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A New Framework for Multijet Predictions and its application to Higgs Boson production at the LHC

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We present a new framework for calculating multi-jet observables through a resummation of hard emission. The framework is based on the high energy factorisation of amplitudes, with the region of applicability extended by constraining the analytic properties of the scattering amplitudes according to known all-order results. As an example of application we study predictions for Higgs Boson production through gluon fusion at the LHC in association with at least two jets. The result is matched to the tree level matrix elements for 2 and 3 jet production in association with a Higgs boson, and implemented in a Monte Carlo generator.

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I. INTRODUCTION

Events with multiple jets form the backbone of many search strategies at the LHC for physics both within and beyond the Standard Model (SM) of particle physics. However, the calculation of the SM contribution to such channels beyond even the tree-level approximation is notoriously difficult, and has been performed in only a few of the relevant cases.

In this study we will develop a new formalism, which resums to all orders in α_s the dominant contribution to hard jet production for a general multi-jet observable. The formalism is first implemented for the production of a Higgs boson in gluon fusion mediated through a top loop in association with multiple (at least two) jets. We take as a starting point a factorised form for the scattering amplitudes, which formally applies in a certain kinematic limit. We extend the domain of applicability of the amplitudes from Asymptotia into the region of relevance for the LHC by using known all-order constraints of scattering amplitudes. We validate the approach by comparing the approximations to the results obtained in full, fixed order perturbation theory, where these are available.

Furthermore, the resulting estimate for the n-parton final state (which includes some virtual corrections) is matched to the known tree level results for hjj and hjjj. Finally, we implement the description in a Monte Carlo event generator for Higgs + multiparton production, and present a sample of results. This is done both for the new approach advocated here, and for the traditional implementation using the BFKL equation[1, 2, 3], see e.g. Ref. [4, 5, 6]. Based on the dramatic improvement obtained in the description (and the poor performance of the BFKL approximations even after energy and momentum conservation is implemented), we propose that the method described here should completely replace the use of the BFKL equation in the description of jet physics, e.g. in the study of Mueller-Navelet dijets[7] etc.

We choose to implement our new framework first for Higgs boson production in association with hard jets, since this channel is important not just for the possible discovery of a Higgs boson, but also offers an important possibility for measuring the couplings to the electroweak bosons of any Higgs boson candidate[8]. In order to measure these couplings, it is necessary to suppress the gluon fusion contribution to the production of a Higgs boson in association with two jets. This is achieved[9] to some degree by applying the so-called *weak boson fusion*-cuts in Table I. However, only by calculating higher order corrections is it possible to estimate the efficiency of such cuts in suppressing the gluon fusion contribution. The effect of further radiation beyond what is presently calculable in full fixed order perturbation theory for this process[10] has previously been approximated[11] using parton shower algorithms. In this letter we examine a different approach, and consider how to best estimate hard jet emission in Higgs production via gluon fusion. This will help in illuminating the multi-parton dynamics and topologies, and answer questions as to the efficiency of various cuts in suppressing the gluon fusion channel.

$p_{c_\perp}, p_{d_\perp}, p_{j_\perp} > 40$ GeV	$y_c \cdot y_d < 0$
$y_j < 5$	$\lvert y_c - y_d\rvert > 4.2$
$s_{cd} > (600\text{ GeV})^2$	$y_c \leq y_h \leq y_d$

TABLE I: The cuts used in the following analysis which bias the Higgs boson plus dijet sample towards the WBF production process. The suffices c, d label the tagged jets, j any (possibly further) jet in the event, and h the Higgs boson.

II. HIGH ENERGY FACTORISED MATRIX ELEMENTS AND INCLUSIVE JET SAMPLES

A. The FKL Factorised Amplitude and Fixed Order Results

Our aim is to estimate the matrix elements for Higgs boson production with multiple hard jets. As a starting point, we use the FKL factorised $(2 \to n+2)$-gluon amplitudes[1] adapted to include also the production of a Higgs boson. For the Higgs boson produced between the jets (in rapidity) these amplitudes take the form:

$$
\begin{aligned}
i\mathcal{M}_{\rm HE}^{ab\to p_0\ldots p_j h p_{j+1} p_n} &= 2i\hat{s}\left(ig_s f^{ad_0c_1} g_{\mu_a\mu_0}\right)\\
&\cdot \prod_{i=1}^{j}\left(\frac{1}{q_i^2}\exp[\hat\alpha(q_i)(y_{i-1}-y_i)]\left(ig_s f^{c_id_ic_{i+1}}\right)C_{\mu_i}(q_i,q_{i+1})\right)\\
&\cdot \left(\frac{1}{q_h^2}\exp[\hat\alpha(q_i)(y_j-y_h)]C_H(q_{j+1},q_h)\right)\\
&\cdot \prod_{i=j+1}^{n}\left(\frac{1}{q_i^2}\exp[\hat\alpha(q_i)(y'_{i-1}-y'_i)]\left(ig_s f^{c_id_ic_{i+1}}\right)C_{\mu_i}(q_i,q_{i+1})\right)\\
&\cdot \frac{1}{q_{n+1}^2}\exp[\hat\alpha(q_{n+1})(y'_n-y_b)]\left(ig_s f^{bd_{n+1}c_{n+1}} g_{\mu_b\mu_{n+1}}\right),
\end{aligned}
\tag{1}
$$

where g_s is the strong coupling constant ($\alpha_s=\frac{g_s^2}{4\pi}$); f^{abc} colour structure constants; q_i, q_h are the 4-momentum of gluon propagators (e.g. $q_i = p_a - \sum_{k=0}^{i-1} p_k$ for $i<j$). We have also introduced the *Lipatov effective vertex*:

$$
C^{\mu_i}(q_i,q_{i+1}) = \left[(q_i+q_{i+1})_\perp^{\mu_i} - \left(\frac{\hat{s}_{ai}}{\hat{s}} + 2\frac{t_{i+1}}{\hat{s}_{bi}}\right)p_b^{\mu_i} + \left(\frac{\hat{s}_{bi}}{\hat{s}} + 2\frac{\hat{t}_i}{\hat{s}_{ai}}\right)p_a^{\mu_i}\right], \tag{2}
$$

and the effective vertex for the production of a Higgs boson C_H, as calculated in Ref. [12]. The factors $\hat\alpha(q_i)$ arise from the *Lipatov Ansatz* for the Reggeisation of the gluon propagator, and encode virtual corrections (see e.g. Ref [13]):

$$
\hat\alpha(q_i) = -\frac{g_s^2\ C_A\ \Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}}\frac{2}{\varepsilon}\left(|q_{\perp i}|^2/\mu^2\right)^{\varepsilon}. \tag{3}
$$

The colour factors in equation (1) are for incoming gluons. The form of the amplitude is the same for initial state quarks, apart from different colour factors such that the net effect of all parton species is reproduced by using the *effective parton density function* [14, 15]:

$$
f(x,q^2) = g(x,q^2) + \frac{C_F}{C_A}\sum_f\left(q_f(x,q^2)+\bar{q}_f(x,q^2)\right). \tag{4}
$$

The approximation to the full matrix elements in Eq. (1) formally applies in the limit of Multi Regge Kinematic (MRK), which can be expressed in terms of the rapidities $\{y_i\}$ of the outgoing partons and their transverse momenta $\{p_{i\perp}\}$:

$$
y_0 \gg y_1 \gg \ldots \gg y_{n+1};\quad p_{i\perp}\simeq p_{i+1\perp};\quad q_i^2\simeq q_j^2; \tag{5}
$$

which is particularly well suited for studies within the WBF cuts of Table I, since a large rapidity span of the event is then guaranteed. We see that in the limit of MRK, $\hat{s}\to\infty, |p_\perp|$ fixed. However, crucially no strict cut on the rapidity separation between further radiation can be applied. We will therefore need to ensure that the amplitudes give sensible results also when evaluated outside the kinematical limit in which they become exact.

