

Theoretical Tools

*Report of the LoopVerein
and
the Event Generators Group*

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- 2 Multi-loop calculations — progress on $2 \rightarrow 2$ and $1 \rightarrow 3$ reactions
- 3 Radiative corrections to $2 \rightarrow 3, 4, \dots$ processes
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1 Introduction

Experiments at LEP/SLC/Tevatron

- confirmation of **Standard Model as quantum field theory** (quantum corrections significant)
- top mass m_t **indirectly constrained** by quantum corrections
↔ in agreement with m_t **measurement** of Tevatron
- Higgs mass M_H **indirectly constrained** by quantum corrections
↔ impact on Higgs searches

Great success of precision physics

↔ **even greater potential at future e^+e^- linear collider**

- **GigaZ/MegaW**: precision increases by factor ~ 10 w.r.t. LEP/SLC
EXP: $\Delta \sin^2 \theta_{\text{eff}}^{\text{lept}} \sim 0.00001$, $\Delta M_W \sim 6 \text{ MeV}$
TH: go from a few 10^2 to a few 10^4 (more complicated) diagrams
- **high-energy runs** ($\sqrt{s} = 350, 500, 800, \dots \text{ GeV}$):
various cross sections at %-level, SUSY particle production (?)

LC physics program demands precision calculations

- progress on higher-order corrections
 - ◇ 2-loop accuracy for $2 \rightarrow 2$ and $1 \rightarrow 3$ reactions
 - ◇ full 1-loop calculations for $2 \rightarrow 3, 4, \dots$ processes
- progress on simulation tools
 - ◇ Monte Carlo generators for multi-particle final states

This talk: **summary of developments** (more topical than comprehensive)
achieved since spring 2001 (TESLA TDR) in

– **the LoopVerein**

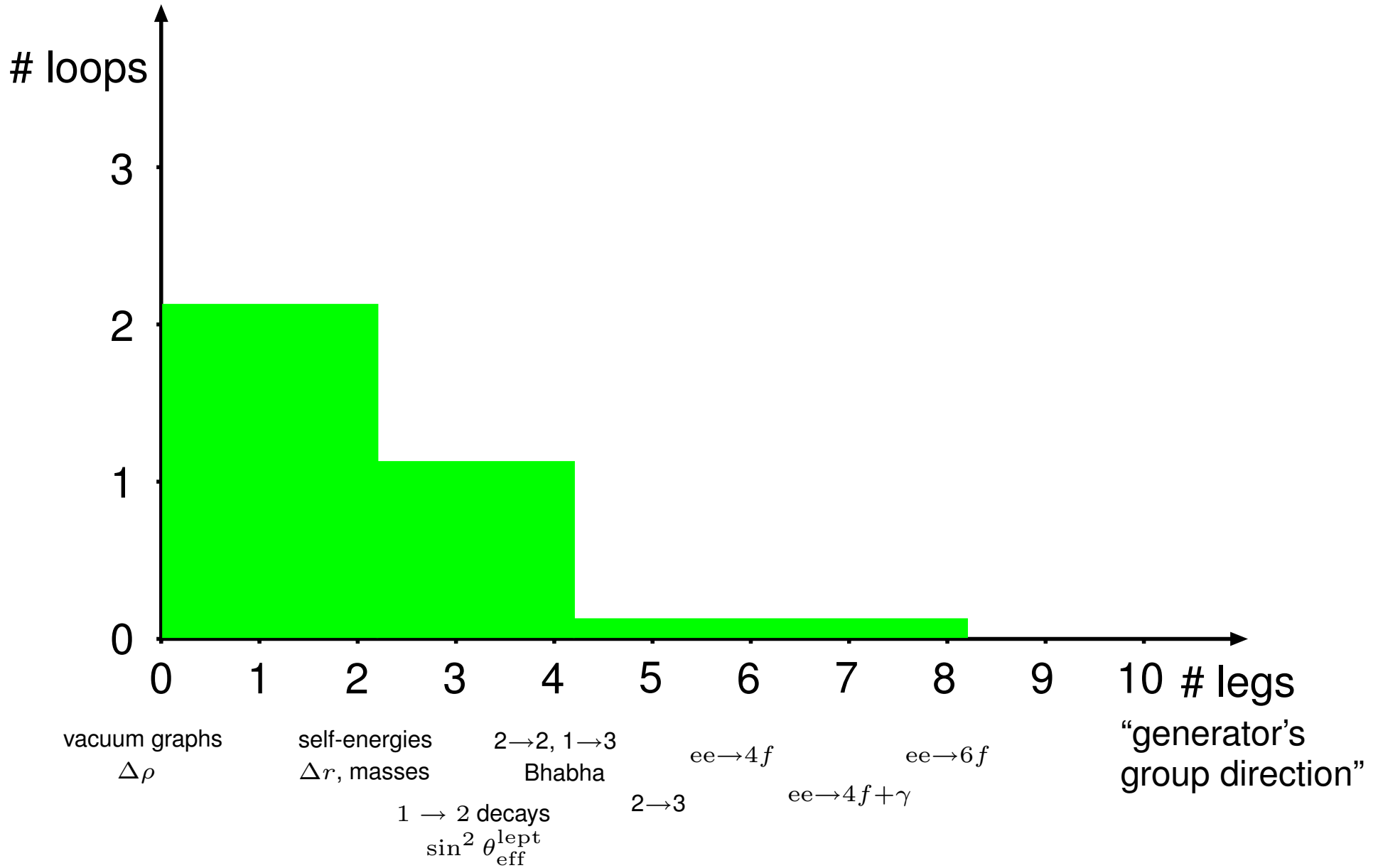
(conveners: S. Dittmaier (replacing F. Jegerlehner), W. Hollik)

– **the Event Generators Working Group**

(conveners: M. Antonelli, S. Jadach, S. Moretti)

“LoopVerein direction”

