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The universality of the shift of the Chern-Simons parameter for a general class of BRS invariant regularizations

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We consider a biparametric family of BRS invariant regularization methods of $SU(N)$ Chern-Simons theory (the parameters defining the family taking arbitrary values in $\mathbb{R}^2$) and show that the shift $k \to k + \text{sign}(k)\, N$ of the Chern-Simons parameter k occurs for arbitrary values of the family defining parameters. This supports irrefutably the conjecture that the shift of k is universal for BRS invariant regulators.

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The interest of perturbative quantization of $SU(N)$ Chern-Simons theory lies on the possibility of finding a series expansion of the Jones Polynomial [1] and of its generalizations [2], as well as of other topological invariants of three-manifolds, *e.g.* the partition function [3]. This program would procure a definition of topological invariants by means of techniques widely used in quantum field theory. It is worth mentioning along this line the discovery [4] of integral representations for the coefficients of the HOMFLY polynomial [2]. There are other reasons that support perturbative quantization of Chern-Simons theory; among them, the following two. First, perturbative quantization provides an alternative to canonical quantization as used in Ref. [3], where reduction to the physical degrees of freedom is prior to quantization [5]. It is important to emphasize in this regard that there is no a priori reason why perturbative quantization and canonical quantization as used in Ref. [3] should yield the same quantum theory so that agreement is not guaranteed, least trivial. Secondly, the perturbative framework constitutes a very useful tool for studying models with Chern-Simons gauge fields coupled to matter [6].

An important issue currently subject to debate in perturbative quantization of the theory is the meaning of radiative corrections to the Chern-Simons parameter and their dependence on the renormalization scheme one chooses (Ref. [7] versus Refs. [3,8,9]). The purpose of this paper is to provide incontestable evidence that one-loop radiative corrections to the classical Chern-Simons parameter k are universal for all gauge invariant regularization methods, thus validating the position defended in Refs. [3,8,9]. The latter radiative corrections give for k the shift [8,9]

$$k \to k + \operatorname{sign}(k)\, c_V \,, \tag{1}$$

where c_V denotes the quadratic Casimir operator in the adjoint representation of the gauge group $[\, c_V = N$ for $SU(N)\,]$. Here we do not attempt to give an explanation (still to be found) of this highly non-trivial universality but to exhibit it. We would like to stress that if the parameter k has the same meaning in perturbative quantization as in canonical quantization, the shift in eq. (1) is a necessary condition for the semiclassical evaluation of the partition function [10] to agree with the non-perturbative value as computed with surgery techniques [3].

To better understand what we mean, let us recall very briefly some well known results from standard local perturbative renormalization [11,12]. Let us consider a quantum field theory that is renormalizable by power counting and let us assume that there exists a *regularization method* that explicitly preserves the symmetries of the theory. If we denote by ϕ_B and k_B the bare fields and the bare parameters of the regularized quantum theory, perturbative renormalization ensures that there always exist dimensionless functions Z_ϕ and Z_k of the regulator such that the Green functions of the fields $\phi_R = Z_\phi^{-1}\phi_B$ as functions of the parameters $k_R = Z_k^{-1}k_B$ remain finite and satisfy the Ward identities as the regulator is removed. The fields ϕ_R and the parametres k_R are called renormalized fields and renormalized parameters and are finite as the regulator is removed; the functional that generates finite renormalized Green functions being called the renormalized effective action Γ. It is well known that the requirement of finite renormalized Green functions does not in general fix Z_ϕ nor Z_k, for any further symmetry-preserving finite renormalization still gives rise to finite renormalized Green functions. To eliminate this ambiguity one has to choose what is called a *renormalization scheme*, while preserving the symmetries of the theory. This is as far as one can go with standard local perturbative renormalization. Our conjecture following Ref. [13] is precisely that for Chern-Simons theory there is a natural choice for Z_k, namely

$$Z_k = 1\,. \tag{2}$$

This choice corresponds to taking $k_R = k_B \equiv k$, which is the parameter entering in eq. (1). Prior to the discussion of the reasons why we say the choice $Z_k = 1$ is natural is the observation that $Z_k = 1$ is only valid for finite theories and not for merely renormalizable theories (such as QCD), since for the latter ones $Z_k = 1$ does not take care of UV divergences. We recall in this regard that Chern-Simons theory is finite [14].

