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A PERTURBATIVE APPROACH TO B DECAYS INTO TWO π MESONS

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Abstract

The modified perturbative approach in which transverse degrees of freedom as well as Sudakov suppressions are taken into account, is applied to B decays into two π mesons. The influence of various model parameters (CKM matrix elements, B decay constant, mesonic wave functions) on the results as well as short distance corrections to the weak Hamiltonian are discussed in some detail. The perturbative contributions to the B decays yield branching ratios of the order of $10^{-7} - 10^{-6}$ which values are well below the upper limit for the $\bar{B}^0 \to \pi^+\pi^-$ branching ratio as measured by CLEO.

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I. INTRODUCTION

Recently the rare decay modes of the B meson into light mesons, e.g., $\bar{B}^0 \to \pi^+\pi^-$, attracted much attention of theoreticians although such decays have barely been observed (see, however, ref. [1]). The reason for this interest lies in the expectation that, owing to the large momentum release, such processes are amenable to a perturbative treatment. If that is the case the rates for such B decays can be calculated in a similar fashion as for instance the large momentum transfer behavior of the pion's electromagnetic form factor. The perturbative approach would form an attractive alternative to phenomenological models based on soft physics (e.g., overlaps of wave functions and/or vector meson dominance [2]). A reliable estimate of the rates Γ for exclusive non-leptonic B decays is also of enormous importance in the investigation of CP violation. For the decays of charged B mesons, for instance, one studies asymmetries between the rates of CP conjugated processes. A non-zero rate asymmetry, signaling CP violation, requires phase differences in the interfering amplitudes which may be generated by penguin diagrams.

The first perturbative calculation of exclusive B decays has been carried out by Szczepaniak, Henley and Brodsky [3]. Other authors [4–8] have repeated that analysis using similar methods. The results for the decay widths obtained by the various authors agree with each other to a certain extent. However, there is a technical difficulty which has not been solved in a satisfactory way by these authors. As a consequence of the collinear approximation used in these analyses, i.e. of the neglect of the transverse degrees of freedom, a singularity appears in the end-point regions where one of the partons carries most of the momentum of its parent hadron. It is presumed that a Sudakov factor damps the end-point regions sufficiently and a finite, stable result remains. This conjecture is taken as a justification for cutting off the end-point regions, a procedure which leads to an unwelcome strong dependence of the results on the position of the cut-off.

Botts and Sterman [9] have recently calculated a Sudakov factor, comprising gluonic radiative corrections, for exclusive reactions in next-to-leading log approximation using resummation and renormalization group techniques. This approach necessitates the explicit consideration of the transverse degrees of freedom which, as we mentioned above, are conventionally neglected. The Botts-Sterman approach which may be termed the modified perturbative approach, has already been applied to several exclusive reactions, among them is the pion's and the nucleon's electromagnetic form factor [10–14]. It has turned out that the modified perturbative approach allows a reliable, theoretically self-consistent calculation of the perturbative contributions to electromagnetic form factors. Previous objections [15] against the applicability of the standard perturbative approach do not apply to the modified approach.

The aim of the present paper is to perform a detailed analysis of B decays into two π mesons using the modified perturbative approach. As it will turn out, the above-mentioned singularity disappears, hence there is no need for cutting off the range of integration. A second advantage of the modified approach is that it allows both a simple parameter-free treatment of the strong coupling constant and a choice of an appropriate renormalization scale which keeps under control contributions from higher order perturbation theory. Owing to these two advantages the modified perturbative approach provides a reliable estimate of the perturbative contributions to B decays into two π mesons. This will be elucidated in some detail in the body of the text. Comparison with future experimental results will reveal whether the processes under investigation are indeed dominated by perturbative contributions or whether they are still under control of soft physics. It should be emphasized that of all experimentally accessible exclusive decays into hadronic final states the process $B \to \pi\pi$ has the largest momentum release and can therefore be considered as a hard processes best. If here the perturbative approach fails in comparison with experiment its application to other exclusive non-leptonic decay processes seems dubious.

In Sec. 2 we discuss the calculation of the decay rate for the process $\bar{B}^0 \to \pi^+\pi^-$ within the modified perturbative approach. The numerical results and a discussion of a number of approximations made in the calculation are presented in Sec. 3. Short distance corrections will be discussed in Sec. 4 as well as a calculation of other $B \to \pi\pi$ decay modes. The weak $B \to \pi$ transition form factors can be calculated in a similar fashion as the $B \to \pi\pi$ decay rates. Results for transition form factors are presented in Sect. 5. The paper terminates with a few concluding remarks (Sec. 6). In an appendix some details about the Sudakov factor are presented.

II. THE MODIFIED PERTURBATIVE APPROACH TO B DECAYS

We start with the process, $\bar{B}^0 \to \pi^+\pi^-$, and consider the unperturbed weak Hamiltonian

$$\mathcal{H}_W = 4\frac{G_F}{\sqrt{2}} v_u (\bar{u}_\alpha \gamma_\mu L b_\alpha)(\bar{d}_\beta \gamma^\mu L u_\beta) \tag{2.1}$$

where α and β are color labels. L is the left-handed projection operator. G_F is the usual Fermi constant and $v_u = V_{ud}^* V_{ub}$ where the V_{ij} denote the CKM matrix elements. Because of the large momentum release in the decay of the B meson into two light mesons it seems possible to apply perturbative QCD. For the same reason one may also neglect long-range final state interactions. On these premises the decay amplitude $\mathcal{M} = \langle \pi^+\pi^- | \mathcal{H}_W | \bar{B}^0 \rangle$ can be calculated along the same lines as for instance the pion's electromagnetic form factor. With the help of the factorization formula of perturbative QCD for exclusive reactions the decay amplitude is written as a convolution of meson wave functions and a hard scattering amplitude T_H to be calculated from the effective Hamiltonian (2.1) and, to leading order in the strong coupling constant α_S, the exchange of a gluon between the spectator quark and one of the other quarks (see Figs. 1 and 2). In principle there are contributions from higher Fock states too where additional quarks and/or gluons appear in the mesons. Such contributions will be ignored by us since they are suppressed by powers of α_S/M_B^2 (M_B denotes the mass of the B meson). In order to calculate the decay amplitude we utilize the modified perturbative approach. The crucial element for incorporating the Sudakov corrections in the calculation of that amplitude within the modified perturbative approach is the explicit dependence on the transverse degrees of freedoms in the convolution of wave functions and hard scattering amplitude. This convolution formula can formally be derived by using the methods described in detail by Botts and Sterman [9]. Adapting Li and Yu's result for the semi-leptonic $B \to \pi$ transition matrix element [16] to our case of the $B \to \pi\pi$ decay amplitude, the convolution formula can be written in the form

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} v_u \int [\mathrm{d}x] \left[\frac{\mathrm{d}^2\mathbf{b}}{4\pi}\right] \hat{\Psi}^*_{\pi_1}(x_1, \mathbf{b}_1)\, \hat{\Psi}^*_{\pi_2}(x_2, \mathbf{b}_2)\, \hat{T}_H(\{x\}, \{\mathbf{b}\}, M_B) \hat{\Psi}_B(x, -\mathbf{b}) \times \exp\left[-S(\{x\}, \{b\}, M_B)\right]. \tag{2.2}$$

x (x_i, $i = 1, 2$) denotes the usual fraction of the p_B^+ (p_1^+, p_2^-) component of the $B(\pi_1 = \pi^+$, $\pi_2 = \pi^-)$ meson's momentum the quark carries. The antiquarks carry the fractions

$1-x$ and $1-x_i$ respectively. $\mathbf{b}$ ($\mathbf{b}_i$) is the quark-antiquark transverse separation in the B (π_i) meson. $[\mathrm{d}x]$ is short for the product $\mathrm{d}x\mathrm{d}x_1\mathrm{d}x_2$ and $\{x\}$ stands for the set of variables x, x_1, x_2; analogous definitions are used for the $\mathbf{b}$ variables. Last $\hat{\Psi}_h (h = B, \pi_i)$ is the Fourier transform of the momentum space wave function Ψ_h:

$$\hat{\Psi}_h(x_h, \mathbf{b}_h) = \left(\frac{1}{2\pi}\right)^2 \int \mathrm{d}^2 k_{\perp h} \Psi_h(x_h, \mathbf{k}_{\perp h})\, e^{-i\mathbf{k}_{\perp h}\cdot\mathbf{b}_h} \tag{2.3}$$

where the Fourier transform variable $\mathbf{k}_{\perp h}$ is the quark's transverse momentum defined with respect to the momentum of the meson h. Strictly speaking the meson wave function represents only the soft part, i. e. the full wave function with the perturbative tail removed from it. The full meson state is described by the product of the (scalar) wave function Ψ_h, a color and a flavor function of obvious form and of the spin wave function which is written in a covariant fashion

$$\Gamma_h = \frac{1}{\sqrt{2}} \left(\not{p}_h + M_h\right) \left(1 + \frac{g_h(x_h)}{M_h}\, \not{K}_h\right) \gamma_5. \tag{2.4}$$

K_h^μ is the relative momentum of quark and antiquark forming the meson h and M_h is the meson's mass. Note that the relative momentum is only determined up to a multiple of the hadron's momentum p_h^μ. The term $\sim \not{K}_h$ in (2.4) takes into account the fact that quark and antiquark do not move collinear with their parent hadron; it represents the first term of an expansion over powers of K. The function $g_h(x_h)$ is controlled by soft physics as is the wave function Ψ_h; it cannot be calculated from first principles at the time being. Therefore, one has to rely on models if its explicit form is needed. Fortunately, as will become clear in a moment, we have not to specify g_h to the order we are working.