In the true limit of MRK, the squared 4-momenta fulfil $q_i^2\to -q_{\perp i}^2$, and the square of the Lipatov vertices fulfil $-C_{\mu_i}C^{\mu_i} \to 4\frac{|q_{\perp i}|^2|q_{\perp i+1}|^2}{|k_{\perp i}|^2}$. With these simplications, one can perform the sum over n in Eq. (1) and integrate over the full phase space of emitted gluons by solving two coupled BFKL equations. This framework is formally applicable in the phase space limit of:

$$
y_0 \gg y_1 \gg \ldots \gg y_{n+1};\ p_{i\perp}\simeq p_{i+1\perp};\ q_{\perp i}^2\simeq q_{\perp j}^2, \tag{6}
$$

although the kinematical approximations are applied everywhere in phase space when the BFKL equation is solved.

While both the FKL and BFKL expressions are valid to the same logarithmic accuracy when Eq. (6) applies, we can extend the applicability of the FKL results to the region of interest for particle physics phenomenology by adhering to the following guidelines:

1. Obey Energy and Momentum Conservation: When the BFKL limit of invariants is substituted into the amplitudes of Eq. (1), these become independent on the longitudinal component of the incoming momenta. This has led some to advocate the use of BFKL partonic cross sections with the Bjorken x's evaluated only to leading logarithmic accuracy in $\hat{s}$, using only the contribution from the partons furthest apart in rapidity. This violates energy and momentum conservation. However, energy and momentum conservation is clearly a significant physical constraint, which influences the BFKL evolution (and even stops it since obviously $\hat{s}<s<\infty$) before the formal MRK limit is reached. Furthermore, the constraint is easily implemented in a Monte Carlo event generation context.

2. Do not modify the analyticity of the amplitudes unnecessarily: Using the expression in Eq. (1) corresponds to removing some divergences from the full scattering amplitude (the collinear divergences). This is different to the BFKL approach, where the analytic properties of the amplitude are changed further, when the limits of the kinematic invariants are used. By using the kinematic invariants in Eq. (1) instead of their BFKL limits, important corrections to the BFKL evolution entering first at next-to-leading logarithmic accuracy are taken into account *beyond* any logarithmic accuracy. This connection will be elaborated further in Ref.[16].

3. Do not apply the formalism where it fails: We veto emission within the *small* region of phase space where the expression of Eq. (1) results in unphysical (negative) differential cross sections. This happens when the effective Lipatov vertex is applied to momentum configurations very far from the limit of MRK, where it is possible to obtain $-C_{\mu_i}C^{\mu_i}<0$. It is perhaps interesting to note that restricting the region of phase space where the

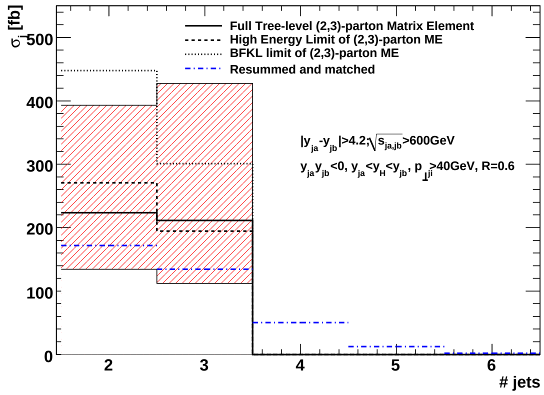

FIG. 1: The 2 and 3 parton cross-sections calculated using the known LO matrix elements (solid), and the estimate gained from the modified high energy limit (dashed). The uncertainty band on the LO result corresponds to scale variation by a factor of two. Also shown is the all orders result described in the text (dot dashed).

formalism is applied is similar to the *kinematic constraint* of Ref.[17, 18, 19]. This connection will be further elaborated in Ref.[16].

In Figure 1 we compare the prediction for the production of a Higgs boson in association with two and three partons (in a hard two-jet and three-jet configuration respectively) within the WBF cuts of table I, obtained using both the full matrix element (extracted from `MADEvent/MADGraph`[20]) and the relevant expression of Eq. (1) for two and three parton production, with the virtual corrections set to zero ($\hat{\alpha}(q_i) = 0$) (i.e. an order-by-order comparison of the results obtained in the approximation with the full fixed order results). We choose renormalisation and factorisation scales in accordance with the study of Ref.[11], and the following values for the mass parameters

$$m_H = 120\text{GeV}, \quad v = 246\text{GeV}, \quad m_t = 174\text{GeV}. \qquad (7)$$