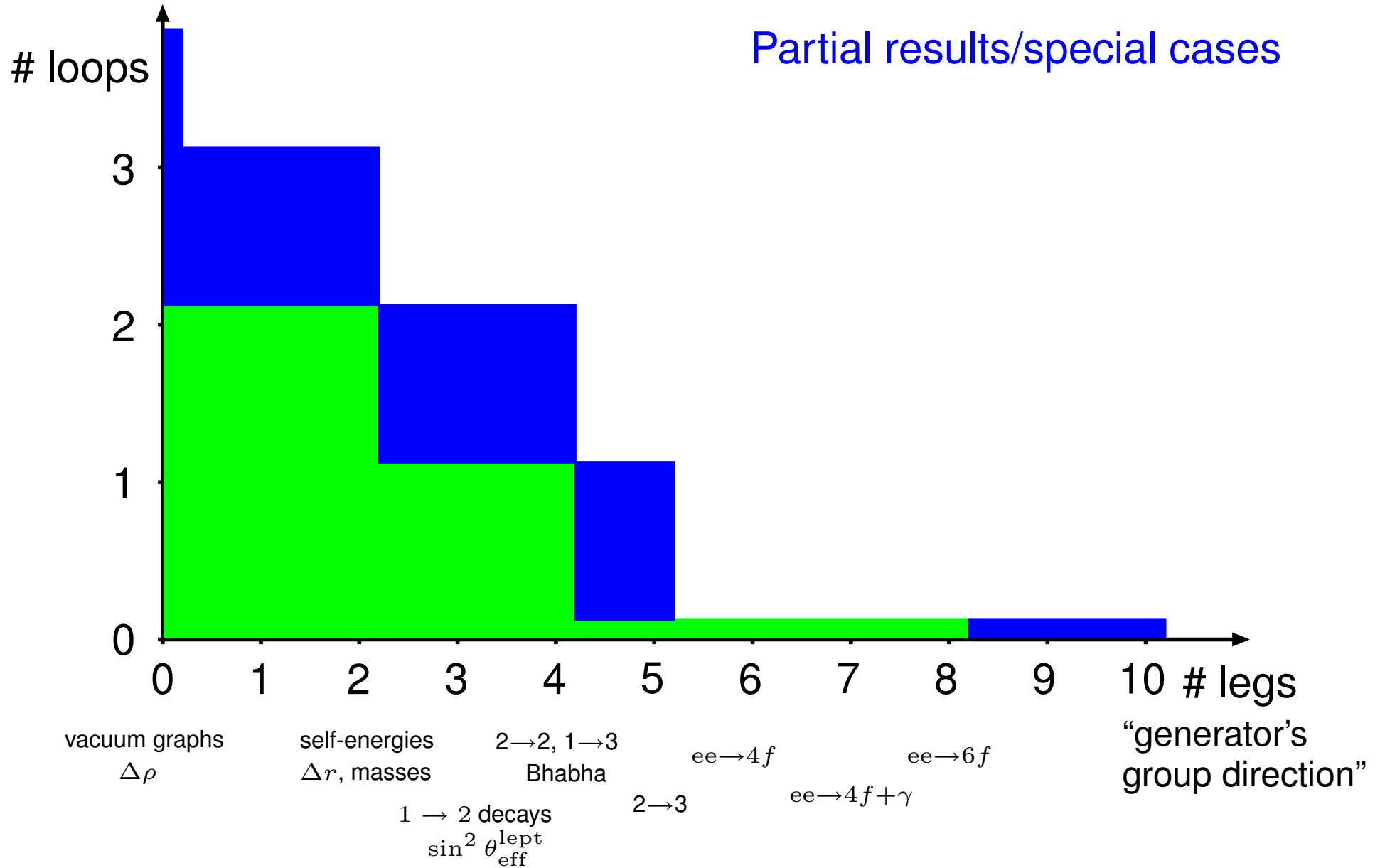
Technique well established



“LoopVerein direction”

Technique well established

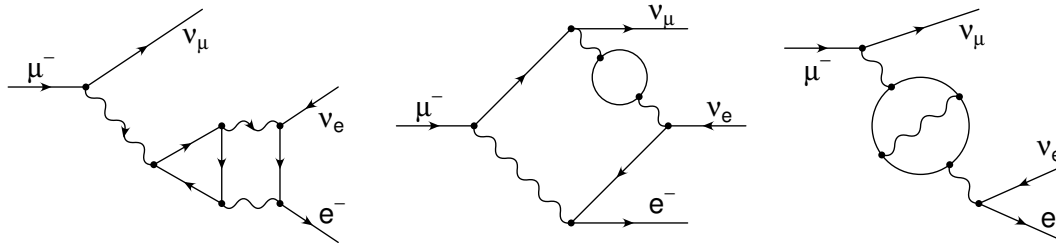
Partial results/special cases



2 Multi-loop calculations — progress on $2 \rightarrow 2$ and $1 \rightarrow 3$ reactions

2.1 Precision calculations for μ decay

- 2-loop contribution of closed fermion loops



Freitas, Hollik, Walter, Weiglein '00

$$\Delta M_W = \mathcal{O}(50 \text{ MeV})$$

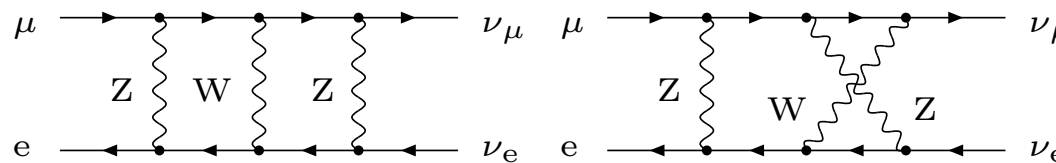
NEW result by
Awramik, Czakon '03

- diagram generation by **FeynArts** Böhm, Denner, Eck, Küblbeck '90-'92; Hahn '98-'02
- algebraic reduction by **TwoCalc** Weiglein
- semianalytical evaluation of 2-loop master integrals

- M_H dependence of 2-loop diagrams

Freitas, Hollik, Walter, Weiglein '02

- Complete 2-loop calculation of boson loops



Awramik, Czakon '02

Onishchenko, Veretin '02

$$\Delta M_W = \mathcal{O}(1 \text{ MeV})$$

- diagram generation by **Diagen** (AC) and **DIANA** (OV)
- algebraic reduction by **FORM** Fleischer, Tentyukov '99-'02
- semianalytical evaluation (AC) and asymptotic expansion (OV)

Recent progress: 3-loop contributions to $\Delta\rho$

Faisst, Kühn, Seidensticker, Veretin '03

- corrections of $\mathcal{O}(G_\mu^2 m_t^4 \alpha_s)$ and $\mathcal{O}(G_\mu^3 m_t^6)$
 $\hookrightarrow \Delta M_W = \mathcal{O}(5 \text{ MeV})$ $\hookrightarrow \Delta M_W = \mathcal{O}(0.5 \text{ MeV})$