In (2.2) $\hat{T}_H$ represents the Fourier transform of the hard scattering amplitude. To lowest order perturbation theory, it is to be calculated from the Feynman graphs shown in Fig. 2. In order to perform that calculation it is convenient to work in the rest frame of the B meson where the hadronic momenta in light-cone coordinates read

$$p_B^\mu = (M_B/\sqrt{2}, M_B/\sqrt{2}, \mathbf{0}_\perp), \qquad p_1^\mu = (M_B/\sqrt{2}, 0, \mathbf{0}_\perp), \qquad p_2^\mu = p_B^\mu - p_1^\mu. \tag{2.5}$$

The quark momenta are specified in Fig. 1. In a possibly crude approximation, yet well in line with the basic ideas of the parton model, we chose the relative momenta K_h^μ in such a way that they have only transverse components $\mathbf{k}_{\perp h}$ and, hence, the K_h^μ are all orthogonal to the mesonic momenta. The wave function for the B meson is known to have a pronounced peak at $x = x_0 = m_b/M_B$ (m_b being the b-quark mass). It is clear therefore that only regions close to the peak position contribute to any degree of significance. This implies, to a very good approximation, that the b-quark mass equals xM_B. In accord with the heavy quark effective theory it also implies equal velocities of the b-quark and of the B meson up to corrections of order $k_\perp/M_B$. Since the r.m.s. transverse momentum is of the order of 300 to 400 MeV these corrections can safely be ignored. The masses of the light quarks and mesons are neglected in the kinematics.

Given these assumptions and approximations the denominators of the internal quark and gluon propagators in the graphs Fig. 2a and 2b read

$$\begin{aligned} D_b &= q_b^2 - m_b^2 = (1-2x+x_1)\ M_B^2 - \mathbf{k}_{\perp 1}^2 \\ D_1 &= q_1^2 = -(1-x)\ M_B^2 - \mathbf{k}_\perp^2 \\ q_G^2 &= -(1-x)(1-x_1)\ M_B^2 - (\mathbf{k}_\perp - \mathbf{k}_{\perp 1})^2 \end{aligned} \qquad (2.6)$$

where, following previous authors, e.g., [3–8], we neglect terms $\sim (1-x)^2$. If one would ignore the $(1-x)$-term in D_b which is equivalent to the assumption $m_b = M_B$, the denominators of the internal partons are of the same type as in the case of the pion's electromagnetic form factor with the B-meson mass playing the rôle of the momentum transfer. It has been shown [10,12] that for momentum transfers of the order of the value of the B-meson mass perturbation theory can be applied self-consistently. In our numerical evaluations of the decay width we, however, take into account the $(1-x)$-term in D_b. It leads to a pole within the range of integration which is handled in the usual way by using the prescription $1/(x-i\varepsilon) = P(1/x) + i\pi\delta(x)$. This pole which corresponds to the situation of the b-quark propagator going on-shell, is not a pinched singularity and is, therefore, not associated with long distance propagations of the b quark. Hence, a perturbative treatment of the pole contribution is justified [17]. For a detailed discussion of this pole we refer to [18].

The hard scattering amplitude T_H can now be easily worked out as traces of products of the mesonic spin wave functions and spinor expressions representing the graphs shown in Fig. 2. From the color structure of the operator (2.1) it is clear that the graphs shown in Figs. 2c and 2d do not contribute to the process $\bar{B}^0 \to \pi^+\pi^-$. For graph 2a, one finds

$$T_b \sim Tr\left\{\bar{\Gamma}_{\pi_1}\gamma_\mu L(\not{q}_b + m_b)\gamma_\nu \Gamma_B \gamma^\nu\right\} Tr\left\{\bar{\Gamma}_{\pi_2}\gamma^\mu L\right\} \tag{2.7}$$

where, as usual, $\bar{\Gamma} = \gamma_0 \Gamma^\dagger \gamma_0$. Working out the traces we find the simple results

$$T_b(\{x\},\{\mathbf{k}_\perp\}) = 16\sqrt{2}\pi\alpha_s(\mu)\frac{(2x - x_1)\; M_B^4}{D_b q_G^2}\left[1 + \mathcal{O}(k_\perp/M_B, k_{\perp 1}/M_B)\right] \tag{2.8}$$

and, since we neglect the pion mass, for the graph 2b

$$\frac{|T_1|}{|T_b|} = \mathcal{O}(k_\perp/M_B, k_{\perp 1}/M_B). \tag{2.9}$$

μ, the renormalization scale, will be chosen subsequently. A color factor is not included in T_b. The Fourier transform of T_b reads

$$\begin{aligned} \hat{T}_b(\{x\},\{\mathbf{b}\}) = & \frac{4\sqrt{2}}{\pi}\alpha_s(\mu)M_B^4(2x - x_1)K_0(\sqrt{(1-x)(1-x_1)}M_B b) \\ & \times K_0(\sqrt{2x - x_1 - 1}M_B|\mathbf{b} + \mathbf{b}_1|)\delta(\mathbf{b}_2) \end{aligned} \tag{2.10}$$

where K_0 is the modified Bessel function of order zero. Note that for $1 + x_1 > 2x$ the second Bessel function in (2.10) has an imaginary argument. In this case we use the relation $K_0(iz) = -i/2H_0^{(1)}(z)$ where $H_0^{(1)}$ is a Hankel function. This treatment of the Fourier transform of D_b is in accord with the $i\varepsilon$ prescription mentioned earlier.

The last item in (2.2) to be specified, is the Sudakov factor $\exp[-S]$ which comprises those gluonic radiative corrections not taken into account by the usual QCD evolution. On the basis of previous work by Collins et al. [19], Botts and Sterman [9] have calculated the Sudakov factor using resummation techniques and having recourse to the renormalization group. Adapting their result to the case at hand, $B \to \pi\pi$, we write

$$S(x_1, x_2, b_1, b_2, M_B, \mu) = S_{\pi_1}(x_1, b_1, M_B, \mu) + S_{\pi_2}(x_2, b_2, M_B, \mu) \tag{2.11}$$

$$S_{\pi_i}(x_i, b_i, M_B, \mu) = s(x_i, b_i, M_B) + s(1 - x_i, b_i, M_B) - 4/\beta_0 \ln\left(\frac{\ln(\mu/\Lambda_{QCD})}{\ln(1/(b_i\Lambda_{QCD}))}\right) \tag{2.12}$$

where $\beta_0 = 11 - 2n_f/3$; n_f is the number of the active flavors, which is 4 in our case. The last term arises from the application of the renormalization group. The Sudakov function s, which includes leading and next-to-leading logarithms, is given explicitly in the appendix. Contrary to [16] we do not allow for a Sudakov factor for the B meson. We present the arguments for the ansatz (2.11) in the appendix. Although our treatment of the B meson differs from that of [16], the numerical results are practically the same since the Sudakov factor for the B meson used in [16], only yields a tiny additional suppression of the perturbative contribution of the order of a few percent.

Following other authors [10,16] we choose as the renormalization scale the largest mass scale appearing in the process

$$\mu = \max\left\{\sqrt{(1-x)(1-x_1)}M_B, 1/b, 1/b_1\right\}. \tag{2.13}$$

The evaluation of the $\bar{B}^0 \to \pi^+\pi^-$ amplitude requires the integration over the transverse separations. This implies two angle integrations, say the integration over the directions of $\mathbf{b}$, simply providing a factor of 2π, and the integration over the relative angle φ between $\mathbf{b}$ and $\mathbf{b}_1$. The latter integration is non-trivial since the relative angle also appears in the argument of one of the Bessel functions (see (2.10)) but it can be carried out analytically by means of Graf's theorem

$$\begin{aligned} f(y,b,b_1) &\equiv \frac{1}{2\pi}\int \mathrm{d}\varphi K_0(y|\mathbf{b} \mp \mathbf{b}_1|) \\ &= \Theta(b-b_1)K_0(yb)I_0(yb_1) + \Theta(b_1-b)K_0(yb_1)I_0(yb) \end{aligned} \tag{2.14}$$

where Θ is the step function. $I_0(x) = J_0(ix)$, where J_0 is the Bessel function of order zero. As an inspection of (2.10) and (2.11) reveals the x_2 and $\mathbf{b}_2$ integrals factorize. With the help of the δ-function appearing in (2.10) the $\mathbf{b}_2$ integration can directly be carried out, yielding a factor

$$\frac{1}{4\pi}\int \mathrm{d}x_2 \hat{\Psi}^*_{\pi_2}(x_2, 0). \tag{2.15}$$

The value of this integral is fixed by the $\pi^- \to \mu^-\bar{\nu}_\mu$ decay and equals $f_\pi/(2\sqrt{6})$ [20] where f_π (= 130.7 MeV) is the pion decay constant. The fact that the π_2-part in (2.2) factorizes implies the factorization of non-leptonic decay amplitudes into a product of two current matrix elements

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} v_u \langle\pi^-(p_2)|J^\mu_W|0\rangle\langle\pi^+(p_1)|J^W_\mu|\bar{B}^0(p_B)\rangle \tag{2.16}$$

with the first matrix element being parameterized in the usual way as

$$\langle\pi^+(p_2)|J^\mu_W|0\rangle = f_\pi p^\mu_2. \tag{2.17}$$

Frequently (2.16) is used as a hypothesis in soft models for decay amplitudes.

