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Free Fermion Orientifolds

Elias Kiritsis[1], **Michael Lennek**[2] **and Bert Schellekens**[3,4,5]

[1] *Department of Physics, University of Crete, 71003 Heraklion, Greece*

[2] *CPHT, Ecole Polytechnique, 91128, Palaiseau, France, (UMR du CNRS 7644).*

3*NIKHEF, Kruislaan 409, 1009DB Amsterdam, The Netherlands*

[4] *IMAPP, Radboud Universiteit Nijmegen, The Netherlands*
[5] *Instituto de Física Fundamental, CSIC, Madrid, Spain*

Abstract: We investigate a class of orientifold models based on tensor products of 18 Ising models. Using the same search criteria as for the comparable case of Gepner model orientifolds we find that there are no three-family standard model configurations with tadpole cancellation. Even if we do not impose the latter requirement, we only find one such configuration in the special case of complex free fermions. In order to allow a comparison with other approaches we enumerate the Hodge numbers of the type-IIB theories we obtain. We provide indications that there are fermionic IIB vacua that are not $Z_2 \times Z_2$ orbifolds.

Contents

1. Introduction

In the last decade, there has been substantial progress in the construction of semi-realistic, standard-model-like string spectra using orientifolds. It was realized early on that orientifolds are successfully tuned to allow bottom up constructions of the SM spectrum using D-branes, [1, 2]. This has led to a separation of the problem of the construction of SM-like vacua to that of a local problem (engineering the SM stack of branes) and a global problem (tadpole cancellation).

Two classes of approaches have been applied to the construction of orientifold vacua, namely geometric and algebraic. The former starts with torus compactifications, to which orbifold and orientifold projections are applied. The latter starts with some rational conformal field theory (RCFT) to which boundary and crosscap states are added. In general, geometric constructions have the advantage that the moduli space of a solution is under much better control, whereas the algebraic approach probes deeper into the landscape of possibilities. The geometric approach has so far been applied mainly to $\mathbf{Z}_2 \times \mathbf{Z}_2$, $\mathbf{Z}_6$ and $\mathbf{Z}'_6$ orientifolds (see [3, 4] and references therein).

The algebraic approach has been applied successfully to Gepner Models [5]. It gave the richest class of SM-like vacua without chiral exotics [6]. Moreover it also gave the richest class of possibilities of embedding the SM spectrum into Chan-Paton groups, [7]. For other work on Gepner orientifolds see [8]-[13].

In principle, the geometric and algebraic (RCFT) approaches are not strictly separated. Here we will consider a class of orientifold vacua that is accessible from both of these directions, namely orientifolds of free-fermionic theories. From the algebraic point of view, this class is obtained by tensoring 18 Ising models in order to obtain the required central charge of 9, and imposing a world-sheet supersymmetry constraint. Geometrically, it is known that such theories are closely related to $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifolds. Our hope is, on the one hand, to find a standard-model-like configuration that can be studied from both perspectives. On the other hand, such a configuration might allow an explicit computation of couplings in a realistic example. While this is in principle possible for tensor products of $\mathcal{N} = 2$ minimal models (*i.e.* Gepner models), the required formalism is in practice only available for the simplest RCFT, the Ising model, or for the free boson.

The fermionic construction of string compactifications was pioneered in the heterotic context, [14]-[19]. It has proved a very practical tool and the phenomenologically most successful heterotic vacua were found in this context, [20]-[22]. It allowed for an algorithmic search of vacua using computers, and a rather straightforward algorithmic computation of the superpotential that has been exploited up to eighth order in the fields [23]. The fermionic approach to the heterotic string has been revived recently, [24]-[26]. It has been also used for statistical studies of the heterotic landscape, [27]-[30].

The art of free-fermion model building consists of simultaneously satisfying three requirements: world-sheet supersymmetry, modular invariance and, if desired, space-time supersymmetry. The first and the latter condition are essentially always satisfied in the same way. World-sheet supersymmetry is imposed by using a realization of the world-sheet supercurrent first presented in [14], leading to a "triplet constraint" on the free fermions, which in the language of conformal field theory results in extending the chiral algebra by certain currents of spin 3. Space-time supersymmetry always amounts to an extension of the chiral algebra by a definite spin-1 current. However, there are various ways of dealing with the third constraint, modular invariance. The most general one, proposed in [15] and [16] is to derive conditions on the boundary conditions of fermions on non-contractible cycles on the torus and higher genus surfaces (dealing with higher loop modular invariance is not entirely straightforward, however [31]). The second one is to consider the special situation where free fermion and free boson constructions overlap, *i.e.* complex free fermion pairs, in which case one may use the covariant lattice construction [32], and modular invariance at arbitrary genus can be derived using Lorentzian self-dual lattices. The third method is to use simple current modifications of diagonal partition functions, in which case consistency is guaranteed by general theorems [33]. The choice of method is limited by the requirement of being able to perform an orientifold projection on the result.

For the first method this problem was studied in [34, 35, 36], but so far no fully general method has been formulated. For the other two methods such a method *does* exist. As we shall see in the next section, the simple current method in combinations with the requirement of space-time supersymmetry does require bosonization of some, but not all of the fermions. It is thus somewhat less general than the full free fermion construction, but more general than a free boson construction, and it is, to the best of our knowledge, the most general method currently available for free fermion orientifold constructions.

Although the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifolds relevant for the fermionic constructions have been successful in the heterotic context the associated results for $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifolds have not been very encouraging so far, [37, 38]. It should be however be appreciated that the searches done so far concern a rather small set of possible standard model realizations. In [7] a minimally biased set of requirements was formulated, which allows many more – although sometimes rather exotic – realizations of the standard model. Basically, the only requirement (constraint) is that all quarks and leptons originate from a maximum of four participating branes, and that the strong and weak gauge groups are not diagonally embedded in multiple brane stacks. This allows for example arbitrary embeddings of the weak hypercharge Y and quarks and leptons originating from rank two tensors (that were classified), as well as various kinds of gauge unification. Here we will use exactly the same set of requirements. For a more detailed description we refer to [7].

Our main conclusion regarding standard model spectra is that even with these much broader search criteria, the set of free fermion orientifolds (and hence presumably the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifolds) is an extremely poor region in the orientifold landscape, in comparison to orientifolds of interacting CFT's, in particular of Gepner models. Since we use identical search criteria in both cases this is a fair comparison. Indeed, in the present search we did not find any solution to the tadpole conditions that contains the standard model spectrum. Even keeping only the condition that the spectrum is right, before trying to find a tadpole canceling hidden sector, we found just a handful of solutions of two different chiral types (which, however, are remarkably simple and elegant). By contrast, in the case of Gepner models both problems (finding the standard model spectrum with or without tadpole cancellation) had a huge number of solutions: the number of distinct chiral types in that search was more than 19000 [7], in comparison with just two in the present search.

This paper is organized as follows. In the next section we will describe the free fermion CFT's we are considering. In section three we discuss the closed sector of these CFT's, and present a list of Hodge numbers for comparison with other work. This list should in particular be useful to determine the precise scope of our search. Since we do not have any formalism to deal with free fermion orientifolds in full generality (*i.e.* for 18 unpaired real fermions), it would be interesting to know the full list of Hodge data for the general case, and compare with ours. Despite the more than twenty years of history of the subject, apparently such a list is not available at present. Finally, in section four we will present the standard model search results. The appendix contains a more detailed list of Hodge data, including results on heterotic singlets and the number of boundary states.

2. CFT considerations

Our basic building block is the Ising CFT, which has three primaries 0, ψ and σ with conformal weights 0, $\frac{1}{2}$ and $\frac{1}{16}$ respectively. Since its central charge is $\frac{1}{2}$ one can tensor 18 copies in order to obtain a $c = 9$ "internal" CFT for a compactified type-II string theory. Fermionic string theory consistency requires an $\mathcal{N} = 1$ world-sheet supersymmetry. Unlike the building blocks used in Gepner models, the $\mathcal{N} = 2$ minimal models, the Ising building blocks are not supersymmetric. But it has been known for a long time [14, 15, 18] how to realize world-sheet supersymmetry on a triplet of Ising models. The world sheet supercurrent is simply the product of the three fermionic currents of the factors, $\psi_1\psi_2\psi_3$. Having realized supersymmetry on a triplet of fermions, we still have to impose it on products of supersymmetric building blocks, so that their NS and R sectors are properly aligned. This is done, as in the case of Gepner models, by extending the chiral algebra with all products of the supercurrents of the building blocks, including the space-time NSR factor. These products are sometimes called "alignment currents". They have spin 3, because they are products of spin-$\frac{3}{2}$ of the separate factors, or the supercurrent $X^\mu\partial\psi_\mu$ of the NSR factor of the theory. Extending the algebra by these currents implies a projection on the spectrum, which in the special case of free fermionic models is called the "triplet constraint".

In the case of interest, one divides the 18 Ising models into six groups of three to impose this constraint. The result is a fermionic string theory, which in general has a spectrum without space-time supersymmetry. To obtain space-time supersymmetry we have to perform another extension of the chiral algebra, by a spin-1 current that is spinorial in the NSR sector. The resulting projection on the spectrum is of course the GSO-projection. This current consists of an NSR spin fields with weight $\frac{5}{8}$ combined with six Ising spin fields σ, so that the total conformal weight is 1. Locality with the alignment currents requires that there be an odd number of σ fields in each fermionic triplet, and then obviously the only solution is to choose precisely one per triplet.