- diagrams generated by QGRAF and asymptotically expanded in FORM

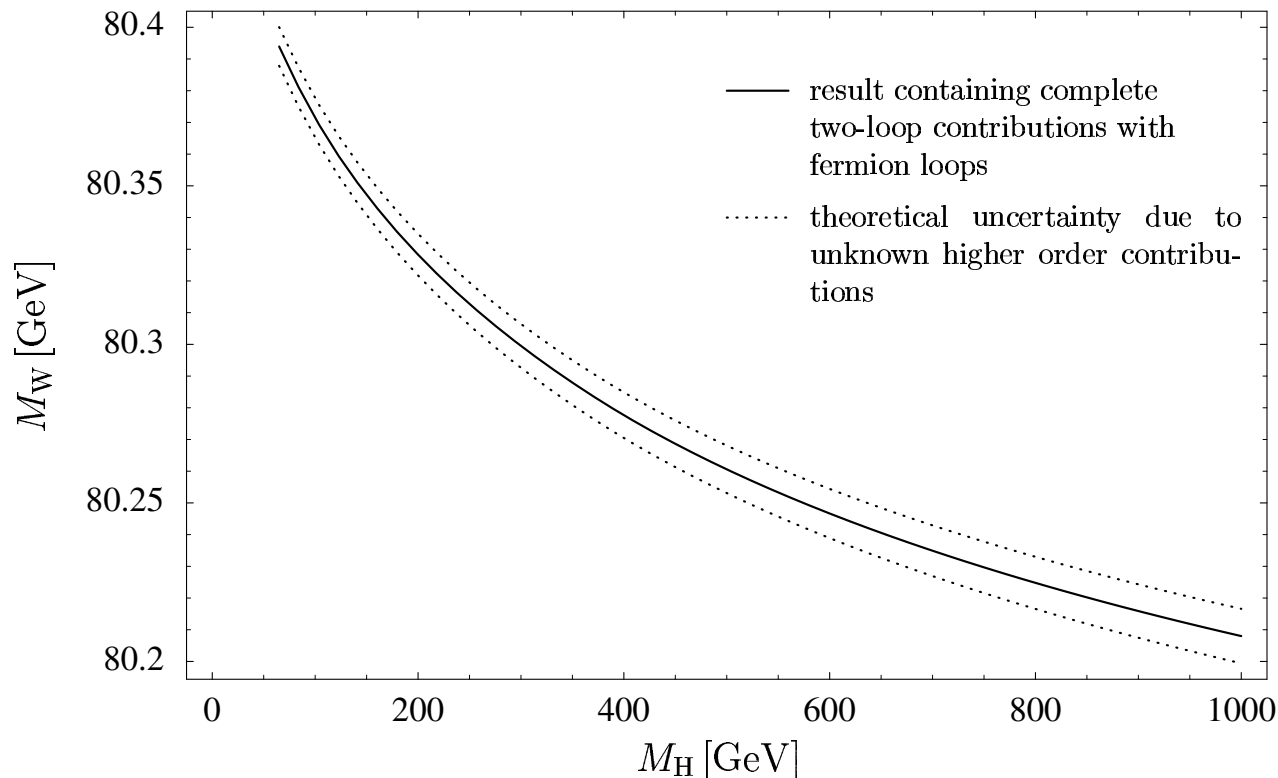
Nogueira '93

- \hookrightarrow tadpole diagrams computed with MATAD

Steinhauser '00

Prediction of M_W from muon decay:

Freitas, Hollik,
Walter, Weiglein '00



Theoretical uncertainty:

status 2000:

$$\Delta M_W \sim 6 \text{ MeV}$$

status 2003:

$$\Delta M_W \sim 2 \text{ MeV}$$

2.2 Recent results from the 2-loop frontier

Genuine 2-loop corrections to $2 \rightarrow 2$, $1 \rightarrow 3$ processes

- **Algebraic reduction to master integrals** Anastasiou, Gehrmann, Glover, Laporta, Oleari, Remiddi, Smirnov, Tausk, Veretin '00
by integration by parts, Lorentz invariance identities
↪ calculation of master integrals by Mellin–Barnes technique,
Anastasiou, Smirnov, Tausk, Tejada-Yeomans '99-'02
differential equations, numerical techniques
Gehrmann, Remiddi '00, '01 Binoth, Heinrich '00
- **Direct reduction** of full 2-loop amplitudes Moch, Uwer, Weinzierl '02
↪ higher transcendental functions → nested harmonic sums
- **Explicit algebraic results:**
 - ◇ 2-loop amplitudes for **massless $2 \rightarrow 2$ processes**
(Bhabha, QCD parton scattering, etc.) Anastasiou, Bern, v.d.Bij, De Freitas, Dixon, Ghinculov, Glover, Oleari, Schmidt, Tejada-Yeomans, Wong '01, '02
 - ◇ 2-loop QCD amplitudes for **$e^+e^- \rightarrow 3\text{jets}$** Garland, Gehrmann, Glover, Koukoutsakis, Moch, Remiddi, Uwer Weinzierl '02

Further activities:

- $\mathcal{O}(\alpha)$ correction to $e^+e^- \rightarrow f\bar{f} + \gamma$ Jadach, Melles, Ward, Was, Yost '99-'01
Czyz, Kühn, Rodrigo '02
- **Numerical approach** to 2-loop diagrams
↪ self-energies, some vertex corrections Passarino, Uccirati '01, '02
- Progress on **universal QED corrections**
to polarized e^\pm scattering in higher orders Blümlein, Kawamura '02
- Part of 2-loop renormalization of **massive Bhabha process**:
(1-loop graphs) \times (1-loop counterterms) Fleischer, Riemann, Tarasov, Werthenbach '02
- 2-parton bremsstrahlung for $e^+e^- \rightarrow 2\text{jets}$ in NNLO
leading colour terms for **dipole subtraction approach** Weinzierl '03

2.3 Multi-loop calculations in the MSSM

Precision observables in the MSSM

- known: – precision observables in 1-loop order since '94-'96
– $\mathcal{O}(\alpha\alpha_s)$ corrections to $\Delta\rho$
Djouadi, Gambino, Heinemeyer, Hollik, Jünger, Weiglein '97, '98
- new: corrections to $\Delta\rho$ of $\mathcal{O}(y_t^2)$, $\mathcal{O}(y_t y_b)$, $\mathcal{O}(y_b^2)$ Heinemeyer, Weiglein '02
obtained with *FeynArts / TwoCalc*
↪ induced corrections: $\Delta M_W \sim 2 \text{ MeV}$, $\Delta \sin^2 \theta_{\text{eff}}^{\text{lept}} \sim 0.00001$

Precision calculations for MSSM Higgs masses and widths

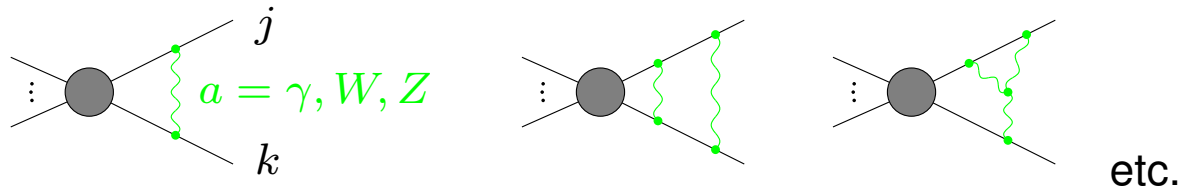
- Higgs masses: progress in *FeynHiggs* Frank, Hahn, Heinemeyer, Hollik, Weiglein '98-'03
– subleading 2-loop corrections
– Δm_b resummation
– new 1-loop renormalization (mainly $\tan\beta$)
- Higgs decays: progress in *HDECAY* and *FeynHiggsDecay* Djouadi, Kalinowski, Spira '97-'03 Heinemeyer, Hollik, Weiglein '00-'03
↪ Higgs and SUSY working groups

Renormalization of the MSSM

- no regularization respects gauge invariance *and* SUSY
↳ particular care needed \Rightarrow **algebraic renormalization**
- **symmetry identities, proof of renormalizability, non-renormalization theorems** Hollik, Kraus, Roth, Rupp, Sibold, Stöckinger '02
↳ construction of symmetry-restoring counterterms possible