Putting all together we have the following representation of the $\bar{B}^0 \to \pi^+\pi^-$ amplitude

$$\mathcal{M} = -\frac{\sqrt{3}G_F C_F}{\sqrt{2}} v_u M_B^4 \Omega_b \tag{2.18}$$

where

$$\Omega_b = \frac{f_\pi}{2\sqrt{3}\pi} \int \mathrm{d}x\mathrm{d}x_1 b\mathrm{d}b b_1 \mathrm{d}b_1 \hat{\Psi}^*_{\pi_1}(x_1, \mathbf{b}_1) \hat{\Psi}_B(x, -\mathbf{b}) \exp\left[-S_{\pi_1}(x_1, b_1, M_B, \mu)\right]$$
$$\times \alpha_s(\mu)[2x - x_1] K_0(\sqrt{(1-x)(1-x_1)} M_B b) f(\sqrt{2x - x_1 - 1} M_B, b, b_1) \tag{2.19}$$

The color factor is $\sqrt{3}C_F$ where $C_F(= 4/3)$ is the Casimir operator of the fundamental representation of $SU(3)_c$. The remaining four dimensional integral has to be carried out numerically. Before presenting the numerical results (see Sec. 3) two remarks are in order:
i) Owing to the behavior of the K_0 function the hard scattering amplitude has a singularity of the type $\ln(1 - x_1)$ for $x_1 \to 1$, x fixed. Since the pion wave function provides a factor $1 - x_1$ in that limit (see Sec. 3) the integral in (2.19) is regular. This result appears as a consequence of the fact that the transverse degrees of freedom are explicitly considered in the modified perturbative approach. In the standard approach [3–8], on the other hand, the neglect of the transverse momenta (and the assumption $m_b = M_B$) leads to a $1/(1 - x_1)^2$ behaviour of the hard scattering amplitude (see (2.6)). Hence the decay amplitude $\mathcal{M}$ is logarithmically singular. That property of $\mathcal{M}$ forced previous authors to cut off the x_1-integral at $x_1 = 1 - \varepsilon$ (with ε being chosen to lie in the range 0.05-0.1) under the argument that the Sudakov factor will suppress the end-point region. It should be clear from our discussion that the singularity of the amplitude is avoided when the transverse degrees of freedom are taken into account. The Sudakov factor provides only further suppression of the end-point regions. In contrast to [3–8] our results are therefore insensitive to the end-point regions. Moreover, they do not depend on a cut-off parameter.
ii) For $b(b_1)\Lambda_{QCD} \to 1$, $\alpha_s(\mu)$ is singular. However, as an inspection of (A1) reveals, the Sudakov factor tends to zero faster than any power of $\ln(1/b(b_1)\Lambda_{QCD})$ compensating the α_s singularity and rendering the integral in (2.19) finite. This is an attractive feature the modified perturbative approach possesses; it allows to choose the renormalization scale in such a way that large logs from higher order perturbation theory are avoided and a finite result for

the amplitude $\mathcal{M}$ is nevertheless obtained without introducing an external parameter (such as e.g. a gluon mass) to regularize α_s. We will take up this point again.

III. NUMERICAL EVALUATION OF THE $\bar{B}^0 \to \pi^+\pi^-$ DECAY WIDTH

The first step in the evaluation of that decay width is to specify the wave functions. The pion wave function is rather well-known since it is constrained by the decay processes $\pi^- \to \mu^-\bar{\nu}_\mu$ and $\pi^0 \to \gamma\gamma$ [20]. In [14] a parameterization of that wave function is given which respects these two constraints. The soft transverse configuration space wave function is written as

$$\hat{\Psi}_{\pi_i}(x_i, \mathbf{b}_i) = \frac{f_\pi}{2\sqrt{6}}\, \Phi_{\pi_i}(x_i)\, \hat{\Sigma}_{\pi_i}(\sqrt{x_i(1-x_i)}\,\mathbf{b}_i). \tag{3.1}$$

For the distribution amplitude Φ we use

$$\Phi^{AS}_{\pi_i}(x_i) = 6\, x_i(1-x_i) \tag{3.2}$$

and, as an alternative, we use a distribution amplitude originally proposed by Chernyak and Zhitnitsky [21]

$$\Phi^{CZ}_{\pi_i}(x_i) = 30\, x_i(1-x_i)\,(2x_i-1)^2. \tag{3.3}$$

The b-dependent part $\hat{\Sigma}$ is assumed to be a simple Gaussian

$$\hat{\Sigma}_{\pi_i}(\sqrt{x_i(1-x_i)}\,\mathbf{b}_i) = 4\pi\; \exp\left(-x_i(1-x_i)\, b_i^2/4a_\pi^2\right). \tag{3.4}$$

More complicated forms than (3.4) (e. g., a two-humped shape of the momentum space wave function) are proposed in [22] on the basis of dispersion relations and duality. At large transverse momentum, however, the soft momentum space wave function should behave like a Gaussian [22]. The examination of a number of examples corroborates our expectation that forms of $\hat{\Sigma}_{\pi_i}$ other than (3.4) will not change the results and the conclusions presented in our paper markedly. The parameter a_π appearing in the Gaussian has a value of $2.02\,\mathrm{GeV}^{-1}$ for both the distribution amplitudes.

In [14] it is also shown that the asymptotic (AS) distribution amplitude combined with the Gaussian (3.4) leads to a reasonable description of the $\pi \to \gamma$ transition form factor within the modified perturbative approach while the CZ distribution amplitude seems to be in conflict with the data. We nevertheless retain that distribution amplitude since we cannot exclude it definitely and in order to examine the influence of the form of the distribution amplitude on the final results.

For the B meson we take the Bauer-Stech-Wirbel wave function [2] which has been proven to be useful in weak decays. Writing that wave function in a fashion similar to (3.1), we have

$$\hat{\Sigma}_B = 4\pi \ \exp\left(-b^2/4a_B^2\right) \tag{3.5}$$

and

$$\Phi_B(x) = A\, x(1-x)\ \exp[-a_B^2 M_B^2 (x-x_0)^2]. \tag{3.6}$$

The distribution amplitude Φ_B exhibits a pronounced peak, its position is approximately at $x_0 = m_b/M_B = 0.93$ for $m_b = 4.9\,\mathrm{GeV}$. This property of the B-meson wave function parallels the theoretical expected and experimentally confirmed behaviour of heavy meson fragmentation functions. The two parameters appearing in the B-meson wave function, namely A and a_B, are fixed by normalizing the wave function to unity - the neglect of higher Fock states seems to be a reasonable assumption for heavy hadrons - and by taking a value of 180 MeV for the B-meson decay constant f_B. This value has been found in a recent lattice gauge theory analyses [23]. Under these conditions A and a_B have the values 63.05 and $1.491\,\mathrm{GeV}^{-1}$ respectively. One may be tempted to use a non-factorizing form for Σ_B similarly to that of Σ_π in (3.4). However, we favor the function (3.5) since the non-factorizing form has theoretical deficiencies in the formal limit $M_B \to \infty$ and is, therefore, in conflict with the heavy quark effective theory as has been shown in [24].

Using a value of 0.975 for the CKM matrix element V_{ud} and 200 MeV for Λ_{QCD}, we find the following numerical results for the decay width of the process $\bar{B}^0 \to \pi^-\pi^+$ when either the AS distribution amlitude (3.2) or the CZ one (3.3) is employed:

$$\Gamma(\bar{B}^0 \to \pi^+\pi^-) = \frac{1}{16\pi M_B}|\mathcal{M}|^2 = \left(\frac{V_{ub}}{0.005}\right)^2 \times 10^{-10}\mathrm{eV} \begin{cases} 1.04 & (AS) \\ 2.58 & (CZ) \end{cases} \tag{3.7}$$

Since the CKM matrix element V_{ub} is still poorly known we tentatively take for it the value 0.005 and show V_{ub} explicitly in (3.7). We repeat that we favor the result obtained with the asymptotic distribution amplitude and believe that the CZ distribution amplitude perhaps leads to an overestimate of the perturbative contribution to the $\bar{B}^0 \to \pi^+\pi^-$ decay. The predictions for the decay width are also subject to errors due to uncertainties in the model parameters and assumptions leaving aside the difference between the results for the two possibilities for the pion's distribution amplitude. The influence of small modifications in the wave functions like the use of the BHL factor $\exp[-a_i^2 m_q^2/(x_i(1-x_i))]$ in the distribution amplitudes (where m_q is a constituent mass) [20] or changes in the b dependence is rather mild, the decay width is typically altered by $5-10\%$. The largest uncertainty in our results arises from the error of 50 MeV for f_B quoted in [23]. Although this error is very large and, moreover, the decay width depends quadratically on f_B , the resulting error for the width only amounts to about 25%. This happens because of the wave function normalisation: A change in f_B is accompanied by a corresponding change in a_B, both changes compensate each other to a large extent. Combining all uncertainties in the model parameters we estimate the error of the prediction to amount to about 35%. This errors applies to both the results presented in (3.7).

The results (3.7) are comparable in magnitude, although in general smaller, with previous estimates of the perturbative contribution [3–8] with two exceptions where substantially larger values are quoted. Simma and Wyler [4] obtain their large value by adjusting the perturbative contribution to the semi-leptonic decay $B \to De\bar{\nu}$ to the data and applying the normalization factor obtained by that procedure also to non-leptonic B decays. Carlson and Milana [6,18] use the so-called peaking approximation advocated for in [24]. In that approximation the B-meson distribution amplitude is assumed to be $\sim \delta(x - x_0)$ which is with respect to the shape of (3.6). While the peaking approximation allows one to discuss the qualitative features of the results in a rather simple fashion it numerically is not very reliable in some cases. Using it in the evaluation of the decay width within our approach, we find a result which is larger by a factor of 2.7 (for the AS wave) function than the value quoted in (3.7).