There is an important difference between the alignment currents and the space-time supercurrent. The former consists entirely of simple currents, whereas the latter involves the Ising field σ, which is not a simple current. The boundary state formalism we want to use is the one of [39], which includes the most general available extension of earlier work of the Rome group [40, 41], which in its turn is based on the classic paper by Cardy [42]. This formalism produces the complete set of boundary states for all simple current extensions of the chiral algebra. Unfortunately it cannot be applied to extensions that are not simple current related, like the space-time supercurrent we encounter here.

But there is a way out of this in some cases. An Ising model corresponds to a real (Majorana) free fermion. If we combine a pair of them into a complex free fermion, then the spinor current turns out to be a simple current. Such a pairing implies that the two fermions have the same boundary conditions on any cycle on any Riemann surface, and hence is a restriction on the total number of possibilities. This can be achieved by extending the chiral algebra of the theory with the spin-1 current $\psi_i\psi_j$, where i and j label the fermions to be paired. In order to use the simple current boundary state formalism we have to group the six fermions participating in the space-time supercurrent into three pairs. This yields then a type-II theory built out of three complex fermions (with standard, periodic and anti-periodic boundary conditions) and twelve real fermions. We may consider pairing some of the remaining real fermions as well. Such a pairing replaces the real fermion pair by a free boson compactified on a circle of radius $R^2 = 4$. The resulting CFT has central charge 1 and may be thought of as the extrapolation of the D_n affine Lie algebras to $n = 1$. Therefore we will denote it as D_1. Hence the resulting $c = 9$ CFT is in general built out of a combination of Ising models and free bosons. This should not be confused with the case studied in [43], the 2^6 Gepner model. This models are also tensor products of free fermions and free bosons, but in this case the bosons are on a circle of radius $R^2 = 8$, and are not straightforwardly related to free fermions.

It may seem that there is no advantage to pairing two real fermions into a boson. Normally, the pairing of fermions reduces the number of options for choosing fermion boundary conditions, and hence the largest number of free-fermionic CFT's is obtained by leaving all boundary conditions free and independent. Indeed, the pairing of two fermions amounts to an extension of the chiral algebra. In general, there are two ways of dealing with such extensions. The first is to extend the chiral algebra directly, and work with the reduced set of characters this implies. The second is to implement the extension as a modular invariant partition function (MIPF), which has the form of a sum squares of linear combinations of characters. These linear combinations correspond to the reduced set of characters of the extended chiral algebra, and indeed this MIPF is identical to the diagonal partition function of the extended theory. These two methods therefore yield identical closed string sectors. We will refer to these two cases as a *direct extension* and a *MIPF extension* henceforth. Although they yield identical closed strings, there is an important different between these two cases when open strings are considered, using the formalism of [39]. In the case of a direct extension, only boundary states are allowed that respect the extended symmetry, whereas in the case of a MIPF extension only the original chiral algebra is required to be respected. Hence in that case there are boundary states that respect the extension, but also additional ones that do not respect it. Therefore it is in general advantageous to implement an extension as a MIPF, unless the extended symmetries are themselves required for the physics of the problem under consideration. The latter is true for world-sheet supersymmetry, sometimes for space-time supersymmetry, but not for the pairing extension discussed above.

However, there is one exception to the foregoing if the formalism of [39] is used. This exception occurs when the extended CFT has simple currents that result from fixed point resolution. In that case working directly in the extended CFT allows us to use these simple currents to build MIPFs that cannot be obtained as simple current MIPFs in the unextended theory. In the unextended theory those MIPFs are exceptional invariants, to which the general formalism of [39] does not apply, and for which ad-hoc formalisms must be developed, as was done for example for the E-type invariants of $SU(2)$ [44].

This can most easily be studied in the tensor product of two fermions. This has a total of nine primaries, four of which are simple currents. If we extend the chiral algebra by the spin-1 current $\psi_1\psi_2$, we get a new CFT with four primaries. Two of these are the identity $(0,0)+(\psi,\psi)$ and the free fermion $(0,\psi)+(\psi,0)$. The other two originate from the combination (σ,σ). This turns out to be a fixed point of the extension current $\psi_1\psi_2$, which is resolved into two separate fields in the extended CFT. In rare cases it may happen that such a resolved fixed point field becomes a simple current in the extended CFT, and this is such a case. If we consider the MIPF obtained by using the simple current $\psi_1\psi_2$, we get a total of six boundary states. Four of these respect the extended symmetry, and two of them do not. If we work instead directly in the extended CFT, *i.e.* D_1, we only see the four boundary states the respect the extension. So here the MIPF has the advantage over the direct extension. To see the opposite, consider 16 free fermions. We can pair these, using a direct extension, into 8 free bosons. This CFT, $(D_1)^8$, has a simple current MIPF corresponding to D_8, which is also a simple current MIPF of $(\text{Ising})^{16}$, but it also has a MIPF corresponding to E_8, which is *not* a simple current MIPF of $(\text{Ising})^{16}$. Although we would not be able to obtain this E_8 theory with simple current MIPFs of only Ising models, it can be obtained with the method we use in the present paper, namely combinations of Ising models and free bosons. In fact, although we would expect that examples exist which can only be obtained using the full free fermion construction, and not by means of simple currents in combinations of free boson and free fermion CFT's, we are not aware of any such example.

The conclusion is that to maximize the number of cases we are able to consider with the formalism at our disposal, we should consider all possible options for pairings of the 12 remaining free fermions. The starting point is the completely unpaired case. This is a CFT that is a tensor product of a four-dimensional NSR model, three free bosons ϕ labeled a,b,c, and twelve free fermions ψ labeled $1,\ldots,12$. The chiral algebra is extended by the following alignment currents

$$\partial X_\mu \psi^\mu e^{i\phi_a}\psi_1\psi_2 \qquad \partial X_\mu \psi^\mu e^{-i\phi_a}\psi_3\psi_4 \tag{2.1}$$

plus four more with labels $(b,5,6),(b,7,8),(c,9,10),(c,11,12)$. Here ψ^μ are the NSR fermions. The space-time supersymmetry current is $S_\alpha\sigma_a\sigma_b\sigma_c$, where S_α denote the NSR spin fields combined with the usual contribution from the bosonized superghosts, and σ_a, σ_b and σ_c are the D_1 spinors. The latter three are simple currents, and for all practical purposes, so is the NSR spin field. The nicest way of dealing with it explicitly as a simple current is to use the covariant lattice method of [32], where it becomes a spinor of D_5. Note that there are two choices available for each of the factors of the space-time supersymmetry current, but all these choices are equivalent.

All other options are obtained from this starting point by adding pairing currents $\psi_i\psi_j$, with $i,j = 1,\ldots 12$. These pairing currents are always local with respect to the alignment currents, the space-time susy current and with respect each other, so they can be added without any constraint. However, we have to close the algebra after adding any such current, which may lead to undesirable consequences.

Let us first consider the special case where we only add pairing currents for the first four fermions. If we add just one pairing current, the distinct possibilities (taking permutations into account) are $\psi_1\psi_2$ and $\psi_1\psi_3$. The former choice, when combined with (2.1), implies an extension of the chiral algebra with $\partial X_\mu \psi^\mu e^{i\phi_a}$, which means that the four-dimensional NSR model is extended to a six-dimensional one. Hence all theories we get this way are torus compactifications of a six-dimensional theory. This is of no interest, since in such a theory all characters are non-chiral in space-time and hence there is no possibility for obtaining the standard model from boundary states.[1] If we add the current $\psi_1\psi_3$, then closure of the algebra with the two currents in (2.1) implies that also the combination $\psi_2\psi_4$ is in the chiral algebra. Hence there are just two options that are of interest, namely no pairing, and the pairing $(1,3)(2,4)$.

One may continue this procedure to eight fermions. Obvious solutions are no pairings, two pairings (either (1,3)(2,4) or (5,7)(6,8), which are equivalent under permutation), or four pairings, (1,3)(2,4)(5,7)(6,8). But in addition to these three distinct possibilities one may also consider pairings between the first and the second group of four. Here one also encounters some possibilities that are of no interest. For example the pairing $(1,5)(1,6)$ has the effect of combining the three fermions (1,5,6) into $SO(3)$. But this has no advantages, because $SO(3)$ has no simple currents on top of those of the original free fermions. Similarly, extensions to other $SO(N)$ groups with $N \geq 3$ need not be considered. Taking this, as well as all permutations, into account, we arrive at a total of 11 possibilities, including the three described above.

We proceed in a similar way with the case of twelve fermions. Here we obtain a total of 62 distinct cases, including all combinations of four and eight fermion subcases. There is some overcounting in this set, because it turns out that some purely

[1]Note that we are considering direct extensions here. If, on the other hand, we implement the extension by $\psi_1\psi_2$ as a MIPF, there is a possibility of having a six-dimensional bulk CFT but space-time chiral boundary states that do not respect the bulk symmetry.

free boson (complex fermion) cases are extensions of others by currents of spin two or three, which is clearly a direct extension that has no advantages over a MIPF extension. There may be other such "useless" extensions for mixtures of real and complex fermions, but there is an important caveat here: there are examples that look like extensions of cases that are not on the list of 62 themselves. We found that if this happens the "de-extended" combination does not have a valid world-sheet supersymmetry realization, even though the extended combination does. This can happen because the aforementioned spin two or three current may be combinations of pairing currents and world-sheet supersymmetry currents, and removing it may therefore destroy world-sheet supersymmetry. Since the superfluous cases are anyway among the easiest to deal with in terms of computer time, it was not worthwhile to eliminate them.