2.4 Electroweak radiative corrections at high energies

Sudakov logarithms induced by soft gauge-boson exchange



+ **sub-leading logarithms** from mass singularities

Typical impact on $2 \rightarrow 2$ reactions at $\sqrt{s} \sim 1$ TeV:

$$\delta_{LL}^{1\text{-loop}} \sim -\frac{\alpha}{\pi s_W^2} \ln^2\left(\frac{s}{M_W^2}\right) \simeq -26\%, \quad \delta_{NLL}^{1\text{-loop}} \sim +\frac{3\alpha}{\pi s_W^2} \ln\left(\frac{s}{M_W^2}\right) \simeq 16\%$$

$$\delta_{LL}^{2\text{-loop}} \sim +\frac{\alpha^2}{2\pi^2 s_W^4} \ln^4\left(\frac{s}{M_W^2}\right) \simeq 3.5\%, \quad \delta_{NLL}^{2\text{-loop}} \sim -\frac{3\alpha^2}{\pi^2 s_W^4} \ln^3\left(\frac{s}{M_W^2}\right) \simeq -4.2\%$$

\Rightarrow **Corrections still significant at 2-loop level**

NOTE: **differences to QED / QCD** where Sudakov log's cancel

- massive gauge bosons W, Z can be reconstructed
 \hookrightarrow no need to add “real W, Z radiation”
- non-Abelian charges of W, Z are “open” \rightarrow Bloch–Nordsieck theorem violated

\Rightarrow **Aim:** **universal prescription for improving (e.g. 1-loop) calculations**

Progress at the 2-loop level

Explicit calculations for

- LL level Melles '00; Hori, Kawamura, Kodaira '00; Beenakker, Werthenbach '00, '01
- LL + angular-dependent NLL level Denner, Melles, Pozzorini '03

Structure of result at ang.-dep. NLL level:

$$\mathcal{M} \sim \mathcal{M}_0 \otimes \left(\delta_{\text{ew}} + \delta_{\text{em}} + \frac{1}{2} \delta_{\text{ew}}^2 + \delta_{\text{ew}} \otimes \delta_{\text{em}} + \frac{1}{2} \delta_{\text{em}}^2 \right)$$

with

$$\delta_{\text{ew}} = \frac{\alpha}{4\pi} \left\{ -\frac{1}{2} \sum_j C_j^{\text{ew}} \ln^2 \frac{M_{\text{W}}^2}{s} + \sum_{k \neq j} \sum_{a=\gamma, Z, W^\pm} I_j^a I_k^{\bar{a}} \ln \frac{s}{|2p_j p_k|} \ln \frac{M_{\text{W}}^2}{s} \right\}$$
$$\delta_{\text{em}} = \frac{\alpha}{4\pi} \left\{ -\frac{1}{2} \sum_j Q_j^2 \left[2 \ln \frac{m_j^2}{s} \ln \frac{m_\gamma^2}{M_{\text{W}}^2} - \ln^2 \frac{m_j^2}{M_{\text{W}}^2} \right] + \sum_{k \neq j} Q_j Q_k \ln \frac{s}{|2p_j p_k|} \ln \frac{m_\gamma^2}{M_{\text{W}}^2} \right\}$$

Agreement with proposed resummations

based on a **symmetric SU(2) × U(1) theory at high energies**
matched with QED at electroweak scale

$$\mathcal{M} \sim \mathcal{M}_0 \otimes \exp(\delta_{\text{ew}}) \otimes \exp(\delta_{\text{em}}) \quad \text{Fadin et al. '99; Melles '00, '01; Kühn, Moch, Penin, Smirnov '01}$$

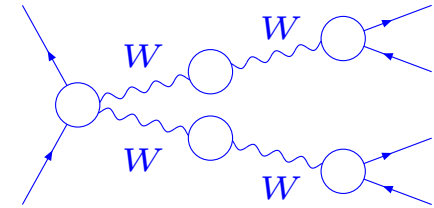
Progress in the MSSM

Explicit 1-loop results with proposed exponentiations Beccaria et al. '01, '02

3 Radiative corrections to $2 \rightarrow 3, 4, \dots$ processes

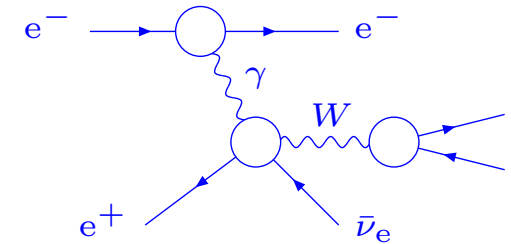
3.1 Progress on $e^+e^- \rightarrow 4f$ since TESLA TDR

W-pair production $e^+e^- \rightarrow WW \rightarrow 4f(+\gamma)$



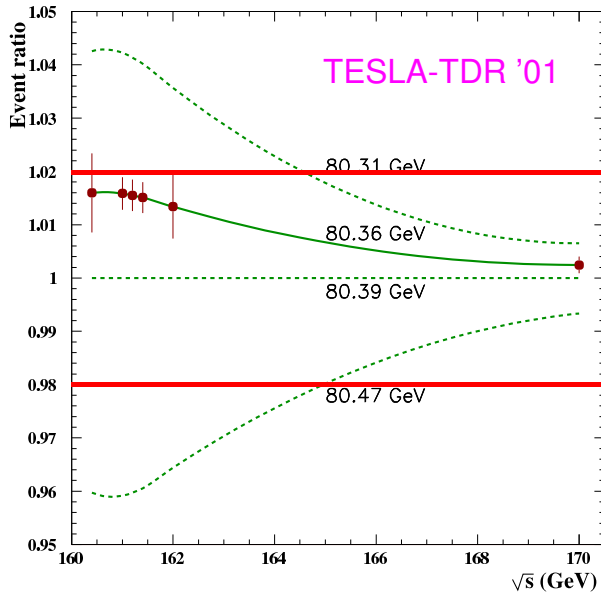
- *RacoonWW* (Denner, S.D., Roth, Wackerath)
 - anomalous triple and quartic gauge couplings
 - unweighting procedure and interface to *Pythia*
 - *YFSWW* (Jadach, Płaczek, Skrzypek, Ward)
 - combination with *KoralW* to concurrent MC code *KandY*
↔ cross-talk via FIFO files under UNIX/LINUX
 - Reliable **estimates of theoretical uncertainties** (TU) for
 - M_W reconstruction (Jadach et al. '01) $\Delta M_W \lesssim 5 \text{ MeV}$
 - bounds on anomalous TGC λ (Brunelière et al. '02) $\Delta \lambda \lesssim 0.005$
- ↔ State-of-the-art generators include
- full lowest-order matrix elements for $e^+e^- \rightarrow 4f(+\gamma)$
 - non-universal electroweak corrections in “double-pole approximation”
 - improvements by leading higher-order corrections

Single-W production $e^+e^- \rightarrow e\nu_e W \rightarrow e\nu_e + 2f(+\gamma)$



