The reaction $e^- e^+ \to hh$ recomputed.

J.J. Lopez-Villarejo[a], J.A.M. Vermaseren[b]

[a] Departamento de Física Teórica, C-XI,
Universidad Autónoma de Madrid,
Canto Blanco,
E-28034 Madrid, SPAIN.

[b] NIKHEF, P.O. Box 41882,
1009 DB, Amsterdam, The Netherlands

Abstract

We notice that the existing literature about the reaction $e^- e^+ \to hh$ suffers from a mistake in the relative sign between the t-channel and u-channel graphs. Correcting this lowers the crosssections by about an order of magnitude.

1 Introduction

The reaction $e^-e^+ \to hh$ is an interesting one from the theoretical viewpoint. Because the coupling of the higgs to electrons is proportional to the electron mass, the tree graphs are so small that the one loop contribution gives the dominant part of the crosssection. This has been recognized in the past [1, 2] and it has been found that the crosssections are rather small. Only for special higgs particles in for instance SUSY models can the crosssection be larger, mainly due to a bigger tree graph contribution. In principle this would finish the subject if it were not for the fact that during a project in which we were testing methods we recalculated this reaction and found both papers to have a number of shortcomings. Correction of these altered the final numbers significantly.

Of course this doesn't mean that now this reaction will become important at a future ILC, but at least reports about the variety of reactions that may or may not be of interest can quote the correct numbers. And if there exist extra higgs particles the errors of the past can be avoided in the future.

The major problem with one loop calculations is that they involve much work. Hence we like to use symmetry arguments as much as possible to minimize the number of diagrams to compute. A traditionally safe symmetry to use is either Bose or Fermi statistics when there are identical particles involved. Because there are two higgs particles involved, both previous papers have used this symmetry, but both got the relative sign of the diagrams wrong. This is because they used an indirect argument that for identical particles the amplitude should be maximal at the region of overlap and the scattering at 90 degrees was seen as the region of overlap. They adjusted the relative sign for this. We will show in the next section that this is not the case and that actually at 90 degrees the amplitude is zero. This diminishes the crosssection considerably. We have verified this result with the use of the FeynArts [3] system and it agrees with our findings.

This letter is arranged as follows. We start with explaining the relative sign of the graphs in the formulas, using the tree graphs as an example. Then we show the diagrams to compute and how the sign of the tree graphs transfers to the loop graphs. Because the calculation itself can be done in many ways and can even be done automatically with systems like FeynArts, we don't go into unnecessary details. Finally we present the new numbers and show a few distributions.

2 The calculation

The lowest order of the reaction $e^-e^+ \to hh$ has in principle three diagrams, which we will label the t-channel, the u-channel and the s-channel graphs.

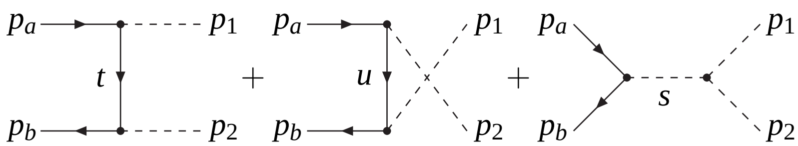

Fig. 1: The three tree graphs for the reaction $e^- e^+ \to hh$

Because of the very small coupling of the higgs to the electron these diagrams are suppressed by powers of $(m_e/M_W)^2$ which is a number that is less than 10^{-10}. The t-channel and u-channel graphs have two of such powers and the s-channel graph has only one. Let us however, for study purposes, look at the t-channel and u-channel graphs assuming that the coupling is a fixed constant c that doesn't depend on the electron mass. In that case the amplitude for the t-channel graph becomes:

$$A_t = c^2\, \overline{v}(p_b) \frac{\gamma_\mu p_a^\mu - \gamma_\mu p_1^\mu + m_e}{t - m_e^2} u(p_a) \tag{1}$$