We also include a factor multiplying the effective Higgs Boson vertices accounting for finite top-mass effects [21]:

$$K(\tau) = 1 + \frac{7\tau}{30} + \frac{2\tau^2}{21} + \frac{26\tau^3}{525}, \quad \tau = \frac{m_H^2}{4m_t^2}. \qquad (8)$$

We choose the k_t-jet algorithm as implemented in Ref.[22] with $R = 0.6$, and the parton distribution functions of Ref. [23]. As tagging jets we choose the two furthest apart in rapidity, which satisfy the cut in transverse momentum. One notes two things. Firstly, the tree level cross section for the production of a Higgs boson in association with 3 jets is similar to the one for the production of a Higgs boson in association with two jets (full lines in Fig. 1). The large size of the three-jet rate was already reported in Ref.[24], and clearly demonstrates the necessity of considering hard multi-parton final states in order to describe correctly the expected event topology and to answer questions on e.g. the effectiveness of a central jet veto in suppressing the gluon fusion channel. Secondly, the approximation to the jet rates obtained in this approach is good to within 10%, well within the scale uncertainty of the known tree level results (dashed line). We have therefore explicitly shown that the approximation obtained using the FKL amplitudes of Eq. (1) supplemented by the rules 1-3 is a very good approximation. This holds true for all sub-channels, and point-by-point in phase space. The results obtained in the BFKL approach, however, fare much worse, as indicated by the dotted line in Fig. 1. If energy and momentum conservation is dropped, the situation worsens further, and the results are out by up to a factor of 15 compared to the full fixed order results! It would seem difficult to attach any confidence to results built on such an approximation. While no analytic expression for the fully inclusive partonic cross section based on the amplitudes in Eq. (1) can be obtained, contrary to the case in the BFKL approach, the implementation of the amplitudes offers only little additional complications compared to the implementation of energy and momentum conservation in the BFKL approach based on Ref.[4, 5, 6].

B. All Order Results And Matching

The aim of the formalism is obviously not just to reproduce the results which could be obtained in the standard fixed order approach, but rather to also combine the real and virtual corrections to any order. The divergence in Eq. (1) obtained when any $p_i \to 0$ is regulated by the divergence of the virtual corrections encoded in $\hat{\alpha}$. By implementing the regularisation through phase space slicing it becomes possible to obtain the fully inclusive any-parton sample by summing Eq. (1) over all j, n. This is very efficiently implemented by following the method for phase space generation outlined in Ref.[25]. Furthermore, since we can trivially expand the expressions to any order in α_s, it is possible to implement matching to the known two and three parton tree level results at each point in two and three-jet phase space respectively. We choose to implement $\ln R$-matching at the amplitude-level for channels and rapidity configurations which have a contribution in the high-energy limit (e.g. $ug \to hug$ and $gg \to hggg$), and R-matching for those which do not (e.g. $gg \to hu\bar{u}$ and $u\bar{u} \to hggg$).

The distribution of final state jets subject to the cuts of table I is shown with the dot-dashed histogram in Figure 1. One sees a significant number of events with more than 3 hard ($p_\perp > 40$GeV) jets. More importantly though, the method outlined in this paper allows for an estimate of the emissions of partons not quite hard enough to be classified as jets, but still causing sufficient decorrelation between jets. We especially note that the 2-jet cross section is not significantly reduced by higher order corrections.

	A_ϕ
LO 2-jet	0.50
$\sum_n$ n-parton, $=$ 2-jet	0.27
LO 3-jet	0.23
$\sum_n$ n-parton, $\geq$ 2-jet	0.16

TABLE II: The angular decorrelation parameter given by equation (9), subject to the cuts of table I. Note that the 2 and 3-jet values are obtained from matrix elements matched to the known tree level results.