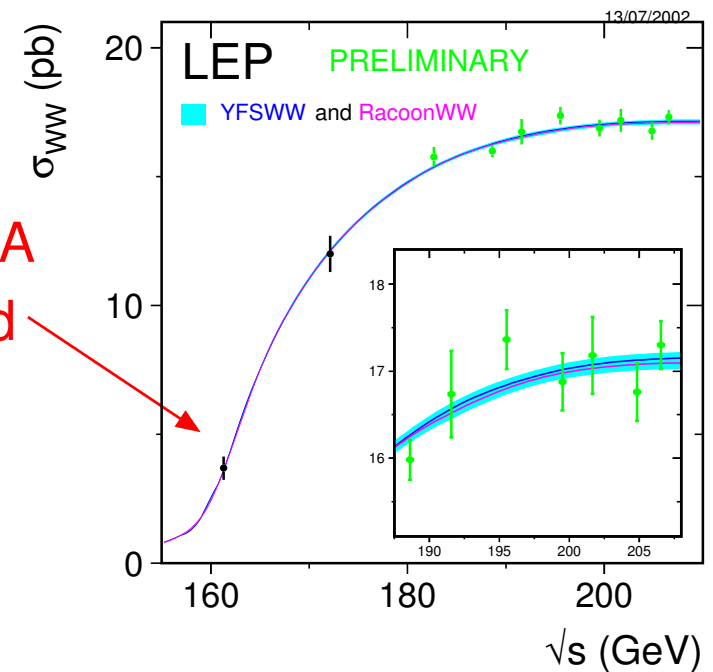
- *KoralW* (Jadach, Płaczek, Skrzypek, Ward, Was)
 - inclusion of ISR–FSR interference (“electric charge screening”) by reweighting
 - improvement of running $\alpha(Q^2)$ in t -channels
- \hookrightarrow TU \sim O(5%) as in other state-of-the-art programs *grc4f* (Kurihara et al.), *NEXTCALIBUR* (Berends et al.), *SWAP* (Montagna et al.), *WPHACT* (Accomando et al.), *WTO* (Passarino)

WW-threshold scan at MegaW:

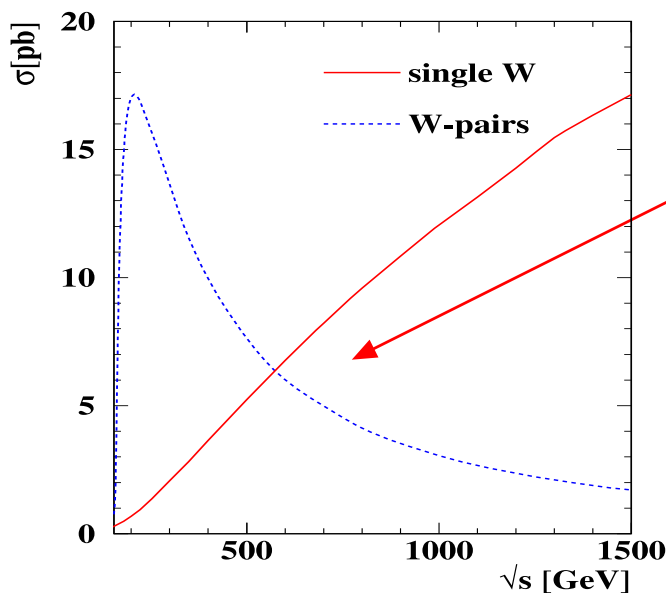


$\pm 2\%$ TU of IBA
near threshold

WW cross section at LEP2:



σ_{WW} and σ_W at LC energies:



Measurements at %-level

Requirements for LC physics:

- Full $\mathcal{O}(\alpha)$ correction for $e^+e^- \rightarrow 4f$
- Leading corrections beyond $\mathcal{O}(\alpha)$

3.2 Electroweak corrections to $e^+e^- \rightarrow \nu\bar{\nu}H$

- Fermion-loop corrections (+sfermion loops in MSSM)

Eberl, Majerotto, Spanos '02; Hahn, Heinemeyer, Weiglein '02

- Full $\mathcal{O}(\alpha)$ corrections calculated by different groups

Jegerlehner, Tarasov '02 (only virtual corrections, no numerics)

– diagrams generated by DIANA and reduced with FORM/MAPLE

Bélanger, Boudjema, Fujimoto, Ishikawa, Kaneko, Kato, Shimizu '02

– diagrams generated and calculated with GRACE

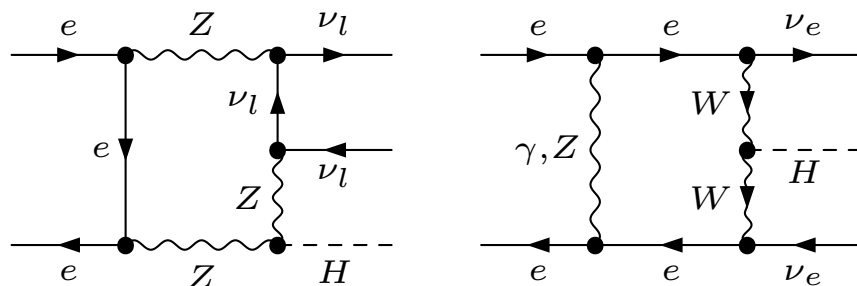
Denner, S.D., Roth, Weber '03

– diagrams generated by FeynArts

– two independent calculations in Mathematica (one partially with LoopTools)