Of course, for the choice $Z_k = 1$ to make sense, all regularization methods preserving gauge invariance should give the same corrections to the parameter k. If this is the case, the parametrization eq. (2) is natural in the sense that gauge invariance, if preserved at the regularized level, fixes all ambiguities introudced by regularization. We would like to observe at this point that eq. (1) holds for all gauge invariant regularization methods used as yet [8,9] if $Z_k = 1$, and remind that the latter shift (1) is a necessary condition for the partition function evaluated non-perturbatively using surgery techniques [3] to agree with its semi-classical expression [10]. The parametrization $Z_k = 1$ is also natural in the sense that the value of k_R in terms of k_B needs not to be calculated at each order in perturbation theory. In the sequel we show that one-loop radiative corrections to the parameter k are the same for a biparametric family of gauge invariant regularization methods, pattern which has been observed for isolated gauge invariant regulators [3,8,9]. It is important to realize that uniqueness of quantum corrections to k does not mean that the renormalized effective action is unique. As a matter of fact, there are infinitely many different renormalized effective actions, each of them corresponding to a different choice of renormalization constants Z_ϕ for the fields. What happens is that wave-function renormalizations preserving BRS invariance are cohomologically trivial in the BRS sense and hence do not contribute to gauge invariant radiative corrections. Let us now move on to the description of our calculation.

The Chern-Simons classical action is given by

$$S_{\rm CS} = -\frac{ik}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho}\left(\frac{1}{2}\, A^a_\mu \partial_\nu A^a_\rho + \frac{1}{3!}\, f^{abc} A^a_\mu A^b_\nu A^c_\rho\right) ,$$

with obvious notation and k for convenience chosen to be positive. Our regularization method consists in adding to the classical action a higher covariant derivative term so that the action becomes

$$S_\Lambda = S_{\rm CS} + \frac{k}{4\pi}\int d^3x\,\left[\frac{u}{4\Lambda}\,F^a_{\mu\nu}F^{a\,\mu\nu} - \frac{iv}{2\Lambda^2}\,\epsilon^{\mu\nu\rho}F^a_{\mu\sigma}(D_\nu F_\rho{}^\sigma)^a + \frac{1}{4\Lambda^3}\,(D_\rho F_{\mu\nu})^a(D^\rho F^{\mu\nu})^a\right]$$
$$+\int d^3x\,\left[-b^a\partial A^a + (J^{a\mu}-\partial^\mu\bar{c}^a)\,(D_\mu c)^a - \frac{1}{2}\,f^{abc}H^a c^b c^c\right]\,,$$

where we have already included the standard gauge fixing term in the Landau gauge $\partial A^a=0$. Here u and v are arbitrary real parameters with $uv\neq 1$ if u is negative. As usual, the higher covariant derivative terms here added do not completely regularize the theory, for there is a finite number of 1PI Feynman diagrams still divergent by power counting. We regularize the latter diagrams by using 't Hooft-Veltman's dimensional regularization method for theories involving parity violating objects [12]. There are other dimensional regularization prescriptions but they are either algebraically inconsistent or violate BRS invariance (we refer the interested reader to Ref. [8] for details). In this way we define renormalized Chern-Simons theory in the scheme $Z_k=1$ as the limit $\Lambda\to\infty$ of the limit $D\to 3$ of the dimensionally regularized theory whose classical action is S_Λ. The calculation of the one-loop renormalized Chern-Simons effective action in the scheme eq. (2) is then performed as follows:

Step 1. We begin recalling the expression of the local part of the renormalized effective action up to one loop for Chern-Simons theory:

$$\Gamma^{\rm local} = -\frac{i(k+\alpha)}{4\pi}S_{\rm CS} + \int d^3x\,\left\{-b^a\partial A^a + \Delta\left[\,\beta\,(J^{a\mu}-\partial^\mu\bar{c}^a)A^a_\mu - (1+\gamma)\,H^a c^a\,\right]\right\}\,,$$

α, β and γ being arbitrary coefficients of order $\hbar$ and Δ the Slavnov-Taylor operator for the theory (see Ref. [8] for details). There are two types of radiative corrections: gauge invariant radiative orrections, labelled by α, and gauge dependent radiative corrections, labelled by β and γ. We are going to show that gauge invariant corrections do not depend neither on u nor v. However, gauge non-invariant corrections will depend on u and v but, being cohomologically trivial with respect to the Slavnov-Taylor operator, can be absorbed by a wave-function renormalization thus not contributing to the observables. Using that the renormalized effective action generates renormalized 1PI Green functions, the coefficients α, β and γ can be uniquely determined by computing the vacuum polarization tensor $\Pi^{ab}_{\mu\nu}$, the ghost self-energy Ω^{ab} and the Hcc-vertex V^{abc} at one loop. To calculate the latter Green functions we evaluate the limit $\Lambda\to\infty$ (step 3) of the limit $D\to 3$ (step 2) of their regularized counteparts $\Pi^{ab}_{\mu\nu}(D,\Lambda)$, $\Omega^{ab}(D,\Lambda)$ and $V^{abc}(D,\Lambda)$.

