written as a polynomial in $\tilde\xi^2$ with coefficients $F_r(\tilde\xi^{\dot\alpha\alpha},\tilde\xi^{\dot\alpha})$ (F_0 vanishes due to (A.2)):

$$f=\delta_-h+F,\quad F=\sum_{r\geq 1}(\tilde\xi^2)^r F_r(\tilde\xi^{\dot\alpha\alpha},\tilde\xi^{\dot\alpha}). \tag{A.9}$$

$\delta_-f=0$ requires

$$\delta_-F_r(\tilde\xi^{\dot\alpha\alpha},\tilde\xi^{\dot\alpha})=0\quad\forall r \tag{A.10}$$

since δ_--invariant functions of different degree in the $\tilde\xi^\alpha$ have to be separately invariant (thus in fact one can assume without loss of generality that the sum $\sum_r$ in (A.9) contains only one nonvanishing contribution). By means of (A.1) one concludes from (A.10)

$$F_r(\tilde\xi^{\dot\alpha\alpha},\tilde\xi^{\dot\alpha})=Q_r(\vartheta^\alpha,\tilde\xi^{\dot\alpha}). \tag{A.11}$$

Thus F is a linear combination of δ_--invariant monomials $(\tilde\xi^2)^r(\tilde\xi^{\dot 1})^s(\tilde\xi^{\dot 2})^t(\vartheta^1)^v(\vartheta^2)^w$ where $v,w\in\{0,1\}$ since the ϑ^α anticommute. By means of the identities

$$\tilde\xi^2\tilde\xi^{\dot\alpha}=-\tfrac{i}{4}\,\delta_-\tilde\xi^{\dot\alpha 2},\quad \tilde\xi^2\vartheta^2=\tilde\xi^2\tilde\xi^{\dot\alpha 2}\tilde\xi_{\dot\alpha}=-\tfrac{i}{8}\,\delta_-(\tilde\xi^2{}_{\dot\alpha}\tilde\xi^{\dot\alpha 2}), \tag{A.12}$$

$$(\tilde\xi^2)^2\vartheta^1=(\tilde\xi^2)^2\tilde\xi^{\dot\alpha 1}\tilde\xi_{\dot\alpha}=-\tfrac{i}{4}\,\delta_-(\tilde\xi^2\tilde\xi^2{}_{\dot\alpha}\tilde\xi^{\dot\alpha 1}-\tfrac12\,\tilde\xi^1\tilde\xi^2{}_{\dot\alpha}\tilde\xi^{\dot\alpha 2}) \tag{A.13}$$

one concludes that these monomials are δ_--trivial except for those which satisfy $s=t=w=0$ and $r<2$. In fact F contains only one monomial with these properties since (A.2) excludes the values $r=0$ and $v=0$ in the case $s=t=w=0$ and one concludes

$$F=\sum_{r\geq 1}(\tilde\xi^2)^r Q_r(\vartheta^\alpha,\tilde\xi^{\dot\alpha})=\delta_-Y-2k\,\tilde\xi^2\vartheta^1. \tag{A.14}$$

The proof of (4.4) is completed by means of the identity

$$\tilde\xi^2\vartheta^1=\tfrac12(\tilde\xi^2\vartheta^1-\tilde\xi^1\vartheta^2)+\tfrac12(\tilde\xi^2\vartheta^1+\tilde\xi^1\vartheta^2)=-\tfrac12\,\Theta-\tfrac{i}{8}\,\delta_-(\tilde\xi^2{}_{\dot\alpha}\tilde\xi^{\dot\alpha 1}). \tag{A.15}$$

B Decomposition of $\tilde\omega^{5,\tau}_{nonabel}$

It is has been mentioned already that (3.21) does not depend on the gravitino. This remarkable fact holds since the gravitino depending contributions to $\tilde\xi^\alpha$, $\tilde\xi^{\dot\alpha}$ and $F_{ab}{}^i$ cancel in $\tilde{\mathcal{F}}$. Namely evaluating $\tilde{\mathcal{F}}$ explicitly by means of (2.20) and (2.6) one obtains

$$\tilde{\mathcal{F}}=\hat d\hat A+\hat A^2+\hat L,\quad \hat d=\hat C^n\partial_n,\quad \hat A=\hat C^n A_n{}^i T_i,\quad \hat L=\hat C^n(\xi\sigma_n\Lambda^\dagger-\Lambda\sigma_n\bar\xi) \tag{B.1}$$

with $\hat C^n$ as in (2.21). Using (B.1) and $\tilde C=C+\hat A$ one easily determines the decomposition (4.1) of (3.21):