3. The closed string sector and the associated geometry

As discussed earlier, we are utilizing a purely algebraic approach in order to construct these models. The fact that there are no explicit geometric considerations that enter into the model construction method makes examining the resulting compactification geometries interesting. The primary constraints on what compactification geometries result stem only from the CFT considerations discussed in Sect. 2 and stringy consistency conditions. We shall start our discussion of the compactification geometries at the "global" level by discussing all of the compactification geometries found.

In the course of this study, we found thirty-two different compactification manifolds. These manifolds are differentiated solely on the basis of their Hodge numbers (namely h_{11} and h_{12}) and the amount of space-time supersymmetry preserved. The full list of manifolds is presented on Table 1. As the structure of the table suggests, we find that each model has a mirror, but we shall defer a discussion of mirror symmetry until later. The table does illustrate that we find a wide variety of different Hodge numbers and find that within this set of manifolds there is a large variation in the amount of supersymmetry preserved.

As discussed in Sect. 2, our study consisted of sixty-two different model classes. The manifolds listed on Table 1, were distributed amongst these different model classes. We shall now examine how these manifolds were distributed amongst the different model classes. This can give some idea how generically these manifolds may be found in this context. There is a large variation between different compactification manifolds with respect to the number of model classes that realize them. The manifold preserving $\mathcal{N} = 4$ supersymmetry, which most likely corresponds to a toroidal compactification, is found in every single model class. There were also two manifolds preserving $\mathcal{N} = 2$ supersymmetry (namely $(h_{11}, h_{12}) = (13, 13), (5, 5)$) which were each found in over fifty of the model classes. There are many model classes which *only* realize these three common manifolds. On the other extreme, there are three

Hodge Numbers (h_{11}, h_{12})	Amount of $d=4$ SUSY		
	$\mathcal{N}=1$	$\mathcal{N}=2$	$\mathcal{N}=4$
(51,3) and (3,51)	X		
(31,7) and (7,31)	X		
(27,3) and (3,27)	X		
(25,1) and (1,25)	X		
(21,9) and (9,21)	X		
(19,7) and (7,19)	X		
(17,5) and (5,17)	X		
(15,3) and (3,15)	X		
(12,6) and (6,12)	X		
(21,21)		X	
(19,19)	X		
(15,15)	X		
(13,13)	X	X	
(11,11)	X		
(9,9)	X	X	X
(7,7)	X		
(5,5)	X	X	
(3,3)	X		
(1,1)		X	

Table 1: The compactification manifolds found in this study along with the amount of space-time supersymmetry that they preserve.

manifolds, ($\mathcal{N}=1$ $(25,1),(1,25),(13,13)$) that are only realized in one model class each. These manifolds only are found in the case of the extension only involving powers of D_1 (that is, all fermions paired). Most of the manifolds are realized in a relatively small number of model classes with twenty-four of the manifolds only being realized in less than a third of the available model classes.

Another quantity that can be utilized to differentiate between compactification manifolds is the number of so-called "heterotic singlets". The model construction method utilized allows for the counting of the number of massless states which transform as singlets under an E_6 factor within the chiral algebra[2]. The name "heterotic singlets" derives from the following fact: any type-II partition function with (1,1) space-time supersymmetry can be uniquely mapped to a heterotic vacuum with N=1 space-time susy with E_6 symmetry via a modular invariance preserving map first de-

[2]For toroidal compactifications, the E_6 factor is enhanced to an E_8. Thus, for these compactifications we count the number of E_8 singlets instead of E_6 singlets. For this study, this only affects the manifolds preserving $\mathcal{N}=4$ supersymmetry.

scribed in [45, 32] and applied to map type-II strings to heterotic ones in [5]. (this map is sometimes called the “bosonic string map” or the “Gepner map”). In this related heterotic ground state, there is a number of singlets under the E_6 group. Their number depends on the topology of the CY manifold and its tangent bundle, and is a useful quantity for distinguishing MIPFs.

Using this information along with the Hodge numbers and the amount of preserved space-time supersymmetry to differentiate between compactification manifolds, we find that there were 421 different compactification manifolds. The full list of these manifolds is in Appendix A. In addition, we find that 58 of these manifolds exhibit extended supersymmetry. There is a single manifold which preserves $\mathcal{N} = 4$ supersymmetry. Unlike the case earlier, it is relatively rare that two manifolds have exactly the same Hodge numbers and number of singlets and yet preserve different amounts of space-time supersymmetry. This was observed in five cases, which is about one percent of the total sample.

As discussed in Sect. 2, there were sixty-two different model classes considered in this study. The 421 distinct manifolds were distributed amongst these model classes. Ninety of these manifolds were found in exactly one model class. This represents a factor of thirty increase from the earlier case of three manifolds being found in only one model class. Interestingly, only three of the thirty-two manifolds are not represented in these ninety. They are $\mathcal{N} = 4$ $(9, 9)$, $\mathcal{N} = 2$ $(21, 21)$ and $(1, 1)$. There are again three very common manifolds. The $\mathcal{N} = 4$ $(9, 9)$ is again found in every model class. The other two common manifolds from earlier remain very common. This suggests that these three manifolds represent very symmetric cases. This stems from the fact that they are both very common and the extra differentiation into singlets did not seem to have an effect upon their ubiquity. The general behavior for the rest of the manifolds is that they are found in a very limited number of different model classes with more than three quarters of all of the manifolds being found in five or fewer different model classes.

As the entries in Table 1 suggest, this method of constructing models seems to preserve mirror symmetry in the sense that, for each model which appears in the set, the mirror is also in the set. However, we did not strictly check that each model actually has a mirror, only that another model with the correct Hodge numbers and the same number of singlets appeared in the set. That is, we did not explicitly construct a map from one model to the proposed mirror. With that warning in mind, we shall examine the appearance of mirror symmetry within this set of models.

We shall start at the level of looking at the entire set of realizable models. This is the broadest level possible, as we do not worry about from which model class each model comes from. At this level, we define the mirror of a model to be an identical model with the appropriately flipped Hodge numbers (*e.g.* $(h_{11}, h_{12}) \rightarrow (h_{12}, h_{11})$, the same amount of supersymmetry preserved, and the same number of singlets. Using this definition, we find that *every* model has an appropriate mirror within the model set. This comes with the caveat that we allow for the situation that a model is actually invariant under a mirror transformation. That is, we allow manifolds with $h_{12} = h_{11}$ to have odd multiplicities. This occurs rarely, but it does occur. In addition, we can consider each model class separately. Mirror symmetry even holds using this more stringent division.

In addition to considering Hodge numbers and the number of heterotic singlets, one can also differentiate manifolds by considering how many boundary states are consistent with the model. We shall not discuss this very much except to note that, the apparent mirror symmetry is broken with models where $h_{11} \neq h_{12}$. There exist models for which no corresponding mirror with the same number of boundary states, number of singlets, and appropriate Hodge numbers is in the set. This does not come as a surprise, since in the similar case of T-duality for circle compactifications, the number of boundary states also is not preserved by the duality: for radius $R^2 = 2N$, the T-duals have 2 or $2N$ boundary states.

Thus far, we have only discussed what compactification manifolds we have found in our scan over the full sixty-two model classes. However, it is also potentially interesting to examine what is the minimum set of these sixty-two model classes for which every single compactification manifold is contained. In other words, if one just wanted to classify all of the manifolds realizable in this specific construction, what is the minimum set of extensions that must be considered? Clearly, this question depends on how one differentiates between compactification manifolds. If we simply utilize Hodge numbers, then we would find every distinct manifold considering only two model classes of models. These are, in some sense, the extreme cases, which are every fermion paired and every fermion unpaired. However, if we consider the singlet data as well as Hodge numbers then we find that although we increase the number of distinct manifolds from 32 to 421, we only require four different model classes in order to find every manifold. In fact, if we only took the two model classes required in the earlier differentiation method we would only miss four manifolds. Thus, the two extra model classes only provide these few missing manifolds. This is not to say that searching through the sixty-two different model classes for the Standard Model would be fruitless only that the sum total of all of the different compactification manifolds will have been found after only searching through these four different model classes.

We would also like to compare our results to other methods. There are two related questions in this context. The first is how our construction algorithm is related to the traditional fermionic construction, [15, 18]. Although this search was never done to our knowledge in the IIB string we can argue that our vacua fall within the conventional definition of fermionic constructions as these were described in [15, 18]. The reason is that the simple current extension technique we use to generate MIPFs from a reference MIPF, is preserving the fermionic nature of MIPFS. More precisely if it acts on a MIPF that is a sesquilinear form of θ-functions or Ising characters, it still gives MIPFs that can be written as sesquilinear forms of θ-functions or Ising characters. Therefore the IIB vacua described here is a large subset of all fermionic IIB vacua.