We emphasize that in the modified perturbative approach in which the transverse degrees of freedom and Sudakov corrections are taken into account, the soft end-point regions are strongly suppressed. Therefore, there is no need for an cut-off parameter in the x_1 integration as it is the case in the previous perturbative calculations of the $\bar{B}^0$ decay width [3–8]. The extreme sensitivity of the results to the region near $x_1 = 1$ found by these authors has completely disappeared in our approach. There is second advantage in our approach: we can make use of the standard one-loop formula for the strong coupling constant. Moreover, we can choose the renormalization scale as to avoid large logs from higher order perturbation theory (see (2.13)). Still the integral appearing in (2.19) is regular since the Sudakov factor compensates the α_S singularity. Actually the suppression of the end-point regions is so strong that the bulk of the perturbative contribution is accumulated in regions where α_S is small. In order to demonstrate that we cut off in (2.19) regions where α_S exceeds a given value α_c and plot Ω_b as a function of α_c. As can be seen from Fig. 3 50% of the result is already accumulated in regions where α_S is smaller than about 0.5. The result saturates for $\alpha_S \simeq 0.8$, there are practically no contributions from regions where α_S is larger than that value. Therefore, we consider the perturbative contribution to the decay amplitude as theoretically self-consistent. Note that the regions of large values of α_S correspond to large quark-antiquark separations in the mesons. The use of the one-loop formula for α_S (as well as the parameterization of the parton propagators) in the end-point regions may be questioned. However, the saturation property of the perturbative result tells us that α_S can be frozen in at a value $\simeq 0.8$ or regularized by, say, the introduction of a gluon mass without changing the final results. In order to avoid the introduction of such external parameters we keep the standard α_S parameterization, being conscious of the fact that the actual parameterization is of no account in the soft end-point regions.

IV. SHORT DISTANCE CORRECTIONS

It has been shown (see, for instance, [25,26] and references therein) that leading order short distance corrections lead to an effective Hamiltonian at the scale μ

$$\mathcal{H}_W^{eff} = 4\frac{G_F}{\sqrt{2}} v_u \sum_{i=1,2} C_i(\mu)\mathcal{O}_i + 4\frac{G_F}{\sqrt{2}} v_t \sum_{i=3}^{6} C_i(\mu)\mathcal{O}_i, \tag{4.1}$$

replacing the weak Hamiltonian (2.1) ($v_t = V_{td}^* V_{tb} = -v_u - v_c$). The operators $\mathcal{O}_i$ are given as follows

$$
\begin{aligned}
&\mathcal{O}_1 = (\bar{d}_\alpha \gamma_\mu L b_\alpha)(\bar{u}_\beta \gamma^\mu L u_\beta) && \mathcal{O}_2 = (\bar{d}_\beta \gamma_\mu L b_\alpha)(\bar{u}_\alpha \gamma^\mu L u_\beta) \\
&\mathcal{O}_3 = (\bar{d}_\alpha \gamma_\mu L b_\alpha) \sum_q (\bar{q}_\beta \gamma^\mu L q_\beta) && \mathcal{O}_4 = (\bar{d}_\beta \gamma_\mu L b_\alpha) \sum_q (\bar{q}_\alpha \gamma^\mu L q_\beta) \\
&\mathcal{O}_5 = (\bar{d}_\alpha \gamma_\mu L b_\alpha) \sum_q (\bar{q}_\beta \gamma^\mu R q_\beta) && \mathcal{O}_6 = (\bar{d}_\beta \gamma_\mu L b_\alpha) \sum_q (\bar{q}_\alpha \gamma^\mu R q_\beta)
\end{aligned} \tag{4.2}
$$

where R denotes the right-handed projection operator and q is running over the quark flavors being active at the scale μ. The Wilson coefficients at the scale $\mu = M_W$ are $C_k(M_W) = \delta_{k2}$. The renormalization group evolution from the scale M_W to $\mu = m_b$, relevant to the B decays, leads to the following values of the Wilson coefficients at the latter scale [26]:

$$
\begin{aligned}
&C_1 = -0.229, \qquad C_2 = 1.097, \qquad C_3 = 0.021, \\
&C_4 = -0.039, \qquad C_5 = 0.007, \qquad C_6 = -0.029.
\end{aligned} \tag{4.3}
$$

At the scale $\mu = M_W$ (4.1) reduces to the Hamiltonian (2.1); after Fierz reordering the operator $\mathcal{O}_2$ is the one appearing in (2.1).

Taking into account the Hamiltonian (4.1) in the calculation of B decays into two pions, we have to consider many contributions. There is, on the one side, the class of direct contributions where the d quark together with the spectator quark forms one of the pions. A second class contains the exchanged contributions where the d quark forms a pion together with one of the other light quarks. Due to Fierz reordering, however, the exchanged contributions can be brought into the form of the contributions from the first class, with eventually different color factors and/or Wilson coefficients. Consequently, we have only to work out three Dirac traces and hence three integrals corresponding to the graphs shown in Figs. 2a, 2c and 2d. As we mentioned in Sect. 2 the graph 2b leads to a vanishing contribution for zero pion mass. The relevant trace and the integral for graph 2a are given in (2.7) and (2.19).

Making use of the same set of approximations as in Sect. 2, we find for the contributions from the graphs 2c and 2d:

$$
\begin{aligned}
\Omega_2 = \frac{\sqrt{2}}{4\pi^2} \int & [\mathrm{d}x] b \mathrm{d}b b_2 \mathrm{d}b_2 \hat{\Psi}^*_{\pi_1}(x_1, -\mathbf{b}) \hat{\Psi}^*_{\pi_2}(x_2, \mathbf{b}_2) \hat{\Psi}_B(x, -\mathbf{b}) \\
& \times \exp\left[-S_{\pi_1}(x_1, b, M_B, \mu) - S_{\pi_2}(x_2, b_2, M_B, \mu)\right] \alpha_s(\mu)[x_2 + x - 1] \\
& \times K_0(\sqrt{(1-x)(1-x_1) - (x - x_1)x_2} M_B b) f(\sqrt{(1-x)(1-x_1)} M_B, b, b_2)
\end{aligned} \tag{4.4}
$$

and

$$\Omega_3 = \frac{\sqrt{2}}{4\pi^2}\int[\mathrm{d}x]b\mathrm{d}bb_2\mathrm{d}b_2\hat{\Psi}^*_{\pi_1}(x_1,-\mathbf{b})\hat{\Psi}^*_{\pi_2}(x_2,\mathbf{b}_2)\hat{\Psi}_B(x,-\mathbf{b})$$
$$\times\exp\left[-S_{\pi_1}(x_1,b,M_B,\mu)-S_{\pi_2}(x_2,b_2,M_B,\mu)\right]\alpha_s(\mu)[x_1+x_2-2x]$$
$$\times K_0(\sqrt{1-x_1x_2-x(2-x_1-x_2)}M_Bb)f(\sqrt{(1-x)(1-x_1)}M_B,b,b_2). \qquad (4.5)$$

In the renormalization scale expression (2.13) b_1 is to be replaced by b_2. Obviously, the contributions from the penguin operators $\mathcal{O}_3$ and $\mathcal{O}_4$ lead to the same integrals, (2.19), (4.4) and (4.5) as the operators $\mathcal{O}_1$ and $\mathcal{O}_2$ respectively. The contributions from the operators $\mathcal{O}_5$ and $\mathcal{O}_6$, involving right-handed quarks, are either zero or lead again to the integrals (2.19), (4.4) and (4.5).

There is a third class of contributions, namely those from annihilation topologies (see Fig. 4). In principle such topologies are generated by all six operators $\mathcal{O}_i$ and in each of these topologies a gluon can be exchanged between the quark line connecting π_1 and π_2, and any of the other quarks, i. e., there are again four graphs contributing similar to those shown in Fig. 2. The contributions from the factorizing graphs are, however, zero. Only the contributions from the graphs where the gluon is exchanged between one of the quarks forming the B meson and the quark connecting π_1 and π_2 are non-zero provided the color structure is appropriate. The two non-zero contributions read

$$\Omega^a_b = \frac{\sqrt{2}}{4\pi^2}\int[\mathrm{d}x]b\mathrm{d}bb_2\mathrm{d}b_2\hat{\Psi}^*_{\pi_1}(x_1,\mathbf{b}_2)\hat{\Psi}^*_{\pi_2}(x_2,\mathbf{b}_2)\hat{\Psi}_B(x,-\mathbf{b})$$
$$\times\exp\left[-S_{\pi_1}(x_1,b_2,M_B,\mu)-S_{\pi_1}(x_2,b_2,M_B,\mu)\right]\alpha_s(\mu)[x_1-1]$$
$$\times K_0(\sqrt{(1-x)(1-x_1-x_2)-x+x_1x_2)}M_Bb)f(\sqrt{(1-x_1)(1-x_2)}M_B,b,b_2)$$
$$\Omega^a_1 = \frac{\sqrt{2}}{4\pi^2}\int[\mathrm{d}x]b\mathrm{d}bb_2\mathrm{d}b_2\hat{\Psi}^*_{\pi_1}(x_1,\mathbf{b}_2)\hat{\Psi}^*_{\pi_2}(x_2,\mathbf{b}_2)\hat{\Psi}_B(x,-\mathbf{b})$$
$$\times\exp\left[-S_{\pi_1}(x_1,b_2,M_B,\mu)-S_{\pi_2}(x_2,b_2,M_B,\mu)\right]\alpha_s(\mu)[x-x_1]$$
$$\times K_0(\sqrt{(x-x_1)(x-x_2)}M_Bb)f(\sqrt{(1-x_1)(1-x_2)}M_B,b,b_2). \qquad (4.6)$$