Hahn '98-'02

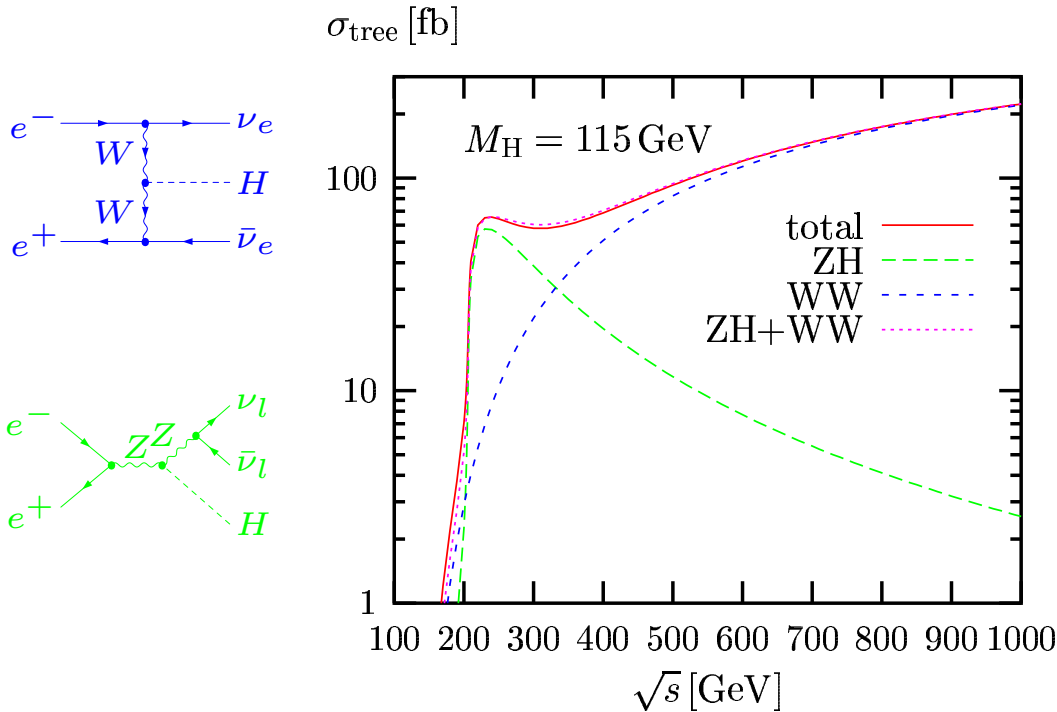
Main complications: pentagon diagrams



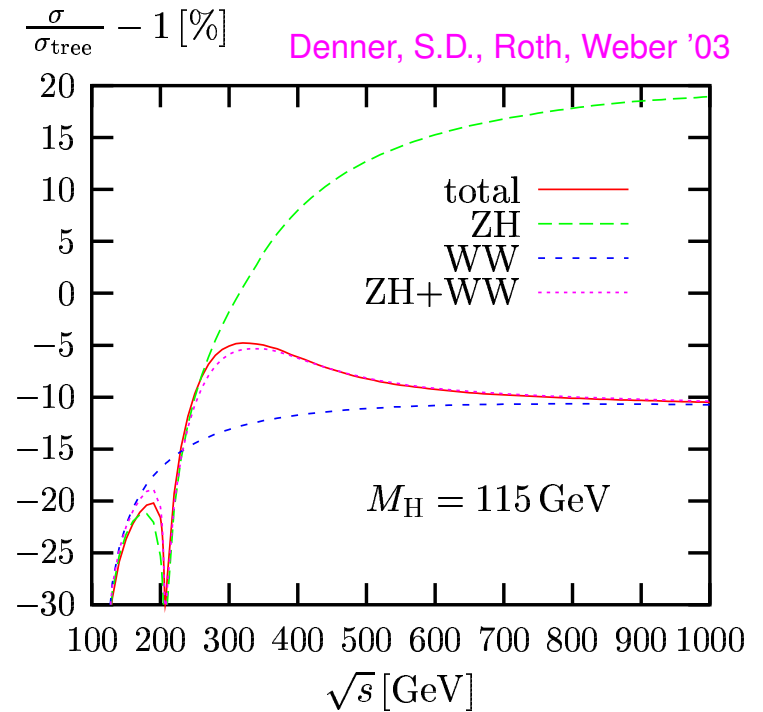
Computational techniques improved

Total cross section for $e^+e^- \rightarrow \nu\bar{\nu}H$

Lowest order:



Relative corrections (G_μ scheme):



Comparison of results: ($\sqrt{s} = 500 \text{ GeV}$, $\alpha(0)$ scheme)

M_H [GeV]	σ_{tree} [fb]	σ [fb]	$\sigma/\sigma_{\text{tree}} - 1 [\%]$	
150	61.074(7)	60.99(7)	-0.2	Bélanger et al.
	61.076(5)	60.80(2)	-0.44(3)	Denner et al.
300	10.758(1)	10.30(1)	-4.2	Bélanger et al.
	10.7552(7)	10.282(4)	-4.40(3)	Denner et al.

Agreement
within $\mathcal{O}(0.2\%)$

4 Event generators for multi-particle final states

4.1 Generic generators for parton level

Basic idea: flexible event generator for a large set of final states

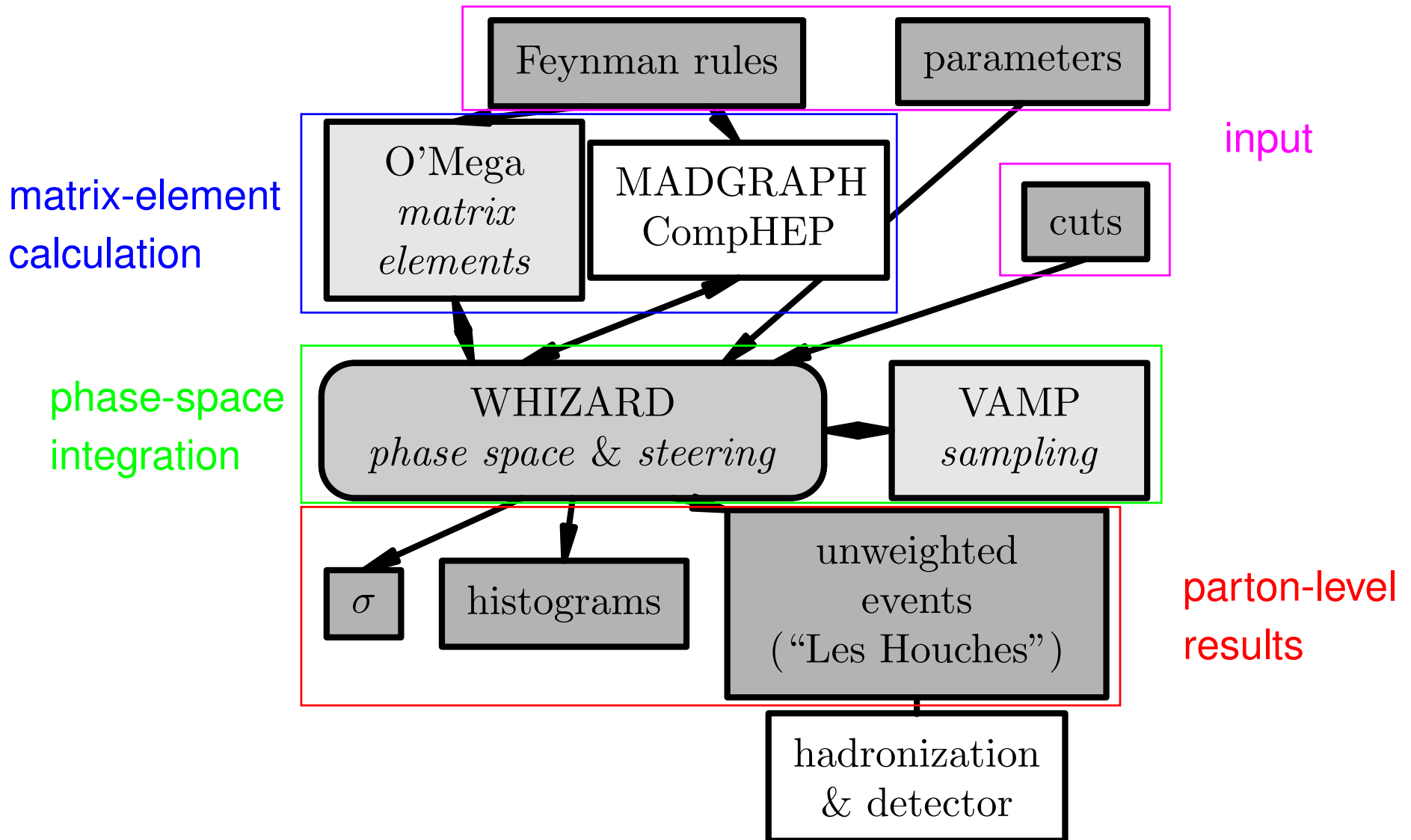
Existing programs:

- *AMEGIC++*
Krauss, Kuhn, Schumann, Soff '01,'02
- *MadEvent + Madgraph*
Maltoni, Stelzer '02; Stelzer, Long '94
- *Whizard + Omega/Madgraph/CompHEP*
Kilian '01; Moretti, Ohl, Reuter '01 / Stelzer, Long '94 / Boos et al.'89-'02
- *GRACE*
MINAMI-TATEYA group '92
- *PHEGAS + HELAC*
Papadopoulos '00; Kanaki, Papadopoulos '00

Typical features – differences to process-specific generators:

- lowest-order predictions
improved by universal corrections (parton shower, LL ISR, etc.)
- + many final states, uniform setup for different process classes, ideal for studying the physics potential of colliders
- long codes, improvements by radiative corrections problematic / limited
↪ usually not high-precision tools

Structure of *Whizard* – a typical flow chart of a generic generator



4.2 Event generators and results for $e^+e^- \rightarrow 6f$

- *eett6f* for $e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b} + 4f$ Biernacik, Kołodziej '02
no e^\pm in final state, massive fermions, not a generator

- *Lusifer* S.D., Roth '02
all $6f$ final states, massless fermions

- *SIXFAP* Carloni Calame, Gangemi, Montagna, Moretti, Nicosini, Piccinini '97-'03
all $6f$ final states, massive fermions

- results from generic generators

AMEGIC++, GRACE, PHEGAS+HELAC, Whizard+Omega/Madgraph

all $6f$ final states, massive fermions

Tuned comparison in progress

for various final states including $6f, 4f+2g, 2f+4g, 4f+\gamma, 8f, 4f+2\tilde{\chi}, \text{etc.}$

↪ *MC-tester* project Golonka, Pierzchała, Was '02

Some results on $e^+e^- \rightarrow t\bar{t} \rightarrow 6f$

- Full set of diagrams vs. signal diagrams vs. narrow-width approximation (NWA) for $e^+e^- \rightarrow \nu_\mu\mu^+\mu^-\bar{\nu}_\mu b\bar{b}$:

\sqrt{s} [GeV]	σ_{full} [fb]	σ_{signal} [fb]	σ_{NWA} [fb]
360	4.416(6)	4.262(1)	4.624(2)
500	6.705(6)	6.354(2)	6.400(7)
800	3.538(29)	3.058(2)	2.973(4)

eett6f

Biernacik, Kołodziej '02

↔ Full calculations necessary (at least for proper signal definition)

- Comparison of various calculations ($\sqrt{s}=500$ GeV, cuts applied, $m_f=0$):

σ_{full} [fb]	<i>AMEGIC++</i>	<i>eett6f</i>	<i>Lusifer</i>	<i>PHEGAS</i>	<i>SIXFAP</i>	<i>Whizard</i>
$\nu_e e^+ e^- \bar{\nu}_e b\bar{b}$	5.879(8)		5.853(7)	5.866(9)	5.854(3)	5.875(3)
$\nu_e e^+ \mu^- \bar{\nu}_\mu b\bar{b}$	5.827(4)		5.819(5)	5.822(7)	5.815(2)	5.827(3)
$\nu_\mu \mu^+ \mu^- \bar{\nu}_\mu b\bar{b}$	5.809(5)	5.807(3)	5.809(5)	5.809(5)	5.804(2)	5.810(3)
$\nu_\mu \mu^+ \tau^- \bar{\nu}_\tau b\bar{b}$	5.800(3)	5.820(3)	5.800(4)	5.798(4)	5.798(2)	5.796(3)
$\nu_\mu \mu^+ d\bar{u}b\bar{b}$	17.209(9)	17.275(28)	17.171(24)	17.204(18)		
without QCD:	17.097(8)		17.095(11)	17.107(18)	17.096(4)	17.103(8)

↔ Good agreement

4.3 “True” Monte Carlo event generators

PYTHIA (Sjostrand et al.), *HERWIG* (Corcella et al.), *ISAJET* (Baer et al.)

- Produce **actual events as they appear in detector**, by including parton shower, fragmentation/hadronization, hadron decays
- Mainly based on **tree-level $2 \rightarrow 1$, $2 \rightarrow 2$ hard scatterings**
- **Matching of QCD matrix-elements** (ME) for one-gluon real emission **to parton shower** (e.g., $t \rightarrow bW^+g$, $q\bar{q} \rightarrow Vg$ in HERWIG & PYTHIA)
- Recent developments (Extended Study) for LC physics:
 - ◇ complete **$2 \rightarrow 2$ MSSM processes** available (ALL)
 - ◇ **$Q\bar{Q}^{(\prime)}$ + Higgs** (SM and MSSM)
 - ◇ **R-parity violating SUSY** in HERWIG & cMSSM in PYTHIA
 - ◇ **spin correlations \oplus 3,4-body decay MEs** (HERWIG)
- Future/current developments:
 - ◇ some **NLO QCD processes** (HERWIG, soon PYTHIA)
 - ◇ progress in **C++**: *HERWIG++* and *PYTHIA 7*

5 Conclusions

LC precision physics is great challenge

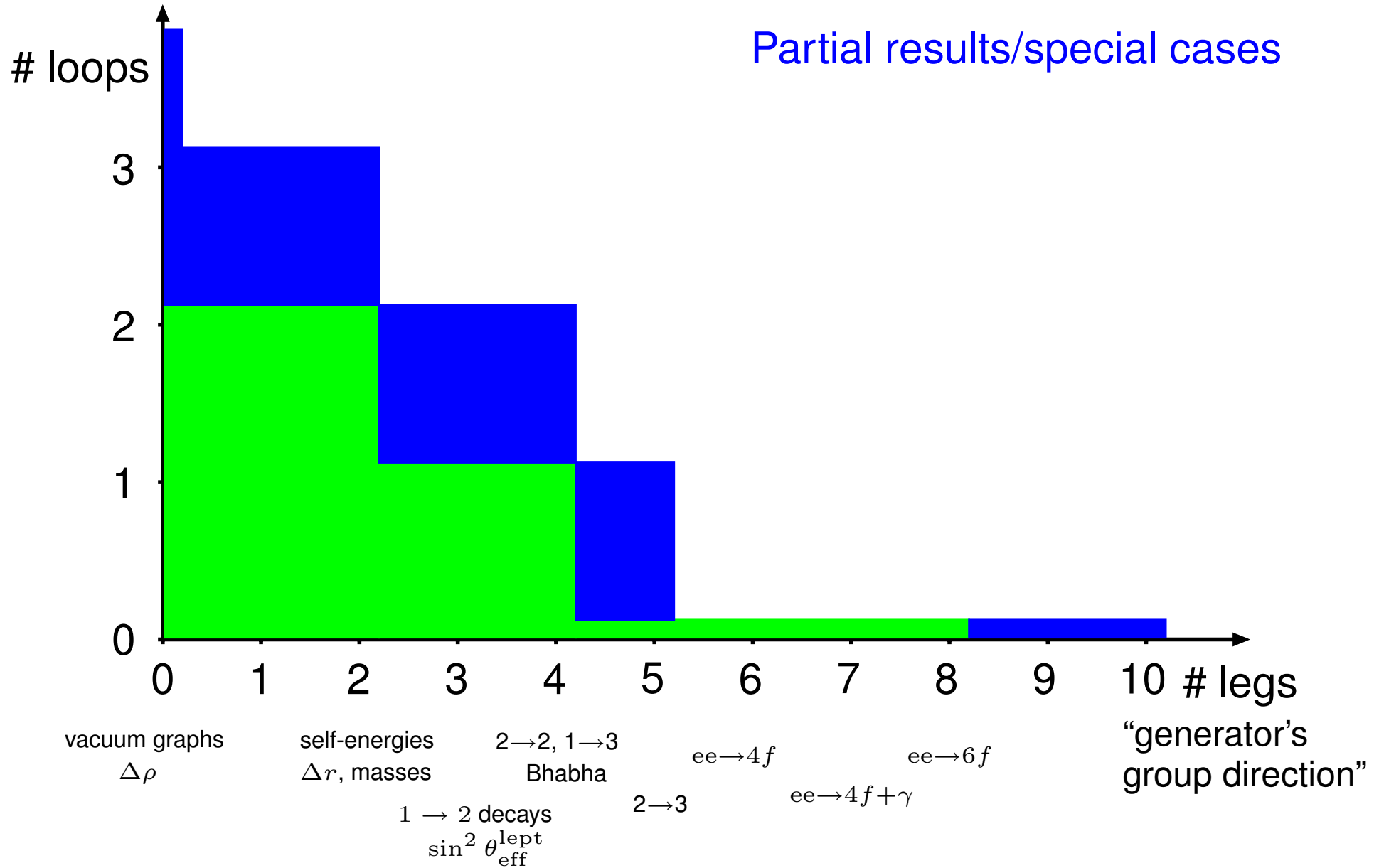
Good progress since TESLA TDR (3/2001) in various directions:

- 2-loop techniques for $2 \rightarrow 2$ and $1 \rightarrow 3$ reactions
- μ decay in full 2-loop accuracy and improved beyond
- 1-loop calculations for $2 \rightarrow 3$ processes ($e^+e^- \rightarrow \nu\bar{\nu}H$)
- multi-purpose Monte Carlo generators
(*AMEGIC++*, *GRACE*, *PHEGAS*, *Whizard*)
- studies of and generators for $e^+e^- \rightarrow 6f$ (*eett6f*, *Lusifer*, *SIXFAP*)
- leading electroweak 2-loop corrections at high energies
- etc.

“LoopVerein direction”

Technique well established

Partial results/special cases



“LoopVerein direction”

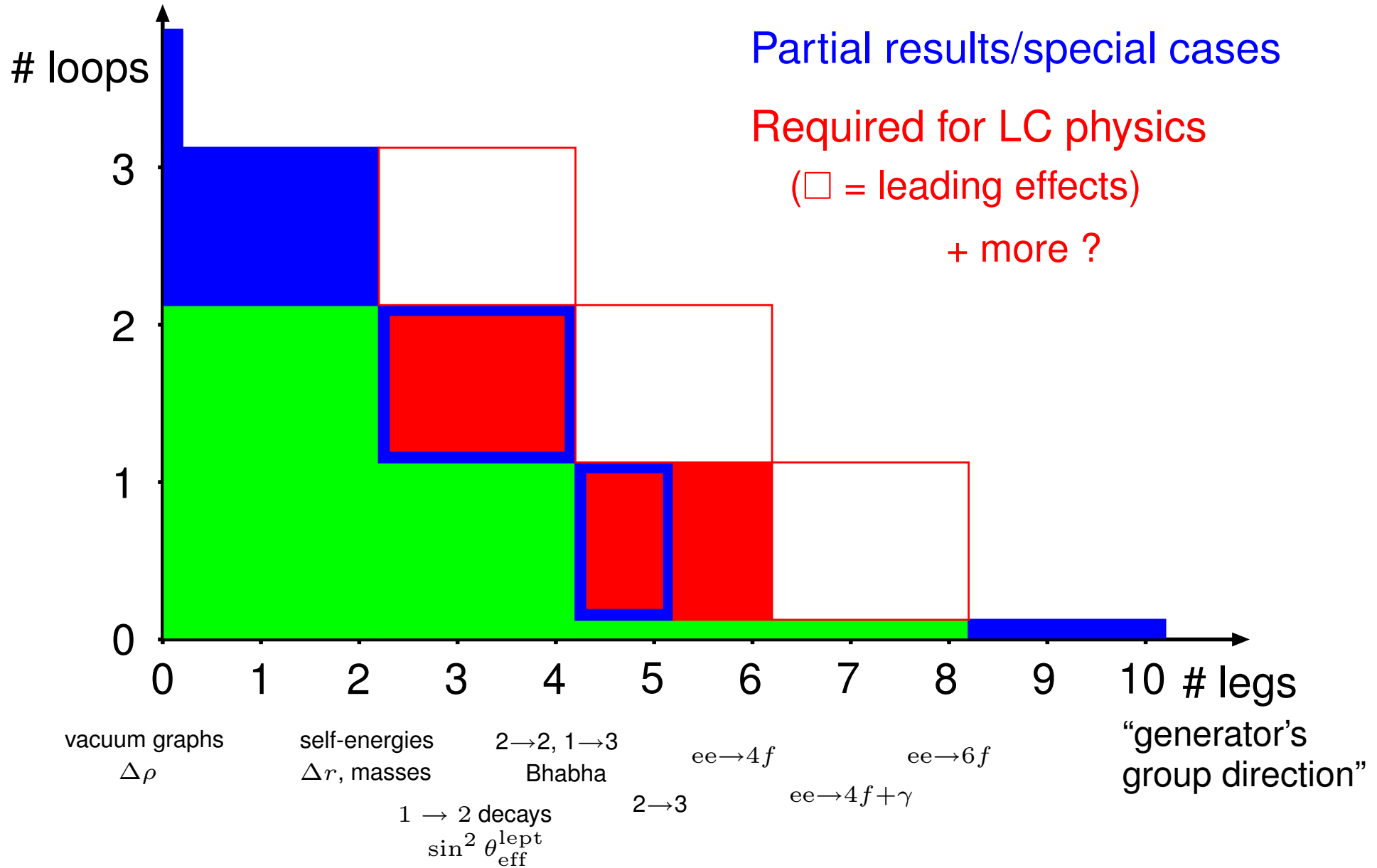
Technique well established

Partial results/special cases

Required for LC physics

(□ = leading effects)

+ more ?



Many issues for the future:

- complete **2-loop calculations for $1 \rightarrow 2, 3$ and $2 \rightarrow 2$ reactions**
 - ◇ Z line shape and Z-pole pseudo-observables ($\sin^2 \theta_{\text{eff}}^{\text{lept}}$, etc.)
 - ◇ Bhabha scattering, $e^+e^- \rightarrow 3\text{jets}$, etc.
- complete **1-loop calculations for $e^+e^- \rightarrow 4f$**
- **improvements in Monte Carlo generators**
 - ◇ inclusion of radiative corrections
 - ◇ unweighting procedures
 - ◇ matching of matrix elements with parton showers in NLO
- general issues
 - ◇ resummation of leading higher-order corrections
 - ◇ treatment of unstable particles
 - ◇ parallelization / automatization / standardization

⇒ **Many long-termed projects — continuous support needed !**