Another set of vacua fermionic theories are usually compared to is to $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifolds. There even seems to be a "folk theorem" stating that the two sets are equivalent[3] The second question therefore is to what extend this is true. A first attempt was made to classify all $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifolds in [46]. This task was recently completed in [47]. In that paper, the authors classified all $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold actions on $(T^2)^3$ including shifts and discrete torsion. The list of Hodge data obtained matches our table 1 with one exception: our list contains in addition the Hodge numbers (25,1) and its mirror. It is not exactly yet clear what is the origin of this mismatch. It is expected however that the simple current method is related to the orbifold method (or its inverse). In particular, the (25,1) models in our list are constructed from a $\mathbf{Z}_4$ simple current extension and it is plausible that this is the reason it is not found in [47]. This seems to suggest that the folk theorem stating that fermionic constructions are equivalent to $\mathbf{Z}_2 \times \mathbf{Z}_2$ seems to fail. This observation requires further study.

It is noteworthy that we have found one Hodge number pair in addition to those of $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifolds, but that on the other hand none of the Hodge number pairs of [47] is missing from our list. This seems to suggest that either we are covering most, if not all, free-fermionic theories, or the aforementioned folk theorem is not even close to being correct. To check this, it would be very interesting to have a complete list of Hodge numbers and heterotic singlets for all free fermionic type-IIB theories.

A partial list of closed string data for $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifolds appears in [48]. These authors do not only present the Hodge numbers, but also the number of singlets and additional vector bosons in heterotic strings. The latter number is two in all cases, in other words the heterotic gauge group is $E_6 \times E_8 \times U(1)^2$. Although we have not included information on the number of $U(1)$'s in this paper, we did compute this information, so that we can compare results. In our construction, two is the lowest number of additional gauge bosons encountered, and it only occurs in the case of twelve unpaired free fermions (which is easily understandable, since any pairing introduces an additional $U(1)$ factor). Of the eight spectra published in [48], three match exactly with ours (namely (3,51,252), (3,27,132) and (7,31,172)), whereas the five others have a remarkably small number of singlets outside our range. In addition, four out of seven cases with $h_{11} = h_{12}$ match with ours (these were not published in [48], but communicated to us by the authors). It is not clear to us if all of the spectra of [48] can be obtained with the original free fermionic construction of [14, 15], but if

[3]This equivalence is between free fermionic theories and special points in the moduli spaces of $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifolds. The precise statement of such a theorem could be that for every free fermionic type-IIB theory there is a point in the moduli space of a type-IIB $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold matching it, while every type-IIB $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold has at least one point in its moduli space that can be described by free fermions.

they can, then this would be the first examples were the mixed fermion/boson simple current construction we use here misses some case. On the other hand, if we impose the condition that the number of additional $U(1)$'s is exactly 2, we get a total of 83 distinct $(h_{11}, h_{12}, \text{singlets})$ cases, compared to a total of 15 in [48]. There are also some differences in cases with extended supersymmetry, and furthermore the (25,1) models absent in [47] are also absent in [48]. This is not entirely surprising, since our (25,1) spectra have 8 additional $U(1)$'s and are apparently outside the scope of [48]. All of this just underscores the need for a systematic comparison of the different approaches.

4. Standard Model Search

One of the main goals of this study is to find a string vacuum with a low-energy limit that consists of, at least, a semi-realistic MSSM. Such a vacuum could be studied from both the geometric and the algebraic perspectives. In particular, its realization as a free-fermion CFT will make the process of evaluating the effective action simpler and amenable to an algorithmic/computer treatment. This is necessary for a detailed scan of the region in the neighborhood of the vacuum found, as electroweak symmetry breaking, supersymmetry breaking mass generation and other important effects are expected to be triggered by the local effective potential.

The search methodology utilized in the present study was detailed in Ref. [7]. The methodology is an implementation of the bottom-up approach [1, 2], implemented in the context of RCFT orientifolds [39] and amended with an algorithm of tadpole cancellation [6]. This procedure first constructs a **top-down** spectrum that matches the SM by utilizing the BCFT boundary states. This is what we call a "top-down solution". Such a solution can be promoted to a bona-fide string vacuum by solving the tadpole conditions. This is achieved when possible by adding an appropriate "hidden sector". Since our use of the terminology "top-down" may be confusing, let us summarize the three distinct classes of spectra that enter the discussion. A "bottom-up configuration" is any combination of unitary, orthogonal or symplectic gauge groups with bi-fundamental or rank-2 tensor matter that is free of all relevant anomalies, and which might therefore be realized with a set of intersecting branes or a set of boundary states. If such a realization is found in an explicit model, we speak of a "top-down solution". If in addition a tadpole canceling hidden sector can be found (or if no hidden sector is needed to cancel all tadpoles), we call the result a "string vacuum".

We will describe now this procedure in a bit more detail. Full details can be found in [7] where the search criteria were developed and where a general characterization of hypercharge embeddings was found. The first step in the search consists of dividing the full set of boundary states (branes) present in the model into observable and hidden sectors. The observable sector is defined as the set of branes where Standard

Model matter resides. This sector also gives rise to all of the Standard Model gauge symmetries. There are some criteria that can be placed only on the observable sector. These include the requirement that the $SU(3)$ and $SU(2)$ gauge symmetries each arise from single stacks of branes. This eliminates the possibility that these groups arise from the diagonal combination of two branes. We do not make any further assumptions about the symmetry breaking mechanism if these gauge symmetries are embedded in larger groups. Hypercharge is allowed to arise from any massless linear combination of $U(1)$ factors arising from observable sector stacks of branes. Next we require that there be the matter content consistent with the three generation MSSM present in the observable sector, and no chiral exotics (see below). Furthermore we require that the observable sector consists of no more than four distinct stacks of branes, in order to keep the search manageable. With more stacks of branes, the number of ways of embedding the hypercharge Y increases drastically, and one may also obtain quarks and leptons from several distinct bi-fundamentals. On the other hand, the number of options for chiral exotics increases. It is not clear which of these competing effects dominates.

As our definition of what constitutes a chiral exotic may differ a bit from that usually used in the literature, we shall now define chiral exotics for these models. We do not put any constraints on matter that is not charged under the observable Chan-Paton group, *i.e.* we allow for any amount of chiral matter that is limited to the hidden sector. If the chiral matter is in the observable sector we require it either be part of the MSSM spectrum or at least non-chiral with respect to all of the Standard Model gauge groups. Apart from the standard three families of quarks, charged leptons and left-handed neutrinos, this definition does allow a few more particles that are chiral with respect to the observable part of the Chan-Paton group.[4] It allows for right-handed neutrinos that are chiral with respect to an extension of the standard model (the most common case being a broken or unbroken gauged $B - L$ symmetry). It also allows Higgs pair candidates that are chiral with respect to a $U(2)$ group, which contains $SU(2)_{\text{Weak}}$ (in this case the additional $U(1)$ is broken by axion mixing). Among the less desirable particles in this category are mirror pairs of quarks and leptons that are chiral with respect to the Chan-Paton group, but non-chiral with respect to the standard model gauge group. Note that although the latter particles are exotic and chiral with respect to the full Chan-Paton group, we do not call them "chiral exotics" because they are not chiral with respect to $SU(3) \times SU(2) \times U(1)$.

Apart from chiral observable and chiral hidden matter, a third category is chiral observable-hidden matter. Such matter may be subject to symmetry breaking or confinement in the hidden sector, and is therefore not necessarily fatal. Furthermore,

[4]The precise definition of the "observable part of the Chan-Paton group" is those factors of the original Chan-Paton group that contain parts of $SU(3) \times SU(2) \times U(1)$, before taking breaking through axion mixing into account.

there are several kinds of chiral observable-hidden matter that nevertheless fulfill the requirements stated in the previous paragraph, *i.e.* that they are non-chiral with respect to the standard model gauge group. Nevertheless, in the previous searches [6] and [7] chiral observable-hidden matter was not accepted. In other words, boundary states with a chiral intersection with the standard model branes were given Chan-Paton multiplicity zero. This has the advantage of limiting the scope of the search to the *a priori* most attractive models. In a few cases where this requirement was lifted, this resulted in an explosion of the number of solutions by several orders of magnitude. In situations where the main search result is negative, it is natural to remove this requirement. This is indeed what we have done in the present paper.

Because of the existence of several possible definitions of chiral exotics, we wish to emphasize that most spectra obtained in previous searches are free of chiral exotics, for *any* definition of the latter. For example, apart from three chiral right-handed neutrinos that are chiral with respect to $B-L$, but not "exotic" by any standard, about 85% of the about 200.000 spectra collected in [6] have have no extra chiral matter at all, 12.5% has a chiral hidden sector, and about 2% have a $U(2)$ chiral Higgs pair and/or chiral mirror pairs.

Utilizing the criteria outlined above, we found that only 1 of the 62 model classes yielded any top-down solutions. This model class was the case of all fermions paired (that is, simple current extensions only involving powers of D_1), or in other words a compactification that can be realized entirely using free bosons and self-dual lattices [32]. No other model classes yielded models that satisfied these criteria. The search was done without any constraint on the number of boundary states. In [7] an upper limit of 1750 was used. In the present case the number of boundary states goes up to 3040, but in most cases already the first step in the search (looking for three quark doublets) failed. Thus because of lack of results, larger numbers became accessible.