$$\begin{aligned}
\tilde\omega^{5,\tau}_{nonabel}&=\sum_{p=0}^{4}\hat\omega_p,\\
\hat\omega_4&=Tr\{C\hat d(\hat A\hat d\hat A+\tfrac12\hat A^3)+(\hat L\hat A+\hat A\hat L)\hat d\hat A+\tfrac32\hat L\hat A^3-3\,\Xi\,(\xi\Lambda\,\Lambda^\dagger\Lambda^\dagger+\bar\xi\Lambda^\dagger\,\Lambda\Lambda)\},\\
\hat\omega_3&=Tr\{-\tfrac12\,(C^2\hat A+C\hat AC+\hat AC^2)\hat d\hat A-\tfrac12\,C^2\hat A^3+(\hat LC+C\hat L)\hat d\hat A\\
&\quad+\tfrac12\,(C\hat A^2-\hat AC\hat A+\hat A^2C)\hat L+\hat A\hat L^2\},\\
\hat\omega_2&=\tfrac12\,Tr\{-C^3\hat d\hat A+\hat AC\hat AC^2-(C^2\hat A+C\hat AC+\hat AC^2)\hat L+2C\hat L^2\},\\
\hat\omega_1&=\tfrac12\,Tr(C^4\hat A-C^3\hat L),\\
\hat\omega_0&=\tfrac{1}{10}\,Tr(C^5).
\end{aligned}$$

C Useful identities

$$
\begin{aligned}
&\Xi = -\tfrac{1}{24}\,\varepsilon_{abcd}\,\tilde{\xi}^a\tilde{\xi}^b\tilde{\xi}^c\tilde{\xi}^d = -\tfrac{i}{48}\,\tilde{\xi}^{\dot{\beta}\alpha}\tilde{\xi}_{\alpha\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}\beta}\tilde{\xi}_{\beta\dot{\beta}} && \text{(C.1)}\\
&\vartheta^\alpha = \tilde{\xi}_{\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}\alpha}, \quad \bar{\vartheta}^{\dot{\alpha}} = \tilde{\xi}^{\dot{\alpha}\alpha}\tilde{\xi}_\alpha, \quad \Theta = \tilde{\xi}_{\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}\alpha}\tilde{\xi}_\alpha && \text{(C.2)}\\
&\eta^\alpha = -\tfrac{i}{6}\,\vartheta^\beta\tilde{\xi}_{\beta\dot{\alpha}}\tilde{\xi}^{\dot{\alpha}\alpha}, \quad \bar{\eta}^{\dot{\alpha}} = \tfrac{i}{6}\,\tilde{\xi}^{\dot{\alpha}\alpha}\tilde{\xi}_{\alpha\dot{\beta}}\bar{\vartheta}^{\dot{\beta}} && \text{(C.3)}\\
&\delta_-\Xi = -\tfrac{1}{3}\,\vartheta^\alpha\tilde{\xi}_{\alpha\dot{\alpha}}\bar{\vartheta}^{\dot{\alpha}}, \quad \delta_-\eta^\alpha = 2\,\vartheta^\alpha\,\Theta = -\vartheta^\beta\vartheta_\beta\tilde{\xi}^\alpha, \quad \delta_-\bar{\eta}^{\dot{\alpha}} = 2\,\bar{\vartheta}^{\dot{\alpha}}\,\Theta = \bar{\vartheta}_{\dot{\beta}}\bar{\vartheta}^{\dot{\beta}}\tilde{\xi}^{\dot{\alpha}} && \text{(C.4)}\\
&\delta_-(\Xi\,\tilde{\xi}^{\dot{\alpha}}) = \tfrac{1}{2}\,\vartheta^\alpha\vartheta_\alpha\bar{\vartheta}^{\dot{\alpha}}, \quad \delta_-(\Xi\,\tilde{\xi}^\alpha) = \tfrac{1}{2}\,\bar{\vartheta}_{\dot{\alpha}}\bar{\vartheta}^{\dot{\alpha}}\vartheta^\alpha && \text{(C.5)}\\
&\tilde{\mathcal{D}}_{\dot{\alpha}}\Xi = -2i\,\bar{\eta}_{\dot{\alpha}} = -\tfrac{i}{3}\,\varepsilon_{abcd}\,\tilde{\xi}^\alpha{\sigma^a}_{\alpha\dot{\alpha}}\tilde{\xi}^b\tilde{\xi}^c\tilde{\xi}^d = -\tfrac{i}{3}\,\varepsilon_{abcd}\,\xi^\alpha{\sigma^a}_{\alpha\dot{\alpha}}\tilde{\xi}^b\tilde{\xi}^c\tilde{\xi}^d - 2i\,\Xi\,{\psi_a}^\alpha{\sigma^a}_{\alpha\dot{\alpha}} && \text{(C.6)}\\
&\tilde{\mathcal{D}}_{\dot{\alpha}}\tilde{\mathcal{D}}^{\dot{\alpha}}\Xi = -4i\,\bar{\vartheta}_{\dot{\alpha}}\bar{\vartheta}^{\dot{\alpha}} - 2M\Xi && \text{(C.7)}\\
&\qquad = 8i\tilde{\xi}^a\tilde{\xi}^b\xi\sigma_{ab}\xi + 16i\tilde{\xi}^a\tilde{\xi}^b\tilde{\xi}^c\xi\sigma_{ab}\psi_c + \Xi\,(16\psi_a\sigma^{ab}\psi_b - 2M) && \text{(C.8)}
\end{aligned}
$$

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