Using the Dirac equation in the limit that the electron mass goes to zero this gives

$$A_t = -c^2\, \frac{1}{t}\, \overline{v}(p_b)\, \gamma_\mu p_1^\mu\, u(p_a) \tag{2}$$

For the u-channel diagram we have to exchange p_1 and p_2 and hence also t and u which gives

$$\begin{aligned} A_u &= -c^2\, \frac{1}{u}\, \overline{v}(p_b)\, \gamma_\mu p_2^\mu\, u(p_a) \\ &= +c^2\, \frac{1}{u}\, \overline{v}(p_b)\, \gamma_\mu p_1^\mu\, u(p_a) \end{aligned} \tag{3}$$

in which we replaced p_2 by $p_a + p_b - p_1$ and the p_a and p_b can be removed with the Dirac equation. It is this change in sign, which at first looks counter intuitive, that caused all the problems because this way the sum of the t-channel and u-channel graphs becomes proportional to $t - u$:

$$A_t + A_u = c^2\, \frac{t-u}{tu}\, \overline{v}(p_b)\, \gamma_\mu p_1^\mu\, u(p_a) \tag{4}$$

and the result is that at 90 degrees this part of the amplitude vanishes. The complete matrix element is still invariant under the exchange of t and u as $(t-u)^2$ and (tu) are invariant under this exchange and all t and u dependence can be written in terms of these two variables. Their contribution to the full matrix element becomes:

$$|A_t + A_u|^2 = 2\, c^4 \frac{(t-u)^2}{tu} (1 - \frac{m_h^2}{tu}) \tag{5}$$

We leave out the s-channel graph because the equivalent one loop graphs are not important: they always vanish in the limit that the electron mass is put to zero.

Let us see how this works out at the one loop level.

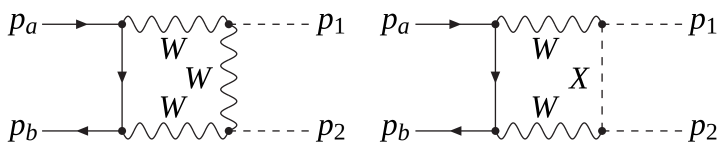

Fig. 2: Some one loop graphs in the reaction $e^- e^+ \to hh$

There are two types of diagrams: the ones with the vector bosons and the ones with the ghosts. As it turns out, the ones with the vector bosons are the easier ones as they involve only one power of the loop momentum in the numerator. Due to the Dirac equation and the fact that the loop momentum gives us, after integration, only Lorenz tensors that involve the external momenta only the box diagrams are nonzero in the limit that the electron mass becomes zero. If the terms in a diagram have only an odd number of gamma matrices (γ_5 counts as an even number for these purposes), eventually the diagram becomes of the type

$$A \;=\; \bar{v}(p_b)\,\gamma_\mu p_1^\mu\,(F_1(s,t,u) + F_2(s,t,u)\gamma_5)\,u(p_a) \qquad (6)$$

and when exchanging (p_1, t, u) and (p_2, u, t) we get the same minus sign as before. This is different for an even number of gamma matrices where we have a relative plus sign. In the case of the standard model however we have only diagrams with an odd number of gamma matrices when we put the electron mass equal to zero.

While doing the calculation we ran into another problem with reference [1]. Their figures show a rather strange and unphysical behaviour when the higgs mass is varied. We could track this down to the exchange of two indices in a tensor in their equation 8. Once this is corrected we get a much more physical behaviour although we still don't get exactly the same result. There are however not enough details to investigate this further. We don't understand the statement in reference [2] that they agree with the previous paper as they seem to have only a problem with the relative sign of the diagrams.

For the rest the calculation was rather standard. We evaluated the standard model higgs reaction first using FORM [4] to reduce to scalar loop integrals according to the methods in ref [5]. Then we did the scalar loop integrals by standard techniques. Eventually we also used FeynArts as an extra check as one should take disagreement with two papers not lightly. We would also have used the GRACE [6] system as a check, but unfortunately the version we had wasn't ready for it yet and for the version that might have handled it we didn't have the proper supporting programs.