We did have to impose a limitation on the scope of the MIPF search. The most difficult case, twelve real and three complex fermions, has 534700 MIPFs. As explained earlier, not all of these are distinct. The vast majority of this large number comes from the discrete torsion signs of large simple current subgroups. Since the simple current group in this case is $(\mathbf{Z}_2)^7$, the largest subgroup, the simple current group itself, admits 21 such signs (they form an anti-symmetric 7×7 matrix [49]). This leads to 2^{21} possibilities, still subject to identification by permutations. It turns out that these in principle distinct MIPFs produce very few distinct Hodge numbers. For this reason we have searched the MIPFs originating from large simple current subgroups by taking a random sample of 100 discrete torsion sign choices per subgroup[5].

The top-down solutions we found were of a chiral type already encountered in [7] for Gepner models. The simplest of them is a Pati-Salam type of spectrum, which

[5] This kind of sampling was only done for the Standard Model search. The Hodge number scan was done completely, and gave rise to many degeneracies for a given simple current subgroup.

is remarkably simple. The Chan-Paton group is $U(4) \times U(2) \times U(2)$, with all $U(1)$ symmetries broken by axion mixing (note that Y is the $U(4)$ generator $\frac{1}{6}(1,1,1,-3)$). The spectrum consists of the following left-handed particles (with "V" for vector and "V^*" for conjugate vector)

$$\begin{array}{rc} 2\times & (V,V,0) \\ & (V,V^*,0) \\ 2\times & (V^*,0,V^*) \\ & (V^*,0,V) \\ 2\times & (0,V,V^*) \end{array}$$

which represent respectively three $SU(4)$-unified quark and lepton doublets, three $SU(4)$ unified anti-quark and charged lepton singlets, and 2 particles with the quantum numbers of a MSSM Higgs pair. Therefore, apart from the $U(4)$ baryon-lepton unification and the extra Higgs pair this is precisely the MSSM spectrum. We emphasize that the multiplicities given above are the exact multiplicities of left-handed particles, and not the net number (left minus right). Hence the additional Higgs pair is the only exotic, there are no mirror quarks or leptons whatsoever, not even fully non-chiral ones. This is extremely rare, and we do no know any such example in the entire set of spectra obtained from Gepner models[6]

The second chiral type we found is essentially the same as the foregoing, but with the $SU(4)$ stack split in three baryon and one lepton stack. This spectrum has one additional exotic, a non-chiral set of leptoquarks originating from the gaugino corresponding to the broken generators of $SU(4)$.

However, even after relaxing the observable-hidden chirality constraint, as explained above, we were unable to obtain a solution to the tadpole conditions for any of these models.

As an extra check on the top-down model search algorithm, we relaxed the requirement that there be exactly three generations and found numerous examples of one and two generation models in many different model classes. We tried this on a total of 65 MIPFs, a small fraction of the total, and found top-down configurations in 62 of them. Tadpole solution were found for some one-family models, but not for two-family models. Due to the limited number of cases considered, no conclusions with regard to family statistics should be drawn from these observations. But this does reinforce the finding that there are only very few models with three generations in the entire set of models constructed. Despite the need for statistical sampling

[6]Note however that the available spectra in the Gepner model search, [6], are free of tadpoles; there is no databases of exact spectra of top-down solutions prior to tadpole cancellation. The search performed in [7] focused more on chiral types than on tadpole solutions, but the chiral types were collected modulo non-chiral exotics, so that there is no such database in that case either.

mentioned above, it seems extremely unlikely to us that any three family models were missed.

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Appendices

A. Full List of Manifolds

The following table contains the full list of compactification manifolds found during this search. We have organized the table in such a way as to include the Hodge numbers the number of E_6 singlets (listed as heterotic singlets on the table), the amount of space-time supersymmetry preserved, and the number of boundary states. The boundary state information includes the following values: the maximum value that the number of boundary states took, the minimum value for the number of boundary states, and the total number of different values for the number of boundary states. For a more complete discussion of all of this information see Ref. [6].

Hodge numbers	Heterotic	$d=4$	Boundary States		
(h_{11}, h_{12})	Singlets	Susy	Maximum	Minimum	Distinct
$(51,3)$	258	$\mathcal{N}=1$	2048	32	14
$(51,3)$	256	$\mathcal{N}=1$	2272	160	16

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d = 4$ Susy	Boundary States Maximum	Minimum	Distinct
$(51, 3)$	254	$\mathcal{N} = 1$	2528	448	12
$(51, 3)$	252	$\mathcal{N} = 1$	3040	2048	5
$(3, 51)$	258	$\mathcal{N} = 1$	1280	32	11
$(3, 51)$	256	$\mathcal{N} = 1$	1504	128	15
$(3, 51)$	254	$\mathcal{N} = 1$	1760	416	12
$(3, 51)$	252	$\mathcal{N} = 1$	2272	1280	5
$(31, 7)$	254	$\mathcal{N} = 1$	1216	32	20
$(31, 7)$	252	$\mathcal{N} = 1$	1376	128	21
$(31, 7)$	230	$\mathcal{N} = 1$	1376	160	18
$(31, 7)$	228	$\mathcal{N} = 1$	1552	496	15
$(31, 7)$	209	$\mathcal{N} = 1$	1600	152	23
$(31, 7)$	208	$\mathcal{N} = 1$	1376	128	22
$(31, 7)$	207	$\mathcal{N} = 1$	1952	592	15
$(31, 7)$	206	$\mathcal{N} = 1$	1552	416	17
$(31, 7)$	190	$\mathcal{N} = 1$	1600	320	16
$(31, 7)$	188	$\mathcal{N} = 1$	1952	1312	10
$(31, 7)$	174	$\mathcal{N} = 1$	1600	256	22
$(31, 7)$	172	$\mathcal{N} = 1$	1952	1088	12
$(7, 31)$	254	$\mathcal{N} = 1$	704	32	14
$(7, 31)$	252	$\mathcal{N} = 1$	992	128	13
$(7, 31)$	230	$\mathcal{N} = 1$	992	128	16
$(7, 31)$	228	$\mathcal{N} = 1$	1168	416	11
$(7, 31)$	209	$\mathcal{N} = 1$	1216	152	21
$(7, 31)$	208	$\mathcal{N} = 1$	992	64	18
$(7, 31)$	207	$\mathcal{N} = 1$	1568	592	13
$(7, 31)$	206	$\mathcal{N} = 1$	1168	256	14
$(7, 31)$	190	$\mathcal{N} = 1$	1216	304	14
$(7, 31)$	188	$\mathcal{N} = 1$	1568	928	10
$(7, 31)$	174	$\mathcal{N} = 1$	1216	224	18
$(7, 31)$	172	$\mathcal{N} = 1$	1568	704	12
$(27, 3)$	270	$\mathcal{N} = 1$	448	8	16
$(27, 3)$	240	$\mathcal{N} = 1$	1024	40	16
$(27, 3)$	234	$\mathcal{N} = 1$	1024	32	15
$(27, 3)$	216	$\mathcal{N} = 1$	1184	128	16
$(27, 3)$	213	$\mathcal{N} = 1$	1184	110	18
$(27, 3)$	212	$\mathcal{N} = 1$	1024	128	12

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d = 4$ Susy	Boundary States Maximum	Minimum	Distinct
$(27, 3)$	200	$\mathcal{N} = 1$	1184	128	21
$(27, 3)$	198	$\mathcal{N} = 1$	1360	392	13
$(27, 3)$	189	$\mathcal{N} = 1$	1504	172	18
$(27, 3)$	188	$\mathcal{N} = 1$	1184	496	8
$(27, 3)$	182	$\mathcal{N} = 1$	1360	304	23
$(27, 3)$	180	$\mathcal{N} = 1$	1664	1312	6
$(27, 3)$	167	$\mathcal{N} = 1$	1504	440	16
$(27, 3)$	166	$\mathcal{N} = 1$	1360	224	27
$(27, 3)$	164	$\mathcal{N} = 1$	1664	1168	11
$(27, 3)$	148	$\mathcal{N} = 1$	1664	992	15
$(27, 3)$	132	$\mathcal{N} = 1$	1664	896	16
$(3, 27)$	270	$\mathcal{N} = 1$	256	8	12
$(3, 27)$	240	$\mathcal{N} = 1$	608	32	14
$(3, 27)$	234	$\mathcal{N} = 1$	448	32	9
$(3, 27)$	216	$\mathcal{N} = 1$	800	128	11
$(3, 27)$	213	$\mathcal{N} = 1$	800	86	18
$(3, 27)$	212	$\mathcal{N} = 1$	320	64	8
$(3, 27)$	200	$\mathcal{N} = 1$	800	64	12
$(3, 27)$	198	$\mathcal{N} = 1$	976	392	10
$(3, 27)$	189	$\mathcal{N} = 1$	1120	172	18
$(3, 27)$	188	$\mathcal{N} = 1$	800	304	7
$(3, 27)$	182	$\mathcal{N} = 1$	976	256	17
$(3, 27)$	180	$\mathcal{N} = 1$	1280	928	6
$(3, 27)$	167	$\mathcal{N} = 1$	1120	344	16
$(3, 27)$	166	$\mathcal{N} = 1$	976	128	18
$(3, 27)$	164	$\mathcal{N} = 1$	1280	784	11
$(3, 27)$	148	$\mathcal{N} = 1$	1280	608	15
$(3, 27)$	132	$\mathcal{N} = 1$	1280	608	15
$(25, 1)$	230	$\mathcal{N} = 1$	256	32	4
$(1, 25)$	230	$\mathcal{N} = 1$	64	32	2
$(21, 9)$	172	$\mathcal{N} = 1$	1184	64	26
$(21, 9)$	170	$\mathcal{N} = 1$	1504	160	19
$(21, 9)$	169	$\mathcal{N} = 1$	1120	152	22
$(21, 9)$	167	$\mathcal{N} = 1$	1312	496	13
$(21, 9)$	166	$\mathcal{N} = 1$	1184	304	12
$(21, 9)$	164	$\mathcal{N} = 1$	1504	1088	8

