

Construction of a new electroweak sphaleron

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Abstract

We present a self-consistent ansatz for a new sphaleron in the electroweak standard model. The resulting field equations are solved numerically. This sphaleron sets the height of the energy barrier for the global $SU(2)$ anomaly.

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1 Introduction

In a series of papers [1, 2, 3] we have argued for the existence of a new static, but unstable, classical solution S^* in the electroweak standard model. The existence of this sphaleron S^* would be due to the presence of non-contractible spheres in configuration space. Here we give the explicit construction of this new classical solution. The self-consistent ansatz for S^* turns out to be a direct generalization of the configuration at the top of the non-contractible sphere as constructed in our previous papers. The same method can probably be used to discover other solutions, as will be explained later on.

The outline of this article is as follows. In sect. 2 we present the ansatz for the sphaleron S^* . The resulting expressions for the energy and field equations are discussed in sect. 3. The field equations are then solved numerically and some preliminary results are given in sect. 4. Section 5 contains further remarks on the sphaleron S^* and its physical interpretation.

2 Ansatz

We consider static classical fields in the bosonic sector of the electroweak standard model. These are the $SU(2)$ gauge fields $W_m \equiv W_m^a \sigma^a / (2i)$, the $U(1)$ hypercharge gauge field B_m and the complex Higgs doublet Φ . The $SU(2) \times U(1)$ Yang-Mills-Higgs theory has an energy functional

$$E = \int_{\mathbb{R}^3} d^3x \left[\frac{1}{4g^2} (W_{mn}^a)^2 + \frac{1}{4g'^2} (B_{mn})^2 + |D_m \Phi|^2 + \lambda (|\Phi|^2 - v^2/2)^2 \right] \quad (1)$$

and field equations

$$\begin{aligned} D_m W_{mn} &= g^2 \left[(D_n \Phi)^\dagger \frac{\sigma^a}{2i} \Phi - \Phi^\dagger \frac{\sigma^a}{2i} (D_n \Phi) \right] \frac{\sigma^a}{2i} \\ \partial_m B_{mn} &= g'^2 \left[(D_n \Phi)^\dagger \frac{1}{2i} \Phi - \Phi^\dagger \frac{1}{2i} (D_n \Phi) \right] \\ D_m D_m \Phi &= 2\lambda \left[\Phi^\dagger \Phi - v^2/2 \right] \Phi, \end{aligned} \quad (2)$$

with the following definitions for the field strengths and covariant derivatives

$$\begin{aligned} W_{mn} &\equiv W_{mn}^a \frac{\sigma^a}{2i} \equiv \partial_m W_n - \partial_n W_m + [W_m, W_n] \\ B_{mn} &\equiv \partial_m B_n - \partial_n B_m \\ D_k W_{lm} &\equiv \partial_k W_{lm} + [W_k, W_{lm}] \\ D_m \Phi &\equiv \left(\partial_m + W_m^a \frac{\sigma^a}{2i} + B_m \frac{1}{2i} \right) \Phi, \end{aligned}$$

where the indices run over the values 1, 2, 3, and σ^a are the standard Pauli matrices. The semiclassical masses of the W^\pm and Z^0 vector bosons are $M_W = \frac{1}{2} g v$ and $M_Z = M_W / \cos \theta_w$, with the weak mixing angle θ_w defined as $\tan \theta_w \equiv g'/g$. The mass of the single Higgs scalar is $M_H = \sqrt{8 \lambda/g^2} M_W$. The classical Yang-Mills-Higgs theory depends only on the two coupling constants λ/g^2 and θ_w .