Putting all together, we find the following expressions for the decay amplitudes of the three $\pi\pi$ channels

$$\mathcal{M}(\bar{B}^0\to\pi^+\pi^-) = -\frac{G_FC_F}{\sqrt{6}}v_uM_B^4\{[C_2-v_t/v_uC_4]3\Omega_b$$
$$+[C_1-v_t/v_uC_3][\Omega_b+\Omega_2+\Omega_3]+[C_2-2v_t/v_uC_4][\Omega^a_b+\Omega^a_1]\}$$
$$\mathcal{M}(B^-\to\pi^0\pi^-) = -\frac{G_FC_F}{2\sqrt{3}}v_uM_B^4[C_1+C_2]\,[4\Omega_b+\Omega_2+\Omega_3)] \qquad (4.7)$$

and

$$\mathcal{M}(\bar{B}^0 \to \ \pi^0 \, \pi^0) = 1/\sqrt{2}\mathcal{M}(\bar{B}^0 \to \pi^+\pi^-) - \mathcal{M}(B^- \to \pi^0\pi^-) \tag{4.8}$$

i. e. the amplitudes satisfy the familiar isospin relation for $B \to \pi\pi$ decays [27]. The contributions generated by the operators $\mathcal{O}_5$ and $\mathcal{O}_6$ are either zero or so small (for annihilation topologies) that they can safely be neglected.

Numerical evaluation of the amplitudes and the corresponding rates using the wave functions discussed in Sect. 3, leads to the final results for the decay widths for the three B decay processes shown in Table 1. The results shown in the table are evaluated with $V_{ub} = 0.005$ and $v_t = -0.0138 + i0.0050$. The uncertainties due to the wave functions and the B-decay parameter amount to about 35%, see the discussion in Sect. 3. As an inspection of Table 1 reveals the QCD corrections are substantially and, in total, amount to a reduction of the leading term generated by the Hamiltonian (2.1) of $20 - 25\%$. An exceptional rôle plays the $\bar{B}^0$ decay into two uncharged pions because for that reaction there is no direct contribution from the Hamiltonian (2.1) and the exchanged contribution is suppressed by color (a factor 1/3 in the amplitude). The other contributions are therefore relatively strong. The penguin contributions provide corrections of the order of 20% to the process $\bar{B}^0 \to \pi^+\pi^-$. Since v_t is complex the CP conjugated B^0 decay rates slightly differ from the $\bar{B}^0$ rates presented in the table (by $\simeq 0.2\%$). We stress that our aim is to estimate, as accurate as possible, the strength of the perturbative contributions to the B decay rates. For a calculation of such subtle effects as CP violations a more refined treatment of the penguin contributions, taking into account $\mathcal{O}(\alpha_S^2)$ corrections, is required [4,7,28]. Our conclusions about the strength of the perturbative contributions to the B decay rates will not be altered substantially by a refined treatment of the penguin contributions. Note that in other processes, in particular if $b \to s$ transitions are involved, the rôle of the penguin graphs may be more important (see e. g. [4]). There are also reactions in which the annihilation contributions are more important than in the $B \to \pi\pi$ decays, e. g., in $B^- \to K^0 K^-$ where the direct and exchanged contributions are only generated by the penguin operators.

Our final predictions for the branching ratios are:

$$BR(\bar{B}^0 \to \pi^+\pi^-) = \left(\frac{V_{ub}}{0.005}\right)^2 \begin{Bmatrix} 0.16 & (AS) \\ 0.44 & (CZ) \end{Bmatrix} \times 10^{-6} \pm 35\%,$$

$$BR(\bar{B}^0 \to \pi^0\pi^0) = \left\{ \begin{array}{ll} 0.69 & (AS) \\ 0.71 & (CZ) \end{array} \right\} \times 10^{-8},$$

$$BR(\bar{B}^- \to \pi^0\pi^-) = \left(\frac{V_{ub}}{0.005}\right)^2 \left\{ \begin{array}{ll} 0.15 & (AS) \\ 0.36 & (CZ) \end{array} \right\} \times 10^{-6} \pm 35\%. \tag{4.9}$$

We emphasize again that we favor the results obtained with the AS wave function for reasons explained in [14] yet the other results obtained with the CZ wave function cannot definitely be excluded for the time being. The errors quoted in (4.9) do not contain the uncertainty in the value of V_{ub} but it is shown explicitly in (4.9). Since the penguin operators provide only small corrections also the branching ratio for the first reaction is (approximately) proportional to V_{ub}^2. Since the rate for the $\pi^0\pi^0$ channel is much smaller than the other rates we refrain from quoting an error for it.

While the decay of the $\bar{B}^0$ meson into two charmless mesons has been observed experimentally the results for individual branching ratios still suffer from large statistical and systematical errors. In fact only an upper limit of 2.9×10^{-5} for the process $\bar{B}^0 \to \pi^+\pi^-$ has been quoted by the CLEO collaboration. Our predictions are an order of magnitude below that limit, tempting us to conclude that the perturbative contributions to the $B \to \pi\pi$ decays are likely be too small, soft physics may still dominate these processes. Yet a definite conclusion cannot be drawn at present.

For comparison we also quote the relative rates for $\bar{B}^0$ decays into two-pion states with isospin 2 and 0

$$\frac{|\langle \pi\pi, I=2 \mid \mathcal{H}_W^{eff} \mid \bar{B}^0\rangle|^2}{|\langle \pi\pi, I=0 \mid \mathcal{H}_W^{eff} \mid \bar{B}^0\rangle|^2} = \left\{ \begin{array}{ll} 0.84 & (AS) \\ 0.71 & (CZ) \end{array} \right. \tag{4.10}$$

We observe that perturbation theory slightly favors $\Delta I = 1/2$ transitions over $\Delta I = 3/2$ transitions in B decays.

V. B-π TRANSITION FORM FACTORS

Our calculations presented in Sect. 2 and 3 implicitly contain an evaluation of the B-π transition form factors at $p_2^2 = 0$. These calculations can easily be generalized to other values of p_2^2. The transition form factors are defined as

$$\langle \pi^+(p_1)|J^W_\mu|\bar{B}^0(p_B)\rangle = F_+\left(p_2^2\right)(p_B+p_1)^\mu + F_-\left(p_2^2\right)(p_B-p_1)^\mu. \tag{5.1}$$

We write the energy of the π^+ meson in the B meson rest frame as

$$E_{\pi_1} = \eta M_B/2 \tag{5.2}$$

where $0 \le \eta \le 1$. Then the momentum transfer p_2^2 equals $M_B^2(1-\eta)$. Calculating the two form factors from the graphs 2a and 2b, we find

$$\begin{aligned} F_+ &= \frac{1}{2}\frac{C_F M_B^2}{\pi}\left(\frac{2\sqrt{3}\pi}{f_\pi}\Omega_b - (1-\eta)\Omega_1\right) \\ F_- &= \frac{1}{2}\frac{C_F M_B^2}{\pi}\left(-\frac{2\sqrt{3}\pi}{f_\pi}\Omega_b + (1+\eta)\Omega_1\right) \end{aligned} \tag{5.3}$$

where the integral Ω_b is given in (2.19) with the replacements: M_B by $\sqrt{\eta}M_B$ and $[2x-x_1]$ by $[2x-1+(1-x_1)\eta]$. The contribution from the graph 2b leads to an integral similar to Ω_b

$$\begin{aligned} \Omega_1 = \int & \mathrm{d}x\mathrm{d}x_1 b\mathrm{d}b b_1 \mathrm{d}b_1 \hat{\Psi}^*_{\pi_1}(x_1,\mathbf{b}_1)\hat{\Psi}_B(x,-\mathbf{b})\exp\left[-S_{\pi_1}(x_1,b_1,M_B,\mu)\right] \\ & \times\alpha_s(\mu)[1-x]K_0(\sqrt{(1-x)(1-x_1)\eta}M_B b) f(\sqrt{(1-x)\eta}M_B,b,b_1). \end{aligned} \tag{5.4}$$

The results for the transition form factors obtained with the wave functions (3.1), (3.2) and (3.4) for the pion and (3.5), (3.6) for the B meson, are shown in Fig. 5. The results can be trusted for values of η between 0.2 and 1. For smaller values of η the perturbative contribution is inconsistent in the sense that more than 50% of the perturbative contribution is built up in regions where $\alpha_s \ge 0.7$. The perturbative contribution to the differential decay rate for the semileptonic $B \to \pi$ decay does not match at $\eta = 0.2$ with the soft pion result obtained in [29] and, integrated from $\eta = 0.2$ to $\eta = 1.0$ is tiny in comparison with the current experimental limit [30] on the branching ratio of $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$.

Li and Yu [16] have also calculated these form factors within the modified perturbative approach. While we agree with their analytical expressions for the form factors, our numerical results are about an order of magnitude smaller than theirs. We suspect that Li and Yu normalised the B state wrongly.

Several other predictions for the form factor F_+ at $\eta = 1$ are to be found in the literature (BSW overlap model [2], QCD sum rules [31,32], lattice gauge theory [33]). The predicted values from these non-perturbative approaches range between 0.24 and 0.33. For the form factor F_+ at $\eta = 1$ the decay width $\Gamma(B^0 \to \pi^+\pi^-)$ can be estimated, assuming factorization (2.16) to hold for soft physics. One finds

$$\Gamma(\bar{B}^0 \to \pi^+\pi^-) = \frac{G_F^2 |v_u|^2}{32\pi} f_\pi^2 \, M_B^3 \, |F_+(0)|^2. \tag{5.5}$$

and using the quoted values for $F_+(\eta = 1)$, the predictions for the branching ratio range from 0.9×10^{-5} to 1.8×10^{-5}, i.e., they are rather close to the upper limit measured by the CLEO collaboration [1] in contrast to the perturbative contribution.