3 Results

Because in the old calculations the amplitude reached a maximum at 90 degrees and the new calculation gives zero there, it should come as no surprise that the total cross-section is now significantly smaller. We give some numbers:

CM energy(GeV)	Higgs Mass(GeV)	Crosssection(fb)
500	120	0.0157
500	150	0.0116
1000	120	0.00964
1000	150	0.00924
1000	200	0.00791
1000	300	0.00439

Corrected crosssections for the reaction $e^- e^+ \to hh$ in the standard model.

In the table we show a number of crosssections for different energies and masses. This can be used as a reference to test programs. For the coupling constants we used the running values. This is not automatic in the FeynArts system.

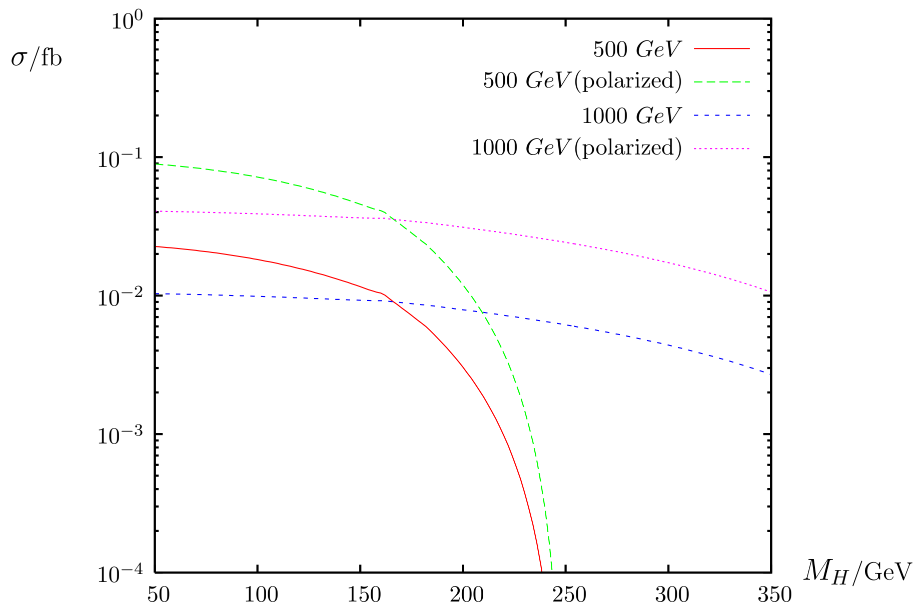

Fig. 3: The crosssection as a function of the mass of the higgs.

In figure 3 we present the crosssections as a function of the mass of the higgs. We notice that for purely polarized beams we can get a four times larger crosssection.