Before we present the ansatz we introduce some notation. The standard cylindrical coordinates ρ , z and ϕ and spherical coordinates r , θ and ϕ are defined in terms of the cartesian coordinates by

$$(x_1, x_2, x_3) \equiv (\rho \cos \phi, \rho \sin \phi, z) \equiv (r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta).$$

We also define three matrices

$$\begin{aligned} \underline{u} &\equiv +\cos \phi \frac{\sigma^1}{2i} + \sin \phi \frac{\sigma^2}{2i} \\ \underline{v} &\equiv -\sin \phi \frac{\sigma^1}{2i} + \cos \phi \frac{\sigma^2}{2i} \\ \underline{w} &\equiv \frac{\sigma^3}{2i}, \end{aligned}$$

which take values in the Lie algebra of $SU(2)$. These matrices have several useful properties [4], such as $\underline{u} \cdot \underline{v} = -\underline{v} \cdot \underline{u} = \frac{1}{2} \underline{w}$ and further on cyclically.

We are now ready to present the ansatz. The $SU(2)$ and $U(1)$ gauge fields and the Higgs doublet are given by

$$\begin{aligned} W_1 &= \frac{\alpha_1}{\rho} \cos \phi \underline{v} + \frac{\alpha_2}{\rho} \sin \phi \underline{u} + \frac{\alpha_3}{\rho} \sin \phi \underline{w} \\ W_2 &= \frac{\alpha_1}{\rho} \sin \phi \underline{v} - \frac{\alpha_2}{\rho} \cos \phi \underline{u} - \frac{\alpha_3}{\rho} \cos \phi \underline{w} \\ W_3 &= \frac{\alpha_0}{z} \underline{v} \\ B_1 &= \tan^2 \theta_w \frac{\alpha_4}{\rho} \sin \phi \\ B_2 &= -\tan^2 \theta_w \frac{\alpha_4}{\rho} \cos \phi \\ B_3 &= 0 \\ \Phi &= \left[\beta_1 (\cos \phi i\sigma^1 + \sin \phi i\sigma^2) - \beta_2 i\sigma^3 \right] \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \end{aligned} \quad (3)$$

with

$$\begin{aligned} \alpha_0 &= \frac{4\rho z}{a} f_0 \\ \alpha_1 &= \frac{4\rho z}{a} f_1 \\ \alpha_2 &= \frac{4\rho z}{a} f_2 \end{aligned}$$

$$\begin{aligned}
\alpha_3 &= \frac{4\rho^2}{a} f_3 \\
\alpha_4 &= \frac{4\rho^2}{a} f_4 \\
\beta_1 &= \frac{2\rho z}{a} h_1 \\
\beta_2 &= h_2
\end{aligned} \tag{4}$$

and

$$a \equiv \rho^2 + z^2 + r_a^2,$$

where r_a is an arbitrary scale parameter. The axial functions $f_\mu = f_\mu(\rho, z)$ and $h_\nu = h_\nu(\rho, z)$, with $\mu = 0, 1, 2, 3, 4$ and $\nu = 1, 2$, are non-singular and have reflection symmetry

$$\begin{aligned}
f_\mu(\rho, z) &= f_\mu(\rho, -z) \\
h_\nu(\rho, z) &= h_\nu(\rho, -z),
\end{aligned} \tag{5}$$

Neumann boundary conditions on the z -axis

$$\begin{aligned}
\partial_\rho f_\mu(0, z) &= 0 \\
\partial_\rho h_\nu(0, z) &= 0,
\end{aligned} \tag{6}$$

and Dirichlet boundary conditions at infinity

$$\lim_{|x| \rightarrow \infty} \begin{pmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -\cos 2\theta \\ 1 + \cos 2\theta \\ 0 \\ 1 \\ -\cos 2\theta \end{pmatrix}. \tag{7}$$

The additional condition

$$f_1(0, z) = f_2(0, z) \tag{8}$$

ensures having regular gauge field configurations on the z -axis. The ansatz (3, 4) is self-consistent, as will be explained in sect. 3. It remains to solve for the axial functions f_μ and h_ν of the ansatz. This will be done numerically and the results will be presented in sect. 4.