VI. CONCLUDING REMARKS

We have calculated the rates for $B \to \pi\pi$ decays in the modified perturbative approach in which the transverse degrees of freedom as well as Sudakov effects, comprising gluonic radiative corrections, are taken into account. We believe that the perturbative contributions to these processes are reliably estimated: The hard scale is provided by M_B^2 and the Sudakov factor suppresses the soft end-point regions strongly so that the bulk of the perturbative contributions is accumulated in regions where α_S is sufficiently small. Therefore our estimate of the perturbative contribution can be considered as theoretically self-consistent. The difficulties previous authors [3–8] had with the singular behaviour of the hard scattering amplitude disappear in our approach, there is no need for a cut-off and correspondingly no need for an additional external free parameter. Also the extreme sensitivity of the perturbative contribution to the (soft) end-point regions disappears completely. The phenomenological input into our calculation, namely the mesonic wave functions - which are controlled by long distance physics and are, therefore, not calculable at present to a sufficient degree of accuracy - is fairly well constrained. The influence of various parameters and corrections on our results is discussed in some detail. The major uncertainties arise from the CKM matrix element V_{ub}, the B-meson decay constant f_B and from the pion's distribution amplitude (although we favor the results obtained with the AS distribution amplitude). The experimental upper bound for the $\bar{B}^0 \to \pi^+\pi^-$ branching ratio is rather large as compared with the magnitude of the perturbative contribution. Although definite conclusions cannot be drawn as yet, we have to be aware that $B \to \pi\pi$ decays are perhaps controlled by soft physics. Estimates of rate asymmetries based on perturbation theory may be misleading.

In a similar fashion as the $B \to \pi\pi$ decays we can also calculate the rates for B decays into other light mesons involving K, ρ and/or K^* mesons. Depending on the particular channel considered the rôle of the operators in (4.1) other than $\mathcal{O}_2$, may be more important than in the $\pi\pi$ channel. Thus, as shown in [4], the penguin graphs provide substantial corrections to the process $B^- \to K^-\pi^0$. We expect similarly small perturbative contributions to the rates of other light meson channels. Of utmost phenomenological interest are B decays into channels involving D mesons since the branching ratios for these processes are typically two orders of magnitude larger than the upper bound for the $\bar{B}^0 \to \pi^+\pi^-$ branching ratio. This enhancement is essentially due to the fact that now the CKM matrix element V_{bc} is involved instead of the much smaller matrix element V_{ub}. A treatment of such channels within the modified perturbative approach is justified since, in contrast to the standard approach, the relevant parton virtualities are sufficiently large, and correspondingly α_S is small enough, thanks to the transverse degrees of freedom (see (2.6) and (2.13)). In the light of our experience with the $\pi\pi$ channels and backed by explorative studies we do not expect that the perturbative contributions to the channels involving D mesons are large enough to account for the experimental values.

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APPENDIX:

In this appendix we present a few details on the Sudakov factor used in our calculation. The Sudakov factor comprises those parts of gluonic radiative corrections which are not taken into account by the usual QCD evolution. Characteristic of it are double logs produced by overlapping collinear and soft divergencies (for almost massless quarks). Examples of one-loop graphs responsible for radiative corrections are shown in Fig. 6. In axial gauges the two-particle reducible graphs, like the examples shown in Fig. 6a, give rise to double logs, whereas the non-reducible graphs (Fig. 6b) only lead to single logs. The double logs from higher order loops can be resummed using the techniques developed by Collins et al. [19]. The resummation results in the exponentiated one-loop corrections.

We analyse the process under consideration, $B \to \pi\pi$, in the B meson rest frame (see(2.5)). In this frame the pions have momenta with either a large + or a large − light-cone component, while the other components are zero. The same uneven kinematical situation holds for the quarks the pions are made up. This is exactly the kinematical situation for which Collins et al. [19] derived the Sudakov factor. For exclusive processes, like $B \to \pi\pi$, the Sudakov factor or more precisely the Sudakov function reads [9]

$$\begin{aligned} s(\xi, b, Q) = & \frac{8}{3\beta_0}\left(\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right) - \hat{q} + \hat{b}\right) \\ & + \frac{4\beta_1}{3\beta_0^3}\left[\hat{q}\left(\frac{\ln(2\hat{q})+1}{\hat{q}} - \frac{\ln(2\hat{b})+1}{\hat{b}}\right) + \frac{1}{2}\left(\ln^2(2\hat{q}) - \ln^2(2\hat{b})\right)\right] \\ & + \frac{4}{3\beta_0}\ln\left(\frac{e^{2\gamma-1}}{2}\right)\ln\left(\frac{\hat{q}}{\hat{b}}\right) + A^{(2)}\frac{4}{\beta_0^2}\left[\frac{\hat{q}-\hat{b}}{\hat{b}} - \ln\left(\frac{\hat{q}}{\hat{b}}\right)\right] \end{aligned} \tag{A1}$$

where the definitions

$$\hat{q} \equiv \ln\left(\xi Q/\sqrt{2}\Lambda_{QCD}\right); \qquad \hat{b} \equiv \ln\left(1/b\Lambda_{QCD}\right) \tag{A2}$$

and

$$A^{(2)} \equiv \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{2\beta_0}{3}\ln\left(e^{\gamma}/2\right) \tag{A3}$$

are used ($\beta_1 = 102 - 38n_f/3$; γ is the Euler constant). $Q/\sqrt{2}$ is the large (+ or −) component of the pion's momentum for an appropriate choice of the gauge vector. In the case of $B \to \pi\pi$ (see (2.5)) Q equals M_B and for the B-π transition form factor $Q = \eta M_B$. ξ is the relevant momentum fraction x_i or $(1 - x_i)$, $i = 1, 2$. Note that the coefficients in the second line of (A1) differ from previous publications [9–11]. In the case of the pion form factor the effect of these corrections to the Sudakov function results in a reduction of the perturbative contribution of about 2% as compared to previously reported results [12].

The range of validity of (A1) for the Sudakov function is limited to not too small values of the transverse separation of quark and antiquark in the meson. Whenever $b \le \sqrt{2}/\xi Q$ (i.e.,

$\hat{b} \geq \hat{q}$), the gluonic corrections are to be considered as higher-order corrections to the hard scattering amplitude and, hence, are not contained in the Sudakov factor but are absorbed in $\hat{T}_H$. Therefore,

$$s(\xi, b, Q) = 0 \qquad \text{if} \quad \hat{b} \geq \hat{q} \tag{A4}$$

is assumed. Similarly, the complete Sudakov factor $\exp[-S]$ is set to unity, whenever it exceeds unity, which is the case in the small b region. As b increases the Sudakov function increases as well, tending to infinity with $b \to 1/\Lambda_{QCD}$. Consequently, the Sudakov factor $\exp[-S]$ drops to zero. For b_1 larger than $1/\Lambda_{QCD}$, the true soft region, the Sudakov factor is zero.

For the B meson the situation is completely different from that of the π mesons. Due to the large b-quark mass the radiative corrections only produce soft divergencies but no collinear ones. Consequently, double logs do not appear and, hence, the Sudakov function for the b-quark is zero. For the light quark there are still both soft and collinear divergencies. However, because of the pronounced peak the B-meson wave function exhibits, the bulk of the perturbative contribution is accumulated in regions where the momentum of the light quark is soft, i.e., has no large component at all. As has been shown in [19] the divergencies do not overlap in these situations. Thus, also for the light quark the Sudakov function is zero.

In contrast to us, Li and Yu [16] also associate a Sudakov function to the light quark contained in the B meson. Because of the shape of the B-meson wave function only values of x close to x_o are relevant for which there is only a weak or even no Sudakov suppression. Thus, wether or not a Sudakov function for the light quark in the B meson is taken into account is irrelevant in praxis, the numerical values for the decay rates differ only by 5%.

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TABLES

TABLE I. The rates for $B \to \pi\pi$ decays in units of 10^{-10} eV as predicted from the effective Hamiltonian (4.1). Column A: Contributions from the operator $\mathcal{O}_2 (C_2 = 1)$, annihilations neglected. Column B: Contributions from $\mathcal{O}_1$ and $\mathcal{O}_2$, annihilations neglected. Column C: As B but penguin contributions included. Column D: Full result with annihilations included.

		A	B	C	D
$\bar{B}^0 \to \pi^+\pi^-$	AS	1.04	1.11	0.92	0.81
	CZ	2.58	2.88	2.30	2.21
$\bar{B}^0 \to \pi^0\pi^0$	AS	0.050	0.015	0.021	0.034
	CZ	0.089	0.012	0.043	0.035
$B^- \to \pi^0\pi^-$	AS	0.86	0.65	0.65	0.65
	CZ	2.03	1.53	1.53	1.53

FIGURES

FIG. 1. The basic graph for the decay of a B meson into two pions. The circle stands for the effective weak Hamiltonian. The quark momenta are specified.

FIG. 2. Lowest order Feynman graphs for B decays into two pions. The internal quark and gluon momenta are indicated.

FIG. 3. Saturation of the perturbative contribution: Ω_b vs. α_c (see text). Solid (dashed) line represents the perturbative contribution with (without) the Sudakov factor.

FIG. 4. The annihilation topology.