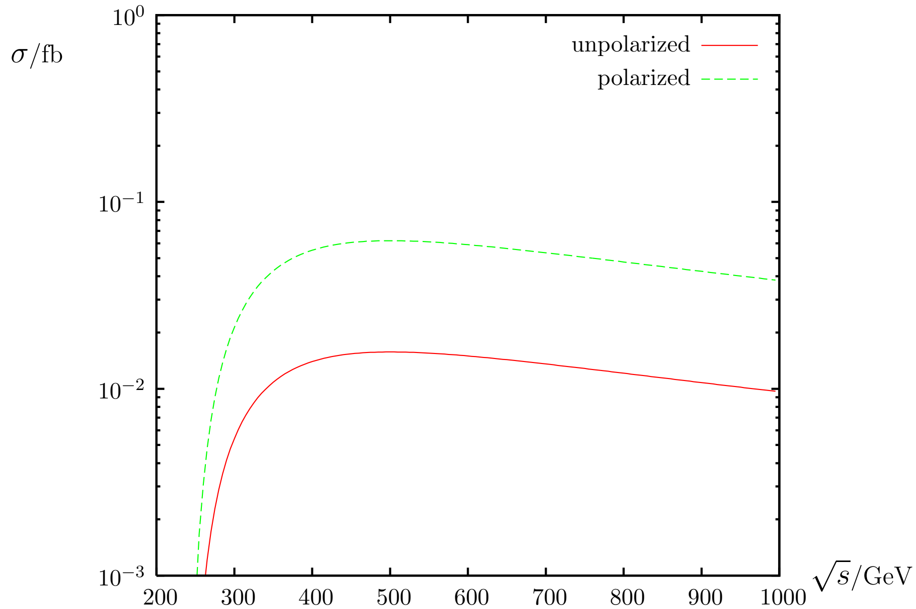

Fig. 4: The polarized and unpolarized crosssection as a function of the CM energy. The mass of the higgs is 120 GeV.

In figure 4 we show the crosssections as a function of the CM energy for a 120 GeV higgs mass. We see a slowly declining crosssection when we are away from the threshold. This must clearly be a loop effect, as the tree graphs would show a slow (logarithmic) rise.

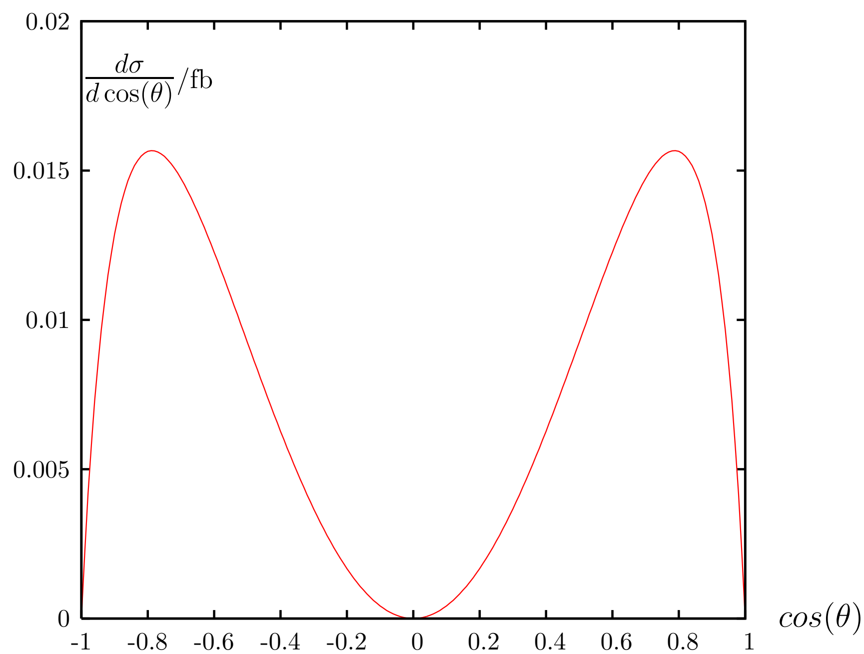

Fig. 5: The differential crosssection w.r.t. theta in the CM frame. The mass of the higgs is 120 GeV and the total CM energy is 500 GeV.

In figure 5 we see the differential distribution in terms of the cos of the CM azimuthal angle. As mentioned before, this distribution should be zero at 90 degrees when $\cos\theta = 0$. The immediate consequence of this is that if the energy is much above threshold, one should look for these events in a different region of phase space than the one that the previous papers would indicate.

We have also redone some of the MSSM calculations of ref [2]. For this we used solely the FeynArts system [3, 7]. In this case the situation is potentially more complicated, but the diagrams with even numbers of gamma matrices between the spinors are much smaller again and hence the same sign problem makes that the crosssections becomes correspondingly smaller than in the original calculation. This means that the ratio between the signals from the standard model and the MSSM model doesn't change significantly.

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