The ansatz (3, 4), restricted to two independent axial functions $f(\rho, z)$ and $h(\rho, z)$, reproduces, up to a global gauge transformation, the field configurations

at the top of the non-contractible sphere as constructed in [2]. Specifically, the restricted axial functions f_μ and h_ν are

$$\begin{aligned}
f_0 &= \frac{(d^2/4 + \rho^2 + z^2) a}{b} f \\
f_1 &= \frac{(d^2/4 - \rho^2 - z^2) a}{b} f \\
f_2 &= \frac{(d^2/4 + \rho^2 - z^2) a}{b} f \\
f_3 &= \frac{2 z^2 a}{b} f \\
f_4 &= 0 \\
h_1 &= \frac{a}{\sqrt{b}} h \\
h_2 &= \frac{d^2/4 + \rho^2 - z^2}{\sqrt{b}} h, \tag{9}
\end{aligned}$$

with

$$b \equiv (\rho^2 + (z - d/2)^2) (\rho^2 + (z + d/2)^2)$$

and the boundary conditions

$$\begin{aligned}
f(0, \pm d/2) &= 0 \\
h(0, \pm d/2) &= 0 \\
\lim_{|x| \rightarrow \infty} f, h &= 1.
\end{aligned}$$

The parameter d of these configurations sets the distance between the two points on the z -axis where the Higgs fields vanishes, as will be discussed in the next section. The present ansatz (3, 4) is thus the axisymmetric generalization of a particular maximum configuration of a non-contractible sphere, with the number of independent axial functions increased from two to seven. The ansatz (3) has indeed the most general axisymmetric form, as discussed by Manton [4], whose notation we follow, up to some trivial modifications. The essential dynamics resides in the structure (4–8) for the axial functions α_μ and β_ν , see sect. 3 below.

At this point we can make three general remarks. First, we note that the behaviour at infinity of the $SU(2)$ gauge and Higgs fields of S^* , in the radial gauge, is essentially the same as for the well-known sphaleron S [5, 6], except for the dependence on the polar angle θ , which runs twice as fast (i.e. 2θ replacing θ) [1]. This suggests the interpretation of S^* as a bound state of two sphalerons S , or, more precisely, a sphaleron S and an anti-sphaleron \bar{S} . Secondly, we remark that the $U(1)$ field of our ansatz is consistent with having two magnetic dipoles aligned along the

z-axis, with the behaviour towards infinity $(B_1, B_2, B_3) \propto \tan^2 \theta_w r^{-3} (x_2, -x_1, 0)$. This would be a consequence of the interpretation of S^* as an $S-\bar{S}$ bound state and the fact that the sphaleron S itself has a magnetic dipole field [6]. Thirdly, we observe that the fields (3, 4) of S^* respect parity reflection symmetry, which is not the case for the configurations of the non-contractible sphere in general (the same holds for the sphaleron S and the corresponding non-contractible loop). In fact, there are only two configurations on the minimal non-contractible sphere [2] that have parity reflection symmetry, namely the vacuum and the sphaleron S^* . This completes, for the moment, our discussion of the ansatz.

3 Energy and field equations

It is a straightforward, but tedious, exercise to insert the ansatz (3, 4) into the energy functional (1). In terms of dimensionless distances (defined by $\tilde{x}_m \equiv x_m M_W$, and dropping the tildes) we find for the energy of the ansatz

$$E = \frac{4\pi v}{g} \int_0^\infty dz \int_0^\infty d\rho \rho e(\rho, z), \quad (10)$$

where the dimensionless energydensity $e(\rho, z)$ is an even function of z . Specifically, the energydensity is given by

$$e = e_{\text{WKIN}} + e_{\text{BKIN}} + e_{\text{HKIN}} + e_{\text{HPOT}} + e_{\text{GFIX}}, \quad (11)$$