FIG. 5. The B-π transition form factors vs. η. The solid (dashed) and dash-dotted (dotted) lines represent the results for the form factor F_+ (F_-) obtained with the AS and CZ wave functions respectively.

FIG. 6. Radiative corrections.

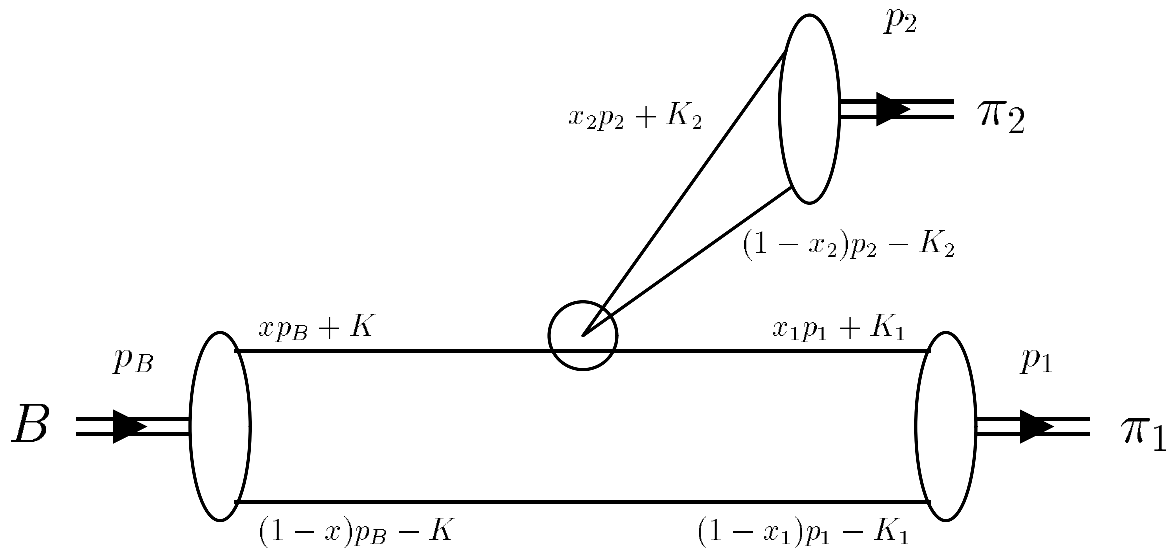


Fig. 1

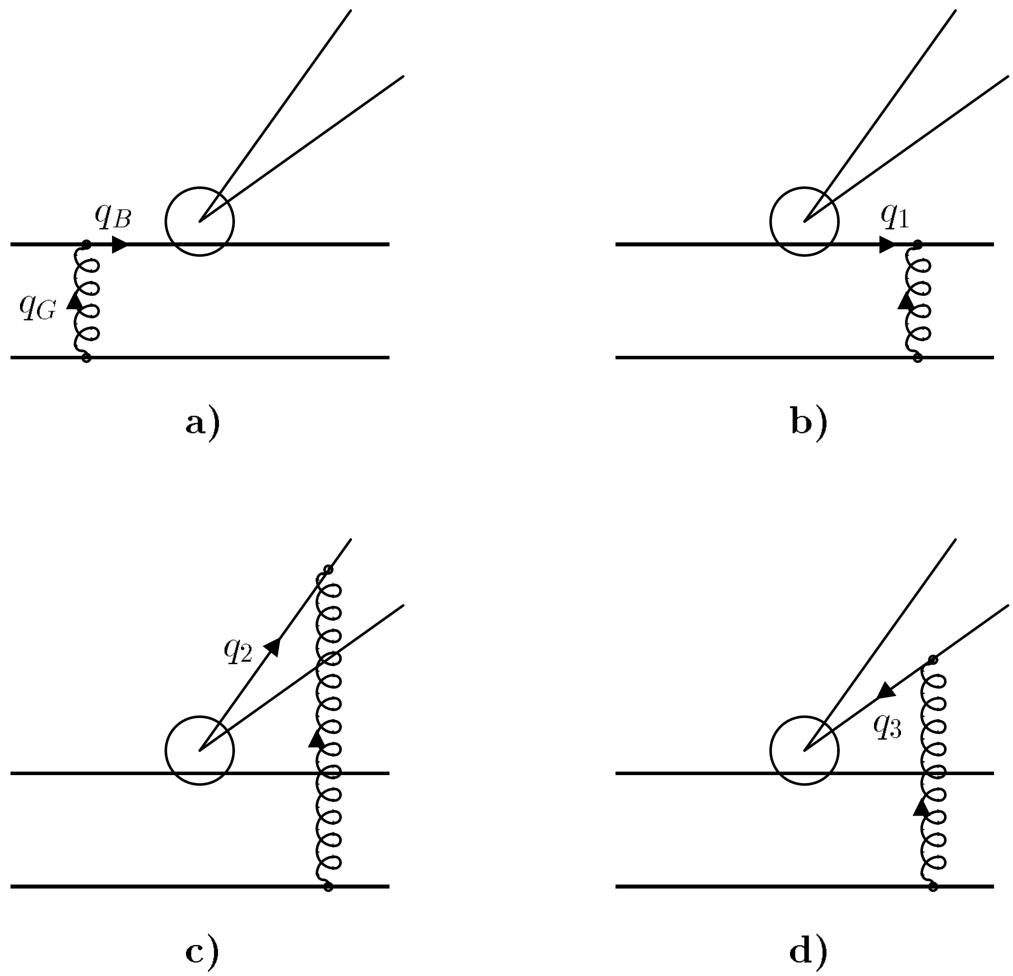


Fig. 2

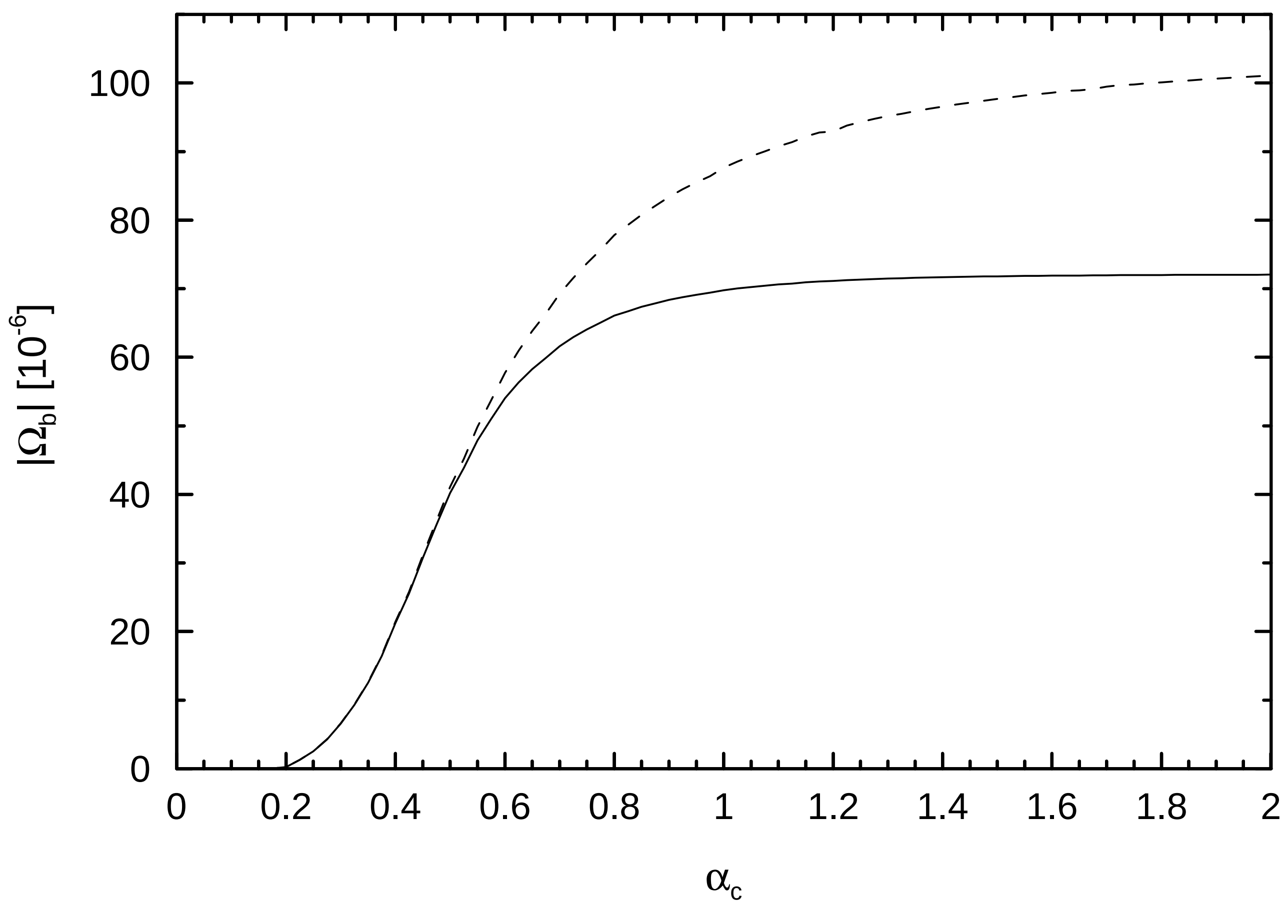
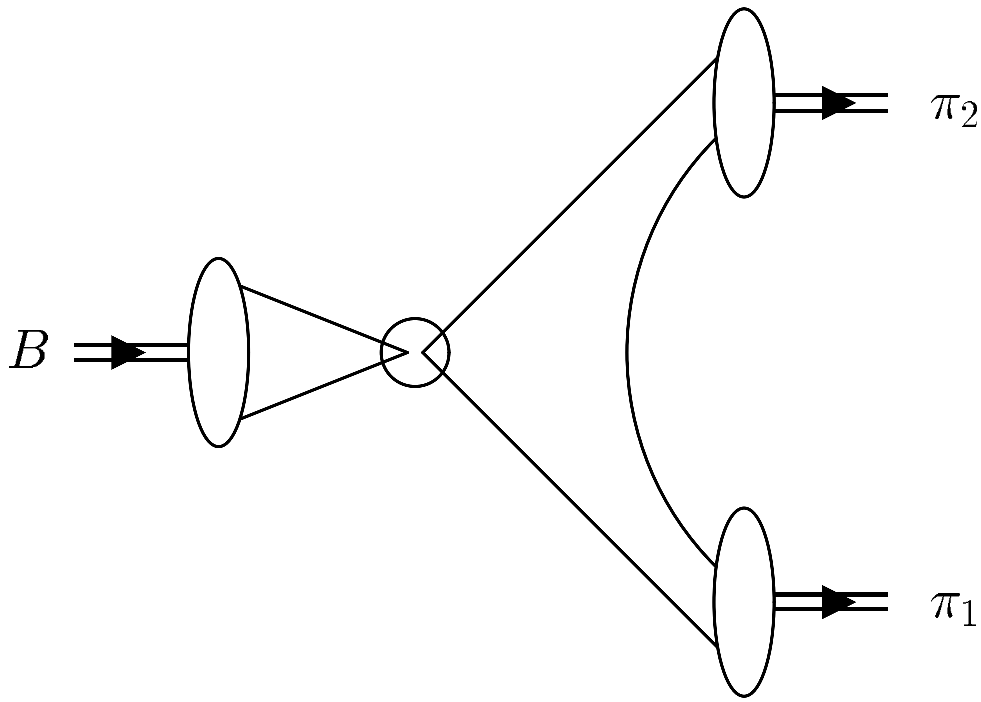