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d = 4$ Susy	Boundary States Maximum	Minimum	Distinct
$(9, 21)$	172	$\mathcal{N} = 1$	992	32	23
$(9, 21)$	170	$\mathcal{N} = 1$	1312	160	18
$(9, 21)$	169	$\mathcal{N} = 1$	928	124	21
$(9, 21)$	167	$\mathcal{N} = 1$	1120	400	13
$(9, 21)$	166	$\mathcal{N} = 1$	992	296	10
$(9, 21)$	164	$\mathcal{N} = 1$	1312	896	8
$(19, 7)$	208	$\mathcal{N} = 1$	832	128	17
$(19, 7)$	202	$\mathcal{N} = 1$	832	128	20
$(19, 7)$	196	$\mathcal{N} = 1$	832	128	13
$(19, 7)$	187	$\mathcal{N} = 1$	992	172	18
$(19, 7)$	184	$\mathcal{N} = 1$	992	392	14
$(19, 7)$	181	$\mathcal{N} = 1$	992	152	21
$(19, 7)$	178	$\mathcal{N} = 1$	992	392	11
$(19, 7)$	168	$\mathcal{N} = 1$	992	304	20
$(19, 7)$	166	$\mathcal{N} = 1$	1264	196	21
$(19, 7)$	163	$\mathcal{N} = 1$	1264	448	15
$(19, 7)$	162	$\mathcal{N} = 1$	992	304	18
$(19, 7)$	160	$\mathcal{N} = 1$	1264	1072	5
$(19, 7)$	147	$\mathcal{N} = 1$	1264	392	23
$(19, 7)$	144	$\mathcal{N} = 1$	1264	896	9
$(19, 7)$	128	$\mathcal{N} = 1$	1264	736	12
$(7, 19)$	208	$\mathcal{N} = 1$	320	128	6
$(7, 19)$	202	$\mathcal{N} = 1$	608	64	14
$(7, 19)$	196	$\mathcal{N} = 1$	608	64	10
$(7, 19)$	187	$\mathcal{N} = 1$	800	172	10
$(7, 19)$	184	$\mathcal{N} = 1$	800	304	11
$(7, 19)$	181	$\mathcal{N} = 1$	800	152	17
$(7, 19)$	178	$\mathcal{N} = 1$	800	304	9
$(7, 19)$	168	$\mathcal{N} = 1$	800	224	15
$(7, 19)$	166	$\mathcal{N} = 1$	1072	196	16
$(7, 19)$	163	$\mathcal{N} = 1$	1072	392	13
$(7, 19)$	162	$\mathcal{N} = 1$	800	224	14
$(7, 19)$	160	$\mathcal{N} = 1$	1072	880	5
$(7, 19)$	147	$\mathcal{N} = 1$	1072	304	20
$(7, 19)$	144	$\mathcal{N} = 1$	1072	704	9
$(7, 19)$	128	$\mathcal{N} = 1$	1072	544	12

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d = 4$ Susy	Boundary States Maximum	Minimum	Distinct
$(17, 5)$	238	$\mathcal{N} = 1$	128	32	3
$(17, 5)$	176	$\mathcal{N} = 1$	832	32	25
$(17, 5)$	173	$\mathcal{N} = 1$	800	86	18
$(17, 5)$	167	$\mathcal{N} = 1$	800	172	10
$(17, 5)$	164	$\mathcal{N} = 1$	832	32	25
$(17, 5)$	161	$\mathcal{N} = 1$	896	124	33
$(17, 5)$	158	$\mathcal{N} = 1$	896	296	18
$(17, 5)$	152	$\mathcal{N} = 1$	1088	64	30
$(17, 5)$	149	$\mathcal{N} = 1$	992	152	25
$(17, 5)$	146	$\mathcal{N} = 1$	1088	160	31
$(17, 5)$	143	$\mathcal{N} = 1$	1120	344	27
$(17, 5)$	140	$\mathcal{N} = 1$	1120	896	7
$(17, 5)$	130	$\mathcal{N} = 1$	1088	128	24
$(17, 5)$	127	$\mathcal{N} = 1$	1120	304	31
$(17, 5)$	124	$\mathcal{N} = 1$	1120	736	12
$(5, 17)$	238	$\mathcal{N} = 1$	128	32	3
$(5, 17)$	176	$\mathcal{N} = 1$	640	32	15
$(5, 17)$	173	$\mathcal{N} = 1$	608	62	17
$(5, 17)$	167	$\mathcal{N} = 1$	608	124	9
$(5, 17)$	164	$\mathcal{N} = 1$	640	32	19
$(5, 17)$	161	$\mathcal{N} = 1$	704	124	20
$(5, 17)$	158	$\mathcal{N} = 1$	704	224	14
$(5, 17)$	152	$\mathcal{N} = 1$	896	64	25
$(5, 17)$	149	$\mathcal{N} = 1$	800	152	21
$(5, 17)$	146	$\mathcal{N} = 1$	896	128	23
$(5, 17)$	143	$\mathcal{N} = 1$	928	304	20
$(5, 17)$	140	$\mathcal{N} = 1$	928	704	7
$(5, 17)$	130	$\mathcal{N} = 1$	896	64	20
$(5, 17)$	127	$\mathcal{N} = 1$	928	248	26
$(5, 17)$	124	$\mathcal{N} = 1$	928	544	12
$(15, 3)$	222	$\mathcal{N} = 1$	64	32	2
$(15, 3)$	160	$\mathcal{N} = 1$	160	32	4
$(15, 3)$	138	$\mathcal{N} = 1$	928	16	19
$(15, 3)$	132	$\mathcal{N} = 1$	832	64	23
$(15, 3)$	129	$\mathcal{N} = 1$	928	124	33
$(15, 3)$	126	$\mathcal{N} = 1$	928	128	24

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d = 4$ Susy	Boundary States Maximum	Minimum	Distinct
$(15, 3)$	123	$\mathcal{N} = 1$	928	304	23
$(15, 3)$	120	$\mathcal{N} = 1$	976	736	8
$(3, 15)$	222	$\mathcal{N} = 1$	64	32	2
$(3, 15)$	160	$\mathcal{N} = 1$	64	32	2
$(3, 15)$	138	$\mathcal{N} = 1$	416	16	13
$(3, 15)$	132	$\mathcal{N} = 1$	608	32	17
$(3, 15)$	129	$\mathcal{N} = 1$	736	124	23
$(3, 15)$	126	$\mathcal{N} = 1$	736	64	17
$(3, 15)$	123	$\mathcal{N} = 1$	736	248	17
$(3, 15)$	120	$\mathcal{N} = 1$	784	544	8
$(12, 6)$	129	$\mathcal{N} = 1$	848	62	24
$(12, 6)$	126	$\mathcal{N} = 1$	848	148	30
$(12, 6)$	123	$\mathcal{N} = 1$	848	304	18
$(12, 6)$	120	$\mathcal{N} = 1$	848	688	6
$(6, 12)$	129	$\mathcal{N} = 1$	752	62	20
$(6, 12)$	126	$\mathcal{N} = 1$	752	112	24
$(6, 12)$	123	$\mathcal{N} = 1$	752	272	15
$(6, 12)$	120	$\mathcal{N} = 1$	752	592	6
$(21, 21)$	160	$\mathcal{N} = 2$	1600	16	15
$(21, 21)$	148	$\mathcal{N} = 2$	1760	80	16
$(21, 21)$	144	$\mathcal{N} = 2$	2240	16	31
$(21, 21)$	140	$\mathcal{N} = 2$	2560	64	43
$(21, 21)$	136	$\mathcal{N} = 2$	3392	256	34
$(19, 19)$	242	$\mathcal{N} = 1$	1280	32	15
$(19, 19)$	240	$\mathcal{N} = 1$	1504	128	22
$(19, 19)$	238	$\mathcal{N} = 1$	1760	416	14
$(19, 19)$	208	$\mathcal{N} = 1$	1504	128	14
$(19, 19)$	206	$\mathcal{N} = 1$	1760	448	13
$(19, 19)$	204	$\mathcal{N} = 1$	2272	1472	7
$(19, 19)$	180	$\mathcal{N} = 1$	1472	128	14
$(19, 19)$	178	$\mathcal{N} = 1$	1696	320	12
$(19, 19)$	176	$\mathcal{N} = 1$	1952	128	21
$(19, 19)$	174	$\mathcal{N} = 1$	1760	416	12
$(19, 19)$	172	$\mathcal{N} = 1$	2272	1280	5
$(15, 15)$	270	$\mathcal{N} = 1$	448	32	16
$(15, 15)$	240	$\mathcal{N} = 1$	832	128	18