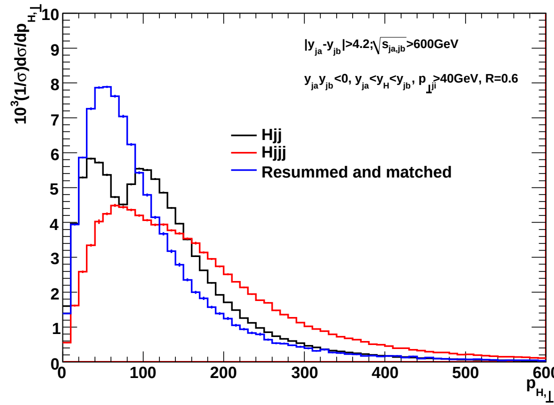

FIG. 2: The transverse momentum spectrum of the Higgs boson in the process $pp \to H + n$jets at tree-level for $n = 2$, $n = 3$, and the transverse momentum spectrum obtained in the resummed approach presented in this paper. The spectra obtained in the lowest orders clearly exhibit unphysical behaviours not present in the resummed result.

The azimuthal angular correlation between the tagging jets has previously been suggested as a good observable for differentiating between the GGF and WBF production modes. Furthermore, the nature of the distribution of the azimuthal angle ϕ between the two tagging jets can potentially be used to determine the nature of the Higgs coupling to fermions [26]. However, the usefulness of this tree-level observation is threatened by emission which acts to decorrelate the tagging jets. As suggested in Ref.[27] the structure of the distribution $d\sigma/d\phi_{j_aj_b}$ can be distilled into a single number A_ϕ given by:

$$A_\phi = \frac{\sigma(\phi_{j_aj_b} < \pi/4) - \sigma(\pi/4 < \phi_{j_aj_b} < 3\pi/4) + \sigma(\phi_{j_aj_b} > 3\pi/4)}{\sigma(\phi_{j_aj_b} < \pi/4) + \sigma(\pi/4 < \phi_{j_aj_b} < 3\pi/4) + \sigma(\phi_{j_aj_b} > 3\pi/4)}. \tag{9}$$

A CP-even coupling for the Higgs boson leads to a positive value for A_ϕ at leading order in perturbation theory, whereas a CP-odd coupling results in a negative value. The Standard Model coupling for the weak gauge bosons to the Higgs boson would lead to $A_\phi \approx 0$. The results using our approach are collected in Table II. Of particular interest is the difference between the first two numbers. The first ($A_\phi = 0.50$) describes the result obtained in the two-jet tree-level calculation. The second ($A_\phi = 0.27$) is the result obtained for events classified as containing only two hard jets, but otherwise completely inclusive in the number of final state partons. Even if all additional jets are vetoed and only the two-jet events are considered, there is still a significant modification to the azimuthal correlation of the tagging jets due to multiple parton emission. This makes the QCD channel appear more WBF-like.

On Fig. 2 we consider the effect of the resummation on the transverse momentum distribution of the Higgs boson. This distribution clearly shows an unphysical behaviour at the lowest orders in perturbation theory in the region of most interest. All such pathological behaviour is absent from the spectrum of the resummed results based on our approach.

III. CONCLUSIONS

We have outlined a new technique for resumming to all orders multiple hard parton emission, and demonstrated its application to Higgs boson production (via GGF) in association with at least two jets. Our starting point is the FKL factorised form of Higgs+multijet amplitudes, which formally applies in multi-Regge kinematics (MRK). We then extend the region of applicability of the formalism by adhering to three well founded rules: (i) energy and momentum conservation is implemented; (ii) full 4-momenta are used in the virtual gluon propagators; (iii) the small phase space region where the squared Lipatov vertex is negative, $-C.C < 0$, is vetoed. We compare the results obtained order by order to those obtained in a fixed order approach and find very good agreement. The approximations agree to within 10% with the full results, well within the uncertainty estimated by varying the renormalisation and factorisation scale by a factor of two in the tree level results. The approximations allow for an all-order resummation of hard emissions.

We have presented example results for the distribution of final state jets, for the azimuthal decorrelation parameter A_ϕ, and for the transverse momentum spectrum of the Higgs boson. We find significant decorrelation arising from additional hard final state radiation not captured by present NLO calculations; significantly more than previously estimated using parton shower algorithms.

The technique outlined here can be extended to e.g. W+jet emission, as well as pure multijet final states. It would be very interesting to interface the final states found here with parton shower algorithms, thus resumming in principle both the number of jets (hard partons) and the structure of each jet (soft collinear radiation). Furthermore, one can implement next-to-leading logarithmic corrections to the emission vertices adopted here.

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