with

$$\begin{aligned} e_{\text{WKIN}} = & + 4 \left[\partial_\rho \left(\frac{\rho f_0}{a} \right) - \partial_z \left(\frac{z f_1}{a} \right) \right]^2 \\ & + 4 \left[\partial_\rho \left(\frac{z f_2}{a} \right) + 4 \frac{\rho z}{a^2} f_1 f_3 - \frac{z}{\rho} \frac{1}{a} (f_1 - f_2) \right]^2 \\ & + 4 \left[\partial_z \left(\frac{z f_2}{a} \right) + 4 \frac{\rho^2}{a^2} f_0 f_3 - \frac{1}{a} f_0 \right]^2 \\ & + 4 \left[\partial_\rho \left(\frac{\rho f_3}{a} \right) - 4 \frac{z^2}{a^2} f_1 f_2 + \frac{1}{a} f_3 \right]^2 \\ & + 4 \left[\partial_z \left(\frac{\rho f_3}{a} \right) - 4 \frac{\rho z}{a^2} f_0 f_2 \right]^2 \end{aligned} \quad (12)$$

$$e_{\text{BKIN}} = + \tan^2 \theta_w \left(4 \left[\partial_\rho \left(\frac{\rho f_4}{a} \right) + \frac{1}{a} f_4 \right]^2 + 4 \left[\partial_z \left(\frac{\rho f_4}{a} \right) \right]^2 \right) \quad (13)$$

$$e_{\text{HKIN}} = + \left[\partial_\rho \left(\frac{2\rho z h_1}{a} \right) - 2 \frac{z}{a} f_1 h_2 \right]^2 + \left[\partial_\rho h_2 + 2 \frac{z}{a} f_1 \frac{2\rho z h_1}{a} \right]^2$$

$$\begin{aligned}
& + \left[\partial_z \left(\frac{2\rho z h_1}{a} \right) - 2 \frac{\rho}{a} f_0 h_2 \right]^2 + \left[\partial_z h_2 + 2 \frac{\rho}{a} f_0 \frac{2\rho z h_1}{a} \right]^2 \\
& + \left[2 \frac{\rho}{a} (f_3 + f_4 \tan^2 \theta_w) \frac{2\rho z h_1}{a} - 2 \frac{z}{a} (h_1 - f_2 h_2) \right]^2 \\
& + \left[2 \frac{\rho}{a} (f_3 - f_4 \tan^2 \theta_w) h_2 - 2 \frac{z}{a} f_2 \frac{2\rho z h_1}{a} \right]^2
\end{aligned} \tag{14}$$

$$e_{\text{HPOT}} = + 2 \frac{\lambda}{g^2} \left[\frac{4\rho^2 z^2 h_1^2}{a^2} + h_2^2 - 1 \right]^2, \tag{15}$$

and, for the moment, e_{Gfix} is set to zero. The energy density is finite everywhere and vanishes at infinity, for the appropriate behaviour of the axial functions.

We turn now to the classical field equations (2). Again, it is a straightforward exercise to insert the ansatz (3, 4). We find that the field equations reduce to seven partial differential equations for the seven functions f_μ and h_ν , which are identical¹ to the variational equations from the ansatz energy (10, 11). In short, our ansatz is self-consistent.

The solution of these partial differential equations for $f_\mu(\rho, z)$ and $h_\nu(\rho, z)$ depends on their boundary conditions (6–8) on the half-plane. The boundary conditions for h_2 , in particular, result in a curve $h_2 = 0$ in the ρ, z -plane that comes in from infinity at an angle $\theta = \pi/4$ and then hits, by necessity, either the z -axis or ρ -axis. With $\beta_1 = 0$ automatically on both axes, these points $\beta_2 = h_2 = 0$ have vanishing Higgs field Φ altogether. The two simplest possibilities correspond to having $\Phi = 0$ at two points on the z -axis or on a ring in the $z = 0$ plane (these two points or the ring may collapse to a single point at the origin). Whether or not such a molecule-like or vortex-like configuration is realized follows from the solution of the field equations. Of course, the ansatz used should be sufficiently general, in order to allow for both possibilities. Before we solve the field equations, there is one technical point that must be clarified.