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d=4$ Susy	Boundary States Maximum	Minimum	Distinct
$(15,15)$	234	$\mathcal{N}=1$	448	32	9
$(15,15)$	216	$\mathcal{N}=1$	992	128	15
$(15,15)$	213	$\mathcal{N}=1$	992	172	22
$(15,15)$	212	$\mathcal{N}=1$	832	64	16
$(15,15)$	200	$\mathcal{N}=1$	1024	64	22
$(15,15)$	198	$\mathcal{N}=1$	1184	392	16
$(15,15)$	190	$\mathcal{N}=1$	992	128	16
$(15,15)$	189	$\mathcal{N}=1$	1312	536	10
$(15,15)$	188	$\mathcal{N}=1$	1168	304	19
$(15,15)$	184	$\mathcal{N}=1$	1024	64	15
$(15,15)$	182	$\mathcal{N}=1$	1216	224	28
$(15,15)$	180	$\mathcal{N}=1$	1568	1120	10
$(15,15)$	169	$\mathcal{N}=1$	1216	152	23
$(15,15)$	167	$\mathcal{N}=1$	1568	440	21
$(15,15)$	166	$\mathcal{N}=1$	1216	224	29
$(15,15)$	164	$\mathcal{N}=1$	1568	896	17
$(15,15)$	160	$\mathcal{N}=1$	1184	64	25
$(15,15)$	158	$\mathcal{N}=1$	1360	208	20
$(15,15)$	150	$\mathcal{N}=1$	1216	304	14
$(15,15)$	148	$\mathcal{N}=1$	1568	800	18
$(15,15)$	138	$\mathcal{N}=1$	1184	256	13
$(15,15)$	136	$\mathcal{N}=1$	1360	896	6
$(15,15)$	132	$\mathcal{N}=1$	1472	704	16
$(13,13)$	230	$\mathcal{N}=1$	256	32	4
$(13,13)$	192	$\mathcal{N}=2$	320	8	10
$(13,13)$	172	$\mathcal{N}=2$	704	40	9
$(13,13)$	160	$\mathcal{N}=2$	896	8	22
$(13,13)$	156	$\mathcal{N}=2$	1024	64	29
$(13,13)$	148	$\mathcal{N}=2$	1024	64	24
$(13,13)$	144	$\mathcal{N}=2$	1184	8	28
$(13,13)$	140	$\mathcal{N}=2$	1024	32	28
$(13,13)$	136	$\mathcal{N}=2$	1184	256	17
$(13,13)$	128	$\mathcal{N}=2$	1024	80	14
$(13,13)$	120	$\mathcal{N}=2$	1216	16	31
$(13,13)$	116	$\mathcal{N}=2$	1472	128	31
$(13,13)$	112	$\mathcal{N}=2$	1472	32	36

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d = 4$ Susy	Boundary States Maximum	Minimum	Distinct
$(13, 13)$	108	$\mathcal{N} = 2$	1984	128	31
$(13, 13)$	104	$\mathcal{N} = 2$	1984	896	12
$(13, 13)$	96	$\mathcal{N} = 2$	448	32	11
$(13, 13)$	84	$\mathcal{N} = 2$	1024	128	19
$(13, 13)$	80	$\mathcal{N} = 2$	1216	32	18
$(13, 13)$	76	$\mathcal{N} = 2$	1472	128	28
$(13, 13)$	72	$\mathcal{N} = 2$	1984	608	18
$(11, 11)$	238	$\mathcal{N} = 1$	704	32	14
$(11, 11)$	226	$\mathcal{N} = 1$	896	32	12
$(11, 11)$	224	$\mathcal{N} = 1$	1120	128	16
$(11, 11)$	214	$\mathcal{N} = 1$	832	128	19
$(11, 11)$	204	$\mathcal{N} = 1$	832	128	16
$(11, 11)$	200	$\mathcal{N} = 1$	704	32	17
$(11, 11)$	193	$\mathcal{N} = 1$	1024	172	21
$(11, 11)$	192	$\mathcal{N} = 1$	1120	64	23
$(11, 11)$	190	$\mathcal{N} = 1$	1376	416	18
$(11, 11)$	180	$\mathcal{N} = 1$	992	392	17
$(11, 11)$	176	$\mathcal{N} = 1$	832	64	29
$(11, 11)$	174	$\mathcal{N} = 1$	1024	304	20
$(11, 11)$	173	$\mathcal{N} = 1$	800	86	26
$(11, 11)$	172	$\mathcal{N} = 1$	800	64	13
$(11, 11)$	170	$\mathcal{N} = 1$	832	32	25
$(11, 11)$	167	$\mathcal{N} = 1$	704	124	18
$(11, 11)$	164	$\mathcal{N} = 1$	1088	64	29
$(11, 11)$	162	$\mathcal{N} = 1$	1312	320	13
$(11, 11)$	161	$\mathcal{N} = 1$	800	124	25
$(11, 11)$	160	$\mathcal{N} = 1$	1120	64	19
$(11, 11)$	159	$\mathcal{N} = 1$	1376	440	21
$(11, 11)$	158	$\mathcal{N} = 1$	1376	224	33
$(11, 11)$	156	$\mathcal{N} = 1$	1888	896	18
$(11, 11)$	152	$\mathcal{N} = 1$	992	160	22
$(11, 11)$	149	$\mathcal{N} = 1$	1120	152	24
$(11, 11)$	148	$\mathcal{N} = 1$	976	304	15
$(11, 11)$	146	$\mathcal{N} = 1$	992	152	29
$(11, 11)$	143	$\mathcal{N} = 1$	1024	86	40
$(11, 11)$	142	$\mathcal{N} = 1$	1024	160	29

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d = 4$ Susy	Boundary States Maximum	Minimum	Distinct
$(11, 11)$	140	$\mathcal{N} = 1$	1376	784	18
$(11, 11)$	131	$\mathcal{N} = 1$	1216	440	14
$(11, 11)$	130	$\mathcal{N} = 1$	1312	128	27
$(11, 11)$	129	$\mathcal{N} = 1$	1024	124	18
$(11, 11)$	128	$\mathcal{N} = 1$	1568	896	12
$(11, 11)$	127	$\mathcal{N} = 1$	1376	296	35
$(11, 11)$	126	$\mathcal{N} = 1$	1376	224	23
$(11, 11)$	124	$\mathcal{N} = 1$	1888	608	27
$(11, 11)$	120	$\mathcal{N} = 1$	992	64	19
$(11, 11)$	118	$\mathcal{N} = 1$	1168	152	28
$(11, 11)$	114	$\mathcal{N} = 1$	1024	16	21
$(11, 11)$	112	$\mathcal{N} = 1$	1216	64	32
$(11, 11)$	108	$\mathcal{N} = 1$	1376	608	18
$(11, 11)$	102	$\mathcal{N} = 1$	1280	256	10
$(11, 11)$	100	$\mathcal{N} = 1$	1504	896	5
$(11, 11)$	98	$\mathcal{N} = 1$	1312	256	11
$(11, 11)$	96	$\mathcal{N} = 1$	1568	704	13
$(11, 11)$	94	$\mathcal{N} = 1$	1376	224	11
$(11, 11)$	92	$\mathcal{N} = 1$	1888	800	11
$(9, 9)$	222	$\mathcal{N} = 1$	128	32	3
$(9, 9)$	160	$\mathcal{N} = 1$	736	296	14
$(9, 9)$	154	$\mathcal{N} = 1$	736	296	12
$(9, 9)$	147	$\mathcal{N} = 1$	704	124	18
$(9, 9)$	144	$\mathcal{N} = 2$	128	32	3
$(9, 9)$	144	$\mathcal{N} = 1$	704	296	11
$(9, 9)$	141	$\mathcal{N} = 1$	704	124	17
$(9, 9)$	139	$\mathcal{N} = 1$	976	368	14
$(9, 9)$	138	$\mathcal{N} = 1$	832	64	30
$(9, 9)$	136	$\mathcal{N} = 1$	976	784	6
$(9, 9)$	132	$\mathcal{N} = 1$	736	32	24
$(9, 9)$	129	$\mathcal{N} = 1$	832	124	29
$(9, 9)$	126	$\mathcal{N} = 1$	976	128	35
$(9, 9)$	123	$\mathcal{N} = 1$	976	272	27
$(9, 9)$	120	$\mathcal{N} = 1$	976	608	11
$(9, 9)$	117	$\mathcal{N} = 1$	896	86	16
$(9, 9)$	114	$\mathcal{N} = 1$	896	152	21