Fig. 3



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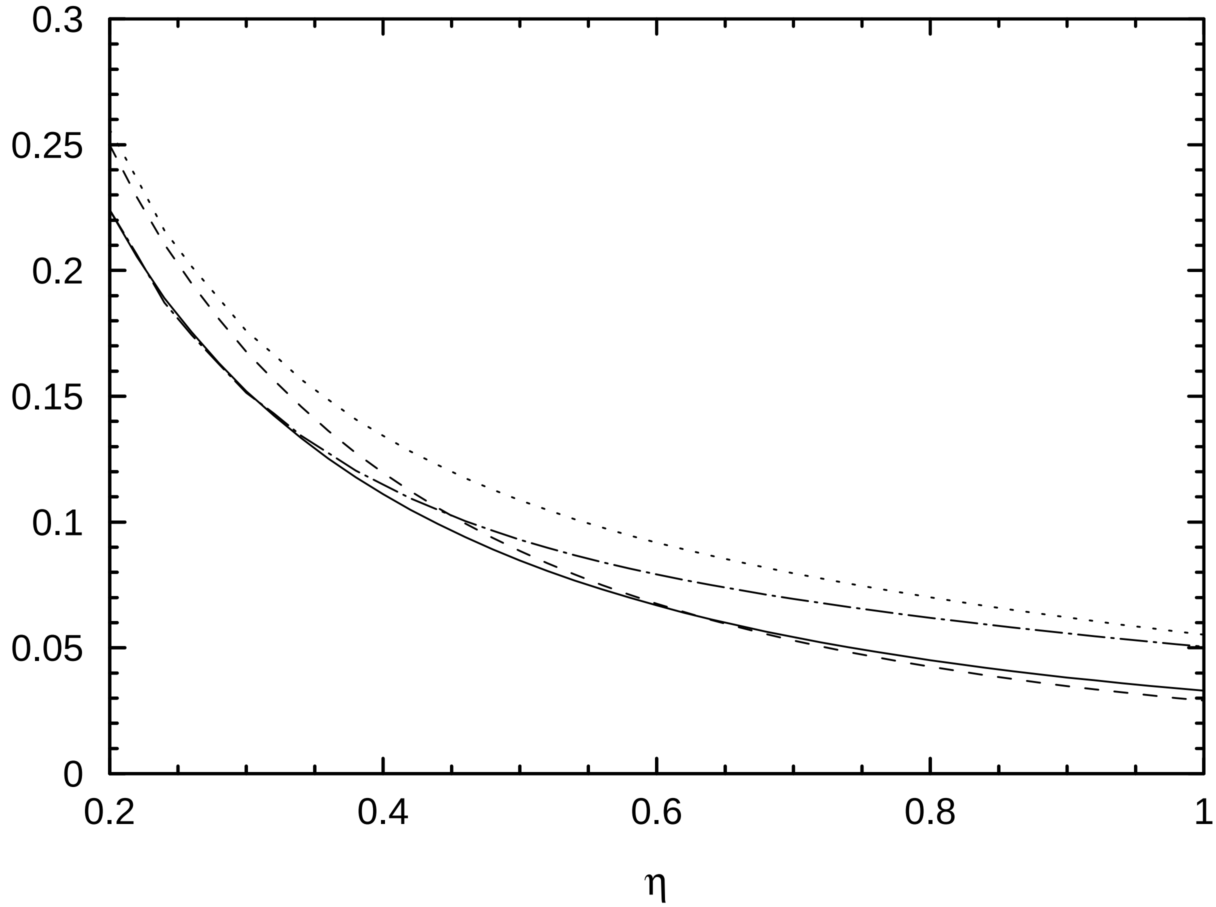

Fig. 5

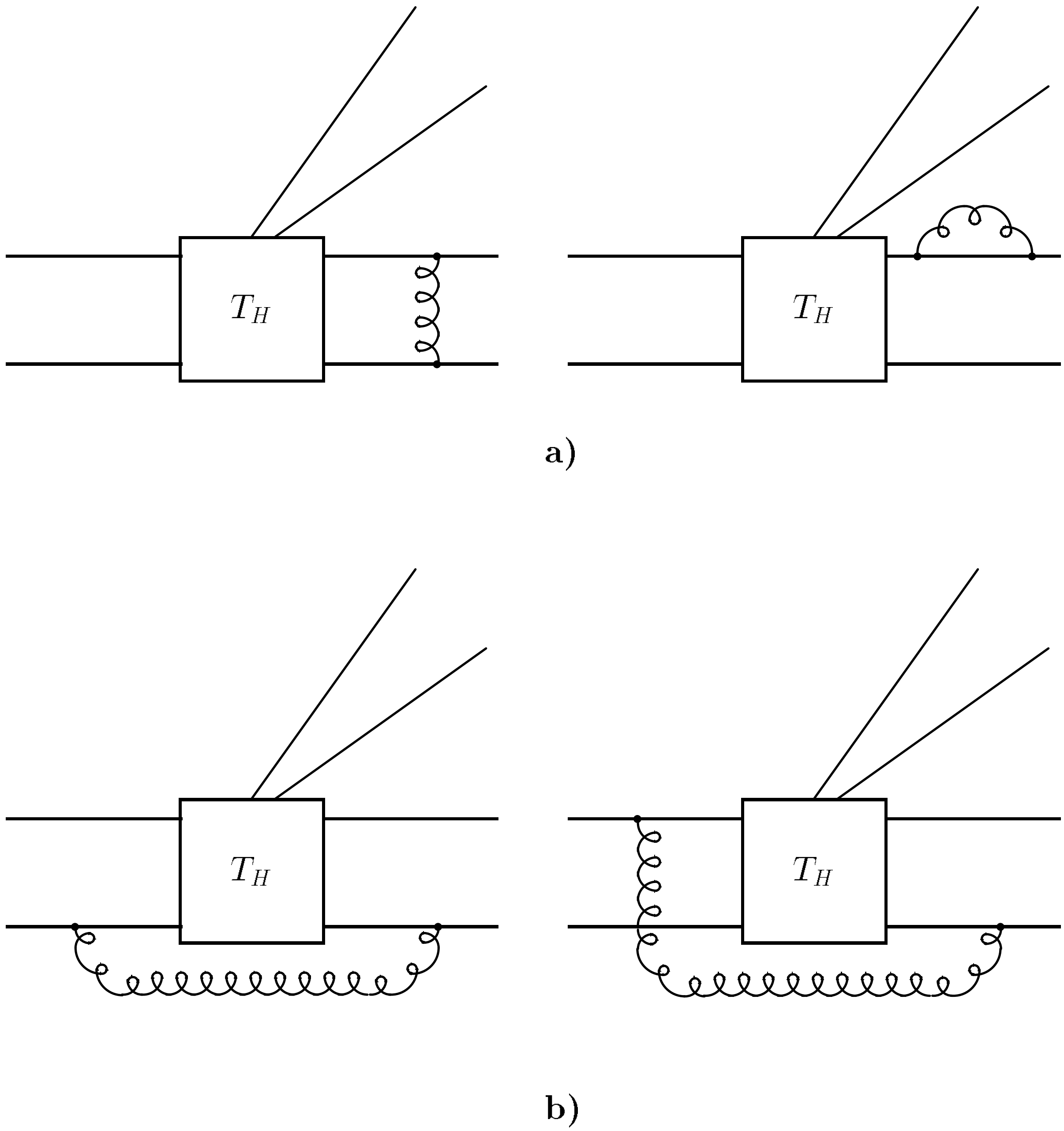

Fig. 6