The ansatz (3) still has a residual $U(1)$ gauge symmetry [4], which is generated by the $SU(2)$ transformation matrix $\exp[\omega(\rho, z) \underline{v}]$. Under these transformations $\tilde{\alpha}_0 \equiv \alpha_0/z$ and $\tilde{\alpha}_1 \equiv \alpha_1/\rho$ behave as 2-dimensional $U(1)$ gauge fields and the four other functions combine into two complex scalars (α_4 is invariant, of course). In order to eliminate this extra degree of freedom we choose the Lorentz gauge for our effective 2-dimensional euclidean gauge theory

$$\partial_z \tilde{\alpha}_0 + \partial_\rho \tilde{\alpha}_1 = 0, \tag{16}$$

¹ This is a manifestation of the so-called principle of symmetric criticality [8], which states that, under certain conditions, it suffices to consider variations that respect the symmetry of the ansatz.

which has also been used (under a different name) for the sphaleron S [7]. This condition can be implemented by adding a gauge fixing term to the energydensity (11)

$$e_{\text{GFIX}} = +\xi \left[\partial_z \left(\frac{\rho f_0}{a} \right) + \partial_\rho \left(\frac{z f_1}{a} \right) \right]^2, \quad (17)$$

with ξ an arbitrary parameter.

To summarize, we have shown that the ansatz (3, 4) solves the field equations, provided the axial functions stationarize the energy integral (10). Concretely, we have to solve the variational equations for f_μ and h_ν , with $\mu = 0, 1, 2, 3, 4$ and $\nu = 1, 2$, that result from variations δf_μ and δh_ν in (10, 11), with the gauge fixing term (17) included. It does not seem possible, however, to obtain an analytical solution for these functions and we have to resort to numerical methods.

4 Numerical solution

In this section we solve numerically the variational equations from the energy integral (10, 11). These variational equations consist of seven coupled non-linear partial differential equations (PDEs). Results are obtained for the case of approximately vanishing Higgs mass $\lambda/g^2 = 1/800$ (or $M_H/M_W = 1/10$) and weak mixing angle $\theta_w = 0$. We also have some preliminary results for the difficult, but more realistic, case $\lambda/g^2 = 1/8$ (or $M_H/M_W = 1$) and $\theta_w = \pi/6$. As to the numerical method, the variational equations are solved by numerical relaxation of the discretized energy (10, 11) and further details can be found in Appendix A of [3].

For the case $\lambda/g^2 = 1/800$ and $\theta_w = 0$, we solve the variational equations for the axial functions f_0, f_1, f_2, f_3, h_1 and h_2 ($f_4 = 0$). The general behaviour of these functions is shown in fig. 1. In particular, we see the $h_2 = 0$ curve coming in from infinity at a polar angle $\theta = \pi/4$ and hitting the z-axis just below $z = 2 M_W^{-1}$. As discussed in the previous section, this means that the solution has two points $z = \pm d^*/2$ on the symmetry axis where the Higgs field Φ vanishes. The resulting estimates for the internal core distance and energy of the sphaleron S^* are

$$\begin{aligned} d^* &\sim (4 \pm 2) M_W^{-1} \\ E_{S^*} &\sim (1.91 \pm 0.02) E_S, \end{aligned} \quad (18)$$

where $E_S = 1.57 4\pi v/g$ is the energy of the sphaleron S. The corresponding energydensity distribution (fig. 2) resembles that of a diatomic molecule, but the exact solution of the PDEs may very well be tighter than the one obtained numerically.

Note that our earlier approximation of S^* from the non-contractible sphere results in [3] gave essentially the same energy, but a larger core distance ($d_{\text{NCS}} \sim 7 M_W^{-1}$).