Step 2. In computing the limit $D\to 3$ we have to properly take care of evanescent operators [8]. It turns out that evanescent objects do not contribute to the three-dimensional limit so the latter limit can be calculated using the D-dimensional gauge field propagator

$$D^{ab}_{\mu\nu}=\frac{4\pi}{k}\,\frac{\delta^{ab}}{q^2\,P(q^2,\Lambda;u,v)}\;\left[\Lambda^4\,(\Lambda^2+vq^2)\,\epsilon_{\mu\rho\nu}q^\rho+\Lambda^3\,(u\Lambda^2+q^2)\,(q^2g_{\mu\nu}-q_\mu q_\nu)\right]\,,$$

where $\epsilon_{\mu\rho\nu}$ in D dimensions is understood in the 't Hooft-Veltman sense [8] and

$$P(q^2,\Lambda;u,v)=\Lambda^2\,(\Lambda^2+vq^2)^2+q^2\,(u\Lambda^2+q^2)^2\,.$$

Note that u and v are such that $P(q^2,\Lambda;u,v)$ is positive definite for any q^μ thus ensuring that power counting and positivity of the real part of the action S_Λ hold. The calculation of the limit $D\to 3$ of the Green functions we are interested in (and of any other Green function of the regularized theory) only involves elementary properties of dimensional regularization [12] and is performed straightforwardly, the result being finite and Λ-dependent. This implies perturbative finiteness for the family of theories defined by S_Λ and constitutes a necessary condition for the computation of the large Λ limit to make sense.

Step 3. We next calculate the limit $\Lambda\to\infty$ of the three-dimensional Green functions $\Pi^{ab}_{\mu\nu}(\Lambda)$, $\Omega^{ab}(\Lambda)$ and $V^{abc}(\Lambda)$ obtained in step 2. The large Λ limit of the Feynman integrals entering these Green functions can be evaluated using the m-theorem in Ref. [8], the result having the form

$$\Pi^{ab}_{\mu\nu}(p) = c_V\, J(u,v)\epsilon_{\mu\rho\nu}p^\rho\, \delta^{ab} \qquad \Omega^{ab}(p) = -\frac{c_V}{k} I(u,v)p^2\,\delta^{ab} \qquad V^{abc}(p_1,p_2) = 0,$$

where p^μ, p_1^μ and p_2^μ are external momenta,

$$J(u,v) = -\frac{2}{3\pi}\int_{-\infty}^{\infty} dt\, \frac{F(t^2;u,v)}{P^2(t^2,1;u,v)} \qquad\qquad I(u,v) = \frac{2}{3\pi}\int_{-\infty}^{\infty} dt\, \frac{u+t^2}{P(t^2,1;u,v)}$$

and

$$\begin{aligned} F(t^2;u,v) &= v\,t^{10} - 2\,(uv + 2\,v^3 + 4)\,t^8 - (3\,u^2v - 3\,uv^3 + 25\,u + 25\,v^2)\,t^6 \\ &\quad - 2\,(11\,u^2 + 4\,uv^2 + 13\,v)\,t^4 - (5\,u^3 + 13\,uv + 5)\,t^2 - 2\,u\,. \end{aligned}$$

Note that $J(u,v)$ and $I(u,v)$ are non-trivial functions of u and v. To convince oneself that they are not constants, it is enough to take *e.g.* $uv = 1$ with u positive to obtain $J(1/v,v) = (7{+}3v^{3/2})/3(1{+}v^{3/2})$ and $I(1/v,v) = 2/3(1{+}v^{3/2})$. Taking now into account the arbitrariness in the choice of Z_ϕ and introducing the notation $Z_\phi \equiv 1 + z_\phi$, it is straightforward to derive the following values for α, β and γ:

$$\alpha = c_V\,[\,J(u,v) - 2\,I(u,v)\,] \qquad \beta + z_A = \frac{4}{3}\,\frac{c_V}{k}\,I(u,v) \qquad \gamma + 2\,z_c + z_H = 0\,.$$

We remind that Z_ϕ, hence z_ϕ are finite and can take arbitrary values as far as they preserve BRS invariance, that is as far as they satisfy $Z_A Z_{\bar c} = Z_c Z_H$, or equivalently $z_A + z_{\bar c} = z_c + z_H$. To derive a more compact expression for α, we evaluate $J(u,v)$ and $I(u,v)$ in terms of the roots of the polynomial $P(t^2,1;u,v)$. We do the latter by writing u and v in terms of the roots, by using residue techniques and by recalling that $P(t^2,1;u,v)$ is positive definite so the integrand in $J(u,v)$ and $I(u,v)$ does not have poles on the real axis. After some lengthy calculations we finally obtain that

$$\alpha = c_V \, .$$

In summary, the renormalized effective action in the scheme $Z_k = 1$ depends in general on the parameters u and v but its gauge invariant part never does. This exhibits the universality of the one-loop shift $k \rightarrow k + c_V$ for the family of gauge invariant regulators we have considered.

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