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d = 4$ Susy	Boundary States Maximum	Minimum	Distinct
$(9, 9)$	111	$\mathcal{N} = 1$	896	392	9
$(9, 9)$	108	$\mathcal{N} = 1$	896	800	4
$(9, 9)$	107	$\mathcal{N} = 1$	976	296	20
$(9, 9)$	104	$\mathcal{N} = 1$	976	608	9
$(9, 9)$	100	$\mathcal{N} = 2$	352	40	8
$(9, 9)$	98	$\mathcal{N} = 1$	896	112	32
$(9, 9)$	96	$\mathcal{N} = 2$	896	16	26
$(9, 9)$	92	$\mathcal{N} = 1$	896	640	7
$(9, 9)$	88	$\mathcal{N} = 2$	1024	16	31
$(9, 9)$	84	$\mathcal{N} = 2$	1024	32	33
$(9, 9)$	80	$\mathcal{N} = 2$	1472	32	35
$(9, 9)$	76	$\mathcal{N} = 2$	1184	64	25
$(9, 9)$	72	$\mathcal{N} = 2$	1472	608	11
$(9, 9)$	0	$\mathcal{N} = 4$	4864	8	43
$(7, 7)$	184	$\mathcal{N} = 1$	704	64	10
$(7, 7)$	174	$\mathcal{N} = 1$	608	64	13
$(7, 7)$	168	$\mathcal{N} = 1$	352	64	9
$(7, 7)$	166	$\mathcal{N} = 1$	832	304	13
$(7, 7)$	156	$\mathcal{N} = 1$	736	64	18
$(7, 7)$	153	$\mathcal{N} = 1$	832	124	28
$(7, 7)$	150	$\mathcal{N} = 1$	832	224	20
$(7, 7)$	144	$\mathcal{N} = 1$	832	64	19
$(7, 7)$	140	$\mathcal{N} = 1$	800	296	15
$(7, 7)$	134	$\mathcal{N} = 1$	832	128	26
$(7, 7)$	133	$\mathcal{N} = 1$	608	62	16
$(7, 7)$	132	$\mathcal{N} = 1$	1184	688	15
$(7, 7)$	130	$\mathcal{N} = 1$	608	8	25
$(7, 7)$	122	$\mathcal{N} = 1$	1024	128	22
$(7, 7)$	119	$\mathcal{N} = 1$	1184	304	27
$(7, 7)$	118	$\mathcal{N} = 1$	800	224	14
$(7, 7)$	116	$\mathcal{N} = 1$	1184	592	18
$(7, 7)$	112	$\mathcal{N} = 1$	800	32	28
$(7, 7)$	110	$\mathcal{N} = 1$	992	196	21
$(7, 7)$	109	$\mathcal{N} = 1$	928	124	27
$(7, 7)$	108	$\mathcal{N} = 1$	784	224	9
$(7, 7)$	106	$\mathcal{N} = 1$	832	32	35

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d=4$ Susy	Boundary States Maximum	Minimum	Distinct
$(7,7)$	104	$\mathcal{N}=1$	1024	784	8
$(7,7)$	103	$\mathcal{N}=1$	832	62	38
$(7,7)$	102	$\mathcal{N}=1$	800	128	9
$(7,7)$	100	$\mathcal{N}=1$	1184	32	42
$(7,7)$	97	$\mathcal{N}=1$	704	62	18
$(7,7)$	94	$\mathcal{N}=1$	896	128	28
$(7,7)$	91	$\mathcal{N}=1$	1024	304	21
$(7,7)$	90	$\mathcal{N}=1$	1024	160	15
$(7,7)$	88	$\mathcal{N}=1$	1024	608	13
$(7,7)$	87	$\mathcal{N}=1$	1184	248	21
$(7,7)$	84	$\mathcal{N}=1$	1184	592	17
$(7,7)$	82	$\mathcal{N}=1$	992	160	15
$(7,7)$	78	$\mathcal{N}=1$	976	152	21
$(7,7)$	76	$\mathcal{N}=1$	992	800	4
$(7,7)$	72	$\mathcal{N}=1$	1024	32	32
$(7,7)$	68	$\mathcal{N}=1$	1184	608	9
$(7,7)$	60	$\mathcal{N}=1$	992	704	5
$(7,7)$	56	$\mathcal{N}=1$	976	800	3
$(5,5)$	160	$\mathcal{N}=2$	320	16	8
$(5,5)$	152	$\mathcal{N}=1$	80	64	2
$(5,5)$	148	$\mathcal{N}=2$	704	64	10
$(5,5)$	144	$\mathcal{N}=2$	896	16	18
$(5,5)$	140	$\mathcal{N}=2$	1280	32	28
$(5,5)$	136	$\mathcal{N}=2$	1600	128	20
$(5,5)$	112	$\mathcal{N}=1$	784	592	7
$(5,5)$	99	$\mathcal{N}=1$	784	272	14
$(5,5)$	96	$\mathcal{N}=2$	896	32	13
$(5,5)$	96	$\mathcal{N}=1$	784	592	6
$(5,5)$	90	$\mathcal{N}=1$	736	148	17
$(5,5)$	86	$\mathcal{N}=1$	784	112	15
$(5,5)$	84	$\mathcal{N}=2$	1024	64	22
$(5,5)$	84	$\mathcal{N}=1$	736	608	5
$(5,5)$	83	$\mathcal{N}=1$	784	272	12
$(5,5)$	80	$\mathcal{N}=2$	1472	8	25
$(5,5)$	80	$\mathcal{N}=1$	784	592	5
$(5,5)$	77	$\mathcal{N}=1$	704	62	18

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d = 4$ Susy	Boundary States Maximum	Minimum	Distinct
$(5, 5)$	76	$\mathcal{N} = 2$	1792	128	30
$(5, 5)$	74	$\mathcal{N} = 1$	704	148	13
$(5, 5)$	72	$\mathcal{N} = 2$	2624	8	33
$(5, 5)$	71	$\mathcal{N} = 1$	704	296	9
$(5, 5)$	68	$\mathcal{N} = 1$	736	32	19
$(5, 5)$	64	$\mathcal{N} = 2$	608	64	11
$(5, 5)$	62	$\mathcal{N} = 1$	736	152	11
$(5, 5)$	56	$\mathcal{N} = 2$	704	16	22
$(5, 5)$	56	$\mathcal{N} = 1$	736	704	2
$(5, 5)$	52	$\mathcal{N} = 2$	1088	40	29
$(5, 5)$	48	$\mathcal{N} = 2$	1088	32	28
$(5, 5)$	44	$\mathcal{N} = 2$	1600	64	26
$(5, 5)$	40	$\mathcal{N} = 2$	1600	8	31
$(5, 5)$	36	$\mathcal{N} = 2$	1024	32	31
$(5, 5)$	32	$\mathcal{N} = 2$	832	32	9
$(5, 5)$	28	$\mathcal{N} = 2$	1024	32	31
$(5, 5)$	24	$\mathcal{N} = 2$	1408	32	28
$(5, 5)$	20	$\mathcal{N} = 2$	1664	128	26
$(5, 5)$	16	$\mathcal{N} = 2$	1984	32	23
$(5, 5)$	12	$\mathcal{N} = 2$	1792	128	26
$(5, 5)$	8	$\mathcal{N} = 2$	2624	608	14
$(3, 3)$	210	$\mathcal{N} = 1$	320	32	7
$(3, 3)$	176	$\mathcal{N} = 1$	704	128	11
$(3, 3)$	148	$\mathcal{N} = 1$	704	64	9
$(3, 3)$	144	$\mathcal{N} = 1$	608	64	11
$(3, 3)$	142	$\mathcal{N} = 1$	992	224	20
$(3, 3)$	126	$\mathcal{N} = 1$	608	224	7
$(3, 3)$	114	$\mathcal{N} = 1$	928	224	15
$(3, 3)$	113	$\mathcal{N} = 1$	640	124	14
$(3, 3)$	110	$\mathcal{N} = 1$	992	256	15
$(3, 3)$	108	$\mathcal{N} = 1$	1504	544	18
$(3, 3)$	104	$\mathcal{N} = 1$	608	32	16
$(3, 3)$	98	$\mathcal{N} = 1$	704	16	13
$(3, 3)$	92	$\mathcal{N} = 1$	992	544	13
$(3, 3)$	86	$\mathcal{N} = 1$	896	256	8
$(3, 3)$	82	$\mathcal{N} = 1$	928	128	10

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Hodge numbers (h_{11}, h_{12})	Heterotic Singlets	$d = 4$ Susy	Boundary States Maximum	Minimum	Distinct
$(3, 3)$	80	$\mathcal{N} = 1$	1184	592	11
$(3, 3)$	79	$\mathcal{N} = 1$	992	248	19
$(3, 3)$	78	$\mathcal{N} = 1$	992	224	9
$(3, 3)$	76	$\mathcal{N} = 1$	1504	544	17
$(3, 3)$	70	$\mathcal{N} = 1$	800	148	20
$(3, 3)$	69	$\mathcal{N} = 1$	736	124	12
$(3, 3)$	64	$\mathcal{N} = 1$	832	64	23
$(3, 3)$	63	$\mathcal{N} = 1$	608	62	17
$(3, 3)$	60	$\mathcal{N} = 1$	992	32	24
$(3, 3)$	52	$\mathcal{N} = 1$	1120	608	8
$(3, 3)$	51	$\mathcal{N} = 1$	832	248	14
$(3, 3)$	48	$\mathcal{N} = 1$	1184	592	10
$(3, 3)$	47	$\mathcal{N} = 1$	992	304	12
$(3, 3)$	44	$\mathcal{N} = 1$	1504	704	7
$(3, 3)$	42	$\mathcal{N} = 1$	800	128	12
$(3, 3)$	38	$\mathcal{N} = 1$	784	112	13
$(3, 3)$	36	$\mathcal{N} = 1$	832	64	16
$(3, 3)$	32	$\mathcal{N} = 1$	800	32	15
$(3, 3)$	24	$\mathcal{N} = 1$	1088	704	3
$(3, 3)$	20	$\mathcal{N} = 1$	1120	800	3
$(3, 3)$	16	$\mathcal{N} = 1$	1184	608	5
$(3, 3)$	12	$\mathcal{N} = 1$	1504	992	3
$(1, 1)$	144	$\mathcal{N} = 2$	64	16	3

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