For the case $\lambda/g^2 = 1/8$ and $\theta_w = \pi/6$, we solve the variational equations for all seven axial functions f_μ and h_ν . The preliminary results are $d^* \sim (6 \pm 3) M_W^{-1}$ and $E_{S^*} \sim (1.99 \pm 0.02) E_S$. Clearly, the numerical accuracy has to be improved to obtain reliable numbers. On physical grounds, though, we expect an energy value below $2E_S$ and a finite core distance, as will be explained in the next section.

To summarize, the solution of the field equations for our ansatz appears to be a rather difficult numerical problem. Still, we have managed to obtain a first estimate of the solution, in particular for the case $\lambda/g^2 \sim 0$ and $\theta_w = 0$.

5 Discussion

We have constructed in this article a self-consistent ansatz for a new sphaleron S^* in the electroweak standard model. The axial functions of the solution can only be determined numerically. Whether or not the core distance d^* between the points of vanishing Higgs field remains finite is a dynamical question. In fact, the sphaleron S^* may be considered as a bound state of a sphaleron S and an anti-sphaleron \bar{S} , each with vanishing Higgs field at the center. Physically, we have for small values of the distance d between S and \bar{S} repulsion from the Yang-Mills fields and for large values of d attraction from either the Higgs scalar (if $M_H < M_W$) or from the parallel magnetic dipoles of the photon field (provided $\theta_w > 0$). Hence we expect, for generic values of M_H/M_W and θ_w , a finite core distance d^* of the sphaleron S^* , or, in other words, a localized solution.

In this paper we have employed a simple procedure to arrive at a new self-consistent ansatz, namely by generalizing the maximum configuration of a non-contractible sphere, while keeping the symmetries of the configurations of the non-contractible sphere. It is important to note that our maximum configuration is distinguished by an additional (discrete) symmetry. This construction method, which is obviously inspired by equivariant Morse theory [9], can probably also be used to discover other classical solutions. In particular, it should be possible to obtain a self-consistent ansatz for a new constrained instanton I^* , for which we have found a non-contractible loop of 4-dimensional euclidean configurations [3]. There is once more a rotational symmetry, which must be maintained when the number of axial functions is increased for the maximum configuration of the non-contractible loop.

² The resulting field equations will have to be solved numerically. Clearly, this will be quite demanding technically, but, in principle, we see no obstacle to the explicit construction of I^* . Note that, loosely speaking, the sphaleron S^* corresponds to a constant time slice of the instanton I^* , as explained in [3].

This brings us back to the sphaleron S^* and its possible physical relevance. The crucial observation seems to be that S^* sets the minimum height of the energy barrier for Witten's global $SU(2)$ anomaly, in the hamiltonian formulation [10]. This can be explained as follows. In hamiltonian $SU(2)$ gauge theory there exist non-contractible loops of 3-dimensional gauge transformations (based on non-trivial maps $S_1 \times S_3 \rightarrow SU(2) \sim S_3$) that may interfere with the implementation of Gauss' law, thereby giving rise to the anomaly. Consider now a non-contractible orbit of an arbitrary vacuum configuration (energy $E = 0$). This loop can be made contractible, simply by lifting it out of the vacuum and pulling it over an energy barrier (fig. 3a). The sphaleron S^* sits at the top of this barrier and $E_{\text{barrier}} = E_{S^*}$. Fixing the gauge completely (assuming the anomalies to cancel) results in a non-contractible sphere of 3-dimensional configurations (fig. 3b), with the unique vacuum V as its lowest point ($E = 0$) and the sphaleron S^* as its highest ($E = E_{S^*}$). This non-contractible sphere has spectral flow of the eigenvalues of the Dirac operator [10, 11]. Most likely, a pair of eigenvalues crosses through zero precisely at the top of the barrier, and the sphaleron S^* may be expected to have fermion zero modes.

The role of S^* as the barrier height for the global $SU(2)$ anomaly is similar to that of the standard sphaleron S for the chiral $U(1)$ anomaly. Both anomalies arise from the spectral flow of Dirac eigenvalues, over non-contractible spheres for the case of S^* and over non-contractible loops for the case of S . The physical consequences of these two sphalerons are, however, entirely different, since in the standard model only the chiral anomaly is coupled to a global quantum number ($B + L$). Moreover, the standard model has no $SU(2)$ anomaly to begin with, because the number of left-handed fermion doublets is even. For this reason we expect the new sphaleron S^* to play a role in the consistency and dynamics of the electroweak theory, rather than produce qualitatively new physical effects. Indeed, the most important application of the electroweak sphaleron S^* , or more appropriately its related constrained instanton I^* [3, 12], may turn out to be for the asymptotics of perturbation theory [13] and multiparticle production [14, 15]. These fundamental problems deserve further study

² In the ansatz of [3], at the top ($\omega = 0$) of the non-contractible loop, we first set the additional functions $g_{\pm} = 1$, then replace the functions $f(r, \tau)$, $h(r, \tau)$ by the more general ones $f(\rho, z, \tau)$, $h(\rho, z, \tau)$, and finally multiply their number to $f_{\mu}(\rho, z, \tau)$, $h_{\nu}(\rho, z, \tau)$.

and perhaps some progress can be made with our new classical solutions.

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References

- [1] F. Klinkhamer, Z. Phys. **C29** (1985), 153
- [2] F. Klinkhamer, Phys. Lett. **246B** (1990), 131
- [3] F. Klinkhamer, Existence of a new instanton in constrained Yang-Mills-Higgs theory, preprint NIKHEF-H/93-02 (March, 1993), hep-ph/9306208
- [4] N. Manton, Nucl. Phys. **B135** (1978), 319
- [5] R. Dashen, B. Hasslacher and A. Neveu, Phys. Rev. **D10** (1974), 4138
- [6] F. Klinkhamer and N. Manton, Phys. Rev. **D30** (1984), 2212
- [7] J. Kunz, B. Kleihaus and Y. Brihaye, Phys. Rev. **D46** (1992), 3587
- [8] R. Palais, Comm. Math. Phys. **69** (1979), 19
- [9] L. Sibner, R. Sibner and K. Uhlenbeck, Proc. Nat. Acad. Sci. USA **86** (1989), 8610
- [10] E. Witten, Phys. Lett. **117B** (1982), 324
- [11] P. Nelson and L. Alvarez-Gaumé, Commun. Math. Phys. **99** (1985), 103
- [12] F. Klinkhamer, Nucl. Phys. **B376** (1992), 255
- [13] J. Le Guillou and J. Zinn-Justin (eds.), *Large-order behaviour of perturbation theory*, North-Holland, Amsterdam, 1990
- [14] H. Aoyama and H. Goldberg, Phys. Lett. **188B** (1987), 506
- [15] A. Ringwald, Nucl. Phys. **B330** (1990), 1

Figure captions

Fig. 1 : Equidistant contours for the axial functions ($f_4 = 0$) in the ansatz (3, 4) of the sphaleron S^* . These functions give the numerical solution of the field equations, evaluated for the ansatz. The scale parameter of the ansatz is $r_a = M_W^{-1}$ and the coordinates ρ and z are also in units of M_W^{-1} . The coupling constants of the theory are $\lambda/g^2 = 1/800$ and $\theta_w = 0$.

Fig. 2: Energydensity $e(\rho, z)$ (with arbitrary normalization) of the sphaleron S^* , corresponding to the axial functions of fig. 1. The coordinates ρ and z are in units of M_W^{-1} and the complete configuration is obtained by reflection of z .

Fig. 3 : (a) Sketch of the energy surface over configuration space. A non-contractible loop in the vacuum, with fixed basepoint, becomes trivial by sliding over an energy barrier. The height of this energy barrier is given by the sphaleron S^* . A more realistic picture of configuration space has antipodal points on the disk shown identified and the vacuum circle replaced by the $SO(3)$ manifold, in order to exhibit the cyclicity of the first homotopy group.

(b) Non-contractible sphere in configuration space, with the gauge completely fixed.